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Using Graph Theory to Create a 3D Miniscaped, Non-Linear Level

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Abstract

This study focuses on how to use the Graph Theory and the Dominion Theory to design a three-dimensional (3D) non-linear, miniscaped level layout in a video game. Using these theories, the researcher aimed to aid players in navigating non-linear levels (which are notoriously difficult to traverse). Consequently, the researcher created a methodology outlining the best practices for constructing a level using Graph Theory and Dominion Theory. The researcher constructed a game level in *The Elder Scrolls V: Skyrim* to explore the effectiveness of this methodology. Testers played the level and provided feedback regarding their experiences. The researcher analyzed this data to confirm or deny their methodology’s accuracy.

Keywords

Graph Theory, Dominion Theory, Non-linear Level, Miniscape, Hakoniwa, Level Design, *The Elder Scrolls V: Skyrim*

1 INTRODUCTION

In three-dimensional (3D) single-player games, non-linear levels can provide a better sense of freedom for the player compared to linear levels and allow them to feel more in control. Yet, non-linear levels, in comparison to linear levels, are notoriously difficult to navigate and players have a hard time maintaining a sense of direction. As a result, non-linear levels are more difficult to design in video games. This thesis focuses on using the Graph Theory and the Dominion Theory to build a design methodology for non-linear levels in 3D single-player games. The goal of this study is to assist level designers in making non-linear levels, in which the player does not get lost and maintains a good sense of exploration.

2 TERMS, DEFINITIONS, & RESEARCH

2.1 Non-linear level

A non-linear level is a video game level that is designed to encourage players to freely maneuver and explore space according to their interests [1].

Figure 1 A non-linear level example in *Pacman* [2]

Figure 1 shows an example of a non-linear level from the video game, *Pacman*. In this level, the player controls the yellow Pacman character. To complete the level, the player must collect all the white dots before the ghosts (colored purple, red, and light blue) touch Pacman. Every dot in the level is an objective that the player needs to reach. In contrast, the purple, red, and light blue ghosts are objectives that the player needs to avoid. This is a non-linear level because the player has full control of where the Pacman character moves, and the player determines where to explore next.

Figure 2 A non-linear level example in *Hitman 2* [3]

Figure 2 illustrates a non-linear level from *Hitman 2*. The player acts as a hitman and tries to kill the mission target in a nearby building. Once the target is killed, the player must escape. This is a non-linear level because the player can decide how they want to enter and exit the building. The level layout does not force the player to move through a particular route. The player can also explore rooms and spaces as they please.

2.2 Miniscape

Miniscape is a word translated from the Japanese word “Hakoniwa.” Hakoniwa refers to a distinctive interior decorative dish garden made with materials that do not require water to grow [4]. The concept of “Hakoniwa” is the idea of making an encapsulated room elaborate enough to tell its purpose and give visitors a deep impression.

In reference to video games, Shigeru Miyamoto, the creator of the *Super Mario* series at Nintendo, compared designing a video game level to building an elaborate Hakoniwa (miniscape) for the player to explore [5].

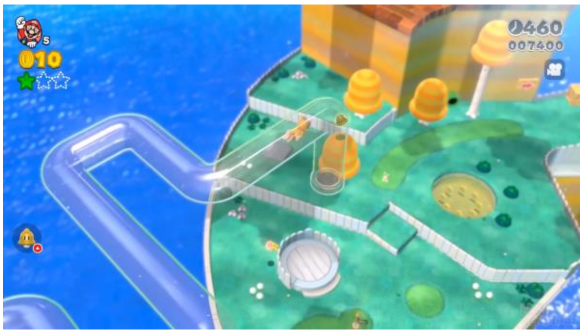


Figure 3 World 1-1 level in the *Super Mario 3D World* [6]

Figure 4 World 1-5 level in *Super Mario 3D World* [6]

Both **Figures 3** and **4** depict levels from *Super Mario 3D World*. These two levels are in the same overworld in the game but have two distinctive decorative themes. They are miniscapes because each level has a different elaborate theme .

Figure 5 A boss fight room in *Dark Souls: III* [7]

The Dark Souls series, like the *Super Mario* series, is famous for its miniscaped levels. At "CEDEC KYUSHU 2021 ONLINE" conference, the developers at *FromSoftware* introduced a method to mitigate player's navigational confusion in their non-linear levels. This method outlines how to create a distinctive decorative theme for rooms and spaces so that they leave a deep impression on the player [7]. The impression allows the player to "mentally map the space". Mental mapping is the act of reconstructing a space from memory.

Hence, "miniscape," in terms of level design, refers to levels, or rooms in a level, with distinctive themes that can assist the player in mentally mapping the level structures.

2.3 Dominion Theory

Dominion Theory is a level design methodology originally used by *Half-Life 2*'s developers. It refers to a principle in level design that states that the level layout must give the player as much time and space as possible between gameplay areas (or dominions) to replenish, explore, and get loot. The Dominion Theory also implies that gameplay areas may be represented by vertices. The vertices have "an area of effect," and each vertex's area of effect should not overlap with other vertices' area of effect to avoid the player being exhausted [8].

2.3.1 Experiential Density

Experiential Density is a term coined by Ken Birdwell at Valve when the studio was developing the original *Half-Life* game. The designer found that the game experience needed to be organized based on distance rather than time. As a result, Experiential Density emphasizes a denser, traversal experience, rather than a time-based (time-limited) experience. The player is given the choice of when to enter high intensity gameplay areas. This agency allows the player to spend as much time as they want resting, gaining loot, or exploring the game world [8].

Figure 6 Dominions in a level in *Half-Life 2* [8]

Figure 6 is a level overview map which shows how Valve's designers created a level using dominions that promoted Experiential Density. The player starts at point A and moves along the coastline. As they progress, the player finds 6 settlements (Points B, C, D, E, F, and G). Each settlement has a heavy concentration of gameplay. The player's experience is measured by the distance traveled between all these points – rather than the time it takes to arrive at each point.

All red circles in **Figure 6** are defined as dominions by Valve's developers. The inner, smaller solid circles (with letter labels) specify the sizes of a gameplay space. The outer rings communicate that the corresponding space is reserved for looting and exploration. In general, the larger the outer ring of a red dominion, the great number of opportunities the player has to explore that dominion.

However, between two dominions, there is often space that is not covered by any dominions' outer red radius. These spaces are designed to encourage the player to take a rest from the high-paced gameplay.

2.4 Landmark

A landmark is a highly visible level design element that stands out in the level layout. A landmark usually has a unique appearance so that the player can use it as a reference for navigation [9].

Figure 7 A landmark in the *Elden Ring* [10]

Figure 8 Landmarks in *The Legend of Zelda: Breath of the Wild* [11]

In *Elden Ring*, the Golden Tree is the most unique landmark in the game world. Although the player is unable to reach the top of the tree, it serves as the player's reference point and helps the player to correctly navigate the world.

In *The Legend of Zelda: Breath of the Wild*, the Death Mountain volcano, the occupied Hyrule Castle on the volcano's left side, and the tower on the right side of the picture are all landmarks. Hyrule Castle is the overall goal of the whole game. The Death Mountain volcano is one of the long-term goals where the player can make great progress in the game's main story. The tower serves as a short-term goal, which the player can reach at the start of the game. By strategically positioning these landmarks, game developers can relay important navigational information to players.

2.5 Graph Theory

Graph Theory focuses on studying graphs connected by vertices and edges. Vertices represent objects or entities. Edges connect vertices to represent the interrelationship among those vertices [12].

2.5.1 Graph, Vertex, and Edge

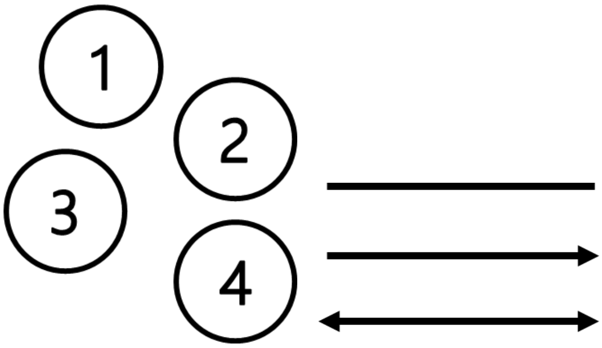


Figure 9 Vertices (Left) and Edges (Right) [13]

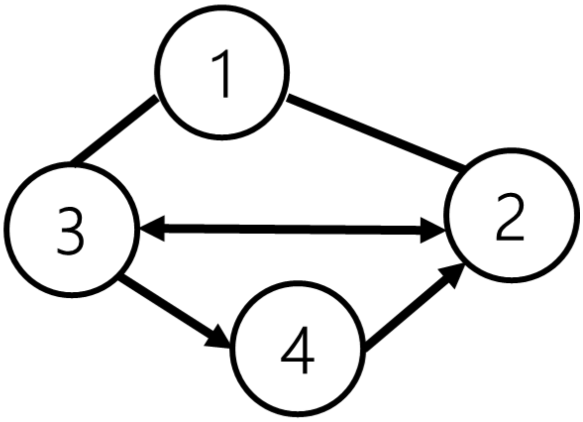


Figure 10 A graph in the Graph Theory [13]

A graph is a group of given elements and the relationships between each element pair. These elements are represented by vertices or nodes while the interrelationships among them are represented by edges [14]. **Figure 9** shows a group of vertices $(1,2,3,4)$ and three edges. **Figure 10** depicts a graph formed by those same vertices $(1,2,3,4)$ and edges.

2.5.2 Unidirectional & Bidirected Edges

An edge can have a directionality. The number of arrows on an edge implies the directionality of the interrelationship between the connected vertices. The number of arrows and their meanings are outlined below:

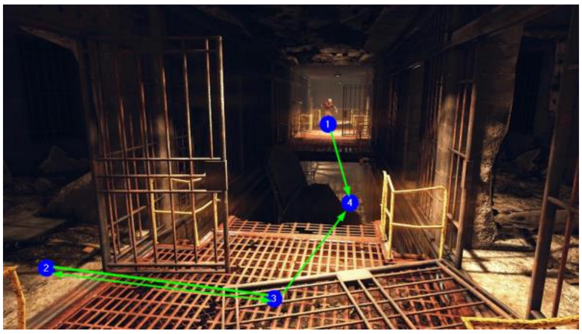


Figure 11 A level design example of an edge's directionality in *Fallout 76* [15]

1. **No arrow or two arrows at both sides**:
   The relationship is **bidirected** or **bidirectional**. It implies a two-way relationship between two vertices. In **Figure 11**, the relationship between room 2 and platform 3 is bidirected because the player can freely step into room 2 and return to platform 3.
2. **One arrow at either side**:
   The relationship is **unidirectional**. It implies a one-way relationship between two vertices. In **Figure 11**, the relationship between platform 3 and the lower floor 4 is unidirectional because the player can only jump down to the lower floor 4 from platform 3 but cannot move in the opposite direction.

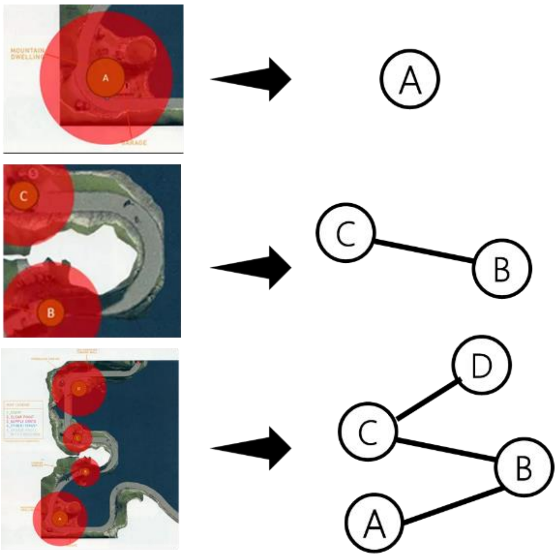


Figure 12 Abstracting vertices and edge from dominions and transitions [8] [13]

A dominion, where gameplay is concentrated, can be represented by a vertex. A transition area between two dominions can be represented by either a unidirectional or a bidirected edge.

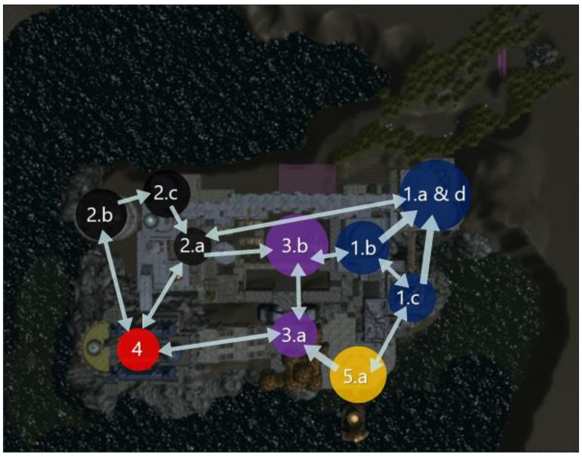


Figure 13 The top-down snapshot of the artifact [16]

Figure 13: demonstrates how the researcher used unidirectional, and bidirected edges to organize the initial interrelationships among dominions in the artifact.

These interrelationships may change when the player plays the artifact because a unidirectional edge may become bidirected if the player resolves an obstacle or a barred door restricting its directionality. For instance, there is a barred door between dominion 1.c and dominions 1.a & d in **Figure 13**. The door prevents does now allow the player to move from dominions 1.a & d to the dominion 1.c. If the player unbars the door between them, the connecting edge becomes bidirected.

2.5.3 Leaf

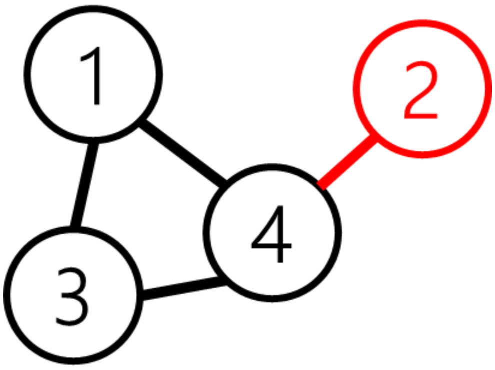


Figure 14 An example of Leaf [13]

A leaf is a vertex with no connecting vertices [17]. In other words, a leaf is a vertex that connects to its one neighbor via one edge. In **Figure 14**, vertex 2 is a leaf because vertex

4 is the only connecting, neighboring vertex. In level design, a leaf usually represents a dead end.

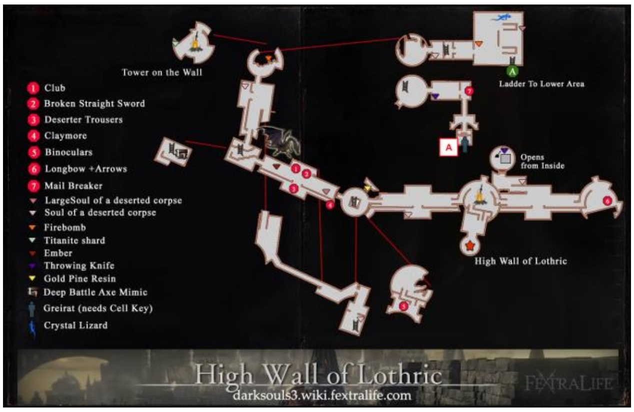


Figure 15 The level map of the *High Wall of Lothric* in the *Dark Souls III* [18]

For example, in the High Wall of Lothric in *Dark Souls III,* shown in **Figure 15**, the room A where the player meets Greirat is a leaf because there is only one path connecting to it. There is no other way for the player to get access to this room.

2.5.4 Chain

A "chain" is a continuous path that is formed by a series of vertices and edges in a graph [19].

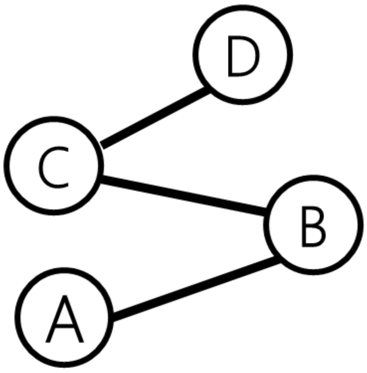


Figure 16 A example of a chain [13]

Figure 16 depicts a simple chain. A and D are connected by a path formed by (A, B, C, D) and the edges between $(A, B), (B, C)$, and (C, D). In addition, the path starting from B to D (B, C, D) is also a chain in this example.

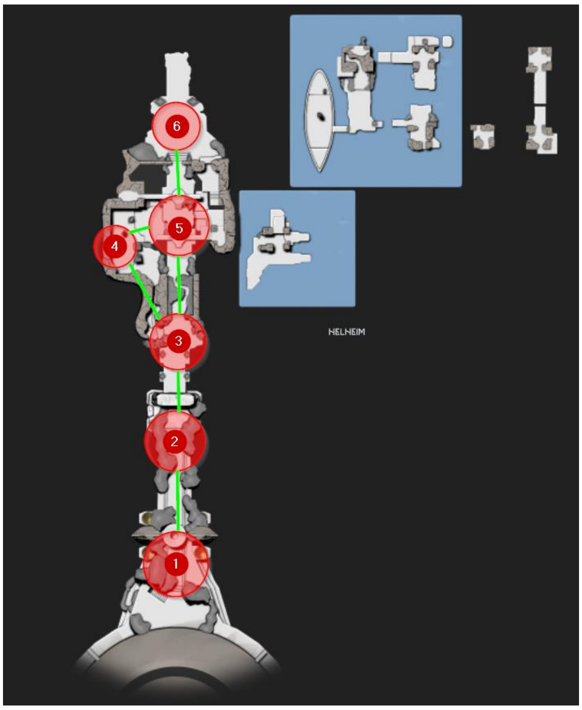


Figure 17 The level map of the *Helheim* in *God of War (2018)* [20]

In terms of level layout, a chain is a sequence of rooms or hallways. **Figure 17** is a map of Helheim in *God of War (2018).* There are six dominions on the map. The two paths (built of rooms and hallways) which connect dominions 1 and 6 are chains. These chains are $(1,2,3,5,6)$, and $(1,2,3,4,5,6)$.

2.5.5 Subgraph

A graph is said to be a "subgraph" if it is a part of another graph [21]. A subgraph is also referred to as an embedded graph.

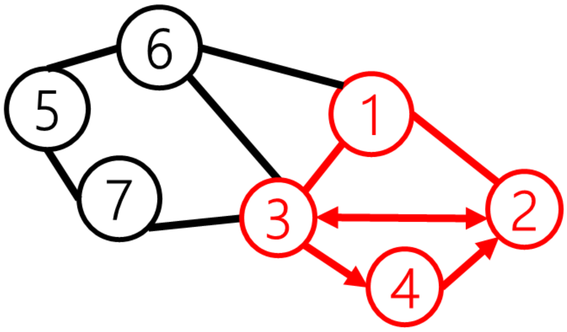


Figure 18 An example of Subgraph [13]

The graph, in **Figure 18**, consists of vertices $(1,2,3,4,5,6,7)$ and the vertices' connecting edges. By a graph's definition, vertices $(1,2,3,4)$ also constitute as a graph. In this case, the graph, made up of vertices $(1,2,3,4)$, is a subgraph of the graph made up of vertices $(1,2,3,4,5,6,7)$.

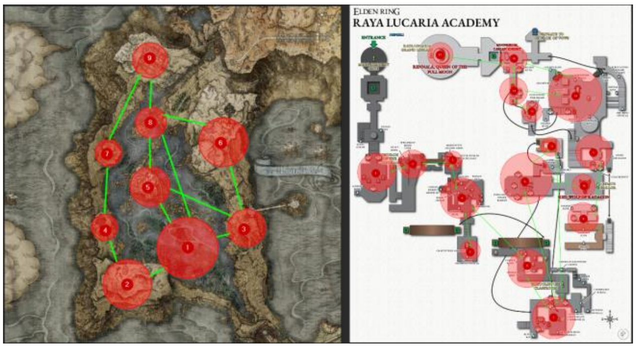
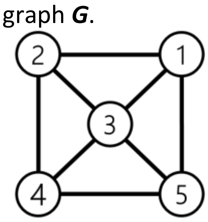


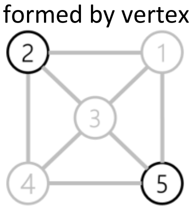
Figure 19 Analysis of dominions and connectedness in the world map of the *Liurnia* and the level map of the *Raya Lucaria Academy* in the *Elden Ring* [20] [22]

In terms of world design and level design, **Figure 19** shows an example of how a subgraph represents game spaces in *Elden Ring*. The red circles in the map represent dominions while the green lines represent edges. The left image is the world map of the Liurnia. The right image is the level map of the Raya Lucaria Academy, located at dominion 5 on the Liurnia. The Academy graph is a subgraph of the Liurnia graph.

2.5.6 Connected & Connectivity

A connected graph is a graph in which each vertex pair has a path between them [17]. In mathematics, connectivity is defined as follows:

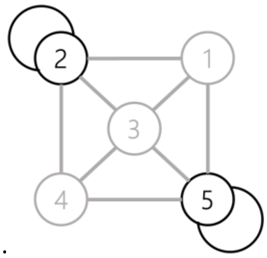
1. Given a graph ***G***.
    a. 

    (Graph ***G***) [13]
2. Determine if there exists one way to remove ***k*** vertices so that the connectedness in the resulting graph breaks. In other words, try to remove ***k*** vertices from this graph to break it into two or more graphs.
    a. For ***G***, by removing vertices $(1,3,4)$ when ***k = 3*** *vertices*, ***G*** will be broken into graph ***G'*** consisting of vertex 2 and ***G''*** formed by vertex 5
    b. 

    (Graph ***G' and Graph G''***) [13]
3. Attempt to remove ***k-1*** vertices and see if the connectedness in the resulting graph breaks.
    a. If the connectedness is broken, make ***k = k-1*** and repeat step 2 and 3.
    b. If there exist no more ways to break the graph's connectedness, then ***k*** is the connectivity of the graph ***G***.
    c. By removing **k – 1** *(vertices)* in our example, which results in **2** vertices, the graph's (***G***) connectedness cannot be broken further. Hence, the connectivity of graph ***G*** is **3**.

A connected graph is a graph in which all vertices are connected. As a result, if a graph becomes "unconnected," meaning the vertices are no longer all connected, then this graph is broken into two or more subgraphs.

In addition, a single vertex without any edge connecting to its neighboring vertex, such as vertices 2 and 5 in step 2.b, is still a connected graph because a vertex can connect to itself with an edge as the following image depicts.



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2.5.7 Directed & Undirected Graphs

A graph can be directed or undirected. As mentioned in section 2.5.2, an edge can be unidirectional or bidirectional.

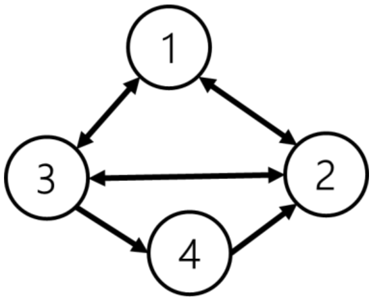


Figure 20 An example of a directed graph [13]

If at least one unidirectional edge exists in a graph, this graph is "directed."

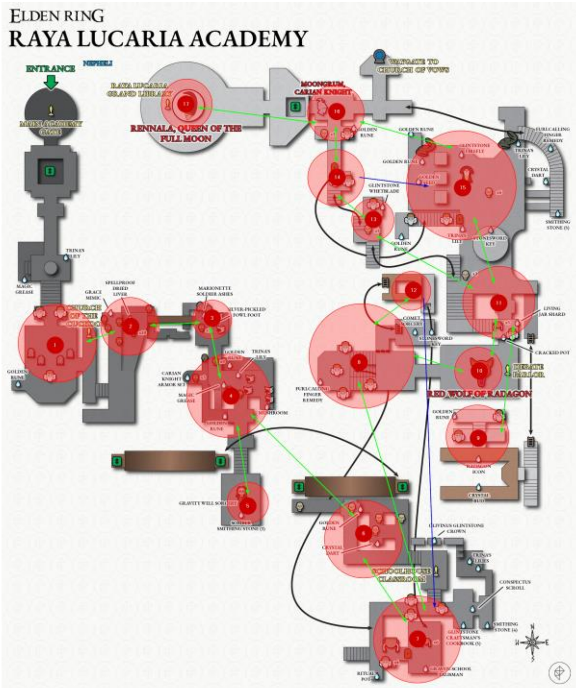


Figure 21 Analysis dominions and connectedness of the level map of the *Raya Lucaria Academy* in the *Elden Ring* [22]

The map depicted in **Figure 21** is a directed graph because there exist two unidirectional edges (marked in blue).

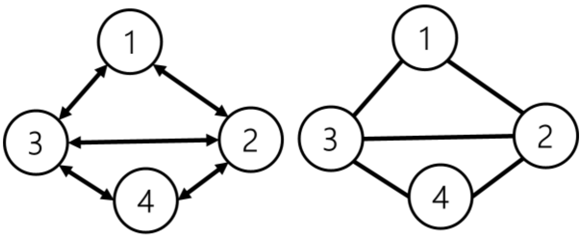


Figure 22 An example of two forms of an undirected graph [13]

In contrast, **Figure 22** shows two forms of undirected graphs. If all edges in a given graph are bidirected, this graph is undirected. In this case, all arrows can be removed from the graph because the directionalities of all edges are not important to analyzing the graph. In other words, the two graphs in **Figure 22** are equivalent when all edges are bidirectional.

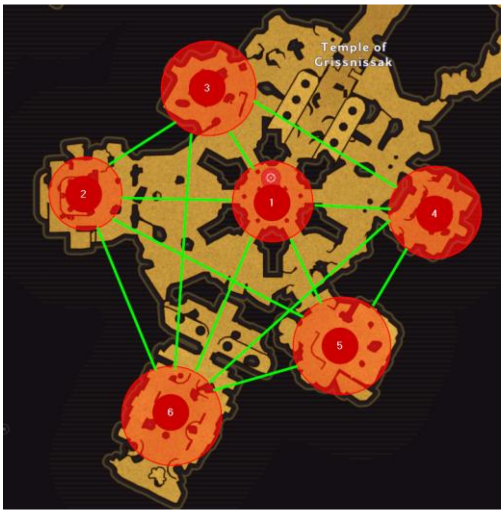


Figure 23 The level map of Temple of Grissnissak in *Tiny Tina's Wonderlands* [23]

An undirected graph usually corresponds to arena-based levels. **Figure 23** shows an arena for a side quest in *Tiny Tina's Wonderlands*. In this level, players can freely move in this temple and defeat the enemies in dominions (2,3,4,5,6) in any order. Subsequently, the players will fight with a boss in dominion 1 to finish the quest. In this case, the directionalities of edges are meaningless. The player may navigate back and forth through areas at will.

2.5.8 Loop, Cycle, and N-vertex Cycle

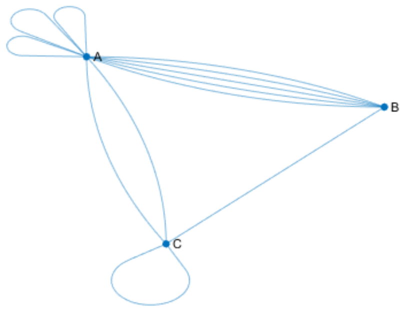


Figure 24 An example of loops [24]

A loop is a vertex with an edge connecting to itself. A cycle refers to a structure where only the first and the last vertex are equivalent [17]. In a cycle, you can return to the first vertex after you go through at least one additional vertex. In **Figure 23**, vertices A and B loop while vertices (A, B, C) and (A, B) are cycles.

In terms of level design, both loop and cycle are when the player eventually returns to the start point without backtracking. For the purpose of this study, the research defined a cycle having N vertices as an "N-vertex cycle." In **Figure 22**, dominions (2,3,4,5,6) form a cycle and it is a 6-vertex cycle.

2.6 Fixed Vertex and Fixed Edge

A fixed vertex is a vertex that is less likely to be removed from a graph because it is special and important to the graph structure [25]. In level design, a fixed vertex represents an area which have anything that is important to gameplay. It is usually a start point, a save point, or a boss room. For example, in **Figure 21**, vertices $(1,7,10,17)$ are all fixed because they contain either a save point or a boss room.

Similarly, a fixed edge is an edge having directionality [25]. In a level, a fixed edge usually refers to a doorway locked by a puzzle, a one-way doorway, or a pit on the floor. In the example in **Figure 11**, the edges from platforms 1 to 4 and from 3 to 4 are fixed edges because they are all unidirectional. A fixed edge is less likely to be removed from a graph because it encourages the player to think about the level structures. For example, if a player is in a hallway with a barred door (a fixed edge), the directionality encourages the player to think about how to reach the other side.

2.7 Minimal Logic Graph

A minimal logic graph is the simplest graph representing the simplest logic of how different gameplay areas connect with each other. A minimal logic graph must satisfy the following conditions: [25]

1. The resulting graph must have a minimum of 3-vertex cycles.
2. Any 2-vertex cycles are removed or simplified.
3. The graph allows loops.
4. Leaves, chains, and subgraphs do not exist unless the connectivity of the graph breaks when they are removed.

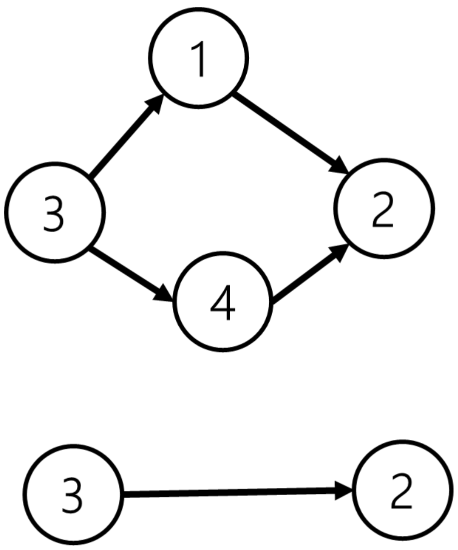


Figure 25 An example of simplifying fixed edges [13]

In addition, while removing graph elements, fixed vertices and fixed edges should be preserved, if possible. It doesn't mean that all fixed edges should be saved. If a vertex has two unidirectional edges leading to the same vertex, simplifying it is feasible. **Figure 25** shows fixed edges that can be simplified. Because paths $(3 \rightarrow 1 \rightarrow 2)$ and $(3 \rightarrow 4 \rightarrow 2)$ both represent a flow from 3 to 2, the graphs could be simplified to a single fixed edge $(3 \rightarrow 2)$.

Similarly, for a fixed vertex, if the vertex becomes a leaf, it now does not meet the first minimal logic graph condition. This means the fixed vertex may be removed. For example, in **Figure 26**, vertices 16 and 17 are both fixed because they both contain a boss battle. It is feasible to combine the two vertices into a single vertex because vertex 17 is a leaf of vertex 16.

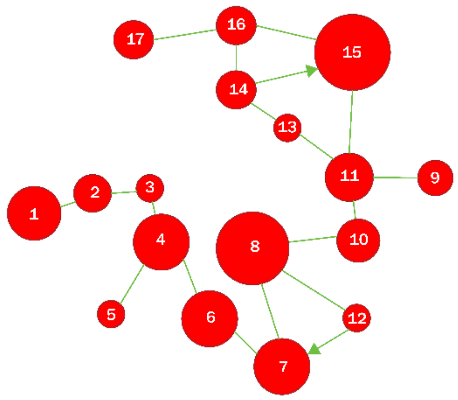


Figure 26 Graph of *Raya Lucaria Academy* in *Elden Ring* [13]

If you extract the dominions and edges from the graph in **Figure 21**, the graph becomes like **Figure 26**.

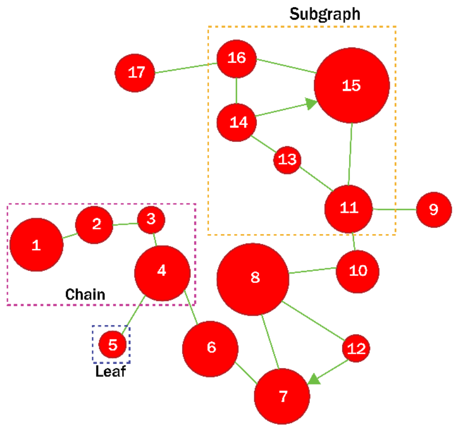


Figure 27 Examples of graph elements that could be modified from Figure 26 [13]

Certain graph elements in **Figure 27** may be identified and modified, according to the minimal logic graph conditions:

1. $(1,2,3,4)$, $(4,6,7)$, $(8,19,11)$, and $(11,13,14)$ are chains that can be simplified to a single dominion.
2. Vertices 5, 9, and 17 are leaves that may be merged with their single neighbor.
3. $(7,8,12)$ and $(13,14,15,16)$ are subgraphs that can be simplified into single larger dominions.

However, because $(12 \to 7)$ and $(14 \to 5)$ are two fixed edges (they are two one-way doorways that imply level structure logic), they cannot be removed or simplified. Moreover, $(1,7,10,16)$ are fixed vertices that need to be preserved. Vertex 1 is the start point of the level. Vertex 7 contains a save point. Vertex 10 and 16 are boss rooms. After removing vertices $(2,3,4,5,6)$, vertex 1 with becomes a leaf of vertex 7. In this case, combining fixed vertices 1 and 7 is feasible.

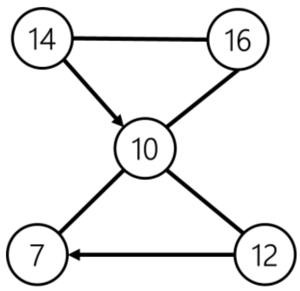


Figure 28 The minimal logic graph of the *Raya Lucaria Academy* in the *Elden Ring* [13]

Consequently, the ***Raya Lucaria Academy*** graph is reduced into its minimal logic graph, as shown in **Figure 28**.

2.8 $Cheeger$ Number

A $Cheeger$ number shares a similar definition with **Connectivity**, outlined in 2.5.6. **Connectivity** is the discrete, minimum number of vertices that are removed. Removing the same number of vertices as a graph's connectivity breaks the given graph's connectedness.

On the other hand, a $\boldsymbol{Cheeger}$ number represents a range of number of vertices that are removed. The $\boldsymbol{Cheerger}$ number reflects how a graph's connectedness changes along with removing different number of vertices. A $\boldsymbol{Cheeger}$ number also represents the ratio of the ways to disconnect a given graph by removing any number of vertices to the total number of ways to remove any number of vertices from the graph. The mathematical definition of $\boldsymbol{Cheeger}$ number, denoted as $\boldsymbol{\lambda_k}$, is as follows:

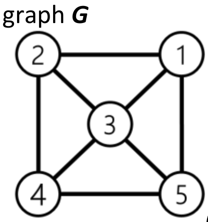
$$\boldsymbol{\lambda_k = 1 - \frac{n}{C_m^k}}$$

The $\boldsymbol{C_m^k}$ represents the binomial coefficient of ***k*** and ***m*** as follows:

$$C_m^k = \frac{k!}{m!\,(k-m)!}$$

The exclamation mark represents the factorial of the number in front of it. For example, $\boldsymbol{k! = k \times (k-1) \times (k-2) \times \cdots \times 1}$

There exist ***n*** ways to remove ***k*** vertices from a given graph ***G*** with ***m*** vertices so that the resulting graph is not a connected graph [25].

1. Given a graph ***G***
   a. 

   (Graph ***G***) [13]
2. According to ***G***, m = 5 because the graph has 5 vertices.
3. The only way to break ***G***'s connectedness is to either removing vertices $(1,3,4)$ or vertices $(2,3,5)$.
   a. Consequently, n = 2 and k = 3. There exist 2 ways to remove 3 vertices from graph ***G*** to break its connectedness.

Hence, if n = 2, k = 3, and m = 5, the $Cheeger$ number is

$$\boldsymbol{\lambda_k = 1 - \frac{2}{C_5^3} = 1 - \frac{2}{10} = 0.8}$$

$Cheeger$ numbers with $k \leq 3$ need to be calculated. For graph ***G***, its $Cheeger$ numbers are:

- $\lambda_1 = 1 - \frac{0}{\binom{5}{1}} = 1$ (k=1, n=0)
- $\lambda_2 = 1 - \frac{0}{\binom{5}{2}} = 1$ (k=2, n=0)
- $\lambda_3 = 1 - \frac{2}{\binom{5}{2}} = 0.8$ (k=3, n=2)

2.9 Stability Factor

The Stability Factor $\boldsymbol{\gamma}$ is a parameter measuring how well spaces are connected in a level or a map [25]. In terms of level design, it represents how easily the player will be able to memorize and understand the layout or structure of spaces. The Stability Factor is a parameter that combines concepts from the Graph Theory and the Dominion Theory by treating dominions as vertices and transitions as edges in a graph and analyzing their relationships. For a given minimal logic graph, the stability factor can be calculated by the following steps:

1. Calculate the $\boldsymbol{Cheeger}$ numbers of a given graph with $k \leq 3$
2. Calculate the stability factor with the following formula:

$$\gamma^{(3)} = \lim_{m \to 3} \frac{\sum_{i=1}^{m} \frac{1}{i!} \lambda_i}{\sum_{i=1}^{m} \frac{1}{i!}} = \frac{1}{e} \lim_{m \to 3} \frac{1}{i!} \lambda_i$$
$$= \frac{3}{5}\left(\lambda_1 + \frac{1}{2}\lambda_2 + \frac{1}{6}\lambda_3\right)$$

In this study, the stability factor is used to verify if the current minimal logic graph of the artifact is constructed in such a way as to help players memorize the layout. According to an analysis of all non-linear levels in the *Dark Souls* series and other Dark-Soul-like games, the standard value of the Stability Factor is **0.94** [25].

3 METHODOLOGY

3.1.1 Summary

The researcher crafted a methodology for creating a minimal logic graph with a strong stability factor. Using this minimal logic graph, the researcher hoped to create a 3D, non-linear, and miniscaped level that the players can easily navigate without getting lost. The researcher created a single-player level named “Lunaric Parchments” in *The Elder Scrolls V: Skyrim.* The level requires the player to collect 7 magical parchments in the game world. The player may also stumble across many hidden chests within the game level.

The researcher’s methodology is as follows:

1. Design a minimal logic graph with a stability factor larger than or equal to 0.94.
2. Expand the minimal logic graph for gameplay purposes by adding leaves, loops, chains, and subgraphs. In addition, for each dominion that is in the minimal logic graph, assign a distinctive decorative theme to it.
3. For each vertex in the graph, treat it like a dominion and design the layout and appropriate gameplay.
4. Design the transition areas for each edge connecting two dominions.
5. Block the level layout and evaluate the stability factor of the level.
6. Fill all dominions with the designed gameplay.
7. Decorate all dominions according to their themes decided in step 3. For each dominion expanded in step 2, investigate the nearest neighboring dominion that is included in the initial minimal logic graph and decorate the expanded dominion in the same theme.

3.2 Preproduction

3.2.1 Design a Minimal Logic Graph

In the preproduction stage of the artifact, the researcher created the following minimal logic graph.

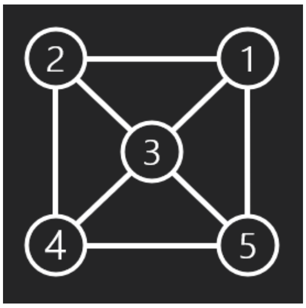


Figure 29 Minimal Logic Graph of the artifact level [26]

For this minimal logic graph, the researcher calculated its $\boldsymbol{Cheeger}$ number as follows:

1. Specify the parameter ***n***
   a. If $\boldsymbol{k = 1}$.
      No matter how the vertex is removed, the resulting graph is still connected.
   b. If $\boldsymbol{k = 2}$.
      No matter how the two vertices are removed, the resulting graph is still connected.

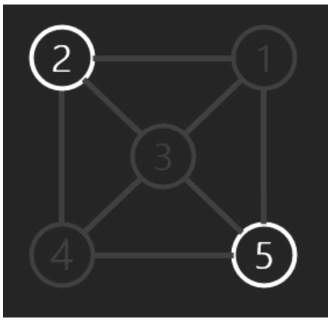


Figure 30 The resulting graph of the artifact's minimal logic graph if vertices (1,3,4) are removed at once [27]

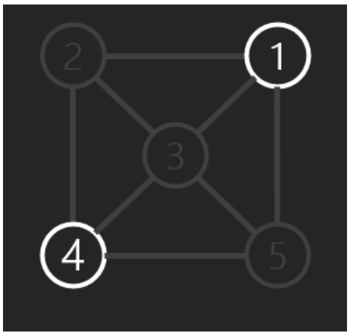


Figure 31 The resulting graph of the artifact's minimal logic graph if vertices (2,3,5) are removed at once [27]

   c. If $\boldsymbol{k = 3}$.
      If vertices $(\boldsymbol{1, 3, 4})$ or vertices $(\boldsymbol{2, 3, 4})$ are removed, as shown in **Figure 30** and **Figure 31**, the connectivity of the resulting subgraph is broken. Hence, there exists **2** ways to break the resulting graph’s connectivity if **3** vertices are removed.
   d. If $\boldsymbol{k = 4}$.
      Technically, a single vertex is still a connected graph. As a result, the resulting graph is still connected no matter how the four vertices are removed.
   e. The following result set is retrieved:
      $$\begin{cases} k = 1\ m = 5, n = 0 \\ k = 2\ m = 5, n = 0 \\ k = 3\ m = 5, n = 2 \end{cases}$$
2. By using the formula $\boldsymbol{\lambda_k = 1 - \frac{n}{C_m^k}}$, the $\boldsymbol{Cheeger}$ numbers of this minimal logic graph are determined as:
   a. $\lambda_1 = 1 - \frac{0}{\binom{5}{1}}$=1 (k=1 m=5, n=0)
   b. $\lambda_2 = 1 - \frac{0}{\binom{5}{2}}$=1 (k=2 m=5, n=0)
   c. $\lambda_3 = 1 - \frac{2}{\binom{5}{2}} = \frac{4}{5}$ (k=3 m=5, n=2)

3. Using the formula of stability factor:
    a. $\gamma^{(3)} = \lim_{m\to 3} \frac{\sum_{i=1}^{m}\frac{1}{i!}\lambda_i}{\sum_{i=1}^{m}\frac{1}{i!}} = \frac{1}{e}\lim_{m\to 3}\frac{1}{i!}\lambda_i = \frac{3}{5}\left(\lambda_1 + \frac{1}{2}\lambda_2 + \frac{1}{6}\lambda_3\right)$
4. The graph has a final stability factor $\gamma^{(3)}$ of:
    a. $\gamma^{(3)} = \frac{\sum_{i=1}^{3}\frac{1}{i!}\lambda_i}{\sum_{i=1}^{3}\frac{1}{i!}} = \frac{3}{5}\left(\lambda_1 + \frac{1}{2}\lambda_2 + \frac{1}{6}\lambda_3\right) = 0.9636$

In conclusion for the minimal logic graph in **Figure 29**, its stability factor is **0.9636** which is larger than the recommended 0.94. This factor indicates that this minimal logic graph is well-suited for the player to explore its dominions without worrying about losing their sense of direction.

3.2.2 Expand the Minimal Logic Graph

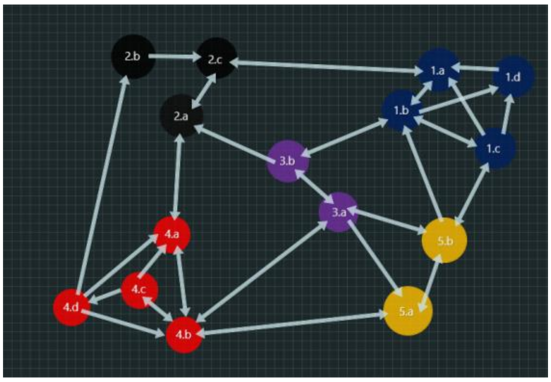


Figure 32 The expanded graph of the artifact's level [28]

To ensure that the player had enough spaces for gameplay, the researcher further expanded the minimal logic graph to a new graph as shown in **Figure 32**. In addition to adding leaves, chains, and subgraphs, the researcher also designed the directionality of edges in the graph. The directional edges helped to add and outline the level flow.

To further test out the methodology, extra embedded subgraphs (having a stability factor larger than 0.94) were added in the red and blue dominions.

The researcher also decided to make this level a castle. Each dominion in **Figure 32** is assigned a theme.

Dominion	Theme
Blue	Castle Sentry Tower
Yellow	Sunk Garden Ruins
Black	Lofty Magic Tower
Purple	Castle Central Courtyard
Red	Hill Chapel

3.2.3 Design Gameplay for Each Dominion

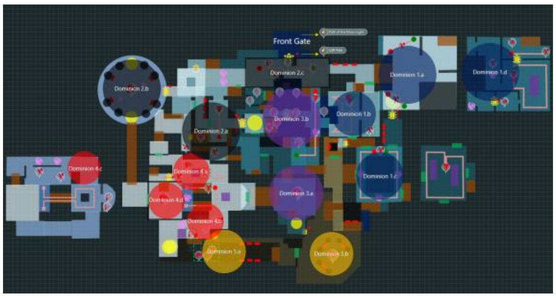


Figure 33 The level layout with dominion labels [29]

Figure 34 The level layout without dominion labels [29]

Figure 35 The legend of the level map [29]

After expanding the minimal logic graph and assigning proper distinctive themes to all dominions, the researcher designed the detailed level layout following the expanded graph as shown in **Figures 33**, **34**, and **35**.

3.3 Level Production

3.3.1 Build the First Past of Level Layout

The researcher initially built the rough layout of the level map in the game editor. In addition to assigning different decorative themes for each dominion, the researcher also implemented different asset themes for each dominion from the available asset themes in *The Elder Scrolls V: Skyrim.*

Figure 36 The top-down level blockout of the artifact's level without dominion labels [30] [31]

Figure 37 The top-down level blockout of the artifact's level with dominion labels [30] [31]

In this blockout stage, the researcher built the level layout with placeholder assets to test out the level scale, the sightlines, and the visibility of landmarks. According to the feedback from early playtesters, the researcher found that the player was easily lost in the red dominion (Hill Chapel).

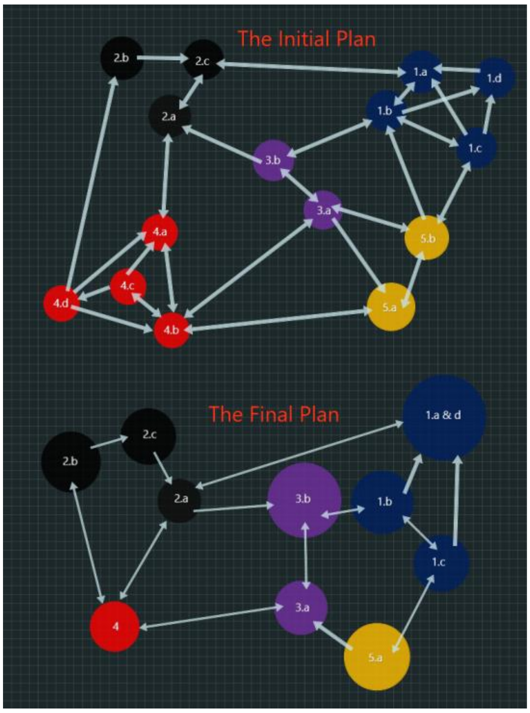


Figure 38 The Second Pass of the Expanded Graph [28]

The researcher believed the reason of this confusion was largely due to dominion 4 being an interior space. When players enter the space, they are initially met with a loading screen. Once they are inside the interior space, the player cannot see any landmarks in the exterior game world. Moreover, staying in the interior space too long causes the player to forget the exterior level layout.

Consequently, the researcher decided to remove all interior cells, which required a loading screen, in the red dominion and the yellow dominion. After addressing this issue, the graph of the artifact level is currently as it is depicted at the bottom of **Figure 38.**

3.3.2 The Final Pass of the Artifact

After modifying the expanded graph, the researcher replaced placeholder assets and put in more eye-catching landmarks to help the player navigate in the level.

Figure 39 Landmark designed for Dominion 1 (blue) [31]

Figure 39 shows a landmark the researcher placed in Dominion 1. The landmark is a majestic giant tree that the player can see wherever in the castle area. All Dominion 1's interior space are decorated using assets from

Whiterun City in *The Elder Scrolls V: Skyrim* as shown in **Figure 39.**

In addition to placing landmark, the researcher also created a custom landmark using the quest's main quest item/objective. To complete the quest, the player needs to collect a total of 7 parchments. Each parchment is accompanied a tall beam of purple light shooting into the sky **(Figure 40)**. This light is highly visible and bright, contrasting with the dark, night sky.

Figure 40 Whiterun City interior theme of sub-dominion in Dominion 1 (Blue) [31]

Figure 41 Imperial exterior theme with snow overlay in Dominion 2 (Black) [31]

Figure 42 Use snow overlay to help the player distinguish altitude [31]

Figure 41 shows how the landmark was designed for dominion 2. Both the lofty tower and the golden statue of Mara are landmarks to help the player memorize this dominion.

The researcher also added additional unique decorative elements to each dominion. In Dominion 2, the researcher added an overlay of snow on the Imperial Dungeon assets. This snow helped the player to better understand contrast and depth in this area.

Figure 43 Labyrinthian exterior theme in Dominion 3 (Purple) [31]

Figure 43 shows Dominion 3's layout design and decorative theme. Dominion 3 is a courtyard in the center of the castle. The existence of a large landmark would be quite ill-fitting. To solve this problem, the researcher added ascending stairs with giant dragon statues from the Labyrinthian Tomb theme.

Figure 44 Solitude City exterior theme in Dominion 4 (Red) [31]

Figure 44 shows the exterior decorations in dominion 4. This dominion is no longer a chapel as it was originally in the preproduction stage due to a lack of available assets. Dominion 4 was subsequently decorated in the Solitude City theme. The famous Solitude Blue Palace is used as a landmark in the area. Meanwhile, the player may navigate through the courtyard in the dominion's center.

Figure 45 Dwemer ruins exterior theme in Dominion 5 (Yellow) [31]

Figure 45 illustrates the decorative theme in dominion 5. The dominion was intended to be a sunken garden. A sunk garden concept allowed this area to be unique in comparison to all the other dominions. However, placing a landmark for this area was challenging due to its low altitude. To mitigate this issue, the researcher placed a soaring golden statue with a burning brazier. Meanwhile, the whole dominion is decorated in the Dwemer Ruins theme from *The Elder Scrolls V: Skyrim*.

3.3.3 Guiding the Player with Level Design Techniques

Although the methodology helps the player to memorize spaces in the level, it cannot guarantee that the player will know how navigate to the quest objectives. The researcher implements various level design guidance techniques in the level. These techniques help the player to understand how to progress.

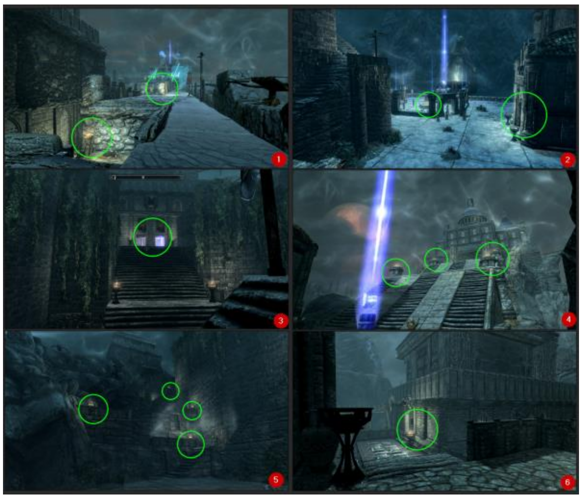


Figure 46 Using lights hinting where to go [31]

- Lighting

Because the whole quest happens at night, lights provide additional contrasts to the dark surroundings. (**Figure 46**)

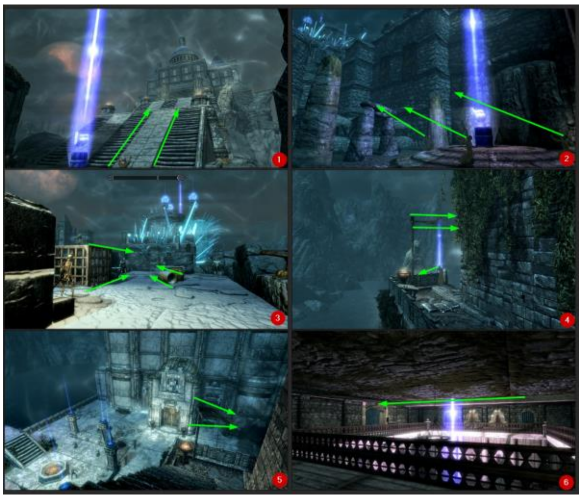


Figure 47 Use carpets to lead the player [31]

- Statues Guiding the Player's Sightline

In **Figure 47**, the researcher lined NPC statues up to face in the same direction. The player can follow the statue's line of sight to see a quest objective.

Figure 48 Use rewarding items to attract the player [31]

- Breadcrumbing and Framing

The researcher used Breadcrumb items (rewarding items) in the artifact to attract the player to go in a specific direction. In Figure 48, items (green circles), such as potions, readable notes, and sacks (containing loot) sits at the end of the hallway to attract the player forward.

As depicted in images 3, 4, and 5 (Figure 48), the researcher also used framing (obscuring part of the image and leaving only the important thing that needs focus) to highlight certain key objects or locations. This creates a focus on the target and directs the player's eye.

4 RESULTS AND DATA ANALYSIS

4.1 Playtester Count

The researcher recruited 17 participants to playtest the artifact. All playtesters completed a pre-test survey, prior to playtesting the level. Once the playtesters finished the level, they completed a post-test survey.

4.2 Demographic Data

Sixteen of the seventeen participants said that they played 3D first- or third-person video games regularly. The remaining eleven participants said they did not often play first or third-person video games. Four of the seventeen participants had never played *The Elder Scrolls V: Skyrim* before the playtest. Thirteen participants had played *The Elder Scrolls V: Skyrim* prior to the playtest session, but notably, six participants had played *The Elder Scrolls V: Skyrim* for over 100 hours.

4.3 Playtest Session Notes

Before the playtest, the researcher advised the participants to take a Quantic Foundry Player Motivation Profile test to help them identify their player type. In the post-test survey, all participants were asked about their player type according to the Quantic Foundry Player Motivation Profile result.

The playtester played the artifact using the researcher's laptop. While the participants were playing with the artifact, the researcher observed their actions and reactions.

The researcher's goal was to use the survey to test the participant's ability to remember the map layout. The participants' responses would help to verify the effectiveness of the methodology.

4.4 Data Analysis

4.4.1 Quiz Questions Testing Mental Mapping Ability

Figure 49 Testing methodology question example [32]

The Qualtrics XM post-survey had 15 questions, a mix of qualitative and quantitative. Several of these questions were quiz-like, in that the participant was meant to respond with a correct answer. The researcher provided the tester with a map of the level (Figure 49). The map showed each of the level's primary dominion, denoted by a unique color. The participants then had to match the dominion name with the correspondingly colored region, based on the provided map image. The researcher reviewed the participants' ability to mentally map the level. Mental mapping means that, given a reference (pictures, keywords, etc.) of an area in a level, the player is able to picture the area's location and come up with a route to this area. The result reflected the participants' ability to mentally map the level.

4.5 Dominion Identification Correctness

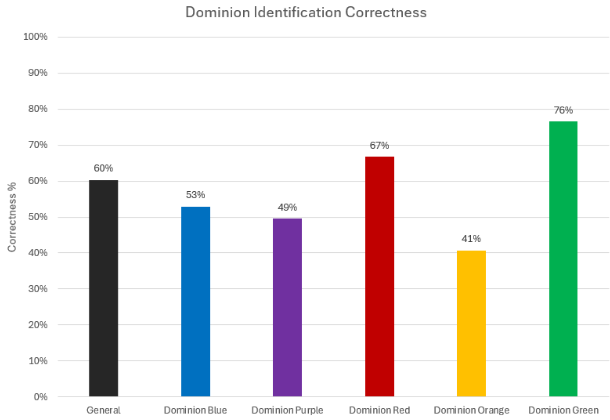


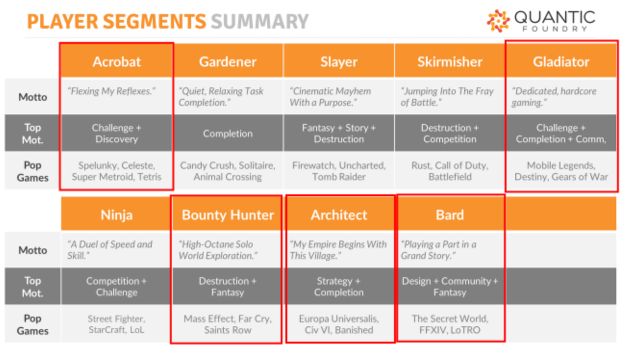
Figure 50 Dominion Identification correctness [32]

Figure 50 depicts the general correctness for all the playtesters. The general correctness (the correctness of all these questions regardless of what dominions a question refers to) is 60%. The playtesters most accurately identified the Green and the Red Dominions in comparison to other dominions. In contrast, the Orange Dominion was the least accurately identified dominion. This result matches the expectation that:

1. The Green and the Red Dominions are the most memorable areas in the level.
2. The Orange Dominion is the least memorable area due to its low altitude.

The Greed and the Red dominions are expected to be most impressive because they both contain a lofty and giant landmark. The player can see them in every place in the level. In contrast, the Orange Dominion is the least impressive because it does not have a landmark on a similar scale as the Red and Green Dominions. Moreover, the Orange Dominion is at the lowest altitude in the whole level, making it harder to see in many of the areas on the level.

4.6 Discovery Player Style with other Styles


PLAYER SEGMENTS SUMMARY — QUANTIC FOUNDRY

	Acrobat	Gardener	Slayer	Skirmisher	Gladiator
Motto	"Flexing My Reflexes."	"Quiet, Relaxing Task Completion."	"Cinematic Mayhem With a Purpose."	"Jumping Into The Fray of Battle."	"Dedicated, hardcore gaming."
Top Mot.	Challenge + Discovery	Completion	Fantasy + Story + Destruction	Destruction + Competition	Challenge + Completion + Comm,
Pop Games	Spelunky, Celeste, Super Metroid, Tetris	Candy Crush, Solitaire, Animal Crossing	Firewatch, Uncharted, Tomb Raider	Rust, Call of Duty, Battlefield	Mobile Legends, Destiny, Gears of War

	Ninja	Bounty Hunter	Architect	Bard	
Motto	"A Duel of Speed and Skill."	"High-Octane Solo World Exploration."	"My Empire Begins With This Village."	"Playing a Part in a Grand Story."	
Top Mot.	Competition + Challenge	Destruction + Fantasy	Strategy + Completion	Design + Community + Fantasy	
Pop Games	Street Fighter, StarCraft, LoL	Mass Effect, Far Cry, Saints Row	Europa Universalis, Civ VI, Banished	The Secret World, FFXIV, LoTRO	

Figure 51 The 9 quantic gamer types [33]

According to the Quantic Survey Player Segments Summary depicted in **Figure 51**, players can be categorized into 9 different types. Among these 9 types, Acrobat, Gladiator, Bounty Hunter, Architect, and Bard consider exploring the game world one of their main objectives. The researcher categorized these 5 player types as Discovery style for this study. Notably, the participants were allowed to select their player types in the post-test survey. By allowing them to self-identify, the researchers accidentally allowed the participants to incorrectly identify themselves. For example, if the playtester was found to be a Bard, they might disagree with it then fill in the survey saying they are a Gladiator.

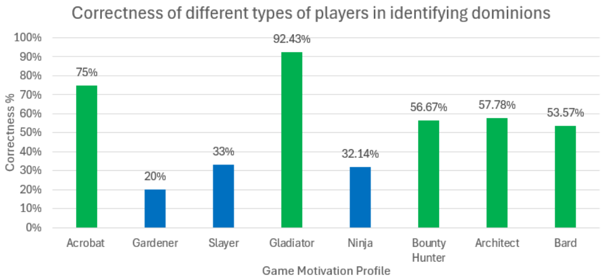


Figure 52 Quiz questions correctness with respect to player type [32]

As depicted in **Figure 52**, a Discovery style (Acrobat, Gladiator, Bounty Hunter, Architect, and Bard) player generally has a higher correctness than other player types. The researcher believes that players who engage in exploration are more likely to engage with the level. As a result, they are able to map the level layout more accurately. The methodology is, therefore, more likely to impact an exploration type player.

4.7 Mental Mapping Level Layout

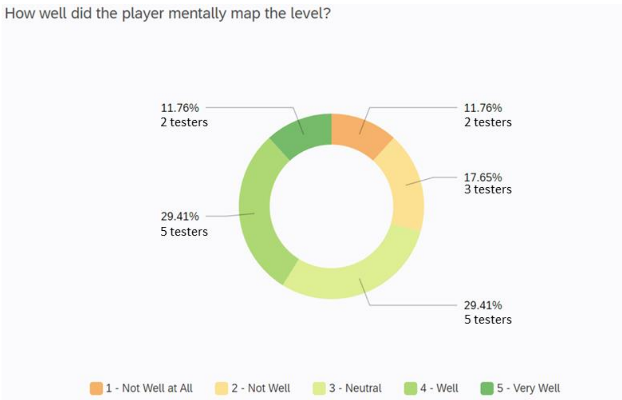


Figure 53 Mental mapping level layout with respect to the 15 questions [32]

Figure 51 shows how well the participants felt about mentally mapping the level layout from their perspective. 12 out of 17 testers (70.58%) felt that they remembered the level layout well to some extent.

4.8 Good Sense of Direction

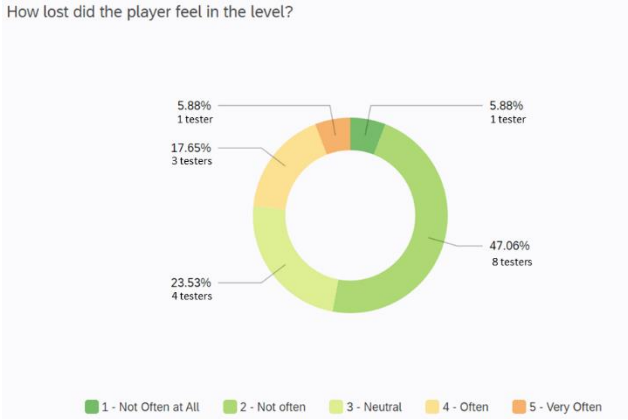


Figure 54 Feeling of lostness for *all participants* [32]

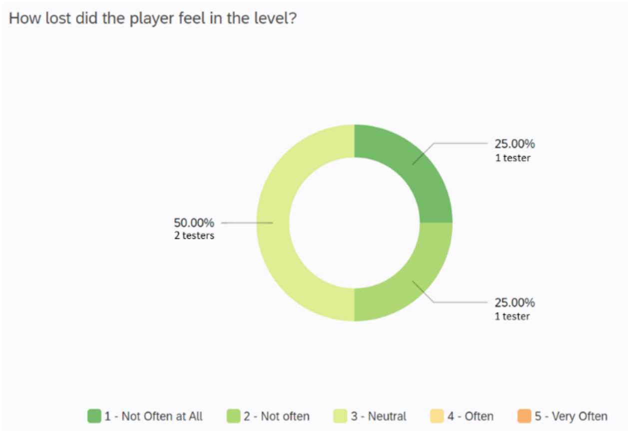


Figure 55 Feeling of lostness for participants *who have not played Skyrim before* [32]

As depicted in **Figure 54**, 13 out of 17 participants (76.47%) felt that they did not feel lost (Not often at All, Not often, Neutral) in the level. The 4 participants who felt lost are all Discovery Type players. They had a general 60.12% correctness of the quiz questions, which indicates that they mentally mapped the map well. The researcher believed these 4 participants might either fill in their player type incorrectly or judge their lostness incorrectly.

During the playtests, the researcher found that the participants who had not played *The Elder Scrolls V: Skyrim* verbally and physically expressed frustration with feeling lost. This reaction may be due to the fact that the testers were overwhelmed by the game's mechanics (controls, combat, etc.). However, according to **Figure 55**, all 4 participants who had not played *Skyrim* before did not feel lost while navigating through the level. Based on the responses, the researcher believes that the methodology strongly improves new players' ability to mentally map the level layout.

4.9 Enjoyable Exploration

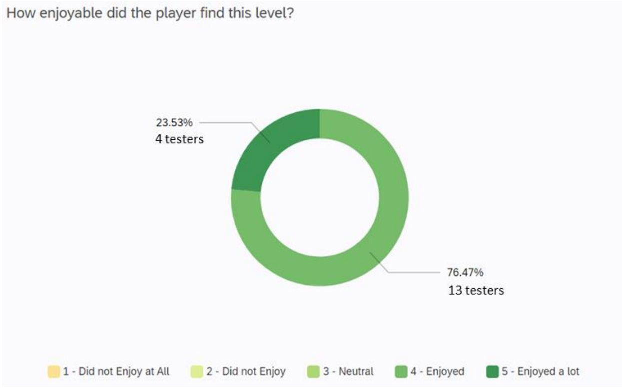


Figure 56 Enjoyment of playing the artifact for all participants [32]

As shown in **Figure 56**, all participants, even those who were often lost, enjoy exploring the artifact layout. The methodology succeeded in creating an enjoyable exploring experience for the players.

5 CONCLUSION

In conclusion, the results preliminarily prove the methodology's effectiveness in designing an enjoyable, easily memorized, non-linear level layout. This methodology does well in:

- Helping players, especially new players, navigate in a non-linear level layout without their sense of direction and their objectives.
- Maintaining players' engagement with the level space by exploring chances when feeling lost.

To further test and prove the methodology, the following points should be considered in subsequent tests or studies:

- Create two levels: one level acts as the control level and the other acts as the test level. Both levels are similarly sized, non-linear miniscaped environments. However, the control level does not use the methodology.
- Recruit more participants so that each Quantic Foundry Player Motivation style has sufficient samples.
- Create an experience designed around evaluating the Quantic Foundry Player Motivation styles.

Lesson

After finishing testing and analyzing the results, the researcher gained some experience from creating non-linear miniscaped levels.

For future research on top of this study, the researcher advises to consider the following points:

- Landmarks in different dominions should have equally impressive landmarks or dominion features (like the crystals in Dominion 2). All dominant landmarks should be placed at a similar altitude so that the player can see all of them from any place in the level.
- While incorporating the Quantic Foundry Player Motivation Profile survey to a study, recording this survey result and the participant's self-identified result in the post-survey is essential.
- To help different player type keep engaging with a level, the corresponding content with respect to different player type should be added.

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