THREE ESSAYS IN MACROECONOMICS

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THREE ESSAYS IN MACROECONOMICS

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THREE ESSAYS IN MACROECONOMICS

A Dissertation Presented to the Graduate Faculty of the

Dedman College
Southern Methodist University

in
Partial Fulfillment of the Requirements
for the degree of
Doctor of Philosophy
with a
Major in Economics

by
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December 19, 2020
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For making my PhD story possible I am deeply thankful to my school math, physics and economics teachers and mentors, but the most importantly I am thankful to my parents and my sister for being forever supported and loved. This dissertation is dedicated to them.
This dissertation investigates several key macroeconomic topics with the use of econometric and machine learning techniques. The dissertation includes three chapters.

In the first chapter, I develop a medium-scale DSGE model with real wage rigidities and financial intermediaries experiencing endogenous capital constraint and liquidity mismatch. Labor market frictions help to mitigate an endogenous labor supply insurance against fluctuations in consumption. Anticipated idiosyncratic bank runs and possibility of direct investment fix cyclical properties of financial leverage and allows explaining banks’ balance sheet dynamics over crisis. The probability of a bank run depends on bank balance sheet and endogenous assets prices. Anticipations of a bank run affect both asset returns and the economy even no crisis periods.

In the second chapter, I investigate nonlinearity in the Effects of Large Oil Shocks on Macroeconomic Activity. There is a broad debate on nonlinearity of oil price shock effects on macroeconomic variables. Previous literature focused attention on the ”Net oil price” variable to test for asymmetry of the responses to positive and negative oil price shocks. So far, most of the nonlinear empirical models of oil prices and economic activity are based on oil price changes without identifying the source of oil price change. For linear models, the use of structural vector autoregressions (SVAR) to estimate the effect of oil supply and demand shocks on macroeconomic activity has become common in recent years. However, none of these models consider nonlinear responses to structural shocks. I propose a SVAR model, based on second order Taylor expansion of a general
nonlinear model, that allows me to identify structural shocks. To handle the large number of parameters, I apply Bayesian methods with priors on the parameters in the spirit of the Minnesota priors used for linear VARs. For Bayesian inference, I embed a particle-filter-based likelihood. The impulse response analysis suggests no significant asymmetry in the responses for “small” shocks, but a significant asymmetric response in economic activity to large oil shocks, regardless of the source of the oil shock.

In the third chapter, I develop a ”human-free” text analysis of the Beige Book that can easily be updated with the new information and used with other macroeconomic data to nowcast current quarter GDP growth. I employ a Latent Dirichlet Allocation to extract topic word groupings from the Beige Book documents; the optimal number of topics is determined by a consensus of four Gibbs sampling methods. GDP growth nowcasting is done by means of supervised learning methods (support vector regression) and a comparison of nowcasting power of text word topics is provided. I find that the Beige Book information about current economic activity is significant for in sample and out of sample forecasting. The prediction power increases with the month of the quarter. Nonlinear kernel SVR analysis show that qualitative information about economic activity has nonlinear nature.
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This is dedicated to my parents and my sister, a doctor in her own right.
Chapter 1

Financial Intermediary Bank Runs in Context of a DSGE Model

1.1. Introduction and literature review

Low participation of households in the stock market is one of the puzzles of modern economics. The standard argument is that given positive excess return of a stock market and with finite relative risk aversion, each household with positive investment wealth should hold some positive proportion of wealth in stocks. I am not trying to explain it, as Janecek (2004) does by using very high risk aversion coefficients and modest transaction costs for participation in the stock market. Instead, I use this puzzle as an argument for financial institutions pricing assets, i.e. the role of deal-brokers as the marginal investors. My goal is to develop a medium-size DSGE model that delivers both financial and real macroeconomic facts and features both financial accelerator effects and bank runs.

Recently, He, Kelly, and Manela (2017) has explored that shocks to the equity ratio of financial intermediaries have strong explanatory power for cross sectional variation in expected returns of asset classes variety. They defined equity ratio as an aggregate value of market equity divided by aggregate market equity plus aggregate book debt of New York Fed primary dealers. This finding together with households’ comparative absence of trading assets expertise, particularly in trading sophisticated financial instruments, led to the idea of the use of financial intermediaries as marginal investors in asset markets.

Following He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014) intermediaries equity ratio or net worth is considered as a proxy for their marginal value of wealth and can be used as a pricing kernel for many security classes. Although my model proceeds in a different way from theirs, I do not build continuous time finance model with value at risk constraint. I will also use the bank’s marginal value of wealth as the asset pricing kernel.
He, Kelly, and Manela (2017) replaces the inexpert households’ first-order condition with that of sophisticated financial intermediaries. Unfortunately, their equity premium estimates imply that the intermediary equity ratio is pro cyclical or, equivalently intermediary leverage is counter cyclical. This goes against the robust evidence for cross sectional pro cyclicality of financial intermediary leverage, initially provided by Adrian, Etula, and Muir (2014) and time series pro cyclicality explored by Adrian, Moench, and Shin (2013). He et al. (2017) explains this discrepancy by difference in data used. Federal Reserve Flow of Funds data, used by Adrian et al. (2014) only contains information about broker-dealer subsidiaries of conglomerates and standalone US broker-dealers. He et al. (2017) uses data at holding company level. The latter one creates a noise in financial intermediary equity rate or, equivalently, leverage, because it compounds external capital funding of financial holding companies with the holding-wide internal funding. Moreover, Nuño and Thomas (2017) found that correlation of leverage ratios with GDP are positive and vary around 0.12 to 0.36, all are statistically significant despite the fact that they are small. So, you can find the evidence of pro cyclical financial leverage to be robust, that is carefully elaborated by Adrian et al. (2013). The Adrian et al. (2014) builds an empirical model and does a number of robustness checks for a single-factor asset pricing model that uses shocks to financial intermediary leverage to construct an intermediary stochastic discount factor (SDF).

Over last few years, there have been a number of attempts to build theories that relate balance sheet capacity of financial intermediary to asset price dynamics. One of the most prominent of them is Brunnermeier and Pedersen (2009). They construct a model, in which even with risk-neutrality, the funding constraint gives rise to nontrivial state pricing since it places higher value on states in which funding constraints are tighter. These are the states when financial intermediaries have very high marginal value of wealth. At states, when funding constraints tighten, intermediaries are forced to deleverage by selling off assets that they can no longer afford. Adrian et al. (2014) get a "justification" for their empirical approach and tests since the leverage is used as a proxy for funding conditions or, put it different, since funding constraints are always binding, financial leverage directly measures these constraints and, thus, measures the marginal value of wealth in Brunnermeier and Pedersen (2009). Unfortunately, this paper as well as, He and Krishnamurthy
(2013) which models leverage as a signal of the financial system’s wealth, feature counter cyclical financial leverage.

A model that matches the aforementioned empirical findings most closely is Adrian and Boyarchenko (2013). They give rise to a pricing kernel with shocks to intermediary leverage. Moreover, it experiences an empirically confirmed finding across asset classes and time periods that the price of risk of intermediary leverage is always positive. Its pricing kernel has aggregate output as the additional risk factor, with a price of risk that fluctuates between positive and negative. Also, its model makes predictions about the evolution of the price of risk over time. These predictions have been further studied by Adrian et al. (2013). The most interesting one is that equity premium relates negatively to the de-trended level of financial leverage. This is a time-series analog of leverage pro cyclicality: increase in leverage leads to an increase in asset prices, lowering expected returns and it corresponds to a low market price of risk. However, the banker in Adrian and Boyarchenko (2013) maximizes the discounted sum of its wealth, not a shareholder value, under Value at Risk constraint and it is not clear if the objective function would matter under its constraint.

I use intermediary’s leverage constraint, created by costly state verification in a banking system or an agency problem coming from bankers ability to divert a fraction of deposits for personal use. My endogenous capital constraint differs from all the aforementioned finance models. This kind of constraint comes from Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). The costly state verification problem on bank’s liability side generates endogenous leverage constraint. This feature illustrates mostly the shadow banking, which includes all financial intermediaries that operated outside the Federal Reserves regulatory framework and did not have strict exogenous leverage requirements.

I do not assume that deposit rate of return is risk-free, as Li (2013), Mimir (2016) as most asset pricing papers, built on Gertler and Kiyotaki (2010) model, do. Bank runs create riskiness of deposit returns. This is one of the reasons why bank runs are needed for asset pricing implications in my model.
Gertler and Karadi (2011) discusses on how adverse shocks may shut down credit supply through the effect on bank equity capital, and how quantitative easing interventions could mitigate the effects of the shocks on economic activity.

Gertler and Kiyotaki (2010) calibrates an economy-wide leverage ratio to be four, explaining it to be a crude attempt of leverage averaging across sectors with utterly different financial structures. It is hard to find a financial intermediary type giving so low intermediary leverage. Commercial banking sector has leverage of 10-15 and shadow banking leverage is even higher and reaches 25-30. Modelling a representative bank would give financial leverage around 13 with an increase to twenty or twenty five during expansion periods. Therefore, the model’s capital constraint is always binding and uses the leverage three times less, than the average financial leverage in a financial system. One reason Gertler and Kiyotaki (2010) needs very low leverage is to impose an assumption that the constraint is always binding around the steady state and to use a local approximation method to solve the model.

Occasionally binding capital constraint is closely related to nonlinearity of financial crises. Evidence of such nonlinearity is provided by Krishnamurthy, Nagel, and Orlov (2014). The crises usually feature a simultaneous collapse in asset prices and increase in excess return. Sharp declines in output are followed by slow recovery, which is obstructed by deleveraged financial system. In order to capture this nonlinearity Li (2013) uses a global method for a model solution, allowing for the possibility that the balance sheet constraint does not always bind. Financial crises are periods when the constraint binds, causing a sharp contraction in economic activity.

Li (2013) shows asset pricing implications of the original Gertler and Kiyotaki (2010) paper and concludes that an endowment economy model with the agency problem can produce realistic excess returns only under recursive preference specification and fails to do so with CRRA utility. However, recursive preferences allow for a separation between the intertemporal elasticity of substitution (IES) and risk aversion. Thus, such preferences permit both parameters to be simultaneously larger than 1 and they alone, without broker-dealers as the marginal investors, succeed in asset pricing.
Gertler and Kiyotaki (2015) proposes bank runs to simulate financial crises. Runs on the shadow banking system were a crucial part of the crisis with Lehman Brothers bankruptcy in September 2008 and collapse of some money market funds and the investment banking sector. They have emerging bank run equilibrium as a sunspot event. I model bank run similarly to Gertler and Kiyotaki (2015), but do not assume that bank runs happen to the entire financial system shifting it to a bank run equilibrium. Also, my modelling differs from sequential service constraint of Diamond and Dybvig (1983) that implies that depositors get either full non-contingent return or zero depending on a place in line.

Since the runs happen for shadow banks not only during crisis periods, I assume each bank perceives a probability $p_t$ of experiencing a liquidity run and a share of $p_t$ banks actually running each period $t$. Idiosyncratic bank runs and and their anticipation help to explain dynamic properties of banks balance sheets over the periods of financial and real side crises. Also, the runs help to deal with the nonlinearity of balance sheet constraint in the marginal investor asset pricing model and to make deposit returns risky.

The framework of real wage rigidities helps to mitigate an endogenous labor supply insurance channel against fluctuations in consumption. Without the rigidities RBC models suffer from labor insurance device, which prevents generating high equity premium. In a case of endogenous economic choices, e.g., consumption and labor in a production economy, agents typically have the possibility to isolate the risk sensitive dimensions of their preferences against aggregate risk, e.g. by adjusting labor supply on a frictionless labor market to smooth their consumption.

1.2. The baseline model

The economy consists of four types of agents: households, bankers, non-financial firms, and capital producers. The financial intermediation sector is modeled similar to Gertler and Kiyotaki (2015).

Households consist of two types of agents – households and bankers – each type with a continuum of measure unity. I need two types of agents to assume that the deposits of the household are put in financial intermediaries (bankers) that it doesn’t own. So banker does not maximize house-
holds’s expected discounted life-time utility, but rather its expected discounted terminal wealth when it exits the financial intermediation sector.

Non-financial firms acquire capital in each period and sell shares to financial intermediaries. Capital producers are incorporated into the model in order to introduce capital adjustment costs in a tractable way. Bankers are experts in making loans. They intermediate funds between households and non-financial firms. Households may also make these loans directly, but are less efficient in monitoring investment projects than bankers. Financial intermediaries are limited in their ability to borrow from households due to an agency (costly enforcement) problem, which I describe further. Moreover, I allow anticipated bank runs to occur in order to account for financial crises nonlinearities.

There are two goods, a nondurable good and a durable asset, capital. Households and bankers hold capital and the total holding of capital is equal to the total supply:

\[ K_t^b + K_t^H = K_t \] (1.1)

where \( K_t^H \) be the amount held by households and \( K_t^b \) is the total capital held by bankers. I do not fix capital supply to be unity as to Gertler and Kiyotaki (2015) does, because I implement production and labor frictions to my model. My modelling of households, capital producers and non-financial firms differs from Gertler and Kiyotaki (2015) as well.

When households hold capital directly at time \( t \) for a payoff at \( t+1 \) they must pay a management cost of \( f(K_t^H) \) units of goods at \( t \). The management cost reflects the household’s lack of expertise relative to the bankers in screening and monitoring investment projects. I suppose the management cost is increasing and convex in the quantity of capital. Convexity means it is increasingly costly at the margin for households to absorb capital directly.

\[ f\left(K_t^H\right) = \frac{\bar{\alpha}}{2}(K_t^H)^2, \quad \bar{\alpha} > 0 \] (1.2)
1.2.1. Households

Let us suppose a unit mass of identical households. Households are indefinitely-lived with the following preferences over consumption and leisure ($l_t$):

I use the following preferences for households:

$$E_t \left[ \sum_{i=0}^{\infty} \beta^i (\log(c_{t+i}) + A l_{t+i}) \right]$$

(1.3)

where $c_t$ and $l_t$ are consumption and leisure of a particular household and $0 < \beta < 1$, and $A$ are parameters. $C_t$ and $L_t$ are aggregate average levels of households’ consumption and leisure: in equilibrium, $C_t = c_t$ and $L_t = l_t$.

The total time endowment is normalized to unity. Total labor supply:

$$n_t = 1 - L_t$$

(1.4)

Households save either by lending funds to competitive financial intermediaries or by holding capital directly.

Bank deposits held from $t$ to $t + 1$ are one period bonds that promise to pay the non-contingent gross rate of return $\overline{R}_{t+1}$ in the absence of a bank run. If a bank run occurs, depositors only receive a fraction $x_{t+1}$ of the promised return, where $x_{t+1}$ is the liquidation value of bank assets per unit of promised deposit obligations. So, a household’s return on deposits is:

$$R_{t+1} = \begin{cases} \overline{R}_{t+1} & \text{if no bank run} \\ x_{t+1} \overline{R}_{t+1} & \text{if bank run occurs} \end{cases}$$

(1.5)

Diamond and Dybvig (1983) use sequential service constraint on deposit contracts that relates payoffs in a run to a depositor’s place in a line. I assume depositors receive the same pro rata share of liquidated assets in case of a run.

Let $R^w_t$ be the market return of direct capital holding. The household makes decision on consumption, labor supply, bank deposits $d_t$, and direct capital holdings $K^w_t$ to maximize expected
utility subject to the budget constraint:

\[ c_t + d_t + q_t K_t^H + f \left( K_t^H \right) = w_t n_t + R_t d_{t-1} + R_t^K q_t K_{t-1} \]  

(1.6)

The household’s first-order condition for labor supply yields the marginal rate of substitution, which is the frictionless wage:

\[ w_f^t = A c_t \]  

(1.7)

I assume not all labor supply reaches the market due to some labor market friction, which I do not model explicitly. Thus the steady state supply of labor is fixed at some exogenously given level below the level of the frictionless economy. The real wage rigidity is similar to Blanchard and Gálı (2007). Around the steady state, households are willing to supply labor at market wage, expressed by:

\[ w_t = (w_{t-1})^\nu (e^{\omega} w_f^t)^{1-\nu} \]  

(1.8)

The parameter \( \omega \) is average wage markup to ensure that \( w > w_f \) locally around the steady state, and that therefore the labor market is (typically) demand constrained. The frictionless scenario is a special case, when \( \nu = 0 = \omega \).

If households assign zero probability of a bank run, from the first order condition for deposits the risk-free rate is:

\[ E_t (M_{t,t+1}) R_{t+1} = 1 \]  

(1.9)

where the stochastic discount factor \( M_{t,t+i} \) satisfies:

\[ M_{t,t+i} = \beta^i \frac{\lambda_{t+i}}{\lambda_t} \]  

(1.10)
where $\lambda_t$ is the Lagrange multiplier given by:

$$\lambda_t = 1/c_t \quad (1.11)$$

Risk premium arises from investigating the Lucas asset pricing equation from the first-order condition for direct capital holdings:

$$E_t(M_{t,t+1}R_{t+1}^H) = 1 \quad (1.12)$$

with the households gross marginal rate of return from a direct capital holding: $R_{t+1}^H = \frac{R_{t+1}^K q_{t+1}}{q_t + f'(K_t^H)}$, where $f'(K_t^H) = \bar{r}K_t^H$. As long as the household has at least some capital held directly, the first order condition (1.12) helps to determine the market price of capital.

1.2.2. Non-financial Firms

I use a continuum of unit mass of non-financial firms that produce the final output in the economy. The production technology at time $t$ is constant returns to scale function with an aggregate TFP realization $z_t$:

$$Y_t = F(K_t, n_t) = z_t K_t^\alpha n_t^{1-\alpha} \quad (1.13)$$

where $K_t$ is the firm’s purchase of physical capital from capital producers and $n_t$ is the amount of labor a non-financial firm is hiring.

Non-financial firms acquire capital $K_{t+1}$ at the end of period $t$ to produce the final output in period $t+1$. Then, the firm can sell the capital on the open market in the end of period $t+1$. Non-financial firms purchase their capital in each period by issuing equities and selling them to financial intermediaries. Firms issue $K_t$ units of state-contingent claims (equity). The financial contract between a financial intermediary and a non-financial firm is an equity contract or equivalently, a state contingent debt contract. The non-financial firm pays a state-contingent rate of return equal to the ex-post return on capital $R_{t+1}^K$ to the banker. The non-financial firms set their capital demand $K_t$ taking such stochastic repayment into consideration. At the beginning of the next period $t+1$,
when shocks are realized and output becomes available, non-financial firms obtain resources $Y_{t+1}$ and use them to make repayments to shareholders (or financial intermediaries). The firm prices each financial claim at the price of a unit of capital $q_t$.

There are no frictions for non-financial firms in obtaining funds from financial intermediaries by issuing shares. The financial intermediary has perfect information about the firm and there is perfect enforcement. Therefore, in the current model, only financial intermediaries face endogenous borrowing constraints in obtaining funds. These constraints directly affect the supply of funds to the non-financial firms.

Non-financial firms choose $K_t$ and $n_t$ in order to maximize their profits. The profit maximization problem solved by a representative non-financial firm is given by:

$$\max_{K_t, n_t} F(K_t, n_t) + q_t (1 - \delta) K_t - R^K_t q_{t-1} K_t - w_t n_t$$

(1.14)

Profit maximization with respect to $n_t$ and $K_t$ gives us the following labor and capital demand conditions:

$$R^K_t = \frac{F_K(K_t, n_t) + q_t (1 - \delta)}{q_{t-1}} = \frac{\alpha z_t K_t^{\alpha-1} n_t^{1-\alpha} + q_t (1 - \delta)}{q_{t-1}}$$

(1.15)

$$w_t = F_n(K_t, n_t) = (1 - \alpha) z_t K_t^{\alpha} n_t^{-\alpha}$$

(1.16)

Condition (1.15) states that the real rate of return on capital is equal to the marginal product of capital plus the capital gain from the asset price change net depreciation. Condition (1.16) states that the market wage rate is equal to the marginal product of labor.

1.2.3. Capital Producers

I follow the literature on financial accelerator and incorporate capital producers into the model. I use capital producers to introduce capital adjustment costs in a tractable way. Introduction of some variation in the price of capital requires capital adjustment costs. Otherwise the price of capital does not respond to the changes in capital stock and always equals to one. I choose the functional form of the capital adjustment cost following previous literature, in particular Mimir
I assume that households own capital producers and receive any profits. At the end of period $t$, competitive capital producers buy capital from non-financial firms to repair the depreciated capital and to build new capital. Then they sell both the new and repaired capital. The cost of replacing the depreciated capital is unity; thus the price of a unit of new capital or repaired capital is $q_t$. The profit maximization problem of the capital producers is given by:

$$\max_{k_t} q_t K_{t+1} - q_t K_t - I_t$$  \hspace{1cm} (1.17)$$

Subject to:

$$K_{t+1} = (1 - \delta) K_t + \Phi \left( \frac{I_t}{K_t} \right) K_t$$  \hspace{1cm} (1.18)$$

where $I_t$ is capital producer’s total investment and $\Phi \left( \frac{I_t}{K_t} \right)$ is the capital adjustment cost function given by:

$$\Phi \left( \frac{I_t}{K_t} \right) = I_t K_t - \varphi \left[ \frac{I_t}{K_t} - \delta \right]^2$$  \hspace{1cm} (1.19)$$

The optimality condition gives the following ”$q$” relation for investment:

$$q_t = \left[ K_t \Phi_t \left( \frac{I_t}{K_t} \right) \right]^{-1} = \frac{1}{1 - \varphi \left[ \frac{I_t}{K_t} - \delta \right]}$$  \hspace{1cm} (1.20)$$

where $\Phi_t \left( \frac{I_t}{K_t} \right)$ is the partial derivative of the capital adjustment cost function with respect to investment at time $t$. Fluctuations in investment expenditures create volatility in the price of capital. A fall in investment at time $t$ reduces the price of capital at the same time period.

1.2.4. Bankers

I use the banking sector similar to Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). Bankers correspond best to the shadow banking system, which does not have strict capital regulations. The banks in the model are completely unregulated, hold long-term securities, and issue short-term debt. Thus, banks may experience liquidity mismatch and, as a consequence, are subject to runs. Each banker manages a financial intermediary. Bankers use their own equity $e_t$ and, as the marginal investors, transfer the funds that they obtain from households as deposits $d_t$ to the
non-financial firms. Bankers are constrained in their ability to collect deposits from households due to the financial market frictions. Thus bankers save their retained earnings and accumulate them in order to move toward full equity financing and avoid the financing constraint. To avoid full equity financing I assume that bankers have a finite expected lifetime. Each banker has an i.i.d. probability $\sigma$ of surviving until the next period and a probability $1 - \sigma$ of exiting. Then the expected lifetime of a banker is $1/(1 - \sigma)$.

I assume that a household puts its deposit in banks that it does not own. This setup ensures that bankers do not maximize households expected discounted lifetime utility and motivates "dividend payouts" from the banking system to ensure that banks use leverage in equilibrium by assuming that bankers are risk neutral and enjoy utility from terminal wealth in the period they exit. The expected utility of a continuing banker at the end of period $t$ is given by:

$$V_t = E_t \left[ \sum_{i=0}^{\infty} \beta^i (1 - \sigma) \sigma^{i-1} \tilde{e}_{t+i} \right]$$

where $\tilde{e}_{t+i}$ is the terminal wealth of an exiting banker at time period $t + i$ and $(1 - \sigma)\sigma^{i-1}$ is the probability of exiting at date $t + i$.

I assume banks can only accumulate net worth via retained earnings. Banker’s balance sheet identity:

$$q_t k^b_t = \epsilon_t \tilde{e}_t + d_t$$

I use the i.i.d. equity (financial) shock $\epsilon_t$ to capture exogenous movements in the net worth of bankers. This shock is similar to net worth shock from Mimir (2016) and financial shock from Meh and Moran (2010). They state that these shocks reflect periods of financial distress and weakness in financial markets. This shock makes financial intermediary asset pricing model to be much closer to the data. According to Brunnermeier and Pedersen (2009) this kind of shock arises from activities like investment banking. They provide with a recent example of Bear Stearns, whose clients terminated their brokerage relationships and ran on the investment bank in March 2008.

In my model, $\epsilon_t$ represents any kind of shock to a bank’s net worth such as asset write-downs, loan losses, and shifts in public sentiment about the value of financial intermediaries or the financial
system. An example of this shock could be a collapse of the market for mortgage-based securities or termination of brokerage relationships between banks and their clients. Further, I think of $e_t\tilde{e}_t$ as the effective net worth of the financial intermediary and denote $e_t\tilde{e}_t$ as $e_t$. So, $e_t$ is the net worth of the banker at the beginning of period $t$ after the financial shock occurs.

Since only the fraction $\sigma$ of the banks survive each period, the net worth of “surviving” bankers is the gross return on assets net the cost of deposits:

$$
\tilde{\epsilon}_{t+1}^s = \sigma[R^K_{t+1} q_t k^b_t - R_{t+1} d_t] \quad (1.23)
$$

Equivalently,

$$
\tilde{\epsilon}_{t+1}^s = \sigma \left[(R^K_{t+1} - R_{t+1}) q_t k^b_t + R_{t+1} e_t\right] \quad (1.24)
$$

For new bankers at $t$, their net worth $\tilde{\epsilon}^n_t$ is composed by the transfers from the exiting bankers’ households, $e_t^e$. The fraction of each household’s transfer is $\epsilon_t^e / (1 - \sigma)$ of its final period net worth. The total final period net worth of exiting bankers at time $t$ is equal to $(1 - \sigma) e_t$. Thus, the net worth of entering financial intermediaries is:

$$
\tilde{\epsilon}_{t+1}^n = \epsilon e_t \quad (1.25)
$$

Assuming the risk premium is always positive, the banker has an incentive to buy shares of non-financial firms and she always wants to obtain additional funds (deposits) from the households.

I introduce costly enforcement (agency) problem to motivate a limit on the bank’s ability to issue deposits. After accepting deposits and buying assets at the beginning of period $t$ the banker can operate honestly, which means holding assets until the payoffs are realized in period $t + 1$ and then paying deposits back with a promised rate of return. Also, the banker may choose to divert a fraction $\theta$ of available funds from its shares of non-financial firms and give them to the household of which the banker is a member or sell the fraction $\theta$ of assets secretly on a secondary market in order to obtain funds for personal use. To remain undetected, banker can only sell the fraction $\theta$ of the assets and she can only sell these assets slowly. For this reason the banker must decide whether to divert at $t$, prior to the realization of uncertainty at $t + 1$. As a result of the banker’s diversion the
deposits force the intermediary into bankruptcy at the beginning of the next period.

The banker’s decision at $t$ is to compare the franchise value of the bank $V_t$, which measures the present discounted value of future payouts from operating honestly, with the profit from diverting funds, $\theta q_t k^b_t$. Therefore, for the households to have an incentive to supply funds to the banker, the financial agreement between the bank and its depositors must satisfy the following incentive constraint:

$$\theta q_t k^b_t \leq V_t$$

(1.26)

This incentive constraint implies the effective net worth $e_t$ must be positive for the bank to operate since the franchise value $V_t$ turns out to be proportional to $e_t$. I choose parameters and shock variances that keep $e_t$ non-negative in “no-bank” run equilibrium. Similar to Diamond and Dybvig (1983), I use the payoff on deposits if there is no bank run as risk-free return, which requires that bank’s effective net worth must be positive without a run. But, the bank run may force $e_t$ to become zero.

When the bankers exit, they consume their net worth. The recursive representation of bank franchise value is the expected discounted value of the sum of net worth conditional on exiting and the value conditional on continuing:

$$V_t = E_t \left[ \beta (1 - \sigma) e_{t+1} + \beta \sigma V_{t+1} \right]$$

(1.27)

Absent a bank-run the banker’s optimization problem would be to choose $(k^b_t, d_t)$ each period to maximize the franchise value (1.27) subject to the incentive constraint (1.26) and the balance sheet constraints (1.22) and (1.23).

I express the growth rate of net worth from the balance sheet constraints:

$$\frac{e_{t+1}}{e_t} = \frac{\alpha Y_t / K_t + q_t (1 - \delta) q_t k^b_t e_{t+1} - R_{t+1} d_t}{e_t}$$

$$= \left[ \left( R^b_t - R_{t+1} \right) \phi_t + R_{t+1} \right] e_{t+1}$$

(1.28)
with
\[
R_{t+1}^b = R_{t+1}^K = \frac{\alpha Y_{t+1} / K_{t+1} + q_{t+1} (1 - \delta)}{q_t},
\]
(1.29)

\[
\phi_t \equiv \frac{q_t k_t}{e_t},
\]
(1.30)

I denote the realized rate of return on bank assets from date $t$ to $t+1$ as $R_{t+1}^b$ and the ratio of assets to net worth, i.e. the "leverage multiple", as $\phi_t$. As I mention above, the growth rate of bank equity is an increasing function of the leverage when the realized rate of return on bank asset is higher than the deposit rate. So, I assume $R_{t+1}^b > R_{t+1}$.

Since the objective and constraints of the banker are constant returns to scale, I express the bank optimization problem as a linear programming problem to choose the leverage to maximize its marginal value of wealth or "Tobin’s $q$ ratio", given by the franchise value per unit of current period equity, $V_t/e_t$. Unlike Gertler and Kiyotaki (2015), I call $V_t/e_t \equiv \psi_t$ the intermediary’s marginal value of wealth. Even though expression for $\psi_t$ describes average, not marginal value of wealth, I call it further marginal value of wealth, because Hayashi (1982) shows conditions for a firm, where Tobin’s average and marginal $q$ are directly related and shows their equality, if the firm is a price-taker with constant returns to scale in both production and installation of assets. I have "price-taking” bank constant returns to scale in the objective and constraints. Moreover, most studies use average $q$ as a proxy for a marginal one. Average $q$ ratios are preferred due to their comparative ease of computation.

Using equations (1.29) and (1.30), the bank’s problem is:

\[
\psi_t = \max_{\phi_t} E_t \left\{ \beta (1 - \sigma + \sigma \psi_{t+1}) \epsilon_{t+1} \left[ (R_{t+1}^b - R_{t+1}) \phi_t + R_{t+1} \right] \right\}
= \max_{\phi_t} \{ \mu_t \phi_t - v_t \}
\]
(1.31)

subject to the incentive constraint:

\[
\theta \phi_t \leq \psi_t = \mu_t \phi_t - v_t,
\]
(1.32)
with
\[\mu_t = E_t \left[ \beta \Omega_{t+1} \left( R^b_{t+1} - R_{t+1} \right) \right]\]  \hspace{1cm} (1.33)

\[\nu_t = E_t (\beta \Omega_t) R_{t+1},\]  \hspace{1cm} (1.34)

where
\[\Omega_{t+1} = \left(1 - \sigma \right) \lambda_t + \sigma \psi_{t+1} \epsilon_{t+1}\]  \hspace{1cm} (1.35)

Expression for \(\nu_t\) could be interpreted as the marginal cost of deposits and let us define the risk-free rate of return, given no bank-run. \(\mu_t\) is the excess return of assets over deposits. Also, \(\Omega_{t+1}\) is a new ”stochastic discount factor” and a probability weighted average of the marginal values of net worth to exiting and to continuing bankers at \(t + 1\). The marginal value of an additional unit of net worth is unity for an exiting banker at \(t + 1\) (which occurs with probability \(1 - \sigma\)), since the banker just consumes her net worth. For a continuing banker (which occurs with probability \(\sigma\)), the marginal value is \(\psi_{t+1}\). Intermediary Tobin’s \(q\), may exceed unity due to the banker’s capital constraint.

Expressions for marginal value of bank deposits and the excess return of assets over deposits could be formulated in the following recursive forms:
\[\mu_t = E_t \left[ (1 - \sigma) \frac{\lambda_{t+1}}{\lambda_t} \left( R^b_{t+1} - R_{t+1} \right) + M_{t,t+1} \sigma \frac{k^b_{t+1}}{k^b_t} \mu_{t+1} \right] \]  \hspace{1cm} (1.36)

\[\nu_t = E_t \left[ (1 - \sigma) \frac{\lambda_{t+1}}{\lambda_t} R_{t+1} + M_{t,t+1} \sigma \frac{e_{t+1}}{e_t} \nu_{t+1} \right] \]  \hspace{1cm} (1.37)

From the banker’s value maximization the incentive constraint is binding if and only if the excess return or excess marginal value from honestly managing assets \(\mu_t\) is positive but less than the marginal gain from diverting \(\theta\) units of assets:
\[0 < \mu_t < 0\]  \hspace{1cm} (1.38)
Given condition (1.38) is satisfied, the incentive constraint leads to the following endogenous limit on intermediaries leverage multiple:

\[ \phi_t = \frac{\psi_t}{\theta} = \frac{\nu_t}{\theta - \mu_t} \]  

(1.39)

This constraint limits the portfolio size to the point where the bank’s gain from diverting funds (per unit of equity) \( \theta \phi_t \) is exactly equal to the cost of losing the franchise value, measured by the intermediary Tobin’s \( q \). Thus, the costly enforcement problem leads to an endogenous constraint on the size of the bank’s portfolio and incorporates financial intermediary’s leverage into a stochastic discount factor. Re-expressing (1.39) gives a relation of excess returns, risk-free asset pricing kernel and intermediary’s marginal value of wealth:

\[ \mu_t = \theta - \frac{\theta \nu_t}{\psi_t} \]  

(1.40)

In the absence of the incentive constraint, bankers intermediate all the capital and the economy resemble frictionless financial system economy. In such economy the financial structure is irrelevant to real activity and the bank runs can never happen. The unlimited arbitrage by banks would push discounted excess returns to zero \( (\mu_t = 0) \).

However, when the incentive constraint binds, limits to arbitrage emerge, leading to positive expected excess returns in equilibrium, \( \mu_t > 0 \), and to the shadow value of bank net worth exceeding unity, \( \psi_t > 1 \). The banker’s portfolio is constrained by its own equity. Fluctuations in net worth lead to fluctuations in investment made through an intermediary, showing the conventional financial accelerator effects. Moreover, since a bank cannot operate with negative net worth, a bank run equilibrium may be possible. I use the device of anticipated bank runs to account for non-linearity in excess return as well as real macro-variables caused by deep crises.

A run may occur if after the realization of productivity shock or equity shock at the beginning of period \( t \), all depositors choose not to roll over their deposits, which are banker’s short-term liabilities. Bank run could be considered the result of a liquidity mismatch between banker’s short-term liabilities and long-term assets.
Also, the possibility of a bank run justifies the further assumption that capital constraint is always-binding around the steady state: when a negative productivity shock hits, the banker’s rate of return drops in the same period, leading to a financial sector deleveraging and making the capital constraint unbind. The same time, negative productivity shock decreases bank’s coverage rate and increases the possibility of a bank run during recessions (I show this later). A bank run is an utterly unwanted event for a banker, since if it occurs, the banker loses all the retained earnings. Thus, non-zero probability of bank runs increases financial sector discipline, i.e, banks rely more on their equity and have higher capital adequacy or lower leverage during all future periods. So, a non-zero probability of bank-runs smooths a jump in bank’s marginal value of wealth during recessions and works on making the capital constraint always binding.

1.3. Equilibrium

Given that the intermediary leverage $\phi_t$ is independent of individual bank-specific factors and assuming the incentive constraint is binding in equilibrium, we can aggregate across banks to obtain the relation between total equity $e_t$ an total assets held by the banking system $q_t K^b_t$:

$$q_t K^b_t = \phi_t e_t$$  \hspace{1cm} (1.41)

The evolution of bankers’ aggregate equity $\bar{e}_{t+1}$ by summing over surviving and entering bankers is:

$$\bar{e}_{t+1} = \sigma \left[ (R_{t+1} - R_t) q_t K^b_t + R_{t+1} e_t \right] + \varepsilon e_t$$  \hspace{1cm} (1.42)

Total output $Y_t$ either used for management costs, or consumed by households and spent on investment:

$$Y_t = f \left( K^H_t \right) + C_t + I_t$$  \hspace{1cm} (1.43)
1.4. Anticipated Bank Runs

I implement possibility of expected bank runs into my model. Suppose households acquire deposits at $t$ that mature in $t + 1$ and attach probability $p_t$ to a possibility of a run at $t + 1$. Then, at time period $t + 1$ they decide whether to roll over deposits for the next period or not. If depositors of a particular bank all together decide not to roll over their deposits, then the run happens for this bank. It is important to note that liquidity runs are idiosyncratic. In other words, each bank perceives a probability $p_t$ of experiencing a liquidity run but only a share $p_t$ actually suffers from this.

I model bank run as sunspot run events similar to Gertler and Kiyotaki (2015), but do not assume that bank runs happen to the entire financial system shifting it to bank runs. The “sweep” of bank runs depends on the condition of a bank balance sheet and an endogenously determined asset liquidation price. It could be the case that a bank run cannot occur in normal times, but a severe recession can lead to a sharp nonlinear drop in asset prices and jump in asset returns. For a bank to continue to operate it must have positive net worth $e_t > 0$. It is individually rational for a depositor not to roll over its deposits, if it believes that other depositors will do the same, forcing a bank into liquidation and such forced liquidation pushes the bank to insolvency with $e_t = 0$.

I assume that at the beginning of period $t$, after the realization of productivity shock, households decide whether to roll over their deposits. If depositors choose to run, the bankers liquidate their capital and sell assets to households who own the capital with their less efficient technology. Let $q^*_t$ be the price of capital in the event of a forced liquidation of the banking system. Then a run on banks happens if the liquidation value of banks assets $(\alpha Y_t/K_t + q^*_t (1 - \delta)) K_{t-1}^b$ is smaller than the liability to the depositors, $R_t D_{t-1}$. This is the case when the banker’s equity is washed away and the recovery rate in the event of a bank run $x_t$ has to satisfy the following condition:

$$x_t = \frac{(\alpha Y_t/K_t + q^*_t (1 - \delta)) K_{t-1}^b}{R_t D_{t-1}} < 1 \quad (1.44)$$

With production in a model, the condition for a bank run depends not only on the liquidation price of capital $q^*_t$ and bank’s balance sheets, but also on the marginal product of capital and
implicitly on a productivity shock. This condition can be expressed as:

\[ x_t = \frac{R^b_t}{R_t} \frac{\phi_{t-1}}{\phi_{t-1}} < 1, \]  

where \( R^b_t = (\alpha Y_t/K_t + q_t (1 - \delta))K^b_{t-1}/q_{t-1} \) is the return on bank assets and \( \phi_{t-1} \) is bank leverage at time period \( t - 1 \).

A bank run happens if the realized rate of return on bank assets \( R^b_t \) is sufficiently low relative to the gross interest rate on deposits \( R_t \) and last period intermediaries’ leverage \( \phi_{t-1} \) is high enough to satisfy condition (1.45).

Since \( R^b_t, R_t \) and \( \phi_{t-1} \) are the same for all intermediaries in equilibrium, the condition for a run does not depend on individual bank specific factors. All these three variables are endogenous. Thus, the possibility of a bank run varies with macroeconomic conditions.

Finally, within my framework the distinction between a liquidity shortage and insolvency is more subtle than is often elaborated in popular literature. Whether banks are insolvent or not depends on equilibrium asset prices which in turn depend on the liquidity in the banking system; and this liquidity can change abruptly in the event of a run. As a real world example of this phenomenon consider the collapse of the banking system during the Great Depression. As Friedman and Schwartz (1963) point out, what was initially a liquidity problem in the banking system (due in part by inaction of the Fed), turned into a solvency problem as runs on banks led to a collapse in long-term security prices and in the banking system along with it.

### 1.5. Asset Pricing Implications

In this section I proceed in solving banker’s maximization problem with non-negative probability of a bank run and consider asset pricing implications. When depositors anticipate a bank run occurs with a positive probability, the promised rate of return on deposits \( \bar{R}_{t+1} \) of each bank from date \( t \) to \( t + 1 \) has to satisfy the household’s first-order condition for deposits:

\[ 1 = E_t [(1 - p_t) + p_t x_{t+1}] M_{t+1} \bar{R}_{t+1} \]  

(1.46)
In the event of a run the depositor recovery rate \( x_{t+1} \) depends on \( R_{t+1} \):

\[
x_{t+1} = \min \left[ 1, \frac{(\alpha Y_{t+1}/K_{t+1} + q_{t+1} \ (1 - \delta)) k_t^b}{R_{t+1} d_t} \right]
\]

\[
= \min \left[ 1, \frac{R_{t+1}^b}{R_{t+1} \phi_t - 1} \right]
\]

(1.47)

From equation (1.46) it follows that \( R_{t+1} \) is increasing with the probability of a run as long as \( E_t(M_{t,t+1} x_{t+1}) < E_t(M_{t,t+1}) \). The banker’s decision problem differs from the case of no-run in a way that the deposit rate now is increasing with the intermediary’s leverage, because the recovery rate is decreasing with the leverage. The bank must offer a higher promised deposit rate to its creditors, when bank run is possible. You can see it by combining (1.46) and (1.47):

\[
\bar{R}_{t+1} = \frac{1 - p_t E_t \left( M_{t,t+1} R_{t+1}^b \right)}{(1 - p_t) E_t \left( M_{t,t+1} \right)}
\]

(1.48)

Thus, the bank’s equity evolves in the following way:

\[
\bar{\varepsilon}_{t+1} = \sigma \left[ (R_{t+1}^K - \bar{R}_{t+1}) q_t K_t^b + \bar{R}_{t+1} e_t \right] + \varepsilon e_t
\]

(1.49)

With a possibility of a run, the bank chooses its balance sheet \((k_t^b, d_t)\) to maximize the same objective \( V_t \), subject to the existing constraints (1.22) and (1.26), the modified expression for \( \bar{\varepsilon}_{t+1} \), (1.49) and the constraint on \( \bar{R}_{t+1} \). So, I have one more constraint (1.48).

The most prominent difference in the solution is that now the banker’s behavior depends on the probability of a run. In particular, the relation of excess return, deposit return pricing kernel and intermediary’s marginal value of wealth remains the same:

\[
\mu_t = \theta - \frac{\theta v_t}{\psi_t}
\]

(1.50)
However, unlike the no-run case, the excess value of bank assets over deposits now is a decreasing
function of bank run probability:

\[ \mu_t = E_t \left\{ \beta \Omega_t \left[ R^b_{t+1} - R^f_{t+1} - p_t \left( R^b_{t+1} - R^f_{t+1} E_t \left( M_{t,t+1} R^b_{t+1} \right) \right) \right] \right\} \]  

(1.51)

with \( R^f_{t+1} = 1/E_t (M_{t,t+1}) \) being the risk-free rate under no bank run.

The excess value of bank assets over deposits, \( \mu_t \), is decreasing in \( p_t \). Therefore, an increase
in the probability of a run tightens the leverage constraint, reduces intermediary’s leverage and the
franchise value of the bank, \( V_t \), which tightens the incentive constraint. This crucial technical detail
allows me to assume always binding banker’s incentive constraint and to use local approximation
methods. I omit the discussion about the individual bank incentives and that the local solution is a
global, because it is exactly the same as in Gertler and Kiyotaki (2015).

Equation (1.51) could be expressed in the following recursive form:

\[ \mu_t = E_t \left[ \left( 1 - \sigma \right) \frac{\lambda_{t+1}}{\lambda_t} \left( R^b_{t+1} - R^f_{t+1} - p_t \left( R^b_{t+1} - R^f_{t+1} E_t \left( M_{t,t+1} R^b_{t+1} \right) \right) \right) + M_{t,t+1} \sigma \frac{q_{t+1} k_{t+1}}{q_t k_t} \mu_{t+1} \right] \]  

(1.52)

My way to determine the probability of a bank run is very similar to Gertler and Kiyotaki
(2015), but I offer an alternative functional form that relates \( p_t \) to the aggregate recovery rate. I
do it to account for non-linearity of the economy fundamentals during crises. A bank run is a
"sunspot" outcome with the probability of the "sunspot" depending on the aggregate recovery rate
\( x_t \), which is the key fundamental determining whether a bank run equilibrium exists:

\[ p_t = g \left( E_t (x_{t+1}) \right) \]  

(1.53)

with \( g' (\cdot) < 0 \ \forall \ t \ (x_{t+1}) \in [0; 1] \)

One may use the following functional forms:

\[ g \left( E_t (x_{t+1}) \right) = 1 - E_t (x_{t+1}) \]  

(1.54)
A linear relation or a cosine relation:

$$g(E_t(x_{t+1})) = \cos\left(\frac{\pi}{2} E_t(x_{t+1})\right)$$

(1.55)

This functional form (Figure 1.1) makes small drops below unity in perceived expectation dangerous and generates the probability of a run 0.45, when $E_t(x_{t+1}) = 0.7$ and probability of a run 0.7, when $E_t(x_{t+1}) = 0.5$, which is 40-50% higher, then just using a linear relation.

An increase in a fraction of running shadow banks $p_t$ reduces $e_{t+1}$. As a likelihood of a very unwanted event for bankers increases, the leverage multiple $\phi_t$ reduces and the cost of funds $\bar{R}_{t+1}$ raises. Thus, the endogenously determined bank run probability amplifies the effects of aggregate disturbances to the economy even more together with the financial accelerator amplification.

The bank capital channel distributes shocks in the following way: a negative productivity shock reduces the profitability of bank lending, making it hard for banks to attract deposits. Bankers must therefore finance a larger share of non-financial firms from their own net worth (equity), which requires a decrease in their leverage. Since bank equity consists of retained earnings, it cannot adjust immediately and bank lending together with aggregate investment drop. These initial declines distribute the shock to future periods, since lower investment depresses bank earnings, which translates into lower bank capital in future periods and thus further decreases in aggregate investment. As a result, the drop in investment leads to a sharp drop in prices of capital and assets and a non-linear jump in excess return.

1.6. Calibration

I calibrate the model to match financial sector and real activity facts. Financial sector is characterized by the average value of financial intermediaries leverage around 12. This value is achieved by choosing bank’s seizure rate $\theta = 0.08$.

Frisch elasticity of labor supply is an important indicator of labor market. It is defined as the elasticity of labor supply to frictionless wages by holding the marginal rate of consumption
constant. In my model specification Frisch elasticity is represented by:

\[ \tau = \frac{U_n}{\bar{n}[U_{nn} - U_{nc}^2/U_{cc}]} = \frac{A}{\bar{n}} \] (1.56)

I assume that the steady state level of steady state level of leisure is \( \bar{l} = 2/3 \) and the leisure utility parameter is set \( A = 2.3 \) to calibrate the steady state level of labour \( \bar{n} = 1/3 \).

The real business cycle literature is used to assume a relatively high Frisch elasticity of two or more (Prescott (1986); King, Plosser, and Rebelo (1988)). However, Justiniano and Primiceri (2008) and more recent papers of DSGE model estimation found far smaller values (0.25 to 0.5) for the Frisch elasticity in a New Keynesian model framework. Small values are in line with the micro-based studies, such as Pistaferri (2003), which argue for small values in a range between 0 and 0.7. Since the model in use is RBC model, Frisch elasticity is set \( \tau = 3.45 \). Intuitively, this value indicates high enough labor market rigidities. The real wage rigidities is an important ingredient to explain a high risk premium in DSGE models.

The whole list of calibrated parameter values is provided in Table 1.1.
<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Households</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quarterly discount factor</td>
<td>$\beta$</td>
<td>0.9942</td>
<td>Annualized real deposit rate</td>
<td>2.37%</td>
</tr>
<tr>
<td>Relative utility weight of leisure</td>
<td>$A$</td>
<td>2.3</td>
<td>Hours worked</td>
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</tr>
<tr>
<td>Frictionless wage markup</td>
<td>$\varpi$</td>
<td>1.15</td>
<td>Average wage markup</td>
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</tr>
<tr>
<td>Degree of real wage stickiness</td>
<td>$\nu$</td>
<td>0.3</td>
<td>RBC literature</td>
<td>N/A</td>
</tr>
<tr>
<td>Households management cost parameter</td>
<td>$\tilde{\alpha}$</td>
<td>0.008</td>
<td>Kliem and Uhlig (2013)</td>
<td>0.008</td>
</tr>
<tr>
<td><strong>Production</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Share of capital in output</td>
<td>$\alpha$</td>
<td>0.36</td>
<td>Labor share of output</td>
<td>0.64</td>
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<tr>
<td>Capital adjustment cost parameter</td>
<td>$\phi$</td>
<td>3.6</td>
<td>Relative volatility of price of investment</td>
<td>0.37</td>
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<tr>
<td>Depreciation rate of capital</td>
<td>$\delta$</td>
<td>0.025</td>
<td>Average annual ratio of investment to capital</td>
<td>10%</td>
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<tr>
<td>Steady-state total factor productivity</td>
<td>$z$</td>
<td>1</td>
<td>Normalization</td>
<td>N/A</td>
</tr>
<tr>
<td><strong>Financial System</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steady-state fraction of assets that can be diverted</td>
<td>$\theta$</td>
<td>0.0713</td>
<td>Leverage ratio of commercial banks</td>
<td>12</td>
</tr>
<tr>
<td>Proportional transfer to the entering bankers</td>
<td>$\varepsilon$</td>
<td>0.001</td>
<td>0.1% of aggregate net worth</td>
<td>N/A</td>
</tr>
<tr>
<td>Survival probability of the bankers</td>
<td>$\sigma$</td>
<td>0.972</td>
<td>Average survival time of banks in years</td>
<td>10</td>
</tr>
<tr>
<td>Steady-state level of equity shock</td>
<td>$\epsilon$</td>
<td>1</td>
<td>Normalization</td>
<td>N/A</td>
</tr>
<tr>
<td><strong>Shock Processes</strong></td>
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<td></td>
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<tr>
<td>Persistence of TFP process</td>
<td>$\rho_z$</td>
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<td>Quarterly persistence of TFP process</td>
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<td>Standard deviation of productivity shock</td>
<td>$\sigma_z$</td>
<td>0.0064</td>
<td>Quarterly standard dev. of TFP shock</td>
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<tr>
<td>Persistence of equity shock process</td>
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<td>Quarterly persistence of equity shock process</td>
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</tr>
<tr>
<td>Standard deviation of equity shock</td>
<td>$\sigma_\epsilon$</td>
<td>0.0512</td>
<td>Quarterly standard deviation of equity shock</td>
<td>0.0512</td>
</tr>
</tbody>
</table>

Table 1.1. Model Calibration
1.7. Numerical Results

I provide the quantitative predictions and the comparison of the baseline model with the bank run model by examining the results of numerical simulations of an economy calibrated to quarterly U.S. data. I present the dynamics of the models in response to productivity and financial (equity) shocks. The figures provided in the appendix show the percentage deviations from steady state values as a result of negative one standard deviation shocks.

Figures 1.2-1.4 present the impulse response functions to a one-time, one-standard deviation negative shock to total factor productivity for a baseline model without anticipated idiosyncratic bank runs. The negative TFP shock reduces the price of investment goods produced by capital producers by 0.4% on impact. This decreases the value of firms’ shares and makes purchase of the shares less profitable for banks, which can be observed from 0.5% fall in the return to capital. As a result, bankers have difficulty in obtaining deposits from households since their equity investment becomes less lucrative. This reduces the return to deposits by 0.1% and induces a lagged countercyclical credit spread with its rise by 0.1%. Since the instant fall in external financing is only 0.1%, but banks’ assets fall much sharper the bankers experience a sharp fall in their equity by 4%, which means an instant jump in leverage to 50% instantly and a sharp deleveraging in the next periods with maximum fall in deposits by 7%. Hence, the model with productivity shocks generates a countercyclical leverage ratio. Because banks cannot adjust their net worth immediately and the lower price of capital reduces the value of their net worth, their financing conditions tighten, inducing aggregate investment to shrink by 1.5%. Finally, hours fall by 0.4% and output declines by 1.9%.

Figures 1.5-1.7 present the impulse response functions to a one-time, one-standard deviation negative shock to shadow banks’ equity for a baseline model without anticipated idiosyncratic bank runs. Due to the negative financial shock net worth of banks shrinks immediately by roughly 8%. To compensate the decline in their internal financing and avoid fire sale of assets, banks need to finance a larger share of their purchases of equities from deposits. As a result, we observe jumps in the leverage ratio by about 80% and deposits by 4% on impact. Therefore, the model driven by net worth shocks also generates a countercyclical leverage ratio. Although bankers have to finance
a greater fraction of their equity investment from deposits, their ability to do so is diluted by fall in their equity, leading deposits to decline after 6 quarters. Furthermore, the fall in banks’ equity weakens their ability to buy firms’ shares and leads to an instant drop in total capital stock by 1% with further decrease down to 4% over 5 quarters after the shock. Also, firms cut their investment by about 1%. This drop in investment lowers the price of capital roughly by 0.3%. Hours worked fall by 0.3% and output drops by about 0.9% on impact. Finally, consumption rises on impact after the shock hits, which was observed at the beginning of the recent financial crisis. In the context of both baseline and run models, this outwardly unappealing result could be explained by the following: on the intra-temporal margin, the fall in aggregate demand caused by lower investments translates into a reduction in the demand for labor, which eventually leads to both a drop in hours worked and wages, because wages are relatively flexible (degree of real wage stickiness is only 0.3). However, the rise in excess return on impact raises banks’ profits. Since households own banks, the rise in their profits helps households sustain their consumption after the financial shock hits. Therefore, roughly over first 4 quarters the rise in bank profits dominates the reduction in wages, pushing consumption up.

Figures 1.8-1.10 present the impulse response functions to a one-time, one-standard deviation negative shock to total factor productivity for a model with anticipated idiosyncratic bank runs. The main difference with the no-run model is that the deposit rate now depends on the probability of bank run. Thus, the negative TFP shock reduces the price of investment goods produced by capital producers by 0.2% on impact. We still observe 0.2% fall in the return to capital. However, the return to deposits and risk free do not decrease, but jump by about 0.07%, because the bankers have to offer a higher promised deposit rate to its creditors, when bank run probability increases due to a decline in the banks’ assets coverage rate. Therefore, the credit spread is pro-cyclical and declines by 1.6% in annual accounting. Now, it is even more difficult for banks to attract deposits from households since their equity investment becomes less lucrative and they have to pay higher interest for the deposits. As a consequence, external financing falls instantly by 0.3% with further decline over the next 15 quarters down to 0.4%. Since banks’ assets fall, but not very sharp, the bankers experience an instant increase in their equity by about 1.8%, which means an
deleveraging by 2% instantly. However, since the capital stock keeps falling for 20 quarters banks’ equity starts decreasing after 15 quarters too. Hence, the model with productivity shocks generates a pro-cyclical leverage ratio, which is consistent with the data and most of leverage asset pricing literature. Because banks’ financing conditions tighten, it induces aggregate investment to shrink by 2%. Finally, hours fall by 0.5%, and output declines by 1%. We observe an instant drop of 2% in the capital stock, invested by households directly without banks’ intermediation. After 8 periods of decline, households’ capital stock rises by about 1% over the next 20 quarters due until banks’ “reputation” renews.

Figures 1.11-1.13 present the impulse response functions to a one-time, one-standard deviation negative shock to shadow banks’ equity for a model with anticipated idiosyncratic bank runs. The negative financial shock leads to two effects: an immediate shrink in banks’ equity and a jump in capital invested by households directly by 5%. This jump constitutes to the deleveraging by 2%, decrease in deposits by 0.2% and deposit rate by less than 0.1%. Therefore, the effect of jump in capital invested by households dominates over a direct effect of negative equity shock and we observe an increase in the equity by 2% and an increase in the equity before it’s the shock by 8%. The difference of 6% between these increases is the net direct effect of an immediate equity destroyed by financial shock. Therefore, the model driven by net worth shocks generates a procyclical leverage ratio as well as with TFP shock. Due to the households ability to invest directly, 0.2% cut in the investment is smaller, then 1% for no-run model. The drop in investment lowers the price of capital only by 0.02%. Hours worked fall only by 0.06% and output drops by about 0.04% on impact, which is much smaller drop, than 0.9% for no-run model. Finally, the excess return jumps by about 5% on impact over the financial crisis simulation, which is consistent with the recent crisis.

1.8. Conclusions and extensions

The paper constructs a DSGE model with endogenously capital constrained financial intermediaries and allows their liquidity mismatch. I combine two approaches of banking modeling in the medium-size macroeconomic model. Bank’s agency problem together with the likelihood of bank
runs allows distinguishing risky deposit returns from risk free rates.

Labor market frictions help to mitigate an endogenous labor supply insurance channel against fluctuations in consumption. This insurance device prevents DSGE models from generating high equity premium.

I show that anticipation of bank runs fixes the cyclical property of leverage. The possibility of liquidity mismatch through bank runs depends on bank balance sheet and endogenous assets prices. The anticipation of a bank run affects both asset returns and real economic activity.

One of the findings of my paper is that bank capital channel amplifies and propagates the effects of productivity and equity shocks on output and investment. Since the model contains several features in addition to financial frictions, such as idiosyncratic bank runs and wage rigidities, these results indicate that accounting for the role of bank capital is important when building medium-scale asset pricing DSGE models.

Another finding is that bank’s equity shock, which causes exogenous decreases in bank capital, leads to a moderate decline in bank lending, investment and output. Financial intermediaries as the marginal investors can not only propagate and amplify shocks, but also be an independent source of shocks with important consequences for asset returns as well as real economic activities.

There are several extensions for further research. First, it is worth checking the assumption of always binding capital constraint and how the inclusion of bank run probability together with equity shock changes the frequency of the events, when the constraint binds during numerical simulations. Second, it could be interesting to bring this model to the data by estimating higher order approximations. This will allow looking at distributions of the first and second moments of asset returns as well as the Sharpe ratio, which is the market price of risk. Furthermore, I can elaborate long-run risk not only by introduction of a predictable component of expected growth into the consumption dynamics, which is done by Bansal and Yaron (2004), but also by the endogenous channel of leverage constraint.
Chapter 2
Nonlinearity in the Effects of Large Oil Shocks on Macroeconomic Activity

2.1. Introduction

Large recessions seem to occur, when oil price increases unexpectedly, but only small economic expansions seem to follow the unexpected declines in the price of oil. Hamilton (1996) argues that 7 out of the 8 postwar U.S. recessions have been preceded by sharp oil price increases. However, periods of high economic growth do not appear to follow the declines in oil prices.

Standard models of oil price shocks transmission that imply symmetric effects of oil price increases and decreases have difficulties explaining large declines in aggregate economic activity in response to positive oil price shocks. SVAR models of macroeconomic aggregates and oil price increases have become widely accepted on the grounds that they produced “better looking” impulse responses (Bernanke, Gertler, and Watson (1997)), than linear and symmetric models, which cannot generate quantitatively grounded effects of oil price shocks on macroeconomic aggregates. However, beyond quantifying the nonlinearity in the oil price effect, identifying the source of the oil market shocks is important and achievable in a SVAR framework.

In order to address the nonlinearity of oil price effects and to account of the shocks’ source, I consider a general nonlinear structural vector autoregressive model (SVAR). SVAR models have been commonly used to estimate the effect of oil price shocks on economic activity. SVAR imposes specific identification assumptions in order to recover economic shocks from observables and then study the effect of these shocks on the economy. An advantage of SVARs is that they allow the researcher to identify the source of the shock and obtain estimates of the impact of a shock of interest.

I propose a SVAR model, based on second order Taylor expansion of a general nonlinear model, that allows me to identify structural shocks of oil supply, oil specific demand, inventory ”specula-
tive” demand shock and a shock to macroeconomic activity. The identification of contemporaneous linear part of the model is based on Baumeister and Hamilton (2019), that belongs to a popular class of the linear structural vector autoregressive models, which are commonly used to estimate the effect of oil supply and demand shocks on macroeconomic activity. However, none of these SVAR models consider nonlinear responses to structural shocks. To handle the large number of parameters, I apply Bayesian methods with priors on the parameters in the spirit of the Minnesota priors used for linear VARs. For Bayesian inference, I embed a particle-filter-based likelihood approximation into a random walk Metropolis-Hastings algorithm. Such method belongs to the class of particle Markov chain Monte Carlo (MCMC) algorithms.

I examine the impulse response function from the model for possible asymmetries. I find little evidence of asymmetry in responses to small oil market shocks. However, the responses in economic activity to large oil supply and large global demand shocks suggest the presence of statistically significant asymmetry. Furthermore, the nonlinearity is a function of both the value of structural shocks and the current state of the economy. The interaction of the state of economy with structural shock is important. When the parameters corresponding to state of the economy are shut down, the sum of the responses shows no significant asymmetry.

2.2. Literature review

2.2.1. Oil prices and nonlinearity

A large number of studies examined the asymmetry in the responses of economic activity to oil market shocks. That is, output may respond differently to positive versus negative oil price movements. Mork (1989) was the first, who provided the empirical evidence that the real oil price increases had far more important effects on U.S. real GDP than real oil price decreases. He noticed that 1985 - 1986 oil price declines failed to lead to an economic boom in oil importing economies and suggested a measure of oil price increases, obtained by censoring oil price changes to exclude all oil price decreases.
Alternatively, Lee, Ni, and Ratti (1995a) argued that oil price decreases may matter too. However, it matters only if the observed changes are a surprise, i.e. an oil price changes have greater impact on real GNP under stable oil prices, than under frequent and erratic oil price movements. They offered an oil price shock variable, that reflects both the unanticipated component and the time-varying conditional variance of oil price change and found it to be significant in explaining economic growth over different sample periods, even when matched against various economic variables and other functions of oil price.

Afterwards, the proposal of Mork (1989) to focus only on oil price increases has been refined by Hamilton (1996), who introduced the “net oil price” variable. This variable distinguishes between oil price raises that set new highs relative to recent prices and increases that merely reverse recent price decreases. Hamilton’s variable is constructed as the maximum of zero and the difference between the log-level of the crude oil price for the current month and the previous year’s maximum.

\[ \Delta p_t^{net} = \max(0, p_t - p_t^{year}), \]  

(2.1)

where \( p_t \) denotes the log of the real price of oil and \( p_t^{year} \) is the maximum log real price of oil over the last year. If the oil price in current month or quarter is lower, than its maximum was over the last year, the variable is set to be zero for this period. Hamilton (1996) found this variable to have a statistically significant and stable negative relationship with U.S. real GDP and made an argument that this variable predicts declines in U.S. real GDP. Numerous other researchers used Hamilton net oil price variable in subsequent empirical work (N. Balke, Brown, and Yücel (2002), Hamilton (2003), Kilian and Vigfusson (2017) and Omolade, Ngalawa, and Kutu (2019)).

Kilian and Vigfusson 2011a and Kilian and Vigfusson (2011b) challenge the consensus that oil price changes have asymmetric effects. They conduct a slope-based and an impulse-response-based symmetry tests for the net increase model and revealed no evidence against the null of symmetric response functions in U.S. real GDP data. Based on a structural model that encompasses both symmetric and asymmetric models as special cases, they state that correctly computed impulse responses have similar magnitude in either direction, consistent with formal tests for symmetric
responses. Their result suggests that energy price shock is not the key factor that contributes to recessions. Moreover, interpreting the data since mid-2008, they state that the effects of oil price declines are not negligible.

In response, Hamilton (2011b) argues that the use of OLS with both price change and the net oil price variables is optimal to estimate the forecasting regression. He reminds that, for Gaussian errors, OLS produces asymptotically efficient maximum likelihood estimates and OLS F-test is the likelihood ratio test with well-known desirable properties. Therefore, Hamilton puts doubt on the statement that the new Kilian and Vigfusson test could be more powerful than the standard OLS test, and even if the OLS test rejects and the new test does not, the reconciliation should not be based on the proposition that the Kilian and Vigfusson test is more powerful. Acknowledging that there is not much evidence of the nonlinear responses to small shocks in the Kilian and Vigfusson dataset and specification, Hamilton reconciles that the data is quite consistent with nonlinear consequences of larger movements in oil prices.

A few channels for asymmetric effect of oil price changes have been described in the literature. The first channel is suggested by Davis (1987), Bresnahan and Ramey (1993) and Davis and Haltiwanger (2001). Costly intra- and inter-sectoral reallocation of capital and labor has a recessionary impact. For instance, reduced expenditures on energy-intensive durable goods, such as vehicles, may cause a reallocation of capital and labor away from the automobile sector in response to unexpectedly high real oil prices. Since the value of energy-intensive purchases may be large relative to the value of the energy they use, even small changes in oil price may have large effects on aggregate demand. Another type of reallocation disturbance is proposed by Hamilton (1988). Unemployment and production decline result from workers who, given a positive belief that conditions will improve in their sector, choose not to relocate even if they are offered a job in a different sector at a wage that exceeds their marginal utility of leisure. Also, if capital and labor are sector or product specific, i.e. they cannot move fast for another use, these inter- and intra-sectoral reallocations will cause capital and labor to be idle and unemployed, resulting in drops of output and employment that go beyond changes in households’ purchasing power. The reallocation effects arise regardless of the direction of the oil price change. In the case of an unexpected
real oil price increase, the reallocation disturbance reinforces the recessionary effect of the loss of purchasing power, generating a much larger recession than for standard linear models. In case of a negative oil price shock, the reallocation disturbance partially offsets the increased expenditures driven by the gains in purchasing power, causing a smaller economic expansion than implied by linear models. This means that in the presence of a reallocation effect, the responses of real output to oil are necessarily nonlinear in unanticipated oil price increases and unanticipated oil price decreases. Quantitatively this channel is determined by the extent to which expenditures change in response to real oil price shock and by how pervasive frictions in capital and labor markets are. For example, Edelstein and Kilian (2009) state that the magnitude of the reallocation disturbance depends on the size of a domestic automobile industry (as measured by shares in real output and employment) and on imported cars’ substitutability for domestic cars.

The second channel of nonlinear response is associated with uncertainty about future oil price. Real options theory suggests that increased oil price uncertainty leads firms to delay investments, causing investment to drop, assuming that unexpected changes in oil price are associated with increased uncertainty about the future price of oil. In practice, uncertainty is measured by the expected volatility of the real price of oil over the relevant investment horizon. Any oil price shock may be associated with higher expected volatility, no matter the real price of oil goes up or down, the uncertainty effect may serve to amplify the effects of unexpected oil price increases and to offset the effects of unexpected oil price declines, resulting in nonlinear response of macroeconomic activity. Bernanke (1983), using a partial equilibrium model, points out that high volatility in oil price may lead agents to postpone their purchases of durable goods, leading to economic contraction. This effect is even stronger for irreversible expenditures. Elder and Serletis (2010) show statistical significance of oil price volatility effect on aggregate output, investment, and durable goods consumption. Plante and Traum (2012) construct a DSGE model with precautionary saving and find that output declines in response to higher oil price volatility. Baštaya, Hülägü, and Küçük (2013) build a small open economy model with incomplete asset markets where households and firms demand imported oil at an exogenously determined price. They get a disruptive effect of oil price volatility increase, but smaller than the effect of increased oil price level on output under
Edelstein and Kilian (2009) posit a third channel for nonlinear oil price effects. Specifically, increased uncertainty about employment prospects in the wake of oil price shock may reduce consumer expenditures or increase precautionary savings. It then leads to a demand-driven drop of production. Through this channel, uncertainty may affect not only energy-intensive consumer durables such as automobiles, but other consumer expenditures as well. Taking into account lifetime consumption smoothing, this effect can be substantial for high oil price shocks, which are associated with an increased likelihood of becoming unemployed.

Monetary policy response justifies the fourth channel of nonlinearity. According to Bernanke et al. (1997), in the face of oil price shock Federal Reserve raises the interest rate in response to inflationary pressure. It amplifies the economic contraction, but the Federal Reserve does not respond so sharply to negative oil price shocks. There is no theoretical model underlying this explanation of asymmetry. N. Balke et al. (2002) show that monetary policy cannot solely account for the asymmetry in the real economy and that asymmetry is transmitted to GDP through market interest rates rather than monetary policy rates.

2.2.2. Structural vector autoregressions

Much of the oil market literature, that allows the identification of oil demand and oil supply shocks, uses linear structural vector autoregressive models. Initial literature is built on Kilian (2009), who proposes exclusion restrictions imposed on the impact multiplier matrix to decompose the real oil price shocks into three components: oil supply shocks, shocks to global demand for all industrial commodities including crude oil, and the demand shocks that are specific to the crude oil market. Later, several authors suggest sign restrictions on the implied responses of the real oil price, global crude oil production and global real economic activity to oil demand and oil supply shocks. This literature includes Peersman and van Robays (2009) and Baumeister and Peersman (2013). However, Kilian and Murphy (2012) demonstrate that imposing sign restrictions alone is not sufficient to know the relative importance of oil demand and oil supply shocks and strengthen the sign restrictions with the additional economically motivated inequality restrictions, bounding
the magnitude of the oil supply elasticity on impact.

To address and rule out the speculative trading explanations of the 2003 - 2008 oil price surge, Kilian and Murphy (2014) add the speculative oil demand shock and identify it with the help of oil inventories data. This model provides with the evidence that 2003 - 2008 oil price surge was caused by unexpected world oil consumption increases, driven by the global business cycle and that the speculative demand shocks played an important role during earlier oil price shocks. Finally, Baumeister and Hamilton (2019) call the traditional structural identification of SVARs, as all-or-nothing approach to the use of prior information, since it treats some features of the underlying structure as known with certainty, while being completely ignorant about other features. They generalize identification with a less restrictive formulation that incorporates uncertainty about the identifying assumptions. They relax Kilian (2009) assumption on knowing with certainty that there is no short-run response of oil supply to the price and Kilian and Murphy (2012) assumption that the short-run price elasticity of oil supply should be strictly less than 0.0258. Furthermore, they illustrate how one can relax the strong assumptions on oil supply and supplement it with imperfect information about demand and other features of the economic structure. To do that, they use very uninformative Student $t$ priors, truncated by sign restrictions. I proceed with the identification of structural shocks by the informative priors on linear structural parameters, used in Baumeister and Hamilton (2019).

2.3. Model

My point of departure is to take a nonlinear version of the SVAR of Baumeister and Hamilton (2019). Consider the general nonlinear SVAR model:

$$Y_t = G(Y_{t-1}, U_t),$$

(2.2)

where $Y_t$ is an $(4 \times 1)$ vector of observed variables:

$$Y_t = \begin{bmatrix} q_t & y_t & p_t & \Delta i_t \end{bmatrix}$$

(2.3)
As in Baumeister and Hamilton (2019), the vector of endogenous variables $Y_t$ includes:

1. $q_t$: growth rate of quarterly global crude oil production (in million barrels/day)

2. $y_t$: a measure of global real activity (industrial production for OECD and six major non-member economies, such as Brazil, China, India, Indonesia, the Russian Federation and South Africa)

3. $p_t$: real WTI spot oil price (deflated by US CPI)

4. $\Delta i_t$: change in proxy for OECD crude oil inventories as a fraction of previous period’s oil production

$U_t$ is $(4 \times 1)$ vector of structural innovations:

$$U_t = [u_{1t} \, u_{2t} \, u_{3t} \, u_{4t}]'$$

The structural shocks are as follows:

1. $u_{1t}$: oil supply shock. It incorporates oil supply disruptions associated with unexpected decisions by OPEC members or other politically motivated events in oil-producing countries.

2. $u_{2t}$: global economic activity shock. This shock captures unexpected fluctuations in the global business cycle.

3. $u_{3t}$: oil specific demand shock. It corresponds to idiosyncratic oil shifts, that are not accounted for by change in global oil demand or expected imbalances between oil supply and demand. Such imbalances are captured by the next shock.

4. $u_{4t}$: "speculative demand" shock or inventories shocks. The examples of oil specific demand shocks could be sharp weather changes or politically motivated releases of the U.S. Strategic Petroleum Reserve.

One can take a second order Taylor approximation of equation (2.2) to get the baseline dynamic structural model of the interest:

$$Y_t = M + A_6 D^{1/2} U_t + A_1 Y_{t-1} + A_2 Y_{t-1} \otimes Y_{t-1} + A_3 Y_{t-1} \otimes U_t + A_4 U_t \otimes U_t, \quad (2.5)$$
where $A_i, i = 0, ..., 4$, denotes the coefficient matrices and $M$ is $(4 \times 1)$ intercept vector. $A_0$ is $(4 \times 4)$ matrix of parameters, associated with contemporaneous linear responses to the structural shocks. $A_1$ is $(4 \times 4)$ matrix of parameters, associated with lagged series. $A_2$ is $(4 \times 10)$ matrix of parameters, associated with Kronecker product of the lagged series itself. $A_3$ is $(4 \times 16)$ matrix of parameters, associated with Kronecker product of the lagged series with the structural shocks. $A_4$ is $(4 \times 10)$ matrix of parameters, associated with Kronecker product of the structural shocks itself. This second order approximation implies that in general shocks could have asymmetric effects (through $A_2$ and $A_4$ parameters). It also implies that the effects could be conditional on the current state of the economy (through $A_2$ and $A_3$ parameters). I assume $u_{it}, i = 1, ..., 4$ are serially uncorrelated and distributed $N(0, I)$. $D$ is the diagonal matrix, corresponding to the variance of structural shocks in Baumeister and Hamilton (2019):

$$ D = \begin{bmatrix} d_{11} & 0 & 0 & 0 \\ 0 & d_{22} & 0 & 0 \\ 0 & 0 & d_{33} & 0 \\ 0 & 0 & 0 & d_{44} \end{bmatrix} $$

2.3.1. Identification of Structural Shocks

An important question is how to distinguish each of four structural shocks. Since the model has two parts: linear and nonlinear, the benchmark for identification of linear contemporaneous responses to the shocks is obtained from Baumeister and Hamilton (2019), that uses informative priors for linear elasticities.

Considering linear analogue of the system, there are the following exclusion restrictions. Oil supply equation:

$$ q_t = \alpha_{qp} p_t + u_{1t}^* $$

(2.7)

where $\alpha_{qp}$ is a linear analogue of price elasticity of supply. $u_{1t}^*$ is a supply shock and is analogues
to a shift in oil supply curve.

Linear relationship between oil price and economic activity is given by:

\[ y_t = \alpha_{yp} p_t + u^*_2 + u^*_2 \]  \hspace{1cm} (2.8)

where \( \alpha_{yp} \) a linear analogue of price elasticity of economic activity. \( u^*_2 \) is an economic activity shock.

From a Bayesian perspective, it is tantamount to assuming with certainty that oil production has no direct effect on economic activity or inventories change, but that oil price may have a direct effect on economic activity and inventories.

Oil demand equation is given by:

\[ q_t = \beta_{qy} y_t + \beta_{qp} p_t + \chi^{-1} \Delta i_t + u^*_3 - \chi^{-1} e_t, \]  \hspace{1cm} (2.9)

where \( \beta_{qy} \) is a linear analogue of income elasticity of oil demand. \( \beta_{qp} \) is a linear analogue of price elasticity of oil demand. \( \chi < 1 \) is a fraction of U.S. inventories in OECD crude oil inventories and \( e_t \) is the measurement error in inventory stocks, which is taken to be serially uncorrelated and uncorrelated with the observed shocks \( u^*_t, i = 1, ..., 4 \) by assumption.

Oil inventory demand is specified as follows:

\[ \Delta i_t = \psi_1 q_t + \psi_2 y_t + \psi_3 p_t + \chi u^*_4 + e_t \]  \hspace{1cm} (2.10)

where \( u^*_4 \) is a shock to inventory demand and has been interpreted as a speculative demand shock by Kilian and Murphy (2014). The inventories are assumed to depend on income only through the effects of income on quantity or price, i.e. set \( \psi_2 = 0 \) to help with identification.

Combining the above equations (2.7) - (2.10) in a linear system one gets:

\[ \tilde{A} Y_t = \tilde{U}_t, \]  \hspace{1cm} (2.11)
where \( \tilde{A} \) is \((4\times4)\) matrix summarizing linear contemporaneous structural relations used in Baumeister and Hamilton (2019):

\[
\tilde{A} = \begin{bmatrix}
1 & 0 & -\alpha_{qp} & 0 \\
0 & 1 & -\alpha_{yp} & 0 \\
1 & -\beta_{qy} & -\beta_{qp} & -\chi^{-1} \\
-\psi_1 & -\psi_2 & -\psi_3 & 1
\end{bmatrix}
\]  \tag{2.12}

The vector of observed shocks is the following:

\[
\tilde{U}_t = \begin{bmatrix}
\tilde{u}_1^t \\
\tilde{u}_2^t \\
\tilde{u}_3^t - \chi^{-1}e_t \\
\chi(1 - \rho/\chi)e_t + e_t
\end{bmatrix},
\]  \tag{2.13}

Due to the presence of measurement error in inventory stocks, the observed shocks \( \tilde{U}_t \) are contemporaneously correlated. The diagonalization of structural error variance is done with the help of the auxiliary matrix \( \Gamma \) (explored in diagonalization appendix B.1). Thus, the structural shocks, \( U_t \), are related to the observed shocks, \( \tilde{U}_t \), as follows:

\[
U_t = \begin{bmatrix}
\tilde{u}_1^t \\
\tilde{u}_2^t \\
\tilde{u}_3^t - \chi^{-1}e_t \\
\chi(1 - \rho/\chi)e_t + e_t
\end{bmatrix},
\]  \tag{2.14}

where \( \rho \) is the negative of a coefficient from a regression of \( \tilde{u}_{4t} \) on \( \tilde{u}_{3t} \):
\[ \rho = \frac{\chi^{-1}\sigma^2_e}{d_{33}^* + \chi^{-2}\sigma^2_e} \]  

(2.15)

Then, I obtain the linear parameters matrix in the way, close to Baumeister and Hamilton (2019):

\[ A_0 = (\Gamma \tilde{A})^{-1}D^{1/2}, \]  

(2.16)

with an auxiliary matrix \( \Gamma \):

\[
\Gamma = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & \rho & 1 \\
\end{bmatrix}
\]  

(2.17)

2.4. Data

The sample period starts at the first quarter of 1975 and ends at the last quarter of 2018. Monthly world oil production data measured in thousands of barrels of oil per day is obtained from the U.S. Energy Information Administration’s (EIA) Monthly Energy Review for the period of January 1975 to December 2018. The nominal spot oil price for West Texas Intermediate (WTI) is retrieved from the Federal Reserve Economic Data (FRED) database maintained by the St. Louis FED (OILPRICE). Prior to 1982 this equals the posted price. This series was discontinued in July 2013. From August 2013 onwards data is obtained from the EIA website. For the deflation of the nominal spot oil price, the FRED U.S. consumer price index is used (CPIAUCSL: consumer price index for all urban consumers: all items). The index from 1982 to 1984 is set to be 100.

The measure for global economic activity is the industrial production index for OECD countries and six major non-member economies (Brazil, China, India, Indonesia, the Russian Federation and South Africa) obtained from the OECD Main Economic Indicators (MEI) database in 2011. The index covers the period 1975:M1 to 2011:M10 and was subsequently discontinued. To extend the
data set after October 2011, the same methodology used by the OCED is applied. Specifically, I use OECD industrial production and industrial production for the individual non-member countries which are available in the MEI database and apply the weights reported by the OECD to aggregate those series into a single index. The source of the weights data is the International Monetary Fund’s World Economic Outlook (WEO) database. The weights are updated on a yearly basis and a link to a document containing the weights can be found at the OECD web-site.

Monthly U.S. crude oil stocks in millions of barrels (which include the Strategic Petroleum Reserve) are available from EIA for the entire period 1975:M1-2018:M12. I obtain an estimate for global stocks as in Baumeister and Hamilton (2019) and Kilian and Murphy (2012)) by multiplying the U.S. crude oil inventories by the ratio of OECD inventories of crude petroleum and petroleum products to U.S. inventories of petroleum and petroleum products. Given that OECD petroleum inventories only start in January 1988, I assume that the ratio before January 1988 is the same as in January 1988. In order to calculate the proxy for $\Delta i_t$, the change in OECD inventories as a fraction of last period’s oil production, I convert the production data into millions of barrels per quarter by multiplying the million barrels of crude oil produced per day by 91.

2.5. Estimation

I estimate the model with the Bayesian method. To estimate the parameters for the nonlinear VAR at hand I apply Random-Walk Metropolis-Hastings algorithm, which works well in high dimensional spaces.

To compute the posterior distribution of the parameter values one must specify prior distributions of the parameter values and the likelihood of the model. Because the structural shocks are not directly observed the likelihood cannot be computed analytically for the non-linear model, but only approximated empirically with a particle filter (PF). The idea of PF is is to produce a sample of independent random variables, ”particles”, distributed according to the posterior distribution of the state in a dynamic system. The likelihood of the data is estimated, based on the structural model (NL-SVAR) fit, which includes the particles as the realization of structural shocks.

For the model at hand, the prior distributions play two key roles:
1. the priors for \( \bar{A} \) parameters help to identify the structural shocks as in Baumeister and Hamilton (2019);

2. the priors for the parameters, associated with the nonlinear terms, help to handle the large number of parameters, corresponding to Kronecker cross-products of the four variables, four shocks and their intersection.

2.5.1. Informative priors for contemporaneous linear parameters

According to the long sequence of studies, the absolute values of the short-run linear demand elasticity \( \beta_{qp} \) and the short-run linear supply elasticity \( \alpha_{qp} \) should not be bigger than 0.5. That is represented with a prior for \( \beta_{qp} \) that is a truncated to be negative Student \( t(c_{\beta_{qp}}, \sigma_{\beta_{qp}}, \nu_{\beta_{qp}}) \) with mode at \( c_{\beta_{qp}} = 0.1 \), scale parameter \( \sigma_{\beta_{qp}} = 0.2 \), \( \nu_{\beta_{qp}} = 3 \) degrees of freedom. This combination of parameters gives a prior probability of 10% that \( \beta_{qp} < 0.5 \). Similarly, the prior for \( \alpha_{qp} \) is truncated to be positive Student \( t(c_{\alpha_{qp}}, \sigma_{\alpha_{qp}}, \nu_{\alpha_{qp}}) \) with mode \( c_{\alpha_{qp}} = 0.1 \), scale parameter \( \sigma_{\alpha_{qp}} = 0.2 \) and \( \nu_{\alpha_{qp}} = 3 \) degrees of freedom.

The use of a conventional measure of industrial production allows making the use of historical evidence about the income elasticity of oil demand. Following Csereklyei, Rubio-Varas, and Stern (2016), that founds the income elasticity of energy demand to be stable across countries and time with a value around 0.7, the prior for linear analogue of income elasticity of oil demand \( \beta_{qy} \) is a truncated positive Student \( t(c_{\beta_{qy}}, \sigma_{\beta_{qy}}, \nu_{\beta_{qy}}) \) distribution with mode \( c_{\beta_{qy}} = 0.7 \), scale parameter \( \sigma_{\beta_{qy}} = 0.2 \) and \( \nu_{\beta_{qy}} = 3 \) degrees of freedom.

Given the small dollar share of crude oil expenditures compared to total GDP, I follow Baumeister and Hamilton (2019), who expect the linear effect of oil prices on economic activity \( \alpha_{yp} \) to be small. This is represented with a truncated negative Student \( t(c_{\alpha_{yp}}, \sigma_{\alpha_{yp}}, \nu_{\alpha_{yp}}) \) distribution with mode \( c_{\alpha_{yp}} = -0.05 \), scale parameter \( \sigma_{\alpha_{yp}} = 0.1 \) and \( \nu_{\alpha_{yp}} = 3 \) degrees of freedom.

The priors for \( \psi_1 \) and \( \psi_3 \) are relatively uninformative, having unrestricted Student \( t \) distribution and centered at 0 with scale parameter 0.5, and 3 degrees of freedom. The parameter \( \chi \) reflects the fraction of total world inventories held in OECD countries. OECD countries account for roughly 60% of world petroleum consumption on average over our the sample, the expectation is that they
also account for about 60% of global inventory. Thus, a prior mean for the fraction of total world inventories $\chi$ is 0.6. Taking into account that $\chi$ is a fraction between 0 and 1, the Beta distribution with parameters $\alpha_\chi = 15$ and $\beta_\chi = 10$ is used. This combination of parameters gives a standard deviation of about 0.1.

The parameter $\rho$, used for diagonalization of structural shock variance, captures the importance of inventories measurement error and should be between 0 and $\chi$ by construction. This justifies the use of Beta-distributed prior, conditional on $\chi$, that is $\chi$ times a variable with parameters $\alpha_\rho = 3$ and $\beta_\rho = 9$. This combination of parameters gives a prior a mean of $0.25\chi$ and a standard deviation of $0.12\chi$.

The priors for an intercept $(4 \times 1)$ vector $M$ have a Gaussian distribution with the mean and standard deviation for each of four parameters taken from a corresponding univariate series. Given the zero prior mean values of nonlinear parameters, the accurate prior mean is:

$$M = (I - A_1)E(Y_t)$$

(2.18)

The priors for the reciprocals of the structural shocks variance $D$ have conditional Gamma distributions:

$$d_{ii}^{-1}\mid A \sim \Gamma(\kappa, \tau_i(A)),$$

(2.19)

where $A = \Gamma \tilde{A}$ and $\tau_i(A) = \kappa \ast \tilde{A}^\prime (i, :) \ast \hat{S} \ast A(i, :)$, with $\kappa = 2$. Such value gives the priors a weight of about 4 observations in the sample. The $i$th row of $A$ is denoted by $A(i, :)$.

The scale of the data is measured by the standard deviation of univariate autoregressions fit to the 4 elements of $Y_t$. The estimated variance-covariance matrix of univariate residuals is denoted by $\hat{S}$.

2.5.2. Informative priors for lag parameters

Since the data is the changes of oil production, economic activity, oil prices, and inventories, the prior expectation is a zero forecast. Thus, the mean of most coefficients for predicting the first-
differences \( A_1 \) are zero. Following Baumeister and Hamilton (2019), I allow a linear analogue of one period lag response of supply or demand to a price change to be similar to the contemporaneous linear responses, i.e. the prior mean of \( a_{13} \) is 0.1 and \( a_{13}^{33} \) taken to be -0.1.

For the prior variance of lag parameters, the so-called Minnesota type Doan, Litterman, and Sims (1984) priors are used. Coefficients on higher lags have greater confidence to be zero. The general form of the prior variance for \( A_1 \) is:

\[
Var(A_{1,i,j}^{l}) = \gamma * g(l) * f(i,j) * S(i)/S(j), 
\]

(2.20)

where \( \gamma \) is an overall tightness parameter. The values in the literature is taken between 0.1 and 0.2. The value, used in the model is relatively "loose" \( \gamma = 0.2 \).

For the purpose of standardization the scope of each of four univariate series, a standard deviation of each series residual, \( S(_) \), is computed via univariate OLS regressions. The number between one and four \( i \) is the ordered number of explanatory variable and \( j \) is the number of dependent variable.

I choose the reciprocal functional form for discounting lag parameters: \( g(l) = 1/l \), where \( l \) is the number of lags.

The tightness of variable \( j \) in equation \( i \) relative to that on variable \( i \) is set to be a "symmetric" type. This is the simplest type and requires only one free hyperparameter. It gives the relative weight \( w = 0.5 \) applied to all the off-diagonal variables in the system:

\[
f(i,j) = \begin{cases} 
1 & \text{if } i = j \\
 w & \text{otherwise} 
\end{cases} 
\]

(2.21)

The use of the symmetric relative tightness is justified by the relative small size of the system. It is suggested to use this type for systems of five or fewer equations.

2.5.3. Informative priors for nonlinear parameters

I handle the large number of parameters, corresponding to Kronecker cross-products of the
four variables, four shocks and their intersection by extending the idea of Minnesota priors on the parameters $A_2$, $A_3$ and $A_4$. The original idea was to make the prior variance of parameters for lagged variables tighter with the number of lags. I come up with the prior variance shrinkage on cross-variable and cross-shock products and put even tighter priors, when the regressors are different from the explanatory variable in SVAR, i.e. the regression includes 3 different series of variables or shocks. Moreover, the term of second order approximation have higher order of overall tightness parameter, $\gamma^2$ instead of $\gamma$ for linear lag terms.

The priors for matrices of nonlinear parameters $A_2$, $A_3$ and $A_4$ have the zero mean and the variances, that account for univariate series’ scales, lag discounting, general and relative tightness. For the regression of $i^{th}$ variable on lag $l$ of $j^{th}$ variable and lag $m$ of $k^{th}$ variable, the prior variance for matrix of parameters $A_2$, responsible for Kronecker product of lagged series with lagged series:

$$Var(A_2) = \gamma^2 \ast f(i,j,k) \ast g(l) \ast g(m) \ast S(i)S(j)S(k)$$  \hspace{1cm} (2.22)

For the non-linear parameters, the overall tightness is set to be $\gamma^2$ and the relative tightness of variables $j$ and $k$ in equation $i$ relative has a modified three dimensional fomr of of a “symmetric” type:

$$f(i,j,k) = \begin{cases} 
1 & \text{if } i = j = k \\
\frac{w^2}{w} & \text{if } i \neq j \neq k \\
w & \text{otherwise} 
\end{cases}$$  \hspace{1cm} (2.23)

For the regression of $i^{th}$ variable on lag $l$ of $j^{th}$ variable and structural shock of $k^{th}$ variable, the prior variance for matrix of parameters $A_3$, responsible for Kronecker product of lagged series with structural shocks:

$$Var(A_3) = \gamma^2 \ast f(j,k)g(l) \ast S(i)S(j)$$  \hspace{1cm} (2.24)

For the regression of $i^{th}$ variable on structural shock of $j^{th}$ variable and structural shock of $k^{th}$ variable, the prior variance for matrix of parameters $A_4$, responsible for Kronecker product of
structural shocks with structural shocks:

$$Var(A_4) = \gamma^2 \ast f(j, k) \ast S(i)$$  \hspace{1cm} (2.25)

2.5.4. Likelihood Approximation

In order to estimate the posterior distributions for the model parameters one needs to evaluate the priors and approximate the likelihood for the given values of parameters. Since the model is nonlinear and the structural shocks, $U_t$, are unobserved, I employ a particle filter to estimate the log-likelihood. This involves augmenting our nonlinear model (2.5) with a $(4 \times 1)$ vector of auxiliary ”measurement noise”, $V_t \sim N(0, \Sigma_v)$:

$$Y_t = M + A_0 U_t + A_1 Y_{t-1} + A_2 Y_{t-1} \otimes Y_{t-1} + A_3 Y_{t-1} \otimes U_t + A_4 U_t \otimes U_t + V_t,$$  \hspace{1cm} (2.26)

The state transition system is trivially a system of structural shocks, $U_t$ (2.4). Particle filter, also known as sequential importance resampling (SIR) is applied to the state space representation above. On the importance sampling step, I draw 10,000 particles from a proposal distribution for $U_t$ that reflects the distribution of the states in each period. Then, I evaluate the incremental weights of particles for each time period, based on Gaussian kernel in the measurement equation 2.26. Finally, I evaluate the approximation of the likelihood as the sum across all time periods of the average normalized weight for each period.

2.5.5. Bayesian inference for nonlinear structural vector autoregressions

For the Bayesian estimation of model parameters I use Random-Walk Metropolis-Hastings algorithm with 1 million draws, including first 500,000 burned-in draws. I evaluate the likelihood with a particle filter. Since the prior variance for linear and non-linear parameters varies greatly and the non-linear parameters are much more concentrated around the prior means, the block-wise Metropolis-Hastings Random-Walk sampler is implemented. Also known as ”block updating”, it refers to splitting a multivariate vector into groups called blocks, and each block is sampled
A general logic for the use of blocks is to group such parameters that are as correlated as possible within a block and keep parameters between blocks as independent as possible. Such strategy retains as much of the parameter correlation as possible for block-wise sampling, as opposed to component-wise sampling, where parameter correlation is ignored. The disadvantages of block-wise MCMC is that I loose some of the correlations, that probably exist between parameters of different blocks, and each block is updated, holding the other blocks constant. Without simultaneous updating, the Markov chain may converge slowly. However, for the state-space model of big size it may be best, when everything is not taken into account at once. Assuming the second half of the chain has converged to the target distribution, I test if the first 10% of the chain can be treated as burn-in. I use Geweke (1992) convergence diagnostic test for linear and nonlinear parameters, that mimics the simple two-sample test of means. The parameters’ mean values of the first 10% are not significantly different from the last 50% of the sample. So, the posterior distributions converged in the first 10% of the chain for the given chain.

The greatest advantage of block-wise sampling is that the acceptance of a newly proposed state is likely to be higher than sampling from all target distributions at once, especially for high dimensions. Also, large proposal variance-covariance matrices can be reduced in size, which is most helpful in high dimensions. The acceptance ratios for linear and nonlinear blocks are between 40% and 50%.

There is a trade-off between higher acceptance when splitting on more blocks, and the computational efficiency when the likelihood is evaluate at every block updates. Thus, the whole vector of parameters is divided into only two blocks: nonlinear \((A_2, A_3 \text{ and } A_4)\) parameters and and all other linear parameters, including contemporaneous, lag, intercept and the reciprocals of the structural shocks variance.

2.6. Nonlinear Impulse Responses

Policy makers, researchers and practitioners are specifically interested in the response of the economy over time to an unexpected oil shocks of different nature: supply, demand or inventory
To conduct an impulse response analyses, it is necessary to be precise about what is meant by this term. The non-linear impulse response is the revision in the conditional forecast of $Y_t$, given a primitive impulse $u_{it}$ and the current information set $\Omega_{t-1}$:

$$E[Y_{t+h}|u_{it}, \Omega_{t-1}] - E[Y_{t+h}|\Omega_{t-1}]$$ (2.27)

The construction of impulse responses for nonlinear models, such as the structural non-linear model proposed in (2.5), is not as straightforward, as in traditional VAR. Impulse responses for nonlinear models, as the structural model proposed in (2.5), depend on the history of the observations $\Omega_{t-1}$ as well as on the magnitude of the shock $u_{it}$. For most of the IRF experiments I set the initial conditions to the sample mean of the corresponding series.

I adapt the existing methods of constructing nonlinear impulse responses to the structural non-linear model proposed in (2.5) and demonstrate how the magnitude of a disruption may matter for a response. Given a particular draw from the posterior distribution of parameters, I proceed with the following steps:

1. Calculate the mean of each of the four series of interest:
   $$\bar{Y}_t = \begin{bmatrix} \bar{q}_t & \bar{y}_t & \bar{p}_t & \Delta \bar{r}_t \end{bmatrix}'$$
   Use these sample means as a history $\Omega_{t-1}$.

2. Given $\Omega_{t-1}$, simulate the time paths for $Y_{t+h}|u_{it}, \Omega_{t-1}$ for $h = 0, ..., H$. Generate the series of time paths for every shock $u_{i,t+h}$, assigning the value of each shock at a time for the first time period $u_{i,t}$, $i = 1, ..., 4$ to be a size of interest (one or two standard deviation). The values of other shocks at the first period $u_{j,t}$ for $j \neq i$ and the values of all shocks for the next periods $u_{i,t+h}$, $h = 0, ..., H$ are drawn from the corresponding empirical distributions. Since the structural errors $u_{i,t}$ and $u_{j,t}$, $j \neq i$ are by definition uncorrelated, they are drawn as independent.

3. Generate $Y_{t+h}|\Omega_{t-1}$ for $h = 0, ..., H$, given $\Omega_{t-1}$ with the values of $u_{i,t+h}$, drawn from the empirical distribution of every shock independently.
4. To eliminate any asymmetry that may arise from sampling variation in the estimation of the conditional expectation, do the two aforementioned steps for \(-u_{i,t+h}\) and average \(Y_{t+h|u_{i,t}, \Omega_{t-1}}\) and \(Y_{t+h|\Omega_{t-1}}\) over positive and negative random shocks.

5. Calculate the difference between the time paths \(Y_{t+h|u_{i,t}, \Omega_{t-1}}\) and \(Y_{t+h|\Omega_{t-1}}\) for \(h = 0,...,H\).

6. Repeat this 100 times and take the average.

I construct 68% and 95% credible bands for the impulse responses as well as find the 50th percentile based on the sample of estimated parameters.

2.7. Empirical results

2.7.1. Interpretation for posterior distributions of contemporaneous responses

The prior distributions of \(\tilde{A}\) parameters, used for the contemporaneous, linear coefficients, are denoted by the solid green curves and the posterior frequencies are denoted by the dark blue histogram regions in Figure 2.1.

The posterior median of linear analogue for short-run price elasticity of oil supply, \(\alpha_{qp}\), is 0.04, which is lower, than anticipated by the prior and estimates in Caldara, Cavallo, and Iacoviello (2019), 0.15, and Baumeister and Hamilton (2019), 0.15. The values above 0.1 are substantially less probable after seeing the data than anticipated by the prior. The posterior median of the linear effect of oil prices on economic activity \(\alpha_{yp}\), -0.003, is smaller, than expected by the prior. However, such posterior is very close to Baumeister and Hamilton (2019) results, who noticed that the data favor a positive value, causing the posterior to bunch near values just below zero.

Unlike Baumeister and Hamilton (2019), data gives us information about the linear analogue for short-run income elasticity of oil demand, \(\beta_{qy}\). Its posterior median is 1.84, which is significantly greater, than anticipated by the prior. The posterior median of linear analogue for short-run price elasticity of oil demand, \(\beta_{qp}\), -0.22, is a little more elastic than anticipated by the prior, but less elastic, than estimates in Baumeister and Hamilton (2019), -0.35. The data for the fraction of total world inventories held in OECD countries and the importance of inventories measurement error do
not provide much more information than the priors, which is similar to Baumeister and Hamilton (2019) results. The posterior median for $\chi$, 0.57, is slightly less, than expected by the prior and Baumeister and Hamilton (2019). The posterior distribution of $\rho$ is very close to Baumeister and Hamilton (2019) with the median, 0.12, smaller, than expected by the prior.

The prior of $A_1$ parameters, used for the lag, linear coefficients, are denoted by the solid green curves and the posterior frequencies are denoted by the dark blue histogram regions in Figure 2.2. Matrix $A_1$ has 14 prior zero-mean and 2 with prior non-zero means estimated parameters. 7 parameters have 95% posterior credibility interval outside of prior mean zero. The 2 parameters, associated with giving a little more prior information to distinguish supply and demand shocks, have posterior distributions closer to zero, than corresponding the prior distributions.

The prior of $A_2$ parameters, associated with the Kronnecker products of lagged series itself, are denoted by the solid green curves and the posterior frequencies are denoted by the dark blue histogram regions in Figure 2.3. Matrix $A_2$ has 40 parameters with zero-mean priors, and only 3 of them have 95% credible set outside of zero. The prior of $A_3$ parameters, associated with the Kronnecker products of lagged series with structural shocks, are denoted by the solid green curves and the posterior frequencies are denoted by the dark blue histogram regions in Figure 2.4. Matrix $A_3$ has 64 parameters with zero-mean priors, and only 4 of them have 95% credible set outside of zero. All of these parameters are in the second row and associated with macroeconomic activity series. The posterior distribution for $a_{1,7}^3$ turns out to be bi-modal with one mode at zero and another mode at $-0.3$. This parameter is associated with the oil production response to a product of lagged industrial production and the inventory shock.

The prior of $A_4$ parameters, associated with the Kronnecker products of structural shocks itself, are denoted by the solid green curves and the posterior frequencies are denoted by the dark blue histogram regions in Figure 2.5. Matrix $A_4$ has 40 parameters with zero-mean priors, and only 3 of them have 95% credible set outside of zero. The posterior distribution for $a_{2,8}^4$ is the most different from the prior distribution. This parameter is associated with the macroeconomic activity response to a "square" of the global demand shock.
2.7.2. Quantification of asymmetry in oil market shocks responses

The median simulated impulse-responses to the small, one-standard deviation shocks, are plotted in Figure 2.6. The median responses to the negative shocks are plotted by the dotted lines.

A negative small oil supply shock, displayed in the first row, is associated with an immediate jump in real oil price and drop of oil production on impact. The positive oil supply shock lowers real oil price and raises oil production on impact. The median effects of small positive and negative oil supply shocks on oil price and production are symmetric.

A negative small oil consumption demand shock, displayed in the third row, lowers both oil production and real oil price. The positive oil demand shock raises both oil production and real oil price. The median effects of small positive and negative oil demand shocks on oil price and production are symmetric as well.

A negative small oil supply shock is associated with a significant reduction in economic activity at the moment and after one and two periods. This is something different than a result, found in a large number of studies, that see a significant decline only after a lag of a number of months. This is due to the use of quarterly data for the current study. The positive small oil supply shock is associated with an expansion of the world industrial production at the moment and after one and two periods. The median effects of small positive and negative oil supply shocks on economic activity are symmetric.

A negative small shock to oil consumption demand seems to have no instant effect on economic activity and relatively small negative effect at the subsequent periods. The positive oil demand shock has no instant effect on economic activity and relatively small positive effect at the subsequent periods. The median effects of small positive and negative oil consumption demand shocks on economic activity are symmetric.

A positive small inventory or speculative demand shock is associated with a relatively small and persistent drop in economic activity and an immediate small jump in oil production, real oil price, and oil stocks itself. The negative oil inventory shock is associated with a small jump in economic activity and an immediate small drop in oil production, real oil price, and oil stocks itself. The median effects of small positive and negative oil inventory demand shocks on economic activity are symmetric.
activity, oil production, oil price and oil stocks are symmetric.

Figure 2.7 provides with the graphical test of asymmetry, by plotting the sum of responses to positive and negative small shocks accordingly. Also, it plots the median responses with one and two standard deviation credibility sets. If the responses are symmetric, the sum should be exactly zero. For the small, one-standard deviation shocks, there appears to be no asymmetry even for one standard deviation credibility sets.

The model implies that large shocks can have substantially different effects than small shocks. To explore this feature of the model, I examine the responses to big, two-standard deviation shocks. The figure 2.8 suggests that some of the responses for positive and negative shocks are not mirror images of one another. The most prominent ”negative” asymmetry is in the world industrial production response to big oil supply shocks. By ”negative” asymmetry is meant that the response to a negative shock is significantly greater, than to the same positive structural shock. The negative big oil supply shock has a negative and persistent accumulative effect on global production, but the positive effect of oil supply increase weakens after 2-3 quarters. However, test of asymmetry in figure 2.9 shows insignificance of asymmetry even for 68% credibility region.

Although, the median effects of the big global demand shocks on macroeconomic activity looks as mirror images and symmetric on figure 2.8 (due to scale difference of subplots), but the graphical asymmetry test shows the biggest credible ”negative” asymmetry in the response of world activity to a global demand shock, which is significant even for 95% credibility region. The response of real oil price to a global demand shock shows a relative ”small” asymmetry, which is significant only for 68% credibility region starting at the first quarter after the shock occurs.

After the second quarter the response in real oil price to oil supply shock shows a ”positive” asymmetry, significant for 68% credibility region. This shows that oil price increases are bigger and more persistent for big negative oil supply disruptions.

Both positive and negative big inventory demand shocks have relatively small and persistent negative median effects on economic activity. The only difference is that negative shock decreases the global production less, than the positive shock. The drop in economic activity, associated with a big decline of oil inventories, could be explained by the expectations of future oil price drop.
However, this asymmetry in the effects of big oil inventory demand shocks on economic activity is not significant even for 68% credibility region. The asymmetry in the other responses is not significant.

2.7.3. Does the size of shock matter?

Since the model has implications of nonlinearity in a broader sense, than only asymmetry discussion, I provide with the graphical test of the size importance for oil market shocks. Figure 2.10 plots the difference of responses to negative big, 2 standard deviation and, magnified by 2 responses to small, 1 standard deviation shocks. The idea is that this difference should be zero, if one could linearly extrapolate the response to big oil market shocks, based on the responses to the small shocks.

Although, this test shows the zero difference for most of the responses, but the world industrial production response to oil supply and global demand shocks have ”negative” skewness, persistently significant for 68% credibility region and instantly significant for 95% credibility region.

2.7.4. Understanding the source of nonlinearity

Figures 2.11 to 2.20 provide with the experiments of setting parameters in $A_2$, $A_3$ and $A_4$ equal to zero. This way, we can begin to understand which parameters are important for generating the nonlinearity. The exclusion of $A_4$ parameters makes the response of world production to oil supply shock ”positively” asymmetric for 68% credibility region. This suggests that the ”negative” asymmetry in the full model is due to the $A_4$ parameters, corresponding to the ”square” of structural shocks. This could be seen in the experiment of excluding both $A_2$ with $A_3$ parameters, which gives even negative global production response to big oil supply shock up to the first quarter and later a positive response. However, the sum of the responses shows that this asymmetry is not significant even for 68% credibility set, when parameters $A_2$ with $A_3$ are shut down. This points out that the shock interaction with the lagged variables is important, i.e. the historical changes matter for reproducing a credible asymmetry.
2.7.5. Historical Impulse Responses

Previous experiments provide us with the evidence that nonlinear impulse responses depend on the initial conditions \( Y_{t-1} \). Elaborating deeper the historical changes, I consider two major exogenous oil market shocks in the post-1973 period. I examine two historical episodes that have been associated with oil supply shocks. I compare responses to these shocks starting from historical initial conditions and compare that with the response when the initial condition is the sample mean of variables.

First, I examine the start of the Iranian Revolution (December 1978-January 1979). The estimated model provides me with a big negative (-2.18 st. dev.) oil supply shock and a relatively small negative (-1.13 st. dev.) oil demand shock at the first quarter of 1979. Speculative oil demand and economic activity shocks are less than 1 standard deviation at this quarter.

Figure 2.21 compares the median responses to the historical 1st quarter 1979 shocks, starting at the last quarter of 1978 with the same size shocks’ responses, starting at the sample mean of the times series. The world economic activity drops deeper in the response to oil supply and demand decline for the historical start, than for the start at the sample mean. However, figure 2.22 suggests that only historical oil supply shock’s effect on economic activity is significantly different from the sample mean response for 68% credible set. These results suggest that the economy was more vulnerable to an oil supply shock at the last quarter of 1978 than in usual periods.

The second historical shock of interest is Iraq’s invasion of Kuwait in August 1990. This even led to 45% increase in the price of oil in the fall of 1990 and was followed by the ninth postwar recession in 1990 - 1991. The estimated model evaluates 1990, quarter 3 shocks as a big negative (-1.67 st. dev.) oil supply shock and a big positive (2.85 st. dev.) oil demand shock. Speculative oil demand and economic activity shocks are less than 1 standard deviation at this quarter.

Figure 2.23 compares the median responses to the historical 3rd quarter 1990 shocks, starting at the 2nd quarter of 1990 with the same size shocks’ responses starting at the sample mean of the time series. The world economic activity drops deeper in the response to oil supply decline for the historical start, than for the start at the sample mean. Moreover, the economic activity increase in the response on oil demand jump is lower for the historical start, than for the start at the sample
mean. However, figure 2.24 suggests that only historical oil supply shock’s effect on economic activity is significantly different from the sample mean response for 68% credible set. Both of these responses suggest that the global activity was more vulnerable to an oil supply shock at the middle of 1990 than in usual periods.

To have a complete analyses of the historical shocks of interest, I compare these shocks with the magnified (by the historical size of each shock) one-sigma counterfactual shocks of the same sign. Figure 2.25 shows the median responses to the historical, 1979, quarter 1, and one-sigma counterfactual shocks. The economic activity drop, associated with the historical oil supply shock is much deeper on impact and gets deeper after the year, than for 1-sigma counterfactual. Figure 2.26 provides with the difference of the responses to 1979, quarter 1 shocks and magnified one-sigma counterfactual shocks. The economic activity drop, associated with the historical shock, turns out to be significant only for 68% credibility set.

Figure 2.27 shows the median responses to 1990, quarter 3 and magnified responses to one-sigma counterfactual shocks. The economic activity drop, associated with the historical 1979, quarter 1 oil supply shock is deep on impact and gets deeper after the year. However, the magnified response of economic activity to one-sigma counterfactual oil supply shock is positive on impact, gets bigger after one quarter and moves close to zero after 2-3 quarters. Figure 2.28 provides with the difference of the responses to the historical and magnified response to one-sigma counterfactual shocks. The economic activity instant drop, associated with the historical shock, is significant for 95% credibility region and after a quarter is significant only for 68% credibility region. After two quarters the difference is not significant for 68% credibility region. These results suggest that the interaction of shocks and initial conditions is important.

2.8. Conclusions

The asymmetry is only a special type of nonlinearity. To account for a nonlinear responses in oil market in a broad sense, I propose the parsimonious 1-lag generic nonlinear model, based on second order Taylor expansion. The large number of parameters are handled with the unique nonlinear parameters’ priors. The estimated posterior distributions of parameters, related to the
linear, instant responses are, in general, comparable with Baumeister and Hamilton (2019) results and the literature.

The impulse response analyses suggests no significant nonlinearity for small oil market shocks and significant asymmetric response in economic activity to large oil supply shocks and global demand shocks. The parameter shut down experiments underscore quantitative importance of interaction of shocks with the historical changes. The comparison of responses to big shocks with the magnified responses to small shocks provides with the evidence for the presence of more comprehensive nonlinearity, than asymmetry. This nonlinearity is particularly strong at the instantaneous responses of global production to oil supply and global demand shocks.

My econometric model suggests that the history matters. The effect of shocks depends not only on the size of the shock, but interaction of the shock with the current state of the economy. The historical analysis provides with the evidence that the economy was more vulnerable to an oil supply shocks at the end of 1978 and middle of 1990 than in usual periods.
3.1. Introduction and literature review

Digital text provides a rich repository of information about macroeconomic activity. Modern statistical tools provide researchers with the opportunity to extract the information and encode it in a quantitative form susceptible to causal or descriptive analysis. A long history of literature has been analyzing central banks communication and its influence on macroeconomic activity. As the Federal Reserve must make monetary policy decisions under a great deal of uncertainty, it relies not only on quantitative, but also qualitative information about current economic conditions.

Federal Open Market Committee (FOMC) statements, Financial Stability Reports (FSR), speeches given by central bank governors and the Beige Books are the sources of macroeconomic verbal communications widely used by researchers. The Beige Books have written description of economic conditions in each of the twelve district banks of the Federal Reserve System and a summary of national economic conditions in sectors such as retailing, manufacturing, construction, real estate, agriculture and natural resources. They have been publicly released two weeks before the FOMC meeting since June 1983, that occur eight times a year. Before that it was known as the Red Book and was not publicly available.

Some of these communication means have not been published until nineties. The FOMC meeting transcripts have not been published until November 1993, but following a policy shift transcripts became public with a time lag. Increased transparency has both potential costs and benefits. It allows more informed and efficient debate due to increased accountability of policy makers. However, transparency can make committee members be inclined to group thinking or more cautious and biased toward the status quo. Hansen, McMahon, and Prat (2017) use this change in
disclosure policy to study how FOMC transparency affects debate during meetings. They come up with the topic model with the outcome of interest being the proportion of member-meeting language devoted to one of the topics, estimated from the fitted topic model. Using a difference-in-differences estimator the authors find that, after the transparency shift, inexperienced members discuss a wider range of topics and make more references to data, discussing economic conditions. That is called accountability effect. It appears to be stronger, as inexperienced members’ topics get more influence in shaping deliberation after transparency. Also, these members speak more like Chairman Greenspan during policy discussions that is considered to be consistent with increased conformity.

In order to forecast fluctuations in treasuries, the content of FOMC statements is used by Lucca and Trebbi (2009). To quantify the direction and intensity of each FOMC statement they use two different dictionary-based methods: Google and Factiva semantic orientation scores. The Google score is used to count a number of search hits of phrases in the statement adding one of the words from a list of antonym pairs signifying positive or negative sentiment (e.g., “hawkish” versus “dovish”). Then, they difference the frequency of positive and negative searches and average over all phrases. Using a vector autoregression with the Feds sentiment proxies, they forecast Treasury yields and conclude that changes in Fed’s sentiment is the main driver of changes in interest rates.

The speeches given by central bank governors and Financial Stability Reports (FSR) are used by Born, Ehrmann, and Fratzscher (2013). They construct a financial stability sentiment index to study the effect of central bank sentiment on returns and volatility of stock market. Using a dictionary-based approach to assign optimism scores, they show that FSRs with optimistic sentiment increase equity prices and reduce market volatility during the subsequent month.

The early attempt to quantify Beige Books has been made by N. Balke and Petersen (2002). They ”manually” read and scored between -2 and 2 by 0.5 increments each of the 1983-1997 Beige Books. The Balke-Petersen index is compared with real GDP growth and allow authors to state that the Beige Book has predictive content for current quarter real GDP above and beyond other quantitative information available to analysts at the time the Beige Book is released. However, N. Balke and Petersen (2002) find that the predictive power of the Beige Book for next quarter real
GDP growth is substantially lower than for current quarter real GDP growth.

The Beige Book has also been studied with human read and score by Rolnick, Runkle, and Fettig (1999), N. S. Balke and Yucel (2000), Ginther and Zavodny (2001) and Zavodny and Ginther (2005). In generally, all these papers find that the Beige Book contains information about current economic conditions both at the national and district level but that Beige Book loses its informational advantage pretty fast with a release of other more recent data.

As the dictionary based text analyses became more popular, there have been a number of its applications to the Beige Book. Payne (2001) has developed a list of key word combinations. Giving a numerical score between -1 and 1 by 0.5 increments to each word, he counts the number of the word combination occurrences in each district summary sentence. His conclusion is a strong predictive content of the Beige Book for current quarter real GDP growth and for the coincident and leading economic indicators. Later Armesto, Hernandez-Murillo, Owyang, and Piger (2009) use the word dictionary in Diction, the off-the-shelf text analytics software. They construct an optimism and a pessimism indices by counting a frequency of words associated with ”optimism increasing” and ”optimism decreasing”. The irregular spacing of the Beige Book is accounted by mixed frequency estimation (MIDAS). The conclusion on predictive content of the Beige Book is made for both aggregate and district economic activity. Sadique, In, Veeraraghavan, and Wachtel (2013) also use a dictionary-based text analytics software, GeneralInquirer, to extract positive or negative and increasing or decreasing ”tone” of the discussions in the Beige Book. They find the business cycle property of the Beige Book’s ”tone” as well as its relation with financial market volatility and volume.

Recent comprehensive text analysis of the Beige Books is done by N. Balke, Fulmer, and Zhang (2017). They encompass dictionary-based, atomic fact, linear regression sub-setting and principal components to get an ensemble index, which is then included into a dynamic factor index model with commonly used quantitative macroeconomic data. They find that at the time the Beige Book is released, it has information about current economic activity not contained in other quantitative data. The finding is a relatively short-lived informational advantage of Beige Book release, which evaporates within three weeks after its release date as other quantitative data is released.
Based on the existing literature one can state that the Beige Book information could be useful for "nowcasting" of macroeconomic activity, which is the prediction of the present or the very near future state of the economy. A great share of text analytics macroeconomic applications keeps relying on ad hoc dictionary methods rather than deploying more sophisticated feature selection methods of unsupervised and supervised learning. Dictionary methods are appropriate in cases where strong prior information and limited availability of appropriately labeled training data are available. However, research in other fields suggests that modern methods outperform ad hoc approaches in a substantial share of applications.

In this paper, I seek an easily updated "human-free" Beige Book analysis that can be used with other macroeconomic data for nowcasting current quarter GDP growth. Staying away from use of human scores or dictionary-based methods, I apply both unsupervised and supervised machine learning for Beige Book quantification. An unsupervised topic model belonging to the Latent Dirichlet Allocation (LDA) class is used to extract topic word groupings from the Beige Book documents. The optimal number of topics is defined by a consensus of four Gibbs sampling methods. GDP growth nowcasting is done by means of such supervised learning methods as linear and support vector regressions (SVR) with nonlinear radial-basis (RBF) kernel. The ability of the Beige Book topic indices to nowcast real GDP is compared with five macroeconomic indicators: jobless claims, industrial production, personal income less transfers, employment, and manufacturing trade growth. Both an in-sample and out-of-sample examination of nowcasting performance are implemented. I find that the Beige Book information about current economic activity is significant for in-sample and out-of-sample nowcasting. The significance of text data increases with the month of the quarter. Nonlinear kernel SVR analysis show that qualitative information about economic activity has nonlinear nature.

3.2. Beige Books text mining

The Beige Book is Federal Reserve System report that describes current economic conditions in each of the twelve Federal Reserve Districts. The Beige Books information is gathered by each of the individual District Banks from contacts in their district.
Information collection methods vary across the Banks and may include face-to-face interactions and formal survey of business or members of Bank Board of Directors. Also, the Beige Book information may be based on news reports and even officially released economic data. The report information is summarized in the form of a written document. In addition to districts’ summaries, there is a national summary of the Beige Book. It is apparently based on the twelve district write-ups and is written by one of the district Banks in rotation.

3.2.1. Preprocessing of text data

Text data of only the National Summary of the Beige Book is used for this research. Usually, it is four to five pages long. The times series of Beige Books consists of 448 documents and starts at January 1970 and ends at June 2020. There are over 14,000 unique character terms in these collection, but it significantly reduced by preprocessing algorithm I describe further.

First, I strip the Beige Books of punctuation and then extract specific parts-of-speech from each sentence. Second, I extract only four parts-of-speech: nouns, verbs, adjectives and adverbs. Finally, instead of the raw character strings, I use the stems of the words in order to reduce the dimensionality of the character terms. This procedure allows to assure that character terms are "unique" by combining morphologically similar words into word concept groupings. Therefore, character strings like "decrease", "decreased", "decreases", "Decreasing", "decreasing" are collapsed to the term "decreas". Such data ingestion allows to reduce the number of unique character terms by a factor of three. The remaining terms allow one to create corpora, a collection of processed Beige Books, that will be used for statistical analyses to in order to quantify the information from the Beige Book.

3.2.2. Unsupervised learning

An unsupervised learning is a machine learning algorithm that discovers an underlying data structure without any given target labels (variables) to learn from. For collections of numerous unlabeled documents topic modeling have become a standard tool in data analysis. Topic models have been used in information retrieval and classification. Such algorithms facilitate analyses of
large corpora by revealing their underlying themes and how each document exhibits them.

The task is to choose an appropriate method of topics identification. Within the set of atheoretical means to identify topics I chose an approach that divides words into topic groups based on certain statistical properties of the corpus. It is Latent Dirichlet Allocation (LDA) approach that was first proposed by Blei, Ng, and Jordan (2003). The term “Dirichlet” stands for a conjugate Dirichlet prior for a topic mixture. The term “Latent” stands for a latent structure, topics of a document, produced by an observed data, a combination of words.

The LDA is a generative probabilistic model that allows words to appear in more than one topic area with some probability. It is a three-level hierarchical Bayesian model, in which each item of a collection is modeled as a finite mixture over an underlying set of topics. Each topic is, in turn, modeled as an infinite mixture over an underlying set of topic probabilities. In the context of text modeling, the topic probabilities provide an explicit representation of a document, Blei et al. (2003).

To put it straightforward, given a corpus, the LDA assumes that every document is a distribution of topics and each topic is a distribution of words. Every word in a document is generated by first sampling a topic from the distribution of topics associated with the document and then sampling a word from the distribution of words associated with a given topic.

The LDA can capture word correlations in a Beige Book corpus with a low-dimensional set of multinomial distribution, that is defined as "topics". However, by definition, the LDA cannot capture the correlations between topics for the independency assumption underlying Dirichlet distribution. So, it is crucial to select the appropriate number of topics for the problem at hand.

Several approaches tackle the problem of automatically finding the right number of LDA topics contained in a set of documents. (Arun, Suresh, Veni Madhavan, and Narasimha Murthy (2010) and Cao, Xia, Li, Zhang, and Tang (2009)) follow the same idea of computing distances or dissimilarity between pairs of topics over models with a different number of topics. The optimal amount of topics is reached when the overall dissimilarity between topics achieves its maximum value. In this research, the optimal number of topics is defined based on a consensus of four Gibbs sampling algorithms and perplexity measure. All existing Gibbs sapling methods train LDA models with
multiple number of topics to select one with the best performance.

Figure 3.1 displays the Arun et al. (2010) and Cao et al. (2009) metrics, applied to the Beige Book corpora. These metrics minimize the value functions to find the optimal number of topics. The Deveaud, Sanjuan, and Bellot (2014) and Griffiths and Steyvers (2004) metrics are maximizing their value functions to determine the optimal number of topics for the Beige Book corpora. These methods suggest that the optimal can be in a range of 4-7 topics. In order to check this assumption on the test data, I will use perplexity measure for hold-out data set.

Perplexity is a commonly used measurement in information theory. It evaluates how well a statistical model describes a dataset. The lower perplexity denotes a better probabilistic model. Perplexity captures how surprised a model is with new data it has not seen before. It is measured as the normalized log-likelihood of a hold-out test set.

Formally, for a given document $D$, the perplexity is defined as:

$$perplexity(D) = 2^{-\mathcal{L}(D)}, \quad (3.1)$$

where log-likelihood ($\mathcal{L}$) is an average probability assigned to each word ($w_d$) in the test set by the model ($\Theta$):

$$\mathcal{L}(D) = \frac{\sum_D \log_2 p(w_d; \Theta)}{\text{count of tokens}} \quad (3.2)$$

Based on a rule of thumb, I stay with the second "fracture" for a hold-out data, which is 7 topics.

Figure 3.3 provides with the Beige Books word distributions for 7 topics. Some topics are relatively easy to interpret and provide relevant information about state of the economy. For example, topic 1 could be associated with "continue" and topic 2 represents more "new" in respect of "activity". Topic 3 could be interpreted as "weak" "activity". It may be hard to distinguish topics 4 and 7, that deals with activity in construction sector. These topics have relatively high probabilities for both "new" and "continue" of a "strong" "construct". Topic 6, in turn, reflects moderate "growth". This is an example, when a relatively uniform algorithm, which does formal word stem-
ming fails to unify "modest" with "moder" representing the same phenomenon or economy state. Nevertheless, the distinction between these topics are still catch-able by human's eye.

3.3. Supervised modelling

In order to quantify the information contained in the report I have to proceed with some of the supervised learning methods. Having in mind that each report has a similar number of sentiment related words that describe recessions and expansions of different magnitude, I consider a nonlinear relationship with GDP growth as a target variable. One of parsimonious nonlinear supervised learning algorithms is support vector regression (SVR). A linear benchmark for comparison with nonlinearity is ordinary least squares (OLS).

SVR allows one to define how much error is acceptable in a model and finds an appropriate hyperplane to fit the data. In contrast to OLS, the objective function of SVR is to minimize the l2-norm of the coefficient vector — not the squared error. The error terms are handled in the constraints, where absolute errors are set to be less than or equal to specified margins, called slack variables or maximum error, $\xi_j$. This is hyperparameter that forges excursions of a few points into the margin and by tuning which the desired accuracy of the model is gained. The objective is to minimize the total amount of slack (3.3) while maximizing the width of margin (3.4):

$$\min_{\beta, \beta_0} \frac{1}{2} ||\beta||^2 + C \sum_j \xi_j$$  \hspace{1cm} (3.3)

subject to

$$y_j (X_j \cdot \beta + \beta_0) \geq (1 - \xi_j) \text{ for } j = 1, \ldots, N$$  \hspace{1cm} (3.4)

$$\xi_j \geq 0 \text{ for } j = 1, \ldots, N$$

The dependent variable $y$ is log-growth of quarterly GDP. $N$ is the number of observations and $\beta$ is the vector of parameters to estimate. Constant $C$ is the "cost" of the slack. It is a hyperparameter to tune and when $C$ is small, it is efficient to allow more points into the margin in order to achieve a larger margin.
Even though the LDA topics appear to describe current economic activity, most literature does not use the Beige Book in isolation. So, I follow N. Balke et al. (2017) in choosing the following five leading macroeconomic indicators to compare and combine with the Beige Books’ LDA topics as features in nowcasting quarterly GDP growth. Macroeconomic indicators include monthly aggregate of weekly initial claims for unemployment insurance (JC), monthly industrial production growth (IP), growth in real personal income less transfer receipts (I), monthly real manufacturing and trade sales (MT), and monthly employees and non-agricultural payrolls (EM). Similar to N. Balke et al. (2017), I log difference all the variables except initial jobless claims.

$$X_t^M = (JC_t, \Delta IP_t, \Delta I_t, \Delta MT_t, \Delta EM_t)'$$ (3.5)

The macroeconomic benchmark model to be estimated includes a vector of macroeconomic indicators and their lags:

$$y_t = \beta_0 + X_t^M \cdot \beta^M + X_{t-1}^M \cdot \beta^{M, lag}$$ (3.6)

Since $y_t$ is log-growth of quarterly GDP and is the same for three months of each quarter, the competing alternative for this benchmark is to include month of the quarter indicator variables:

$$y_t = \beta_0 + X_t^M \cdot \beta^{M, 1} \cdot I_{m=1} + X_t^M \cdot \beta^{M, 2} \cdot I_{m=2} + X_t^M \cdot \beta^{M, 3} \cdot (1 - I_{m=1} - I_{m=2})$$

$$+ X_{t-1}^M \cdot \beta^{M, 1, lag} \cdot I_{m=1} + X_{t-1}^M \cdot \beta^{M, 2, lag} \cdot I_{m=2} + X_{t-1}^M \cdot \beta^{M, 1, lag} \cdot (1 - I_{m=1} - I_{m=2})$$ (3.7)

The indicator variable for month one of the quarter is:

$$I_{m=1} = \begin{cases} 1 & \text{if } t \in \{\text{January, April, July, October}\} \\ 0 & \text{otherwise} \end{cases}$$ (3.8)
The indicator variable for month two of the quarter is:

\[ I_{m=2} = \begin{cases} 
1 & \text{if } t \in \{\text{February, May, August, November}\} \\
0 & \text{otherwise} 
\end{cases} \] (3.9)

There is a mixed frequency challenge since the dependent variable is quarterly, all macroeconomic indicators are monthly and there are 8 to 12 Beige Books per year. I approach this problem by bringing all observations to monthly basis. The monthly GDP growth is the assumed to be the same over month of the quarter. The Beige Books are counted based on the available information in a given month: if there is a Beige Book in the current month, then current month LDA features are taken, if not - then the previous month LDA features are taken as the most recent Beige Book information available. Thus, a vector of LDA features for month \( t \), \( X_{t}^{LDA,v} \) is given by:

\[ X_{t}^{LDA,v} = \begin{cases} 
LDA_{t}^{i} & \text{if } \exists LDA_{t}^{i} \\
LDA_{t-1}^{i} & \text{otherwise}, 
\end{cases} \] (3.10)

where \( i = 1, \ldots, 6 \) since by definition of LDA distribution the sum of 7 topics is equal to one. \( LDA_{t}^{i} \) is a probability of topic \( i \) describes a period \( t \) Beige Book.

The combined model to be estimated includes a vector of macroeconomic indicators with their lags and a vector of the Beige Books’ 6 LDA topics:

\[ y_t = \beta_0 + X_t^M \cdot \beta^M + X_{t-1}^M \cdot \beta^{M, \text{lag}} + X_t^{LDA} \cdot \beta^{LDA} \] (3.11)

3.4. Empirical results

3.4.1. Linear Regression

I start with the OLS estimation of linear benchmark model of five macro indicators with their lags. The estimation results in Table 3.1 show significance for current period of all variables apart growth in real personal income less transfer receipts, \( I \), but its lag is significant.
### Table 3.1. OLS Estimates of Benchmark Macroeconomic Model

|                     | Estimate | Std. Error | t value | Pr(>|t|) | Significance |
|---------------------|----------|------------|---------|----------|--------------|
| Intercept           | 0.0027   | 0.0002     | 11.15   | 0.0000   | ***          |
| JC                  | -0.0340  | 0.0034     | -10.03  | 0.0000   | ***          |
| IP                  | 0.1315   | 0.0372     | 3.54    | 0.0004   | ***          |
| I                   | 0.0250   | 0.0356     | 0.70    | 0.4831   |              |
| MT                  | -0.1476  | 0.0252     | -5.85   | 0.0000   | ***          |
| EM                  | -0.3536  | 0.0677     | -5.22   | 0.0000   | ***          |
| JC.lag              | 0.0206   | 0.0038     | 5.46    | 0.0000   | ***          |
| IP.lag              | 0.0965   | 0.0367     | 2.63    | 0.0088   | **           |
| I.lag               | 0.1064   | 0.0351     | 3.03    | 0.0026   | **           |
| MT.lag              | -0.0628  | 0.0264     | -2.38   | 0.0177   | *            |
| EM.lag              | 0.4373   | 0.0755     | 5.79    | 0.0000   | ***          |

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1

The OLS estimation results of the linear "month-of-quarter" macroeconomic model (3.7) of five economic indicators with their lags are reported in Table C.1. The significance of macroeconomic variables for the first month of the quarter seems to be higher, than for the third and some of the third month variables have smaller p-values than the second month corresponding variables such as $EM_m = 3$ and $JC_m = 3$.

Even though "month-of-quarter" macroeconomic model (3.7) have fewer significant variables than the benchmark macro model (3.6), its performance is better in terms of out-of-sample forecasting.

I compare models’ performance based on in-sample joint tests (Table 3.4) for the LDA features significance and out-of-sample forecasting errors (Table 3.2) for all models’ comparison. The out-of-sample forecasting results are provided by computing mean and confidence interval for root mean squared errors (RMSE) of 25-times repeated 10-fold cross-validation. The confidence interval for RMSE is derived from the following $\chi^2$ distribution:
So, the "month-of-quarter" macroeconomic model (3.7) outperforms the benchmark macro model (3.6) based on RMSE of out-of-sample forecasting.

\[
\frac{N \cdot \text{RMSE}^2}{\sigma_{\text{RMSE}}^2} \sim \chi^2_N \quad (3.12)
\]

Table 3.2. Repeated 10-fold Cross Validation Results

<table>
<thead>
<tr>
<th>Model</th>
<th>RMSE (10^{-3})</th>
<th>RMSE, 95% CI (10^{-3})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark Macro OLS</td>
<td>6.54</td>
<td>6.312 ... 6.769</td>
</tr>
<tr>
<td>Macro month-of-quarter OLS</td>
<td>5.51</td>
<td>5.387 ... 5.629</td>
</tr>
<tr>
<td>Beige Book OLS</td>
<td>5.48</td>
<td>5.376 ... 5.588</td>
</tr>
<tr>
<td>Macro month-of-quarter &amp; Beige Book OLS</td>
<td>5.29</td>
<td>5.187 ... 5.397</td>
</tr>
<tr>
<td>Macro &amp; Beige Book (both) month-of-quarter OLS</td>
<td>5.43</td>
<td>5.318 ... 5.537</td>
</tr>
<tr>
<td>Macro month-of-quarter SVR</td>
<td>4.24</td>
<td>4.104 ... 4.394</td>
</tr>
<tr>
<td>Macro month-of-quarter &amp; Beige Book SVR</td>
<td>3.87</td>
<td>3.674 ... 4.054</td>
</tr>
</tbody>
</table>

The next model to compare includes the Beige Book LDA variables only to nowcast the current quarter GDP growth. OLS Estimates of this model are given in Table 3.3. Almost all LDA topics have statistically significant estimates apart from the LDA topic number 6. In Table 3.2 one can see that the "Beige Book OLS" model outperforms the "Benchmark Macro OLS" (3.6) significantly, but outperforms the "Macro month-of-quarter OLS" (3.7) slightly based on the mean of RMSE, and has overlaying confidence intervals with the benchmark.

The OLS estimates of the combined model with both macroeconomic and Beige Book variables having "month-of-quarter" indicators are given in Table C.2. Although, it has more explanatory variables than the macroeconomic month-of-quarter & Beige Book OLS given in Table C.3, the larger number of variables to estimate seems to over-fit since it outperforms the "Macro month-of-quarter & Beige Book OLS" based on mean of RMSE and has almost not overlay in confidence intervals of RMSE in Table 3.2.

In-sample analysis of the LDA features’ importance is executed based on joint heteroskedasticity-robust F-test of \( LDA_i = 0 \). The results are displayed in Table 3.4. The LDA indicators turn out to
be significant only in a parsimonious "Macro monthly & Beige Book" model.

|             | Estimate | Std. Error | t value | Pr(>|t|) | Significance |
|-------------|----------|------------|---------|----------|--------------|
| Intercept   | 0.0052   | 0.0007     | 7.64    | 0.0000   | ***          |
| LDA1        | -0.0042  | 0.0013     | -3.30   | 0.0010   | **           |
| LDA2        | -0.0047  | 0.0013     | -3.59   | 0.0004   | ***          |
| LDA3        | -0.0023  | 0.0011     | -2.23   | 0.0264   | *            |
| LDA4        | -0.0030  | 0.0010     | -3.04   | 0.0025   | **           |
| LDA5        | -0.0042  | 0.0011     | -3.68   | 0.0003   | ***          |
| LDA6        | 0.0002   | 0.0011     | 0.20    | 0.8388   |              |

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1

Table 3.3. OLS Estimates of the Beige Book LDA Model

<table>
<thead>
<tr>
<th>Model</th>
<th>Joint Test $LDA_i = 0$, p-val</th>
</tr>
</thead>
<tbody>
<tr>
<td>Macro monthly &amp; Beige Book</td>
<td>0.0018</td>
</tr>
<tr>
<td>Macro monthly &amp; Beige Book monthly</td>
<td>0.1123</td>
</tr>
</tbody>
</table>

Table 3.4. In-Sample Model Comparison

In a setup with macroeconomic monthly variables, the results of the Beige Book relative significance for month-of-quarter are reported in Table 3.5. Instead of using month-of-quarter indicator variables, I estimate three regression for first, second and third months of the quarter. Based on both in-sample joint tests of $LDA_i = 0$ and out-of-sample RMSE, the significance of the Beige Book data in nowcasting current quarter GDP growth increases with the month of the quarter.

<table>
<thead>
<tr>
<th>$LDA_{Month (i)} = 0$</th>
<th>In Sample Test, p-val</th>
<th>RMSE (10^{-3})</th>
<th>RMSE, 95% CI (10^{-3})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Month 1</td>
<td>0.162</td>
<td>7.12</td>
<td>7.011 ... 7.252</td>
</tr>
<tr>
<td>Month 2</td>
<td>0.0798</td>
<td>6.74</td>
<td>6.653 ... 6.834</td>
</tr>
<tr>
<td>Month 3</td>
<td>0.0717</td>
<td>6.52</td>
<td>6.416 ... 6.637</td>
</tr>
</tbody>
</table>

Table 3.5. Month of Quarter Significance
3.4.2. Support Vector Regression

To examine nonlinear nature in the Beige Book quantification I estimate Support vector regression with radial-basis kernel. The kernel is essentially a mapping function that transforms a given space into higher dimensional space. There are two hyperparameters to tune. \( C \) is inverse of the strength of regularization. As its value increases the model becomes over-fitted. Another parameter is \( \epsilon \). As its value increases the model becomes over-fitted as well.

The parameters’ cross validation grid search of \( C \) in the range \([2^5; 2^{15}]\) and \( \epsilon \) in the range \([2^{15}; 2^3]\) gives the ”Macro month-of-quarter SVR” (equation 3.7) with \( C = 2^3 \) and \( \epsilon = 2^2 \) parameter values. The best ”Macro month-of-quarter & Beige Book SVR” has \( C = 2^4 \) and \( \epsilon = 2^2 \) hyperparameters.

The the out-of-sample errors are reported in Table 3.2. The model with combined the macro and the LDA indicators outperforms the one with just macro explanatory variables. The comparison of prediction goodness are shown in Figure 3.4 for the best linear model, Figure 3.5 for the SVR with the macro and Beige Book variables and Figure 3.6 for the SVR with macro variables only. This analysis leads to a conclusion that there is nonlinearity in relationship of Beige Book and GDP growth, which is not accounted by linear regression. Moreover, the Beige Book information, processed with the LDA topics, gives a considerable boost in macroeconomic prediction within the nonlinear SVR model.

3.5. Sensitivity Analysis

Sensitivity analysis determines how target variable is affected by changes of input variables. I use a visualization approach, proposed by Cortez and Embrechts (2013) to extract human understandable knowledge from supervised learning black box data mining models, such as SVR.

Input importance bar plot is depicted in Figure 3.7 to show input importance measures, sorted from the highest to the lowest values. This method considers a given input vector that contains the mean values of each input. The Beige Book LDA topics have the lowest input importance for their mean values. The highest input importance have employees and non-agricultural payrolls of 2nd month-of-quarter, EM.m=2. The second highest importance has a lag of employees and non-
agricultural payrolls of 1st month-of-quarter, EM.m=1.lag. The third highest input importance have initial claims for unemployment insurance of 1st month-of-quarter, JC.m=1.

However, the input importance measure may not be enough to make any conclusion on input importance since it is computed for the mean values of each input. To present the different values’ impact of a given explanatory variable in the model, the Variable Effect Characteristic (VEC) curve is used. It plots the feature values (x-axis) versus the gdp growth responses (y-axis) and the feature values frequency histogram. So, one can see whether high variable importance is achieved at the most frequent values or in the tail of its distribution.

Figures 3.8 and 3.9 suggest that the highest variable effect of EM.m=2 and EM.m=1.lag is achieved for low frequency values and for their most frequent values their VEC are 2-3 times lower than corresponding VEC values. For the third highest input importance variables, initial claims for unemployment insurance of 1st month-of-quarter, the highest variable effect is reached at its highest frequency values. This is depicted in Figure 3.10.

The Beige Book LDA topics 1 – 5 have the highest variable effects at their highest frequency values. This is depicted in Figures 3.11 - 3.15. The LDA topic 6 have an opposite VEC curve with its highest variable effect at low frequency probabilities close to 1. This is shown in Figure 3.16. This means that the LDA topics 1 – 5 variable effects are high for a large number of their values. However, among the most important variables, based on input importance, only the initial claims for unemployment insurance of 1st month-of-quarter has a large number of values with high variable effects. Therefore, despite low importance mean values measures of the LDA topics, their highest frequency variable effects (VEC) are higher than VEC of three macroeconomic variables with the largest average importance.

3.6. Conclusions and extensions

In this study, I propose an algorithm for "human-free" analysis of Beige Books text data that can be an easily updated and adjusted to be used with other economic data to predict current quarter GDP growth. I quantify the Beige Books by applying both unsupervised and supervised machine learning techniques. The unsupervised topic modeling is done by constructing the LDA word-topic
and topic-Beige Books distributions and defining the optimal number of topics. The supervised learning methods is implemented with linear and support vector regressions with nonlinear RBF kernel. These methods are applied to a variety of model specifications to nowcast GDP growth and test the Beige Book data significance both in-sample and out-of-sample.

Empirical evidence, resting on a number of model specifications and tests, highlights that there is informational gain in accounting for a month-of-quarter of five monthly macroeconomic indicators to predict quarterly GDP. In particular, empirical evidence suggests that month-of-quarter model performance is better than the economic benchmark in terms of out-of-sample forecasting and the significance of economic variables for the first month of the quarter seems to be higher, than for the third one. Even thought the significance of Beige Book text data increases with the month-of-quarter, accounting for the Beige Book document’s month of the quarter does not give any benefit in terms of economic forecasting.

The linear model with Beige Book information outperforms both the benchmark model with five macroeconomic indicators and their lags (3.6) and the macroeconomic model with month-of-quarter indicators (3.7). Moreover, it is worth pointing out that in-sample testing show the Beige Book significance within the best linear model of combined macro and LDA indicators. Finally, the nonlinear SVR analysis show that qualitative information about economic activity has nonlinear nature. On the one hand, the Beige Book topics have the lowest input importance measured for the mean values each variable. On the other hand, the five LDA topics’ variable effects (VEC), associated with the highest value frequency, are higher than VEC of the three economic indicators with the largest average importance.

The findings in this paper can be potentially useful to the work that attempts to quantify text data accounting for mixed frequency problem. Furthermore, the algorithm can be easily fed into with the Beige Book’s industry chapters or any of twelve districts’ summary information to construct regional, sector specific qualitative indices.
A.1. Bank Run Functional Form

Figure 1.1. A Plot of $(1 - x)$ and $\cos\left(\frac{x}{2}\right)$ for $x \in [0, 1]$
A.2. Numerical Simulations of a Baseline Model (No Bank Runs)

Figure 1.2. Impulse Responses to a negative one-standard-deviation tfp shock
Figure 1.3. Impulse Responses to a negative one-standard-deviation tfp shock
Figure 1.4. Impulse Responses to a negative one-standard-deviation tfp shock
Figure 1.5. Impulse Responses to a negative one-standard-deviation equity shock
Figure 1.6. Impulse Responses to a negative one-standard-deviation equity shock
Figure 1.7. Impulse Responses to a negative one-standard-deviation equity shock
A.3. Numerical Simulations of a Model with Idiosyncratic Bank Runs

Figure 1.8. Impulse Responses to a negative one-standard-deviation tfp shock
Figure 1.9. Impulse Responses to a negative one-standard-deviation tfp shock

Figure 1.10. Impulse Responses to a negative one-standard-deviation tfp shock
Figure 1.11. Impulse Responses to a negative one-standard-deviation equity shock
Figure 1.12. Impulse Responses to a negative one-standard-deviation equity shock

Figure 1.13. Impulse Responses to a negative one-standard-deviation equity shock
B.1. Diagonalization of the structural shocks

Diagonalization of the structural shocks’ variance matrix is required due to a measurement error in inventory stocks’. A measure of global crude oil inventories is obtained based on the data on U.S. crude oil inventories and monthly OECD refined-product inventories, multiplying the U.S. crude oil inventories by the ratio of OECD inventories of crude petroleum and petroleum products to U.S. inventories of petroleum and petroleum products, as in Kilian and Murphy (2014). So, the estimate of the relative change in OECD crude-oil inventories is:

\[ \Delta i_t = \chi \Delta i^*_t + e_t, \]  

(B.1)

where \( \Delta i^*_t \) is the true change in global inventories, \( \chi < 1 \) is a fraction of U.S. inventories in OECD crude oil inventories and \( e_t \) is the measurement error, which is taken to be serially uncorrelated and uncorrelated with the observed shocks \( U^*_t \) by assumption.

Now, the observed shocks are contemporaneously correlated:

\[ \tilde{U}_t = \begin{bmatrix} u^*_{1t} \\ u^*_{2t} \\ u^*_{3t} - \chi^{-1} e_t \\ \chi u^*_{4t} + e_t \end{bmatrix} \]  

(B.2)

The observed residuals \( \tilde{u}_{3t} \) and \( \tilde{u}_{4t} \) are contemporaneously correlated with a variance-covariance matrix for:
Baumeister and Hamilton (2019) come up with a transformation $D = \Gamma \tilde{D} \Gamma'$, that transforms $\tilde{D}$ into a representation with serially uncorrelated shocks $D = diag(d_{11}, d_{22}, d_{33}, d_{44})$, and a matrix $\Gamma$:

$$\tilde{D} = \begin{bmatrix}
    d^*_{11} & 0 & 0 & 0 \\
    0 & d^*_{22} & 0 & 0 \\
    0 & 0 & d^*_{33} + \chi^{-2}\sigma^2_e & -\chi^{-1}\sigma^2_e \\
    0 & 0 & -\chi^{-1}\sigma^2_e & \chi^2 d^*_{44} + \sigma^2_e
\end{bmatrix}$$  \hspace{1cm} (B.3)

The matrix $\Gamma$ is a function of $\rho$, the negative of a coefficient from a regression of $\tilde{u}_{4t}$ on $\tilde{u}_{3t}$:

$$\rho = \frac{\chi^{-1}\sigma^2_e}{d^*_{33} + \chi^{-2}\sigma^2_e}$$  \hspace{1cm} (B.5)

Pre-multiplication $U_t = \Gamma \tilde{U}_t$ gives $(4 \times 1)$ vector of structural disturbances is:

$$U_t = \begin{bmatrix}
    u^*_{1t} \\
    u^*_{2t} \\
    u^*_{3t} - \chi^{-1}e_t \\
    \chi u^*_{4t} + \rho u^*_{3t} + (1 - \rho/\chi)e_t
\end{bmatrix}$$  \hspace{1cm} (B.6)

Although the role of a measurement error in contributing to linear responses is explicitly modeled, the nonlinear analysis of a measurement error is greatly simplified by specifying the lagged and nonlinear dynamics of the structural system directly in terms of the observed variables.
B.2. Graph Appendix

Figure 2.1. Baseline prior (solid light green curves) and posterior (dark blue histograms) distributions concerning the contemporaneous linear coefficients in A.
Figure 2.2. Baseline prior (solid light green curves) and posterior (dark blue histograms) distributions concerning the lagged linear coefficients in $A_1$. 
Figure 2.3. Baseline prior (solid light green curves) and posterior (dark blue histograms) distributions of coefficients in $A_2$. 
Figure 2.5. Baseline prior (solid light green curves) and posterior (dark blue histograms) distributions of coefficients in $A_4$. 
Figure 2.6. Responses to positive and negative 1 st. dev. shocks. Solid lines: Bayesian posterior median response to positive shocks; dotted lines: Bayesian posterior median response to negative shocks.
Figure 2.7. Sum of Responses to positive and negative 1 st. dev. shocks. Solid lines: Bayesian posterior median; shaded region: 68% posterior credible sets; dotted lines: 95% posterior credible sets.
Figure 2.8. Responses to positive and negative 2 st. dev. shocks. Solid lines: Bayesian posterior median response to positive shocks; dotted lines: Bayesian posterior median response to negative shocks.
Figure 2.9. Sum of responses to positive and negative 2 st. dev. shocks. Solid lines: Bayesian posterior median; shaded region: 68% posterior credible sets; dotted lines: 95% posterior credible sets.
Figure 2.10. Difference of responses to big (2 st. dev.) and magnified (by 2) responses to small (1 st. dev.) negative shocks. Solid lines: Bayesian posterior median; shaded region: 68% posterior credible sets; dotted lines: 95% posterior credible sets.
Figure 2.11. Responses to positive and negative 1 st. dev. shocks (Excluding A1). Solid lines: Bayesian posterior median response to positive shocks; dotted lines: Bayesian posterior median response to negative shocks.
Figure 2.12. Sum of responses to positive and negative 1 st. dev. shocks (Excluding $A_4$). Solid lines: Bayesian posterior median; shaded region: 68% posterior credible sets; dotted lines: 95% posterior credible sets.
Figure 2.13. Responses to positive and negative 2 std. dev. shocks (Excluding A). Solid lines: Bayesian posterior median response to positive shocks; dotted lines: Bayesian posterior median response to negative shocks.
Figure 2.14. Sum of responses to positive and negative 2 st. dev. shocks (Excluding $A_4$). Solid lines: Bayesian posterior median; shaded region: 68% posterior credible sets; dotted lines: 95% posterior credible sets.
Figure 2.15. Difference of responses to big (2 st. dev.) and magnified (by 2) responses to small (1 st. dev.) negative shocks (Excluding A4). Solid lines: Bayesian posterior median; shaded region: 68% posterior credible sets; dotted lines: 95% posterior credible sets.
Figure 2.16. Responses to positive and negative 1 st. dev. shocks (excluding $A_2$ and $A_3$). Solid lines: Bayesian posterior median response to positive shocks; dotted lines: Bayesian posterior median response to negative shocks.
Figure 2.17. Sum of responses to positive and negative 1 st. dev. Shocks (excluding $A_2$ and $A_3$). Solid lines: Bayesian posterior median; shaded region: 68% posterior credible sets; dotted lines: 95% posterior credible sets.
Figure 2.18. Responses to positive and negative 2 st. dev. shocks (Excluding $A_2$ and $A_3$). Solid lines: Bayesian posterior median response to positive shocks; dotted lines: Bayesian posterior median response to negative shocks.
Figure 2.19. Sum of responses to positive and negative 2 st. dev. shocks (Excluding $A_2$ and $A_3$). Solid lines: Bayesian posterior median; shaded region: 68% posterior credible sets; dotted lines: 95% posterior credible sets.
Figure 2.20. Difference of responses to Big (2 st. dev.) and magnified (by 2) responses to small (1 st. dev.) negative shocks (excluding $A_2$ and $A_3$). Solid lines: Bayesian posterior median; shaded region: 68% posterior credible sets; dotted lines: 95% posterior credible sets.
Figure 2.21. Responses to historical 1979 Q1 shocks: Solid lines: Bayesian posterior median response with the start at 1978 Q4; dotted lines: Bayesian posterior median response with the start at the sample mean.
Figure 2.22. Difference of responses to historical 1979 Q1 shocks with the start at 1978 Q4 and at the sample mean. Solid lines: Bayesian posterior median; shaded region: 68% posterior credible sets; dotted lines: 95% posterior credible sets.
Figure 2.23. Responses to historical 1990 Q3 shocks. Solid lines: Bayesian posterior median response with the start at 1990 Q2; dotted lines: Bayesian posterior median response with the start at the sample mean.
Figure 2.24. Difference of responses to historical 1990 Q3 shocks with the start at 1990 Q2 and at the sample mean. Solid lines: Bayesian posterior median; shaded region: 68% posterior credible sets; dotted lines: 95% posterior credible sets.
Figure 2.25. Responses to historical 1979 Q1 shocks vs magnified (by historical size of each shock) responses to 1-Sigma counterfactual shocks with the start at 1978 Q4. Solid lines: Bayesian posterior median response to historical 1979 Q1 shocks; dotted lines: Bayesian posterior median response to 1-Sigma counterfactual shocks.
Figure 2.26. Difference of responses to historical 1979 Q1 shock and magnified (by historical size of each shock) responses to 1-Sigma counterfactual shocks. Solid lines: Bayesian posterior median; shaded region: 68% posterior credible sets; dotted lines: 95% posterior credible sets.
Figure 2.27. Responses to historical 1990 Q3 shocks vs magnified (by historical size of each shock) responses to 1-Sigma counterfactual shocks with the start at 1990 Q2. Solid lines: Bayesian posterior median response to historical 1990 Q3 shocks; dotted lines: Bayesian posterior median response to 1-Sigma counterfactual shocks.
Figure 2.28. Difference of responses to historical 1990 Q3 shock and magnified (by historical size of each shock) responses to 1-Sigma counterfactual shocks. Solid lines: Bayesian posterior median; shaded region: 68% posterior credible sets; dotted lines: 95% posterior credible sets.
Appendix C
Appendix of Chapter 3

C.1. Table Appendix

<p>|                | Estimate | Std. Error | t value | Pr(&gt;|t|) | Significance |
|----------------|----------|------------|---------|----------|--------------|
| Intercept      | 0.0020   | 0.0002     | 9.32    | 0.0000   | ***          |
| JC.m=1         | -0.0623  | 0.0048     | -13.02  | 0.0000   | ***          |
| IP.m=1         | 0.0880   | 0.0575     | 1.53    | 0.1263   |              |
| I.m=1          | 0.1046   | 0.0926     | 1.13    | 0.2592   |              |
| MT.m=1         | -0.1338  | 0.0379     | -3.53   | 0.0004   | ***          |
| EM.m=1         | -1.1738  | 0.1843     | -6.37   | 0.0000   | ***          |
| JC.m=1.lag     | 0.0114   | 0.0073     | 1.57    | 0.1163   |              |
| IP.m=1.lag     | -0.0030  | 0.0537     | -0.06   | 0.9553   |              |
| I.m=1.lag      | 0.0965   | 0.0510     | 1.89    | 0.0588   | .            |
| MT.m=1.lag     | -0.0558  | 0.0404     | -1.38   | 0.1678   |              |
| EM.m=1.lag     | 1.5032   | 0.1943     | 7.74    | 0.0000   | ***          |
| JC.m=2         | 0.0152   | 0.0076     | 2.00    | 0.0458   | *            |
| IP.m=2         | 0.0559   | 0.0557     | 1.00    | 0.3163   |              |
| I.m=2          | 0.0583   | 0.0633     | 0.92    | 0.3570   |              |
| MT.m=2         | -0.0054  | 0.0412     | -0.13   | 0.8967   | *            |</p>
<table>
<thead>
<tr>
<th>Variable</th>
<th>Lags</th>
<th>Coefficient 1</th>
<th>Coefficient 2</th>
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Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1

Table C.1: OLS Estimates of ‘month-of-quarter’ Macroeconomic Model
<p>|                                | Estimate | Std. Error | t value | Pr(&gt;|t|) | Significance |
|--------------------------------|----------|------------|---------|----------|--------------|
| Intercept                      | 0.0032   | 0.0006     | 5.65    | 0.0000   | ***          |
| JC.m=1                         | -0.0610  | 0.0048     | -12.59  | 0.0000   | ***          |
| IP.m=1                         | 0.1162   | 0.0581     | 2.00    | 0.0461   | *            |
| I.m=1                          | 0.0725   | 0.0964     | 0.75    | 0.4520   |              |
| MT.m=1                         | -0.1386  | 0.0386     | -3.59   | 0.0004   | ***          |
| EM.m=1                         | -1.2431  | 0.1882     | -6.61   | 0.0000   | ***          |
| JC.m=1.lag                     | 0.0103   | 0.0073     | 1.42    | 0.1573   |              |
| IP.m=1.lag                     | 0.0021   | 0.0536     | 0.04    | 0.9688   |              |
| I.m=1.lag                      | 0.0955   | 0.0514     | 1.86    | 0.0639   | .            |
| MT.m=1.lag                     | -0.0504  | 0.0407     | -1.24   | 0.2157   |              |
| EM.m=1.lag                     | 1.3655   | 0.2007     | 6.80    | 0.0000   | ***          |
| JC.m=2                         | 0.0155   | 0.0077     | 2.02    | 0.0436   | *            |
| IP.m=2                         | 0.0553   | 0.0557     | 0.99    | 0.3215   |              |
| I.m=2                          | 0.0526   | 0.0650     | 0.81    | 0.4185   |              |
| MT.m=2                         | -0.0098  | 0.0411     | -0.24   | 0.8122   |              |
| EM.m=2                         | 0.4727   | 0.2215     | 2.13    | 0.0333   | *            |
| JC.m=2.lag                     | 0.0131   | 0.0069     | 1.90    | 0.0580   | .            |
| IP.m=2.lag                     | 0.0880   | 0.0569     | 1.55    | 0.1223   |              |
| I.m=2.lag                      | 0.0259   | 0.0488     | 0.53    | 0.5965   |              |
| MT.m=2.lag                     | -0.0287  | 0.0392     | -0.73   | 0.4634   |              |</p>
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Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1

Table C.2: OLS Estimates of “month-of-quarter” for both Macroeconomic and Beige Book Model
<p>|                  | Estimate | Std. Error |   t value | Pr(&gt;|t|) | Significance |
|------------------|----------|------------|-----------|----------|--------------|
| Intercept        | 0.0032   | 0.0006     | 5.69      | 0.0000   | ***          |
| JC.m=1           | -0.0609  | 0.0048     | -12.77    | 0.0000   | ***          |
| IP.m=1           | 0.1009   | 0.0572     | 1.76      | 0.0782   | .            |
| I.m=1            | 0.0836   | 0.0924     | 0.91      | 0.3655   |              |
| MT.m=1           | -0.1466  | 0.0378     | -3.88     | 0.0001   | ***          |
| EM.m=1           | -1.1621  | 0.1838     | -6.32     | 0.0000   | ***          |
| JC.m=1.lag       | 0.0114   | 0.0072     | 1.58      | 0.1151   |              |
| IP.m=1.lag       | 0.0048   | 0.0533     | 0.09      | 0.9286   |              |
| I.m=1.lag        | 0.0937   | 0.0506     | 1.85      | 0.0646   | .            |
| MT.m=1.lag       | -0.0617  | 0.0401     | -1.54     | 0.1248   |              |
| EM.m=1.lag       | 1.4355   | 0.1948     | 7.37      | 0.0000   | ***          |
| JC.m=2           | 0.0135   | 0.0075     | 1.80      | 0.0732   | .            |
| IP.m=2           | 0.0511   | 0.0552     | 0.93      | 0.3551   |              |
| I.m=2            | 0.0499   | 0.0630     | 0.79      | 0.4289   |              |
| MT.m=2           | -0.0103  | 0.0409     | -0.25     | 0.8004   |              |
| EM.m=2           | 0.4120   | 0.2148     | 1.92      | 0.0557   | .            |
| JC.m=2.lag       | 0.0134   | 0.0068     | 1.96      | 0.0510   | .            |
| IP.m=2.lag       | 0.0834   | 0.0563     | 1.48      | 0.1391   |              |
| I.m=2.lag        | 0.0273   | 0.0481     | 0.57      | 0.5710   |              |
| MT.m=2.lag       | -0.0271  | 0.0389     | -0.70     | 0.4862   |              |</p>
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Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1

Table C.3: OLS Estimates of "month-of-quarter" Macroeconomic and Beige Book Model
C.2. Graph Appendix

Figure 3.1. Four Gibbs sampling methods to define the optimal number of topics
Figure 3.2. Perplexity of training (green) and hold-out (red) data for the optimal number of topics
Figure 3.3. Word distributions for 7 LDA topics
Figure 3.4. Linear model for Macroeconomic & Beige Book Indicators
Figure 3.5. Nonlinear SVR for Macroeconomic & Beige Book Indicators
Figure 3.6. Nonlinear SVR for Macroeconomic Indicators only
Figure 3.7. SVR Input Importance
Figure 3.8. VEC plot for employees and non-agricultural payrolls of 2nd month-of-quarter, EM.m=2
Figure 3.9. VEC plot for lag of employees and non-agricultural payrolls of 1st month-of-quarter, EM,m=1.lag
Figure 3.10. VEC plot for initial claims for unemployment insurance of 1st month-of-quarter, JC.m=1
Figure 3.11. VEC plot for the Beige Book LDA topic 1, LDA1
Figure 3.12. VEC plot for the Beige Book LDA topic 2, LDA2
Figure 3.13. VEC plot for the Beige Book LDA topic 3, LDA3
Figure 3.14. VEC plot for the Beige Book LDA topic 4, LDA4
Figure 3.15. VEC plot for the Beige Book LDA topic 5, LDA5
Figure 3.16. VEC plot for the Beige Book LDA topic 6, LDA6


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