Budgeting Capital for R & D: An Application of Option Pricing

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AN APPLICATION OF OPTION PRICING

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by

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This paper presents the insight that R&D investments are "natural" options, and examines the extent to which they can be valued using currently available option pricing models. The nature of the industrial research effort determines whether the appropriate model is based on a diffusion process or a jump process. The option model based on the former is sufficiently developed to satisfactorily deal with the direct benefits of R&D, while the model based on the latter presents several problems. Furthermore, indirect benefits are not captured in these models. These applications and shortcomings are examined in detail in the hope of not only indicating a new direction for analysis of R&D investment decisions, but also pointing out the need for further scholarly research to deal more fully with this very interesting problem.
Introduction:

Deciding where and how much to invest in industrial research and development is not only a very interesting problem, but also one of great importance -- in 1977 the total U.S. R&D pricetag topped $40 billion.\(^1\) In 1976 the top three private spenders -- General Motors, IBM, and Ford -- alone accounted for $3.2 billion.\(^2\) R&D investment is big, and it is hard to picture a vital, dynamic economy without such venturing.

We have ample evidence that there is a systematic relationship between profitability and the level of commitment to R&D;\(^3\) however, there is a serious shortcoming in the way of rigorous methods to show what level is best for a particular situation, as even the most sophisticated optimization models suffer when benefit estimates are soft.\(^4\)

There are several mathematical models available, some very complex, for optimizing the R&D project mix, but as pointed out by Baker & Freeland [4] in their review of the literature, measuring the benefits from R&D is a critical area for further research. The best of the available approaches depend on an estimate of cash flows from the successful project. However, it is difficult or impossible to analyze R&D expenditure using any of the standard capital budgeting techniques which discount expected future cash flows from the project. The very reason for research is that much is unknown about the fledgling product or technology; for example, it might be unreasonable to make an estimate of cash flows prior to embarking on research into, say, commerical applications of solar-powered electric generation. Even if such estimates were made, it would be equally difficult to estimate the appropriate required rate of return to be used as the decision criterion. In many cases it is
difficult to confidently estimate an appropriate beta, or deal with the correlations between the new project and existing projects within the firm's portfolio. Moreover, it is the nature of research that new information is being sought which may have a profound impact on the value of the project.

What is being bought with research money is opportunity -- opportunity to exploit any marketable results of the research. In its simplest aspect, the decision facing the business strategist is whether or not the value of the opportunity exceeds the cost. Fortunately, a tool exists which may allow the necessary cost-benefit comparison to be made in a disciplined way: the Option Pricing Model (OPM). An R&D opportunity is a "natural" option; the funding of research is the purchase of an option to exploit any product which might result.

Discussion of the Problem:

Consider the executive faced with deciding whether to fund a particular project. He probably has fairly reliable estimates of the cost of research, but no guarantee that any return will come from "casting his bread upon the waters." If the potential is great enough, he will give the go-ahead knowing that if only one out of every ten of these high-flyers pays off, the company will still come out ahead. He also knows that if the business doesn't take risks, it will go stale.

The direct benefit from a successful R&D effort would come from the opportunity to exploit a new product with competition held at bay by patents or delays in developing their own substitutes. This can be dealt with as an option. The option framework is intuitively a very attractive place to look for a solution, as the decision faced by the executive above is similar to that faced by an investor considering purchase of a call option. The executive
must pay a price to get into a game in which there may be no payoff at all; however, the attraction is the chance for a really big payoff.

The price of conducting research buys the investor the opportunity to exploit any product which might result, and can be thought of as an option to make an investment in production. This concept could be modeled in such a way that the underlying thing of value is the present value of revenues from production, the exercise price is the cost of initiating production, and the expiration date is the time the money runs out from the initial research investment. Although this approach raises the complication of a stochastic exercise price, the problem is addressable in a straight-forward manner as shown by Stanley Fischer [18].

An appropriate model would incorporate an initial cost of the option equal to the present value of the cost of conducting research over a particular period of time, discounted at the risk-free rate; an expiration date coinciding with exhaustion of the fund set up by the initial investment, as well as an underlying thing of value and exercise price which follow stochastic processes through time. Upon exhaustion of the original money invested, further research or development might appear attractive; and this could be treated as a separate investment in a new option.

The decision variables for executives are the size of the initial investment and the length of time to be funded at each decision point (in other words, the number of times the research effort must be reevaluated).

Initiating product research is the setting in motion of an information-gathering process. At the outset, the research management team brings to bear all the currently available information about the prospects for the research, and this information is reflected in their initial estimate of the project's value. Once research is under way, new information surfaces continuously and
the value estimate is revised accordingly. It is a matter of uncertainty exactly where the research will lead, but the range of uncertainty can be estimated ahead of time and the process modeled statistically.

The option investor however can do something our corporate executive cannot so easily do -- the option investor can hedge. By judiciously selling short the underlying stock, the holder of a call option can theoretically eliminate risk altogether. However, when the underlying thing of value is an undeveloped product or technology, such hedging is not available. This point is important primarily from the standpoint of modeling. Many, perhaps most, option investors play the game because options offer them a way to take the high levels of risk that justify the long-run expectation of high reward. These folks are not interested in forming riskless hedges. However, the possibility of doing so allows arbitrage, and the hedge is key to the Black-Scholes market equilibrium solution to the value of an option.

Although the stochastic calculus used in the OPM is sophisticated, the basic idea is simple. Because there is a direct relationship between the value of a call option and the value of its underlying security, there is perfect negative correlation between returns to a call and returns to a short position in the stock. It is therefore possible to form a riskless hedge position, and in capital market equilibrium the return on the hedge must be the same as the return on other risk-free investments. By expressing this hedge in a dynamic continuous time set of equations, Black and Scholes [6] were able to derive a solution for the value of the call option. The finance literature contains several good reviews of option pricing, for the reader interested in full details.6

The lack of hedging opportunities puts the R&D executive's problem in a different light; the company will not necessarily have to pay the full
equilibrium value to get in on the action. Not only is the company not buying into the gamble on an active and efficient securities market, but even if that were the case, there would be no opportunity for arbitrageurs to enforce the going market reward for risk by the method of forming hedges. The option pricing model is nevertheless applicable, because the active option market represents an alternative opportunity to provide risk-bearing services of the same kind. The OPM can give insight into what the market equilibrium value should be for a particular venture, and if it could be bought for that amount or less, it would be a good investment.

**Diffusion Process Models:**

The assumptions made at the outset (about the appropriate stochastic process underlying the project value) determine which option pricing model will be used. Research intended to improve on an existing product or process might fit into the framework of a diffusion process. The essence of a diffusion process is that it represents a continuous random walk around a trend and, at least in the short run, seldom offers sudden surprises. The basic assumption of the Black-Scholes OPM is that the value of the underlying security follows a log-normal diffusion process expressed as,

\[ \frac{dS}{S} = \mu dt + \sigma_S dz_S, \tag{1} \]

where \( S \) is the value of the underlying security, \( \mu \) is the drift term, \( \sigma_S \) is the instantaneous standard deviation around the drift, and \( dz_S \) is a Weiner process. A Weiner process describes Brownian motion, which can be illustrated by the movement of a very small particle suspended in a fluid and bombarded at random by moving molecules. Each movement is small, the bombardment is continuous, and the two-dimensional analogue of the path is a random walk.
There are many research projects whose value would change through time in the same way. This would be the case if the product or process being researched were already well-developed, the goal of the research well-defined, and the obstacles to be overcome fairly well-known in advance. The majority of R&D money is spent on such projects. In short, such research would be expected to produce a steady upward trend, with random shocks along the way which individually would most likely be small. In the short run, no great surprises would be expected; but even so, the final outcome could not be known in advance with certainty.

The nature of the random shocks themselves is worth further discussion. Random upward boosts of course could result from fortuitous discoveries, and unforeseen bottlenecks could produce downward shoves of enough magnitude to dampen or cancel out the upward drift. Would it, however, be realistic to leave the model free to capture an actual decline in the value of the product or process? In other words, it seems fair to ask whether the time path should be restricted to upward movement only. It could be possible that the research would reveal previously unknown flaws in the idea, and it is also desirable to allow for external events (such as competition or the possibility of product liability suits) which could have an adverse impact on the value of the product. It therefore seems reasonable to leave as is the specification presented in equation (1).

From equation (1), Black and Scholes developed a formula for the equilibrium value of a call option with a known exercise price. If the cost of implementing the changes resulting from the research (the striking price for exercising the research option) were known with certainty beforehand, the basic Black-Scholes model could be used unmodified. It is stated as follows:
\[ C = S \cdot N \left\{ \frac{\ln \left( \frac{S}{X} \right) + [r + (\sigma^2/2)] T}{\sigma \sqrt{T}} \right\} - e^{-rT} \cdot N \left\{ \frac{\ln \left( \frac{S}{X} \right) + [r - (\sigma^2/2)] T}{\sigma \sqrt{T}} \right\} \] (2)

Notation:
- \( C \) = call option price
- \( S \) = current stock price
- \( X \) = exercise price
- \( r \) = default-free interest rate
- \( \sigma^2 \) = instantaneous variance of return on the stock
- \( T \) = time to expiration
- \( N(\cdot) \) = cumulative normal distribution function

However, it may be more realistic to suppose that the exercise price is not known with certainty beforehand. In such a case it would be possible to make an initial estimate of it, with stochastic changes anticipated. In the case where the exercise price also follows a diffusion process, a solution exists as presented by Stanley Fischer [18]. Where new information impacting on the initial estimate of exercise price is assumed to come in random, continuous small jolts, the process generating the time path for it could be described by the following equation:

\[ \frac{dX}{X} = \alpha_X dt + \sigma_X dz_X \] (3)

where \( X \) is the exercise price, \( \alpha_X \) is a drift term, \( \sigma_X \) is the instantaneous standard deviation, and \( dz_X \) is a Weiner process.

In the R&D case, it would be valid to assume that no drift would be expected, so that \( \alpha_X \) would be zero. Modifying Fischer's equation accordingly, the market equilibrium value would be,
\[
C = S \cdot N \left\{ \frac{\ln(S/X) + [r_h + (\sigma^2 / 2)]T}{\sigma \sqrt{T}} \right\} - xe^{-rt} \cdot N \left\{ \frac{\ln(S/X) + [r_h - (\sigma^2 / 2)]T}{\sigma \sqrt{T}} \right\}
\]

where \( \sigma^2 = \sigma_s^2 - \sigma_x^2 - 2\sigma_s\sigma_x\rho_{sx} \). The parameter \( \sigma^2 \) is the instantaneous proportional variance of change in the ratio \( S/X \). The parameter \( r_h \) is the rate of return on the (possibly imaginary) security used to hedge away the risk from the fluctuating exercise price. The expected return on this hypothetical security would be given by:

\[
r_h = r + b
\]

where \( r \) is the risk-free rate and \( b \) is the appropriate risk premium. Applying the Capital Asset Pricing Model to establish the appropriate value for \( b \) results in the following:

\[
b = \rho_{mx} \sigma_x / \sigma_m (r_m - r)
\]

where the subscript \( m \) denotes the market as a whole. Thus, when \( \rho_{mx} = 0 \), \( b \) is also zero. In most cases of R&D, the correlation between random shocks to the stock market and random shocks to the exercise price would be nonexistent. That is, the risk associated with the exercise price in the R&D case is in all likelihood based on technological uncertainties which are completely unsystematic and therefore diversifiable. Thus, \( r_h \) would be, in this case, the risk-free rate.

It may be that in the real world, opportunities do not exist to form the hedges assumed in the derivation of the option pricing models; nevertheless, the models are valuable and applicable.
Surely the basic justification for the existence of a business organization is that it can exploit imperfections in the markets for goods and services which the individual investor cannot do alone. Unless the managers can find investments which offer a return to risk ratio at least as favorable as that available to the capital market investor, the organization cannot pay the freight and will not long exist. Security valuation models can therefore be used as a benchmark for evaluating the corporation's investments, in that they show the appropriate reward for a given kind and level of risk.

The wide availability of software for the Black-Scholes OPM (even for hand-held calculators) makes its adaptation for estimating the benefits from R&D projects very attractive. It is a model with a proven track record, which is reassuring to decision-makers. To set up for its use, the analyst needs estimates of only seven inputs:

\[
\begin{align*}
S &= \text{an initial estimate of the present value of cash flows from the product or process to be developed} \\
X &= \text{an initial estimate of the cost of undertaking production} \\
r &= \text{the risk-free interest rate} \\
\delta_s &= \text{the instantaneous standard deviation around the trend line for } S \\
\sigma_X &= \text{the standard deviation around the trend line for } X \\
\rho_{sx} &= \text{the correlation between random shocks to } S \text{ and } X \\
T &= \text{time to expiration of the research effort}
\end{align*}
\]

These estimates could be made subjectively for each project, or objectively on the basis of historical data for similar projects previously undertaken. The making of these estimates, especially that for \( S \), interfaces the model with the expertise of the R&D planning staff. The inability of existing mathematical models to do this adequately has been an area of past criticism.
Not only is equation (4) useful for estimating the value of initiating research, it can also give insight into the decision about the length of time for which the research should be funded. One of the results from option pricing theory is that the value of the option increases the longer the time to expiration.\textsuperscript{10} Intuitively, this can be explained because the longer the time the process has in which to operate, the greater the potential spread between exercise price and the value of the underlying security. Managers could use the model to see the results of various decisions about $T$, and could choose accordingly.

An Implication of the Diffusion Model

One remarkable fact from option pricing theory is that because of the limited liability of the option (so that the downside is truncated) the option is more valuable the higher the variance of return on the underlying security. Increasing the variance, ceteris paribus, means an increase in upside potential, but the limited liability prevents downside risk from increasing proportionately — thus the increase in the value of the option. Translated into the R&D field, this means that the greater the uncertainty about what the research will discover, the greater the value of that research. The somewhat troubling implication which naturally follows is that so long as this holds true, society need have little worry about the prospects for continued technological progress. This runs contrary to the malaise perceived by many to be currently afflicting industrial research in the United States.

Barring an unlikely lack of research opportunities (resting on the foolish notion that there is little left to invent), the malaise could be traced within the confines of the model to low perceived value for the underlying securities. This would be the natural result of long delays imposed by
government regulatory bodies between the time research has produced a product and the time that product can be marketed. In the drug industry, for example, such delays are necessary for testing the safety of new drugs. Also of concern would be a political climate hostile to the apparently large profits of successful development efforts, which is an especially important possibility in the fields of food, fuel, and medicine. Finally, the increased risk of product liability litigation in the current business environment certainly adds a dimension capable of greatly reducing the expected value of going ahead with production, therefore reducing the incentive for research.

In order for a society to progress, it is necessary that risks be taken, and that risk-takers be rewarded. During the formative years of this nation, large numbers of very ordinary people were willing to take the ultimate risk necessary to settle the frontiers. Progress depends not only on the taking of financial risks, but also on the taking of technological and even physical risks. Those willing to bear these risks serve an essential role, and are compensated by the hope of a better life. Yet, as more people share in a high-quality life, it may be that fewer are willing to bear non-financial risk; and we more often call into question the fairness of the circumstances out of which such risk-taking arises.

Not only business firms, but also governments and nations must be competitive. It is to be expected that the less risk-averse groups will be the centers of innovative activity, and that they will tend to progress more rapidly.

The Problem of Indirect Benefits

The benefits from R&D are complex and subtle. Besides the direct benefit, indirect benefits from an ongoing R&D effort could come from chance
discoveries or perhaps more importantly from the know-how in place to respond to breakthroughs in technology achieved elsewhere. Because the option model just presented does not capture these indirect benefits, it is possible that research projects which should be accepted would be erroneously rejected if the OPM were used as the sole criterion.

Another interesting problem with indirect benefits concerns the model's basic underlying assumption about the value of the information which creates the changes in the price of the underlying security. When dealing with stocks, the use of a log-normal diffusion process grows out of the assumption of an efficient market, within which the security's price changes in response to new information (which comes at random). Once the new information has been captured in the price of the underlying stock, the information has no more value. The initiation of industrial research sets in motion a similar information generating process and this information adjusts the estimated value of the underlying security, but it is not necessarily true that the information then becomes worthless. Any residual value of the information is another indirect benefit not captured in straightforward application of the OPM to the R&D problem. If the value of direct benefits were enough to justify the initiation of research, consideration of indirect benefits would be a moot point for the decision-maker (although not for the securities analyst). From the decision-maker's point of view, the OPM can only give a clear "go" indication for the R&D decision, but cannot be relied upon alone for a "no-go" choice. Even though the problem of indirect benefits thus limits its usefulness, the OPM still can give a clear indication in one direction, and so is not without value in application to the R&D problem. Moreover, despite its drawbacks, it captures more of the value of R&D than any of the discounted cash flow techniques which represent the current state-of-the-art.
Placing a value on these indirect benefits is a pregnant area for further scholarly research.

Problems with the Jump Process Model:

In the framework of the diffusion process, it is possible to deal with situations characterized by slow, steady change. There are no sudden leaps expected, although they can occur and are allowed for in the variance of the diffusion process (although most moves cluster around the mean, allowance is implicitly made for the occasional very large jump). There are R&D situations, however, in which sudden leaps are the essence rather than the exception. In fact, the exciting projects dealing with things on the frontiers of science and technology would not fit very neatly into the diffusion process scenario. In this exciting world of high-technology research, days -- even weeks -- can go by with no apparent progress; then suddenly a barrier comes down and a great leap is accomplished in but a moment of time. There is a continuous time stochastic process, the jump process, which captures this. Unfortunately, application of the available jump process option model to R&D is much less satisfying than application of diffusion process models.

Cox and Ross [14] have worked out market equilibrium solutions to option values under a variety of stochastic processes, including jump processes. If \( x \) denotes the current state of the world, then a general form for a Markov jump process can be stated as:

\[
\begin{align*}
\frac{dS}{S} &= \mu(x)dt + \lambda(x)dt \frac{k(x)-1}{1 - \lambda(x)dt}, \\
\end{align*}
\]

(7)

Notation:

- \( \mu(x) \) = drift term
- \( \lambda(x) \) = probability of jump during time interval \( dt \), or the process intensity
- \( k(x)-1 \) = jump amplitude
Cox and Ross showed that the diffusion process of equation (1) is actually a special case of the more general process of equation (7). Unlike the R&D scenario which fit equation (1), the case now being dealt with is one which contains no drift. As noted above, a long time may go by with no apparent progress. Thus, the drift term would drop out. However, several other, more restrictive assumptions are necessary to derive a workable solution. By assuming that any discoveries would be good news, the jump direction would have to be up and the term \([k(x) - 1]\) can be confined to positive values. This is necessary to prevent violation of the limited liability condition. Further, if it were assumed that all knowledge about the current state of the world were captured in the initial estimate of the project's value, \(S\), then \(\lambda(x)\) and \(k(x)\) could be restated as \(\lambda(S)\) and \(k(S)\). Finally, if the process intensity \(\lambda(x)\) were specialized to be proportional to \(S\), \(\lambda S\), and the amplitude assumed to be independent of \(S\), the process would be refined to one for which Cox & Ross have accomplished a market equilibrium solution. It is a pure birth process without drift,

\[
dS = \begin{cases} \lambda Sdt & \text{if } 1, \\ 1-\lambda Sdt & \text{if } 0. \end{cases}
\]

Unfortunately, a solution has not yet been found for the situation in which the jump amplitude can take more than one value, because it greatly complicates formation of the hedge necessary to specify the market equilibrium condition, and because it allows the possibility that the limited liability constraint might be violated. Even so, there is no practical difference between a single large jump and a rapid-fire series of small jumps, which could occur under the above specification. The valuation formula derived under the pure birth process is given by Cox & Ross as follows:
\[ C = S \cdot \sum_{j > [X/(k-1) + 2]} B \left( j; \frac{S}{k-1 + 1}, e^{-r(T-t)} \right) 
- xe^{-r(T-t)} \cdot \sum_{j > [E/(k-1) + 1]} B \left( j; \frac{S}{k-1}, e^{-r(T-t)} \right) \] (9)

Notation:

- \( C \): value of option
- \( S \): value of underlying security
- \( X \): exercise price
- \( k \): jump amplitude
- \( T \): time of expiration
- \( t \): current time
- \( r \): risk-free rate

\[ B(j;p,q) = \binom{j-1}{p-1} q^p (1-q)^{j-p}, \] the negative binomial distribution, with values for \( j \) and \( p \) adjusted to the largest integer not exceeding the original value.

To implement the use of this technique, the analyst would need point estimates for \( S, X, k, \) and \( r \) based on the best currently available knowledge. Unfortunately, a solution has not yet been worked out which allows the exercise price to be stochastic.

As Cox and Ross pointed out, the market equilibrium solution is independent of \( \lambda \), the process intensity. This is because within the hedge portfolio of stock and option postulated for the arbitrageur, jumps cancel each other out. It is not the frequency of jumps but the size of them which determines the option's value within such a hedge. This presents a problem when there is uncertainty about the jump amplitude. As pointed out in the discussion of
FOOTNOTES

1. Source: National Science Foundation report, "National Patterns of R&D Resources, 1953-1977." The 1977 total was estimated at $40.8 billion, up 9% from the $37.3 billion in 1976. Of the total, $5.2 billion was for basic research, $9.0 billion for applied research, and $26.6 billion for development. Of the total estimate, 53% was for federally-funded projects—the majority devoted to space and defense.


3. See Branch [7], Clarkson [9], Grabowski & Mueller [24], Leonard [29], Scherer [43], Schwartzman [47], Severn & Laurence [48], and Worley [53].

4. See Baker and Freeland [4], p. 1169, "In summary, despite the large number of benefit measurement models in the literature, relatively little is known about the performance of these models when applied within an R&D environment. This is a critical area for future research."

5. The risk-free rate is recommended here because risk is being handled through the OPM. The money committed to research represents a voluntary obligation, and the amount of the outflow is assumed to be fixed ahead of time with certainty.

6. Perhaps the best published review is that by Clifford Smith [51].

7. See data in note 1. Of the 1977 total, 65% was for product development; only 12.7% was for basic research.

8. The uncertainty about the exercise price arises from technological matters. There is little a priori reason to believe that there should be a systematic relationship between such things and the market as a whole.

9. See Baker & Freeland [4], p. 1165, where available models were criticized for "no explicit recognition and incorporation of the experience and knowledge of the R&D manager."

10. See Smith [51] for proof.

11. That is, all the company stands to lose is the research investment.


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