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Three Essays in Industrial Organization

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THREE ESSAYS

IN

INDUSTRIAL ORGANIZATION

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in
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This dissertation includes two papers about issues in two-sided markets and one paper talking about Hierarchical Stackelberg model.

Chapter 1 analyzes endogenous horizontal product differentiation in a two-sided market with two platforms based on a two-stage Hotelling model. When the two sides are both single-homing, the platforms will choose maximal product differentiation in the equilibrium. However, when one side is multi-homing, and the multi-homing side views the platforms indifferent, the platforms choose minimal product differentiation in the equilibrium.

Chapter 2 investigates horizontal mergers in a two-sided market between three horizontally differentiated platforms. This chapter provides a theoretical analysis of impacts on price and welfare based on merger cost savings and cross-group externalities. The existence of the cross-group externalities weakens the role of the strong cost savings playing in the price decline after the merger. Consumer welfare may increase even when the prices of the merged platform increase. If the platforms have different costs, a merger between the less efficient platforms can lead to higher consumer surplus than a merger between the more efficient platforms. However, the latter generates higher social welfare.

Chapter 3 investigates the equilibrium of a hierarchical Stackelberg oligopoly model that firms choose output sequentially with possibly heterogeneous firms. This chapter also incorporates the comparisons of the equilibrium outcomes between the Hierarchical Stackelberg model and the standard Cournot model. The results demonstrate that the equilibrium prices

in the hierarchical Stackelberg model with heterogeneous firms are lower regardless of entry sequences. The total welfare loss relative to the optimal social situation with the most efficient entry sequence in the hierarchical Stackelberg model is lower than that in the Cournot model.

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This is dedicated to my parents and my cat for all the love they gave to me.

Chapter 1

PRODUCT DIFFERENTIATION IN TWO-SIDED MARKETS

1.1. Introduction

Two-sided markets refer to the markets with two distinct sides of agents, and each side values the interactions with the other side. Two-sided markets are prevalent in real life, such as computer operating systems, payment schemes, media markets, and search engines.[1] Typically, consumers cannot obtain great value without a platform in such markets. For example, advertisers benefit from TV channels to reach and gain the attention of a greater audience and advertising through those platforms. Google or Facebook, allows firms to identify better and target their potential clients. Hence, in two-sided markets, products usually are the service that platforms provide to let different sides of users meet or trade on them and consumer surplus is generated when the two sides have interaction with each other. The externalities that benefit to one side originate in the number of participants from the other side. Therefore, this cross-group externalities can be viewed as a significant characteristic of two-sided markets. For instance, video game developers value video game consoles more if there are more video game players and vice versa. For men, the value of a heterosexual dating club increases with the number of women in the club and vice versa. Cardholders value a payment card more if the more merchants have a point-of-sale terminal that accepts that payment card. Thus the markets for video consoles, heterosexual dating clubs, and payment cards are two-sided markets characterized by two positive cross-group externalities.[2]

[1] Rochet and Tirole (2003,2006) and Evan (2003) provide extensive examples of two-sided markets.

[2] Negative cross-group externalities occur instead when the value obtained by one side of customers decreases with the number of customers on the other side. For instance, although advertisers are likely to value a TV channel more if the more viewers it has, the viewers are generally annoyed by TV advertising. I focus on two-sided markets characterized by two positive cross-group externalities.

The purpose of this chapter is to explore the endogenous choice of platforms on horizontal product differentiation in a two-sided market. In reality, platforms that are in the same industry usually provide very similar services or products to consumers. For instance, Master card and Visa provide comparable payment services to consumers; Uber and Lyft are usually treated as substitute riding services. However, there still exists some extent of product differentiation among those similar services. For instance, Lyft allows customers to reserve a ride up to 7 days in advance, and Uber lets customers schedule a ride up to 30 days in advance; Uber serves hundreds of cities in ten countries, while Lyft operates only in the United States. The general reason that platforms choose to develop different features of their products/services is to avoid intensive price competition against their rivals so that they can keep their loyal consumers who appreciate their distinctive features. However, there are also some examples that platforms which are used to provide very distinct services choose to develop similar services to compete with its rivals directly. For instance, Meituan, an original Chinese food delivery application, announced to establish its car-hailing service; and DiDi Chuxing, a car-hailing application, also decided to begin its meal delivery business recently. These two companies, which used to exert market powers in their own industries, now choose to compete directly in the two sectors by providing similar services. Therefore, what is the optimal degree of product differentiation that multiple platforms should provide under fierce price competition is a question arising from the above situations.

Economists usually use the two-stage Hotelling linear city model to investigate the endogenous product differentiation problem. In traditional one-sided markets, by locating at the endpoints of the unit interval, two firms will choose maximal product differentiation when they enter into a market to avoid intensive price competition. In other words, the gain from the increasing demand of a firm cannot dominate the loss from the price decline due to fierce competition by providing relatively similar products. Given that two-sided markets are different from traditional one-sided markets because of cross-group externalities, we need to confirm whether the logic of horizontal product differentiation in traditional one-sided markets can also be directly applied to two-sided markets as well. In two-sided markets, the increasing participation on one side in a platform will attract more participation on the other side to join this platform due to the cross-group externalities. If a platform decides to provide more similar service as the rival's, the effect of increasing demand is supposed to be stronger in the two-sided market than that in traditional one-sided markets. Hence the chapter aims to investigate what is the choice of product differentiation in the two-sided market; in other words, what are the impacts that cross-group externalities have on horizontal product differentiation problem. To be specific, the research questions are that when two platforms enter and compete with each other in a two-sided market, what choice of endogenous product differentiation will the platforms make and what role that cross-group externality will play in the equilibrium.

The results demonstrate that the choices of product differentiation are different if the restrictions on agents in the two-sided market are different. When the agents are only allowed to single-home (agents in the market can only join at most one platform), the two platforms in the two-sided market will choose the maximal product differentiation at the first stage, which is consistent with the conclusions in traditional one-sided markets. However, Two results in the two-sided market resulted from the cross-group externalities cannot be obtained in one-sided markets. First, when the rival is located at a location in the unit interval of Hotelling model, a platform can take over the entire market by choosing a specific location, which means a platform can steal the loyal agents from the competitor due to the cross-group externalities. Secondly, unlike the situation in a one-sided market that the profit of a firm is always decreasing as the firm locates closer to its rival, in the two-sided market, the profit of the platforms may increase as locating closer to the rival. In other words, the effect of increasing demand can dominate price competition at some point in the two-sided market, but the former is always dominated by the latter in traditional one-sided markets.

When the agents are allowed to multi-home (I focus on the situation that one side is single-homing, and the other side is multi-homing), the original model is modified that the multi-homing side views the platforms homogeneous, the single-homing side treats the platforms differently, and the platforms choose product differentiation to the single-homing side. The result reveals that the two platforms will choose minimal product differentiation in the equilibrium. The platforms have to compete on the single-homing side so intensively that they will offer free service to this side. Otherwise, the platforms will lose all the multi-homing side and then lose part of the single-homing side as well. Hence, with free service to the sing-homing side, the platforms choose to provide very similar products/services to strive for the maximal market share.

Besides, in each situation being discussed, the social optimal conditions are identical that one platform locates at $\frac{1}{4}$ and the other locates at $\frac{3}{4}$. Thus, for the equilibrium that the two platforms choose to maximize product differentiation by locating at the two endpoints of the unit interval, there is too much product differentiation from the perspective of social planner; and for the equilibrium that the platforms choose to minimize product differentiation by locating at $\frac{1}{2}$, there is too little product differentiation. In the last section, there are several extensions of the single-homing case being discussed.

1.1.1. Related Literature

The analysis builds on the burgeoning literature on platforms competition in two-sided markets. Caillaud and Jullien (2003) analyze a model of imperfect price competition between intermediaries service provider. They investigate how competition between two intermediaries may lead to different market structures in equilibrium (dominant platform or market sharing equilibria). They also demonstrate that divide-and-conquer pricing strategies may solve the chicken-and-egg coordination problem, thus explaining the unbalanced pricing structure. Armstrong (2006) studies the platforms competition with single-homing restrictions in two-sided markets based on the Hotelling model. He uses the classical one-stage Hotelling model to analyze price competition in a two-sided market that two platforms which are exogenously determined to locate at the two endpoints of a unit interval decide the prices to the agent in the market. By assuming exogenous maximal product differentiation on both sides, he concludes that the model with two single-homing sides has a unique symmetric equilibrium. Armstrong and Wright (2007) focus on the combination of the two above cases–Armstrong (2006) and Caillaud and Jullien (2003), where one side views the platform as homogeneous, while the other views the platforms as heterogeneous. Moreover, they also consider the use of exclusive contracts that prevent the agents from multi-homing.

However, all the above literature assumes there is no endogenous product differentiation on either side of the market, and the amount of related literature on endogenous product differentiation in two-sided markets is very scant. Several papers discuss vertical product differentiation in two-sided markets, such as Gabszewicz and Wauthy (2014). They demonstrate that in a market with membership externality, the prices set by platforms elicit participation on either side and thereby simultaneously determine platform quality.

To the best of my knowledge, this paper is the first one to study the endogenous horizontal product differentiation in two-sided markets.

The chapter is organized in the following way. Section 1.2 illustrates the primary model of the two-stage Hotelling game in a two-sided market. Section 1.3 presents the subgame Nash equilibrium of single-homing situation. Section 1.4 describes the equilibrium of multi-homing situation. Section 1.5 is the extension part. All proofs are in the Appendix.

1.2. The Model

The basic model is originated from Armstrong (2006). A two-sided market is comprised of agents on two sides and two platforms.

Agents. All the agents in the two-sided market belong to two sides, side S and B. I refer to the agents on the side S as sellers and the agents on the side B as buyers. Both the sellers and buyers are consumers of the platforms in the two-sided market. The agents on one side are willing to participate in a platform to approach the agents on the other side on the platform.[3] Let all the agents uniformly distributed along each side of the market. They all have inelastic demand, consuming at most one unit of the good/service.[4] The sellers and buyers in the market obtain the respective utilities u_i^S, u_i^B if they join platform i ($i = 1, 2$).

[3] To focus on the cross-group externalities, I rule out the possibility that the agents care about the presence on the same side on the platform.

[4] In this section, I restrict single-homing condition to all the agents that they are only allowed to join one platform in the equilibrium. This restriction will be relaxed in the section of multi-homing.

Their utilities u_i^S, u_i^B are determined in the following way:

$$u_i^S = v_S + \alpha_S n_i^B - p_i^S - t_S(d_i^S)^2; \quad u_i^B = v_B + \alpha_B n_i^S - p_i^B - t_B(d_i^B)^2. \tag{1.1}$$

v_S, v_B are the intrinsic valuations assumed to be platform independent, and they are large enough that the entire market can be fully covered. p_i^S, p_i^B are the respective prices charged by platform i to the two sides. Thus I focus on the platforms that charge fix payments to agents.[5] n_i^S and n_i^B indicate the amounts of the sellers and buyers participating in platform i. α_S (α_B) is the cross-group externality that the extra benefit that a seller (buyer) can obtain when joining platform i if one more buyer (seller) joins this platform. For simplicity, I assume that α_S and α_B do not depend on which platform an agent participates in but only on which side the agent is on. t_S and t_B are the transportation costs to the sellers and buyers which can be considered as the product differentiation parameters for the two sides that describe the competitiveness of the market.[6] d_i^S and d_i^B are the distance that a participating agent locates from platform i.[7]

Platforms. Platforms in two-sided markets can be considered as firms in traditional markets which provide products or services to agents. I assume that the two platforms are competing in the two-sided market. Notice that the "product" mentioned in the chapter refers to the services that the platforms provide to the buyers and sellers, not the transactions that may occur between the sellers and buyers. The platforms earn profits by charging fixed membership fees. By assuming that the platforms do not incur any cost for attracting the agents for simplicity, the profits of platform i are

$$\pi_i = n_i^S p_i^S + n_i^B p_i^B.$$

[5] I assume prices being non-negative. This is a reasonable assumption for many examples of two-sided markets that people pay to participate in the platform, such as TV channels or newspapers. Platforms may charge free or even subsidy to one group, like the yellow page, but there may be the adverse selection and moral hazard problems.

[6] The transportation cost can be considered as the physical costs of reaching the platform, costs of signing up for a service, or the initial set-up costs that consumers face for learning about a new product or service.

[7] I apply quadratic form for measuring transportation costs in the utility equations since the linear form would lead to no equilibrium in the two-stage Hotelling game.

The timeline of the game is as the following: at the first stage, which is also called the locations game stage, the two platforms determine the horizontal product differentiation level by choosing their locations a and $1 - b$ simultaneously in the unit interval;[8] at the second stage, the two platforms determine their prices to the agents on the two sides.

1.3. Single-homing

In this section, I focus on investigating the subgame perfect equilibrium of the single-homing situation, in which all the agents are only allowed to join at most one platform in the market. To explore the subgame perfect equilibrium of this two-stage game, I first need to elicit the optimal prices that the two platforms charge to the two sides at the second stage according to the backward induction.

1.3.1. Price Equilibrium

According to the Hotelling linear city model, the demand functions of the two platforms on each side are,

$$n_1^S = \frac{u_1^S - u_2^S}{2t_S(1-b-a)} + \frac{1-b+a}{2}; \quad n_1^B = \frac{u_1^B - u_2^B}{2t_B(1-b-a)} + \frac{1-b+a}{2}$$

$$n_2^S = \frac{u_2^S - u_1^S}{2t_S(1-b-a)} + \frac{1-a+b}{2}; \quad n_2^B = \frac{u_2^B - u_1^B}{2t_B(1-b-a)} + \frac{1-a+b}{2}.$$

Substituting u_1,u_2 with Eq.1.1, and inserting $n_2^S = 1 - n_1^S, n_2^B = 1 - n_1^B$, the implicit functions of the market shares of platform 1 are

$$n_1^S = \frac{\alpha_S(2n_1^B - 1) + (p_2^S - p_1^S)}{2t_S(1-b-a)} + \frac{1-b+a}{2} \tag{1.2}$$

[8] The distance between the two locations can be considered as the product differentiation degree they choose. The more far away they decide to locate from each other, the higher product differentiation they will impose. I restrict that $0 \le a \le 1-b \le 1$ without loss of generality; Without the restriction $a < 1-b$, the equilibrium has a symmetric counterpart. This is merely a question of labeling the firms.

$$n_1^B = \frac{\alpha_B(2n_1^S - 1) + (p_2^B - p_1^B)}{2t_B(1-b-a)} + \frac{1-b+a}{2} \quad (1.3)$$

$$n_2^S = \frac{\alpha_S(2n_2^B - 1) + (p_1^S - p_2^S)}{2t_S(1-b-a)} + \frac{1-a+b}{2} \quad (1.4)$$

$$n_2^B = \frac{\alpha_B(2n_2^S - 1) + (p_1^B - p_2^B)}{2t_B(1-b-a)} + \frac{1-a+b}{2}. \quad (1.5)$$

If $p_2^B = p_1^B$, according to Eq.1.3, $n_1^B = \frac{\alpha_B(2n_1^S-1)}{2t_B(1-b-a)} + a + \frac{1-b-a}{2}$. a is the captive agents or loyal agents to the backyard of platform 1, and $\frac{1-b-a}{2}$ is half of the agents between a and $1-b$ which is the competition zone. These features are identical as the two-stage Hotelling model in traditional one-sided markets. Besides, Eq.1.3 indicates that an extra seller joining on this platform will attract further $\frac{\alpha_B}{t_B(1-b-a)}$ buyers to participate if keeping its price to the buyers fixed. This is the role that the cross-group externalities play in the market. Moreover, the less product differentiation that the platforms choose to provide,[9] the much more significant the effect of the cross-group attraction will be. If the two platforms provide relatively similar products/services, the agents will put more weight on the participation of the other side in the platform instead of their product tastes when considering joining this platform. On the contrary, if the two platforms provide distinct products/services to the agents, some agents may choose the platform which provides the product that is more close to their taste even though the platform may have fewer agents on the other side.[10]

Assumption 1.1 $\alpha_S = \alpha_B = \alpha$, $t_S = t_B = t$.

To have tractable and straightforward results, I assume that the two sides are identical in terms of the cross-group externalities and the transportation costs.

[9] Namely $1-b-a$ is smaller, which means the two platforms choose to locate closer toward each other in the Hotelling unit interval.

[10] For example, if two game platforms offer the same game genres, say adventure game, game players would like to play on the platform which provides a more significant amount of the adventure game. If the two platforms offer the different game genres, some players will pick the platform which provides the game genre they prefer to play.

By solving the simultaneous Eq.1.2 to 1.5, the market shares of the two platforms are

$$n_1^S = \frac{\alpha(p_2^B - p_1^B) + t(1-b-a)(p_2^S - p_1^S)}{2[t^2(1-b-a)^2 - \alpha^2]} + \frac{t(1-b-a)(1-b+a) - \alpha}{2[t(1-b-a) - \alpha]}$$

$$n_1^B = \frac{\alpha(p_2^S - p_1^S) + t(1-b-a)(p_2^B - p_1^B)}{2[t^2(1-b-a)^2 - \alpha^2]} + \frac{t(1-b-a)(1-b+a) - \alpha}{2[t(1-b-a) - \alpha]}$$

$$n_2^S = \frac{\alpha(p_1^B - p_2^B) + t(1-b-a)(p_1^S - p_2^S)}{2[t^2(1-b-a)^2 - \alpha^2]} + \frac{t(1-b-a)(1-a+b) - \alpha}{2[t(1-b-a) - \alpha]}$$

$$n_2^B = \frac{\alpha(p_1^S - p_2^S) + t(1-b-a)(p_1^B - p_2^B)}{2[t^2(1-b-a)^2 - \alpha^2]} + \frac{t(1-b-a)(1-a+b) - \alpha}{2[t(1-b-a) - \alpha]}.$$

Assumption 1.2 *t is sufficiently larger than α.*

The brand preference t must be larger than the cross-group effect α to makes sure that no single platform obtains the whole market in the equilibrium.[11] So, in this case, t must be larger than α sufficiently to make sure that $t(1-b-a) - \alpha > 0$ holds in the equilibrium.[12] In addition, the profit functions are concave if and only if $t(1-b-a) - \alpha > 0$ holds.

The equilibrium prices with the first order conditions of the platforms' profit maximization are

$$p_1^S = p_1^B = p_1(a,b) = \frac{t(1-b-a)(3+a-b)}{3} - \alpha$$

$$p_2^S = p_2^B = p_2(a,b) = \frac{t(1-b-a)(3+b-a)}{3} - \alpha.$$

[11] If the cross-group externalities are larger than the brand preferences, the agents will be much easier to be attracted to one platform which has more agents on the other side on, and ignore their original brand preference, which leads to the consequence that there could only exist equilibrium that one platform owns the entire market on both sides.

[12] According to the demand functions, the number of agents on each side in one platform, for instance, n_1^S, decreases with the price that they have to pay, p_1^S, and decreases with p_1^B, the price that the other side has to pay to this platform, which corresponds to the law of demand. If the cross-group externalities are stronger or the brand preference is weaker, e.g., $t_B(1-b-a)^2 - \alpha_B < 0$, the number of agents on one platform will be increasing with the prices. Hence the market would naturally turn to the monopoly.

The first part of the equilibrium price, which is $\frac{t(1-b-a)(3+a-b)}{3}$, is the expression of the equilibrium price of firms in a traditional one-sided market with the two-stage Hotelling linear model. Therefore, the only difference of the equilibrium prices between one-sided markets and two-sided markets is that the platforms remove the cross-group externalities out of the prices in two-sided markets, which means the platforms return the cross-group externalities to the agents under price competition.

Thus the market shares of platform 1 and 2 are

$$n_1^S = n_1^B = n_1(a,b) = \frac{t(1-b-a)[1+\frac{1}{3}(a-b)]-\alpha}{2[t(1-b-a)-\alpha]} \tag{1.6}$$

$$n_2^S = n_2^B = n_2(a,b) = \frac{t(1-b-a)[1-\frac{1}{3}(a-b)]-\alpha}{2[t(1-b-a)-\alpha]}. \tag{1.7}$$

Lemma 1.1 presents how the prices and market shares change with the cross-group externalities when the two platforms are located at a and $1-b$ in the unit interval.

Lemma 1.1 *Consider a two-sided market with two platforms competing in a two-stage Hotelling model by determining their locations in a unit interval, given locations a and $1-b$,*

(i) the equilibrium prices of the two platforms decrease with the cross-group externalities, i.e., $\dfrac{dp_i(a,b)}{d\alpha} < 0$, $(i = 1, 2)$;

(ii) the equilibrium market shares of the platforms increase with the cross-group externalities if the platform relatively locates further from the endpoint than the opponent platform does in the unit interval, i.e., $\dfrac{dn_1(a,b)}{d\alpha} \geq 0$ and $\dfrac{dn_2(a,b)}{d\alpha} \leq 0$ if $a \geq b$; $\dfrac{dn_1(a,b)}{d\alpha} < 0$ and $\dfrac{dn_2(a,b)}{d\alpha} > 0$ if $a < b$.

The equilibrium results demonstrate that the prices always decrease with the cross-group externalities regardless of product differentiation. If α is getting larger, which means that the extra benefit of the buyers (sellers) enjoying when one more seller (buyer) participates in the platform is becoming more significant, the incentive of the buyers (sellers) to join platform i will become stronger. Then the platforms would like to lower their price to the sellers (buyers) to attract more of them as well as obtaining more buyers (sellers).[13]

Part (ii) indicates that n_1 increases with α only when $a > b$ holds and decreases with α if the opposite. The number of agents on the two sides in a platform increases with the cross-group externalities only when the platform has more captive or loyal agents in its backyard. Otherwise, it will decline with the cross-group externalities. In the two-sided market, the number of agents on one side on the platform has a significant impact on the utilities of the agents on the other side. If $b > a$, platform 1 does not have enough loyal agents on both sides to make itself attractive to other agents. Thus the other agents are more willing to participate in platform 2 when the cross-group externalities are larger, and platform 2 will obtain more share of the market from platform 1.

With the functions of the equilibrium prices and demands, the profits of the two platforms are,

$$\pi_1(a,b) = n_1^S p_1^S + n_1^B p_1^B = \frac{[t(1-b-a)(1+\frac{1}{3}a-\frac{1}{3}b)-\alpha]^2}{t(1-b-a)-\alpha}$$

$$\pi_2(a,b) = n_2^S p_2^S + n_2^B p_2^B = \frac{[t(1-b-a)(1+\frac{1}{3}b-\frac{1}{3}a)-\alpha]^2}{t(1-b-a)-\alpha}.$$

Next, it is needed to clarify the optimal choices of the locations which also can be interpreted as the endogenous product differentiation choices of the two platforms.

1.3.2. Locations Game

I focus on the analysis of the best responses of platform 1 given the location $1-b$ of platform 2. The analysis of platform 2 is similar due to the symmetric character in the Hotelling model which can be omitted.

According to the last section, the prices, marker shares and profit functions of platform 1 in the equilibrium, given the rival's location choice $1-b$, are the following,

[13]Literally it can charge a higher price to the other side to compensate for the loss from undercutting the price of one side. Caillaud and Jullien (2003) provide and discuss the idea of "divide and conquer," that platform will charge a lower price to or even subside one side to steal more agents from its opponent, then compensate from charging a higher price to the other side to make profits.

$$p_1(a,b) = t(1-b-a)[1+\frac{1}{3}(a-b)] - \alpha$$

$$n_1(a,b) = \frac{t(1-b-a)[1+\frac{1}{3}(a-b)] - \alpha}{2[t(1-b-a)-\alpha]}$$

$$\pi_1(a,b) = \frac{\{t(1-b-a)[1+\frac{1}{3}(a-b)] - \alpha\}^2}{t(1-b-a)-\alpha}.$$

Lemma 1.2 *If the cross-externality does not exist, that $\alpha = 0$, the equilibrium outcome of a two-sided market coincides with that of a one-sided market.*

According to the above equations, if $\alpha = 0$, a two-sided market turns into two independent traditional markets that each has a side of consumers and two firms. According to Eq.1.1,

$$u_1^S = v_S - p_1^S; \quad u_1^B = v_B - p_1^B.$$

The agents on each side only consider the price when determining to participate in a platform. In other words, platform 1 and 2 compete through the Hotelling model in two independent and separate traditional markets S and B. The equilibrium prices and the market share of the agents of each platform are just equivalent to that of one-sided markets.

Back to the situation that the cross-group externalities are positive. With the equilibrium outcome $p_1(a,b)$ and $n_1(a,b)$, we can obtain the optimal location that platform 1 will choose to achieve the maximal profit in the competition. Recall that in the traditional Hotelling model, given the location choice of the rival, the profit of firm 1 is always decreasing with its location choice a, which measures the distance between the location of firm 1 and the endpoint 0. Therefore, firm 1 should choose the endpoint 0 to locate. In order to explore the optimal locations of the platforms, we need to clarify the available location choices for each platform and how their profits change with these choices.

Proposition 1.1 *Consider a two-sided market with two platforms competing in the two-stage Hotelling linear city model by choosing locations in a unit interval. There exists a* *location for a platform that it can obtain the entire two-sided market, given the rival's location.*

Given the location $1-b$ of platform 2, there exists a location a^*, and $a^* > b$, such that platform 1 can steal the entire market share with positive profits. In other words, platform 1 can become the sole incumbent platform in the market given the rival's location choice. Therefore, the available moving range of platform 1 in the unit interval is $[0, a^*]$.

It is not possible in the traditional one-sided market with the Hotelling model to exclude the competitor. In the one-sided market, the market share of firm 1 is $n_1 = \frac{1}{2} + \frac{a-b}{6}$, given the location choices a and $1-b$. If $a = b$, the two firms split the market into half; if $a > b$, the firm with location a earns more than half. However, no matter what the location that the firm with location a is in the equilibrium, it cannot gain the whole market since $0 < \frac{1}{2} + \frac{a-b}{6} < 1$ always holds. In other words, firm 2 can always keep some loyal agents regardless of firm 1's decisions.

Things are different in the two-sided market. At a particular location a^*, platform 1 can get enough larger share of the market on both sides. The larger share of sellers will make more buyers want to deviate from platform 2 to platform 1, and the larger share of the buyers will make more sellers join 1 and so on so forth. Then platform 1 can achieve the entire market at this location a^*. Therefore, if the competitor does not have options to determine the product differentiation, an entrant with some superiority can choose to exclude the existing competitor and become the sole incumbent in the two-sided market.[14]

Then we can have the optimal location of platform 1 as long as figuring out how the profit of platform 1 change within the range $[0, a^*]$.

Proposition 1.2 *Consider a two-sided market with two platforms competing in the two-stage Hotelling linear city model by choosing locations in a unit interval. Given the rival's location, if the cross-group externalities are strong enough, the profit of a platform will decrease first then increase as locating closer to its rival; otherwise, the profit of the platform*

[14] For instance, the action that Amazon, the typical two-sided platform, acquired Whole Foods company can be considered as the aggressive approach to its retailing rivals.

always decreases as locating closer to its rival.

The change of a platform 1's profit within the range of its location choices is relatively complicated since it depends on the values of cross-group externalities. When α is relatively small, the profit of platform 1 is always decreasing as platform 1 locates closer to platform 2, which coincides with the conclusion in traditional one-sided markets with the two-stage Hotelling model. However, when the cross-group externalities are strong enough,[15] the profit of platform 1 will decrease first, then increase with the moving distance.

When t is larger enough than α, the power of the cross-group externalities is not strong enough to offset price competition; the profit will always decline as platform 1 approaches closer to platform 2. On the contrary, if t is close to or not large enough than a, the cross-group externalities make a significant difference. The closer that platform 1 locates to platform 2, the much more shares of the market on both sides it can steal from 2. At some point, this increasing demand effect can dominate price competition which leads to an increase in the profit.

Proposition 1.3 is summary of the subgame perfect equilibrium of two-stage Hotelling linear city model in the two-sided market.

Proposition 1.3 *Consider a two-sided market with two platforms competing in the two-stage Hotelling linear city model. In subgame perfect Nash equilibrium, the two platforms choose maximal product differentiation. So the platforms split the two-sided market in half, and the equilibrium price to the two sides is*

$$t - \alpha,$$

and the platforms make the profit of

$$t - \alpha.$$

Even though the profit of a platform may be increasing with its moving distance at some point, it is still optimal for the platforms to locate at the two endpoints since the profit of

[15]Such that $10t + 4\alpha + 4bt \geq \sqrt{4(t - 2\alpha - 2bt)^2 + 12t(t - tb^2 - \alpha)} + 3\sqrt{4t^2 + 8bt^2 + 4t^2b^2 + 12\alpha t}$

being sole incumbent is even less with the relatively lower prices. Therefore, in the subgame perfect equilibrium, the two platforms will still choose the two endpoints in the unit interval to soften price competition, and they charge the same price and have the half share of both sides. The intuition behind this is similar with the traditional Hotelling model in a one-sided market: the membership fees charged by the platform 1 to the agents are always decreasing when platform 1 moves closer to platform 2 since the product differentiation is less and price competition is fiercer. The change of equilibrium prices to the two sides are synchronous since they are identical. Under this circumstance, the influence of the profit brought by price competition is still significant although the more agents will be attracted to participate in this platform with the cross-group externalities.

Corollary 1.1 demonstrates the comparative statistics of equilibrium prices and profits with α and t.

Corollary 1.1 *Consider a two-sided market with two platforms competing in the two-stage Hotelling linear city model. The equilibrium prices and profits of the platforms*

(i) increase with the transportation cost t,

(ii) decrease with the cross-group externality α.

With a more substantial transportation cost, the platforms will increase their prices as the agents value their brand preferences more. According to Lemma 1.1, the prices decline dramatically with more massive cross-group externalities, which leads to the decline of the profit.

Proposition 1.4 *Consider a two-sided market with two platforms competing in the two-stage Hotelling model. From a social point of view, the market solution leads to too much differentiation.*

The optimum social solution is the one that minimizes costs (or maximizes utility), and it would be $a = 1/4$ and $1 - b = 3/4$. Therefore, maximal differentiation caused by the market equilibrium yields too much product differentiation compared to what is socially optimal.

In other words, the agents are willing to sacrifice some variety (have the products closer together in product space) for a lower price. Therefore, the social planners are supposed to take some actions to make platforms reduce their product varieties.

1.4. Multi-homing

In this section, I consider the multi-homing situation that the agents are allowed to join more than one platforms in the equilibrium. Armstrong and Wright (2007) investigate the multi-homing situation in detail with the classic one-stage Hotelling model.[16] I apply and extend their analysis to study the issue of endogenous product differentiation when the agents can multi-home. Since it is not reasonable to assume that all the agents choose to multi-home, I focus on the equilibrium in which the agents multi-home on one side and single-home on the other side.[17]

Armstrong and Wright (2007) have proven that a unique equilibrium exists in which all the agents single-home in the classic Hotelling model. Hence I need to modify the model to study the product differentiation problem with the multi-homing situation. According to Armstrong and Wright (2007), I relax the restriction that exogenous product differentiation for both sides by assuming that product differentiation on only one side. To be specific, I consider a market where two platforms are viewed as identical by the sellers and as differentiated by the buyers. Moreover, I assume that the sellers can multi-home and the buyers are only allowed to single-home in the equilibrium.[18]

[16]They have discussed some of the propositions in this section. I mainly extend their propositions to endogenous product differentiation problem.

[17]If all the agents on one side multi-home, agents on the other side will not choose to multi-home with the non-negative price.

[18]"This stands for the situation that in many two-sided markets with the buyers and sellers, the sellers view the competing platforms as more or less homogeneous (controlling for the number of buyers on the platforms), while the buyers have preferences for using a particular platform over the other. One reason for this might be that platforms supply a bundled service to the buyers in which the interaction with the sellers is just one aspect of the platforms service to the buyers. (For instance, newspapers or television channels supply news and entertainment content as well as adverts). By contrast, the sellers will often not care much about the content which is supplied together with their adverts (after controlling for the number of buyers on the platform). Alternatively, the buyers might have to travel to the platform (think of shopping malls or supermarkets), whereas transport costs will be much less of an issue for the sellers. Where product differentiation on the buyer side is high relative to network benefits, the buyers will tend to single-home. This gives an incentive for the sellers to multi-home to maximize their exposure to the buyers."- Armstrong

Assumption 1.3 *$v_S = 0$ and v_B is large enough that every buyer joins a platform in equilibrium.*

Assumption 1.4 *$t_S = 0$, $t_B = t > \alpha_B$.*

Assumption 1.5 *α_S is sufficiently larger that t.*[19]

The timeline of the game is the following: at the first stage, two platforms determine their endogenous product differentiation to the single-homing side; at the second stage, the two platforms determine their prices to the agents on the two sides.

1.4.1. Price Equilibrium

Armstrong and Wright (2007) have proven that all the equilibrium involves the sellers being left with zero surplus in the classic Hotelling model. I apply this conclusion directly to the analysis of the two-stage Hotelling model since the endogenous product differentiation problem does not exist for the sellers.

Lemma 1.3 *Consider a two-sided market with one side single-homing and the other side multi-homing. The multi-homing side is fully extracted by the platforms that they have zero surplus in all the equilibrium.*

Consistent with Armstrong and Wright (2007), I focus on symmetric equilibrium, that either the sellers join the two platforms or none. I can rule out an equilibrium that the platforms do not accept the sellers because of $\alpha_B > 0$. Therefore, I focus on constructing a symmetric equilibrium of the sellers joining all the two platforms. In such equilibrium, the two platforms will split the buyers in the market due to the symmetry of the Hotelling model. According to the model, the utility of the sellers is

$$u_i^S = \alpha_S n_i^B.$$

and Wright (2007)

[19] I have to make sure that $t(1-b-a)(1-\frac{1}{3}b+\frac{1}{3}a)-\alpha_S < 0$ and $t(1-b-a)(1-\frac{1}{3}a+\frac{1}{3}b)-\alpha_S < 0$ are valid in the equilibrium.

Therefore, the prices to the sellers in such an equilibrium are

$$p_1^S = \alpha_S n_1^B; \quad p_2^S = \alpha_S n_2^B.$$

Lemma 1.4 *Consider a two-sided market with one side single-homing and the other side multi-homing. The platforms will charge free to the single-home side in the equilibrium.*

Given that the sellers have zero surplus in the equilibrium, there is always a deviating strategy for the platforms. For instance, if platform 2 charges the proposed equilibrium prices to the two sides when platform 1 undercuts the proposed price to the buyers very slightly, it will cause more of the buyers to join platform 1, and fewer the buyers to join platform 2. Since the sellers have zero surplus with the proposed price by platform 2, the decrease in the number of buyers on platform 2 indicates that the sellers will have a negative surplus when joining platform 2. Therefore, a small decrease in the proposed price p_1^B will lead all the sellers to join platform 1 exclusively, which will then generate a further decrease in the number of buyers on platform 2 due to $\alpha_B > 0$. Hence if and only if the two platforms cannot undercut the price to the buyers, it is not profitable for the platforms to make this deviation in the equilibrium. In other words, p_B is not above 0 in the equilibrium so that it is not profitable for the platforms to induce the sellers to join it exclusively. Since I have restricted that equilibrium prices to be non-negative, so the platforms will provide free service to the single-homing side in the subgame perfect equilibrium.

With the equilibrium price to the two sides, we can investigate the optimal locations that the platforms will choose in the equilibrium.

1.4.2. Location Game

With the sellers joining all the two platforms in the equilibrium, the price is the main determinant to the surplus of the buyers. According to the model, the demand functions of the buyers are

$$n_1^B = \frac{p_2^B - p_1^B}{2(1-b-a)t} + \frac{1-b+a}{2}; \quad n_2^B = \frac{p_1^B - p_2^B}{2(1-b-a)t} + \frac{1-a+b}{2}.$$

Therefore, the profits of the two platforms are

$$\pi_1 = (\frac{1-b+a}{2})\alpha_S; \quad \pi_2 = (\frac{1-a+b}{2})\alpha_S.$$

Taking derivative of the expressions with location choices a and b, we have $\frac{d\pi_1}{da} > 0$ and $\frac{d\pi_2}{db} > 0$. Therefore, in the subgame perfect equilibrium, it is optimal for the two platforms to choose minimal product differentiation in the market, that is $a = 1 - b = \frac{1}{2}$.

Proposition 1.5 *Consider a two-sided market with two platforms competing in the two-stage Hotelling model, one side is single-homing, and the other side is multi-homing. In subgame perfect equilibrium, the platforms choose minimal product differentiation to the single-homing side. The equilibrium price to the multi-homing side is*

$$\frac{\alpha_S}{2},$$

and equilibrium price to the single-homing side is 0. *The profit of each platform is*

$$\frac{\alpha_S}{2}.$$

The sellers care about the presence of the buyers so much that the platforms have to provide free service to the buyers to keep the buyers firmly. Meanwhile, the buyers care about the presence of the sellers with $\alpha_B > 0$; then the platforms have to make sure that the sellers will not join the rival platform exclusively. Therefore, the platforms will charge nothing to the buyers to prevent the rivals to, on the one hand, undercut its price to own the sellers exclusively, and on the other hand, keep their buyers at the same time. So the platforms split the buyers in half, then the sellers will join both platforms to access to all the buyers.

With the fixed equilibrium price to the buyers that are independent of the location choices, a platform will locate as close as possible to its rival in the unit interval to achieve the larger share of captive agents. As a result, the platforms will choose to provide the identical service to the buyers in the equilibrium.

Proposition 1.6 *Consider a two-sided market with two platforms competing in the two-stage Hotelling model. From a social point of view, the market solution leads to too little differentiation.*

The proof is similar to Proposition 1.4. The social optimum solution is that $a = 1/4$ and $1-b = 3/4$. Hence, minimal differentiation yields too little product differentiation compared to the socially optimal situation. In other words, single-homing agents are willing to pay a higher price for various services.

1.5. Extensions

In this section, I focus on some extensions of the single-homing situation.

1.5.1. Intra-group Externalities

First, I want to investigate the role of intra-group externalities in the situation.

Rewriting the model as the following to investigate what if the same side externality is added,

$$u_i^S = v_S + \alpha n_i^B + \beta n_i^S - p_i^S$$

$$u_i^B = v_B + \alpha n_i^S + \beta n_i^B - p_i^B.$$

Then the equilibrium prices of the two platforms to the two sides are,

$$p_1 = p_1^S = p_1^B = \frac{t(1-b-a)(3+a-b)}{3} - (\alpha+\beta)$$

$$p_2 = p_2^S = p_2^B = \frac{t(1-b-a)(3+b-a)}{3} - (\alpha+\beta).$$

And the equilibrium market share for the two platforms are,

$$n_1 = n_1^S = n_1^B = \frac{t(1-b-a)[1+\frac{1}{3}(a-b)] - (\alpha+\beta)}{2[t(1-b-a) - (\alpha+\beta)]}$$

$$n_2 = n_2^S = n_2^B = \frac{t(1-b-a)[1+\frac{1}{3}(b-a)] - (\alpha+\beta)}{2[t(1-b-a) - (\alpha+\beta)]}.$$

According to the above equations, the intra-group externality which is also referred to as network effect has a direct impact on the cross-group externalities in the equilibrium. Hence it will not influence the subgame perfect equilibrium result such that the platforms will choose maximal product differentiation. However, there are some interesting results. For instance, if $\beta > 0$, the agents value the presence of the agents on the same side, which will lead more agents on the other side to join the platform since the two externalities work in the same direction. Meanwhile, the equilibrium price declines a lot as well. When $\beta < 0$ and $\alpha + \beta > 0$, the agents do not value the other agents on the same side to join the same platform, and intra-group externalities weaken the cross-group externalities but not strong enough to eliminate α. If $\beta < 0$ and $\alpha + \beta < 0$, negative intra-group externalities dominate the cross-group externalities that even though some buyers are willing to join the platform when extra sellers are on it, more buyers will exit the platform since too many competitors on the same side on the platform.

1.5.2. Two-dimension

It is reasonable to think about the two-dimensional case since there are two sides of agents in the market. The situation of two dimensions refers to that platforms provide services to two sides separately. For example, TV channels accept advertise from manufacturers and play their TV programs along with the ads to agents. Therefore, TV programs are also the service being valued by the consumer. Moreover, the sellers only care about whether their ads are exposed to those agents. Under this circumstance, platforms are facing two choices of product differentiation to make. To be specific, there are two unit intervals, one for each side. Also, they are uniformly distributed in their interval. Therefore, platforms are supposed to choose their location in each interval. To shed light on this problem, I start with a simple setting that platforms' locations are exogenously fixed at two endpoints in one interval, say the buyers. And they need to determine their product differentiation for the sellers.

Therefore, the expressions of demand as the following,

$$n_1^S = \frac{\alpha(2n_i^B - 1) + (p_j^S - p_i^S)}{2t(1-b-a)} + \frac{1-b+a}{2}$$

$$n_1^B = \frac{\alpha(2n_i^S - 1) + (p_j^B - p_i^B)}{2t} + \frac{1}{2}.$$

Solving out the above equations,

$$n_1^S = \frac{t(p_j^S - p_i^S) + \alpha(p_j^B - p_i^B) + t^2(1-b-a)(1+a-b) - \alpha^2}{2[t^2(1-b-a) - \alpha^2]}$$

$$n_1^B = \frac{\alpha(p_j^S - p_i^S) + t(1-b-a)(p_j^B - p_i^B) + t^2(1-b-a)[\alpha(a-b) + t] - \alpha^2}{2[t^2(1-b-a) - \alpha^2]}.$$

With the first order conditions of the profit maximization, the equilibrium prices are

$$p_1^S = [t(1-b-a) - \alpha] + \frac{(a-b)t(1-b-a)}{3}; \quad p_1^B = t - \alpha$$

$$p_2^S = [t(1-b-a) - \alpha] - \frac{(a-b)t(1-b-a)}{3}; \quad p_2^B = t - \alpha.$$

And the market share of platform 1 is

$$n_1^S = \frac{1}{2} + \frac{(a-b)t^2(1-b-a)}{6[t^2(1-b-a) - \alpha^2]}; \quad n_1^B = \frac{1}{2} + \frac{\alpha(a-b)t(1-b-a)}{6[t^2(1-b-a) - \alpha^2]}.$$

The market share of platform 2 is $1 - n_1$.

For the sellers, the platforms choose their locations in the unit interval. Therefore, a and

b have impacts on equilibrium prices to the sellers and market shares of both platforms. As for the buyers, the equilibrium prices are independent of the location choices, but the market shares of buyers also are influenced by the location choice. The cross-group externality makes the buyers care about product differentiation of the sellers despite fixed prices to them.

If $a < b$, in other words, platform 1 have less captive agents, not only the market share of platform 1 is less than half on both sides, but the price to sellers of platform 1 is also less than that of platform 2. Meanwhile, the market share of the buyers of platform 1 shrinks as well although the price to the buyers is not affected. This simple model directly demonstrates the cross-group externality plays a role in the game.

It is not easy to investigate how the profit change along with the moving distance in this case. So I directly to check whether platforms will choose to locate at endpoints in the equilibrium.

Proposition 1.7 *Consider a two-sided market with two platforms competing in the two-stage Hotelling linear city model. The platforms choose their product differentiation to one side. In the subgame perfect equilibrium, the platforms choose to provide maximal product differentiation to the side. And each platform obtains the half share of each side in the market. The equilibrium prices to the two sides are* $t - \alpha$*, and platforms make the profit of* $t - \alpha$.

To avoid intensive price competition, the platforms will choose maximal product differentiation for the sellers in the equilibrium.

1.6. Conclusion

This chapter analyzes the endogenous product differentiation problem of two platforms in a two-sided market by using the two-stage Hotelling model. In the two-stage Hotelling model, the distance between the locations of the two platforms in a unit interval can depict the endogenous horizontal product differentiation choice. The further that the platforms locate away from each other, the higher product differentiation that the two platforms choose to provide to the agents.

The significant difference between the traditional one-sided markets and two-sided markets is the cross-group externality, which is also the remarkable characteristic of the two-sided market. In traditional one-sided markets, price competition always dominates the increasing demand that firms will choose to provide maximal product differentiation to avoid price competition. Hence I expect the existence of the cross-group externalities may make a difference in the equilibrium because if a platform can steal more agent on one side from the other platform, it will attract a certain amount of agent on the other side in two-sided markets. In other words, I anticipate the effect of cross-group externalities may dominate the impact of fierce price competition which can lead to a different result from the endogenous product differentiation problem in one-sided markets.

The equilibrium results indicate that the platforms in the two-sided market still choose maximal product differentiation in the equilibrium. However, the cross-group externalities do make a difference for the following two aspects. First, in two-sided markets, when the agents on the two sides are single-homing, I prove that the demand effect resulted from the cross-group externalities at some point may stronger than price competition. Besides, a platform can attract loyal agents of the other platform due to the existence of the cross-group externalities. Therefore, when the rival has been located at a point in the unit interval, the platform can select a location to make itself be the sole incumbent in the market.

Moreover, when one side is allowed to be multi-homing, and the other side is still single-homing, the two platforms have to compete so intensively on the single-homing side that they both provide free service to this side. Otherwise, the multi-homing side participates in the rial platform exclusively, and the platform will lose part of the single-homing side as well. As a result, the platforms will choose to provide minimal product differentiation to the single-homing side to obtain the largest market share.

Chapter 2

HORIZONTAL MERGERS IN TWO-SIDED MARKETS

2.1. Introduction

I analyze the effects of horizontal mergers in a two-sided market with a three-platform Salop's circle model. Specifically, I investigate the impacts of horizontal mergers of two out of the three platforms on the equilibrium prices and welfare for the two sides with the cost savings from the merger.

Horizontal mergers are business consolidations that occur between firms who operate in the same industry, often as competitors offering a similar good or service. For instance, over the last decades, there has been a massive consolidation wave in media industries all over the world which has led to the emergence of large cross-media companies. Media markets are different from traditional markets that they serve two sides of agents, namely advertisers and consumers. A media market is a typical example of two-sided markets.

Even though horizontal mergers have been widely investigated in traditional one-sided markets, the analysis in one-sided markets cannot be applied to two-sided markets directly due to the need for two-sided platforms to consider the interests of two different sides of agents to maximize profits. It is possible that platforms in two-sided markets behave in a way that would be not optimal for firms in traditional one-sided markets. For example, given more market power, firms in traditional markets usually increase their prices to gain more profits after a merger which is detrimental to consumer welfare. In two-sided markets, platforms, even monopolists may gain more profits if they choose to lower their prices to at least one side of agents to attract more agents on the other side. Besides, a higher price after the merger maybe not detrimental to consumer welfare in two-sided markets if agents on one side can be provided with more agents on the other side. Hence, the fact that the standard economic analyses do not always hold in two-sided markets is the driving force to seek relevant corresponding research results when confronted with issues of horizontal mergers in two-sided markets.

This chapter addresses the following questions: How prices and welfare change with horizontal mergers based on the merger efficiency it generates? I model that three platforms are equidistantly located on a unit circular model, and all agents are uniformly distributed along each side of the market. To explore the impact of the merger cost savings, I assume that two of the three platforms have a merger and the merging parties obtain efficiency gains from the cost savings generated by the merger.

Regarding the impacts on price, given the significant cross-group externalities, the equilibrium prices of all the platform to both sides can decline after the merger even though the merger cost savings are not significant. In contrast to the merger theory of traditional one-sided markets that only the strong cost savings can make the prices decline, in the two-sided market, the strong cost savings are less necessary for the post-merger prices to decline with the significant cross-group externalities.

As for the impacts on welfare, consumer surplus will still increase when the post-merger prices of the merged platform increase to some extent. Since the merger enables the merged platform to merge their agents, the agents on one side on one of the merged platform can approach the agents on the other side on all the merged parties. Hence, the surplus of the agents may not decrease with higher prices. In contrast to the merger theory of one-sided markets that consumers will be harmed if the merged firm increases their prices, the horizontal merger is beneficial for the agents in the two-sided market even though the merged platform increase its prices to a certain degree. This conclusion confirms the insight in Evans (2003) that mergers that increase the number of consumers on one side increase the value on the other side. Hence, consumer welfare may increase even though the prices increase on one side or in total. Besides, social welfare will increase post-merger with the strong cross-group externalities due to a higher industry profit after the merger.

This chapter also sheds some light on the question of which type of mergers between two of the three platforms is favorable to consumers or the society when all the platforms have different costs. I argue that a horizontal merger between the less efficient platforms can generate higher consumer surplus, but a merger between the more efficient platforms yields higher social welfare due to its higher industry profit. Therefore, the results are useful for antitrust authorities when considering which type of mergers should be given more credit. In the extensions section, I discuss the situations when the main restrictions in the model are relaxed.

In summary, at a theoretical level, I develop the analysis of change on price and welfare with horizontal mergers in a circular market; at a policy level, I provide the necessary and sufficient conditions of lower prices and the sufficient conditions of increasing welfare after the merger for regulators when they evaluate a proposed horizontal merger in two-sided markets.

2.1.1. Related Literature

There is extant theoretical literature on horizontal mergers in traditional one-sided markets with Cournot or Bertrand competitions. For example, based on Cournot competition, Farrell and Shapiro (1990) provide a necessary and sufficient condition for a merger to raise the price; Noker and Whinston (2013) analyze the optimal merger approval policy of an antitrust authority. In spatial models with price competition, Deneckere and Davidson (1985), Levy and Reitzes (1992), Brito (2003, 2005) demonstrate that, in the absence of the cost savings, a horizontal merger between neighbors necessarily generates a market power effect that the merged firms gain more profits by increasing their prices. Farrell and Shapiro (2010) discuss that whether a proposed merger will generate net upward pricing pressure involves comparing two opposing forces: the loss of direct competition between the merging parties, which creates upward pricing pressure, and marginal-cost savings from the merger, which create downward pricing pressure.

Several survey papers discuss horizontal merger issues in two-sided markets. Evans and Schmalensee (2013) point out that the analytic tools for assessing effects from mergers between one-sided firms may yield incorrect assessments insofar as they fail to account for inter-dependencies in demand among the multiple sides. Evans (2003) explains how and why mergers in two-sided markets may not result in the same antitrust concerns as in traditional one-sided markets. He presents that mergers that increase consumer participation on one side increase the value on the other side. Hence, consumer welfare may increase even though price increases on one side or in total. Rysman (2009) indicates that if the merger reduces cost on one side, it will have an impact on the other side. Moreover, a merger can increase the market power on both sides of the market but still lead to price to decline on one side.

To the best of my knowledge, two papers provide theoretical analysis to address optimal pricing strategies with horizontal mergers in two-sided markets. Chandra and Collard-Wexler (2009) discuss horizontal mergers in the newspaper industry by assuming that duopolists merge into a monopolist. Their main result demonstrates that higher market concentration may not lead to higher prices on both sides. The monopolist may choose to set a lower price to consumers at each merged newspaper than competing duopolists. However, their result relies on the assumption that the price is lower than the marginal cost on the reader side. Leonelle (2010) also investigates horizontal mergers in the newspaper industry. The merged monopolist can allow advertisers in one newspaper to advertise in the other newspaper as well after the merger. Hence, advertisers can approach twice the readers with a single price. Leonello (2010) demonstrates that with this advertising bundling, the monopolist will want to charge lower prices on at least one side of the market.

All the above theoretical papers focus on the newspaper industry specifically, and they target the situation that duopolists become a monopolist after a merger. However, it is reasonable to expect that there may still exist price competitions even though powerful platforms have a merger in the industry. This paper is different from these two papers by establishing a general model to focus on the impacts that the merger cost savings have on the optimal pricing strategies of all platforms, including the merging parties and the outside platform in the market. These two parties may take different strategies after the merger. This is essential to antitrust authorities since they need to balance the efficiency gains to the entire market from a merger.

Besides, I also provide a complete welfare analysis of the merger instead of focusing solely on individual price and profit. Welfare is an important indicator that antitrust authorities take into consideration when evaluating a merger case. Different from a merger in one-sided markets, a merger of two-sided platforms would ordinarily increase the size of all customer groups and thereby providing extra benefits. Moreover, the model makes it possible to consider different types of mergers when involving heterogeneous platforms in the market and I can explore which merger is preferable to consumers or the entire society.

The empirical research about mergers in two-sided markets is relatively plentiful, and most of them focus on media industries such as radio and newspaper.[1] For instance, Jeziorski (2014) estimates a structural supply-and-demand model using data from the 1996-2006 merger wave in the U.S. radio industry and taking account of cross-group externalities. He finds that this merger wave harmed advertisers but benefited listeners. He also finds that these mergers increased market-specific product variety, which contributed importantly to consumer benefits.

The rest of the chapter is organized as follows. Section 2.2 provides the details of the model. Section 2.3 provides the equilibrium characterization of pre-merger and post-merger. Section 2.4 presents the comparison of the equilibrium results between pre-merger and post-merger. Section 2.5 analyzes the comparison between different mergers with the heterogeneous platforms. Section 2.6 discusses some extensions on relaxing the assumptions of the primary model. Section 2.7 provides the conclusion. All proofs are provided in the Appendix.

[1] For example, Song (2013), Fan (2013), Wright and Kaiser (2006), Collard-Wexler (2009), Filistrucchi, Klein and Michielsen (2012) provide empirical analyses of mergers in two-sided markets

2.2. The Model

I follow and extend the two-sided market model in Armstrong (2006) to Salop's circle model (1979).[2] To be specific, I consider a two-sided market which is comprised of agents on the two sides and three platforms.

Agents. All the agents in the two-sided market belong to the two sides, side S and B. I refer to all the agents on the side S as sellers and the agents on the side B as buyers. Both sellers and buyers are consumers of the platforms in the two-sided market. The agents on one side are willing to participate in a platform to approach the agents on the other side on the platform.[3] Let all the agents uniformly distributed along each side of the market. They all have inelastic demand, consuming at most one unit of the good/service.[4] The sellers and the buyers in the market obtain the respective utilities u_i^S, u_i^B if they join platform i ($i = 1, 2, 3$). Their utilities u_i^S, u_i^B are determined in the following way:

$$u_i^S = v_S + \alpha_S n_i^B - p_i^S - t_S d_i^S; \quad u_i^B = v_B + \alpha_B n_i^S - p_i^B - t_B d_i^B.$$

v_S, v_B are the intrinsic valuations assumed to be platform independent, and they are large enough that the entire market can be fully covered. p_i^S, p_i^B are the respective prices charged by platform i to the two sides. Thus I focus on the platforms that charge fix payments to agents.[5] n_i^S and n_i^B indicate the amounts of the sellers and the buyers participating in platform i. α_S (α_B) is the cross-group externality that the extra benefit that a seller (buyer) can obtain when joining platform i if one more buyer (seller) joins this platform. For simplicity, I assume that α_S and α_B do not depend on which platform an agent participates in but only on which side the agent is on. t_S and t_B are the transportation costs to the sellers

[2] Armstrong (2006) investigates the optimal pricing strategies of two platforms competing in a Hotelling model.

[3] To focus on the cross-group externalities, I rule out the possibility that the agents care about the presence of the same side on the platform.

[4] In this section, I restrict single-homing condition to all the agents that they are only allowed to join one platform in the equilibrium. This restriction will be relaxed in the section of extensions.

[5] I assume prices being non-negative. This is a reasonable assumption for many examples of two-sided markets that people pay to participate in the platform, such as TV channels or newspapers. Platforms may charge free or even subsidy to one group, like the yellow page, but there may be the adverse selection and moral hazard problems.

and buyers which can be considered as the product differentiation parameters for the two sides that describe the competitiveness of the market. d_i^S and d_i^B are the distance that a participating agent locates from platform i.[6]

Platforms. Platforms in two-sided markets act as firms in traditional one-sided markets, providing services or products to consumers. In two-sided markets, platforms usually provide service by enabling the two sides to make interactions with each other. Assuming three platforms compete in prices and are equidistantly located on the unit circular model. Platform i ($i = 1, 2, 3$) earns its profits by attracting the agents on the two sides and incurs a constant production cost c_i for each agent. So the profit function of platform i can be written as

$$\pi_i = (p_i^S - c_i)n_i^S + (p_i^B - c_i)n_i^B.$$

To facilitate a clean and tractable characterization, applying the symmetric restrictions that $v_S = v_B = v, \alpha_S = \alpha_B = \alpha, t_S = t_B = t, d_i^S = d_i^B = d_i$.

Assumption 2.1 *$t > 4\alpha$.*

Assumption 2.1 indicates that the cross-group externalities are small compared with the differentiation parameter t. Given Assumption 2.1, we can focus on the market-sharing equilibrium. Otherwise, if the cross-group externalities were not sufficiently less than the brand preference, there could only be the equilibrium that one platform corners both sides of the market.

The demand functions of the three platforms, according to Salop's circle model, can be written as

$$n_1^j = \frac{1}{3} + \frac{\alpha(3n_1^{-j} - 1) - 2p_1^j + p_2^j + p_3^j}{2t} \tag{2.1}$$

$$n_2^j = \frac{1}{3} + \frac{\alpha(3n_2^{-j} - 1) - 2p_2^j + p_1^j + p_3^j}{2t} \tag{2.2}$$

[6]Without loss of generality, I assume the linear version of transportation costs since both the linear and the quadratic versions of transportation cost in Salop's circle model generate the same demand functions, which is valid for both one-sided markets and two-sided markets.

$$n_3^j = \frac{1}{3} + \frac{\alpha(3n_3^{-j} - 1) - 2p_3^j + p_1^j + p_2^j}{2t} \tag{2.3}$$

with $j \in \{S, B\}$.

Keeping the price of one side, namely the sellers, fixed, Eq.2.1 shows that an extra buyer on a platform attracts a further $\frac{3\alpha}{2t}$ sellers to that platform.

2.3. Equilibrium Characterization

To investigate the impacts of horizontal mergers on the two-sided market, I first discuss the equilibrium results pre-merger and post-merger.

2.3.1. Pre-merger Equilibrium

Prior to the merger, all the platforms compete with the constant per-agent cost c_i, ($c_i > 0$). Each platform maximizes its profit by setting optimal prices to the two sides.

To determine the pre-merger equilibrium, I first establish the system of demands. By Eq.2.1-2.3, the demand functions of the three platforms can be rewritten as

$$n_1^j = \frac{1}{3} + \frac{6t(p_2^j + p_3^j - 2p_1^j) + 9\alpha(p_2^{-j} + p_3^{-j} - 2p_1^{-j})}{12t^2 - 27\alpha^2}$$

$$n_2^j = \frac{1}{3} + \frac{6t(p_1^j + p_3^j - 2p_2^j) + 9\alpha(p_1^{-j} + p_3^{-j} - 2p_2^{-j})}{12t^2 - 27\alpha^2}$$

$$n_3^j = \frac{1}{3} + \frac{6t(p_1^j + p_2^j - 2p_3^j) + 9\alpha(p_1^{-j} + p_2^{-j} - 2p_3^{-j})}{12t^2 - 27\alpha^2}$$

with $j \in \{S, B\}$.

By Assumption 2.1, the law of demand is satisfied in the two-sided market that the demand on each side of a platform decreases with the platform's prices to the two sides and increases with the rivals' prices. Thus a lower price on one side not only attracts more agents on that side but also leads to more participation on the other side due to the cross-group externalities.

The profit function π_i, that

$$\pi_i = (p_i^B - c_i)n_i^B + (p_i^S - c_i)n_i^S,$$

is concave in p_i by Assumption 2.1.

Lemma 2.1 demonstrates the equilibrium prices from the platforms' profits maximization.

Lemma 2.1 *Consider three platforms competing in prices in a two-sided market, the equilibrium prices of the platforms to the two sides are*

$$p_1^S = p_1^B = \frac{3c_1 + c_2 + c_3}{5} + \frac{t}{3} - \frac{\alpha}{2}$$

$$p_2^S = p_2^B = \frac{c_1 + 3c_2 + c_3}{5} + \frac{t}{3} - \frac{\alpha}{2}$$

$$p_3^S = p_3^B = \frac{c_1 + c_2 + 3c_3}{5} + \frac{t}{3} - \frac{\alpha}{2}.$$

Due to the symmetry of the three platforms on each side of the market, $p_i^S = p_i^B$. The equilibrium price of each platform on each side increases with the transportation cost and the per-agent costs and decreases with the cross-group externality. Hence the agents pay for the platform's costs and the transportation costs, and the platforms pay for the cross-group externalities. When one side values the presence of the other side more, the competition among platforms of attracting the agents is more intensive that they have to cut prices more.

2.3.2. Post-merger Equilibrium

I assume that the merging platforms will merge their agents. So the merger allows an agent in one of the merging platforms to approach more agents on the other side, even though they are connected through one platform.[7] Besides, I assume that the locations of

[7]Merging consumers is very common in two-sided markets. For instance, when the New York Times purchased the Boston Globe, one justification was the ability to sell newspaper advertisements throughout the northeastern United States; Sharelatex and Overleaf, two online document sharing and editing websites, merge all their accounts together after the merger; Iqiyi and Tudou, two video websites in China, integrate

the merging platforms do not change on the unit circle which means the merging platforms will not relocate after the merger. So the merger only changes the ownership of the merging platforms in the market, but will not change the agents preferences. It can also be interpreted as the merging platforms deciding to keep their original brands after the merger.

Before exploring a merger between two out of the three platforms, I first briefly discuss the merger that all the platforms merge into a monopolist. The merged platform will charge the highest prices that it can charge to the two sides to extract maximal consumer surplus. In Salop's circle model, the highest prices that a merged monopolist can charge are the ones that make the indifferent agents have zero surplus. The utilities of the indifferent agents, according to the model, are $v + \alpha n_i^{-j} - p_i^j - \frac{1}{6}t$.

Since $n_i^{-j} = 1$ after the merger, the equilibrium prices of the merged monopolist are

$$p^j = v + \alpha - \frac{1}{6}t$$

with $j \in \{S, B\}$.

Different from the pre-merger equilibrium prices, the equilibrium prices after the merger are positively related to the cross-group externalities. The more that a seller (buyer) values the presence of the buyers (sellers), the more he has to pay to the monopolist to access to the other side. Besides, the equilibrium post-merger prices are irrelevant with the per-agent costs since the monopoly platform will always fully exploit the indifferent agents. In other words, merger cost efficiency, if it is generated, cannot be passed on to the agents without price competition. Although the agents on one side, under this circumstance, can approach all the agents on the other side in the entire market, it can be easily proven than consumer welfare will decrease after the merger given the high monopolistic prices.

In the following content, I focus on the merger between two of the three platforms. The chapter mainly targets the merger cases that the merging platforms gain efficiency by

their videos with each other post-merger. The reasons that merging platforms to merge their consumers are the following: first of all, it is easy to achieve without incurring extra costs, even may generate cost savings for platforms since most of them provide information or services to agents; second, it can boost the competitiveness of the merging platforms since one side values the presence of the other side.

achieving cost savings since I want to explore the relationship between merger efficiency gains and cross-group externalities. Assuming that platforms 1 and 2 have a horizontal merger and they achieve the cost savings that their per-agent cost $c_{1+2} \leq min\{c_1, c_2\}$. Platform 3 which does not participate the merger is denoted as the outside platform. The per-agent cost of platform 3 does not change after the merger. Let $\mathbf{n}_{1+2}^j$ be the total number of the agents on the side j that the merging platforms have. Hence a buyer can be connected to all the sellers on the two platforms which are $\mathbf{n}_{1+2}^S$ if he participates in any one of the two platforms. In the equilibrium, platform 1 and 2 will charge the same prices to their sellers or buyers, which are $\mathbf{p}_{1+2}^j$. Similarly, $\mathbf{n}_3^j$ indicates the total number of the side j that the outside platform has, and $\mathbf{p}_3^j$ is its price to the side j post-merger. Bold letters denote post-merger values of all variables.

Similar to Eq.2.1-2.3, the post-merger demand functions of all the platforms can be written as

$$\mathbf{n}_{1+2}^j = \frac{2}{3} + \frac{\alpha(\mathbf{n}_{1+2}^{-j} - \mathbf{n}_3^{-j}) - \mathbf{p}_{1+2}^j + \mathbf{p}_3^j}{t}$$

$$\mathbf{n}_3^j = \frac{1}{3} + \frac{\alpha(\mathbf{n}_3^{-j} - \mathbf{n}_{1+2}^{-j}) - \mathbf{p}_3^j + \mathbf{p}_{1+2}^j}{t}.$$

with $j \in \{S, B\}$.

With the first order conditions of profits maximization, the post-merger equilibrium results are demonstrated in Lemma 2.2.

Lemma 2.2 *Consider two of three platforms having a merger in a two-sided market, the equilibrium prices to the two sides are*

$$\mathbf{p}_{1+2}^S = \mathbf{p}_{1+2}^B = \frac{2}{3}c_{1+2} + \frac{1}{3}c_3 + \frac{5}{9}t - \alpha; \quad \mathbf{p}_3^S = \mathbf{p}_3^B = \frac{1}{3}c_{1+2} + \frac{2}{3}c_3 + \frac{4}{9}t - \alpha.$$

2.4. Comparisons Results

In this section, I explore the impacts of horizontal mergers on prices, profits, and welfare. To make the comparisons more straightforward and tractable, I assume that all the platforms have homogeneous per-agent cost before the merger, that $c_i = c$, $(i = 1, 2, 3)$. After the merger, the merged platform, platform 1 and 2, achieves the cost savings Δ that their per-agent cost $c_{1+2} = c - \Delta$, $(0 \leq \Delta \leq c)$. The cost of the outside platform-platform 3 is still c after the merger. The post-merger market is hence an asymmetric duopoly due to the merger cost efficiency.

Assumption 2.2 $\Delta < \dfrac{4}{3}t - 3\alpha$.

Assumption 2.2 indicates that the cost savings are not so large that all the platforms earn non-negative profits post-merger in the equilibrium. If Assumption 2.2 is violated, the outside platform will have negative profit in the equilibrium since it does not enjoy the merger cost savings but has to burden part of them in its prices.

Before the merger, according to Lemma 2.1, the equilibrium prices of all the platforms are the same,

$$p^* = c + \frac{t}{3} - \frac{\alpha}{2}. \tag{2.4}$$

After the merger, the prices of the platforms to the two sides, by Lemma 2.2, are

$$\mathbf{p}^j_{1+2} = c - \frac{2}{3}\Delta + \frac{5}{9}t - \alpha; \quad \mathbf{p}^j_3 = c - \frac{1}{3}\Delta + \frac{4}{9}t - \alpha, \tag{2.5}$$

with $j \in \{S, B\}$.

The merger cost savings Δ are passed on to the agents through price competition. Thus not only the agents who are on the merged platform can enjoy the cost savings, but the agents who join the outside platform can also benefit from the cost savings. The difference is that the agents on the merged platform enjoy more massively. Compared with the pre-merger situation, the impacts of the transportation costs and the cross-group externalities on the prices are greater after the merger due to less intensive price competition.

The market shares of the platforms post-merger on each side are

$$\mathbf{n}_{1+2}^{j} = \frac{5t - 9\alpha + 3\Delta}{9(t - 2\alpha)}; \quad \mathbf{n}_{3}^{j} = \frac{4t - 9\alpha - 3\Delta}{9(t - 2\alpha)} \tag{2.6}$$

Notice that the market share of the merged platform on each side is not necessarily larger than $\frac{2}{3}$ which depends on the value of Δ.[8]

Market concentration is one useful indicator of the likely competitive effects of a merger. The antitrust agencies often calculate the Herfindahl-Hirschman Index (HHI) of market concentration. The HHI is obtained by summing the squares of the individual firms market shares. According to the model, before the merger, every platform accounts for 33% of the market share. Hence $HHI_{pre} = 3 * (33)^2 = 3267$. According to the merger guideline, the market is already a highly concentrated market before the merger. After the merger, the HHI index is $HHI_{post} = (33)^2 + (33 + 33)^2 = 5445$. So the change of the HHI is 2178. The merger is anti-competitive since mergers increasing the HHI of more than 200 points will be considered to be likely to enhance market power. However, market shares may not fully reflect the competitive significance of firms in the market or the impact of a merger. They are used in conjunction with other evidence of competitive effects. So it is still needed to explore the impacts on the prices and welfare.

Before the merger, each platform earns the profit of

$$\pi^* = \frac{2}{9}t - \frac{\alpha}{3}. \tag{2.7}$$

After the merger, their profits are

$$\pi_{1+2} = \frac{2(5t - 9\alpha + 3\Delta)^2}{81(t - 2\alpha)}; \quad \pi_3 = \frac{2(4t - 9\alpha - 3\Delta)^2}{81(t - 2\alpha)}. \tag{2.8}$$

[8] In the one-sided market with Salop's circle model, when two adjacent firms have a merger, they would keep at least $\frac{1}{3}$ share of the agents located between them. Whether they can earn more than the other $\frac{1}{3}$ share of the agents located at the rest of the market depends on whether the merged firms charge a lower price than the outside firm. Things are similar in the two-sided market, but the difference is that the merged platform does not have to charge lower than the outside platform. The agents located between the merged platform can attract some agents to join even though the prices of the merged parties are higher than that of the outside platform.

To explore the role that the merger cost savings play in the optimal pricing strategies of all the platforms, I first investigate how the equilibrium post-merger prices and profits change after the merger without the cost savings being generated.

Lemma 2.3 *Consider a horizontal merger between two out of the three symmetric platforms in a two-sided market. If no cost savings are generated, i.e., $\Delta = 0$,*

(i) the prices of the merged platform increase after the merger regardless of the values of the cross-group externalities;

(ii) the prices of the outside platform decrease after the merger if the cross-group externalities are strong enough;

(iii) the profits of all the platforms increase after the merger.

Notice that the merger cost savings create prerequisite motivations for the merging platforms to lower their prices. In other words, if $\Delta = 0$, the equilibrium prices of the merged platform are certainly higher than pre-merger no matter how strong the cross-group externalities are. The intuition behind this is that the merged platform will soften price competition post-merger since they do not need to compete for the agents located between them. Thus the merged platform has monopoly power over these agents.[9] The monopoly power is so dominant that it is optimal for the merged platform to increase their prices despite the cross-group externalities. As for the outside platform, if $\Delta = 0$, it will lower its prices post-merger when the cross-group externalities are strong, i.e. $\alpha > \frac{2}{9}t$. Although its competitor increases prices, the outside platform chooses to decrease its prices to attract more agents on both sides with the significant cross-group externalities. So the merger free rider problem can be avoided with strong cross-group externalities in two-sided markets.[10] The prices of the merged platform and the outside platform are not strategy complements after the merger.

[9] A merger will not affect the equilibrium prices without the cost savings unless it is made between neighboring platforms. The merged platform hopes to gain by softening price competition between them, but this happens only if they are competing for the same agents. Non-adjacent platforms do not compete for the same agents, so there are no effects on the solution to each products maximization problem.

[10] In one-sided markets, if a horizontal merger does not induce any cost efficiency, besides the merged firms, the outside firm will also increase its price since the merged firms are competing less against it, which can be considered as the merger free rider problem.

Moreover, the profits of all the platforms increase after the merger, and the outside platform benefits more from the merger than an insider of the merger if the cross-group externalities are weak enough. The intuition is that the increase in prices brought by the merged platform shifts demand away to the outside platform. Although the merged platform maintains all the agents located between insiders and increases their prices, this situation is not as profitable as the increase in demand that the rival has. The merged platform cannot attract enough agents on the two sides with the weak cross-group externalities to compensate the higher prices resulting from the market power. Therefore, the platforms may be willing to be an outsider over being involved in a merger without cost savings since there is no chance to lose profits being an outsider, which strengthens the motivation to explore horizontal mergers with merger cost savings.

Next, I focus on exploring the impacts of the horizontal mergers with cost savings in the two-sided market. Proposition 2.1 demonstrates the comparison of the equilibrium prices between pre-merger and post-merger by comparing Eq.2.4 and 2.5.

Proposition 2.1 (**Price Comparison**) *Consider a horizontal merger between two out of the three symmetric platforms in the two-sided market,*

(i) if and only if the merger cost savings are sufficiently strong, i.e., $\Delta > \frac{t}{3}$, *regardless of the values of the cross-group externalities, the prices of all the platforms decrease after the merger, and the prices of the merged platform are lower than that of the outside platform;*

(ii) if and only if the merger cost savings are not sufficiently strong but the cross-group externalities are strong enough, i.e., $\Delta \in (\frac{t}{3} - \frac{3}{4}\alpha, \frac{t}{3})$, *the prices of all the platforms decrease after the merger, and the prices of the merged platform are higher than that of the outside platform;*

(iii) if and only if the merger cost savings and the cross-group externalities are not strong enough, i.e., $\Delta \in (\frac{t}{3} - \frac{3}{2}\alpha, \frac{t}{3} - \frac{3}{4}\alpha)$, *the prices of the merged platform increase and the prices of the outside platform decrease after the merger.*

The impact of a horizontal merger on prices depends jointly on the intensities of the cross-group externalities and the cost savings. According to Lemma 2.3, the merger cost savings create primary internal incentives for the merged platform to lower their prices by passing part of efficiency to the agents directly. Thus when the merger cost savings are sufficiently significant that $\Delta > \frac{t}{3} - \frac{3}{4}\alpha$, all the platforms will lower their prices due to intensified competition. Although the outside platform does not achieve any cost savings, it has to lower its prices as well to compete to the merged platform. Because the merged platform has some market power over the agents located between them, their prices are higher than the outside platform upon most occasions of Δ. However, if the cost savings are so large that $\Delta > \frac{t}{3}$, the prices of the merged platform are lower than outside platform since the merged platform burdens a larger part of the cost savings.

The existence of the cross-group externalities makes a difference in the post-merger prices by weakening the role that the cost savings play in the price decline. To be specific, the strong cost savings are only the sufficient condition of the post-merger prices decreasing, but not the necessary condition. If the cost savings are weak, the post-merger prices will also decrease as long as the cross-group externalities are strong enough. For instance, $\Delta > \frac{t}{3} - \frac{3}{4}\alpha$ can be established with a small Δ (i.e. $\Delta < \frac{t}{3}$) if the cross-group externalities are significant. Therefore, all the platforms can still lower their post-merger prices even though the cost savings are weak. In other words, given the significant cross-group externalities, the strong cost savings from the merger are less necessary for the prices to decline. The platforms are provided with extra incentives to lower their prices with the cross-group externalities since they can attract more agents on both sides to participate by decreasing prices to at least one side. When the cross-group externalities are strong, the desire to lower prices can fully dominate the desire to raise prices resulting from increased market powers even with the weak cost savings. On the contrary, the weak cross-group externalities cannot fully offset the market power effect from the merger, and it needs strong cost savings for the merged platform to lower their prices.

Similar to the results in the case of no cost savings, if the merger cost savings and the cross-group externalities are not strong, the merged platform will increase its prices, but the outside platform will decrease its prices to compete to the merged entity.

In summary, although the existence of the merger cost savings is essential for the post-merger prices to decline, the strong cost savings are less necessary with the significant cross-group externalities in the two-sided market since the platforms have an additional stronger internal incentive to lower prices.

Corollary 2.1 *Post-merger prices in the two-sided market can decrease with weak cost savings, which is impossible in traditional one-sided markets that post-merger prices only decrease with strong cost savings.*

If merger cost savings are generated, all platforms in a two-sided market and firms in a one-sided market may charge lower prices after the merger. In the traditional one-sided market, if and only if the significant cost savings can lower the prices of all the firms, say $\Delta > \frac{t}{3}$, and the price of the merged firm is lower than that of the outside firm.[11] In other words, if and only if the price of the merged firm is lower after the merger, the price of the outside firm will decrease its price as well but not as low as the merged firms'. However, in the two-sided market, relying on the cross-group externalities, according to Proposition 2.1, the prices of the merged platform will be lower after the merger even when $\Delta < \frac{t}{3}$. If the cross-group externalities are not so strong that the merged platform increases its prices, the outside platform may still choose to lower prices after the merger.

Comparing Eq.2.7 and 2.8, we have the analysis of profits of all the platforms with the horizontal merger.

Proposition 2.2 (**Profit Comparison**) *Consider a horizontal merger between two out of the three symmetric platforms in the two-sided market,*

(i) the profit of the merged platform is higher than the sum of profits of these two platforms pre-merger regardless of values of cost savings and cross-group externalities;

[11] I model the one-sided market with three perfectly symmetric firms equidistantly located in the Salop circle. All consumers are uniformly distributed along the circle, and they consume at most one unit of products or services that firms provide. The setting can be considered as letting $\alpha = 0$ in the original model. Thus two sides of the agents do not value each other and only care about the content platforms serve. Assuming the platforms do not serve multiple products, the two sides integrate into one group of the consumers in the market.

(ii) the profit of the outside platform after the merger is lower if the cross-group externalities or the merger cost savings are strong enough.

Regardless of the values of the cost savings and the cross-group externalities, the merged platform always earns a higher profit after the merger since the market power of the merged platform is dominant. A merger between neighboring platforms competing in prices is always profitable for them especially the cost savings are involved. The merged platform either benefits from higher prices resulting from the market power or more participation resulting from the lower prices. Meanwhile, it enjoys the cost savings when serving the agents.

As for the outside platform, the comparison result depends on the values of α and Δ.[12] When the cross-group externalities are strong enough, the profit of the outside platform always decreases post-merger. When the cross-group externalities are weak, the outsider platform is harmed by the merger with the significant cost savings. This result is directly related to the impacts of the merger on the prices. The outside platform has to lower its price when the merged platform lowers theirs, especially with the strong cross-group externalities or the cost savings. Since the outside platform does not enjoy the cost savings, their net profit per-agent will decline post-merger.[13]

At the very center of this chapter which is also the focus of antitrust authorities is the question of how horizontal mergers in a two-sided market impact economic welfare. Proposition 2.3 is the comparison of welfare between pre-merger and post-merger.

Proposition 2.3 (**Welfare Comparison**) *Consider a horizontal merger between two out of the three symmetric platforms in the two-sided market,*

(i) consumer welfare is higher after the merger if the cross group externalities or the merger cost savings are strong enough, i.e., $\Delta > \frac{29}{84}t - \frac{27}{28}\alpha$;

[12]The profit of the outside platform post-merger is lower if the inequality $14t^2 + 108\alpha^2 - 81t\alpha + 108\alpha\Delta - 48t\Delta + 18\Delta^2 > 0$ is established.

[13]The equilibrium price of the outside platform has a reduction of $\frac{1}{3}\Delta$. Since its cost does not change after the merger, the impact of the cost savings on its net per-agent profit is negative. Similarly, for the merged platform, the impact of the cost savings on the equilibrium price of the merged platform is the reduction of the amount of $\frac{2}{3}\Delta$. Since the cost is reduced by Δ, the net profit of per-agent participation increase by $\frac{1}{3}\Delta$ post-merger.

(ii) producer surplus is always higher after the merger regardless of the values of the cross-group externalities and the merger cost savings;

(iii) social welfare is higher after the merger regardless of the merger cost savings if the cross-group externalities are strong enough.

The surplus of a consumer on a platform in a two-sided market consists of two primary factors according to the model: the cross-group externality effect-the number of agents on the other side on the platform; and the price effect that the price he pays to the platform. So the changes in both two aspects need to be analyzed when considering the impact of a merger on consumer welfare.

According to Proposition 2.1, if $\Delta > \frac{29}{84}t - \frac{27}{28}\alpha$, the prices of the outside platform are lower after the merger, and its market share is larger than $\frac{1}{3}$ on each side. So the agents on the outside platform always have a higher surplus after the merger. Therefore, the analysis of the change of consumer surplus on the merged platform is essential to the change of consumer welfare in the entire market.

By the assumption that the merged platform merge their agents, the agents who join one of the merged platform can always connect with more agents on the other side after the merger. In other words, a large share of the agents in the market always gain benefits from the cross-group externality effect. Hence, if the cost savings or the cross-group externalities are significant such that $\Delta > \frac{t}{3} - \frac{3}{4}\alpha$, the merged platform, by Proposition 2.1, lowers its prices after the merger, the consumer welfare will increase after the merger. On the contrary, when the cost savings or the cross-group externalities are not that significant such that $\Delta \in (\frac{29}{84}t - \frac{27}{28}\alpha, \frac{t}{3} - \frac{3}{4}\alpha)$, the post-merger consumer welfare increases as well, and according to the result (iii) in Proposition 2.1, the merged platform will increase their prices under this circumstance.[14].

Therefore, if the cross-group externalities are strong enough, the cross-group externality effect and the price effect will work in the same direction. Consumer welfare post-merger increases because not only that the agents on one side can approach more on the other side

[14] In this case if $t < 21\alpha$, $\frac{29}{84}t - \frac{27}{28}\alpha < \frac{t}{3} - \frac{3}{4}\alpha$.

but also the prices decline. If the cross-group externalities are not very strong, consumer welfare can still increase if the cross-group externality effect dominates the price effect. The agents on the merged platform are willing to take higher prices if they can connect with more agents on the other side. This conclusion confirms the argument in Evans (2003) that mergers that increase the number of consumers on one side increase the value on the other side. Hence, consumer welfare may increase even though the price increase on one side or in total. So mergers are beneficial to the agents in the two-sided market with the significant cross-group externalities even though the prices they pay to the merged platform may increase.

As for producer surplus, regardless of the value of cost savings, the industry profit post-merger is always higher because the positive gain from the merged platform can adequately compensate for the possible loss from the outside platform. Since producer surplus is always higher post-merger, social welfare is undoubtedly higher when consumer surplus is higher post-merger. If applying the measurement that total social welfare is equal to the sum of consumer welfare and industry profit, the social welfare will be higher post-merger with strong cross-group externalities, i.e., $t < 6\alpha$. Under this circumstance, consumer surplus is harmed slightly or even increase post-merger.[15]

Corollary 2.2 *Consumer welfare in the two-sided market may increase even when the merged platform increases prices, which is impossible in traditional one-sided markets that consumer welfare increases only if the merged firm decreases price.*

In the traditional one-sided market, because the price is the primary determinant of consumer surplus, consumer welfare will increase only if the merged firm lowers their price due to their larger market share. Different from the one-sided market, the agents in the two-sided market also care about the participation of the other side on platforms, as well as the prices they pay to platforms. According to Proposition 2.3, when the cross-group externalities are significant, mergers are consumer welfare enhancing even though the merged

[15]Consisting with a vast amount of the literature, the welfare of agents and producers should be given equal weight.

platform increases prices. In other words, the agents are willing to pay higher prices to access more agents on the other side after the merger if the extent they care about the other side is strong.

In conclusion, the above propositions provide some policy implications to antitrust authorities. If antitrust authorities are more concerned about price issues, I argue that post-merger prices will decline in the two-sided market due to the strong cross-group externalities. If antitrust authorities consider consumer welfare to a greater extent when evaluating a proposed merger, they may eliminate some merger cases in two-sided markets which may increase prices but will increase consumer welfare if agencies merely apply the price evaluation criteria. Moreover, not many economists would accept that antitrust policy should aim to maximize consumer surplus alone, without considering profits. In a conventional economic view, the appropriate goal of the antitrust policy is to maximize overall welfare. In this case, antitrust agencies may be more positive about mergers in two-sided markets since social welfare always increases with significant cross-group externalities.

2.5. Mergers between Asymmetric Platforms

In the previous section, I discuss how horizontal mergers impact a two-sided market with three symmetric platforms, so mergers of any two platforms will have the same impact on the market. In the real world, platforms may have different costs that merger between any two platforms may have different impacts on consumers or society. If there are multiple proposed mergers in an industry that are all beneficial to consumers under some conditions according to the conclusions in the previous section, antitrust authorities may have to decide which merger is preferable regarding consumer or social welfare. If those platforms are aware of antitrust authorities' preferences, endogenous mergers can be induced that specific platforms will be more willing to participate in mergers.

To analyze different impacts of different mergers, I relax the constraint that all the platforms have the same constant cost by assuming that the platforms have different costs. Instead of comparing the equilibrium results between pre-merger and post-merger, I focus on comparing impacts of different mergers, in other words, whether a merger between more efficient platforms is superior to a merger between less efficient ones from any perspectives.

For simple and tractable comparisons, I assume that the three platforms have the heterogeneous constant costs, given $c_1 = c$, $c_2 = c + \varepsilon$, $c_3 = c + 2\varepsilon$, and $\varepsilon > 0$.[16] The merger generates cost efficiency that the cost of the merged platform is equal to the lowest one between the two merging parties' costs.

Assumption 2.3 $\varepsilon < min\{\frac{5}{9}t - \frac{5}{6}\alpha, \frac{2}{3}t - \frac{3}{2}\alpha\}$.

Similar to Assumption 2.1, Assumption 2.3 avoids the situation that the differences among the costs of all the platforms are so significant that the less efficient platforms will earn negative profits in the equilibrium.

Indicating the most efficient merger is the merger between the most efficient platform and the second most efficient platform. In this case, the most efficient merger is the merger between platform 1 and 2. Thus the constant unit cost of the merged platform is c. The outside platform, platform 3 has the cost $c + 2\varepsilon$.

By Lemma 2.2, the equilibrium prices after the merger are

$$\mathbf{p}^j_{1+2} = c + \frac{2}{3}\varepsilon + \frac{5}{9}t - \alpha; \quad \mathbf{p}^j_3 = c + \frac{4}{3}\varepsilon + \frac{4}{9}t - \alpha \tag{2.9}$$

with $j \in \{S, B\}$.

It is ambiguous about the comparison of the equilibrium prices between the merged platform and the outside platform as two factors are working in opposite directions. On the one hand, the merged platform achieves the monopoly power over the agents located between them. Thus they have incentives to charge higher prices than their competitor which is reflected by a more significant impact of the transportation costs on their prices. On the other hand, the costs of the merged platform are lower than the outside platform

[16]Prior to the merger, each platforms $p^j_1 = c + \frac{3}{5}\varepsilon + \frac{t}{3} - \frac{\alpha}{2}$, $p^j_2 = c + \varepsilon + \frac{t}{3} - \frac{\alpha}{2}$, and $p^j_3 = c + \frac{7}{5}\varepsilon + \frac{t}{3} - \frac{\alpha}{2}$. I do not provide the comparisons between pre-merger and post-merger in this section since the results have no significant difference with those in the previous section. I focus on comparisons between different horizontal mergers.

that they can pass a more substantial part of efficiency to their agents than the rival which is reflected by a smaller impact of ε on its prices.

The market shares can be written as

$$\mathbf{n}_{1+2}^{j} = \frac{5t - 9\alpha + 6\varepsilon}{9(t - 2\alpha)}; \quad \mathbf{n}_{3}^{j} = \frac{4t - 9\alpha - 6\varepsilon}{9(t - 2\alpha)}$$

The market share of the merged platform is larger than that of the outside platform regardless of the value of ε because the merged platform is more efficient.

The least efficient merger is referred to as the least efficient platform merging with the second least efficient platform. In this case, it is the merger between platform 2 and 3. The per-agent cost of the merged platform is $c + \varepsilon$. The outside platform, platform 1, has a constant cost of c. The equilibrium prices post-merger are, by Lemma 2.2,

$$\mathbf{p}_{2+3}^{j} = c + \frac{2}{3}\varepsilon + \frac{5}{9}t - \alpha; \quad \mathbf{p}_{1}^{j} = c + \frac{1}{3}\varepsilon + \frac{4}{9}t - \alpha \tag{2.10}$$

with $j \in \{S, B\}$.

In the case of the least efficient merger, the merged platform always charges higher prices than the outside platform because of the two factors working in the same direction that both the transportation cost and ε have more significant impacts on the merged parties' prices. The market shares are

$$\mathbf{n}_{2+3}^{j} = \frac{5t - 9\alpha - 3\varepsilon}{9(t - 2\alpha)}; \quad \mathbf{n}_{1}^{j} = \frac{4t - 9\alpha + 3\varepsilon}{9(t - 2\alpha)}$$

The merged platform has fewer market shares than the outside platform if their costs are large enough such as $\varepsilon > \dfrac{t}{6}$.

By comparing Eq.2.9 and 2.10, Proposition 2.4 demonstrates the result of price comparison between the two mergers.

Proposition 2.4 (**Price Comparison**) *Consider two horizontal mergers in a two-sided market with three platforms: the most efficient merger that the most efficient platform merges* *with the second most efficient platform, and the least efficient merger that the least efficient platform merges with the second least efficient platform,*

(*i*) *the prices of the merged platforms in the two mergers are same;*

(*ii*) *the prices of the outside platform in the least efficient merger are the lowest one among all the platforms in the two mergers.*

The reason that the prices of the merged parties in the two mergers are the same is that the costs of their rivals are different. According to Lemma 2.2, the merged platform has to burden part of the cost of the outside platform, as well as part of its own costs. Since the outside platform in the most efficient merger is the one with the highest cost but that in the least efficient merger is the one with the lowest cost, the merged platforms in these two mergers will charge the same prices after balancing all these costs. Therefore, the merged platform in the most efficient merger does not achieve price advantages than the merged platform in the least efficient merger. Also, the prices of the outside platform in the least efficient merger are lower than all the platform in the most efficient merger because it has the lowest cost and the cost of its competitor is not very large compared with the most efficient merger.

Proposition 2.5 (**Welfare Comparison**) *Consider two horizontal mergers in a two-sided market with three platforms: the most efficient merger that the most efficient platform merges with the second most efficient platform, and the least efficient merger that the least efficient platform merges with the second least efficient platform,*

(*i*) *consumer welfare is higher with the least efficient merger;*

(*ii*) *producer surplus is higher with the most efficient merger;*

(*iii*) *social welfare is higher with the most efficient merger.*

It may be a little counter-intuitive that the most efficient merger is not superior to the least efficient merger regarding consumer welfare. According to Proposition 2.4 , the merged platform in the most efficient merger does not have price advantage than the merged ones in the least efficient merger. Meanwhile, the prices of the outside platform in the most efficient merger are higher. Hence from the perspective of price structure, the most efficient merger is inferior to the least efficient merger because the cost asymmetry post-merger is more intensive after the most efficient merger. To be specific, with the most efficient merger, costs of the merged platform and the outside platform are c and $c + 2\varepsilon$; with the least efficient merger, costs are $c + \varepsilon$ and c. Although the merged platform in the most efficient merger occupies a larger share than any platform in the least efficient merger, the price of the outside platform in this merger is so high that even the cross-group externalities can not dominate the price loss. Besides, the outside platform in the least efficient merger also serves a large number of agents with the lowest prices. Therefore, consumer surplus is higher with the least efficient merger.

The industry profit is higher in the most efficient merger since the most efficient platforms take over a more substantial portion of the market and the least efficient platform occupies relatively small. As a result, social welfare is higher with the most efficient platform since the gain on producer surplus can compensate for the loss of consumer surplus.

In conclusion, from the perspective of consumer welfare, the least efficient merger is preferable since it lessens the cost asymmetry to a greater extent by improving the efficiency of the least efficient platforms. From the perspective of social welfare, the most efficient merger is better due to the higher industry profit of this merger. Although the cross-group externalities do not make a difference in outcomes, I provide theoretical evidence to prove that when confronted with different types of mergers in a two-sided market, it is essential for antitrust authorities to consider that the less efficient mergers are more preferred than the more efficient mergers if they concern about consumer welfare.

2.6. Extensions

In this section, I relax three assumptions of the primary model in Section 2.3 separately. Firstly, I discuss the situation that what if the merging platforms choose not to merge their agents with each other. It may happen when the merging platforms specialize in different areas; secondly, I allow the merging platform to relocate by assuming that both platforms locate to the opposite of the outside platform in the circle; Lastly, I relax the single-homing assumption by allowing one side can multi-home in the equilibrium.

2.6.1. Separate Operation

I relax the assumption that the merging platforms merge their agents by assuming that platform 1 and 2 may decide not to merge their consumers, especially when they provide substantial heterogeneous services. Therefore, similar to mergers in a traditional market, the merging platforms charge agents with prices that maximize their joint profits.

The profit functions of platforms are

$$\pi_{1+2} = \sum_{j=\{S,B\}} (\mathbf{p}_1^j - c + \Delta)\mathbf{n}_1^j + (\mathbf{p}_2^j - c + \Delta)\mathbf{n}_2^j$$

$$\pi_3 = (\mathbf{p}_3^S - c)\mathbf{n}_3^S + (\mathbf{p}_3^B - c)\mathbf{n}_3^B.$$

Solving out first order conditions, the equilibrium prices are

$$\mathbf{p}_1^S = \mathbf{p}_1^B = \mathbf{p}_1 = c - \frac{2\Delta}{3} + \frac{5}{9}t - \frac{5}{6}\alpha$$

$$\mathbf{p}_2^S = \mathbf{p}_2^B = \mathbf{p}_2 = c - \frac{2\Delta}{3} + \frac{5}{9}t - \frac{5}{6}\alpha$$

$$\mathbf{p}_3^S = \mathbf{p}_3^B = \mathbf{p}_3 = c - \frac{\Delta}{3} + \frac{4}{9}t - \frac{2}{3}\alpha.$$

And the equilibrium market shares are

$$\mathbf{n}_1^S = \mathbf{n}_1^B = \mathbf{n}_1 = \frac{6\Delta + 10t - 15\alpha}{18(2t - 3\alpha)}$$

$$\mathbf{n}_2^S = \mathbf{n}_2^B = \mathbf{n}_2 = \frac{6\Delta + 10t - 15\alpha}{18(2t - 3\alpha)}$$

$$\mathbf{n}_3^S = \mathbf{n}_3^B = \mathbf{n}_3 = \frac{8t - 12\alpha - 6\Delta}{9(2t - 3\alpha)}.$$

Therefore, profits of the merging platforms are

$$\pi_{1+2} = \frac{(10t + 6\Delta - 15\alpha)^2}{81(2t - 3\alpha)}.$$

The comparison results of profits of the merged platforms between this type of merger and the one I discuss in Section 2.3 are ambiguous. Thus I cannot determine which one is preferable for the merging parties regarding the profit. However, if the merging platforms choose not to merge their agents, they will charge lower prices when $\Delta > \frac{t}{3} - \frac{1}{2}\alpha$. Based on Proposition 2.1, when they merge their agents, their prices are lower if $\Delta > \frac{t}{3} - \frac{3}{4}\alpha$. Thus the impact of the cross-group externalities on the equilibrium prices is stronger with full openness. So openness is preferable to consumers concerning consumer surplus.[17]

2.6.2. Relocating

I relax the assumption that locations do not change post-merger. The two merging platforms may choose to integrate horizontally and become one platform in the market. Under this circumstance, I assume that after platform 1 and 2 merging, they decide to keep platform 1 brand and locate at the opposite to platform 3 in the circular model.[18]

The demand functions of platform 1 and 3 are

$$\mathbf{n}_1^j = \frac{1}{2} + \frac{4\alpha(\mathbf{p}_3^{-j} - \mathbf{p}_1^{-j}) + 2t(\mathbf{p}_3^j - \mathbf{p}_1^j)}{2t^2 - 8\alpha^2}$$

$$\mathbf{n}_3^j = \frac{1}{2} + \frac{4\alpha(\mathbf{p}_1^{-j} - \mathbf{p}_3^{-j}) + 2t(\mathbf{p}_1^j - \mathbf{p}_3^j)}{2t^2 - 8\alpha^2}$$

with $j\{S, B\}$.

[17] If this type of merger is applied to the multi-homing situation, we have the same equilibrium price as in the previous merger with merging the agents. Therefore, the platforms will not choose this merger when the sellers are multi-homing since it will result in them to incur double costs from accepting the sellers.

[18] Locations do not impact the demand functions in Salop's circle model with two firms.

Platform 1 enjoys the cost saving after the merger, the profits of platform 1 and 3 are

$$\pi_1 = (\mathbf{p}_1^S - c + \Delta)\mathbf{n}_1^S + (\mathbf{p}_1^B - c + \Delta)\mathbf{n}_1^B, \quad \pi_3 = (\mathbf{p}_3^S - c)\mathbf{n}_3^S + (\mathbf{p}_3^B - c)\mathbf{n}_3^B.$$

Solving out first order conditions, the optimal prices are

$$\mathbf{p}_1^S = \mathbf{p}_1^B = \mathbf{p}_1 = c - \frac{2}{3}\Delta + \frac{t}{2} - \alpha, \quad \mathbf{p}_3^S = \mathbf{p}_3^B = \mathbf{p}_3 = c - \frac{1}{3}\Delta + \frac{t}{2} - \alpha.$$

And the profits of the platforms are

$$\pi_1 = \frac{(3t + 2\Delta - 6\alpha)^2}{18(t - 2\alpha)}, \quad \pi_3 = \frac{(3t - 2\Delta - 6\alpha)^2}{18(t - 2\alpha)}.$$

It can be easily proven that the profit of the merging platform in this case is lower than that in the Section 2.3.[19] It is obvious that the equilibrium prices of the merged platform in this case are lower than that in the original model. Since the merging platforms lose the monopoly power that they cannot gain by charging higher to the agents located between them after the merger, price competition is fiercer after the brand of platform 2 exiting the market. Thus keeping two brands after the merger is more profitable for the merging platforms.

2.6.3. Multi-homing

In this section, I consider the multi-homing situation with the three platforms, in which consumers are allowed to join more than one platforms at the same time. Armstrong and Wright (2007) have investigated multi-homing equilibrium with the classic Hotelling model.[20] I can apply and extend their analysis to study the equilibrium of pre- and post-merger with Salop spatial model. Since it is not reasonable to assume that all the agents choose to multi-home,[21] I focus on the equilibrium that consumers are multi-homing on one side and

[19] $\pi_1 = \frac{t-2\alpha}{2} = \frac{\frac{81}{2}(t-2\alpha)^2}{81(t-2\alpha)} = \frac{2(\frac{9}{2}t-9\alpha)^2}{81(t-2\alpha)} < \frac{2(5t-9\alpha)^2}{81(t-2\alpha)}$.

[20] They construct the equilibrium when two platforms locate at the endpoints of the Hotelling model.

[21] If every agent on one side multi-home, the agents on the other side will not multi-home with non-negative prices.

single-homing on the other side.

Armstrong and Wright (2007) have proven that a unique equilibrium exists in which all agents are single-homing in the classic Hotelling model. Therefore the original model is needed to be modified to investigate equilibrium with the setting of multi-homing. According to Armstrong and Wright (2007), I relax the restriction that exogenous product differentiation for both sides by assuming that product differentiation only exists on one side. I consider a market where the three platforms are viewed as identical by the sellers and as differentiated by the buyers. Moreover, I assume that the sellers can multi-home and the buyers are only allowed to single-home in the equilibrium.

Assumption 2.4 *$v_S = 0$ and v_B is large enough that every buyer joins a platform in equilibrium.*

Assumption 2.5 *$t_S = 0$, $t_B = t > \alpha_B$.*

Assumption 2.6 *$\alpha_S \geq 3c, \Delta \leq \alpha_S - 3c$.*

Assumption 2.6 makes sure that platforms earn non-negative profits pre-and post-merger.

Assumption 2.7 *$t < \frac{3}{2}\alpha_S, \Delta < \frac{3}{2}\alpha_S - \frac{2}{3}t$.*

This assumption makes sure the equilibrium is valid.

Following Armstrong and Wright (2007), we have that all the equilibrium involves the sellers being left with zero surplus in Salop model.[22]

Lemma 2.4 *Consider three symmetric platforms competing in prices in a two-sided market with one side single-homing, and the other side multi-homing. The multi-homing side has zero surplus in all the equilibrium.*

[22] Armstrong and Wright (2007) apply two refinements to restrict the range of possible subgame equilibrium. *Inertia condition* requires that any equilibrium must have the property that if one platform changes its prices and the original demand configuration remains consistent with the deviation, this configuration should be selected. *Monotonicity condition* requires that any deviation by a platform from the equilibrium that weakly reduces both prices must weakly increase that platform's demands from the two group.

No matter how many platforms the sellers choose to participate in the equilibrium, platforms always fully extract their surplus. I focus on equilibrium with the sellers choose to join all the three platforms. With $\alpha_B > 0$, I can rule out an equilibrium that the platforms do not accept the sellers. Therefore, I focus on constructing a symmetric equilibrium of the sellers select to multi-home. Since platforms' strategies are also symmetric in the Salop model without the merger, they will charge $\frac{\alpha_S}{3}$ to the sellers and charge the same equilibrium price p_B to the buyers in such an equilibrium before the merger.

There are two possible deviations in the equilibrium to be considered. First, given that the sellers must be fully extracted their surplus in the equilibrium, there is always a profitable deviation for a platform: if platform 1 undercut the proposed price to the buyers very slightly, it will result in more of the buyers joining platform 1, then fewer buyers to join platform 2 and 3. Since the sellers have zero surplus with the proposed equilibrium prices of platform 2 and 3, the decrease in the numbers of buyers on platform 2 and 3 makes the sellers have negative surplus. Hence the sellers will not choose to join these two platforms with platform 1's deviation. Therefore, an insignificant decrease in the price p_1^B will lead the sellers to join platform 1 exclusively, which will then lead an increase in the number of buyers on platform 1 with $\alpha_B > 0$. To avoid such a profitable deviation in the equilibrium, the prices should be below the cost. And another profitable deviation is that one platform may increase its price to the buyers and decrease the price to the sellers correspondingly that the sellers still choose to multi-home.

Combining the above profitable deviations, the price to the single-homing side in any symmetric subgame perfect equilibrium before the merger is

$$max\{0, \frac{t}{3} + c - \alpha_S\} \leq p_B \leq c.$$

To make a straightforward comparison between pre-merger and post-merger, I select the best equilibrium with the highest price to the buyers. Therefore, the prices to the buyers equal to the platforms' costs in the equilibrium before the merger.

Lemma 2.5 *Consider three symmetric platforms competing in prices in a two-sided market with one side single-homing, and the other side multi-homing, in the best equilibrium, the price to the multi-homing side is* $p_S = \frac{\alpha_S}{3}$*, and the price to the single-homing side is* $p_B = c$.

After the merger, the platforms still fully exploit the surplus of the multi-homing sellers, But the analysis of the prices to the single-homing buyers after the merger is slightly different from pre-merger since the costs of the platforms are no longer identical. Platform 1 and 2 achieve the cost savings Δ post-merger, and platform 3 still has the cost c. Hence, the merging platforms can always undercut platform 3's price if its price to the buyers is higher than $c - \Delta$. Then the sellers will join the merging platforms exclusively. Therefore platform 3 has to charge a price less or equal to $c - \Delta$ to its buyers in the equilibrium. Meanwhile, the equilibrium price of the merging platforms to the buyers should be no larger than $c - \Delta$ as well. Otherwise, the merging platforms can still gain more profits by slightly decreasing their price to the buyers. Therefore, $\mathbf{p}^B_{1+2} \leq c - \Delta$ and $\mathbf{p}^B_3 \leq c - \Delta$ in the equilibrium. Besides, we have to make sure that platforms cannot gain more profits by increasing their price to the buyers and decrease their price to the sellers that the sellers still choose to multi-home. In the subgame perfect equilibrium post-merger, the price to the single-homing side is

$$max\{0, \frac{t}{3} + \frac{1}{2}(c + \mathbf{p}^B_3 - \Delta - \alpha_S)\} \leq \mathbf{p}^B_{1+2} \leq c - \Delta$$

$$max\{0, \frac{t}{6} + \frac{1}{2}(c + \mathbf{p}^B_{1+2} - \alpha_S)\} \leq \mathbf{p}^B_3 \leq c - \Delta.$$

With asymmetric feature post-merger, it is difficult to verify the multiple equilibria. I still select the best equilibrium with the highest prices of all the platforms to the buyers.

Lemma 2.6 *Consider a horizontal merger between two of three symmetric platforms in a two-sided market with one side single-homing, and the other side multi-homing, in the best equilibrium, the price of the merged platform to the single-homing side is* $\mathbf{p}^B_{1+2} = c - \Delta$*, and to the multi-homing side is* $\mathbf{p}^S_{1+2} = \frac{2}{3}\alpha_S$*; the price of the outside platform to the single-homing side is* $\mathbf{p}^B_3 = c - \Delta$*, and to the multi-homing side is* $\mathbf{p}^S_3 = \frac{1}{3}\alpha_S$.

By comparing Lemma 2.5 and 2.6 directly, we can have the comparison result of equilibrium prices before and after the merger.

Proposition 2.6 (**Price Comparison**) *Consider a horizontal merger between two out of three symmetric platforms in a two-sided market with one side single-homing, and the other side multi-homing,*

(i) all the platforms decrease their prices to the single-homing side after the merger;

(ii) the prices to the multi-homing side do not change after the merger.

All the platforms cannot charge a profitable price to the buyers since they are afraid of being cut down by their rivals so that they may lose a large amount of the buyers and all the sellers. Therefore, the platforms have to offer free their services to the buyers without earning profits and get compensations from the sellers by fully extracting all surplus of them. After the merger, the cost of the merged platform reduces to $c-\Delta$, then the outside platform has to keep its price low enough to make sure the merged platform can not undercut its price. Thus all the buyers fully and equally enjoy the cost savings after the merger that all the platforms decrease their prices to the buyers to $c - \Delta$. With the same prices to the buyers after the merger, the price structure to the multi-homing sellers does not change. Hence the merger has no impact on the price of the multi-homing side.

Proposition 2.7 (**Profit Comparison**) *Consider a horizontal merger between two out of three symmetric platforms in a two-sided market with one side single-homing, and the other side multi-homing,*

(i) the merged platform earns more profits after the merger;

(ii) the outside platform earns fewer profits after the merger.

The profit of the merged platform is always larger than the sum of profits of platform 1 and 2 pre-merger regardless of the value of the cost savings. Platform 1 and 2 both incur costs when they accept the sellers before the merger, but the sellers can approach all the buyers on platform 1 and 2 by only joining any one of them after the merger. Therefore, the merged platform not only benefits from the cost savings from per agent but also avoid generating double costs when serving all the sellers. Hence, the profit of the merged platform increases significantly no matter how much the value of cost savings are. The profit of the outside platform is harmed since it does not enjoy cost savings.

Proposition 2.8 (**Welfare Comparison**) *Consider a horizontal merger between two out of three symmetric platforms in a two-sided market with one side single-homing, and the other side multi-homing,*

(i) consumer surplus increases after the merger;

(ii) producer surplus increases after the merger;

(iii) social welfare increases after the merger.

The buyers have a higher surplus because of the lower prices and more sellers on the platforms, and the surplus of the sellers do not change post-merger. Hence, consumer surplus increases post-merger. As for industry profit, it increases as well since the gain from the merged platform dominates the loss from the outside platform. Therefore, social welfare increases post-merger.

2.7. Conclusion

I study the effects of horizontal mergers in a two-sided market with three platforms based on Salop's circle model. Generally speaking, I provide the theoretical analysis of post-merger price and welfare based on the amount of merger cost savings and the size of cross-group externalities in a two-sided market, and develop a model of platform competition that incorporates both insiders and outsiders of a horizontal merger to explore the full impact of the merger on the two-sided market.

To be specific, I demonstrate that the horizontal merger will lead to lower prices of the merged platform and the outside platform if the cost savings or the cross-group externalities are significant. Moreover, the existence of the cross-group externalities makes the strong cost savings less necessary for the prices to decline. In other words, the post-merger prices will decrease in the two-sided market with the weak cost savings if the cross-group externalities are strong enough. Note that in a traditional one-sided market which has a similar model setting as the two-sided market but has only one side of consumers, only the strong merger cost savings lead to lower prices. The agents in the two-sided market care about not only the prices they pay to a platform but also the presence of the agents on the other side on the platform. In other words, the agents on one side are willing to pay higher prices if they can connect with more agents on the other side on the platform, as long as their utilities increase. Since the merged platform will merge their agents on the two sides, the agents on one of the merged platform can approach all the agents on the other side that the two platforms have although they are connected through a single platform. As a result, consumer welfare will still increase although the prices of the merged platform increase to some extent. Therefore, horizontal mergers in a two-sided market are very different from that in a traditional one-sided market from the perspectives of price and welfare.

Therefore, when antitrust authorities evaluate proposed horizontal merger cases in two-sided markets, it is essential for them to consider cross-group externalities. In particular, if they concern about rising price after the merger, it is crucial for them to be aware of that post-merger prices may decline with weak merger cost savings, especially when cross-group externalities are strong. Besides, consumer surplus may be enhanced after the merger even though the merging entity increases its price to some extent. So antitrust authorities may overrule horizontal mergers which are beneficial to consumers if they stick to the price evaluation criteria.

Besides, I investigate which type of mergers is more efficient for the agents or the entire society when the platforms have different costs. I argue that a merger between the more efficient platforms is inferior to a merger between the less efficient platforms regarding consumer welfare since the latter lessens cost asymmetry more, but the former is superior to the latter regarding social welfare since it generates a higher industry profit. Therefore, it is more beneficial to consumers if a horizontal merger between smaller platforms is approved under the premise that the proposed merger will increase consumer welfare.

Chapter 3

HIERARCHICAL STACKELBERG MODEL VERSUS COURNOT MODEL

3.1. Introduction

This chapter examines and compares the equilibrium outcomes and welfare between two different oligopoly models in the industrial organization. One is the hierarchical Stackelberg model in which firms choose output sequentially. The Stackelberg model which was developed by Heinrich von Stackelberg in 1934 is considered as a widely applied model of non-cooperative oligopoly behavior. Various research in the industrial organization has used the Stackelberg model to study multiple critical economic issues, such as entry and strategic pre-commitment (see, e.g., Basu, 1993; Tirole 1988). The other model is the standard Cournot model that firms simultaneously choose output. The Cournot model is also a widely used model in the industrial organization.

Stackelberg models for hierarchical oligopoly markets with a homogeneous product were studied by economists extensively. Two types of this model are mainly discussed. One is that the hierarchical Stackelberg model that each firm chooses its output at a stage sequentially, which is a multi-stage game. The other one is that the Stackelberg model with two stages that multiple leaders choose output simultaneously in the first place, then multiple followers determine their output simultaneously at the second stage, given the leaders' output. This chapter discusses the hierarchical Stackelberg model with multi-stages form. It can be thought that firms enter in sequence because some firms are aware of a profitable market before others or some entrants who have privileges to enter the market in advance than others. Although there is a considerable amount of research studying this type of hierarchical Stackelberg model, most of the existing literature restrict the model with identical firms in the market such that firms have the same marginal costs or the same cost functions.

Besides, fewer papers discuss the comparison of equilibrium outcomes between the hierarchical Stackelberg model and the standard Cournot model with firms of different costs. Since these two models are the two workhorses in the industrial organization and both the quantity competition models, we are always interested in the superiority of one of them from the society's or consumers' point of view. Therefore, this chapter aims to investigate the equilibrium of the hierarchical Stackelberg model with possible heterogeneous firms and compare the equilibrium results and welfare between the standard Cournot model and Stackelberg model.

I assume that the number of firms is exogenously determined which is n ($n \geq 2$) in the hierarchical Stackelberg model and the Cournot model. The firms have constant marginal costs which are possibly different from each other. The market demand is linear, and the firms all produce a positive amount in the equilibrium.

In the hierarchical Stackelberg model, I demonstrate that entry sequences impact the equilibrium outcomes. Among all the possible entry sequences, the equilibrium market output reaches the highest if the firms enter the market according to the sequence from the smallest marginal cost to the largest marginal cost; the equilibrium market output reaches the lowest if the entry sequence is the opposite.

As for the main results of equilibrium comparison between the two models, the equilibrium prices in the hierarchical Stackelberg model are always lower than the equilibrium price in the Cournot model. The exogenously determined entry sequence plays an essential role in the equilibrium price with nonidentical firms. I demonstrate that the equilibrium prices in the hierarchical Stackelberg model are lower regardless of entry sequences when compared with the Cournot model. Besides, I use the total welfare loss relative to the socially optimal level as the measurement of welfare. I demonstrate that the total welfare loss in the hierarchical Stackelberg model with the most efficient entry sequence, which is the order that firms enter the market sequentially from the lowest marginal cost to the highest cost, is less than the loss in the Cournot model. However, the comparison of the welfare loss is ambiguous between the Cournot model and the Stackelberg model with the least efficient entry sequence in which firms enter the market from the highest marginal cost to the lowest marginal cost.

3.1.1. Related Literature

Anderson and Engers (1992), and Boyer and Moreaux (1986) provide and show the existence of the unique equilibrium solutions to the hierarchical Stackelberg model that identical firms choose output sequentially, and the number of firms is exogenous and greater than two. Besides, Anderson and Engers (1992) also compare an m-firm Cournot model with a hierarchical Stackelberg model where m firms choose output sequentially. They conclude that the Stackelberg equilibrium price is lower, the production and total surplus are higher, and overall profits are lower in the Stackelberg model. There is also some literature analyzing the Stackelberg model where the number of firms is determined endogenously with identical firms, such as Robson (1990), Vives (1988) and Church and Ware (1996).

However, most of the existing literature that extends the hierarchical Stackelberg model to include more than two firms always assume identical cost functions to all the firms. Two significant problems may occur under this assumption. First, under this circumstance, the equilibrium output of each firm only relies on its timing of entry and has nothing to do with the number of firms in the market. It is reasonable to believe that the number of firms in the market is supposed to have an impact on a firm's output. Secondly, this assumption is too restrictive that the firms' costs are maybe actually different from each other. In the hierarchical Stackelberg model, firms choose the output sequentially; therefore, some elements such as the production technology or the input price probably different overtime which would lead to the different costs among the firms.

There is indeed some literature discussing the hierarchical Stackelberg model with heterogeneous firms case. For instance, Pal and Sarkar (2001) and Galegov and Garnaev (2008) analyze a Stackelberg oligopoly with nonidentical firms. Pal and Sarkar (2001) mainly study the impact of the entry of a new firm to the existing firms in the market. Galegov and Garnaev (2008) generalize the equilibrium of the hierarchical Stackelberg model composed of M firms arranged into N groups of firms.

This paper is different from the existing relevant literature by the following two aspects: first of all, the paper demonstrates that the exogenously determined entry sequence plays a vital role in the equilibrium outcome of the hierarchical Stackelberg model. Besides, the paper incorporates the comparison of equilibrium outcomes and welfare between the two models with heterogeneous firms.

The chapter is organized in the following way: Section 3.2 demonstrates the equilibrium outcomes of the hierarchical Stackelberg model and the Cournot model. Section 3.3 is the comparison results of the equilibrium outcomes and welfare loss between the two models. Section 3.4 is the conclusion. All proofs are in the Appendix.

3.2. The Model

Consider a market with n ($n \geq 2$) firms where the firms produce homogeneous products. Let q_i be the output produced by firm $i, (i = 1, 2, ..., n)$, and $Q = \sum_{i=1}^{n} q_i$ be the total market output. $p(Q) = a - bQ$ is the inverse demand function, where $a > 0$ and $b > 0$. The linear demand function for the model is well behaved which makes the comparison of the equilibrium tractable and straightforward. Firm i has a constant marginal cost of production, denoted by c_i. For simplicity, assuming zero fixed cost for all the firms.

In the hierarchical Stackelberg model (HSM), the firms sequentially decide how much to produce: firm 1 produces first; firm j produces after firm $j-1, (j = 2, ..., n)$. During its quantity choice, firm j knows the quantities chosen by the firms 1 to $j-1$ and takes them as given. Also, firm k considers how its choice of quantity will influence the quantities chosen by the firms $k+1, (k = 1, ..., n-1)$, to n. After firm n makes its quantity choice, the market price is determined by the inverse demand function. Each firm's objective is to maximize its profit.

3.2.1. Homogeneous Firms

In this subsection, I assume that the firms are identical that they have the same constant marginal cost such that $c_i = c$. Although Anderson and Engers (1992) discuss the HSM with the identical firms, to shed some light and deeply understand the model with the nonidentical firms, I incorporate the case of identical firms as well.

Assumption 3.1 *$a - c > 0$.*

Assumption 3.1 states that the market demand is sufficiently large to make each firm to produce a positive amount of product in the HSM.

Lemma 3.1 states the equilibrium for the HSM with identical firms.

Lemma 3.1 *Consider the hierarchical Stackelberg model with n firms having the same marginal cost c, the firms choose their output sequentially and face a linear inverse demand $p = a - bQ$. In the equilibrium, firm i, $(i = 1, 2, ..., n)$, produces*

$$q_i^{S*} = \frac{a-c}{2^i b}.$$

The market output is

$$Q_S^* = \frac{a-c}{b}(1 - \frac{1}{2^n}),$$

and the equilibrium market price is

$$p_S^* = \frac{a + (2^n - 1)c}{2^n}.$$

In the HSM with identical firms, the first firm entering the market will produce the largest quantity since it can adequately perform the first mover advantage. When the subsequent firms make their decisions, their quantities are decreasing exponentially. In other words, the production vector is a geometric series with a variance of $\frac{1}{2}$. This is a very straightforward result since firm 1 is the monopolist of the entire market and firm 2 is the monopolist of the residual market after firm 1's share, then firm 3 maximizes its profit after the production decisions of firm 1 and firm 2, so do all the subsequent firms. Thus from the perspective of the firms, each one can be seen as the monopolist of the residual market. Moreover, the equilibrium output for each firm only depends on its entry timing, not the total number of firms. However the overall output level, from the perspective of society, is influenced by the total numbers of firms instead of the entry sequence.

Corollary 3.1 *Consider the hierarchical Stackelberg model with n firms having the same marginal cost c, the firms choose their output sequentially and face a linear inverse demand $p = a - bQ$. If the number of firms, n, close to the infinity, the subgame perfect equilibrium in the hierarchical Stackelberg competition coincides with the equilibrium in the perfect competition.*

Proof. According to Lemma 3.1, if $n \to \infty$, then the total production quantity in the market tends to $\frac{a-c}{b}$, and the market price will be $p = c$ which leads to zero profit for each firm.

According to Corollary 3.1, when the number of identical firms that entering the market is sufficiently large, the first mover advantage can not bring more profits to the firms since each firm makes zero profit with the lowest price.

3.2.2. Heterogeneous Firms

In this subsection, I focus on the situation of firms with different marginal costs.

Assumption 3.2 $\frac{1}{b}(\frac{a + \sum_{i=1}^{n} 2^{n-i} c_i}{2^n} - c_i)2^{n-i} > 0.$

Assumption 3.2 states that the market demand is sufficiently large that each firm enters the market with a positive amount of product in the HSM.

Proposition 3.1 demonstrates the subgame perfect equilibrium of the HSM with nonidentical firms.

Proposition 3.1 *Consider the hierarchical Stackelberg model with n firms having constant marginal cost $c_i, (i = 1, 2, ..., n)$, the firms choose their output sequentially and face a linear inverse demand $p = a - bQ$. In the equilibrium, the equilibrium market output and price are*

$$Q_S^* = \frac{(2^n - 1)a - (\sum_{i=1}^n 2^{n-i} c_i)}{2^n b}$$

$$p_S^* = \frac{a + \sum_{i=1}^n 2^{n-i} c_i}{2^n},$$

the equilibrium output of firm i is

$$q_i^{S*} = \frac{1}{b}(p_S^* - c_i)2^{n-i}.$$

The equilibrium output of each firm not only depends on its entry timing but also the number of firms, which is different from the case of identical firms. For instance, the equilibrium output of firm k according to Proposition 3.1 is

$$q_k^{S*} = \frac{a + \sum_{i \neq k} 2^{n-i} c_i - 2^n(1 - 2^{-k})c_k}{2^k b}.$$

The impacts of the entry timing on the individual equilibrium output are the following. First, the earlier a firm enters the market, the larger its equilibrium output is. This is because the firm faces a relative larger residual market, which is consistent with the case of identical firms.[1] Second, the entering timing influences the effect of the firm's marginal cost on its output. It is well known that the marginal cost of a firm hurts its output. For the firms in the HSM, this negative impact is weaker if the firm is an early entrant. Besides, early entrants have more substantial impacts on the equilibrium market output: the marginal cost of firm 1 owns the largest absolute value on coefficient, and the absolute value of coefficient on the marginal costs decreases exponentially. Therefore the equilibrium market output will obtain the maximum if $c_1 \leq c_2 \leq ... \leq c_n$ and the opposite if $c_1 \geq c_2 \geq ... \geq c_n$.

Corollary 3.2 *Consider the hierarchical Stackelberg model with n firms having constant marginal cost $c_i, (i = 1, 2, ..., n)$, the firms choose their output sequentially and face a linear inverse demand $p = a - bQ$,*

[1] It is reflected by the denominator of the equilibrium output.

(i) the equilibrium market output reaches the maximum if the firms enter the market sequentially from the one with the smallest marginal cost to the one with the largest marginal cost;

(ii) the equilibrium market output obtains the minimum if the entry sequence is the opposite.

Assumption 3.3 $c_1 \leq c_2 \leq ... \leq c_n$

Without loss of generality, I assume that $c_1 \leq c_2 \leq ... \leq c_n$ for the firms.[2]

Therefore, to let the market output reach the highest level, the most efficient firm, firm 1, is supposed to enter the market first, then firm 2 enters, followed by firm 3, till firm n. Q^*_{SE} and p^*_{SE} denoted the equilibrium market output and price with this entry sequence are

$$Q^*_{SE} = \frac{(2^n - 1)a - \sum_{i=1}^n 2^{n-i} c_i}{2^n b}$$

$$p^*_{SE} = \frac{a + \sum_{i=1}^n 2^{n-i} c_i}{2^n}.$$

This entry sequence can be considered as the most efficient entry sequence in the HSM regarding the equilibrium market output and price.

The market output will be minimized if the least efficient firms enter first which means firm n enters first, and followed by firm $n-1$, till firm 1. The market output and equilibrium price are Q^*_{SN} and p^*_{SN} such that

$$Q^*_{SE} = \frac{(2^n - 1)a - \sum_{i=1}^n 2^{i-1} c_i}{2^n b}$$

$$p^*_{SN} = \frac{a + \sum_{i=1}^n 2^{i-1} c_i}{2^n}.$$

This sequence can be considered as the least efficient entry sequence in the HSM.

[2]The firms are ranked by their marginal costs instead of the entry sequence.

So these two entry sequences are the extreme cases in the HSM. I will compare the equilibrium outcomes of two entry sequences in the HSM with that in the Cournot model to determine which model is more superior.

3.3. Comparison with the Cournot Competition

In this part, I focus on the comparisons of equilibrium outcomes between the HSM and the Cournot model (CM).

3.3.1. Homogeneous Firms

Proposition 3.2 *Consider the hierarchical Stackelberg model and the Cournot model with n firms having the same constant marginal cost c, and facing linear inverse demand $p = a - bQ$,*

(i) the equilibrium price in the hierarchical Stackelberg model is lower than that in the Cournot model;

(ii) the industry profit of the hierarchical Stackelberg model is less than that in the Cournot model;

(iii) the total welfare in the hierarchical Stackelberg model is greater than that in the Cournot model.

In the CM, each firm takes the output of all the other firms as given. However, an HSM leading firm realizes that if it increases its production, all the subsequent firms will cut back and then accommodate it somewhat. This provides all the firms in the HSM with an extra incentive to increase output at the margin and leads the tendency for the Stackelberg equilibrium output to be higher which means the equilibrium price is lower in the HSM. Also, the first mover advantage also plays a significant role in the Stackelberg equilibrium output that the output increased by the early entrants dominates the decline from the subsequent firms. Therefore, consumer surplus is better off in the HSM than that in the CM with the identical firms.

The industry profit is larger in the CM than that in the HSM. The Stackelberg equilibrium price falls geometrically with the numbers of the firms n according to Lemma 3.1. However, in the equilibrium of the CM, the price declines as the inverse of an arithmetic progression. Therefore, the Stackelberg price falls more rapidly relative to the Cournot price as the number of firms grows larger, and this effect completely dominates the larger output in the Stackelberg. As a result, the industry profit in the HSM is lower.

Since the equilibrium price in the HSM is so small relative to the CM, consumer surplus is so higher that dominates the loss on the industry profit in the HSM relative to the CM. Therefore, the HSM is superior to the CM as well from the perspective of social planners.

3.3.2. Heterogeneous Firms

Proposition 3.3 *Consider the hierarchical Stackelberg model with the most efficient entry sequence and the Cournot model, n firms with possible heterogeneous marginal costs face a linear inverse demand $p = a - bQ$,*

(i) the equilibrium output of the first entrant in the hierarchical Stackelberg model is always larger than its equilibrium output in the Cournot model;

(ii) if for firm i, $(i = 1, 2, ..., n)$, the following inequality holds,

$$2^i \geq n + 1,$$

the equilibrium output of firm i in the hierarchical Stackelberg model is less than its equilibrium output in the Cournot model;

(iii) the equilibrium market output in the hierarchical Stackelberg model is higher than that in the Cournot model.

Firm 1 will benefit the most in the HSM when it is the first leader regarding the individual output, but more firms suffer the loss of the production in the HSM relative to the CM.

Note that Proposition 3.3 (*ii*) is also valid for the case of the homogeneous firms. Firm 1 which has the smallest marginal cost can earn a large amount of market share by hurting the subsequent firms as much as possible through cutting back their output amounts. As a result, most of the firms are worse off in the HSM in terms of individual output.

Proposition 3.3 (*iii*) indicates that the equilibrium market output in the HSM with the most efficient entry sequence is higher than that in the CM. First mover advantage can be so significant since the early entrants who have relatively lower costs can produce more.

Proposition 3.4 *Consider the hierarchical Stackelberg model with the least efficient entry sequence and the Cournot model, n firms with possible heterogeneous marginal costs face a linear inverse demand $p = a - bQ$,*

(i) the equilibrium output of the last entrant in the Stackelberg model is less than its output in the Cournot model;

(ii) the equilibrium market output in the hierarchical Stackelberg model is higher than that in the Cournot model.

For the least efficient entry sequence, the comparison of the individual equilibrium output with the CM is not as clear as the case of the most efficient entry sequence. However, it is certain that the last entrant is worse off in the HSM regarding the output. As for the other firms, the comparisons of the equilibrium output are not straightforward which depend on the distribution of the marginal costs. For instance, consider there are three firms with $c_1 < c_2 < c_3$ competing in the market. For the HSM with the least efficient entry sequence, the leader, firm 3, its output is less than its output in the CM if the marginal cost of firm 3 is larger enough than that of firm 2 and 1. In other words, if the firm is inefficient enough, it will have less output in the HSM than in the CM.

Although the first mover advantage may not lead to higher individual output in the HSM with the least efficient entry sequence, the production gain resulted from the subsequent firms with smaller marginal costs may also play an important role. Some early entrants can benefit from the first mover since it can not cutback following output heavily with the largest cost. Then the gain on the output obtained from these early entrants dominates the loss on the production. Therefore the equilibrium market output in the HSM with the least efficient entry sequence is still larger than in the CM.

Corollary 3.3 *The equilibrium output in the hierarchical Stackelberg model with any entry sequence is larger than that in the Cournot model.*

Proposition 3.3 and 3.4 indicate that regardless of the entry sequences, the total output in the HSM is always less than that in the CM due to the first mover advantage.

Proposition 3.5 *Consider the hierarchical Stackelberg model and the Cournot model, n firms with possible heterogeneous constant marginal costs face a linear inverse demand $p = a - bQ$,*

(i) the welfare loss in the hierarchical Stackelberg model with the most efficient entry sequence is lower than that in the Cournot model;

(ii) the comparison of the welfare loss between the hierarchical Stackelberg model with the least efficient entry sequence and the Cournot model is ambiguous.

It is complicated to compare the industry profit between the two models with n nonidentical firms. Therefore we achieve a comparison of the total welfare by examining the welfare loss relative to the optimal social situation instead.

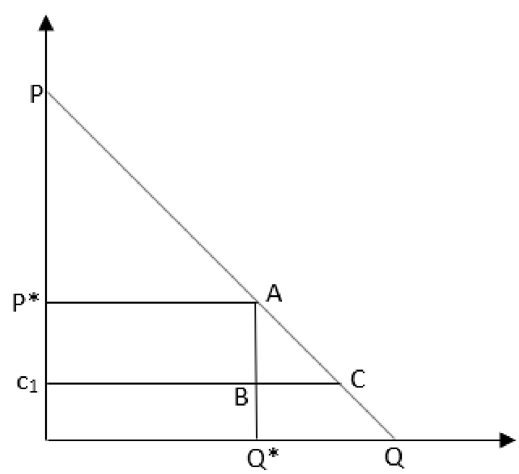

Figure 3.1. Welfare Loss

As Figure 3.1 shows, in the optimal social situation, the firms are supposed to produce at the most efficient level which is c_1 and the market equilibrium price should be c_1 as well.

However, the equilibrium prices are always higher than c_1 in the HSM and CM so that there will exist welfare loss. According to Figure 3.1, the total welfare of the socially optimal level is the triangle $\triangle pc_1C$, and when the market has equilibrium price p^*, the total welfare is the sum of the consumer welfare which is the triangle $\triangle pp^*A$ and the industry profit which is expressed as $p^*Q^* - \sum_{i=1}^{n} c_i q_i^*$. It is obvious that the industry profit $p^*Q^* - \sum_{i=1}^{n} c_i q_i^*$ is smaller than the area of p^*ABc_1. Therefore, the welfare loss relative to the social optimal level consists of two parts: the triangle $\triangle ABC$ and some certain area in p^*ABc_1 which equals to $\sum_{i=1}^{n}(c_i - c_1)q_i$. The $\triangle ABC$ can be considered as the deadweight loss resulted from not serving consumers who are supposed to be served in the optimal situation and $\sum_{i=1}^{n}(c_i - c_1)q_i$ is the product misallocation that the firms produce at higher level costs than the optimal level of productivity.

According to Proposition 3.3, the equilibrium price in the HSM with the most efficient sequence is lower than in the CM. Thus the deadweight loss is smaller in the HSM. Meanwhile, the production misallocation in the HSM with the most efficient entry sequence is also smaller. Therefore, the total welfare loss in the HSM with the most efficient entry sequence is less than in the CM. In the HSM with the most efficient entry sequence, firm 1, the most efficient firm, is the first mover. Therefore, the output of firm 1 accounts for a profoundly large part of the market output. Hence the production misallocation is entirely comprised of products produced by the subsequent firms which do not produce at the most efficient level. Also, those firms produce significantly less in the HSM than they are in the CM. Therefore, the production misallocation in the HSM with the most efficient entry sequence is smaller than that in CM.

In the HSM with the least efficient entry sequence, firm 1 is the last one to enter the market. Therefore, its output occupies a relatively small part of the market output. Also, the firms with the relatively large marginal costs are early entrants, so their output accounts for a relatively large amount of the total output which is higher than their output in the Cournot model. As a result, the production misallocation is worse off in the HSM than in the CM. According to Proposition 3.4, the deadweight loss in the HSM with the least efficient entry sequence is lower. Hence the comparison of the total welfare loss between the HSM with the least efficient entry sequence and the CM is ambiguous with the two factors working in the opposite directions.

3.4. Conclusion

The chapter discusses the hierarchical Stackelberg model with possible heterogeneous firms. Moreover, the chapter systematically compares the equilibrium outcomes between the hierarchical Stackelberg model and the Cournot model.

In the hierarchical Stackelberg model, the existence of the heterogeneity among the firms makes the entry sequence play an essential role in the equilibrium. The most efficient entry sequence is that the firms enter the market according to the sequence from the smallest marginal cost to the largest marginal cost. In this case, the equilibrium price is the lowest among all the possible entry sequences. The equilibrium price will be the highest if the entry sequence is the opposite so it can be considered as the least efficient entry sequence. Therefore, these two entry sequences are the extreme cases in the hierarchical Stackelberg model which can be used to stand for the hierarchical Stackelberg model when compared the equilibrium outcomes with the standard Cournot model.

Generally speaking, the equilibrium prices in the hierarchical Stackelberg model are lower than that in the Cournot model regardless of the entry sequences. As for the welfare comparison, the welfare loss relative to the optimal social situation in the Stackelberg model with the most efficient entry sequence is less compared to the Cournot model. However, the comparison of the welfare loss between the hierarchical Stackelberg model with the least efficient entry sequence and the Cournot model is ambiguous.[3] Therefore, it is unable to summarize the entire comparison results of social welfare between these two models which may need further study.

[3] I do not include the comparison of the industry profit in equilibrium in a different situation due to its complexity.

Appendix A

Proofs in Chapter 1

Proof: [Proof of Lemma 1.1]
(*i*) $\dfrac{\partial p_i}{\partial \alpha} = -1.$
(*ii*) $\dfrac{dn_1}{d\alpha} = \dfrac{t(1-b-a)(a-b)}{6[t(1-b-a)-\alpha]^2}$; $\dfrac{dn_2}{d\alpha} = \dfrac{t(1-b-a)(b-a)}{6[t(1-b-a)-\alpha]^2}$.

Proof: [Proof of Proposition 1.1] According to Eq.1.7, platform 1 can choose its location "a_2" satisfying $t(1-b-a_2)[1-\frac{1}{3}(a_2-b)] = \alpha$. So,

$$a_2 = \frac{4t - \sqrt{4t^2 + 8bt^2 + 4b^2t^2 + 12\alpha t}}{2t}$$

. When platform 1 locates at a_2, $n_2 = 0, n_1 = 1$. And the price of platform 1 is $p_1 = \frac{2}{3}t(1-b-a_2)(a_2-b)$. It can be easily proven that $a_2 > b$, therefore platform 1 can obtain the whole market with positive profit.

Proof: [Proof of Proposition 1.2] Using the platform 1 as the example. Differentiating the platform 1's profit with its location choice a,

$$\frac{d\pi_1}{da} = \frac{-t[t(1-b-a)(1+\frac{1}{3}a-\frac{1}{3}b)-\alpha]\{(3a+1)[t(1-b-a)-\alpha]+bt(1-b-a)-a\alpha\}}{3[t(1-b-a)-\alpha]^2}.$$

Indicating a_1 as $\dfrac{d\pi_1}{da_1} = 0$, and solving out this equation,

$$a_1 = \frac{2t - 4\alpha - 4bt + \sqrt{4(t-2\alpha-2bt)^2 + 12t(t-tb^2-\alpha)}}{6t}.$$

Thus a_1 is the threshold point that for all $a < a_1$, $\dfrac{d\pi_1}{da_1} < 0$; for all $a > a_1$, $\dfrac{d\pi_1}{da_1} > 0$.

As the proof of Proposition 1.1 demonstrates, platform 1 can choose the location a_2 to obtain the whole market. Therefore, the range that platform 1 can move within along the unit interval is $[0, a_2]$.

Comparing the expressions of a_1 and a_2 directly, given the certain transportation cost t, the cross group externality α and the location choice $1-b$ of platform 2.

If

$$10t + 4\alpha + 4bt \geq \sqrt{4(t-2\alpha-2bt)^2 + 12t(t-tb^2-\alpha)} + 3\sqrt{4t^2+8bt^2+4t^2b^2+12\alpha t},$$

we have $a_2 \geq a_1$, which means the profit of platform 1 will increase first then decrease as the moving distance a is larger.

If

$$10t + 4\alpha + 4bt < \sqrt{4(t-2\alpha-2bt)^2 + 12t(t-tb^2-\alpha)} + 3\sqrt{4t^2+8bt^2+4t^2b^2+12\alpha t},$$

we have $a_1 > a_2$. The profit of platform 1 is always decreasing with the moving distance.

Proof: [Proof of Proposition 1.3] According to Proposition 1.2, when the profit is decreasing with the moving distance, it is optimal for platform 1 to locate at the endpoint 0. However, if the profit declines at first, then increases until it becomes the only incumbent, we need to compare the profits of platform 1 when it locates at endpoint 0 and when it is the sole incumbent in the market to determine which location provides more profits. If platform 1 locates at a_2 such that it is able to steal the entire market, so $t(1-b-a_2)[1-\frac{1}{3}(a_2-b)] = \alpha$. In this case, the profit of platform 1 is $\pi_1^m = \frac{4}{3}t(1-b-a_2)(a_2-b)$. If the platform chooses to locate at endpoint 0 given the location $1-b$ of platform 2, the profit of platform 1 is

$$\pi_1^0 = \frac{[t(1-b)(1-\frac{1}{3}b)-\alpha]^2}{t(1-b)-\alpha}.$$

Substituting α with $t(1-b-a_2)[1-\frac{1}{3}(a_2-b)]$,

$$\pi_1^0 = \frac{[t(1-b)(1-\dfrac{1}{3}b)-\alpha]^2}{t(1-b)-\alpha} = \frac{[t(1-b)(\dfrac{1}{3}a_2-\dfrac{2}{3}b)+a_2t(1-\dfrac{1}{3}a_2+\dfrac{1}{3}b)]^2}{\dfrac{1}{3}t(1-b)(a_2-b)+a_2t(1-\dfrac{1}{3}a_2+\dfrac{1}{3}b)}.$$

Then

$$\pi_1^0 - \pi_1^m == \frac{at^2[\dfrac{8}{9}a_2b(1+b-a_2)+\dfrac{4}{9}b(1-b)+\dfrac{1}{9}a_2^3]}{\dfrac{1}{3}t(1-b)(a_2-b)+a_2t(1-\dfrac{1}{3}a_2+\dfrac{1}{3}b)}.$$

It is obviously that denominator and numerator are positive for any b and t.

Therefore,

$$\pi_1^0 - \pi_1^m > 0.$$

So regardless of how profit of platform 1 changing with the location a, it is always optimal for platform 1 to locate at the endpoint 0.

Proof: [Proof of Corollary 1.1]

(i) $$\frac{d\pi_1}{d\alpha} = \frac{-\{[t(1-b-a)(1+\dfrac{1}{3}a-\dfrac{1}{3}b)-\alpha][t(1-b-a)(1-\dfrac{1}{3}a+\dfrac{1}{3}b)-\alpha]\}}{[t(1-b-a)-\alpha]^2} < 0.$$

(ii) $$\frac{d\pi_1}{dt} = \frac{[t(1-b-a)(1+\dfrac{1}{3}a-\dfrac{1}{3}b)-\alpha]^2(1-b-a)}{[t(1-b-a)-\alpha]^2} > 0.$$

Proof: [Proof of Proposition 1.4] According to the model, the surplus of consumer locating at x on the side S is: $v_S+\alpha_S n_1^B - p_1^S - t(x-a)^2$ if he joins platform 1; $v_S+\alpha_S n_2^B - p_2^S - t(1-b-x)^2$ if he chooses platform 2. For each agent on the side S, the platform 1's profit is p_1^S, platform 2's profit is p_2^S. Therefore, the total surplus associated with a given consumer x is $\alpha n_1^B - t(x-a)^2$ if he is on platform 1; $\alpha n_2^B - t(1-b-x)^2$ if he is on platform 2.

To derive the social optimum, we need to derive the indifferent consumer $\tilde{x}$ on the side S, so for the indifferent consumer,

$$\alpha n_1^B - t(\tilde{x} - a)^2 = \alpha n_2^B - t[(1-b) - \tilde{x}]^2,$$

then

$$\tilde{x} = n_1^S = \frac{t(1-b-a)(1-b+a) + \alpha(2n_1^B - 1)}{2t(1-b-a)}$$

Similarly,

$$n_1^B = \frac{t(1-b-a)(1-b+a) + \alpha(2n_i^S - 1)}{2t(1-b-a)}.$$

Solving the above equations,

$$n_1^S = n_1^B = \frac{t(1-b-a)(1-b+a) - \alpha}{2[t(1-b-a) - \alpha]}.$$

The social planner has to maximize total surplus which is the same as minimize transportation costs,

$$min2[\int_0^a t(a-z)^2 dz + \int_a^{\tilde{x}} t(z-a)^2 dz + \int_{\tilde{x}}^{1-b} t(1-b-z)^2 dz - \int_{1-b}^1 t[x-(1-b)]^2 dz]$$

which equals

$$min[\int_0^a (a-z)^2 dz + \int_a^{\tilde{x}} (z-a)^2 dz + \int_{\tilde{x}}^{1-b} (1-b-z)^2 dz - \int_{1-b}^1 [x-(1-b)]^2 dz].$$

Taking the integral,

$$min[a^3 + (\frac{t(1-b-a)^2 - \alpha(1-2a)}{2[t(1-b-a) - \alpha]})^3 + (\frac{t(1-b-a)^2 - \alpha(1-2b)}{2[t(1-b-a) - \alpha]})^3 + b^3].$$

With the first order conditions $\frac{\partial}{\partial a} = 0$ and $\frac{\partial}{\partial b} = 0$, so $a = b = \frac{1}{4}$.

Proof: [Proof of Lemma 1.3] Referring the proof in Armstrong and Wright (2007).

Proof: [Proof of Proposition 1.7] π_1^0 denotes the profit of platform 1 when it is located at the endpoint 0, given the rival's location choice b; and π_1^a is the profit of platform 1 when it is located at some point a in the unit interval other than the endpoint 0.

$$\begin{aligned}
\pi_1^a - \pi_1^0 =& \{\frac{1}{2} + \frac{(a-b)t^2(1-a-b)}{6[t^2(1-a-b)-\alpha^2]}\}[t(1-a-b) - \alpha + \frac{(a-b)t(1-a-b)}{3}] \\
&+ \{\frac{1}{2} + \frac{\alpha(a-b)t(1-a-b)}{6[t^2(1-a-b)-\alpha^2]}\}(t-\alpha) - \{\frac{1}{2} - \{\frac{1}{2} - \frac{\alpha bt(1-b)}{6[t^2(1-b)-\alpha^2]}\}(t-\alpha) \\
&- \frac{bt^2(1-b)}{6[t^2(1-b-a)-\alpha^2]}\}\{t(1-b) - \alpha - \frac{bt(1-b)}{3}\} \\
=& -\frac{at}{2} + \frac{at(1-a-b)}{6} + \frac{(a-b)t(1-a-b)}{6} + \frac{(a-b)^2t^3(1-a-b)^2}{18[t^2(1-a-b)-\alpha^2]} + \frac{bt(1-b)}{6} \\
&- \frac{b^2t^3(1-b)^2}{18[t^2(1-b)-\alpha^2]}.
\end{aligned}$$

Since n_2^S and n_2^B are non-negative in the equilibrium,

$$\frac{(a-b)t^2(1-a-b)}{6[t^2(1-a-b)-\alpha^2]} \leq \frac{1}{2}.$$

When $a \geq b$,

$$\begin{aligned}
\pi_1^a - \pi_1^0 \leq& -\frac{at}{2} + \frac{at(1-a-b)}{6} + \frac{(a-b)t(1-a-b)}{6} + \frac{(a-b)t(1-a-b)}{6} \\
&+ \frac{bt(1-b)}{6} - \frac{b^2t^3(1-b)^2}{18[t^2(1-b)-\alpha^2]} \\
=& -\frac{t(3a^2+ab+b-b^2)}{6} - \frac{b^2t^3(1-b)^2}{18[t^2(1-b)-\alpha^2]} < 0.
\end{aligned}$$

Therefore, there does not exist a location a such that $a \geq b$, given platform 2's location choice b, and platform 1 would earn more when it locates there than it locates the endpoint 0. Choosing a location that $a \geq b$ is a dominated strategy for platform 1.

When $a < b$, if there does not exist a location a such that $a < b$, given platform 2's location choice b, and platform 1 would earn more when it locates there than it locates 0 endpoint. Then platform 1 will locate at 0, and platform 2 locates at 1 in equilibrium. On the contrary, if there does exist a location $a^{'}$ such that $a^{'} < b$ and platform 1 earns the highest profit among all the available locations choices that satisfying $a < b$, platform 1 will locate at $a^{'}$, then platform 2 would deviate to a location at its endpoint 1 since $b > a^{'}$. And platform 1 would deviate to endpoint 0 since $a^{'} > b = 0$.

As a result, in the subgame perfect equilibrium, the platforms will locate at the two endpoints of the unit interval, which means they choose maximal product differentiation.

Appendix B

Proofs in Chapter 2

Proof: [Proof of Corollary 2.1] Consider a one-sided market which has identical settings as the two-sided market in the chapter except that there is only one side of agents and no cross-group externalities. Prior to merger, each firm charges a price $p_i = c + \frac{t}{3}, (i = 1, 2, 3)$. After the merger, the merging firms charge $c - \frac{2}{3}\Delta + \frac{5}{9}t$, and the outside firm charges $c - \frac{1}{3}\Delta + \frac{4}{9}t$. Therefore, if and only if $\Delta > \frac{t}{3}$, all the firms will charge lower prices than pre-merger.

Proof: [Proof of Proposition 2.2] According to Lemma 2.1, the equilibrium profit of each platform prior to the merger is

$$\pi^* = \frac{2t - 3\alpha}{9},$$

and the equilibrium profits of the platforms post-merger are

$$\pi_{1+2} = \frac{2(5t - 9\alpha + 3\Delta)^2}{81(t - 2\alpha)}; \quad \pi_3 = \frac{2(4t - 9\alpha - 3\Delta)^2}{81(t - 2\alpha)}.$$

Thus

$$\pi_{1+2} - 2\pi^* = \frac{14t^2 + 54\alpha^2 - 54\alpha t + 60\Delta t - 108\alpha\Delta + 18\Delta^2}{81(t - 2\alpha)}.$$

With $t > 4\alpha$, $14t^2 + 54\alpha^2 - 54\alpha t = 2[(7t - 9\alpha)(t - 3\alpha) + 3\alpha t] > 0$, and $60t\Delta - 108\alpha\Delta > 0$. So $\pi_{1+2} - 2\pi^* > 0$.

$$\pi_3 - \pi^* = \frac{14t^2 + 108\alpha^2 - 81\alpha t + 108\alpha\Delta - 48\Delta t + 18\Delta^2}{81(t - 2\alpha)}.$$

Therefore, $\pi_3 - \pi^* > 0$ if and only if $14t^2 + 108\alpha^2 - 81\alpha t + 108\alpha\Delta - 48\Delta t + 18\Delta^2 > 0$.

Proof: [Proof of Proposition 2.3] (i) Since surplus of two sides are identical due to the symmetric character. We can compare welfare of one side instead.

Consumer surplus pre-merger is

$$CS = 6\int_0^{\frac{1}{6}}(v + \frac{5\alpha}{6} - c - \frac{t}{3} - tx)dx = v + \frac{5}{6}\alpha - c - \frac{5}{12}t.$$

Consumer surplus post-merger is

$$\widetilde{CS} = 2[\int_0^{x_1}(v + \alpha\mathbf{n_{1+2}} - \mathbf{p_{1+2}} - tx)dx + \int_0^{\frac{1}{6}}(v + \alpha\mathbf{n_{1+2}} - \mathbf{p_{1+2}} - tx)dx \\ + \int_0^{\frac{1}{3}-x_1}(v + \alpha\mathbf{n_3} - \mathbf{p_3} - tx)dx],$$

in which $x_1 = n_{1+2} - \frac{1}{3} = \frac{2t - 3\alpha + 3\Delta}{18(t-2\alpha)}$.

Then

$$\begin{aligned}\widetilde{CS} =& v - c + \alpha + \frac{4}{9}\triangle + x_1\frac{(2t^2 - 3\alpha t + 3\triangle t - 12\alpha\triangle)}{9(t-2\alpha)} \\ &+ \alpha(\frac{1}{3}\mathbf{n_{1+2}} + \frac{2}{3}\mathbf{n_3}) - \frac{67}{108}t.\end{aligned}$$

In which

$$\begin{aligned}&x_1\frac{(2t^2 - 3\alpha t + 3\triangle t - 12\alpha\triangle)}{9(t-2\alpha)} \\ &= \frac{4t^3 + 9\alpha^2 t - 12\alpha t^2 + 9\Delta^2 t + 12\Delta t^2 - 42\alpha\Delta t + 36\alpha^2\Delta - 36\alpha\Delta^2}{162(t-2\alpha)^2} \\ &= \frac{(4t + 12\Delta)(t-2\alpha)^2 + \alpha t(4t - 7\alpha) + 6\alpha\Delta(t-2\alpha) + 9\Delta^2(t-4\alpha)}{162(t-2\alpha)^2},\end{aligned}$$

so

$$x_1\frac{(2t^2 - 3\alpha t + 3\triangle t - 12\alpha\triangle)}{9(t-2\alpha)} > \frac{4t + 12\Delta}{162},$$

Since $\frac{1}{3}\mathbf{n_{1+2}} + \frac{2}{3}\mathbf{n_3} > \frac{1}{3}$, so

$$\widetilde{CS} > v - c + \frac{4}{3}\alpha + \frac{14}{27}\triangle - \frac{67}{108}t + \frac{2}{81}t.$$

The difference of consumer welfare on one side between pre- and post-merger is

$$\Delta CS = \widetilde{CS} - CS > \frac{14}{27}\triangle + \frac{1}{2}\alpha - \frac{29}{162}t.$$

If $\Delta > \frac{29}{84}t - \frac{27}{28}\alpha$, then $\widetilde{CS} > CS$.

(ii) According to the proof of Proposition 2.2, the difference of the industry profit between pre- and post-merger is

$$\begin{aligned}\Delta\Pi &= \pi_{1+2} + \pi_3 - (\pi_1 + \pi_2 + \pi_3) \\ &= \frac{2(2t-5\alpha)(7t-17\alpha) + \alpha(3t-8\alpha) + 12\Delta t + 36\Delta^2}{81(t-2\alpha)}.\end{aligned}$$

With the assumption that $t > 4\alpha$, $\Delta\Pi > 0$.

(iii) Adding up the differences of consumer welfare and industry profit between pre- and post-merger, the change of social welfare is

$$\begin{aligned}\Delta TW &= 2\Delta CS + \Delta\Pi \\ &= \frac{-t^3 + 108\alpha^3 + 34\alpha t^2 - 117\alpha^2 t + 12\Delta t(8t-31\alpha) + 9\Delta^2(5t-12\alpha) + 360\alpha^2\Delta}{81(t-2\alpha)^2}\end{aligned}$$

With assumption that $t > 4\alpha$, $34\alpha t^2 - 117\alpha^2 t > 19\alpha^2 t > 76\alpha^3$.

Thus, if

$$-t^3 + 184\alpha^3 + 12\Delta t(2t - 7\alpha) + 9\Delta^2(5t - 12\alpha) + 360\alpha^2\Delta > 0,$$

$\Delta TW > 0$. Therefore if $t < 6\alpha$, social welfare is higher post-merger.

Proof: [Proof of Corollary 2.2] According to the proof of Corollary 2.1, in the one-sided market, consumer surplus pre-merger is

$$CS = 6\int_0^{\frac{1}{6}} (v - c - \frac{t}{3} - tx)dx = v - c - \frac{5}{12}t.$$

And consumer surplus post-merger is

$$\widetilde{CS} = \frac{4}{9}t + \frac{(2t + 3\Delta)^2}{162t} - \frac{67}{108}t.$$

Then the difference of consumer surplus between pre- and post-merger is

$$\Delta CS = \widetilde{CS} - CS = \frac{-29t^2 + 84\Delta t + 9\Delta^2}{162t}.$$

Therefore, if $\Delta > \frac{t}{3}$, consumer welfare is higher post-merger and all the firms in the one-sided market charge lower prices according to Corollary 2.1. If $\Delta < \frac{t}{3}$, consumer welfare is worse off post-merger and all the platforms charge higher prices.

Proof: [Proof of Proposition 2.5] (i) Consumer welfare with the most efficient merger is

$$CS_{ME} = 2[\int_0^{x_3} (v + \alpha \mathbf{n_{1+2}} - \mathbf{p_{1+2}} - tx)dx + \int_0^{\frac{1}{6}} (v + \alpha \mathbf{n_{1+2}} - \mathbf{p_{1+2}} - tx)dx \\ + \int_0^{\frac{1}{3}-x_3} (v + \alpha \mathbf{n_3} - \mathbf{p_3} - tx)dx],$$

which can be expressed as

$$v + (\alpha \mathbf{n_{1+2}} - \mathbf{p_{1+2}})(x_3 + \frac{1}{6}) - \frac{t}{2}[x_3^2 + (\frac{1}{3} - x_3)^2] + (\alpha \mathbf{n_3} - \mathbf{p_3})(\frac{1}{3} - x_3),$$

in which $x_3 = \frac{2t - 3\alpha + 6\varepsilon}{18(t - 2\alpha)}$.

Similarly, consumer welfare with the least efficient merger is

$$CS_{LE} = v + (\alpha\mathbf{n_{2+3}} - \mathbf{p_{2+3}})(x_4 + \frac{1}{6}) - \frac{t}{2}[x_4^2 + (\frac{1}{3} - x_4)^2] + (\alpha\mathbf{n_3} - \mathbf{p_3})(\frac{1}{3} - x_4),$$

and $x_4 = \frac{2t-3\alpha-3\varepsilon}{18(t-2\alpha)}$.

Inserting all equilibrium results into above equations,

$$CS_{ME} - CS_{LE} = \frac{-3\varepsilon(t-2\alpha)(t-3\alpha)}{2(t-2\alpha)^2} < 0.$$

(ii) With the most efficient merger, the profits of all the platforms are:

$$\pi_{1+2} = \frac{2(5t-9\alpha+6\varepsilon)^2}{81(t-2\alpha)}; \quad \pi_3 == \frac{2(5t-9\alpha-6\varepsilon)^2}{81(t-2\alpha)}.$$

With the least efficient merger, the profits of all the platforms are:

$$\pi_{2+3} = \frac{2(5t-9\alpha-3\varepsilon)^2}{81(t-2\alpha)}; \quad \pi_1 == \frac{2(4t-9\alpha+3\varepsilon)^2}{81(t-2\alpha)}.$$

Hence the difference of the industry profit between these two merger schemes is

$$\Pi_{ME} - \Pi_{LE} = \frac{18\varepsilon(2t+6\varepsilon)}{81(t-2\alpha)} > 0.$$

(iii) The difference of social welfare is the sum of difference of consumer welfare and producer welfare, which is

$$\Delta TW = TW_{ME} - TW_{LE} = \frac{9\varepsilon t^2 - 162\varepsilon\alpha^2 + 63\alpha\varepsilon t + 108\varepsilon^2 t - 216\varepsilon^2\alpha}{\Delta^2}.$$

With $t > 4\alpha$, social welfare is better off with the most efficient merger.

Proof: [Proof of Lemma 2.4] Extending the original proof with two platforms in Armstrong and Wright (2007) to the model with three platforms.

With the assumption that sellers are allowed to choose at least one platform to join, there exist four cases for sellers in equilibrium: sellers choose one platform to join, sellers choose two out of the three platforms to join, sellers choose to join all the platforms and sellers join none platform. Assuming that when sellers are indifferent between joining and not joining a platform, they will join the platform.

Case 1. Sellers multi-home all the platforms

Given that sellers join the three platforms in the equilibrium, the demand function for buyers of each platform is the standard Hotelling formula:

$$n_1^B = \frac{1}{3} + \frac{p_2^B + p_3^B - 2p_1^B}{2t}, \quad n_2^B = \frac{1}{3} + \frac{p_1^B + p_3^B - 2p_2^B}{2t}, \quad n_3^B = \frac{1}{3} + \frac{p_1^B + p_2^B - 2p_3^B}{2t}.$$

Sellers surplus is supposed to satisfy the following inequalities:

$$\alpha_S - p_1^S - p_2^S - p_3^S \geq \alpha_S(n_1^B + n_2^B) - p_1^S - p_2^S \tag{B.1}$$

$$\alpha_S - p_1^S - p_2^S - p_3^S \geq \alpha_S(n_1^B + n_3^B) - p_1^S - p_3^S \tag{B.2}$$

$$\alpha_S - p_1^S - p_2^S - p_3^S \geq \alpha_S(n_2^B + n_3^B) - p_2^S - p_3^S \tag{B.3}$$

$$\alpha_S - p_1^S - p_2^S - p_3^S \geq \alpha_S n_2^B - p_2^S \tag{B.4}$$

$$\alpha_S - p_1^S - p_2^S - p_3^S \geq \alpha_S n_3^B - p_3^S \tag{B.5}$$

$$\alpha_S - p_1^S - p_2^S - p_3^S \geq \alpha_S n_1^B - p_1^S \tag{B.6}$$

$$\alpha_S - p_1^S - p_2^S - p_3^S \geq 0. \tag{B.7}$$

Inequality B.1-B.3 demonstrate that sellers prefer joining all the platforms than joining any two of three platform. B.4-B.6 demonstrate that sellers prefer joining all the platforms than joining any one platform. And inequality B.7 states that sellers prefer joining all the platforms than not joining any platforms. Inequality B.4-B.7 follow from B.1-B.3, and B.1-

B.3 can be rewritten as

$$p_1^S \le (\frac{1}{3} + \frac{p_2^B + p_3^B - 2p_1^B}{2t})\alpha_S \tag{B.8}$$

$$p_2^S \le (\frac{1}{3} + \frac{p_1^B + p_3^B - 2p_2^B}{2t})\alpha_S$$

$$p_3^S \le (\frac{1}{3} + \frac{p_1^B + p_2^B - 2p_3^B}{2t})\alpha_S$$

Then the profits of the platforms are

$$\pi_1 = p_1^S - c + (p_1^B - c)(\frac{1}{3} + \frac{p_2^B + p_3^B - 2p_1^B}{2t})$$

$$\pi_2 = p_2^S - c + (p_2^B - c)(\frac{1}{3} + \frac{p_1^B + p_3^B - 2p_2^B}{2t})$$

$$\pi_3 = p_3^S - c + (p_3^B - c)(\frac{1}{3} + \frac{p_1^B + p_2^B - 2p_3^B}{2t}).$$

Case 2. Sellers multi-home any two of the three platforms.

The example that sellers join platform 1 and 2 can be used to illustrate this case and the analysis of sellers join platform 1, 3 and platform 2,3 is omitted due to the symmetric feature of the model.

The demand functions for buyers of each platform are

$$n_1^B = \frac{1}{3} + \frac{p_2^B + p_3^B - 2p_1^B + \alpha_S}{2t}$$

$$n_2^B = \frac{1}{3} + \frac{p_1^B + p_3^B - 2p_2^B + \alpha_S}{2t}$$

$$n_3^B = \frac{1}{3} + \frac{p_1^B + p_2^B - 2p_3^B - 2\alpha_S}{2t}.$$

Sellers surplus is supposed to satisfy the following inequalities

$$\alpha_S(n_1^B + n_2^B) - p_1^S - p_2^S \ge \alpha_S - p_1^S - p_2^S - p_3^S$$

$$\alpha_S(n_1^B + n_2^B) - p_1^S - p_2^S \ge \alpha_S n_1^B - p_1^S$$

$$\alpha_S(n_1^B + n_2^B) - p_1^S - p_2^S \geq \alpha_S n_2^B - p_2^S \tag{B.9}$$

$$\alpha_S(n_1^B + n_2^B) - p_1^S - p_2^S \geq \alpha_S n_3^B - p_3^S \tag{B.10}$$

$$\alpha_S(n_1^B + n_2^B) - p_1^S - p_2^S \geq \alpha_S(n_2^B + n_3^B) - p_2^S - p_3^S \tag{B.11}$$

$$\alpha_S(n_1^B + n_2^B) - p_1^S - p_2^S \geq \alpha_S(n_1^B + n_3^B) - p_1^S - p_3^S$$

$$\alpha_S(n_1^B + n_2^B) - p_1^S - p_2^S \geq 0.$$

Inequality B.9 and B.11 can conduct the other inequalities and can be rewritten as

$$p_1^S \leq (\frac{1}{3} + \frac{p_2^B + p_3^B - 2p_1^B + \alpha_S}{2t})\alpha_S \tag{B.12}$$

$$p_2^S \leq (\frac{1}{3} + \frac{p_1^B + p_3^B - 2p_2^B + \alpha_S}{2t})\alpha_S \tag{B.13}$$

$$p_3^S \geq (\frac{1}{3} + \frac{p_1^B + p_2^B - 2p_3^B - 2\alpha_S}{2t})\alpha_S.$$

Then the profits of the platforms are

$$\pi_1 = p_1^S - c + (p_1^B - c)(\frac{1}{3} + \frac{p_2^B + p_3^B - 2p_1^B + \alpha_S}{2t})$$

$$\pi_2 = p_2^S - c + (p_2^B - c)(\frac{1}{3} + \frac{p_1^B + p_3^B - 2p_2^B + \alpha_S}{2t})$$

$$\pi_3 = (p_3^B - c)(\frac{1}{3} + \frac{p_1^B + p_2^B - 2p_3^B - 2\alpha_S}{2t}).$$

Case 3. Sellers join one platform

The case that sellers only join platform 1 is the example of this case. The analysis of other cases of sellers joining one platform are similar.

The demand functions for buyers of each platform are

$$n_1^B = \frac{1}{3} + \frac{p_2^B + p_3^B - 2p_1^B + 2\alpha_S}{2t}$$

$$n_2^B = \frac{1}{3} + \frac{p_1^B + p_3^B - 2p_2^B - \alpha_S}{2t}$$

$$n_3^B = \frac{1}{3} + \frac{p_1^B + p_2^B - 2p_3^B - \alpha_S}{2t}.$$

Sellers surplus is supposed to satisfy the following inequalities

$$\alpha_S n_1^B - p_1^S \geq 0 \tag{B.14}$$

$$\alpha_S n_1^B - p_1^S \geq \alpha_S(n_1^B + n_2^B) - p_1^S - p_2^S \tag{B.15}$$

$$\alpha_S n_1^B - p_1^S \geq \alpha_S(n_1^B + n_3^B) - p_1^S - p_3^S \tag{B.16}$$

$$\alpha_S n_1^B - p_1^S \geq \alpha_S(n_2^B + n_3^B) - p_2^S - p_3^S$$

$$\alpha_S n_1^B - p_1^S \geq \alpha_S n_2^B - p_2^S$$

$$\alpha_S n_1^B - p_1^S \geq \alpha_S n_3^B - p_3^S$$

$$\alpha_S n_1^B - p_1^S \geq \alpha_S - p_1^S - p_2^S - p_3^S$$

Inequality B.14-B.16 can elicit other above inequalities, and they can be rewritten as

$$p_1^S \leq (\frac{1}{3} + \frac{p_2^B + p_3^B - 2p_1^B + 2\alpha_S}{2t})\alpha_S \tag{B.17}$$

$$p_2^S \geq (\frac{1}{3} + \frac{p_1^B + p_3^B - 2p_2^B - \alpha_S}{2t})\alpha_S$$

$$p_3^S \geq (\frac{1}{3} + \frac{p_1^B + p_2^B - 2p_3^B - \alpha_S}{2t})\alpha_S.$$

Then the profits of the platforms are

$$\pi_1 = p_1^S - c + (p_1^B - c)(\frac{1}{3} + \frac{p_2^B + p_3^B - 2p_1^B + 2\alpha_S}{2t})$$

$$\pi_2 = (p_2^B - c)(\frac{1}{3} + \frac{p_1^B + p_3^B - 2p_2^B - \alpha_S}{2t})$$

$$\pi_3 = (p_3^B - c)(\frac{1}{3} + \frac{p_1^B + p_2^B - 2p_3^B - \alpha_S}{2t}).$$

Case 4. Sellers join none platforms

Supposing that sellers have positive surplus in any equilibrium, then case 4 is ruled out.

First, supposing that sellers are in the equilibrium of case 1 with $\alpha_S - p_1^S - p_2^S - p_3^S > 0$. Then inequality B.7 does not bind which can leads B.8 does not bind. Thus platform 1 can increase p_1^S until the inequality binds to make more profits which is a contradiction of the equilibrium.

Supposing sellers are in the equilibrium of case 2 with $\alpha_S(n_1^B + n_2^B) - p_1^S - p_2^S > 0$. By the assumption, this can make at least one of B.12 and B.13 does not bind. Therefore, platform 1 or 2 can increase their prices to sellers to make profits. Contradiction.

Supposing sellers are in the equilibrium of case 3 with $\alpha_S n_1^B - p_1^S > 0$. So inequality B.17 dose not bind that platform 1 can increase p_1^S to get more profits. Contradiction.

Hence, sellers are always fully exacted surplus by the platforms in any equilibrium when they are allowed to multi-home.

Proof: [Proof of Lemma 2.5] With $p_B \leq c$, it is not profitable for the platforms to induce sellers to join them exclusively. However there is another possible deviation[1]: platform 1 may increase the equilibrium price p_1^B and decrease its price p_1^S such that sellers still choose multi-home but the platform can gain more profits.

The platforms profit when it sets the prices $p_1^B > p_B$ and $p_1^S = \alpha_S n_1^B$ is

$$\pi_1 = (\frac{1}{3} + \frac{p_2^B + p_3^B - 2p_1^B}{2t})(p_1^B + \alpha_S - c) - c.$$

In equilibrium, $p_2^B = p_3^B = p_B$. If $p_1^B = p_B$, the derivative of the profit function has the sign of $-p_B + c + \frac{t}{3} - \alpha_S$. Therefore, if $p_B < c + \frac{t}{3} - \alpha_S$, the profit of platform 1 increases as p_1^B increases.

[1] According to inertia condition.

Therefore, in equilibrium,

$$max\{0, \frac{t}{3} + c - \alpha_S\} \leq p_B \leq c.$$

So the best equilibrium is that $p_B \leq c$.

Appendix C

Proofs in Chapter 3

Proof: [Proof of Proposition 3.1] First, for the case with two firms, it can be easily proved that $q_1^* = \dfrac{a-c}{2b}$, $q_2^* = \dfrac{a-c}{2^2 b}$. Then, by assuming that $q_k = \dfrac{a-c}{2^k b}$ is valid, it needs to be proved that $q_{k+1}^* = \dfrac{a-c}{2^{k+1} b}$ is also valid.

With $k+1$ firms in the industry, for firm $k+1$, with the first order condition, $q_{k+1}^* = \dfrac{a-c}{2b} - \frac{1}{2}\sum_{i=1}^{k} q_i^*$.

For firm k, the profit is $(a - (\sum_{i=1}^{k+1} q_i^*))q_k^* - cq_k^*$. By substituting q_{k+1} and with the first order condition,

$$q_k^* = \frac{a-c}{2b} - \frac{1}{2}\sum_{i=1}^{k-1} q_i^*.$$

Since $q_k = \dfrac{a-c}{2^k b}$,

$$\frac{1}{2}\sum_{i=1}^{k-1} q_i^* = \frac{a-c}{2b} - \frac{a-c}{2^k b}.$$

Therefore

$$q_{k+1}^* = \frac{a-c}{2b} - \frac{1}{2}\sum_{i=1}^{k} q_i^* = \frac{a-c}{2b} - (\frac{a-c}{2b} - \frac{a-c}{2^k b}) - \frac{1}{2}q_k^* = \frac{a-c}{2^k b} - \frac{a-c}{2^{k+1} b} = \frac{a-c}{2^{k+1} b}.$$

So, for the with n identical firms, $q_i^* = \dfrac{a-c}{2^i b}$ and $Q^* = \sum_{i=1}^{n} q_i^* = \dfrac{a-c}{b}(1 - \dfrac{1}{2^n})$, $p^* = a - bQ^* = a - (a-c)(1 - \dfrac{1}{2^n})$.

Proof: [Proof of Lemma 3.1] By backward induction with the first order conditions,

$$q_n^{S*} = \frac{a-c_n}{2b} - \frac{1}{2}(\sum_{i=1}^{n-1} q_i)$$

$$q_{n-1}^{S*} = \frac{a + c_n - 2c_{n-1}}{2b} - \frac{1}{2}(\sum_{i=1}^{n-2} q_i)$$

$$q_{n-2}^{S*} = \frac{a + c_n + 2c_{n-1} - 2^2 c_{n-2}}{2b} - \frac{1}{2}(\sum_{i=1}^{n-3} q_i)$$

$$\vdots$$

$$q_2^{S*} = \frac{a + \sum_{i=3}^{n} 2^{n-i} c_i - 2^{n-2} c_2}{2b} - \frac{1}{2} q_1$$

$$q_1^{S*} = \frac{a + \sum_{i=2}^{n} 2^{n-i} c_i - 2^{n-1} c_1}{2b}.$$

Assuming that

$$X_n = \frac{a - c_n}{2b}$$

$$X_{n-1} = \frac{a + c_n - 2c_{n-1}}{2b}$$

$$X_{n-2} = \frac{a + c_n + 2c_{n-1} - 2^2 c_{n-2}}{2b}$$

$$\vdots$$

$$X_2 = \frac{a + \sum_{i=3}^{n} 2^{n-i} c_i - 2^{n-2} c_2}{2b}$$

$$X_1 = \frac{a + \sum_{i=2}^{n} 2^{n-i} c_i - 2^{n-1} c_1}{2b}.$$

So the first order conditions of each firm can be simplified as the following,

$$q_n^{S*} = X_n - (\sum_{i=1}^{n-1} \frac{1}{2^{n-i}} X_i)$$

$$q_{n-1}^{S*} = X_{n-1} - (\sum_{i=1}^{n-2} \frac{1}{2^{n-1-i}} X_i)$$

$$q_{n-2}^{S*} = X_{n-2} - (\sum_{i=1}^{n-3} \frac{1}{2^{n-2-i}} X_i)$$

$$\vdots$$

$$q_3^{S*} = X_3 - \frac{1}{2} X_2 - \frac{1}{2^2} X_1$$

$$q_2^{S*} = X_2 - \frac{1}{2} X_1$$

$$q_1^{S*} = X_1.$$

Adding up the above equations,

$$Q_S^* = [1 - (\sum_{i=1}^{n-1} \frac{1}{2^i})] X_1 + [1 - (\sum_{i=1}^{n-2} \frac{1}{2^i})] X_2 + [1 - (\sum_{i=1}^{n-3} \frac{1}{2^i})] X_3 + ... + (1 - \frac{1}{2}) X_{n-1} + X_n$$
$$= \frac{1}{2^{n-1}} (X_1 + 2X_2 + 2^3 X_3 + ... + 2^{n-1} X_n).$$

Substitute $X_{1,} X_2, ... X_n$,

$$Q_S^* = \frac{(2^n - 1)a - c_n - 2c_{n-1} - 2^2 c_{n-2} - ..... - 2^{n-1} c_1}{2^n b} = \frac{(2^n - 1)a - (\sum_{i=1}^n 2^{n-i} c_i)}{2^n b}$$

$$p_S^* = \frac{a + c_n + 2c_{n-1} + 2^2c_{n-2} + ... + 2^{n-1}c_1}{2^n} = \frac{a + \sum_{i=1}^{n} 2^{n-i}c_i}{2^n b}.$$

In addition, from the first order conditions, in the equilibrium each firm will produce:

$$q_1^{S*} = \frac{a + c_n + 2c_{n-1} + 2^2c_{n-2} + ... + 2^{n-2}c_2 - 2^{n-1}(2-1)c_1}{2b}$$

$$q_2^{S*} = \frac{a + c_n + 2c_{n-1} + 2^2c_{n-2} + ... + 2^{n-3}c_3 - 2^{n-2}(2^2-1)c_2 + 2^{n-1}c_1}{2^2 b}$$

$$\vdots$$

$$q_k^{S*} = \frac{a + c_n + 2c_{n-1} + ... + 2^{n-k-1}c_{k+1} - 2^{n-k}(2^k-1)c_k + ... + 2^{n-1}c_1}{2^k b}$$

$$\vdots$$

$$q_n^{S*} = \frac{a - (2^n-1)c_n + 2c_{n-1} + 2^2c_{n-2} + 2^3c_{n-3} + ... + 2^{n-1}c_1}{2^n b}$$

By summarizing,

$$q_i^{S*} = \frac{1}{b}(p_S^* - c_i)2^{n-i}.$$

Proof: [Proof of Corollary 3.2] (i) In this case, the entry sequence indicates that $c_1 \leq c_2 \leq ... \leq c_n$. With the coefficients chosen from the set $1, 2, 2^2, 2^3, ..., 2^{n-1}$, we need to prove that the $c_n + 2c_{n-1} + ... + 2c_2 + 2^{n-1}c_1$ is the smallest if $c_1 \leq c_2 \leq ... \leq c_n$ among all the sums of c_i(i=1,2,...,n) multiplies one coefficient. If we want to prove that $c_n + 2c_{n-1} + ... + 2^{n-2}c_2 + 2^{n-1}c_1$ is the smallest, then we need to prove that for any c_i with the coefficient 2^a and any c_j with the coefficient 2^b, if $c_i < c_j$, then we have $2^a > 2^b$.

Supposing there exists a combination that $c_{k_n} + 2c_{k_{n-1}} + ... + 2^{n-2}c_{k_2} + 2^{n-1}c_{k_1}$ is the minimization of the problem, then for any c_{k_i}, c_{k_j} with coefficients 2^a,2^b, we have $2^a < 2^b$. Then $(2^b c_{k_i} + 2^a c_{k_j}) - (2^a c_{k_i} + 2^b c_{k_j}) = (2^b - 2^a)(c_{k_i} - c_{k_j}) < 0$, which is a contradiction to that $c_{k_n} + 2c_{k_{n-1}} + ... + 2^{n-2}c_{k_2} + 2^{n-1}c_{k_1}$ is the minimization. Therefore, $c_n + 2c_{n-1} + ... + 2c_2 + 2^{n-1}c_1$ is the smallest and the equilibrium output which is $Q_S^* = \dfrac{(2^n-1)a - (\sum_{i=1}^{n} 2^{n-i}c_i)}{2^n b}$ is the largest in this case.

(ii) The proof is similar as (i).

Proof: [Proof of Proposition 3.3] (i) In the HSM with the most efficient entry sequence, the equilibrium output of firm 1 is

$$q_1^{SE*} = \frac{1 + \sum_{i=2}^{n} 2^{i-1}c_i}{2}.$$

In Cournot model, the equilibrium output of firm 1 is

$$q_1^{C*} = \frac{1 - nc_1 + \sum_{i=2}^{n} c_i}{n+1}.$$

Then,

$$q_1^{SE*} - q_1^{C*} = \frac{(n-1) + \sum_{i=2}^{n}[2^{n-i}(n+1) - 2]c_i - [2^{n-1}(n+1) - 2n]c_1}{2(n+1)}.$$

With the assumption that $c_1 \leq c_2 \leq ... \leq c_n$,

$$\begin{aligned}
&(n-1) + \sum_{i=2}^{n}[2^{n-i}(n+1) - 2]c_i - [2^{n-1}(n+1) - 2n]c_1 \\
&> \{(n-1) + \sum_{i=2}^{n}[2^{n-i}(n+1) - 2] - [2^{n-1}(n+1) - 2n]\}c_1 = 0.
\end{aligned}$$

Therefore, $q_1^{SE*} - q_1^{C*} > 0$.

(ii) In the HSM with the most efficient entry sequence, the equilibrium output of firm i is

$$q_i^{SE*} = \frac{1 + c_n + 2c_{n-1} + ... + 2^{n-i+1}c_{i+1} - 2^{n-i}(2^i - 1)c_i + ... + 2^{n-1}c_1}{2^i}.$$

Thus

$$q_i^{SE*} - q_i^{C*} = \frac{-\{(2^n - n - 1) + \sum_{j \neq i}[2^i - 2^{n-j}(n+1)]c_j + [(2^n - 2^{n-i})(n+1) - 2^i n]c_i\}}{2^i(n+1)}.$$

For the certain i, the function $2^i - 2^{n-j}(n+1)$ is increasing with j, and there exist k such that for all $j \leq k$, $2^i - 2^{n-j}(n+1) \leq 0$; for all $j > k$, $2^i - 2^{n-j}(n+1) > 0$.

In addition, since $2^n - n - 1 \geq 0$,

$$\begin{aligned}
q_i^{SE*} - q_i^{C*} = & \frac{-\{(2^i - n - 1) + \sum_{j=1}^{k}[2^i - 2^{n-j}(n+1)]c_j + \sum_{j=k+1}^{i-1}[2^i - 2^{n-j}(n+1)]c_j}{2^i(n+1)} \\
& + \frac{[(2^n - 2^{n-i})(n+1) - 2^i n]c_i}{2^i(n+1)} + \frac{\sum_{j=i+1}^{n}[2^i - 2^{n-j}(n+1)c_j}{2^i(n+1)} \\
< & \frac{-\{(2^n - n - 1)c_{k+1} + \{\sum_{j=1}^{k}[2^i - 2^{n-j}(n+1)]\}c_k}{2^i(n+1)} \\
& + \frac{\{\sum_{j=k+1}^{i-1}[2^i - 2^{n-j}(n+1)] + \sum_{j=i+1}^{n}[2^i - 2^{n-j}(n+1)] + [(2^n - 2^{n-i})(n+1)]\}c_{k+1}}{2^i(n+1)} \\
= & \frac{-[(n+1)(2^n - 2^{n-k}) - k2^i](c_{k+1} - c_k)}{2^i(n+1)}.
\end{aligned}$$

Since $c_{k+1} > c_k$, so $q_i^{SE*} - q_i^{C*} < 0$.

(iii) In the HSM with the most efficient entry sequence, the equilibrium industry quantity is

$$Q_{SE}^{*} = \frac{(2^n - 1)a - c_n - 2c_{n-1} - 2^2 c_{n-2} - ..... - 2^{n-1}c_1}{2^n b}.$$

In the Cournot competition, $Q_C^* = \dfrac{na - \sum_{i=1}^{n} c_i}{(n+1)b}$.

the function $f(x) = a + 1 - 2^x$ is monotone decreasing. So there exists x^* such that for any $x < x^*$, $f(x) \geq 0$; and for any $x > x^*$, $f(x) < 0$.

Therefore, there exists k that for any $x \leq k$, $n+1-2^x \geq 0$; and for any $x^{'} > k$, $n+1-2^{'} < 0$.

Hence,

$$Q_{SE}^* - Q_c^* = \frac{(2^n - n - 1)a - \sum_{i=1}^{k} 2^{n-i}(n+1-2^k)c_k - \sum_{j=k+1}^{n}(n+1-2^j)c_j}{2^n(n+1)b}.$$

Notice that for $c_1, c_2, ..., c_k$, the coefficients are negative; for $c_{k+1}, c_{k+2}, ..., c_n$, the coefficients are positive. Since

$$a > c_n \geq c_{n-1} \geq ... \geq c_{k+1} \geq c_k \geq ... \geq c_2 \geq c_1,$$

so,

$$\begin{aligned}
Q_{SE}^* - Q_c^* &> \frac{(2^n - n - 1)c_{k+1} - [\sum_{i=1}^{k} 2^{n-i}(n+1-2^i)]c_k - [\sum_{i=k+1}^{n} 2^{n-i}(n+1-2^i)]c_{k+1}}{2^n(n+1)} \\
&= [(2^n - n - 1) - \sum_{i=k+1}^{n} 2^{n-i}(n+1-2^i)]c_{k+1} - [\sum_{i=1}^{k} 2^{n-i}(n+1-2^i)]c_k \\
&= [(n+1)(2^n - 2^{n-k}) - 2^n k](c_{k+1} - c_k) > 0.
\end{aligned}$$

Proof: [Proof of Proposition 3.4] (i) In the HSM with the least efficient entry sequence, the equilibrium output of firm 1 is

$$q_1^{SN*} = \frac{a - (2^n - 1)c_1 + 2c_2 + 4c_3 + ... + 2^{n-1}c_n}{2^n b}.$$

Hence,

$$\begin{aligned}
&q_1^{C*} - q_1^{SN*} \\
&= \frac{(2^n - n - 1)a + (2^n - n - 1)c_1 + \sum_{i=2}^{n}[2^n - 2^{i-1}(n+1)]c_i}{2^n(n+1)b}.
\end{aligned}$$

The numerator of the above equation can be decomposed as,

$$
\begin{aligned}
&(2^n-n-1)a+(2^n-n-1)c_1+\sum_{i=2}^{n}[2^n-2^{i-1}(n+1)]c_i\\
=&(2^n-n-1)a-n(2^n-n-1)c_n+(2^n-n-1)\sum_{i=1}^{n-1}c_i\\
&+(2^{n-1}-n)(n+1)c_n-\sum_{i=2}^{n-1}(2^{i-1}-1)(n+1)c_i.
\end{aligned}
$$

Since in the equilibrium of Cournot, firm n should produce positive amount,

$q_n^{C*}>0 \Rightarrow a-nc_n+\sum_{i=1}^{n-1}c_i>0$,

With the assumption that $c_1\leq c_2\leq ...\leq c_n$,

$$
\begin{aligned}
&(2^{n-1}-n)(n+1)c_n-\sum_{i=2}^{n-1}(2^{i-1}-1)(n+1)c_i\\
&\geq[(2^{n-1}-n)(n+1)-\sum_{i=2}^{n-1}(2^{i-1}-1)(n+1)]c_n>0.
\end{aligned}
$$

So, $q_1^{C*}-q_1^{SN*}>0$.

(ii) In the HSM with the least efficient entry sequence, the equilibrium price is

$$p_{SN}^*=\frac{a+c_1+2c_2+...+2^{n-1}c_n}{2^n}.$$

Then,

$$
\begin{aligned}
&p_C^*-p_{SN}^*\\
=&\frac{(2^n-n-1)a+(2^n-n-1)c_1+(2^n-2(n+1))c_2+...+(2^n-2^{n-1}(n+1))c_n}{2^n(n+1)}.
\end{aligned}
$$

By mathematics induction, first it can be easily proven that $p_c^*-p_{SN}^*>0$ for $n=2$. With the equilibrium constrain that $1-2c_2+c_1>0$ which make sure that firm 2 produces positive quantity. Assuming that $p_c^*-p_{SN}^*>0$ is valid for firm k, so

$$(2^k - k - 1) + (2^k - (k+1))c_1 + (2^k - 2(k+1))c_2 + ... + (2^k - 2^{k-1}(k+1))c_k > 0,$$

then

$$2^k(1 + c_1 + c_2 + ... + c_k) - (k+1)(1 + c_1 + 2c_2 + ... + 2^{k-1}c_k) > 0. \tag{C.1}$$

We need to prove that

$$(2^{k+1} - k - 2) + (2^{k+1} - (k+2))c_1 + (2^{k+1} - 2(k+2))c_2 + ... + (2^{k+1} - 2^k(k+2))c_{k+1} > 0,$$

which equals that

$$2^{k+1}(1 + c_1 + c_2 + ... + c_{k+1}) - (k+2)(1 + c_1 + 2c_2 + ... + 2^{k-1}c_k + 2^k c_{k+1}) > 0. \tag{C.2}$$

With C.1,

$$2^{k+1}(1 + c_1 + c_2 + ... + c_k) - (2k+2)(1 + c_1 + 2c_2 + ... + 2^{k-1}c_k) > 0. \tag{C.3}$$

Using left hand side of C.2 subtracts the left hand side of C.3,

$$2^{k+1}c_{k+1} + k(1 + c_1 + 2c_2 + ... + 2^{k-1}c_k - 2^k c_{k+1}) - 2^{k+1}c_{k+1} = k(1 + c_1 + 2c_2 + ... + 2^{k-1}c_k - 2^k c_{k+1}).$$

In the subgame perfect equilibrium of HSM with the least efficient entry sequence, the equilibrium output of the firm $k+1$ is $q_{k+1}^{S*} = \dfrac{1 + c_1 + 2c_2 + ... + 2^{k-1}c_k - 2^k c_{k+1}}{2} > 0$.

Therefore, C.2 holds.

Proof: [Proof of Proposition 3.5] (i) For the first part in the total welfare loss, it is $\dfrac{1}{2b}(p_{SE}^* - c_1)^2$ in the HSM. Similarly, in the Cournot model, it is $\dfrac{1}{2b}(p_C^* - c_1)^2$. And $\dfrac{1}{2b}(p_{SE}^* - c_1)^2 < \dfrac{1}{2b}(p_C^* - c_1)^2$.

For the second part, the difference of the production misallocations between the HSM and Cournot model can be expressed as

$$\sum_{i=1}^{n}(c_i - c_1)(q_i^{SE*} - q_i^{C*}).$$

Since the assumption that $c_1 \leq c_2 \leq ... \leq c_n$, then the above equation can be rewritten as

$$\begin{aligned}
&\sum_{i=1}^{n}(c_i - c_1)(q_i^{SE*} - q_i^{c*}) \\
&= \sum_{i=2}^{n}(c_i - c_1)(q_i^{SE*} - q_i^{c*}) < (c_n - c_1)(Q_{SE}^* - Q_c^* - q_1^{SE*} + q_1^{c*}).
\end{aligned}$$

Since

$$\begin{aligned}
&Q_{SE}^* - Q_C^* - q_1^{SE*} + q_1^{c*} \\
=&\frac{[(2^{n-1}-1)(n+1) - 2^n(n-1)]a + [(n+1)2^{n-1}(2^{n-1}-1) - 2^n(n-1)]c_1}{2^n(n+1)b} \\
&+ \frac{\sum_{i=2}^{n}[(n+1)(2^{n-1} - 2^n - 1)2^{n-i} + 2^{n+1}]c_i}{2^n(n+1)b} \\
<&\frac{[(2^{n-1}-1)(n+1) - 2^n(n-1)]a + [(n+1)2^{n-1}(2^{n-1}-1) - 2^n(n-1)]c_1}{2^n(n+1)b} \\
&+ \frac{\sum_{i=2}^{n}[(n+1)(2^{n-1} - 2^n - 1)2^{n-i} + 2^{n+1}]c_1}{2^n(n+1)b}
\end{aligned}$$

Therefore,

$$Q_{SE}^* - Q_C^* - q_1^{SE*} + q_1^{C*} < \frac{[(2^{n-1}-1)(n+1) - 2^n(n-1)](a - c_1)}{2^n(n+1)b}.$$

Since $a > c_n \geq c_{n-1} \geq ... \geq c_1$, and it can be easily proven that $(2^{n-1}-1)(n+1) - 2^n(n-1) < 0$ for $n \geq 3$., So,

$$\sum_{i=1}^{n}(c_i - c_1)(q_i^{SE*} - q_i^{C*}) < (c_n - c_1)(Q_{SE}^* - Q_C^* - q_1^{SE*} + q_1^{C*}) < 0.$$

(ii) In the HSM with the least efficient entry sequence, the product misallocation is $\sum_{i=1}^{n}(c_i - c_1)q_i^{SN*}$, the difference of production misallocation between the two models is

$$\sum_{i=1}^{n}(c_i - c_1)(q_i^{SN*} - q_i^{C*}) = \sum_{i=2}^{n}(c_i - c_1)(q_i^{SN*} - q_i^{C*}).$$

With the assumption that $c_1 \leq c_2 \leq ... \leq c_n$,

$$\begin{aligned}
&\sum_{i=2}^{n}(c_i - c_1)(q_i^{SN*} - q_i^{C*}) \\
&\geq (c_2 - c_1)\sum_{i=2}^{n}(q_i^{SN*} - q_i^{C*}) = (c_2 - c_1)(Q_{SN}^* - Q_C^* - q_1^{SN*} + q_1^{C*}).
\end{aligned}$$

Since,

$$\begin{aligned}
&Q_{SN}^* - Q_C^* - q_1^{SN*} + q_1^{C*} \\
&= \frac{(2^{n+1} - 2n - 2)a + (2^{n+1} - 2n - 2)c_1 - \sum_{i=2}^{n}[2^i(n+1) - 2^{n+1}]c_i}{2^n(n+1)b}.
\end{aligned}$$

The numerator can be decomposed of the summation of two components:

$$\begin{aligned}
&(2^{n+1} - 2n - 2)a + (2^{n+1} - 2n - 2)c_1 - \sum_{i=2}^{n}[2^i(n+1) - 2^{n+1}]c_i \\
=&\{(2^{n+1} - 2n - 2)a - n(2^{n+1} - 2n - 2)c_n + (2^{n+1} - 2n - 2)\sum_{i=1}^{n-1}c_i\} \\
&+ \{(n+1)[(2^n - 2n)c_n - \sum_{i=2}^{n-1}(2^i - 2)c_i\}.
\end{aligned}$$

It can be easily proven that $(2^{n+1} - 2n - 2) > 0$, and with the assumption $a - nc_n + \sum_{i=1}^{n-1}c_i > 0$

Then,

$$(2^{n+1}-2n-2)a - n(2^{n+1}-2n-2)c_n + (2^{n+1}-2n-2)\sum_{i=1}^{n-1} c_i > 0.$$

Since $c_1 \leq c_2 \leq ... \leq c_n$, and not all equalities holds, then

$$(2^n-2n)c_n - \sum_{i=2}^{n-1}(2^i-2)c_i > ([2^n-2n-\sum_{i=2}^{n-1}(2^i-2)]c_n > 0.$$

Therefore, $\sum_{i=1}^{n}(c_i-c_1)(q_i^{SN*}-q_i^{C*}) > 0$.

BIBLIOGRAPHY

[1] ANDERSON, S. P., AND ENGERS, M. Stackelberg versus cournot oligopoly equilibrium. *International Journal of Industrial Organization 10*, 1 (1992), 127–135.

[2] ARMSTRONG, M. Competition in two-sided markets. *The RAND Journal of Economics 37*, 3 (2006), 668–691.

[3] ARMSTRONG, M., AND WRIGHT, J. Two-sided markets, competitive bottlenecks and exclusive contracts. *Economic Theory 32*, 2 (2007), 353–380.

[4] BASU, K. *Lectures in industrial organization theory.* Blackwell, 1993.

[5] BOYER, M., AND MOREAUX, M. Perfect competition as the limit of a hierarchical market game. *Economics Letters 22*, 2-3 (1986), 115–118.

[6] BRITO, D. Preemptive mergers under spatial competition. *International Journal of Industrial Organization 21*, 10 (2003), 1601–1622.

[7] BRITO, D. Should alternative mergers or acquisitions be considered by antitrust authorities? *International Journal of Industrial Organization 23*, 1-2 (2005), 129–153.

[8] CAILLAUD, B., AND JULLIEN, B. Chicken & egg: Competition among intermediation service providers. *RAND journal of Economics* (2003), 309–328.

[9] CHANDRA, A., AND COLLARD-WEXLER, A. Mergers in two-sided markets: An application to the canadian newspaper industry. *Journal of Economics & Management Strategy 18*, 4 (2009), 1045–1070.

[10] CHURCH, J., AND WARE, R. Delegation, market share and the limit price in sequential entry models. *International Journal of Industrial Organization 14*, 5 (1996), 575–609.

[11] DENECKERE, R., AND DAVIDSON, C. Incentives to form coalitions with bertrand competition. *The RAND Journal of economics* (1985), 473–486.

[12] EVANS, D. S. The antitrust economics of multi-sided platform markets. *Yale J. on Reg. 20* (2003), 325.

[13] Evans, D. S. Some empirical aspects of multi-sided platform industries. *Review of Network Economics 2*, 3 (2003).

[14] Evans, D. S., and Schmalensee, R. The antitrust analysis of multi-sided platform businesses. Tech. rep., National Bureau of Economic Research, 2013.

[15] Fan, Y. Ownership consolidation and product characteristics: A study of the us daily newspaper market. *American Economic Review 103*, 5 (2013), 1598–1628.

[16] Farrell, J., and Shapiro, C. Horizontal mergers: an equilibrium analysis. *The American Economic Review* (1990), 107–126.

[17] Farrell, J., and Shapiro, C. Antitrust evaluation of horizontal mergers: An economic alternative to market definition. *The BE Journal of Theoretical Economics 10*, 1 (2010).

[18] Filistrucchi, L., Klein, T. J., and Michielsen, T. O. Assessing unilateral merger effects in a two-sided market: an application to the dutch daily newspaper market. *Journal of Competition Law and Economics 8*, 2 (2012), 297–329.

[19] Gabszewicz, J. J., and Wauthy, X. Y. Vertical product differentiation and two-sided markets. *Economics Letters 123*, 1 (2014), 58–61.

[20] Galegov, A., and Garnaev, A. How hierarchical structures impact on competition. *AUCO Czech Economic Review 2*, 3 (2008), 227–237.

[21] Jeziorski, P. Effects of mergers in two-sided markets: The us radio industry. *American Economic Journal: Microeconomics 6*, 4 (2014), 35–73.

[22] Kaiser, U., and Wright, J. Price structure in two-sided markets: Evidence from the magazine industry. *International Journal of Industrial Organization 24*, 1 (2006), 1–28.

[23] Levy, D. T., and Reitzes, J. D. Anticompetitive effects of mergers in markets with localized competition. *JL Econ. & Org. 8* (1992), 427.

[24] Nocke, V., and Whinston, M. D. Merger policy with merger choice. *American Economic Review 103*, 2 (2013), 1006–33.

[25] Pal, D., and Sarkar, J. A stackelberg oligopoly with nonidentical firms. *Bulletin of Economic Research 53*, 2 (2001), 127–134.

[26] Robson, A. J. Stackelberg and marshall. *The American Economic Review* (1990), 69–82.

[27] Rochet, J.-C., and Tirole, J. Platform competition in two-sided markets. *Journal of the european economic association 1*, 4 (2003), 990–1029.

[28] Rochet, J.-C., and Tirole, J. Two-sided markets: a progress report. *The RAND journal of economics 37*, 3 (2006), 645–667.

[29] Rysman, M. The economics of two-sided markets. *Journal of economic perspectives 23*, 3 (2009), 125–43.

[30] Salop, S. C. Monopolistic competition with outside goods.

[31] Song, M. Estimating platform market power in two-sided markets with an application to magazine advertising.

[32] Tirole, J. *The theory of industrial organization.* MIT press, 1988.

[33] Vives, X. Sequential entry, industry structure and welfare. *European Economic Review 32*, 8 (1988), 1671–1687.

[34] Wright, J. One-sided logic in two-sided markets. *Review of Network Economics 3*, 1 (2004).