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DEALING WITH THE CAPITAL INVESTMENT DECISON
AND OPERATING RISK USING OPTION PRICING THEORY

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by

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ABSTRACT

This paper presents a model which incorporates the option to abandon as a put option. The purpose of the paper is not only to discuss the advantages of the option approach, but also to use the model for a theoretical analysis of project risk. The option approach captures several dimensions of the investment project which have previously been considered to be intangible, such as asset flexibility, durability, and project innovativeness. Analysis of the dynamics of project value through the model indicates that the systematic risk of the project is a weighted average of several components and that project beta is non-stationery, vibrating around a trend through time.

1. Introduction

Any reasonable approach to the firm's capital investment decision requires an attempt to determine the effect of investment in the project on the market value of the firm's equity. Thus it is an attempt to express the characteristics of the project in terms of a package of equivalent financial assets. Besides components which can be characterized as dividend-paying equities, many capital investment projects also include the option to abandon. Treating this abandonment option as equivalent to a put, a market value can be estimated.

Robichek & VanHorne (1967), Dyl & Long (1969), and VanHorne (1980) have recognized the necessity of dealing with the abandonment option explicitly, and have advocated its inclusion as a contingency in the forecast of cash flows used for calculation of an investment project's net present value or internal rate of return. Bonini (1977) has taken the same approach using dynamic programming.[1] Because the abandonment option is a put option which comes free with the purchase of the assets, however, this approach does not capture many of the factors which affect its value. This paper presents a model of the project with abandonment as an option. In so doing, not only the project's earning power, but also the flexibility of the assets, their durability, and the innovativeness of the use to which they are being put are all captured as contributors to the project's market value. Best of all, the computations required to apply the technique can be accomplished with a hand-held calculator.

Besides improving the capital investment decision process, treating project abandonment as a put option opens the way to new insights into operating risk and the beta of the firm. Operating risk has traditionally been identified with the concept of the risk arising when capital is tied up in the purchase of plant or equipment, especially that designed for a special function. A portion of the invested capital stands at risk that the enterprise might fail, or the economic life of the asset might be shortened by obsolescence. The essence of this risk centers on the attractiveness of the abandonment option: the extent to which the capital could be recovered on abandonment of the enterprise; yet the measure which has been developed for operating risk, the degree of operating leverage (D.O.L.), ignores the abandoment option altogether, and focuses simply on the elasticity of operating earnings with respect to sales. It is shown in section 5 of this paper that the project's systematic risk is a weighted average of the systematic risk of the earnings stream from the project and the systematic risk of the abandonment value, which is determined by the alternative uses of the assets. Furthermore, it is shown that the project's beta is non-stationary, and vibrates around a trend through time. Being a weighted average of project betas, the firm's beta would thus be constantly shifting as existing projects age or terminate and new projects are initiated.

2. The Model

A simple model of an investment project with an abandonment option can be created by conjecturing a project whose life spans two periods: the first period being the time prior to expiration of the abandonment option while the second period is the remaining life of the project after expiration of the abandoment option, with the two periods not necessarily of equal length. In its simplest form, the model would assume the following.

1. The value of the project at any time is equal to the expected present value of the stream of net future real earnings, plus the value of the abandonment option.

2. The abandonment decision will be made on a specific date, so that the abandonment option is a simple European put.

3. On the date of the abandonment decision, the project will be terminated if the abandonment value is greater than the value of continuing the venture.

4. The abandonment value of the project is uncertain. The estimate is continuously revised as new information becomes available.

Whenever possible, conclusions will be drawn without making specific assumptions about the stochastic process involved in the generation of changes in the abandonment value or the values of the future earnings streams. However, when making use of the Black-Scholes Option Pricing Model (OPM), the following additional assumptions will be necessary:

5. The capital market operates continuously with no frictions.

6. The estimates of the values of future earnings streams and the abandonment value follow log-normal diffusion processes through time, with stationary variance rates.

The assumption of log-normal diffusion processes can be defended as reasonable because new information is what produces changes in the estimates of these values, and the diffusion process assumption applies to the process by which this information is produced. It requires that new items of information produce a gradually developing and evolving picture, with no sudden shocking surprises.

The value of the investment project can be formulated as follows:

$$V = S_1 + S_2 + P(S_2, X, t) \tag{1}$$

where:

V = value of the project.

S_1 = present value of the expected net future real earnings stream from the project during period 1.

S_2 = present value of the expected net future real earnings stream from the project during period 2.

$P(S_2,X,t)$ = value of the abandonment option expressed as a put.

X = exercise price of the abandonment option: that is, the project's real abandonment value.

t = time to expiration of the abandonment option, in years.

The expression in equation (1) contains the equivalent of holding a put together with a share of the underlying stock, and Merton (1973) has shown without distributional assumptions that such a position is identical to holding a call with the same exercise price and expiration date as the put along with a bond which pays the exercise price on the termination date of the option.[2] This can be stated formally as follows:

$$V = S_1 + e^{-r_h t} E(X) + C(S_2,X,t) \tag{2}$$

where the new symbols are:

$C(S_2,X,t)$ = value of a call option with the same exercise price and expiration date as the abandonment option.

$E(X)$ = expected real abandonment value.

r_h = appropriate continuous risk-adjusted real discount rate.

e = base of natural logarithms.

Equation (2) makes no specific distributional assumptions, but merely unpackages the project's value. In order to utilize the Black-Scholes OPM, as modified by Fischer (1978) for the case of uncertain exercise price, it is necessary to specify the processes generating changes in S_1, S_2, and X. The following assumptions will be made:

$$dS_1/S_1 = \alpha_1 dt + \sigma_1 dz_1 \tag{3}$$

$$dS_2/S_2 = \alpha_2 dt + \sigma_2 dz_2 \tag{4}$$

$$dX/X = -\alpha_x dt + \sigma_x dz_x \tag{5}$$

where the new symbols are,

α_1 = the expected instantaneous rate of change in valuation of the period 1 earnings stream. This may be negative, to reflect the declining value of remaining earnings as the end of period 1 approaches closer.

α_2 = the expected instantaneous rate of change in valuation of the period 2 earnings stream. This would be positive to reflect the increasing value of this earnings stream as the time of receipt draws nearer.

α_x = the rate at which real value is extracted from the assets as they are used up. The usual case would be one of downward drift, hence the negative sign.

$\sigma_1, \sigma_2, \sigma_x$ = the instantaneous standard deviations of $\alpha 1$, $\alpha 2$, and α_x.

dz_1, dz_2, dz_x = standard Gauss-Wiener processes.

The parameters α_1, $\alpha 2$, α_x, $\sigma 1$, $\sigma 2$, and σ_x are assumed to be stationary. Fischer solved the problem of valuing an option with uncertain exercise price by substituting for the Black-Scholes neutral hedge a new hedge portfolio containing a security to offset the uncertainty arising from the exercise price. This hedge security, H, must have the following dynamics:

$$dH/H = r_h dt + \sigma_x dz_x \tag{6}$$

Fischer showed, using the continuous-time capital asset pricing model (CAPM), that the expected return on this hedge security would be given by:

$$r_h = r + b \tag{7}$$

where r is the zero-beta real rate of return, and b is the risk premium given by,

$$b = \beta_x(r_m - r). \tag{8}$$

β_x = the relative measure of systematic risk for the abandonment value.

r_m = expected instantaneous real return on the market portfolio.

The parameters r, r_m, β_x, and hence b, are assumed to be stationary.

Given the specification of equation (3) the expected abandonment value can be stated as follows:

$$E(X) = Xe^{-\alpha_x t} \tag{9}$$

Substituting equation (9) into equation (2) yields:

$$V = S_1 + e^{-(r_h+\alpha_x)t} X + C(S_2,X,t) \tag{10}$$

It remains only to specify the value of the call to provide a complete specification of V under the Black-Scholes assumptions. Fischer derived a modification of the Black-Scholes OPM for the case of a stochastic exercise price, which is here restated for the case of a declining exercise price.[3]

$$C(S_2,X,t) = S_2 \cdot N\{d_1\} - Xe^{-(r_h+\alpha_x)t} \cdot N\{d_2\} \tag{11}$$

where

$$d_1 = \frac{\ln(S_2/X) + [r_h + \alpha_x + (\hat{\sigma}^2/2]t}{\hat{\sigma} \sqrt{t}}$$

$$d_2 = d_1 - \hat{\sigma} \sqrt{t}$$

$$\hat{\sigma}^2 = \sigma_2^2 + \sigma_x^2 - 2\sigma_2\sigma_x\rho_{2,x}$$

$N\{\cdot\}$ = cumulative normal probability distribution function.

Substituting (11) into (10) yields the completed valuation function.

$$V = S_1 + S_2N\{d_1\} + Xe^{-(r_h+\alpha_x)t} (1 - N\{d_2\}) \tag{12}$$

The intuitive interpretation of equations (10) and (12) is that the value of the project is made up of the expected present value of the real earnings stream prior to the abandonment decision, the expected present value of abandonment, and a call option to purchase the period two real earnings streams by foregoing abandonment. This breakdown depends on the assumption that the abandonment option is a European put; equation (1) is the correct specification for the case of an American put. Because of the early exercise privilege, an American put may be more valuable than a European put, and (10) and (12) would understate project value. However, the conclusions of the remaining sections are not dependent on the assumption of the European put, and the assumption is made to simplify presentation.

3. The Abandonment Option and the Capital Investment Decision

Taking explicit account of the abandonment option captures dimensions of the project that are beyond the reach of discounted cash flow (DCF) techniques. Besides the earning power of the project, which is captured equally well by the DCF techniques, the option approach provides a way to capture the more elusive dimensions of innovativeness and flexibility. To illustrate this, the signs of several important partial derivatives of the valuation equation are stated below, with discussion to follow.

$$\frac{\partial V}{\partial S_1}, \frac{\partial V}{\partial S_2}, \frac{\partial V}{\partial X}, \frac{\partial V}{\partial \sigma_2} > 0; \frac{\partial V}{\partial \rho_{2,x}}, \frac{\partial V}{\partial \alpha_x}, \frac{\partial V}{\partial b} < 0; \frac{\partial V}{\partial \sigma_x} \lesseqgtr 0 \qquad (13)$$

3.1. Innovativeness

The dimension of innovativeness is captured in two ways, through both of which innovativeness adds to the project's measured market value. The variables of interest are σ_2 and $\rho_{2,x}$: the total risk of the period two earnings stream and the strength of the relationship between the return from the intended use and the return from alternative uses for the project's assets. A highly innovative project would be one which puts assets into a new, unproven use which has little relation to other alternative uses.

The total riskiness of the period two earnings stream, σ_2, affects the value of equation (10) only through its last term, the call option. The

Black-Scholes OPM can be called upon to demonstrate that as the total risk of the underlying security increases, so does the value of the call option. This is because the option has limited liability, and the resultant increase in upside potential is not offset by increased downside risk. The point can also be argued without making specific assumptions about underlying distributions, as is necessary with the Black-Scholes model.[4]

The correlation coefficient between the value of period two earnings and the abandonment value clearly also affects only the last term of equation (10), the call. In the abandonment option case, as the correlation between S_2 and X decreases, the variance rate of the ratio $S/X, \hat{\sigma}^2$, increases. The Black-Scholes OPM as modified by Fischer can be called upon to show that a decrease in $\rho_{2,x}$ leads to an increase in the value of the option. This can be understood intuitively by considering the project as a portfolio consisting of the alternative uses of the assets as well as the intended use. The lower the correlation between these components, the less the risk of the portfolio, and the greater its value.

Putting assets into a new, high-risk use which bears little relation to alternative uses is an attractive and valuable thing to do when the option to bail out is available, and the value of this innovativeness is clearly captured only through explicit consideration of the abandonment option as an option.

3.2. Flexibility

The dimension of asset flexibility enters the abandonment option model in several ways. Perhaps the most obvious is through the abandonment value, X. Merton (1973) has shown with distribution-free arguments that the value of a put increases with X; hence through equation (1) it can be seen that project value increases as the value of alternative uses for assets increases.

Several other less obvious aspects of flexibility are also captured by the model. Besides the abandonment value itself, its riskiness is also an important dimension of flexibility. The premium for the systematic risk of the abandonment value, b, enters equation (10) through r_h in both the second and third terms. A decrease in b increases the value of the bond (the second term) and decreases the value of the call. Referring to equation (12) to resolve the net effect, it can be seen that,

$$\frac{\partial V}{\partial b} = -t \cdot e^{-(r+b+\alpha_x)t} X \cdot N\{-d_2\} < 0 \tag{14}$$

Equation (18) shows that the increase in value of the bond more than offsets the decreased value of the call, so that the net effect of a decrease in b is an increase in the value of the project. It is reasonable to interpret a lower systematic risk premium on the security used to hedge out fluctuations in the abandonment value as an indication of greater flexibility for the assets. That is, more flexible assets would have a wider variety of uses and so by their adaptability be less sensitive to systematic forces.

The total risk of the abandonment value, σ_x, on the other hand, has an ambiguous effect on the value of the project. Taking the partial of (10) produces:

$$\frac{\partial V}{\partial \sigma_x} = \frac{dC}{\partial \sigma_x} - (e^{-(r+b+\alpha_x)t}X)(t\rho_{xm}/\sigma_m)(r_m-r) \tag{15}$$

Whether or not the increase in value of the option which results from the added uncertainty is offset by the decrease in the value of the bond depends on the correlation coefficient, ρ_{xm}, the strength of the systematic relationship between the abandonment value and the market portfolio. If that relationship is weak enough, the option effect can swamp out the bond effect, and vice versa. Thus, if uncertainty about the abandonment value arises primarily from technological, unsystematic factors, it may add to the value of the abandonment option.

Students of finance have for years been taught by anecdotal example and intuition that flexibility of assets is an important aspect of the capital investment decision. The above results provide the teacher with rigorous arguments, and lay the foundation for putting an actual dollar value on flexibility.

Finally, the related dimension of durability enters through α_X, the drift term for the abandonment value. Although an increase in α_X would slightly increase $N(d_1)$ and $N(d_2)$, and so the value of the call option in equation (10), the decrease in the value of the bond (the second term of equation (10)) would be more than offsetting. Thus the more durable the asset, the greater the project's measured value.

3.3. Earning Power and the Project's Value

It can be proven, even without making any assumptions about the probability distribution followed by the value of the future revenues, that for positive abandonment values the value of the project exceeds the present value of expected net future earnings streams.[5] This raises questions about the adequacy of techniques which fail to consider the value of the abandonment option. Even those models which include abandonment value in the cash flow stream, such as RVH and DL, do not consider the option aspects of abandonment.

3.4. The Ideal Project

The project's market value is at its height when flexible, widely-used, durable "off-the-shelf" assets are put to innovative and potentially profitable uses. The full range of these considerations is captured in a model which explicitly considers abandonment as an option.

4. Optimizing the Production Technique

Explicit valuation of the abandonment option has the additional advantage that it incorporates the choice of production technique directly into the capital investment decision. One criticism of the existing work on measurement of operating risk[6] is that it provides no clear guidance on the choice of optimal production technology, but simply says that greater reliance on capital as opposed to variable inputs leads to higher risk. Presumably a highly uncertain demand for output would dictate a choice of lower operating risk, while more predictable demand for ouput would allow greater reliance on capital. The problem has remained unsolved of determining whether the operating risk involved in an alternative technique is justified by its expected return. However, when the abandonment option is explicitly valued and added to the expected present value of net future cash flows, the choice between alternative production techniques can be seen to simply involve selecting the alternative which maximizes the difference between the value of the project and its cost: it is simply a matter of maximizing net present value. Greater application of capital will widen the gap between revenue and variable cost, thereby increasing the present value of net real earnings. The result will be an increase in the value of the project's benefits. If the assets have alternative uses, a greater application of capital will also increase the abandonment value, and as has been discussed in section 3.2, this also increases the project's gross value. Additional capital will be applied as long as the cost is offset by an increase in project value. Naturally, the more flexible the assets used, and the more innovative the use to which they are put, the more favorable it will be to intensify the employment of capital in the production mix.

5. Operating Risk and the Abandonment Option

Clifford Smith (1979) provides a well-stated compact presentation of the deeper insight which Galai & Masulis (1976) obtained by applying the option pricing model to measure the financial risk of the firm, and these results can easily be extended to incorporate a refined measure of the firm's operating risk. By considering the equity of a firm to be a call option to buy the firm back from bondholders at the maturity of the firm's debt, it is possible to measure the effect of financial leverage on the beta of the firm's equity. Smith's paper can be referred to for proof that,

$$\beta_E \equiv \frac{\partial E}{\partial V}\frac{V}{E}\,\beta_V \tag{16}$$

where

β_E = the measure of the instantaneous systematic risk for equity.

β_V = the measure of the instantaneous sytematic risk for the firm.

E = the value of equity.

V = the value of the firm.

$\frac{\partial E}{\partial V}\frac{V}{E}$ = $\varepsilon(E,V)$ = the elasticity of the value of equity with respect to the value of the firm.

Smith presents proof that $\varepsilon(E,V)$ is greater than one for the levered firm (there are very few firms, if any, which don't have at least some short-term debt) so that the systematic risk of equity is a magnification of the systematic risk of the firm. Furthermore, because the elasticity is not constant, even if the beta for the firm is stationary, the instantaneous beta for the equity will not be.

The continuous-time capital asset pricing model (CAPM) can be applied to the problem of measuring operating risk by first taking the derivative of equation (10), using Itô's lemma.

$$dV = \frac{\partial V}{\partial S_1} dS_1 + \frac{\partial V}{\partial S_2} dS_2 + \frac{\partial V}{\partial X} dX + \Psi dt \tag{17}$$

Ψ is defined in footnote 7.

The instantaneous rate of return for the project is,

$$r_V = \frac{dV}{V} = \frac{\partial V}{\partial S_1} \frac{S_1}{V} \frac{dS_1}{S_1} + \frac{\partial V}{\partial S_2} \frac{S_2}{V} \frac{dS_2}{S_2} + \frac{\partial V}{\partial X} \frac{X}{V} \frac{dX}{X} + \Psi \frac{dt}{V}$$

$$r_V = \frac{\partial V}{\partial S_1} \frac{S_1}{V} r_1 + \frac{\partial V}{\partial S_2} \frac{S_2}{V} r_2 + \frac{\partial V}{\partial X} \frac{X}{V} r_X + \Psi \frac{dt}{V} \tag{18}$$

The instantaneous beta for the project can be derived from the continuous time CAPM as follows:

$$\beta_V = \frac{Cov(\tilde{r}_V, \tilde{r}_m)}{\sigma^2(\tilde{r}_m)} \equiv \frac{\partial V}{\partial S_1} \frac{S_1}{V} \frac{Cov(\tilde{r}_1, \tilde{r}_m)}{\sigma^2(\tilde{r}_m)} + \frac{\partial V}{\partial S_2} \frac{S_2}{V} \frac{Cov(\tilde{r}_2, \tilde{r}_m)}{\sigma^2(\tilde{r}_m)} + \frac{\partial V}{\partial X} \frac{X}{V} \frac{Cov(\tilde{r}_X, \tilde{r}_m)}{\sigma^2(\tilde{r}_m)}$$

$$\beta_V = \frac{\partial V}{\partial S_1} \frac{S_1}{V} \beta_1 + \frac{\partial V}{\partial S_2} \frac{S_2}{V} \beta_2 + \frac{\partial V}{\partial X} \frac{X}{V} \beta_X$$

$$\beta_V = \varepsilon(V,S_1)\beta_1 + \varepsilon(V,S_2)\beta_2 + \varepsilon(V,X)\beta_X \tag{19}$$

where $\varepsilon(V,\cdot)$ is the elasticity of the project's value with respect to S_1, S_2, and X, respectively.

In the special case where $\beta_1 = \beta_2$, equation (19) can be simplified to

$$\beta_V = \varepsilon(V,S)\beta_S + \varepsilon(V,X)\beta_X \tag{19'}$$

where $S = S_1 + S_2$.

Equation (19) presents a project's systematic risk as a weighted average of the risk associated with the intended use of the assets and the risk of the alternative uses of the assets.[8] As the preferability of the intended use of the assets over the alternatives grows more pronounced and the likelihood of abandonment grows smaller, the beta of the revenues receives heavier weighting while the beta of the abandonment value receives less weight (and vice versa).

A measure of the operating leverage can be obtained by taking the ratio β_V/β_S. As a result of the elasticities summing to unity, it can be seen by simple inspection of (19') that when $\beta_X<\beta_S$, the risk of the project will be less than the risk of the earnings, and the measure of operating leverage will be less than one. If $\beta_X>\beta_S$, the risk of the project would be greater than the risk of the earnings, and the measure of operating leverage would be greater than one. This measure of operating risk is similar in objective, although superior in sophistication, to the well-worn but conceptually weak degree of operating leverage (D.O.L.) still presented in many corporate finance texts. The D.O.L., which is the elasticity of operating income with respect to sales, is weakened by its dependence on accounting data and its focus on earnings within a single accounting period, as well as its assumptions of linearity in product price and variable costs. The operating risk measure here proposed is concerned with economic rather than accounting data, and spans the life of the project. As with D.O.L., the beta-based measure of the degree of operating risk for the firm is a weighted average of the measures for the firm's individual projects.

Furthermore, even if β_1, β_2 and β_X are stationary, β_V will not be. Galai & Masulis (1976) and Clifford Smith (1979) have already argued that even if the beta of the firm is stationary, the beta of equity will not be. Their arguments, along with the argument just presented, are anathema for those trying to measure risk-adjusted return in order to measure portfolio performance, to test capital market efficiency, or to make any test involving cumulative average residuals.

In addition, the intuitive belief that greater flexibility of assets leads to lower risk is reinforced in that high flexibility would translate into a low beta for the abandonment value, and thus would mitigate the risk of the project, especially at the outset.

6. Shortcomings of the Two-Period Model for Application to the Capital Investment Decision

Perhaps the single most annoying assumption made here is that the abandonment decision must be made on a specified day, not before or after. In a real investment project, the abandonment option can be exercised at any time, and there is no certain expiration date. The two obstacles are the problem of valuing the early exercise privilege of the American put and the problem of allowing for an uncertain (although finite) expiration date. The former has been addressed, but the technique is computationally difficult. Given simple reliable solutions to these two problems, a model of the project with abandonment option would be nearly ideal. The two-period model presented here; however, may have a tendency to undervalue the abandonment option, and so err in the direction of rejecting projects that should be accepted. Nevertheless, it is a definite improvement over existing discounted cash flow techniques, as discussed in section 3. Although not perfect, the two-period model is relatively easy to apply, even with hand-held programmable calculators, and produces considerable informational benefits for a low computational cost. To illustrate, a numerical example is presented as an appendix to the paper.

7. Summary

This paper pursued the explicit consideration as an option of the possibility of abandoning an investment project while the assets still can be put to other uses. This approach to project abandonment is superior to past approaches which simply entered abandonment values, weighted by the probability of abandonment, into the estimate of NPV. The option approach was shown to be able to capture important dimensions of the capital investment project which have heretofore been treated as intangibles: flexibility and durability of the assets used, as well as the innovativeness of the use to which the assets are being put in the project.

An additional benefit of the abandonment option model is that it clearly places the choice of production technology (that is, the choice of how much operating risk to bear) directly within the capital investment decision. This point was discussed in Section 4.

The option approach to project abandonment also allowed new insight into operating risk. It was shown that the systematic risk of a project is a weighted average of the systematic risk of the future earnings stream and the systematic risk of the abandonment value, which represents the value of alternative uses for the assets. Furthermore, it was demonstrated that the beta for the project is non-stationary, and vibrates around a trend through time. It can clearly be seen within this approach that a project which ventures into an area with very uncertain earnings streams, and even a high probability of failure, is not necessarily a high risk. If the assets used were flexible, with a fairly stable value, the project's risk would actually be low. An example would be a venture into the commuter airline business on a new route with very uncertain demand and a good chance of failure. The entrepreneur purchasing general aviation aircraft to pursue the venture would be taking considerable risk with regard to the earnings from the specific use to which the airplanes were being put, but would be heavily protected on the downside by the ready market for the airplanes should the project prove unrewarding.

Perhaps the ideal innovative investment project is one which puts readily available "off-the-shelf" assets which are durable and have many alternative uses into a new, potentially profitable but unproven use which is unrelated to alternative uses. The beauty of the option approach is that it can capture much of the market value of all these considerations. It is an exciting addition to the tools of corporate finance.

Notes:

1. Interestingly, Bonini found a positively skewed distribution for project net present value with abandonment (see his figure 2, p. 52). The distribution he presents is very similar to the truncated distribution of net option payoffs.

2. Consider two portfolios: A, which contains the stock with a put, and B, which contains a call (with the same terms as the put) plus a bond paying the exercise price on the expiration date of the option. At expiration, the value of the two portfolios will be the same, regardless of whether the value of the stock, S^*, is above or below the exercise price. If $S^* < X$, the put in A would be exercised to sell the stock, yielding X, while in the case of B the call would expire valueless and the proceeds of the bond, X, retained. If $S^* > X$ the put would expire valueless leaving the value of A equal to S^*; and the proceeds of the bond would be used to exercise the call, leaving the value of B equal to S^* also.

3. The difference between equation (11) and Fischer's result lies in that α_X enters with a positive sign. The same effect could have been achieved by following Fischer exactly, placing a positive sign on α_X in equation (5) and letting it enter equation (11) with a negative sign. Noting that α_X has a negative value in the case of downward drift, we would then be subtracting a negative and Fischer's formulation would be as follows:

$$C(S,X,t) = S \cdot N\{d_1\} - Xe^{-(r_h-(-\alpha_X))t} \cdot N\{d_2\}$$

$$d_1 = \{\ln(S/X) + [r_h - (-\alpha_X) + (\hat{\sigma}^2/2)]t\}/\hat{\sigma}\sqrt{t}$$

As $\hat{\sigma}^2$ is the variance rate for the ratio S/X, it remains unchanged from Fischer's presentation.

4. Consider two spreads, one composed of options on stock A, the other composed of options on stock B. A spread, which consists of simultaneously holding a put and a call with the same terms, earns a risk premium only if the value of the stock at expiration is outside the range defined by $X \pm (P+C)e^{rt}$. Therefore, someone holding a spread is betting that the stock price will move; in other words, betting on the volatility of the stock. The more volatile the stock, the more likely the spread will pay off. Thus if stock A were more volatile than stock B, the spread on A would be more valuable than the spread on B. Since a European put can be written as a portfolio of call, stock, and bond (see Merton [1973]), the European spread can also be stated entirely in terms of call, stock, and bonds with $P + C = 2C + e^{-rt}X - S$. Thus it can be seen that as the value of the spread increases, so does the value of the European call. An American call is always at least as valuable as a European call (again see Merton [1973]); hence even without being specific about the precise stochastic process followed by stock returns, the volatility argument can be made.

5. The point can be proven as follows:

To show that $V > S_1 + S_2$, it is sufficient to show that $S_2 + P(S_2,X,t) > S_2$. Consider the payoffs for two portfolios: A, composed of S_2 alone, and B, composed of S_2 plus $P(S_2,X,t)$. The payoff for various terminal values of S_2 is given below:

	Value of S_2 at expiration	
	$S_2^* < X$	$S_2^* \geq X$
V_A	S_2	S_2
V_B	X	S_2
	$V_A < V_B$	$V_A = V_B$

Since there are some states of the world in which the terminal value of B will be greater than that of A, the value of B prior to expiration should be greater than the value of A, to avoid dominance.

6. Rubinstein (1973) and Lev (1974) addressed the influence of "operating leverage" upon beta. Subrahmanyam & Thomadakis (1980) stated a succinct criticism of that work, quoted here: "This specification is not very enlightening, however, because it fails to relate the optimal choice of margin (i.e., of factor mix and price) to the uncertainty of output."

7.
$$\Psi = \frac{\partial C}{\partial t} + 1/2\left[\frac{\partial^2 V}{\partial S_1^2} S_1^2\sigma_1^2 + \frac{\partial^2 V}{\partial S_2^2} S_2^2\sigma_2^2 + \frac{\partial^2 V}{\partial X^2} X^2\sigma_X^2 + 2\,\frac{\partial^2 V}{\partial S_1 \partial S_2} S_1 S_2 \sigma_1 \sigma_2 r_{12}\right.$$
$$\left. + 2\,\frac{\partial^2 V}{\partial S_1 \partial X} S_1 X \sigma_1 \sigma_X r_{1X} + 2\,\frac{\partial^2 V}{\partial S_2 \partial X} S_2 X \sigma_2 \sigma_X r_{2X}\right]$$

8. The Black-Scholes formulation can be called upon to show that the weights sum to one. Referring to equation (12):

$$V = S_1 + S_2\, N\{d_1\} + Xe^{-(r_h+\alpha_X)t} \cdot N\{-d_2\}$$

$$\frac{\partial V}{\partial S_1} = 1,\quad \frac{\partial V}{\partial S_2} = N\{d_1\},\quad \frac{\partial V}{\partial X} = e^{-(r_h+\alpha_X)t} \cdot N\{-d_2\}.$$

Thus:

$$\frac{\partial V}{\partial S_1}\frac{S_1}{V} + \frac{\partial V}{\partial S_2}\frac{S_2}{V} + \frac{\partial V}{\partial X}\frac{X}{V} = \frac{S_1}{V} + \frac{S_2 \cdot N\{d_1\}}{V} + \frac{Xe^{-(r_h+\alpha_X)t} \cdot N\{-d_2\}}{V}$$

$$= \frac{S_1 + S_2 \cdot N\{d_1\} + Xe^{-(r_h+\alpha_X)t} \cdot N\{-d_2\}}{V} = 1$$

Appendix: Numerical Example

Project cost = \$4,220,000

S_1 = \$1,000,000

S_2 = \$3,000,000

r = 3%

b = .5%

α_X = 5%

σ_2 = .10

σ_X = .04

$\rho_{2,x}$ = .1

$\hat{\sigma}^2$ = .0108

X = \$3,500,000

t = 1 year

Applying equation (12)

$$V = S_1 + e^{-(r+b+\alpha_X)t}\, X \cdot N\{-d_2\} + S_2\, N\{d_1\}$$

$$d_1 = \frac{\ln(S_2/X) + [r + b + \alpha_x + (\hat{\sigma}^2/2]t}{\hat{\sigma}\sqrt{t}}$$

$$d_2 = d_1 - \hat{\sigma}\sqrt{t}$$

Substituting example values:

$$d_1 = \frac{\ln(3/3.5) + [.03 + .005 + .05(.0108/2]1}{\sqrt{.0108}\sqrt{1}} = -\ .6134$$

$$d_2 = d_1 - \sqrt{.0108}\sqrt{1} = -\ .7174$$

$$N\{d_1\} = .2698$$

$$N\{d_2\} = .2366$$

$$V = \$1,000,000 + e^{-.085}\ \$3,500,000(1 - .2366) + \$3,000,000(.2598)$$

$$V = \$4,263,573$$

$$NPV = \$4,263,573 - \$4,220,000 = \$43,573$$

Isolating the value of the abandonment option:

$$P(S_2,X,t) = -S_2 + C(S_2,X,t) + e^{-(r+b+\alpha_X)t}\ X$$

$$P(S_2,X,t) = -S_2 + S_2 \cdot N\{d_1\} - e^{-(r+b+\alpha_X)t}\ X \cdot N\{d_2\} + e^{-(r+b+\alpha_X)t}\ X$$

$$P(S_2,X,t) = \$3,000,000(.2698 - 1) + e^{-.085}\ \$3,500,000(1 - .2366)$$

$$P(S_2,X,t) = \$263,573$$

(Calculations accomplished with a Texas Instruments TI-59 programmable calculator)

Comments:

Without consideration of the abandonment option, the NPV calculated for this example would be -$220,000, indicating rejection. Even with the expected abandonment value included in the cash flow stream with 100% probability of abandonment, the NPV would still be negative. With abandonment treated as an option, however, the NPV is significantly above zero, and the project is revealed as being attractive. For purposes of the example flexibility, durability, and innovativeness were given high values through X, α_X, σ_2, $\rho_{2,X}$ and b. Abandonment value was high, wear rate low, total risk of earnings high, correlation of project earnings with alternatives low, and systematic risk premium for abandonment value low. The abandonment option in the example, as a result, is a very valuable free benefit, the capitalization of which creates wealth in excess of the amount invested when added to the capitalized value of expected net future real earnings.

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