Evaluation Of The Utility Of Informative Priors In Bayesian Structural Equation Modeling With Small Samples

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EVALUATION OF THE UTILITY OF INFORMATIVE PRIORS
IN BAYESIAN STRUCTURAL EQUATION MODELING WITH
SMALL SAMPLES

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EVALUATION OF THE UTILITY OF INFORMATIVE PRIORS IN BAYESIAN STRUCTURAL EQUATION MODELING WITH SMALL SAMPLES

A Dissertation Presented to the Graduate Faculty of Simmons School of Education and Human Development Southern Methodist University in Partial Fulfillment of the Requirements for the degree of Doctor of Philosophy With a Major in Education by Hao Ma

May 16, 2020
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Hao Ma

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ACKNOWLEDGEMENTS

First and foremost, I would like to express my deepest gratitude to my dissertation and research advisor, Professor Akihito Kamata, for his patient directions and mentorship in this Ph.D. program at Southern Methodist University. Without his guidance and unwavering support, it would have been impossible for me to successfully finish the dissertation.

Also, I must sincerely thank my committee members, Professor Paul Yovanoff, Associate Professor Doris L. Baker, and Dr. Yusuf Kara for their friendly suggestions, detailed instructions, and the general collegialities that they provided to me over the past four years.

Meantime, my appreciation goes to Associate Professor Ken Springer, Dr. Annie Wright, and Professor David Rosenfield for help and support that were highly helpful to me to complete my dissertation. Of greatest importance, Denise Patton served as proofreader, a friend, and a big sister to me when the COVID-19 pandemic fiercely hit the whole nation and the state of Texas as well. I appreciate her involvement in this special refinery process.

Let me also recognize all my friends from SMU, and from the Highland Park Chinese Baptist Church, for their sustained assistance, encouragement, and prayers.

Last, but not the least, I would like to acknowledge the countless sacrifices and understandings made by my wife Xuelian; she has been extremely supportive of me throughout this entire process to help me get this point. I also appreciate the support I have received from her family and my family along this journey; I will work hard to make all of you proud in the future.
EVALUATION OF THE UTILITY OF INFORMATIVE PRIORS
IN BAYESIAN STRUCTURAL EQUATION MODELING WITH
SMALL SAMPLES

Advisor: Professor Akihito Kamata


ABSTRACT

The estimation of parameters in structural equation modeling (SEM) has been primarily based on the maximum likelihood estimator (MLE) and relies on large sample asymptotic theory. Consequently, the results of the SEM analyses with small samples may not be as satisfactory as expected. In contrast, informative priors typically do not require a large sample, and they may be helpful for improving the quality of estimates in the SEM models with small samples. However, the role of informative priors in the Bayesian SEM has not been thoroughly studied to date. Given the limited body of evidence, specifying effective informative priors remains challenging for applied researchers. Therefore, a study that investigates performances on the parameter estimates of the SEM models with small samples among the MLE, the Bayesian estimator with informative priors, and the Bayesian estimator with non-informative priors is warranted.

Two Monte Carlo studies were designed for this dissertation: one with a confirmatory factor analysis (CFA) model and another with an SEM model. Both studies replicated 1000 datasets for each of the various experimental conditions. Specifically, they included a) sample sizes (30 and 70), b) the number of items per factor (5 and 10), c) mean factor loadings (.30 and .70), and d) estimators (the MLE, the Bayesian estimator with non-informative priors, the Bayesian estimator with correctly specified informative priors, and the Bayesian estimator with incorrectly specified
informative priors). Results were evaluated by various criteria, including the convergence rate, relative bias, root mean square error (RMSE), standard error (SE). The study on the CFA model focused on the evaluation of factor loadings, while the study on the SEM model concentrated on the evaluation of path coefficients.

Results demonstrated that the Bayesian estimator with informative priors converged with 100% convergence rates even for the CFA models with small sample sizes, as opposed to the ML estimator that had quite low convergence rates. For SEM models, the Bayesian estimator with informative priors displayed high convergence rates when the sample size was large \(N = 70\), while the convergence rates were very low when the sample size was small \(N = 30\) and the mean factor loading was small \(J = .30\). For the other conditions, the differences were not substantially large among estimators. In addition, the Bayesian estimator with correctly specified informative priors outperformed the Bayesian estimator with non-informative priors and the MLE in the recovery of factor loadings, while the Bayesian estimator with incorrectly specified informative priors did not outperform the MLE. Finally, the Bayesian estimator with correctly specified informative priors performed best on the recovery of path coefficients in the SEM models with small sample sizes, compared to the other estimators. Also, it was revealed that the performance between the Bayesian estimator with correctly specified informative priors and the Bayesian estimator with incorrectly specified informative priors was similar in this regard. Thus, it would be practical to use the Bayesian estimator with incorrectly specified informative priors to estimate path coefficients in the SEM models with small samples when researchers specify the prior mean somewhat lower than the mean factor loading, with the belief that the mean factor loading is higher or the mean factor loading is lower but the number of indicators per factor is larger. In contrast, researchers need to choose the location of prior mean in terms of accuracy and precision when the
number of indicators per factor is lower and they believe that mean factor loading is smaller. If researchers specify prior mean somewhat higher or lower than the population value, the estimate of path coefficients would be most accurate or inaccurate with the least or largest bias but the largest or smallest SE and RMSE value.

In addition to the Monte Carlo studies, a real dataset with a small sample size was analyzed. The results were interpreted by reflecting the results of the Monte Carlo simulation studies. Finally, study limitations, practical implications, and future research were discussed.

Keywords: Bayesian statistics; Informative prior; Factor analysis; Structural equation modeling; Small samples
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This is dedicated to my children Zihan and Daniel, and my wife Xuelian.
Chapter

1. INTRODUCTION

1.1. MLE and Parameter Estimates of SEM Model under Frequentist Framework

SEM is a multivariate statistical method that combines factor analysis and regression techniques to model the relationships between latent variables and observed indicators by taking the measurement model and structural model into account to address multiple dependent variables simultaneously (Bentler, 1980). Parameter estimates can be obtained by minimizing the maximum likelihood function of the discrepancy between a sample variance-covariance matrix and the estimated variance-covariance matrix. Though an MLE has advantages for obtaining good quality parameter estimates, its primary usage has been based on normal distribution theory and large sample sizes (Deng, Yang, & Marcoulides, 2018; Jöreskog, 1969). Researchers can resort to an asymptotically distribution free (ADF) estimator when a normal distribution assumption is not satisfied, but this approach works more effectively with large sample sizes (Browne, 1984; Kapalan, 2000, Yuan & Bentler, 1998). Moreover, an ADF tends to overestimate $\chi^2$ and underestimate the SE of parameters in practice (DiStefano, 2002; Hu, Bentler, & Kano, 1992). With that being said, there are many other approaches to manage non-normal distributions, including robust maximum likelihood with the Satorra and Bentler correction (RML), robust weighted least square (RWLS), and generalized least squares (GLS) (Browne, 1984; Mooijaart & Bentler, 1985; Muthén, 1993; Satorra & Bentler, 1988). However, when a small sample size is used with these models, the estimates of parameters are known to be biased.
For example, the MLE tends to produce inaccurate parameter estimates, the GLS estimator typically yields inflated type-I errors when the model is misspecified, and the MLR estimator improves the SE of parameter estimates only (Hwang, Malhotra, Kim, & Tomiuk, 2010; Lee & Song, 2004; Muthén, Muthén, & Asparouhov, 2016; Olsson, Foss, Troye, & Howell, 2000; Olsson, Troye, & Howell, 1999). In addition, small sample sizes typically result in lower power (Liang, 2014; Olsson, Foss, & Troye, 2003).

1.2. Estimators and Parameter Estimates of SEM Model under Bayesian Framework

In contrast, a Bayesian estimator has shown comparative advantages in the SEM models with small samples over the MLE. This estimator is based on a posterior distribution by incorporating information from prior knowledge and data likelihood. By specifying appropriate prior information in a Bayesian SEM, the model is more easily identifiable, with a less frequent occurrence of incorrect solutions and more accurate estimates of parameters (Hayashi & Marcoulides, 2006; Muthén, Muthén, & Asparouhov, 2016; Scheines, Hoijtink, & Boomsma, 1999). Meanwhile, specifying informative priors with small variances for cross-loadings in a Bayesian SEM not only effectively removes the upward biased factor correlation, but also reduces the insufficiency of model fit, signifying an improved model fit (Asparouhov, Muthén, & Morin, 2015; MacCallum, Edwards, & Cai, 2012). In addition, specifying informative priors for residual covariances in a SEM allows researchers to know whether the factor structure is incorrect or whether some factors are not omitted on purpose. (Asparouhov et al., 2015). Moreover, without the restriction on large sample sizes and the assumption of independent and identical sample data, the indeterminacy of covariance and the estimates of structural coefficients is also improved (Muthén & Asparouhov, 2012). Finally, informative priors can be specified for factor loadings, similar to setting up the essentially tau-equivalent model from the classical test
theory perspective. Although the essentially tau-equivalent model assumption is quite restricted, many measurement models and psychometric quantities are based on this assumption such as Cronbach’s alpha and the Rasch model. Therefore, this strategy may be useful to obtain reasonably good estimates for both the Bayesian factor analysis (FA) models and the Bayesian SEM models with small sample sizes. Although the use of informative priors in Bayesian SEM may be justified, specifying an effective informative prior poses many challenges, especially with regards to the challenge of specifying appropriate means of priors (Kruschke, 2014). It is well known that the prior distribution plays a crucial role in estimating a parameter when the sample size is small. The posterior can be obtained by incorporating prior knowledge into the model and specifying informative priors when they are available (Deng et al., 2018; Kruschke, 2014). However, a Bayesian estimator with informative priors is not without limitations, and the use of a Bayesian estimator with inaccurately specified and incorrectly specified informative priors tends to produce large bias and standard error, reduced reliability, and incorrect statistical inferences (Depaoli, 2014; McNeish, 2016).

1.3. Objective and Scope of the Present Study

Based on the literature review, there were few studies that focused on the evaluation of parameter estimates in the CFA models and SEM models with small samples by specifying informative priors for factor loadings only. Therefore, this dissertation mainly investigated whether this Bayesian approach would outperform the MLE in the CFA and SEM analyses with small samples on the recovery of parameter estimates and was expected to provide practical guidelines and cautions for applied researchers in this regard.

This dissertation conducted two Monte Carlo studies and they were designed to compare the quality of parameter estimates in the CFA and SEM models with small samples using the MLE,
the Bayesian estimator with informative priors, and the Bayesian estimator with non-informative priors. The results were evaluated in terms of convergence rate and accuracy. Specifically, the first study on CFA models concentrated on the comparison of the recovery of factor loadings under different sample sizes, mean factor loadings, and number of items per factor. It was expected that factor loadings would not be recovered well by the Bayesian estimator with incorrectly specified informative priors. In contrast, the second study on SEM models focused on the comparison of the recovery of path coefficients under varying experimental conditions. It was anticipated that path coefficients would be recovered reasonably well by the Bayesian estimator with incorrectly specified informative priors. Then, the dissertation analyzed a real dataset with small sample size and interpreted the results by reflecting the results from the simulation studies. Finally, implications for practice, limitations of the present study, and the future research direction were discussed.
Chapter

2. LITERATURE REVIEW

2.1. Methods

This review consists of an introduction, a literature analysis, and a discussion, and it is
organized as follows. First, the purpose of this review is proposed, and an explanation of its
importance is introduced. Second, studies discussing the usefulness of a Bayesian estimator with
informative priors and relevant justifications in the SEM models with varied experimental
conditions are summarized. Finally, the literature is synthesized, and the present study is
proposed.

This literature review has two purposes. The first purpose is to demonstrate the importance of
the role that informative priors play under the Bayesian framework by reviewing the relevant
literature. The second purpose is to compare the performances of different degrees of priors’
variances by specifying effective informative priors so that researchers can understand current
knowledge in this field.

This literature review was conducted with searches in the following databases: Web of
Science, ERIC, PsycINFO, JSTOR, and ProQuest. The following keywords were used to search
without limits: ‘Bayesian CFA,’ ‘Bayesian SEM,’ ‘Bayesian IRT Informative priors,’ ‘Latent
class modeling with informative priors,’ and ‘mixture modeling with informative priors’. The
search results produced overlaps and they were filtered by only including papers with
methodological investigations. This procedure yielded a total of 15 distinctive papers.
This literature review focuses on the usefulness of informative priors’ distribution in SEM analyses in terms of type of model, conditions, parameters of interest, evaluation criteria, and major findings. Within each type of model, each paper is summarized in chronological order. Thereafter, the brief comparison and discussion regarding the performances among different models and estimators lays a solid foundation for further investigation of the usefulness of various degrees of informative priors in SEM analyses and provides guidance for researchers.

The 15 journal articles were categorized into 5 types: confirmatory factor analysis (CFA) models (5 papers), SEM models (3 papers), latent growth curve models (2 papers), mixture models (3 papers), and other types of SEM models (2 papers).

For the CFA model, Heerwegh (2014) was chosen because the researcher conducted the Bayesian factor analysis (FA) models with continuous indicators in small sample sizes, while Liang and Yang (2014) discussed the same topic with emphasis on the CFA models with categorical indicators. In contrast, Natesan (2015) focused on the recovery of interval estimates in the CFA models with categorical indicators among the Bayesian estimator with informative priors and other estimators, and Bailey (2017) examined the comparative performance among the Bayesian estimator with informative priors and other estimators in the CFA models with sparse categorical indicators. Hoofs, Van de Schoot, Jansen, and Kant (2018) evaluated the model fit between the Bayesian-CFA models and the ML-CFA models in large sample sizes.

For the SEM model, Finch and Miller (2019) investigated the feasibility on the recovery of parameter estimates with a Bayesian estimator with incorrectly specified informative priors in multiple indicators, multiple causes (MIMIC) model with a covariate linked with a latent factor in varied sample sizes. As for the multilevel SEM models (MSEM), Depaoli and Clifton (2015) compared the recovery of parameter estimates among varied priors in the MSEM models with
continuous and dichotomous outcomes. Holtmann, Koch, Lochner, and Eid (2016) compared the performances of different estimators in the MSEM models with small sample sizes.

For the latent growth curve model, McNeish (2016) focused on the recovery of parameter estimates with informative priors with small sample sizes. Shi and Tong (2017) did the similar comparisons among informative priors and other estimators with latent basic growth models.

For the mixture model, Root (2007) used a Bayesian estimator with informative priors to examine whether it can help the recovery of parameter estimates in a latent class model. Depaoli (2012) compared the recovery of class separations in mixture CFA models among the Markov Chain Monte Carlo (MCMC), maximum likelihood estimator (MLE), and expectation maximization estimator (EM). Depaoli (2013) examined the usefulness of the Bayesian estimator with informative priors in the mixture class recovery in a Bayesian mixture model.

For the papers beyond the scope described above, Liang, Yang, and Huang (2018) evaluated whether there was an advantage on the recovery of structural coefficients in autoregressive cross-lagged models with informative priors as compared to other estimators, and Lamsal (2015) compared the performances on the parameter estimates of different prior settings for several three-parameter logistic (3-PL) IRT models.

In short, all of the studies selected were more specific explorations because they varied experimental conditions and focused on the comparative performances between the Bayesian estimator with informative priors and other estimators on the estimates of parameter in SEM analyses. Thus, they provide a new perspective for the effective use of a Bayesian estimator with informative priors to estimate the parameters in SEM in practice.
2.2. Summaries of the Literature

This section reviews the use of a Bayesian estimator with informative priors’ distribution in SEM. Results were summarized in chronological order in terms of the model and the estimators used, and the differences and similarities among different scenarios were compared. At the end of this literature review, the present study is proposed as well.

2.2.1. CFA Models

Heerwegh (2014) conducted two simulation studies to compare the performances of parameter estimates among different prior settings for the confirmatory factor analysis (ML-CFA) and the Bayesian confirmatory factor analysis (Bayesian-CFA) with small sample sizes. In the first simulation study, a one-factor CFA model with 4 indicators was investigated. The conditions included: sample sizes (25, 50, 100, and 200), factor loadings (.40, .60, and .80), and estimators ((the MLE, the Bayesian estimator with non-informative priors \(N \sim (0, \infty)\)) for factor correlations, the Bayesian estimator with weakly informative priors \(N \sim (.50, .04)\), and the Bayesian estimator with strong informative priors \(N \sim (.50, .01)\) for factor loadings). The parameters of interest were factor loadings. The results were evaluated in terms of mean square error (MSE), percentage bias, and power.

It was found that the Bayesian estimator with informative priors outperformed the traditional ML estimator on the estimate of factor loadings with the lowest bias and MSE under the weak factor loading across sample sizes or smaller sample size. Also, the research found that there was almost the same performance between the ML-CFA model and the Bayesian-CFA model when either sample sizes were larger or factor loadings were higher. Moreover, the research found that the bias and MSE of the Bayesian-CFA model using the Bayesian estimator with incorrectly specified informative priors were larger than the ML-CFA model.
In contrast, the results of the second simulation study showed that the Bayesian estimator with inaccurately specified informative priors underestimated factor loadings and produced larger bias on the estimates of factor correlations in a two-factor oblique CFA model when experimental conditions were almost the same as the first simulation study except that factor correlation varied between .25 and .40 and the Bayesian estimator with non-informative prior was specified for it.

Liang and Yang (2014) evaluated the performances on the recovery of parameter estimates among a weighted least squares means and variance-adjusted estimation estimator (WLSMV), the Bayesian estimator with informative priors, and the Bayesian estimator with non-informative priors in the context of non-normal underlying distributions in small sample sizes. The conditions included: sample sizes (50 and 200), factor loadings (.50 and .80), distribution of continuous indicators (normal distribution, moderately skewed distribution, and highly skewed distribution), distribution of categorical indicators (homogeneous normal distribution, homogeneous skewed distribution, moderately heterogeneous distribution, and highly heterogeneous distribution), and type of categorical indicator (2 categories and 4 categories). As for the prior specifications, the Bayesian estimator with informative priors $N \sim (.50, .01)$ and $N \sim (.80, .01)$ was specified for factor loadings and the Bayesian estimator with non-informative priors $N \sim (0, 5)$, $IW \sim (1, 3)$, and $IW \sim (0, 3)$ was specified for threshold, factor variance, and factor covariance, respectively, where $N$ represents the normal distribution and $IW$ represents the inverse Wishart distribution. The evaluation criteria included a model-data fit index, point estimate, SE of point estimates, and Bayesian posterior standard deviation.

It was found that the Bayesian estimator with informative priors performed almost the same as other estimators across conditions for the point estimate of parameters under the larger
sample size of 200 or the higher factor loading was of .80. However, the Bayesian estimator with informative priors outperformed the WLSMV estimator on the SE of point estimates of factor loadings and factor correlations under the same conditions, respectively. Although this estimator was less influenced by the distribution of the data as compared to other estimators, it did not demonstrate noticeable advantages on the estimates of factor loadings in small samples over the WLSMV estimator. Furthermore, the WLMSV estimator performed slightly better than the Bayesian estimator with informative priors under the smaller sample size of 50 or lower factor loading of .50. In sum, the Bayesian estimator with informative priors encountered fewer convergence problems than other estimators, outperformed the WLSMV estimator across conditions, and was more robust to non-normality of underlying continuous distributions for overall model fit.

Natesan (2015) compared the recovery of interval estimates among the Bayesian estimators, the RML estimation method, and asymptotically generalized least squares (AGLS) estimation method in a two-factor ordinal CFA model with 5 indicators. The conditions included: factor score shape distributions (multivariate normal and multivariate mild skewed), factor correlations (.20, .50, and .80), sample sizes (42, 63, 84, 105, 210, and 315), and estimators (the Bayesian estimator with informative Uniform prior ~ (0, 1) and the Bayesian estimator with relatively less informative Uniform prior ~ (-1, 1) specified for factor correlations, the RML, the WLS, the robust diagonally weighted least squares (RDWLS), and the robust unweighted least squares (RULS)). The results were evaluated in terms of bias and RMSE.

It was found that the Bayesian estimator with informative priors increased accuracy, with smaller bias and RMSE for the point estimate as compared to other estimators. In addition, this Bayesian approach produced the best coverage, followed by the Bayesian estimator with
relatively less informative priors. Moreover, the Bayesian estimator with informative priors had shorter width intervals than the Bayesian estimator with relatively less informative priors but had longer width intervals than the other estimators across conditions. Furthermore, the Bayesian estimator with informative priors resulted in little difference between standard error estimates and empirical standard errors and had the best negative and positive interval biases as compared to other estimators. Though the Bayesian estimator with informative priors had some advantages, there were no obvious differences on the higher factor correlations across conditions between this estimator and other estimators.

Bainter (2017) compared the performances between the ML estimator and other priors on parameter estimates in a two-factor item factor analysis (IFA) model with 5 binary items per factor. The conditions included: sample sizes (250 and 500), patterns of sparseness (baseline conditions no sparseness, 4 of the 5 items sparse on one factor, and 4 of the 5 items sparse on two factors), probabilities of endorsement (.02 and .04), and estimators (the MLE and the Bayesian estimator). Specifically, the Bayesian estimator with non-informative priors \( N \sim (0, 1000000) \) and the Bayesian estimator with moderate informative priors \( N \sim (0, 49) \) were specified for intercepts and factor loadings, respectively. The results were evaluated in terms of convergence, bias, efficiency, confidence interval, coverage, and statistical power.

It was found that the Bayesian estimator with moderately specified informative priors outperformed the ML estimator on convergence when the indicators had sparse item endorsements and the sample size was greater than 100 across all other conditions. In contrast, the performances of these two estimators on the estimates of intercepts and factor loadings were similar under a baseline condition with moderate sample sizes and endorsement on all items. Though the biases of factor correlations and factor loadings were downward and upward,
respectively, using the Bayesian estimator with moderately specified informative priors removed extreme estimates, improved efficiency, stabilized estimates, and increased statistical power.

Hoofs et al. (2018) evaluated the model fit between the Bayesian-CFA and the ML-CFA with large sample sizes. The conditions varied in model misspecification (no misspecification, light misspecification, and severe misspecification), factor loadings (.50 and .70), number of indicators per factor (6 and 12), number of factors (1 and 2), and sample sizes (50, 100, 250, 500, 1000, 5000, and 10000). Priors used included: the Bayesian estimator with non-informative priors $N \sim (0, \text{Infinity})$ specified for both factor loadings and intercepts, inverse-gamma distribution for factor covariance $IG \sim (-1, 0)$, the Bayesian estimator with informative priors $N \sim (.50, .05)$ and $N \sim (.70, .05)$ specified for factor loadings and intercepts, and the Bayesian estimator with wrong informative priors $N \sim (.30, .05)$ and $N \sim (.30, .05)$ specified for factor loadings. The results used a lower limit of .05 and an upper limit of .08 as evaluation criteria.

It was found that when using the Bayesian root mean square error of approximation (BRMSEA) as an assessment criterion, the models with no or small misspecification were generally accepted, whereas models with moderate or large misspecification were mostly rejected across the number of factors of the reference model if sample sizes increased. There were not any noteworthy differences among different prior settings with large samples. As the sample sizes increased, posterior predictive $p$-values ($ppp$), rejected all models across a degree of misspecification. Only a small proportion of the models with small misspecifications were accepted when the sample size was 100. However, these models were rejected when the sample sizes varied from 5000 to 10000. Like BRMSEA, there were no marked differences among different prior settings for large samples except that some two-factor reference models were rejected when using wrong priors. However, this effect gradually diminished in large samples.
2.2.2. SEM Models

For single-level SEM models, Finch and Miller (2019) investigated if a Bayesian estimator with incorrectly specified informative priors improved the parameter estimates in multiple continuous indicators, multiple cause models (MIMIC) with a covariate. The Experimental conditions included: sample sizes (30, 40, 60, 80, 100, 120, 140, 160, 180, and 200), the unstandardized factor loadings magnitude across indicators (.30 for weak factor loading, .60 for moderate factor loading, and 1.0 for strong factor loading), the numbers of observed indicators variable (5, 10, 20, and 30), the magnitudes of path coefficient (0 for no relationship, .20 for small relationship, .50 for medium relationship, and .80 for strong relationship), estimation methods (the Bayesian estimator and the ML estimator), and Bayesian priors. Furthermore, Bayesian priors were specified for path coefficients and factor loadings in one of the following manners: 1) both parameters were specified with naïve priors or correctly specified informative priors; 2) either one parameter was specified with incorrectly specified 1 SD to 3 SDs informative priors or the other parameter was specified with naïve priors; 3) either one parameter was specified with correctly specified informative priors or the other parameter was specified with naïve priors. The evaluation criteria included convergence and absolute relative bias (ARB).

It was found that all parameters had no convergence problems across conditions. Also, it was found that the Bayesian estimator with fully informative priors specified for factor loadings and path coefficient had the lowest ARB. In contrast, the performance of the ML estimator and naïve priors was similar and almost had the largest ARB with the exception when the sample sizes were set equal to 20 and 30. In addition, the performance of the Bayesian estimator with incorrectly specified informative priors for path coefficient was close to the Bayesian estimator with correctly specified informative priors for both factor loadings and path coefficients.
As for the multilevel SEM models (MSEM), Depaoli and Clifton (2015) investigated the impact of different priors on the parameter estimates and compared the performances of the MSEM models with continuous and dichotomous indicators. The Experimental conditions included: number of clusters (40, 50, 100, and 200), cluster sizes (5, 10, and 20), intraclass correlation coefficients (ICC) (.02, .05, .10, .20, and .40), indicator types (continuous indicator and dichotomous indicator), invariance of loadings (free estimated at each level, and factor loadings held invariant across levels), estimators (ML, WLSM, and Bayes), and prior specifications. Specifically, the Bayesian estimator with non-informative priors, $\Gamma^{-1} \sim (-1, 0)$, $N \sim (0, 10^0)$, $N \sim (0.5)$, and $\Gamma^{-1} \sim (-1, 0)$, was specified for the within-level variance parameters, while for the between-level regression coefficients for models with continuous indicator, categorical indicator, and both types of indicator. In contrast, the Bayesian estimator with informative priors, $N \sim (1, .01)$ and $\Gamma^{-1} \sim (.001, .001)$ was specified for the regression coefficients across levels and the variance parameters at between levels. Finally, the Bayesian estimator with weakly informative priors, $N \sim (1, 5)$, $N \sim (1, .05)$, $N \sim (1, .25)$ and $\Gamma^{-1} \sim (.001, .001)$ was specified for the regression coefficients across levels and the variance parameters between levels. The parameters of interest were the regression coefficients and variances at each level. The results were evaluated in terms of coverage, bias, and efficiency.

It was found that the Bayesian estimator with informative priors outperformed other estimators almost across conditions and levels. Specifically, they produced accurate estimates of the factor loadings and covariate effects for both levels across conditions in the MSEM models with categorical indicators compared, as well as the small values of RMSE and the improved credible interval coverage of path coefficient in cluster-level. Also, the estimates of regression coefficients obtained with informative priors were more accurate than other estimators across
conditions for the models across indicator types. Moreover, $\Gamma^{-1} \sim (0.01, 0.01)$ specified for the cluster-level variances produced more accurate between-level covariate effects for the low to moderate ICC values. Interestingly, placing uniform priors $\Gamma^{-1} \sim (-1, 0)$ also produced the similar results. In addition, the Bayesian estimator with informative prior outperformed other estimators across indicator types in terms of convergence except when there was a small number of clusters ($J = 40$), small cluster size ($N = 50$), and extremely low ICC values. In contrast, the ML estimator had more convergence problems for the models with categorical indicators than the models with continuous indicators.

Holtmann et al. (2016) compared the performances between the ML estimator and other priors in the two-level SEM models with continuous indicators and ordinal indicators. The conditions included: the number of clusters in level 2 (50, 100, 150, and 200), the number of observations in level 1 (2, 4, and 6), indicator types (continuous indicator and ordinal indicator), estimators (the MLE, the WLSMV estimator, and the Bayes estimator), and prior specifications. Specifically, the researcher specified diffuse priors and a Bayesian estimator with strongly informative inaccurate priors for factor loadings across levels, residuals at within-level of continuous indicator models, thresholds in categorical indicator models, intercepts in continuous indicator models, variance parameters across levels, and covariance parameters across levels in both types of indicator models. For diffuse priors, they included $N \sim (0, 10)$ and $N \sim (0, 5)$; $IG \sim (-1, 0)$, $N \sim (0, 10)$ and $N \sim (0, 10)$; $IW \sim (0, -3)$ and $N \sim (1, 3)$; and $N \sim (0, -3)$ and $IW \sim (0, -3)$. For the Bayesian estimator with inaccurately specified strong informative priors, they included $N \sim (0.80, 0.01)$ and $N \sim (1.20, 0.01)$; $IG \sim (-1, 0)$ and $N \sim (0, 10)$; $N \sim (3, 1)$, $IW \sim (0, -3)$ and $IW \sim (1, 3)$; and $IW \sim (0, -3)$ and $IW \sim (0, 3)$. As for the Bayesian estimator with weakly specified informative priors, they included $N \sim (0.80, 0.20)$ and $N \sim (1.20, 0.20)$ for
factor loadings across levels, $IW \sim (1, 3)$ for variance parameters across levels, and $IW \sim (0, 3)$ and $N \sim (0, 10)$ for covariance parameters across levels and thresholds in categorical indicator models. The Bayesian estimator with incorrectly specified weakly informative priors was set the same as the Bayesian estimator with accurately specified weakly informative priors except that the order of the first two priors was swapped. The results were evaluated by relative bias, SE bias, and coverage.

Compared to the MSEM models with categorical indictors, the Bayesian estimators did not have more advantages over the ML estimator in the MSEM model with continuous indicators. Specifically, using a Bayesian estimator with informative priors tended to overestimate the empirical SD of factor loadings across levels, and there was no clear pattern of difference in coverage values between them. However, the Bayesian estimator with accurately specified strongly informative priors outperformed the WLSMV estimator for the models with categorical indicator and led to a coverage of almost 100% for factor loadings and 95% for the remaining parameters. The Bayesian estimator with strongly informative accurate priors also produced the smallest bias for the estimates of factor loading across levels, variances and covariances, and threshold as compared to other estimators. In contrast, the Bayesian estimator with inaccurately specified strongly informative priors yielded severely biased results and very poor coverage even in large samples.

2.2.3. Latent Growth Curve Models

McNeish (2016) compared the performances of the MLE and different prior settings on the parameter estimates for a latent growth curve model with 2 time-invariant predictors. The author simulated 1000 datasets with varied sample sizes of 20, 30, and 50. The estimation methods and estimators included full maximum likelihood, restricted maximum likelihood with a Kenward-
Roger correction, a Bayesian estimator with non-informative improper inverse Wishart priors specified for covariance components $W^{-1}\sim (0, -p - 1)$ (where $p$ is the dimension of the covariance matrix) and marginal priors specified for variances and residual variances of growth parameters $\Gamma^{-1}\sim (0.1, 0.01)$, and a Bayesian estimator with weakly informative priors $W^{-1}\sim \begin{pmatrix} 3 & 0.30^t \cdot 6 \\ 0 & 0.20^t \cdot 25 \end{pmatrix}$ for the covariance components of growth parameters. The criteria included relative bias, coverage, efficiency, and power.

It was found that the recovery of parameter estimates for the Bayesian estimator with informative priors was more advantageous than other estimators due to the fact that the intercept variance and slope variance across sample sizes had the smallest bias and showed the most statistical efficiency as compared with other estimators. Meanwhile, it outperformed the ML estimator in terms of a 95% coverage interval.

Tong and Shi (2017) compared the performances among different priors on the parameter estimates for a latent basis growth model with 4 time points. The conditions included: sample sizes (50, 200, and 500), factor correlations (-.30, 0, and .50), error variances (.26 and .70), and prior specifications. Specifically, prior specifications included the Bayesian estimator with non-informative priors $N\sim (0, 1,000,000)$ and $N\sim (0, 1,000,000)$, the Bayesian estimator with informative priors $N\sim (.10, .01)$ and $N\sim (.30, .01)$, and the Bayesian estimator with weakly informative priors $N\sim (.20, .01), N\sim (.40, .01), N\sim (.30, .01), N\sim (.90, .01), N\sim (.30, .0001)$ and $N\sim (.90, .0001)$. The results were evaluated in terms of bias, MSE, and SE.

The research found that the Bayesian estimator with accurately specified informative priors led to the most accurate and efficient combination of parameter estimates across all conditions as compared to other estimators. Specifically, misspecified models affected the model estimation more than the Bayesian estimator with accurate informative priors affected the model.
estimation. Moreover, when the Bayesian estimator with weakly informative priors was used, the bias for the growth parameter became larger due to the prior mean shifting away from the true mean. However, there were no salient differences in the bias of the estimates of growth parameters between the Bayesian estimator with non-informative priors and the Bayesian estimator with informative priors except that the latter had better power than the former.

2.2.4. Mixture SEM Models

Root (2007) examined whether the use of a Bayesian estimator with informative priors improved the model identification and recovery of parameter estimates in a series of latent class models. More specifically, the researcher was highly concerned with the effect that a Bayesian estimator with correctly specified informative priors and a Bayesian estimator with incorrectly specified informative priors had on the recovery of parameter estimates. The experimental conditions included: probabilities of latent class sizes (an unconstrained three-class 5 item model with probabilities of .25, .35, and .40 and an unconstrained three-class 5 item model with probabilities of .10, .20, and .70), sample sizes (25 and 50 for the strong item-response probability parameter and 25, 50, and 100 for the weak item-response probability parameter), strengths of the measurement parameter (.20:.80 for strong and .40:.60 for weak), and prior specifications on the measurement parameter (a Bayesian estimator with accurately specified informative priors, a Bayesian estimator with inaccurately specified informative priors, and a Bayesian estimator with accurately specified mild informative priors). The results were evaluated in terms of convergence, bias, and coverage.

The research found that the Bayesian estimator with correctly specified informative priors facilitated the model identification of the unconstrained latent class models with small samples and improved the recovery of measurement parameters across levels. However, the findings on
class size parameters were not consistent with the researcher’s expectations. Class size parameters showed reduced bias when the Bayesian estimator with informative priors was specified for the strong measurement condition. However, the pattern was mixed for the weak measurement condition. Furthermore, when the Bayesian estimator with incorrectly specified informative priors was specified for the strong measurement condition, the bias was much larger than the Bayesian estimator with correctly specified informative priors. Surprisingly, the Bayesian estimator with incorrectly specified informative priors reduced bias significantly in the weak measurement condition. In addition, using the Bayesian estimator with mild informative priors mitigated the uncertainty of selecting the optimal model and decreased the risk of producing incorrect parameter estimates.

Depaoli (2012) investigated the biases under two different forms of class separation within a simple structure two-factor mixture CFA model with 10 indicators across estimators. The experimental conditions for the measurement model included factor loadings of .80 for class 1 and factor loadings of .30, .50, and .70 for class 2, with high and low separation. Prior specifications included weak priors $N \sim (0.80, 100)$, a Bayesian estimator with informative priors $N \sim (0.80, 0.01)$ specified for factor loadings, and a Bayesian estimator with informative class proportion priors $D \sim (80, 20)$ for class proportions (where $D$ represented the Dirichlet distribution). For the structural model, factor loadings were set at .80 and .30 for the 2 mixture classes. Factor variances for both factors in class 2 were set at either 1, 3, or 5 to represent either poor, moderate, or high structural class separation, respectively.

The results of the measurement model found that the Bayesian estimator with informative priors outperformed other estimators with the presence of relatively low biases for all separation conditions for .80: .20 and .20: .80 mixture class proportions, where the minority class was
represented for lower or higher factor loadings. In contrast, none of the estimators demonstrated
problematic biases for a half-and-half mixture class proportion. As for the estimates of factor
loadings, none of the results using the Bayesian estimator with informative priors had
problematic bias as compared to the other priors across mixture class proportions. However, both
the Bayesian estimator with informative priors and the Bayesian estimator with weakly
informative priors produced large biases on the estimates of structural parameters across mixture
class proportions. In contrast, the Bayesian estimator with informative priors showed acceptable
bias on the factor loadings and structural parameters in the structural models. In addition, the
results of the structural model showed that prevalent and higher estimate biases, indicating a
poor mixture class separation, had a larger impact on the estimates of the structural model.

Depaoli (2013) examined the mixture class recovery under 4 levels of class separation of
a 3-class growth mixture model. The conditions consisted of mixture class separations (.50, 1.0,
1.5, and 2.0), trajectory shapes (linear and quadratic), mixture class proportions
(.33: .33, .10: .45: .45, and .10: .20: .70), sample sizes (150 and 800), estimators (the MLE
and the EM), and prior specifications (diffuse priors, informative priors, data-driven informative
priors, weakly informative priors, partial informative priors, and inaccurate priors). The
performances were assessed in terms of convergence rates, MSE, and bias.

It was found that both the trajectory shapes and class proportions were accurately
recovered across conditions when a Bayesian estimator with informative priors was specified for
the growth parameters, followed by the partial-knowledge priors. In addition, the ML estimator
and the Bayesian estimator with weakly specified informative priors performed very poorly in
the recovery of relevant parameter estimates. Comparatively speaking, even though the prior
mean was not incorrectly specified, the Bayesian estimator still showed better performance under some higher-class separation levels.

### 2.2.5. Other SEM Models

Liang et al. (2018) examined the estimates of path coefficients in multiple indicators of autoregressive cross-lagged models using different estimators. The conditions included factor structures (8 models in total with varied measurement conditions at 2 or 5 with or without error correlations and cross-loadings), conditions of non-invariance (mixed pattern and decreased pattern), sizes of path coefficients (medium and large), sample sizes (100, 400, and 1000), and prior specifications (The Bayesian estimator with informative priors $N \sim (0, .01)$ and $N \sim (0, .03)$ specified for the selected parameters and the Bayesian estimator with non-informative priors $N \sim (0, \text{Infinity})$ for the other parameters). The results were evaluated with bias and MSE.

The research found that almost all the models across conditions converged to acceptable solutions except under the conditions where the magnitude of non-invariance of factor loadings was large. The highest rejection rates appeared in the models with a small sample size of 10 by specifying a prior variance of .01 and the bias of autoregressive coefficients was generally smaller in five-wave models instead of two-wave models. Models with a decreased pattern of non-invariance factor loadings produced greater bias than the models with mixed patterns of non-invariance factor loadings. In addition, the index MSE produced similar patterns in both models and the bias of cross-lagged coefficients was effectively reduced due to the improved estimates of cross-loadings.

Lamsal (2015) compared the recovery of parameter estimates in a series of three-parameter logistic regression IRT models with different estimators. The experimental conditions varied in sample sizes, test lengths, prior specifications, and actual item parameters. More specifically,
sample sizes varied between 200, 500, 1000, and 2000 and test lengths varied among 10, 20, and 50. For prior specification, three Bayesian estimators were specified for the discrimination parameter $\alpha_i$, the difficulty parameter $\beta_i$, and the guessing parameter $\gamma_i$, respectively. The Bayesian estimator with non-informative priors was specified for $\alpha_i \sim LN(0, 1000)$, $\beta_i \sim N(0, 1000)$, and $\gamma_i \sim Beta(1, 1)$, the Bayesian estimator with informative priors was specified for $\alpha_i \sim LN(0, 4)$, $\beta_i \sim N(0, 4)$, and $\gamma_i \sim Beta(5, 17)$, and the Bayesian estimator with informative priors with much smaller variance was specified for $\alpha_i \sim LN(0, 1)$, $\beta_i \sim N(0, 1)$, and $\gamma_i \sim Beta(5, 17)$. As for the estimation methods, they mainly included marginal maximum likelihood estimation, full Bayesian estimation using MCMC algorithm, and the Metropolis-Hastings Robbin-Monro estimation. The results were evaluated in terms of convergence, bias, and root mean square deviation (RMSD).

It was found the three-parameter logistic regression IRT models had better convergence and relatively accurate parameter estimates across estimators when either a Bayesian estimator with informative priors or informative priors with much smaller variance was used. In contrast, the issue of non-convergence actually took place using a Bayesian estimator with non-informative priors under specific conditions when estimating discrimination parameters. Also, when the actual parameter value approached the mean or mode of the priors, these three estimators performed better than when actual value approached the boundary value. However, it is difficult to say which one was best because each estimator performed differently in various conditions.

2.3. Usefulness of Informative Priors in Bayesian SEMs

The Bayesian SEM has been widely used across fields in recent years, successfully solving a series of problems incurred in the ML-SEM models with small samples. Specifically, applying a Bayesian estimator with informative priors in simulation studies in FA and SEM analyses can
improve parameter estimates (Baldwin & Fellingham, 2013; McNeish, 2016; Stegmueller, 2013; Stenling, Ivarsson, Johnson, & Lindwall, 2015). Here, the usefulness of informative priors in a Bayesian SEM is justified.

2.3.1. Improve the Parameter Estimates of the ML-SEM Model with Small Samples

The ML estimator is tightly linked with asymptotic theory, and multivariate normality is the basic assumption to obtain reasonable parameter estimates in large samples (Baldwin & Fellingham, 2013; Gelman & Shalizi, 2013; Hancock & Mueller, 2012). However, the performance of the ML estimator is poor if using the data with small samples to fit the FA model or the SEM model. According to Bayes’ Theorem, prior knowledge has little influence on the estimates in large samples. So, the results of a Bayesian estimator with non-informative priors are almost the same as the ML estimator. When a Bayesian estimator with weakly informative priors is used, it has no influence on the posterior estimates, and large prior variance only speeds up the algorithm’s convergence (Muthén et al., 2016). However, both the FA model and the SEM model with small samples are often not identified, resulting in improper solutions; thus the estimates of parameters are unreliable (Bartholomew, 1987). In contrast, a Bayesian estimator with informative priors makes the FA and SEM model more easily identifiable, contributes to the formation of the posterior distribution, and gives a reasonable estimate (Lamsal, 2015).

The SEM model provided a platform to explore the relationships among the latent factors and the observed variables (Bollen, 1989). One of the most important relationships among these was the relationship between the latent factors, which was measured by structural coefficients and reflected the magnitude and direction of one factor’s impact on the others (Tomarken & Waller, 2005). ML-based SEM did not effectively overcome a series of challenges in the estimation process. The first challenge was that the restriction on large sample sizes and the
assumption of independent and identical distribution for samples often made sample covariances indeterminate and made the model more likely to obtain inaccurate estimates of structural coefficients in small samples (Hoogland & Boomsma, 1998). In contrast, a Bayesian estimator with informative priors does not have any restrictions on sample sizes and relies on multivariate normality assumptions, which makes it easier to use with small samples and avoids indeterminacy due to direct use of raw data. The second challenge is that including all direct paths in the ML-SEM model tends to result in an unidentified model whereas applying a Bayesian estimator with informative priors makes the model much more identifiable (Müller, 2012). The third challenge is with cross-loadings, residual correlations, and the correlations between observed variables and latent factors; as these parameters were added to the SEM model and hypothesized to be zero, results of the ML-SEM model often produced larger biases on the structural coefficients, and theoretical support for correlating the residual errors to improve the model fit was not strong enough (Nikolov, Coull, Catalano, & Goldleski, 2006; Tsiatis, Degruttola, & Wulfsohn, 1995). In contrast, Muthén and Asparouhov (2012) found that such parameters did not significantly impact the estimate of the structural coefficients, but rather improved its estimates with informative priors in a Bayesian SEM.

One of the challenges for applied researchers is estimating expected quality factor loadings in the SEM model with small sample sizes (Hui & Salarzadeh, 2016; Lee & Song, 2004). An essentially tau-equivalent condition assumes that the true score for all items is the same but with different error variances (Little, Slegers, & Card, 2006). Under the CFA framework, the tau-equivalent condition can be realized by equal factor loadings. Specifying informative priors uniformly to factor loadings would be similar, but less restrictive than the tau-equivalent condition. Therefore, this strategy may be useful to get reasonably expected estimates for the
SEM models with small samples under the Bayesian framework. Under a one-factor FA framework, all factor loadings were the same for measuring the latent construct, and the intercept and the error variances were freely estimated (Graham, 2006).

2.3.2. Effectively Fix the Problems of Model Specifications in ML-SEM Analyses

Cross-loading is not uncommon due to the stochastic errors contained in the observed variables and items measuring multiple factors in SEM analysis (MacCallum et al., 2012). Some frequentists claim that all small cross-loadings should be treated as zero and adding them into the model results in logic errors, including inflated factor correlations and an unidentified model or other issues (Asparouhov & Muthén, 2009; Hayashi & Marcoulides, 2006; Morin, Marsh, & Nagengast, 2013; Marsh et al., 2009, 2010; Marsh, Morin, Parker, & Kaur, 2014; Stromeyer, Miller, Sriramachandramurthy, & DeMartino, 2015). In fact, specifying very small variances on cross-factor loadings makes itself similar to CFA and avoided the identification problem by embedding the useful information into the analysis procedure. Taking cross-loadings into account in a Bayesian SEM reflects the actual situation, thus resulting in non-zero factor loadings and non-biased factor correlations (Asparouhov et al., 2015). Also, specifying very small variances on other parameters rendered some estimates with credibility intervals outside of 0, meaning that there were some model misspecifications. With the model modification indices, the overall model fit can be improved to some extent (MacCallum et al., 2012).

Residual covariance was very common in both FA and SEM model and can be regarded as an instance when the parameters are misspecified and added to the model, meaning indicators with residual correlations were due to the omission of minor factors. For SEM users, substantive evidence was not easily provided to explain residual correlation, while model fits and results were also not as satisfactory as expected (Kline, 2015). Similar to addressing cross-loadings
issues in SEM, the estimate of a full residual covariance matrix can be obtained by specifying informative priors on residual covariance. With this approach, the residual variance-covariance matrix is constrained by the priors. The logic was transforming residual covariance parameters to approach zero, but not the real zero, by using very small variances. Specifically, Asparouhov et al. (2015) used:

$$\theta \sim IW(dD, d)$$  \hspace{1cm} (2 - 1)

as informative priors, where $IW$ was inverse Wishart distribution, $\theta$ was diagonal residual covariance matrix, $d$ was degree of freedom of prior distribution, and $D$ was the diagonal matrix, which is the same as the CFA estimate $\theta$. Prior mean of $\theta$ can be obtained by the formula:

$$\mu_0 = \frac{d}{d-p-1} D$$  \hspace{1cm} (2 - 2)

proposed by Muthén and Asparouhov (2012), where $p$ was the number of observed variables and $d$ was the only parameter that controlled prior variance. When $d$ gradually increased, the prior variance became more restrictive and gradually approached zero. Inputting different $d$ values into the Bayesian SEM sensitivity procedure ensures that the model quickly converges with a $ppp$ value greater than .05. If the $ppp$ value is greater than .05, then the method using informative priors is acceptable. This method helps researchers understand the origins of the differences between a proposed CFA model and the data used, and know if factor structure is stable or if there are some factors ignored, so the optimal model can be finalized.

2.4. Strategy to Specify Informative Priors in SEM Model

Overall, researchers advocated that valid specification of informative priors was involved in the determination of distribution shape, location, variance of priors, and the information about prior knowledge could be obtained via meta-analyses, reviews, empirical studies, and expert opinions in terms of the availability of these sources (O'Hagan et al., 2006; Rupp et al., 2004;
Smid et al., 2019). There were, however, specifically no unanimous trends in the plethora of literature. Different researchers had different opinions due to varied goals.

For shape of prior distribution, researchers varied their claims regarding the different types of parameters in different models. For CFA or SEM models, normal distribution was specified for the means or the intercepts, factor loadings, and regression coefficients of continuous variables. In addition, uniform distribution was specified for factor correlations. Moreover, normal distribution was also specified for cross-loadings and inverse Wishart distribution was specified for residual covariance (Depaoli & Clifton, 2015; Guo et al., 2019; Heerwegh, 2014; Holtmann et al., 2016; Hoof et al., 2018; Liang & Yang, 2014; Muthén & Muthén, 1998-2017). As with Dirichlet distribution, it was specified for the parameters of categorical latent variables. Similarly, normal distribution or uniform distribution was specified for the thresholds of categorical dependent variables. For inverse Gamma distribution, uniform distribution, and log normal distribution, any of them can be specified for the variances or residual variances of observed variables. In contrast, Inverse Wishart distribution was specified for the same parameters when the number of latent variables was more than one. For latent growth curve models, either normal distribution or inverse Wishart distribution was specified for growth parameter (McNeish, 2016; Tong & Shi, 2017). As for mixture models, normal distribution and Dirichlet distribution was specified for factor loadings and class proportions, respectively (Depaoli, 2012). For IRT and Rasch models, log normal distribution or normal distribution was often specified for the discrimination, threshold, and ability parameters (Kim, Cohen, Kwak, & Lee, 2019; Lamsal, 2015; Marcoulides, 2018; Matteucci, Mignani, & Veldkamp, 2012).

For specification of prior mean, it varied among researchers based on subjective judgement, a combination of data and subjective judgement, and data itself due to different accessibility to
available information (Lunn, Jackson, Best, Spiegelhalter, & Thomas, 2012; Marcoulides, 2018). Typically, mean value was used to express the location of prior that follows a normal distribution while the other prior distributions commonly used in practice may not have a defined mean (Finch & Miller, 2019). When prior mean was set equal to the population value, it allowed researchers to incorporate more specific knowledge on parameters to obtain accurate estimates. Usually, prior mean of parameters was approximately placed on unit scale in educational and psychological research, especially in simulation studies (Depaoli, 2012, 2013; Gelman, 2018; Miočević, MacKinnon, & Levy, 2017; Zondervan-Zwijnenburg, Peeters, Depaoli, & Van de Schoot, 2017). In addition, some researchers advocated that the prior mean should be determined in terms of 95% limits for continuous regression parameters (Depaoli & Clifton, 2015).

However, it was also very common that prior mean was not centered on the population value because researchers never knew the exact parameter value in varied models (Baldwin & Fellingham, 2013; Chen, Zhang, & Choi, 2015; Depaoli & Clifton, 2015; Depaoli & Van De Schoot, 2017; Holtmann et al., 2016; Yuan, & MacKinnon, 2009). In view of the facts-mentioned above, more specifically, Finch and Miller (2019) used three incorrectly specified normal distributed informative priors for regression coefficients and factor loadings in MIMIC models linked with a latent factor with small samples by having prior mean 1SD, 2SDs, and 3 SDs away from the population value. The results showed that Bayesian estimator with incorrectly specified informative priors may be helpful to improve parameter estimates.

As for the specification of informative prior’s variance, it reflects the degree of certainty of prior knowledge. Generally, the smaller the prior variance, the stronger the informative prior is. An example was specifying a very small variance of .01 or .001 for factor loadings or cross-loadings, which effectively reduced the bias of estimates in Bayesian SEM with small samples
(Asparouhov, & Muthén, 2009). In contrast, a variance of 1 was viewed as weakly informative prior (Muthén & Asparouhov, 2012). Likewise, Dapoli (2014) adjusted the prior variance either 10%, 20%, or 50% for the intercept and covariate to compare the effects of Bayesian estimator with incorrectly specified informative priors in a mixture model under two sample sizes. The results showed that parameter estimates were still as expected quality when the prior variance was sufficiently large. However, other research found that the more constraints placed on the variance parameter, the larger the bias of the estimate was (Shi & Tong, 2017). So, it is important to judiciously choose the variance of priors based on the relevant theory and the data rather than subjective specification of .01 as the minimum value.

2.5. Discussion

This literature review compared the performances of informative priors and other prior settings or estimators on the parameter estimates of SEM models in fourteen studies. The criteria for selecting these studies included the use of informative priors in a SEM model, the clear description of a comparative study procedure under the ML framework and Bayesian framework, and the precise description of experimental designs. These findings were examined and synthesized, and three key findings came as the result of this literature review. First, the role that informative priors play in Bayesian frameworks has not yet been adequately studied such that researchers cannot draw firm conclusions about the appropriateness of the use of this technique. Second, the effectiveness of varied degrees of prior variance in Bayesian SEM is unclear. Third, specifying effective informative priors still poses challenges when applying this technique.

2.5.1. Role of Informative Priors in Bayesian SEMs

It is important to examine the role of informative priors in gauging the performance on parameter estimates in Bayesian SEM analyses. Due to the varied settings of priors and different
experimental conditions, it is very difficult to draw a unified conclusion from the findings. The prior studies concentrated on the usefulness of informative priors, including parameter estimates, model identifications, and measurement invariance issues in the ML-SEM with small sample sizes (Baldwin & Fellingham, 2013; Helm, Castro-Schilo, & Oravecz, 2017; Müller, 2012; Stegmueller, 2013). Most of these results indicated that use of informative priors was significantly more effective for obtaining robust parameter estimates, improving efficiency, facilitating the model identification, improving overall model fit, and increasing statistical power in small sample settings. This literature review confirmed the important role that informative priors played in the formation of the posterior distribution.

However, these studies also provided conflicting evidence concerning the role that informative priors play in Bayesian SEM models. For example, when specifying informative priors for factor loadings and non-informative priors for factor correlations in the CFA models with small samples, the corresponding estimates of these two parameters were underestimated and overestimated, respectively (Heerwegh, 2014). Similarly, it was found that the empirical SD of the factor loadings both between-level and within-level in MSEM was overestimated when using this Bayesian approach (Holtmann et al., 2016). Different from these findings, Liang and Yang (2014) ascertained that the informative priors performed slightly inferior than the WLSMV estimator when the sample sizes were small and the factor loadings were low. Root (2007) found that biases were monotonically decreased when informative priors were specified for strong measurement parameters in latent class model; however, the bias patterns were mixed when they were specified for weak measurement parameters. Furthermore, Bainter (2017), Natesan (2015), and Tong and Shi (2017) found that the recovery of factor correlations and growth parameters were similar between Bayesian estimator with informative priors and other estimators, without
any noteworthy differences among them. Because there is currently no consensus on the role that Bayesian estimator informative priors play in Bayesian SEM, further exploration should be done from varied perspectives, with particular attention to the impact of combinations of different experimental conditions and various prior settings on the estimates of parameters.

2.5.2. Effectiveness of Different Degrees of Priors Variances

There are three types of priors: non-informative priors, weakly informative priors, and informative priors (Lynch, 2007), reflecting the degree of uncertainty in prior knowledge. Non-informative priors have minimal impact on the posterior distribution. This is because in large sample sizes, the posterior distribution is mainly determined by likelihood and thus the estimates are close to the results when using an ML estimator. However, even within large sample sizes, non-informative priors can impact and bias parameter estimates (Ghosh & Mukerjee, 1992). In contrast, in small sample sizes, posterior distribution is dominated by the prior specifications and thus the ML estimator typically produces biased estimates, lowers the power, and increases the SE of point estimates (Muthén & Asparouhov, 2012; Zyphur & Oswald, 2015). Heerwegh (2014) found that the ML estimator produced underestimated factor correlations and overestimated factor loadings when true factor loadings were high (.80) in the CFA models with small samples. Liang and Yang (2014) found that factor correlations had the largest biases when factor loadings or factor correlations were low in small sample sizes. In contrast, non-informative priors showed expected performance when factor loadings were large in the CFA model with categorical indicators. More specifically, the recovery of parameter estimates and biases were close to the ML estimator under the baseline condition with moderate sample sizes in the IFA models with sparse items and in several latent growth curve models (Bainter, 2017; McNeish, 2016; Tong & Shi, 2017). Similarly, in scenarios with small sample sizes, large cluster
sizes, the use of the categorical indicators model, and high ICC values, the MSEM models produced poor coverage and larger biased parameter estimates across levels (Depaoli & Clifton, 2015; Holtmann et al., 2016). Additionally, incorrect class proportions were found in the growth mixture model (Depaoli, 2013).

Weakly informative priors exclude unreasonable parameter values and thus they may help improve the quality of parameter estimates. (Gelman, 2018; Gelman, Simpson, & Betancourt, 2017). Heerwegh (2014) found that the MSE improved in the one-factor CFA models with small samples with these priors. Natesan (2015) found that the recovery of coverage, width, bias, the point estimate of RMSE, and the point estimate of bias were slightly inferior to the Bayesian estimator with informative priors in the ordinal CFA models, but Bainter (2017) found that these estimates improved in the IFA models with sparse indicators. Furthermore, the percent relative bias, power, coverage interval, and SD were close to the results with informative prior (McNeish, 2016). Similarly, the improved bias, RMSE, the coverage for between-level parts, and the within-level estimates were found in the MSEM models with categorical indicators (Depaoli & Clifton, 2015). However, this estimator may lead to some problems such as decreased statistical power and large biases in the estimates (Depaoli, 2012, 2013; Depaoli & Clifton, 2015).

Incorrect and inaccurate informative priors are still influential in the formation of posterior distribution. The former lead to larger biases and higher MSE of the parameter estimates (Heerwegh, 2014; Holtmann et al., 2016; McNeish, 2016). Depaoli (2014) obtained similar results in growth mixture modeling when using inaccurate informative priors, and the recovery of parameter estimates was poor as the locations of priors were much more inaccurate. The use of inaccurate priors can increase the biases of parameter estimates and lead to incorrect conclusions, especially when priors are bounded above or below (McDonald & Hodgson, 2018).
However, inaccurate priors can be conducive to obtaining robust estimates. Therefore, when using informative priors, a range of factors needs to be considered.

2.5.3. Specifying Effective Informative Priors Remains Challenging

As stated previously, using a Bayesian estimator with informative priors might have a positive impact on the recovery of parameter estimates and the performance of convergence, coverage, and bias intervals in the SEM models with small samples, low factor loadings, and categorical indicators. However, it is almost impossible to obtain precise information about priors in real research settings. Therefore, researchers usually used a Bayesian estimator with inaccurately specified informative priors based on the belief that extensive knowledge about prior information is not mandatory and is often unrealistic for obtaining useful estimates (McNeish, 2016). Also, priors were usually set around the population value even with a large variance and the parameter estimates were still at acceptable levels (Depaoli, 2014; Lasmal, 2015; McNeish, 2016). However, this was not always the case when experimental conditions varied, and some results were contradictory. As suggested in the literature, the Bayesian estimator with inaccurately specified informative priors biased the parameter estimates when the priors were bounded below or above. Therefore, it is inappropriate to use bounded priors until researchers are confident with prior knowledge (McDonald & Hodgson, 2018). Although using informative priors with small variances is conducive to obtaining robust parameter estimate, inclusion of the incorrect prior information incurs incorrect estimates of the posterior distribution. Therefore, this method has presented contradictory results when degrees of precision of priors are varied and remains challenging in practice (Bolstad, 2016).
2.6. Limitations in the Literature

Although this review is based on literature focusing on the positive impact of informative priors on the estimates of parameters, there are fewer studies discussing the mechanism of the negative impact of informative priors (Matteucci, Mignani, & Veldkamp, 2012; Shi, Song, Liao, Terry, & Snyder, 2017). Moreover, most of the literature in this review used informative accurate priors to explore the effectiveness of different degrees of prior variance. Studies that specifically discussed the use of inaccurate informative priors in Bayesian SEM, especially with small sample sizes, were rare. Disputes on the definition, functionality, and specification of inaccurate informative priors still continue among researchers (Depaoli, 2014; Hahn, 2006). Furthermore, researchers have made sustained efforts to increase the knowledge regarding the specification of effective informative priors; however, the results were mixed due to the use of specific model and varying experimental conditions (Heerwegh, 2014).

In addition to these methodological limitations, there were also some limitations on evaluation practices. First, a variety of SEM models were used in the literature, ranging from a one-factor CFA model to more advanced SEM models. As the foundation of the advanced SEM models, there were extensive references on FA models and SEM models available, which provided rich resources to evaluate this body of research. However, different models under different conditions and prior settings naturally led to different results and it was not possible to compare them using a unified benchmark. Second, the experimental design procedure that served specific models and priors typically imposed very restrictive conditions on the parameters to be estimated, such as invariance of variances and the same factor loadings. Such scenarios are not likely to be realized in the real-world and thus are difficult to apply. Third, although the experimental conditions that were used varied in many aspects, there is still a need to add
additional conditions. Fourth, there was little accuracy and precision in the descriptions of specific evaluation methods used in the literature. For example, several studies focused on the estimates of factor loading in CFA models or IFA models by specifying informative priors for factor loadings, which increased the bias of their results. Future studies can use factor score as the evaluation focus (Clark, 2016; Estabrook & Neale, 2013; Grice, 2011).

2.7. Present Study

The ML estimator, the WLSMV estimator, and the MCMC are the most common estimation methods used in practice. They differ from each other, and each has its own advantages and disadvantages. Therefore, the selection of estimation method should be based on the consideration of specific scenarios. Although the use of Bayesian estimator with informative priors can improve the recovery of parameter estimates in CFA and SEM analyses, the relative performance of this Bayesian approach, as compared to the other frequentist estimation methods, is not well understood under different experimental conditions.

2.7.1. Research Purpose

The present study compares the performances between informative priors and other priors or estimators on the parameter estimates in the CFA and SEM models with small samples under varied conditions. Specifically, it investigates the impact of sample sizes, the number of items per factor, mean factor loadings, and estimators on the parameter estimates in both models.

2.7.2. Research Questions

RQ1: How do CFA models perform by the Bayesian estimator with informative priors for convergence rates and factor loadings, compared to the MLE and other estimators?

RQ2: How do SEM models perform by the Bayesian estimator with informative priors for convergence rate and regression coefficients, compared to the MLE and other estimators?
3. METHODS

3.1. Models to be Investigated

The present study investigated the CFA and SEM models with continuous indicators. In this section, the two investigated models are described.

3.1.1. CFA Model with Continuous Indicators

A CFA model is the most important part of a SEM model. It is mainly used to realize a latent construct by a proposed measurement model based on prior research or a specific theory. It reflects how well the observed indicators measure the latent constructs, which is primarily accomplished by estimating the factor loadings linked to the latent construct. A two-factor CFA model is an extension of a one-factor CFA model with a correlation between two latent constructs. Therefore, for simplicity, only the one-factor CFA model is discussed here. It is represented as:

\[ X_A = \tau_{x_A} + \Lambda_{x_A} F_A + \delta_A, \]  

(3-1)

where \( X_A \) is the vector of \( p \times 1 \) continuous indicators, \( F_A \) is the vector of factor score, \( \Lambda_{x_A} \) is a matrix of \( p \times 1 \) factor loadings, \( \delta_A \) is the vector of \( p \times 1 \) indicator errors, and \( \tau_{x_A} \) is the vector of \( p \times 1 \) intercepts of indicators. The model assumes that both the mean value of indicator errors and the latent constructs are 0, and there is no correlation between the indicator errors. The corresponding diagram is shown in Figure 3.1 below.
3.1.2. SEM Model with Continuous Indicators

A SEM model with continuous indicators is defined as:

\[ X_a = \tau_{xa} + \Lambda_{xa} F_a + \delta_a, \]  

\[ F_{2a} = \gamma_a F_{2a} + \zeta_a, \]  

where \( X_a \) represents the vector of \( p \times 2 \) continuous indicators, \( F_a \) is the vector of \( 2 \times 1 \) factor scores for \( F_{1a} \) and \( F_{3a} \), and \( \Lambda_{xa} \) is a matrix of \( p \times 2 \) factor loadings that link the continuous indicators and the corresponding factors, \( \delta_a \) is the vector of \( p \times 2 \) indicator errors, and \( \tau_{xa} \) is the vector of \( p \times 2 \) intercepts of indicators. \( F_{2a} \) is the continuous variable, \( \gamma_a \) is the regression coefficient that is linked to the outcome variable and the factor scores, and \( \zeta_a \) is the error of outcome variable. The assumptions are the same as those of the CFA model with continuous indicators, but there is no correlation between the error of the outcome variable, latent factors, and indicators. The model is graphically depicted in Figure 3.2.
3.2. Research Design

The present study explored the impact of informative priors and other estimators on the parameter estimates in CFA models and SEM models under varied experimental conditions.

3.2.1. Experimental Conditions

Experimental conditions mainly included sample sizes, number of indicators per factor, mean factor loadings, estimation methods, and Bayesian prior specifications.

3.2.1.1. Sample Sizes

There is no consensus in the literature on how to quantify small sample sizes in SEM models and CFA models. For example, Nunnally and Bernstein (1994) proposed that 10 cases per indicator is the lowest adequate sample size in SEM. In contrast, Bayesian SEM
can work well when a total sample size is less than 100 (Lee & Song, 2004). Therefore, the sample sizes in the present study were set at 30 and 70.

3.2.1.2. Numbers of Indicators per Factor

As with sample sizes, there are no definitive criteria for the number of indicators per factor that can be applied to all scenarios. For instance, Marsh and Hau (1999) advocated that a sample size of 50 was large enough for a CFA model having 6 to 12 indicators for each factor. Boomsma (1985) claimed that the lower bound of the number of indicators each factor had should be 3 to 4 in a CFA model with the sample size of 100. In this study, the number of indicators per factor were set at 5 and 10.

3.2.1.3. Mean Factor Loadings

When the factor loading is high, the recovery of parameter estimates in CFA models and SEM models may be satisfactory even in small samples (De Winter, Dodou, & Wieringa, 2009). Therefore, the population mean factor loading values were set as .30 or .70. Specifically, when the number of indicators per factor was set equal to 5, individual factor loadings were set with the range from .24 to .36 and .64 to .76 in increments of .03 that correspond to the mean factor loading of .30 and .70, respectively. Similarly, when the number of indicators per factor was set equal to 10, individual factor loadings were set with the range from .1650 to .4350 and .5650 to .8350 in the same increments that correspond to the mean factor loading of .30 and .70, respectively.

3.2.1.4. Estimation Methods

The present study used both the ML estimator and the Bayesian estimators for data analyses. For the Bayesian estimator, 2 MCMC chains were used to estimate parameters.
Within each chain, the last half of the iterations were used to construct the posterior distributions, while the first half were discarded (Asparouhov & Muthén, 2010).

3.2.1.5. Bayesian Prior Specifications

Priors were specified only for factor loadings in both models. Note, if all factor loadings are set to equal, then a CFA model would be analogous to the tau-equivalent condition. Therefore, if the same informative priors are imposed to all factor loadings, the CFA model would be similar to the tau-equivalent condition, but less restrictive. Therefore, individual factor loading can be set within the range of mean factor loadings in equally small value increments for the CFA and SEM models with continuous indicators in the small samples. In addition, the priors in the present study were classified as a Bayesian estimator with non-informative priors $N\sim(0, \text{Infinity})$, a Bayesian estimator with correctly specified informative priors and a Bayesian estimator with incorrectly specified informative priors in terms of whether or not the prior mean deviates from the population value as well as the distance between them. Specifically, when the prior mean is set equal to the population mean factor loading values of .30 and .70, $N\sim(.30, .01)$ and $N\sim(.70, .01)$ are specified as Bayesian estimators with correctly specified informative priors. In contrast, when the prior mean is lower or higher than the population mean factor loading values of .30 or .70, $N\sim(.20, .01), N\sim(.40, .01), N\sim(.60, .01)$, and $N\sim(.80, .01)$ were specified as Bayesian estimators with incorrectly specified informative priors in terms of the population mean factor loading .30 and .70, respectively.
3.2.2. Data Generation

3.2.2.1. Standardization

To obtain good quality parameter estimates and convergence rates when implementing the Bayesian SEM with small samples using informative priors, the latent factor needs to be standardized with a mean of 0 and a variance of 1 (Asparouhov & Muthén, 2010). Therefore, all data were generated in the standardized scale for this study. In addition, factor loadings and other relevant parameters needed to be standardized as well.

3.2.2.2. Procedure

For the CFA model with continuous indicators, latent factor $F_A$ was generated first from the standard normal distribution. Next, factor loadings ($\lambda$) were set to known values depending on the mean factor loadings and the number of indicators per factor. Then, error variance of each indicator was computed by $1 - \lambda^2$, where $\lambda$ is a factor loading. Then, the residual term of each indicator was generated from the normal distribution with mean $= 0$ and the variance $= 1 - \lambda^2$. Finally, the indicator score was computed by the sum of the intercept, the factor loading multiplied by the factor score, and the residual term. Like the previous selection of the individual population factor loading value, note that the intercepts were set within a range from .02 to .42 and from .02 to .92 with small increments of .10 when the number of indicators per factor was equal to 5 and 10, respectively.

For the SEM model with continuous indicators, the two latent factors $F_1$ and $F_3$ were generated from the standardized multivariate normal distribution for each observation by specifying the correlation between them to be .30. Then, the residual term for the observed variable $F_2$ was generated for each observation with specified error variance of .766 via,

$$\text{Var}(\text{Error}(F_2)) = \text{Var}(F_2) - (r_{31})^2\text{Var}(F_1) + (r_{21})^2\text{Var}(F_3) + 2r_{31}r_{21}\text{Cor}(F_1, F_3). \quad (3-4)$$
where the population values of two regression coefficients $r_{31}$ and $r_{21}$ were specified at .30, reflecting that the magnitude and the direction of the two latent factors impact the observed variable. Third, the observed variable $F_2$ is generated using a multiple regression technique via

$$F_2 = .30F_1 + .30F_3 + \text{Error}(F_2),$$

(3-5)

Note that the process of generating indicator scores and the corresponding residual terms for two latent factors was the same as the CFA model with continuous indicators.

### 3.2.3. Evaluation Criteria

Convergence rates for the ML estimator were computed as the proportion of the model replications which converged without negative residual variance or large standard errors (Heerwegh, 2014). Convergence rates for Bayesian estimators were evaluated by the proportion of the model replications without large 95% CI values or posterior standard deviation. The qualities of the parameter estimates were evaluated by seven criteria: bias, absolute bias, relative bias, absolute relative bias, RMSE, SE, and empirical SE. These seven criteria are shown in the equation as follows.

$\text{Bias} = \frac{\sum_{i=1}^{n}(\hat{\theta}_i - \theta_{\text{True}})}{n},$  \hspace{1cm} (3-6)

$\text{Absolute Bias} = \left| \frac{\sum_{i=1}^{n}(\hat{\theta}_i - \theta_{\text{True}})}{n} \right|,$  \hspace{1cm} (3-7)

$\text{Relative Bias} = \frac{\sum_{i=1}^{n}(\hat{\theta}_i - \theta_{\text{True}})}{\theta_{\text{True}}},$  \hspace{1cm} (3-8)

$\text{Absolute Relative Bias} = \left| \frac{\sum_{i=1}^{n}(\hat{\theta}_i - \theta_{\text{True}})}{\theta_{\text{True}}} \right|,$  \hspace{1cm} (3-9)

$\text{SE} = \sqrt{\frac{(n-1)s^2}{n}},$  \hspace{1cm} (3-10)
Empirical SE = \sqrt{\frac{\sum_{i=1}^{n}(\theta_i - \sum_{i=1}^{n}\theta_i/n)^2}{n-1}}, \quad (3-11)

\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}((\theta_i - \theta_{\text{True}})^2)}{n}}, \quad (3-12)

where \( \theta_i \) is the \( i \)th sample estimate, \( \theta_{\text{True}} \) is the population parameter value, \( n \) is the number of replications (1000 in this study), and \( s \) is the sample variance.

3.2.4. Software

R was used to generate the data with 1000 replications for each condition. Mplus version 8.3 (Muthén & Muthén, 2019) was used to fit the model via MplusAutomation R package version 0.7.3 (Hallquist & Wiley, 2018).
Chapter

4. RESULTS

In this chapter, the results of the two simulation studies are summarized. Note that there were some conditions with extremely low convergence rates, where the mean factor loading was smaller using the MLR estimator. These conditions were excluded from the results.

4.1. Study 1: CFA Models

Study 1 examined CFA models with small sample sizes. Convergence rates and the recovery of factor loadings were reported and evaluated.

4.1.1. Convergence Rate

Convergence rates were summarized in Table 4.1. In the table, the 8 simulation conditions were labeled as C₁ to C₈. The 3 factors to form these 8 conditions were represented by N for the sample size (N = 30 or 70), J for the mean factor loading (J = .30 or .70), and I for the number of indicators (I = 5 or 10). The result demonstrated that the Bayesian estimators with both informative and non-informative priors demonstrated 100% convergence rates, indicating that the Bayesian estimators were beneficial for achieving a convergence even with a small sample for a one-factor CFA model. In contrast, the MLR estimator encountered the convergence problems in some simulation conditions. While the convergence rates were close to 1.0 under C₂ (N = 30, I = 5, J = .70) and C₇ (N = 70, I = 10, J = .30), lower convergence rates of .7369
and .6293 occurred under $C_1 (N = 30, I = 5, J = .30)$ and $C_5 (N = 70, I = 5, J = .30)$. Note these two conditions had smaller mean factor loading and number of indicators. The convergence rates were poor at .0367 and .0510 for $C_3 (N = 30, I = 10, J = .30)$ and $C_4 (N = 30, I = 10, J = .70)$, respectively, where the sample size was smaller and the number of indicators was larger.
### Table 4.1

*Convergence Rates for CFA Models by Estimator and Condition.*

<table>
<thead>
<tr>
<th>Condition</th>
<th>Estimator</th>
<th>Convergence Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>C(_1)</strong> (N = 30; I = 5; J = .30)</td>
<td>MLR</td>
<td>.6293</td>
</tr>
<tr>
<td></td>
<td>N ~ (0, Infinity)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>N ~ (.20, .01)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>N ~ (.30, .01)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>N ~ (.40, .01)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>MLR</td>
<td>.9970</td>
</tr>
<tr>
<td><strong>C(_2)</strong> (N = 30; I = 5; J = .70)</td>
<td>MLR</td>
<td>.0367</td>
</tr>
<tr>
<td></td>
<td>N ~ (0, Infinity)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>N ~ (.60, .01)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>N ~ (.70, .01)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>N ~ (.80, .01)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>MLR</td>
<td>.0051</td>
</tr>
<tr>
<td><strong>C(_3)</strong> (N = 30; I = 10; J = .30)</td>
<td>MLR</td>
<td>.7369</td>
</tr>
<tr>
<td></td>
<td>N ~ (0, Infinity)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>N ~ (.20, .01)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>N ~ (.30, .01)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>N ~ (.40, .01)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>MLR</td>
<td>.9606</td>
</tr>
<tr>
<td><strong>C(_4)</strong> (N = 30; I = 10; J = .70)</td>
<td>MLR</td>
<td>.8606</td>
</tr>
<tr>
<td></td>
<td>N ~ (0, Infinity)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>N ~ (.60, .01)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>N ~ (.70, .01)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>N ~ (.80, .01)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>MLR</td>
<td>.9606</td>
</tr>
<tr>
<td><strong>C(_5)</strong> (N = 70; I = 5; J = .30)</td>
<td>MLR</td>
<td>.0051</td>
</tr>
<tr>
<td></td>
<td>N ~ (0, Infinity)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>N ~ (.20, .01)</td>
<td>1</td>
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<td></td>
<td>N ~ (.30, .01)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>N ~ (.40, .01)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>MLR</td>
<td>.0051</td>
</tr>
<tr>
<td><strong>C(_6)</strong> (N = 70; I = 5; J = .70)</td>
<td>MLR</td>
<td>.0051</td>
</tr>
<tr>
<td></td>
<td>N ~ (0, Infinity)</td>
<td>1</td>
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<tr>
<td></td>
<td>N ~ (.60, .01)</td>
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<td>N ~ (.70, .01)</td>
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<tr>
<td></td>
<td>N ~ (.80, .01)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>MLR</td>
<td>.0051</td>
</tr>
<tr>
<td><strong>C(_7)</strong> (N = 70; I = 10; J = .30)</td>
<td>MLR</td>
<td>.0051</td>
</tr>
<tr>
<td></td>
<td>N ~ (0, Infinity)</td>
<td>1</td>
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<tr>
<td></td>
<td>N ~ (.20, .01)</td>
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<td></td>
<td>N ~ (.30, .01)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>N ~ (.40, .01)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>MLR</td>
<td>.0051</td>
</tr>
<tr>
<td><strong>C(_8)</strong> (N = 70; I = 10; J = .70)</td>
<td>MLR</td>
<td>.0051</td>
</tr>
<tr>
<td></td>
<td>N ~ (0, Infinity)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>N ~ (.60, .01)</td>
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<td></td>
<td>N ~ (.70, .01)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>N ~ (.80, .01)</td>
<td>1</td>
</tr>
</tbody>
</table>

*Note:* N=Sample Size; I=Mean Factor Loading; J=Number of Indicators Per Factor.
4.1.2. Factor Loading Recovery

The results were summarized in Figure 4.1 through Figure 4.3. Because the population factor loadings for all items were different, relative bias was presented and discussed. Bias, absolute bias, and absolute relative bias were evaluated, and they are included in Appendix A.

4.1.2.1. Relative Bias

First, different estimators presented different patterns of relative bias (RB) in Figure 4.1. The Bayesian estimator with correctly specified informative priors $N \sim (0.30, 0.01)$ and $N \sim (0.70, 0.01)$ periodically produced positive and negative RBs under $C_1$ and $C_5$ and $C_2$ and $C_6$, respectively. In contrast, the Bayesian estimator with incorrectly informative priors produced either negative or positive RBs. Specifically, $N \sim (0.40, 0.01)$ and $N \sim (0.80, 0.01)$ demonstrated positive RBs under $C_1$, $C_3$, $C_5$ and $C_7$, while $N \sim (0.20, 0.01)$ and $N \sim (0.60, 0.01)$ produced negative RBs under $C_2$, $C_4$, $C_6$ and $C_8$ with several exceptions. The exceptions were the positive RBs that $N \sim (0.20, 0.01)$ produced at the indicator $X_1$ and $X_2$ under $C_3$ and $C_5$ and $N \sim (0.60, 0.01)$ produced at the indicator $X_1$ under $C_4$ and $C_8$. Note $C_1$ and $C_5$ had smaller mean factor loadings and a smaller number of indicators per factor, while $C_2$ and $C_6$ had larger mean factor loadings and a smaller number of indicators per factor. In contrast, $C_3$ and $C_7$ had smaller mean factor loadings but a larger number of indicators per factor, while $C_4$ and $C_8$ had larger mean factor loadings and a larger number of indicators per factor. As for the Bayesian estimator with non-informative priors, they produced positive RBs with some exceptions. The exceptions were the positive and negative RBs that appeared periodically under $C_1$ and $C_5$. As with the MLR estimator, it demonstrated different patterns from the other estimators—the negative RBs occurred under $C_2$, $C_4$, $C_6$, and $C_8$ where the mean factor loadings were larger, while the positive RBs occurred under $C_1$ and $C_5$, where the mean factor loadings and the number of indicators per
factor were smaller. Additionally, positive and negative RBs appeared periodically under \( C_3 \) and \( C_7 \) where the mean factor loadings were lower but the number of indicators per factor was larger.

Second, the Bayesian estimator with correctly specified informative priors outperformed the MLR estimator and the Bayesian estimator with non-informative priors, if and only if the prior mean was correctly specified. Note each indicator was associated with the smallest relative bias for the prior closest to correct specification. For example, \( N \sim (.30, .01) \) produced the lowest RB equaling almost 0 at the indicator \( X_3 \) under \( C_1 \) and \( C_5 \), where the population factor loadings were set equal to the prior mean. In contrast, the Bayesian estimator with incorrectly specified informative priors \( N \sim (.20, .01) \) and \( N \sim (.40, .01) \), the MLR estimator, and the Bayesian estimator with non-informative priors produced the negative and positive RBs, respectively, and the absolute value of the RBs were larger than 5%. As expected, the Bayesian estimator with incorrectly specified informative priors did not outperform the MLR estimator. In another instance, the RBs that the Bayesian estimator with incorrectly specified informative priors \( N \sim (.60, .01) \) and \( N \sim (.80, .01) \) demonstrated were 10% lower or higher than the MLR estimator at the indicator \( X_6 \) under \( C_4 \) and \( C_8 \), where the mean factor loadings and the number of indicators per factor were larger. In addition, there were great performance differences among different estimators when the mean factor loading was smaller, while the performance difference was not that large among them when the mean factor loading was larger.

Specifically, the Bayesian estimator with incorrectly specified informative priors \( N \sim (.40, .01) \) tended to overestimate factor loading and evidenced the largest RB among all the estimators. The RB was up to 125% and 105% for the sample size of 30 and 70, respectively, while it decreased to be negligible when the population factor loading increased to .40 across number of indicators per factor and sample sizes. However, the Bayesian estimator with incorrectly specified informative
priors $N \sim (0.20, 0.01)$ underestimated factor loadings and the largest RB was up to -.50 for the smaller sample sizes. By contrast, $N \sim (0.30, 0.01)$ overestimated or underestimated factor loading, and the largest RB was up to .75 for the smaller sample size. As for the Bayesian estimator with non-informative priors, it tended to overestimate factor loading and the highest RB was 25% and 0 for the smaller and larger sample size, respectively. The performance of the ML estimator was similar to the Bayesian estimator with non-informative priors.

Third, all RBs in Figure 4.1 decreased as sample size increased with some exceptions. The difference of RBs between the smaller sample size and the larger sample size for each indicator were much larger and the values exceeded 5% under $C_1$, $C_3$, $C_5$, and $C_7$, while the difference of RBs were smaller and lower than 5% under $C_2$, $C_4$, $C_6$, and $C_8$. The exceptions were the increased RBs that $N \sim (0.60, 0.01)$ produced at the indicator $X_2$ and $X_3$ under $C_4$ and $C_8$. As the number of indicators per factor increased, the RBs increased for $N \sim (0.30, 0.01)$, $N \sim (0.40, 0.01)$, $N \sim (0.70, 0.01)$, and $N \sim (0.80, 0.01)$, while the RBs decreased for $N \sim (0.20, 0.01)$ and $N \sim (0.60, 0.01)$ across sample sizes and mean factor loadings.

4.1.2.2. SE

The SEs are summarized in Figure 4.2. The nearly identical values of SEs and empirical SEs indicate that the estimated SE for each of the replicated analysis was a good estimate of the SE. A summary graph for empirical SE is included in Appendix A.

First, although the Bayesian estimator with informative priors produced the lowest SEs across conditions, its performances were different from the MLR estimator and the Bayesian estimator with non-informative priors depending on the mean factor loadings. Specifically, the differences between them were quite large under $C_1$ and $C_5$, while the differences were not that large under $C_2$, $C_4$, $C_6$, and $C_8$ for all sample sizes and the numbers of indicators per factor. Note
that the MLR estimator had the largest SEs among all estimators and the corresponding values were up to .62 and .49 for the smaller sample size and the larger sample size, respectively.

Second, the SEs increased across the Bayesian estimator with informative priors as the sample size and mean factor loading increased. However, they were reduced for the MLR estimator and the Bayesian estimator with non-informative priors as the sample size and mean factor loading increased across mean factor loadings and the numbers of indicators per factor.

4.1.2.3. RMSE

RMSE for each condition are summarized in Figure 4.3. First, the difference of RMSEs between the MLR estimator and the Bayesian estimators displayed a similar trend. Similarly, the difference of RMSEs among the three Bayesian informative priors were close when the number of indicators per factor was smaller. Meanwhile, the difference was larger when the number of indicators per factor was smaller across sample sizes and mean factor loadings. Note, the MLR estimator had the largest RMSE of .62 and .49 for sample sizes of 30 and 70, respectively.

Second, all RMSEs for the Bayesian estimator with incorrectly specified informative priors were lower than the MLR estimator, except for at the indicators $X_1$ through $X_3$ with $N \sim (0.80, 0.01)$ and at the indicators $X_8$ through $X_{10}$ with $N \sim (0.60, 0.01)$ under $C_4$ and $C_8$.

Third, the RMSE for factor loading decreased as the sample size increased for all estimators across mean factor loadings and number of indicators per factor, except for at the indicator $X_5$ using $N \sim (0.80, 0.01)$ and at the indicator $X_2$ and $X_3$ with $N \sim (0.60, 0.01)$ when the mean factor loadings was larger across sample sizes and number of indicators per factor. In contrast, the RMSEs for $N \sim (0.30, 0.01)$ and $N \sim (0.40, 0.01)$ increased as the number of indicators per factor increased except for at the indicators $X_1$ through $X_5$ using $N \sim (0.30, 0.01)$ and $N \sim (0.60, 0.01)$ across sample sizes and mean factor loadings.
Figure 4.1. Factor Loading's RB Plot in CFA Model by Estimator and Condition.
Figure 4. 2. Factor Loading’s SE Plot in CFA Model by Estimator and Condition.
Figure 4.3. Factor Loading's RMSE Plot in CFA Model by Estimator and Condition.
4.2. Study 2: SEM Model

Study 2 examined the SEM models with small sample sizes. In this section, convergence rates and the recovery of the path coefficients are reported and evaluated.

4.2.1. Convergence Rate

As shown in Table 4.2, the Bayesian estimator with non-informative priors demonstrated very high convergence rates (almost close to 1.0) across conditions, indicating it effectively facilitated model convergence for the SEM model with small samples. In contrast, the MLR estimator and the Bayesian estimator with informative priors encountered convergence problems in some simulation conditions. The convergence rates for the MLR estimator are not available under $C_1$ through $C_4$ for the smaller sample size, due to the extremely low convergence rates for them. On the other hand, the convergence rates substantially increased to higher than .75 and even to 1.0 under $C_5$ through $C_8$ for the larger sample size. As for the convergence rates for the Bayesian estimator with informative priors, they were close to 1.0 except for under $C_5$ ($N = 70, l = 5, j = .30$) and $C_7$ ($N = 70, l = 10, j = .70$), where the prior means were greater than or equal to the mean factor loadings. In contrast, the convergence rates almost reached to 1.0 under $C_2$ ($N = 30, l = 5, j = .70$) and $C_4$ ($N = 30, l = 10, j = .70$). Similarly, the convergence rates were close to .90 under $C_1$ ($N = 30, l = 5, j = .30$) and $C_3$ ($N = 30, l = 10, j = .30$), where the prior mean was larger than the mean factor loading. However, the convergence rates were lower at .6831 and .4608, or even decreased to .0209 and .1103, under the same experimental conditions, where the prior mean was equal to or lower than the mean factor loading.
Table 4. 2.
Convergence Rates for SEM Models by Estimator and Condition.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Estimator</th>
<th>Convergence Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>C₁ ( (N = 30; I = 5; J = .30) )</td>
<td>MLR</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>( N \sim (0, \text{Infinity}) )</td>
<td>( .9901 )</td>
</tr>
<tr>
<td></td>
<td>( N \sim (.20, .01) )</td>
<td>( .0209 )</td>
</tr>
<tr>
<td></td>
<td>( N \sim (.30, .01) )</td>
<td>( .4608 )</td>
</tr>
<tr>
<td></td>
<td>( N \sim (.40, .01) )</td>
<td>( .8889 )</td>
</tr>
<tr>
<td></td>
<td>MLR</td>
<td>N/A</td>
</tr>
<tr>
<td>C₂ ( (N = 30; I = 5; J = .70) )</td>
<td>MLR</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>( N \sim (0, \text{Infinity}) )</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>( N \sim (.60, .01) )</td>
<td>( .9132 )</td>
</tr>
<tr>
<td></td>
<td>( N \sim (.70, .01) )</td>
<td>( .9833 )</td>
</tr>
<tr>
<td></td>
<td>( N \sim (.80, .01) )</td>
<td>( .9950 )</td>
</tr>
<tr>
<td>C₃ ( (N = 30; I = 10; J = .30) )</td>
<td>MLR</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>( N \sim (0, \text{Infinity}) )</td>
<td>( .9960 )</td>
</tr>
<tr>
<td></td>
<td>( N \sim (.20, .01) )</td>
<td>( .1103 )</td>
</tr>
<tr>
<td></td>
<td>( N \sim (.30, .01) )</td>
<td>( .6831 )</td>
</tr>
<tr>
<td></td>
<td>( N \sim (.40, .01) )</td>
<td>( .9597 )</td>
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<tr>
<td>C₄ ( (N = 30; I = 10; J = .70) )</td>
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</tr>
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</tr>
<tr>
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<td>( N \sim (.60, .01) )</td>
<td>( .9804 )</td>
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</tr>
<tr>
<td></td>
<td>( N \sim (.80, .01) )</td>
<td>1</td>
</tr>
<tr>
<td>C₅ ( (N = 70; I = 5; J = .30) )</td>
<td>MLR</td>
<td>1</td>
</tr>
<tr>
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<tr>
<td>C₆ ( (N = 70; I = 5; J = .70) )</td>
<td>MLR</td>
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</tr>
<tr>
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<td></td>
<td>( N \sim (.70, .01) )</td>
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</tr>
<tr>
<td></td>
<td>( N \sim (.80, .01) )</td>
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<td>MLR</td>
<td>.9579</td>
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<tr>
<td>C₈ ( (N = 70; I = 10; J = .70) )</td>
<td>MLR</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>( N \sim (0, \text{Infinity}) )</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>( N \sim (.60, .01) )</td>
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<tr>
<td></td>
<td>( N \sim (.70, .01) )</td>
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</tr>
<tr>
<td></td>
<td>( N \sim (.80, .01) )</td>
<td>1</td>
</tr>
</tbody>
</table>

*Note: N=Sample Size; I=Mean Factor Loading; J=Number of Indicators Per Factor. C₁-C₈=Condition 1 through Condition 8; N/A=Not Available."
4.2.2. Path Coefficient Recovery

The relative bias, SE, and RMSE of path coefficients were summarized in Figure 4.4 through Figure 4.6. Like section 4.2.1, bias, absolute bias, absolute relative bias, and empirical SE were evaluated but not reported. They are included in Appendix A.

4.2.2.1. Relative Bias

First, Figure 4.4 showed varied patterns of relative bias (RB) among estimators. Specifically, $N \sim (0, \text{Infinity})$ showed positive RBs for two path coefficients. As for the Bayesian estimator with informative priors, $N \sim (.40, .01)$, $N \sim (.70, .01)$, and $N \sim (.80, .01)$ produced positive RBs while $N \sim (.60, .01)$ produced negative RBs. In contrast, on most occasions $N \sim (.20, .01)$ and $N \sim (.30, .01)$ showed positive RBs except for the negative RB that $N \sim (.20, .01)$ produced under C3 and the negative RB $N \sim (.30, .01)$ produced under C7.

Second, it was demonstrated that the Bayesian estimator with informative priors outperformed the Bayesian estimator with non-informative priors. Also, the performance was similar between the Bayesian estimator with correctly specified informative priors and the Bayesian estimator with incorrectly specified informative priors. Specifically, the Bayesian estimator with non-informative priors displayed the largest RB for both path coefficients among all the estimators. The RB was up to 125% and 105% for the sample size of 30 and 70, respectively. The Bayesian estimators with correctly specified informative priors, namely, $N \sim (.30, .01)$ and $N \sim (.70, .01)$, produced the smallest RBs for both path coefficients across all conditions. As for the Bayesian estimator with incorrectly specified informative priors, when the mean factor loading was smaller, the largest RBs that $N \sim (.20, .01)$ produced were up to 15% and 5% for the sample size of 30 and 70, respectively under C1 and C5. Similarly, the largest RBs that $N \sim (.40, .01)$ produced were up to 11% and 8% for the sample size of 30 and
70, respectively under $C_3$ and $C_7$. In contrast, when the mean factor loading was larger, the largest RBs that $N \sim (.60, .01)$ produced were up to 11% and 10% for the sample size of 30 and 70, respectively under $C_4$ and $C_8$. Similarly, the largest RBs that $N \sim (.80, .01)$ produced were up to 15% and 10% for the sample size of 30 and 70, respectively under $C_3$ and $C_7$. Interestingly, the RBs of the Bayesian estimator with incorrectly specified informative priors were larger or smaller than the Bayesian estimator with correctly specified informative priors, depending on the condition. The RB differences between the Bayesian estimator with incorrectly and correctly specified informative priors were around 5% for the lower mean factor loading, while the RB differences fluctuated within the range 5% to 10% for the larger mean factor loading regardless of sample sizes and number of indicators per factor.

Third, as the sample size increased, the RBs decreased or remained unchanged for the Bayesian estimator with non-informative priors and the Bayesian estimator with informative priors, except for the increased RBs that the prior $N \sim (.20, .01)$ produced for the path coefficient when the mean factor loading and the number of indicators per factor were smaller. As number of indicators per factor increased, the RBs for both path coefficients increased for the Bayesian estimator with non-informative priors, while the RBs decreased or remained unchanged for the Bayesian estimator with correctly specified informative priors. As for the Bayesian estimator with incorrectly specified informative priors, when the mean factor loading was smaller, the RBs that $N \sim (.20, .01)$ produced for both coefficients decreased while the RBs that $N \sim (.40, .01)$ produced for both path coefficients increased or remain unchanged. In contrast, when the mean factor loading was larger, the trend of the RBs that $N \sim (.60, .01)$ and $N \sim (.80, .01)$ produced was opposite to $N \sim (.20, .01)$ and $N \sim (.40, .01)$, respectively. In summary, the RBs that decreased or increased for both path coefficients fluctuated within the
range from 0 to 5%. Therefore, the impact of the number of indicators per factor was not substantially different between the Bayesian estimator with correctly specified informative priors and the Bayesian estimator with incorrectly specified informative priors.

4.2.2.2. SE

SEs were summarized in Figure 4.5. Although empirical SEs are not presented, the minimum difference between SEs and empirical SEs indicated the estimated SE for each of the replicated analyses were as expected. The results showed that the Bayesian estimator with incorrectly specified informative priors \( N \sim (20, .01) \) produced the lowest SEs. The difference from the other Bayesian estimator with informative priors fluctuated within the acceptable range.

Second, the results showed that the SEs decreased or remained unchanged for all estimators as sample size increased, while they increased or remained unchanged as the number of indicators per factor increased across sample sizes and mean factor loadings. Given that the decreasing or increasing amount fluctuated only about 5%, the performance was similar between the Bayesian estimator with correctly specified informative priors and the Bayesian estimator with incorrectly specified informative priors.

4.2.2.3. RMSE

RMSEs were summarized in Figure 4.6. First, the difference between the Bayesian estimator with incorrectly specified informative priors \( N \sim (20, .01) \) and the other Bayesian estimator with informative priors was only 5% to 10%.

Second, the results showed that the trends similar to the SEs were observed in relation to the sample size and the number of indicators per factor. Therefore, it is concluded that the performance was similar between the Bayesian estimator with correctly specified informative priors and the Bayesian estimator with incorrectly specified informative priors.
Figure 4.4. Path Coefficient's RB Plot in SEM model by Estimator and Condition.

Note: The isolated red dots through Figure 4.4 to Figure 4.6 indicate non-convergent results using the ML estimator.
Figure 4. 5. Path Coefficient's SE Plot in SEM Model by Estimator and Condition.
Figure 4.6. Path Coefficient's RMSE Plot in SEM Model by Estimator and Condition.
Chapter

5. APPLICATION TO REAL DATA

To further illustrate the utility of informative priors in the Bayesian SEM analysis with small sample size, I compared the estimates of path coefficients among the Bayesian estimator with informative priors, the Bayesian estimator with non-informative priors, and the MLR estimator with a dataset collected by the Hispanic Families Network (HFN) project (Baker, Ma, & Gallegos, 2019). The main reason to focus on path coefficients is not only that they are the sole focus in the present study, but also the most concerned part when researchers use SEM model. This chapter consists of a description of the data, model specification, specification of the priors and estimators, posterior computation, results, and discussion.

5.1. Data Description

The HFN project is a small sample experimental study, observing the changes on specific parental knowledge subdomains of 32 Hispanic mothers in 3 Dallas neighborhoods. With the objective of disclosing how the demographic factors impacted parental knowledge, a subset of variables was identified out of the original dataset. More specifically, there were five demographic variables: the number of children that parents have (pnkid), parental education level (pedu), whether parents had a computer at home or not (pchome), previous program participation experiences (prpart), and whether parents use social media or not (ussome). In addition, there were three observed variables derived from the original dataset using item parceling techniques: parental early childhood knowledge (pakn1), parental reading and literacy
knowledge (pakn2), and parental information and technology knowledge (pakn3)—which can be considered as the manifestation of the latent variable “parental knowledge”. Moreover, each of the three parental knowledge variables had 3 missing values, and the missing data proportion of 12.5% is generally acceptable in practice. Basic descriptive statistics comprising the mean, standard deviation, and correlations are presented in Table 5.1.

Table 5.1. 
*Means, Standard Deviations, and Correlations for Parental-Level Data.*

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<tr>
<th>variable</th>
<th>M</th>
<th>SD</th>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
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</thead>
<tbody>
<tr>
<td>1. pakn1</td>
<td>4.23</td>
<td>.65</td>
<td>.43*</td>
<td>.28</td>
<td>.35</td>
<td>.07</td>
<td>.33</td>
<td>.33</td>
<td>-.12</td>
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</tr>
<tr>
<td>2. pakn2</td>
<td>4.41</td>
<td>.70</td>
<td>.11</td>
<td>.01</td>
<td>.19</td>
<td>.43*</td>
<td>.23</td>
<td>.06</td>
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<td>3. pakn3</td>
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<td>-.01</td>
<td>.03</td>
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</tr>
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<td>4. pnkid</td>
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<td>5. pedu</td>
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<td>.14</td>
<td>.32</td>
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<td>8. ussome</td>
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<td></td>
<td></td>
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</tbody>
</table>

*Note. *p<.05. 1. pakn1=parental early childhood knowledge; 2. pakn2=parental reading and literacy knowledge; 3. pakn3=parental information and technology knowledge; 4. pnkid=number of the kids that parents have; 5. pedu=the formal schooling year of parents; 6. ptpart=previous program participations; 7. pchome=whether parents have a computer at home or not; 8. ussome=whether parents used social media at home or not.*

5.2. Model Specification

As shown in Figure 5.1, structural equation modeling is used to investigate the relationship between the demographic variables and the latent construct “parental knowledge”. The measurement model is given as:

\[ y_{pakn} = \tau_{pakn} + \Lambda_{pakn} \times pakn + \varepsilon, \] (5-1)
where $y_{pakan}$ is the vector of $3 \times 1$ specific parental knowledge indicators, $pakan$ is the vector of latent construct parental knowledge, $A_{pakan}$ is a matrix of $3 \times 1$ factor loadings, $e$ is the vector of $3 \times 1$ indicator errors, and $\tau_{pakan}$ is the vector of $3 \times 1$ intercepts of indicators. The structural model is represented as:

$$pakan = (\beta_1, \beta_2, \beta_3, \beta_4, \beta_5) \times \begin{pmatrix} pnkid \\ pedu \\ prpart \\ pchome \\ ussonme \end{pmatrix} + \zeta,$$  

(5-2)

where $\beta_1$ through $\beta_5$ is the regression coefficient of five demographic variables to the latent construct, and $\zeta$ is the residual error of the latent construct.

Figure 5. 1. Diagram of the SEM Model of the HFN Project.
5.3. Specification of the Estimators and Priors

The results of a preliminary analysis indicated that the value of skewness and kurtosis for each variable were very close to 2 and 7, respectively, except for the variable “ussome”. Additionally, the multivariate normal distribution (MVN) test results indicated that this dataset did not meet the MVN assumption and the chi-square $Q-Q$ plot further supported this conclusion. Therefore, the MLR estimator was used to estimate parameters (Satorra & Bentler, 1988). As for prior’s specification, the Bayesian estimator with non-informative priors $N \sim (0, \text{Infinity})$ and the Bayesian estimator with informative priors $N \sim (.50, .01)$ were specified for the three factor loadings, respectively. Note that the means of their priors equaling to .50 are the most likely incorrect specifications, as it is impossible to know their population values.

5.4. Posterior Computations

In all the computations, two chains were used for 20,000 iterations, each, after a burn-in of 10,000 iterations, to compute posterior means, posterior SD, and 95% credible intervals (CIs). The trace plots, autocorrelation plots, posterior distribution, and PSR value were used to assess convergence for the MCMC chains.

5.5. Results

First, it was found that the SEM model with small sample in the present study using the Bayesian estimator with informative priors and the MLR estimator did not have any convergence problems. In Figure 5.2, the trace plots showed that there was no systematic pattern for any of the parameters for the last half iterations. The autocorrelation plots showed the value of autocorrelation less than .01 at the end of 30 lags. Also, the posterior distributions of the parameters are smoothly distributed. Furthermore, the PSR value was 1.009 after 20,000 iterations, for which less than 1.1 is considered a good indication of convergence. In contrast, in
Figure 5.3, the plots did not display good indications of convergence. The trace plot did not show good stability across the iterations. Also, the autocorrelations were substantially higher than .01 at the end of 30 lags. The posterior distributions were not smoothly distributed. Furthermore, the PSR value was 1.353 after 20,000 iterations, which was greater than 1.1.

Second, there was no problem on posterior predictive checking for the SEM model using the Bayesian estimator with informative priors because the predictive posterior p-value (ppp) is .586, indicating a good model-data fit. In contrast, the model-data fit was not satisfactory for the SEM model using the MLR estimator because the SRMR value was .098, which was greater than .08 (Hu & Bentler, 1999). In addition, the value of deviance information criterion (DIC) was 174.740 using the Bayesian estimator with informative priors, which was smaller than the BIC value of 186.970 using the MLR estimator, indicating that the model fit using the Bayesian estimator with informative priors was better than the MLR estimator.

Third, the 95% credible interval and the posterior SD of path coefficient $\beta_1$, $\beta_3$, and $\beta_4$ using the Bayesian estimator with informative priors was narrower than the MLR estimator and the Bayesian estimator with non-informative priors, indicating that the precision of the posterior estimate was better. As with the path coefficients of $\beta_2$ and $\beta_5$, although their 95% C.I. and posterior SD were slightly higher than the MLR estimator, the discrepancies were below 5%.
Table 5.2.
Estimates of Path Coefficients among the MLR Estimator, the Bayesian Non-Informative Priors, and the Bayesian Informative Priors.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>MLR</th>
<th>Bayes ( N \sim (0, \infty) )</th>
<th>Bayes I ( N \sim (.50, .01) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Est</td>
<td>95% C.I.</td>
<td>S.E.</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>.166</td>
<td>(.371, .617)</td>
<td>.274</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>.063</td>
<td>(.275, .400)</td>
<td>.172</td>
</tr>
<tr>
<td>( \beta_3 )</td>
<td>.487</td>
<td>(-.048, 1.022)</td>
<td>.273</td>
</tr>
<tr>
<td>( \beta_4 )</td>
<td>.365</td>
<td>(-.083, .813)</td>
<td>.229</td>
</tr>
<tr>
<td>( \beta_5 )</td>
<td>-.191</td>
<td>(-.472, .090)</td>
<td>.144</td>
</tr>
</tbody>
</table>

Note: \( \beta_1 = \beta_{pmkid}; \beta_2 = \beta_{pedu}; \beta_3 = \beta_{prpart}; \beta_4 = \beta_{pchome}; \beta_5 = \beta_{ussome}; \) EST=Estimate; PSD=Posterior SD; Bayes N=Bayesian Estimator with Non-Informative Priors; Bayes I=Bayesian Estimator with Informative Priors.
Figure 5. 2. Diagnostic Plots of Path Coefficients with Bayesian Informative Priors.
Figure 5.3. Diagnostic Plots of Path Coefficients with Bayesian Non-Informative Priors.
5.6. Summary

This chapter compared the quality of estimates of the path coefficients in the SEM model with a small sample among the Bayesian estimator with informative priors, the Bayesian estimator with non-informative priors, and the MLR estimator using the HFN project data.

Even though the sample size of this real data analysis was very small, the analysis results favored the Bayesian estimator with informative priors over the Bayesian estimator with non-informative priors and the MLR estimator. This was because the SEM model successfully converged with the Bayesian estimator with informative priors and the MLR estimator, while the SEM model encountered convergence problems with the Bayesian estimator with non-informative priors. This was partially consistent with the research findings from the current simulation study of the SEM model. Although the SEM model using the Bayesian estimator with informative priors and the MLR estimator converged, the former had satisfactory model-data fit index, while the latter did not. In addition, the 95% credibility interval for most of the path coefficients was smaller than the analysis with the MLR estimator, and the Bayesian estimator with informative priors likely increased the precision of the estimated path coefficients. This was also consistent with the findings from the simulation study, in which the performance of the Bayesian estimator with correctly specified informative priors and the Bayesian estimator with incorrectly specified informative priors were similar for the path coefficients in SEM models.
Chapter

6. DISCUSSION

The goal of this dissertation was to evaluate the utility of a Bayesian estimator with informative priors in the SEM models with small samples for CFA and SEM analyses. In this chapter, I first summarize and discuss advantages of the Bayesian approach with informative priors over the Bayesian estimator with non-informative priors, as well as the ML estimator according to the findings obtained in this study. Then, major research limitations are briefly discussed. Last but not the least, practical suggestions, practical importance, and future research directions are presented.

6.1. Performance of the Bayesian Informative Priors Method

This section mainly discusses the performances of the proposed Bayesian approach with informative priors in the CFA and SEM analyses with small samples in terms of convergence rate, factor loading recovery, and path coefficient recovery.

6.1.1. Convergence Rate

First, the Bayesian estimator with informative priors was expected to outperform the MLR estimator on convergence rates in both models. This expectation was partially supported: in CFA models, the Bayesian estimator with informative priors had 100% convergence rates, whereas the convergence rates were less than 5% under specific experimental conditions using the MLR estimator. This finding was consistent with the results of Heerwegh (2014), showing the
advantages of achieving convergence for the proposed method over the ML estimator on the estimates in the CFA models with continuous indicators. Meanwhile, it conflicted with the results of Liang & Yang (2014), where the Bayesian estimator with informative priors showed fewer convergence problems, compared with other estimators in the simulation studies, indicating that the proposed method cannot completely avoid the occurrence of convergence problem even under the larger sample size. It was most likely due to the addition to the model of complexity of the two latent factors correlating with each other. By contrast, the convergence rates of the Bayesian estimator with informative priors were higher than the MLR estimator in the SEM models when the sample size was smaller, which is also partially consistent with the findings of Finch and Miller (2019), in which all parameters of the MIMIC model successfully converged across experimental conditions. It was evident that smaller sample sizes were more vulnerable to model convergence problems. However, the convergence rate deteriorated when the prior mean was lower than the lowest mean factor loading even for a larger sample size. It implied that the convergence rate in the Bayesian SEM with small samples was tightly tied to the location of the prior mean relative to the population value, in addition to sample size, mean factor loading, the complexity of SEM models, and the number of indicators per factor.

Second, the MLR estimator was expected to have a similar performance to the Bayesian estimator with non-informative priors according to Bayes’ Theorem, which is partially supported as well: in the CFA models, when the sample size and the number of indicators per factor were larger, the convergence rates for both estimators were very similar and close to 1, regardless of the magnitude of mean factor loadings, which was consistent with the prior findings (Bainter, 2017; McNeish, 2016; Tong & Shi, 2017). However, the difference between them on the estimate of factor loadings was more than .95 when the sample size was smaller but the number
of indicators per factor was larger, regardless of the magnitude of mean factor loadings. This is slightly different from Heerwegh (2014) where the larger factor loading of .80 effectively offset the impact of small sample size of 25 given the number of indicators, which made their performances still on par. Correspondingly, the differences between them were still high, up to .50, when both the sample size and factor loading were smaller. In the SEM models, the convergence rate of the Bayesian estimator with non-informative priors was much higher than the MLR estimator, which is different from the study by Finch and Miller (2019) where no estimators experienced any convergence problems. Comparatively speaking, the complexity of the SEM model with smaller sample sizes is more likely to cause model convergence issue.

Based on the analysis above, it was revealed that a model convergence was impacted by multiple factors, which included, but was not limited to sample size, the number of indicators per factor, mean factor loading, and the model complexity. As suggested in the present study, as one of the core elements in Bayesian statistics, specifying an effective prior mean was crucial to solve the hard issue of lower convergence rates in the SEM models with small samples. On one hand, incorrectly specified informative priors led to more occurrences of the convergence problem. When the prior mean was decreased by .10 from the population factor loading value of .30, the convergence rate dropped to .12 and even to .02 under the conditions where the sample size and mean factor loadings were smaller across the number of indicators per factor. On the other hand, incorrectly specified informative priors were helpful to achieve convergence in the Bayesian SEM with small samples. The rates restored around .50 and .80 under the conditions where the sample size and the number of indicators per factor were larger, respectively, even when the prior mean and population value was still set as the same value above. The conflicting results of convergence rates are worth thinking about further. Thus, future
work should deeply investigate how incorrectly specified informative priors impact the convergence rate in the context of SEM modeling under varied conditions.

6.1.2. Factor Loading Recovery

The present study found the Bayesian estimator with correctly specified informative priors outperformed the MLR estimator in the CFA models with small samples across sample sizes and mean factor loadings. As shown in the graphs in the results chapter, estimates of factor loading had the lowest RB, RMSE, and SE when the prior mean was set equal to the population value of the factor loadings, which was consistent with the prior research findings (Heerwegh, 2014). As expected, when the prior mean deviated from population factor loading value, the Bayesian estimator with incorrectly specified informative priors did not outperform the MLR estimator due to the fact that the absolute value of each index above was higher than the MLR estimator, which was partially consistent with the prior findings (Bainter, 2017; Heerwegh, 2014). The former found that the bias of estimate of factor loading using Bayesian estimator with incorrectly specified informative priors was larger than the ML estimator, while the latter found that there were no distinguishable differences on the estimates of factor loading between other estimators and the Bayesian estimator with informative priors under the condition of moderate sample sizes. In addition, sensitivity analyses further illustrated the discrepancy of the impact between these two estimators on the estimates of factor loadings (Zyphur & Oswald, 2015). In light of the facts above, we see that specifying effective informative priors remains challenging.

In sum, it is inevitable that the prior mean would be misspecified in real practice because we never know the population value. Therefore, it would not be recommended to specify an informative prior for factor loadings if the purpose is just to estimate factor loadings. However, if
the research purpose is other than estimating factor loadings (e.g., factor scores), this approach may be still reasonable, although this dissertation did not investigate that purpose.

6.1.3. Path Coefficient Recovery

The results favored the Bayesian estimator with correctly specified informative priors over the other estimators used in the present study due to the fact that it produced the estimated path coefficients with smallest relative biases (RBs), which was fully consistent with the findings of Finch and Miller (2019). Meanwhile, the differences of RBs, SEs, and RMSEs between the Bayesian estimator with correctly and incorrectly specified informative priors were only 5% to 10%. Therefore, it was concluded that their performances in recovering the path coefficients in the SEM models with small samples were similar.

On the one hand, this finding was consistent with some previous research (Muthén & Asparouhov, 2012; Van Erp, Mulder, & Oberski, 2018). On the other hand, it was contradictory to other research findings (Marcoulides, 2018; McNeish, 2016). The main reason for the contradiction was probably that previous research investigated complicated SEM models with varied sample sizes and did not propose reasons strong enough to specify informative priors for regression coefficients (Depaoli & Clifton, 2015; Finch & Miller, 2019; Holtmann et al., 2016; Zitzmann, Lüdtke, Robitzsch, & Marsh, 2016). In contrast, the current study specified informative priors for factor loadings only in SEM models with small samples with two correlated latent factors that regressed on an observed variable, based on the classical test theory of essentially tau-equivalent condition, which made the research more targeted and the results more convincing.

The results of the present study demonstrated that the Bayesian estimator with incorrectly specified informative priors for factor loadings only was valid in recovering the path coefficients
in the SEM models with small samples. However, this Bayesian approach will not always work well under any conditions. Specifically, when the mean factor loading was higher ($J = .70$), irrespective of sample sizes and the number of indicators per factor, using the Bayesian estimator with incorrectly specified informative priors $N \sim (.60, .01)$ produced the estimate of path coefficient with the lowest RB, SE, and RMSE. Similarly, this result still held when using $N \sim (.20, .01)$ under the conditions where the mean factor loading was lower ($J = .30$) and the number of indicators per factor was larger ($I = 10$) across sample sizes. However, when both the mean factor loadings and the number of indicators per factor were smaller ($I = 5; J = .30$), $N \sim (.40, .01)$ produced the estimate of path coefficient with the lowest RB but the highest SE and RMSE. In contrast, $N \sim (.20, .01)$ produced the estimate of path coefficient with the highest RB but the lowest SE and RMSE. Indeed, researchers never know the population value in the real world. So, taken together, in an applied setting, the results above would show that researchers would most likely obtain the best estimate of path coefficients if they believe the mean factor loading is high (e.g. .60) and specify informative priors with the prior mean somewhat lower than the mean factor loading (e.g. .50). Similarly, the results would still hold if researchers believe the mean factor loading is low (e.g. .20) and specify the prior mean somewhat lower than the mean factor loading (e.g. .10) when the number of indicators per factor is 10 irrespective of sample sizes. However, researchers are more likely to obtain the most accurate or inaccurate estimates of path coefficients with the lowest or highest bias and the highest or lowest SE and RMSE under the conditions where both the mean factor loading and the number of indicators per factor are smaller, if they specify informative priors with the prior mean somewhat lower or higher than the mean factor loading.
6.2. Practical Importance

The present study aimed to investigate whether the use of informative priors for factor loadings would improve the quality of parameter estimates in the CFA and SEM analyses with small sample sizes. Applied researchers have consistently faced challenges of unsatisfactory parameter estimates in this regard (Little, Cunningham, Shahar, & Widaman, 2002). Although sustained efforts have been made to overcome the challenge from both frequentist and Bayesian perspectives, the results have not been satisfactory. For example, evidence from previous research has suggested frequentist methods like restricted maximum likelihood (REML) and the Kenward-Roger correction did not work very well for the parameter estimates for SEM models in general (Kenward & Roger, 1997, 2009). By a similar token, though using some Bayesian estimation methods could potentially work well with small samples, they do not work well all the time (McNeish, 2016). A relatively large body of research evidence shows that specifying informative priors still leads to a larger bias of parameter estimates and a low power in SEM models due to the subjective choice of prior mean and variances (Bolstad, 2016; Depaoli, 2013; Heerwegh, 2014). In contrast, the current study imposed very informative priors only for factor loading, which is similar to the essentially tau-equivalent condition for a measurement model. Therefore, it was speculated this restriction may be useful to obtain good estimates for CFA and SEM models with small samples.

As a result of the current study, two recommendations can be provided to applied researchers who utilize the CFA and SEM analyses with small samples. First, it is not advisable to use the Bayesian estimator with informative priors, if the purpose is to estimate factor loadings, as it is nearly impossible to know the population values of factor loadings. As stated earlier, parameter recovery of factor loadings was acceptable only when the prior mean was very
close to the population factor loading. However, if the researcher has sufficient information about the prior knowledge regarding the magnitude of factor loadings, the researcher should use it to specify informative priors for factor loadings. Such knowledge can be derived via various means, such as previous research and meta-analysis.

Second, the Bayesian estimator with informative priors is a reasonable approach if the interest is to estimate the path coefficient. The simulation results demonstrated that the performances on the recovery of path coefficients in SEM models between the Bayesian estimator with informative priors and the Bayesian estimator with incorrectly specified informative priors were similar. Obviously, given the limitations of Monte Carlo simulation study design, we cannot make comments and provide practical suggestions on the performance of these two methods when the sample size is less than 30 or more than 100. However, researchers should be encouraged to use the Bayesian estimator with informative priors with prior mean a little bit lower than the mean factor loading if they believe either the mean factor loading is higher or the mean factor loading is lower but the number of indicators per factor is larger for a comparable SEM model, such as the one used in the present study. In contrast, if researchers believe that the mean factor loading is lower but the number of indicators per factor is larger, the estimate of path coefficients would be most accurate or inaccurate with the lowest or highest bias and highest or lowest SE and RMSE if the prior mean is specified a little bit higher or lower than the mean factor loading. Under this circumstance, researchers need to decide between accuracy and precision when specifying incorrectly specified informative priors.

6.3. Limitations and Future Research

Despite the fact that the results from the current simulation studies partially supported the proposed method, the evidence unfortunately was not sufficient to conclude that Bayesian
estimator with informative priors can always work well in all small sample situations. Thus, it is subject to several specific limitations.

The first limitation is the subjectivity in assigning prior distribution to model parameters. Although the Bayesian estimator with informative priors can have a positive impact on the recovery of factor loadings for a CFA model and path coefficients for an SEM model with small samples, it also can face limitations when its degrees of precision and accuracy are varied. The analysts need to determine the shape of prior distribution, prior mean, and prior variances before starting Bayesian analysis. In the present study, the shape of the distribution was set to normal, the prior mean was varied around the population value, and prior variances were fixed at .01. However, these elements in the real world could be totally different from the present study. In other words, a researcher cannot assert whether the prior knowledge is completely correct. Although the results from the current simulation studies partially supported the use of a Bayesian estimator with incorrectly specified informative priors, how it improved the recovery of parameter estimates in the SEM model with small samples deserves further study.

The second limitation is the use of posterior SD of unstandardized results greater than 1.0 as the criterion for parameter convergence in the present study. Although it is acknowledged that the PSR value close to 1.0 indicates that MCMC chains have reached a stable state, the corresponding value in Mplus output with non-informative priors and informative priors was mixed and would lead to an incorrect conclusion. Admittedly, even though using the value of posterior SD greater than 1 may filter out some non-unusual results, it is better than the mixed results that lead to a wrong conclusion. According to Asparouhov and Muthén (2017), Mplus version 8 did not report prior posterior predictive p-value (pppp) if the SEM model was fitted with a Bayesian estimator with informative priors and did not converge. However, if the SEM
model was fitted with a Bayesian estimator with non-informative priors, the \( pppp \) value will not be presented irrespective of model convergence. Based on the analysis above, future studies should consider inspecting the excessive estimations and use another index like Kolmogorov-Smirnov test value to judge whether the SEM models with small sample sizes converge with a Bayesian estimator with non-informative priors using Mplus software.

Third, the present study only included the CFA models and SEM models under specific experimental conditions based on the feasibility of the study. Future studies could extend to additional types of models (e.g., latent growth curve model, multilevel SEM, etc.) with categorical indicators and estimation methods such as categorical least squares (CLS) and categorical diagonal weighted least squares method (CDWLS), which are more sensitive to sample sizes and normality assumption. In addition, future studies could set threshold symmetry and missing data patterns for further study. Moreover, under a CFA model, if the research purpose is other than estimating factor loadings (e.g., factor scores), a Bayesian estimator with informative prior may still be reasonable without much knowledge of the population values.

Since the present study did not evaluate the quality of factor scores, future research is warranted.

Lastly, the results should not be used for some conditions with the MLE due to an extremely low convergence rate. Obviously, such a lower convergence rate indicated that it is very likely that one would fail to fit the model with a small sample. However, it does not necessarily mean that the MLE performs poorly. It is possible that the MLE actually performs better than the Bayesian approaches when the MLE successfully converges for a given set of small data. Future studies could generate more replication data sets using a high-performance computing (HPC) cluster to run parallel R-code by revising the simulation algorithm to reduce the computing time to improve the computation efficiency.
REFERENCES


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Appendix

A. Indices Graphs Not Reported but Evaluated in the Dissertation

Figure A-1. Factor Loading’s Bias Plot in CFA Models by Estimator and Condition.

Figure A-2. Factor Loading’s Absolute Bias Plot in CFA Models by Estimator and Condition.
Figure A-3. Factor Loading’s Absolute Relative Bias Plot in CFA Models by Estimator and Condition.

Figure A-4. Factor Loading’s Mean Bias Plot in CFA Models by Estimator and Condition.
Figure A-5. Factor Loading’s Empirical SE Plot in CFA Models by Estimator and Condition.
Figure A-6. Factor Loading’s Mean Absolute Bias Plot in CFA Models by Estimator and Condition.

Figure A-7. Factor Loading’s Mean Relative Bias Plot in CFA Models by Estimator and Condition.
Figure A-8. Factor Loading’s Mean Absolute Relative Bias Plot in CFA Models by Estimator and Condition.

Figure A-9. Factor Loading’s Mean SE Plot in CFA Models by Estimator and Condition.
Figure A-10. Factor Loading’s Mean Empirical SE Plot in CFA Models by Estimator and Condition.

Figure A-11. Factor Loading’s Mean RMSE Plot in CFA Models by Estimator and Condition.
**Figure A-12.** Regression Coefficient’s Bias Plot in SEM Models by Estimator and Condition.

**Note:** The isolated red dots through Figure A-11 to Figure A-22 indicate non-convergent results using ML estimator.
Figure A-14. Regression Coefficient’s Absolute Relative Bias Plot in SEM Models by Estimator and Condition.

Figure A-15. Regression Coefficient’s Mean Bias Plot in SEM Models by Estimator and Condition.
Figure A-16. Regression Coefficient’s Empirical SE Plot in SEM Models by Estimator and Condition.
Figure A-17. Regression Coefficient’s Mean Absolute Bias Plot in SEM Models by Estimator and Condition.

Figure A-18. Regression Coefficient’s Mean Relative Bias Plot in SEM Models by Estimator and Condition.
Figure A-19. Regression Coefficient’s Mean Absolute Relative Bias Plot in SEM Models by Estimator and Condition.

Figure A-20. Regression Coefficient’s Mean SE Plot in SEM Models by Estimator and Condition.
Figure A-21. Regression Coefficient’s Mean Empirical SE Plot in SEM Models by Estimator and Condition.

Figure A-22. Regression Coefficient’s Mean RMSE Plot in SEM Models by Estimator and Condition.
B: R-Code of the Present Study

R Code for the CFA Models for Simulation Condition 1

#Load the required packages.
library(tidyverse)
library(mvtnorm)
library(MplusAutomation)
library(ggplot2)
library(plyr)
library(dplyr)
library(hablar)
library(forcats)
library(magrittr)

#Data generation function.
data.gen.fa5.c111 <-
  function(n) {
    fs1 <- rnorm(n, mean=0, sd=1)
    evar <- diag(5)*c(.9424, .9271, .9100, .8911, .8704)
    error <- rmvnorm(n, mean=rep(0, 5), sigma=evar)
    x1 <- .02 +.24*fs1 +error[,1]
    x2 <- .12 +.27*fs1 +error[,2]
    x3 <- .22 +.30*fs1 +error[,3]
    x4 <- .32 +.33*fs1 +error[,4]
    x5 <- .42 +.36*fs1 +error[,5]
    data.frame(x1=x1,x2=x2,x3=x3, x4=x4,x5=x5,fs1=fs1)}

#Fit the model under MLE estimator
model.gen.fa5.c111.mle <-
  function(data) {
    mplusObject(
      ANALYSIS = "type=general;
                  estimator=mlr;
                  iteration=20000;",
      MODEL =  "f1 by x1* x2-x5;
              f1@1;",
      rdata = data)
  }

#Fit the model under Bayesian estimator via R package MplusAutomation.
bay.priors <- c("N(0,infinity)", "N(0.2,0.01)", "N(0.3,0.01)","N(0.4,0.01)")
bay.names <- paste0("bay", 1:4)
for (i in seq_along(bay.priors)) {
  assign(paste0('model.gen.fa5.', bay.names[i]),
         paste0(
            'function(data) {
              mplusObject(
                ANALYSIS =  "type=general;
                            estimator=bayes;
'
          )
})
processor=2;
chains=2;
fbiterations=10000;
thin=1;"
MODEL =
"f1 by x1* (a1);
f1 by x2-x5 (a2-a5);
f1@1.0",
MODELPRIORS = "a1-a5 ~', bay.priors[i], ';",
rdata = data)') %>% parse(text=.) %>% eval()}

#Initialization of the model.
set.seed(67890)
r <-1000
n <-30
n.item <-5
mfl <-0.30
n.bay <- 1:4
n.rep <- numeric()
fa01mle0.out <- list()
for (p in 1:length(n.bay)) {
  assign(paste0('fa01bay', n.bay[p], '.out'), list())
}
bay.names <- paste0('out.bay', 1:4)

#Generate the data, fit the model, and pick out the good results
#under MLE estimator with i-loop.
for (i in 1:r) {
  n.rep[i] <- 0
  repeat {
    n.rep[i] <- n.rep[i] + 1
    fa.c01.dat <- data.gen/fa5.c111(n)
    mle.mod <- model.gen/fa5.c111.mle(data = fa.c01.dat)
    mle.fit <- mplusModeler(mle.mod,
                            modelout = "mle.inp",
                            hashfilename = F,
                            run = T)
    conv <- ifelse(mle.fit$results$errors %>% length() == 0, "GOOD", "BAD")
    if(conv == "GOOD") {break}
  }
  #Save the fitted factor loadings under MLE estimator into lists.
  fa01mle0.out[[i]] <-
    mle.fit$results$parameters$unstandardized %>%
    filter(paramHeader == "F1.BY") %>%
    select(param, est) %>%
    column_to_rownames(var = "param") %>%
    t() %>%
    as.data.frame()
}

#Use same dataset to fit the model under Bayesian Estimator using J-loop
#and pick out the fitted factor loadings and save them into list.
for (j in 1:length(n.bay)) {
  bay.mod <- paste0('model.gen/fa5.bay', n.bay[j], '(data=fa.c01.dat)') %>%
               parse(text=.) %>%
               eval()
  bay.fit <- mplusModeler(bay.mod,
                          modelout = "bay.inp",
                          hashfilename=F, run=T)
  out <- bay.fit$results$parameters$unstandardized %>%
   filter(paramHeader == "F1.BY") %>%
   select(param, est) %>%
   column_to_rownames(var = "param") %>%
   t() %>%
   as.data.frame()
  eval(parse(text = paste0('fa01bay', n.bay[j], '.out[[i]] <- out'))) }}
# R Code for the SEM Models for Simulation Condition 1

```r
#the MLR Estimator.
set.seed(67890)

r             <- 1000
nn            <- 30
n.item        <- 5
mfl           <- 0.30
n.rep         <- numeric()
n.bay         <- 1:4
sem01mle0.out <- list()

for (p in 1:length(n.bay)) {
  assign(paste0('sem01bay', n.bay[p], '.out'), list())
}

bay.names     <- paste0('out.bay', 1:4)

for (i in 1:r) {
  n.rep[i] <- 0
  repeat {
    n.rep[i] <- n.rep[i] + 1
    # Data generation.
    f13      <- rmvnorm(nn, mean=rep(0, 2), sigma = matrix(c(1, 0.3, 0.3, 1), nrow = 2))
    f1       <- f13[,1]
    f3       <- f13[,2]
    error2   <- rnorm (nn, mean=0, sd=sqrt(0.766))
    f2       <- 0.3*f1+0.3*f3+error2
    error1   <- mvnorm(nn, mean=rep(0,5), sigma=diag(5)*c(.9424, .9271, .9100,.8911, .8704))
    error3   <- mvnorm(nn, mean=rep(0,5), sigma=diag(5)*c(.9424, .9271, .9100,.8911, .8704))
    x101     <- .02 +.24*f1 +error1[,1]
    x102     <- .12 +.27*f1 +error1[,2]
    x103     <- .22 +.30*f1 +error1[,3]
    x104     <- .32 +.33*f1 +error1[,4]
    x105     <- .42 +.36*f1 +error1[,5]
    x301     <- .02 +.24*f3 +error3[,1]
    x302     <- .12 +.27*f3 +error3[,2]
    x303     <- .22 +.30*f3 +error3[,3]
    x304     <- .32 +.33*f3 +error3[,4]
    x305     <- .42 +.36*f3 +error3[,5]

data     <- data.frame(f2=f2, x101=x101, x102=x102, x103=x103, x104=x104, x105=x105,
    x301=x301, x302=x302, x303=x303, x304=x304, x305=x305)

#Fit the SEM model using the MLR estimator.
sem5.mle0.mod<-mplusObject(ANALYSIS ="type=general; estimator=mlr;",
MODEL = "f1 by x101*;
f1 by x102-x105;
f01.0;
f3 by x301*;
f3 by x302-x305;
f3@1.0;
f2 @ 0.766;
f2 on  f3*0.3000;
f2 on  f1*0.3000;
f1 with f3*0.3000;",rdata =data)

sem5.mle0.fit<- mplusModeler (sem5.mle0.mod, modelout=sem5.mle0.inp", hashfilename=F, run=T)

#Judge whether the results is good or bad.
resu               <- ifelse(sem5.mle0.fit$results$errors %>% length() == 0, "GOOD", "BAD")
if(resu == "GOOD") {break} }

#Save the "good" regression coefficients.
sem01mle0.out[[i]] <-
  sem5.mle0.fit$results$parameters$unstandardized %>%
  filter(paramHeader == "F2.ON") %>%
  select(param, est) %>
  column_to_rownames(var = "param") %>
  t() %>
  as.data.frame()
```
# Condition-1 using Bayesian Estimator with non-informative priors N~(0,Inf).

```r
set.seed(67890)
r <- 1000
nn <- 30
n.item <- 5
mfl <- 0.30
n.rep <- numeric()
sem01bay1.out <- list()
for (i in 1:r) {
  n.rep[i] <- 0
  repeat {
    n.rep[i] <- n.rep[i] + 1
  }

  # Data generation.
  f13 <- rmvnorm(nn, mean = rep(0, 2), sigma = matrix(c(1, 0.3, 0.3, 1), nrow = 2))
  f1 <- f13[, 1]
  f3 <- f13[, 2]
  error2 <- rnorm(nn, mean = 0, sd = sqrt(0.766))
  f2 <- 0.3 * f1 + 0.3 * f3 + error2
  error1 <- rmvnorm(nn, mean = rep(0, 5), sigma = diag(5) * c(.9424, .9271, .9100, .8911, .8704))
  error3 <- rmvnorm(nn, mean = rep(0, 5), sigma = diag(5) * c(.9424, .9271, .9100, .8911, .8704))
  x101 <- .02 + .24 * f1 + error1[, 1]
  x102 <- .12 + .27 * f1 + error1[, 2]
  x103 <- .22 + .30 * f1 + error1[, 3]
  x104 <- .32 + .33 * f1 + error1[, 4]
  x105 <- .42 + .36 * f1 + error1[, 5]
  x301 <- .02 + .24 * f3 + error3[, 1]
  x302 <- .12 + .27 * f3 + error3[, 2]
  x303 <- .22 + .30 * f3 + error3[, 3]
  x304 <- .32 + .33 * f3 + error3[, 4]
  x305 <- .42 + .36 * f3 + error3[, 5]
  data <- data.frame(f2 = f2, x101 = x101, x102 = x102, x103 = x103, x104 = x104, x105 = x105,
                      x301 = x301, x302 = x302, x303 = x303, x304 = x304, x305 = x305)
```

# Fit the SEM model using Bayesian Estimator with non-informative priors N~(0,Inf).

```r
sem5.bay1.mod <- mplusObject(ANALYSIS = "type=general;
                          estimator=bayes;
                          processor=2;
                          fbiterations=10000;
                          thin=1;",
                          MODEL = "f1 by x101* (a101);
                          f1 by x102-x105 (a102-a105);
                          f1@1.0;
                          f3 by x301* (a301);
                          f3 by x302-x305 (a302-a305);
                          f3@1.0;
                          f2 @ 0.766;
                          f2 on f3*0.3000;
                          f2 on f1*0.3000;
                          f1 with f3*0.3000;",
                          MODELPRIORS = "a101-a305 ~N(0,Inf);",
                          OUTPUT = "TECH8;", rdata = data)
sem5.bay1.fit <- mplusModeler(sem5.bay1.mod,
                              modelout = "sem5.bay1.inp",
                              hashfilename = F,
                              run = T)
```
# Judge whether the results are good or bad.
resu <- conv <-
ifelse(any(sem5.bay1.fit$results$parameters$unstandardized$posterior_sd > 1),
"BAD", "GOOD")
if(resu == "GOOD") {break}

# Save the "good" regression coefficients.
sem01bay1.out[[i]] <-
  sem5.bay1.fit$results$parameters$unstandardized %>%
  filter(paramHeader == "F2.ON") %>%
  select(param, est) %>%
  column_to_rownames(var = "param") %>%
  t() %>%
  as.data.frame()

# Condition 1 using Bayesian Estimator with informative priors N~(.2, .01).
set.seed(67890)
r <- 1000
nn <- 30
mfl <- 0.30
n.rep <- numeric()
sem01bay2.out <- list()
for (i in 1:r) {
  n.rep[i] <- 0
  repeat {
    n.rep[i] <- n.rep[i] + 1
    # Data generation.
    f13 <- rmvnorm(nn, mean = rep(0, 2), sigma = matrix(c(1, 0.3, 0.3, 1), nrow = 2))
    f1 <- f13[, 1]
    f3 <- f13[, 2]
    error2 <- rnorm(nn, mean = 0, sd = sqrt(0.766))
    f2 <- 0.3*f1 + 0.3*f3 + error2
    error1 <- rmvnorm(nn, mean = rep(0, 5), sigma = diag(5) * c(.9424, .9271, .9100, .8911, .8704))
    error3 <- rmvnorm(nn, mean = rep(0, 5), sigma = diag(5) * c(.9424, .9271, .9100, .8911, .8704))
    x101 <- .02 + .24*f1 + error1[, 1]
    x102 <- .12 + .27*f1 + error1[, 2]
    x103 <- .22 + .30*f1 + error1[, 3]
    x104 <- .32 + .33*f1 + error1[, 4]
    x105 <- .42 + .36*f1 + error1[, 5]
    x301 <- .02 + .24*f3 + error3[, 1]
    x302 <- .12 + .27*f3 + error3[, 2]
    x303 <- .22 + .30*f3 + error3[, 3]
    x304 <- .32 + .33*f3 + error3[, 4]
    x305 <- .42 + .36*f3 + error3[, 5]
    data <- data.frame(f2 = f2, x101 = x101, x102 = x102, x103 = x103, x104 = x104, x105 = x105,
                       x301 = x301, x302 = x302, x303 = x303, x304 = x304, x305 = x305)

    # Fit the SEM model using Bayesian estimator with informative priors N~(.2, .01)
    sem5.bay2.mod <- mplusObject(ANALYSIS = "type=general;
                                estimator=bayes;
                                processor=2;
                                fbiterations=10000;
                                thin=1;",
                               MODEL = "f1 by x101* (a101);
                                  f1 by x102-x105 (a102-a105); 
                                  f1@1.0;
                                  f3 by x301* (a301);
                                  f3 by x302-x305 (a302-a305); 
                                  f3@1.0;"}
sem5.bay2.fit <- mplusModeler(sem5.bay2.mod,
  modelout = "sem5.bay2.inp",
  hashfilename = F,
  run = T)

resu <- ifelse(any(sem5.bay2.fit$results$parameters$unstandardized$posterior_sd >1),
  "BAD", "GOOD")

if(resu == "GOOD") {break}}

#Save the "good" regression coefficients.
sem01bay2.out[[i]]<-
  sem5.bay2.fit$results$parameters$unstandardized %>%
  filter(paramHeader == "F2.ON") %>%
  select(param, est) %>%
  column_to_rownames(var = "param") %>%
  t() %>%
  as.data.frame()

#Condition-1 using Bayesian Estimator with informative priors N~(.3,.01).
set.seed(67890)
r <- -1000
nn <- 30
n.item <- 5
mfl <- -0.30
n.rep <- numeric()
sem01bay3.out <- list()
for (i in 1:r) {
  n.rep[i] <- 0
  repeat {
    n.rep[i] <- n.rep[i] + 1
  }

#Data generation.
f13 <- rmvnorm(nn,mean=rep(0, 2), sigma = matrix(c(1, 0.3, 0.3, 1), nrow = 2))
f1 <- f13[,1]
f3 <- f13[,2]
error2 <- rnorm (nn,mean=0,sd=sqrt(0.766))
f2 <- 0.3*f1+0.3*f3+error2
error1 <- rmvnorm(nn, mean=rep(0, 5),sigma=diag(5)*c(.9424, .9271, .9100, .8911, .8704))
error3 <- rmvnorm(nn, mean=rep(0, 5),sigma=diag(5)*c(.9424, .9271, .9100, .8911, .8704))
x101 <- .02 +.24*f1 +error1[,1]
x102 <- .12 +.27f1 +error1[,2]
x103 <- .22 +.30f1 +error1[,3]
x104 <- .32 +.33f1 +error1[,4]
x105 <- .42 +.36f1 +error1[,5]
x301 <- .02 +.24f3 +error3[,1]
x302 <- .12 +.27f3 +error3[,2]
x303 <- .22 +.30f3 +error3[,3]
x304 <- .32 +.33f3 +error3[,4]
x305 <- .42 +.36f3 +error3[,5]
data <- data.frame(f2=f2,x101=x101,x102=x102,x103=x103,x104=x104,x105=x105,
  x301=x301,x302=x302,x303=x303,x304=x304,x305=x305)

#Fit the SEM model using Bayesian estimator with informative priors N~(.3,.01).
sem5.bay3.mod<-mplusObject(ANALYSIS ="type=general;
112
estimator=bayes;
processor=2;
chains=2;
fbiterations=10000;
thin=1;"

MODEL =
"f1 by x101*(a101);
f1 by x102-x105(a102-a105);
f1@1.0;
f3 by x301*(a301);
f3 by x302-x305(a302-a305);
f3@1.0;
f2 @ 0.766;
f2 on f3*0.3000;
f2 on f1*0.3000;
f1 with f3*0.3000;",

MODELPRIORS = "a101-a305 ~N(0.3,0.01);",
OUTPUT = "TECH8;",
rdata =data)

sem5.bay3.fit <- mplusModeler(sem5.bay3.mod,
modelout = "sem5.bay3.inp",
hashfilename = F,
run = T)

#Judge whether the results are "good" or "bad".
resu <- ifelse(any(sem5.bay3.fit$results$parameters$unstandardized$posterior_sd >1),
"BAD", "GOOD")
if(resu == "GOOD") {break} }

#Save the "good" regression coefficients.
sem01bay3.out[[i]]<-
sem5.bay3.fit$results$parameters$unstandardized %>%
filter(paramHeader == "F2.ON") %>%
select(param, est) %>%
column_to_rownames(var = "param") %>% t() %>% as.data.frame() }

#Condition 1 using Bayesian Estimator with informative priors N~(.4,.01).
set.seed(67890)
r <-1000
nn <-30
n.item <-5
mfl <-0.30
n.rep <- numeric()
sem01bay4.out <- list()
for (i in 1:r) {
  n.rep[i] <- 0
  repeat {
    n.rep[i] <- n.rep[i] + 1
    #Data generation.
    f13 <- rmvnorm(nn,mean=rep(0, 2), sigma = matrix(c(1, 0.3, 0.3, 1), nrow = 2))
    f1 <- f13[,1]
    f3 <- f13[,2]
    error2 <- rnorm (nn,mean=0, sd=sqrt(0.766))
    f2 <- 0.3*f1+0.3*f3+error2
    error1 <- rmvnorm(nn, mean=rep(0, 5), sigma=diag(5)*c(.9424, .9271, .9100, .8911, .8704))
    error3 <- rmvnorm(nn, mean=rep(0, 5), sigma=diag(5)*c(.9424, .9271, .9100, .8911, .8704))
    x101 <- .02 +.24*f1 +error1[,1]
    x102 <- .12 +.27*f1 +error1[,2]
    x103 <- .22 +.30*f1 +error1[,3]
    x104 <- .32 +.33*f1 +error1[,4]
x105  <- .42 + .36*f1 + error1[,5]
x301  <- .02 + .24*f3 + error3[,1]
x302  <- .12 + .27*f3 + error3[,2]
x303  <- .22 + .30*f3 + error3[,3]
x304  <- .32 + .33*f3 + error3[,4]
x305  <- .42 + .36*f3 + error3[,5]
data <- data.frame(f2=f2,x101=x101,x102=x102,x103=x103,x104=x104,x105=x105,
x301=x301,x302=x302,x303=x303,x304=x304,x305=x305)

# Fit the SEM model using Bayesian estimator with informative priors N=(0.4,0.01).
sem5.bay4.mod <- mplusObject(ANALYSIS = "type=general;
estimator=bayes;
processor=2;
chains=2;
fbiterations=10000;
thin=1;",
MODEL = "f1 by x101* (a101);
f1 by x102-x105 (a102-a105);
f1@1.0;
f3 by x301* (a301);
f3 by x302-x305 (a302-a305);
f3@1.0;
f2 @ 0.766;
f2 on f3*0.3000;
f2 on f1*0.3000;
f1 with f3*0.3000;",
MODELPRIORS = "a101-a305 ~N(0.4,0.01);",
OUTPUT = "TECH8;",
rdata = data)

sem5.bay4.fit <- mplusModeler(sem5.bay4.mod,
modelout = "sem5.bay4.inp",
hashfilename = F,
run = T)

# Judge whether the results are "good" or "bad".
resu <- ifelse(any(sem5.bay4.fit$results$parameters$unstandardized$posterior_sd >1),
"BAD", "GOOD")
if(resu == "GOOD") {break}

# Save the "good" regression coefficients.
sem01bay4.out[1]
<- sem5.bay4.fit $results$parameters$unstandardized %>%
  filter(paramHeader == "F2.ON") %>%
  select(param, est) %>%
  column_to_rownames(var = "param") %>%
  t() %>%
  as.data.frame()