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Effects of stochastic freshwater flux on the Atlantic thermohaline circulation

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Abstract

The Atlantic thermohaline circulation (THC) transports large amounts of heat from the equatorial region northward toward the polar regions and is responsible for the relatively mild climate in the north Atlantic. Various studies have shown the Atlantic THC can have multiple stable equilibria. It has also been shown that increases in freshwater input into the Atlantic can reduce the strength of the THC, forcing it into its weak state. A stochastic freshwater forcing component has been introduced in simple models as a means to examine the effect short time scale weather events has on transitions between equilibria, which in turn can contribute to sudden climate change. The basic dynamics of such processes are studied here in the context of simple box models, verifying results from previous research.

1 Introduction

The feedback between climate change and changes in global ocean circulation patterns has become a topic of growing interest in climate research. Of particular interest is the Atlantic thermohaline circulation (THC). Numerical simulations and simple box models have shown that the Atlantic THC is highly sensitive to freshwater perturbation fluxes and exhibits multiple stable equilibria. Due to this sensitivity to freshwater perturbations and the THC's effect on climate, there is growing concern that further climate change could lead to a shutdown of the Atlantic THC, which would in turn drastically affect the climate of the north Atlantic. Thus, accurate representations of the dynamics of the Atlantic transport are of interest.

The long time scales of the ocean circulation do not capture effects of short time scale weather events that could affect the freshwater input of the THC. Therefore, a random white-noise salinity flux term has been suggested as a means of capturing these effects in ocean circulation models (see, e.g. [1]). Though highly idealized, this approach allows for a conceptual analysis of how freshwater fluctuations on short time scales can induce fluctuations in the Atlantic THC and cause the THC to jump from one stable steady state to another. Such quick transitions could drastically affect climate processes. This work seeks to verify previous analyses of the multiple equilibria of THC models in the presence of stochastic salinity fluxes.

2 PDE model

Here, we follow the analysis of Eyink [3], which adds a stochastic component to the model of Cessi and Young [2].

2.1 Governing equations

- Start with 2D, Boussinesq equations for a fluid driven by temperature and salinity gradients in a box $0 \le z \le d$, $-l \le y \le l$
- Linear equation of state

$$\rho = \rho_0[1 + \alpha_S(S - S_0) - \alpha_T(T - T_0)]$$

- Introduce a stream function $v = -\partial_z\psi$, $w = \partial_y\psi$, the equations reduce to

$$\partial_t\nabla^2\psi + J(\psi, \nabla^2\psi) = g(\alpha_T\partial_yT - \alpha_S\partial_yS) + \nu\nabla^4\psi$$
$$\partial_tT + J(\psi, T) = \kappa_T\nabla^2T$$
$$\partial_tS + J(\psi, S) = \kappa_S\nabla^2S$$

- Boundary conditions:

$$T(y,d) = \Delta T(\theta(y)), \quad \partial_zS(y,d) = \Delta SF(y)/d,$$
$$\partial_zT(y,0) = 0, \quad \partial_zS(y,0) = 0,$$
$$\partial_yT(\pm l, z) = 0, \quad \partial_yS(\pm l, z) = 0$$

- Introduce the aspect ratio $\epsilon = \frac{\pi d}{l}$, nondimensionalize:

$$P^{-1}[\partial_t\zeta + J(\psi,\zeta)] = \partial_yT - \partial_yS + (\partial_z^2 + \epsilon^2\partial_y^2)\zeta$$
$$\partial_tT + J(\psi, T) = (\partial_z^2 + \epsilon^2\partial_y^2)T$$
$$L^{-1}[\partial_tS + J(\psi, S)] = (\partial_z^2 + \epsilon^2\partial_y^2)S$$

$\zeta = (\partial_z^2 + \epsilon^2\partial_y^2)\psi$, $P = \nu/\kappa_T$ the Prandtl number, $L = \kappa_S/\kappa_T$ the Lewis number

2.2 Stochasticity and multiscale expansion

Stochasticity is added in the salinity forcing term by:

$$\Delta SF(y)/d \to (\Delta S/d)\bar{F}(y,t) + \Sigma_0\tilde{F}(y,t)$$

where $\bar{F}(y,t)$ is the averaged salinity flux and $\tilde{F}(y,t)$ is a random forcing term. The systematic salinity flux $\bar{F}$ is assumed to vary on a slow scale. The boundary conditions for the above equations are then

$$T(y,1) = a\theta(y), \quad \partial_zS(y,t,1) = b\bar{F}(y,\epsilon^2t) + c\tilde{F}(y,t)$$
$$a = \frac{g\alpha_T\Delta Td^3\epsilon^2}{\nu\kappa_T}, \quad b = \frac{g\alpha_S\Delta Sd^3\epsilon^2}{\nu\kappa_T}, \quad c = \frac{g\alpha_S\Sigma_0(\epsilon d)^{5/2}}{\nu\kappa_T^{1/2}}$$

Assuming a scaling [2]

$$a = \epsilon a_1, \quad b = \epsilon^3 b_3$$

and expanding the ψ, T, S fields as

$$(\psi, T, S) = \epsilon(\psi_1, T_1, S_1) + \epsilon^2(\psi_2, T_2, S_2) + \ldots$$

we can analyze the solutions and solvability conditions at each order of the expansion to ultimately determine a stochastic PDE for the salinity field:

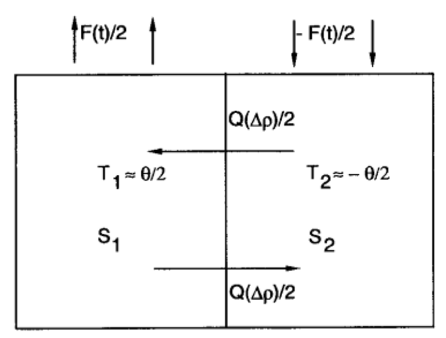
$$\partial_t\chi = \partial_y^2[\mu^2\chi(\chi - \eta)^2 + \chi - \gamma^2\partial_y^2\chi - r\bar{f}(y,\tau)] + \partial_y\tilde{F}(y,\tau)$$

This PDE can be analyzed to determine the dynamics and state transitions of the stream function.

3 Stochasticity in a 2-box model

Making strong assumptions on the flow, the spatial dependence of the PDE model can be reduced so that the dynamics of the flow are governed only by temperature and salinity fields in a simple box model.

This section corroborates the findings of Cessi [1] in the analysis of a simple 2-box model similar to Stommel's original model [4] with stochasticity introduced into the salinity equation.



3.1 Governing equations

- Same linear equation of state as above
- Conservation equations:

$$\dot{T}_1 = -t_r^{-1}(T_1 - \frac{\theta}{2}) - \frac{1}{2}Q(\Delta\rho)(T_1 - T_2)$$
$$\dot{T}_2 = -t_r^{-1}(T_2 + \frac{\theta}{2}) - \frac{1}{2}Q(\Delta\rho)(T_2 - T_1)$$
$$\dot{S}_1 = \frac{F(t)}{2H}S_0 - \frac{1}{2}Q(\Delta\rho)(S_1 - S_2)$$
$$\dot{S}_2 = -\frac{F(t)}{2H}S_0 - \frac{1}{2}Q(\Delta\rho)(S_2 - S_1)$$

Q = exchange function, $F(t)$ = salinity flux
t_r = temp. restoring time, H = depth of model ocean

- Coupled equations, $\Delta T, \Delta S = T_2 - T_1, S_2 - S_1$:

$$\frac{d}{dt}\Delta T = -t_r^{-1}(\Delta T - \theta) - Q(\Delta\rho)\Delta T$$
$$\frac{d}{dt}\Delta S = \frac{F(t)}{H}S_0 - Q(\Delta\rho)\Delta S$$

- Choose $Q = t_d^{-1} + V^{-1}q(\Delta\rho)^2$, nondimensionalize:

$$\dot{x} = -\alpha(x - 1) - x[1 + \mu^2(x - y)^2]$$
$$\dot{y} = p(t) - y[1 + \mu^2(x - y)^2]$$

3.2 Simplification: Multiple scales

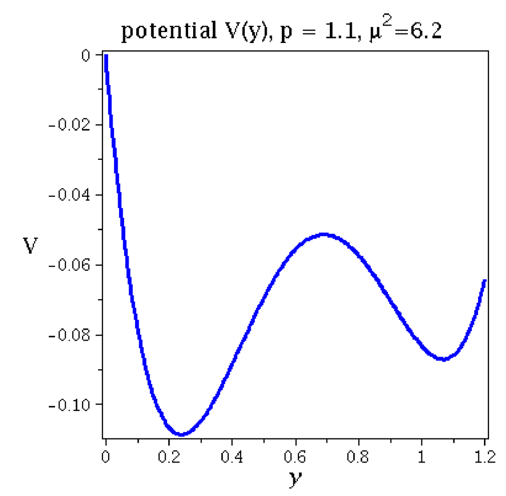
Noting that $\alpha = t_d/t_r$ is very large (typical timescales t_r = 25 days, t_d = 219 years) and all other terms are O(1), we can simplify the system:

$$x = 1 + O(\alpha^{-1})$$
$$\Rightarrow$$
$$\dot{y} = -y[1 + \mu^2(y - 1)^2] + \bar{p} + p'(t) + O(\alpha^{-1})$$

This is the familiar equation for a trajectory in a double-well potential

$$V(y) = \mu^2\left(\frac{y^4}{4} - \frac{2}{3}y^3 + \frac{y^2}{2}\right) + \frac{y^2}{2} - \bar{p}y$$

subject to a Brownian force $p'(t)$. For the non-perturbed case, solutions are found to be $y_a \approx 0.24$, $y_b \approx 0.69$, $y_c \approx 1.07$.



3.3 Deterministic Perturbations

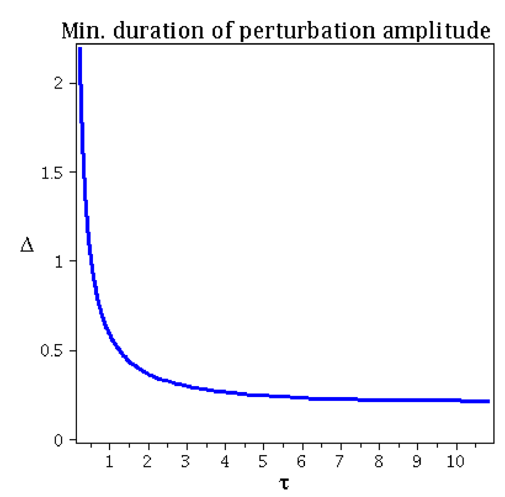
Let $p'(t)$ be defined as

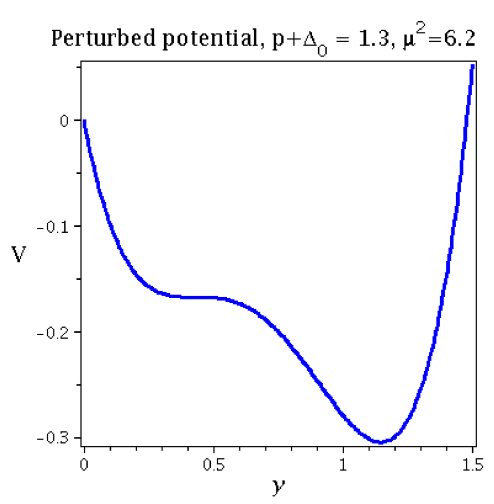
$$p'(t) = \begin{cases} 0, & t \le 0 \\ \Delta, & t \le \tau \\ 0, & t > \tau \end{cases}$$

To find the minimum duration τ of a perturbation necessary to move the solution past the unstable point, integrate:

$$\int_{y_a}^{y_b}[\bar{p} + \Delta - y - \mu^2y(y-1)^2]^{-1}dy = \int_0^\tau dt$$

which gives τ as a function of the perturbation amplitude Δ.





To find Δ_0, the critical perturbation amplitude, perturb the system for an infinite amount of time, and a new steady state y_a' is reached, where y_a and y_b coalesce to $y_a' = y_b'$.

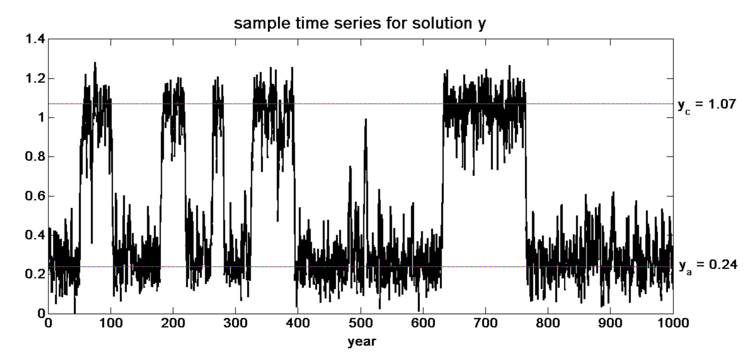
$$\bar{p} + \Delta_0 = \frac{2}{27}\mu^2[1 + (1 - 3\mu^{-2})^{3/2}] + \frac{2}{3}$$

3.4 Stochastic forcing

Stochasticity of the freshwater flux is determined by decomposing the nondimensional freshwater flux into a time-averaged part and a stochastic part:

$$p = \frac{\alpha_S S_0 t_d}{\alpha_T \theta H}F(t) = \bar{p} + p'(t).$$

Here, $p'(t)$ is modeled as stochastic white noise. Discretizing the DE for y in time, the equation is solved using the Euler scheme, with $\bar{p} = 1.1$, $\mu^2 = 6.2$. At each time step, $p'(t)$ is chosen as a random number from a Gaussian distribution with zero mean and standard deviation $\sigma = 3.3$.



- Result: stochastic forcing can cause the circulation to fluctuate between weak and strong steady states. A specific finite duration perturbation is not necessary to cause these transitions as in the deterministic case. Instead, transitions can occur randomly due to freshwater perturbations of varying amplitudes.

4 Discussion and future work

- At this stage, we wish to understand how this topic is generally approached in models and how the existing models may be modified.
- PDE model: Analysis of the multiscale expansion of Eyink is still in progress. We wish to revisit the validity of the approximations, leading to an improved model.
- Two-box model: We clearly see that allowing the freshwater flux term to have a stochastic component, the flow can fluctuate quickly between the two stable equilibria for undetermined magnitudes of the perturbation and lengths of time.
  - We wish to analyze the stochastic bifurcations that arise the stochastic system.
  - Idea: add a slowly varying periodic freshwater term and compare model results with data
- Feedback and suggestions are welcome!

References

[1] P. Cessi. A simple box model of stochastically forced thermohaline flow. *J. Phys. Oceanogr*, 24:1911-1920, 1994.

[2] P. Cessi. and W. R. Young. Multiple equilibria in two-dimensional thermohaline circulation. *J. Fluid. Mech.*, 241:291-309, 1992.

[3] G. L. Eyink Statistical hydrodynamics of the thermohaline circulation in a two-dimensional model. *Tellus*, 57A:100-115, 2005.

[4] H. Stommel. Thermohaline convection with two stable regimes of flow. *Tellus*, 13:224-230, 1961.