Spring 2022

Utility of the Causal Inference Approach to Mediation Analysis With Binary Outcomes

Anh Thu Le
Southern Methodist University, nancythule@gmail.com

UTILITY OF THE CAUSAL INFERENCE APPROACH TO
MEDIATION ANALYSIS WITH BINARY OUTCOMES

Approved by:

Dr. Akihito Kamata
Chair of the Dissertation Committee
Professor and Director of the Ph.D.
Program Education Policy & Leadership

Dr. Candace Walkington
Associate Professor
Department of Teaching & Learning

Dr. Paul Yovanoff
Professor Emeritus
Education of Teaching & Learning

Dr. Yusuf Kara
Senior Data Analyst at
Center on Research and Evaluation

UTILITY OF THE CAUSAL INFERENCE APPROACH TO

MEDIATION ANALYSIS WITH BINARY OUTCOMES

A Dissertation Presented to the Graduate Faculty of

Simmons School of Education Policy and Leadership Department

Southern Methodist University

in

Partial Fulfillment of the Requirement

for the degree of

Doctor of Philosophy

with a

Major in Education

by

Anh Thu Le

M.A., Mathematics

May 6th, 2022

ACKNOWLEDGEMENT

First and foremost, I would like to express my deepest gratitude to my dissertation and research advisor, Dr. Akihito Kamata, for mentoring me in this Ph.D. program at Southern Methodist University. Without his guidance and unwavering support, it would have been impossible for me to successfully finish the dissertation.

I also sincerely thank my committee members, Professor Paul Yovanoff, Associate Professor Candace Walkington, and Dr. Yusuf Kara for their friendly suggestions, detailed instructions, and the general collegialities that they provided me over the past four years.

My appreciation goes to my best friend from Virginia Tech, Yvonne Chang, who serves as my editor and great supporter. I also appreciate writing consultants at Graduate Writing Center at SMU and my other two friends, Brian Bernstein and John Dunsford II, for proofreading and editing my dissertation.

I want to send big thank to Moody School of Graduate and Advanced Studies at SMU for financially supporting me on the last year of my dissertation. I also thank all of my friends and colleagues from SMU, UNT, Appamada, RLC, and Open Door Zen Community for their sustained assistance, deep listening, encouragement, and prayers.

Last but not least, I would love to send my appreciation to my parents and brother for their care and support all these years, especially during the pandemic.

Le, Anh Thu Ph.D., Education, 2022

UTILITY OF THE CAUSAL INFERENCE APPROACH TO MEDIATION ANALYSIS WITH BINARY OUTCOMES

Advisor: Dr. Akihito Kamata

ABSTRACT

Mediation analysis is the valuable technique used to investigate mediation models. One example of the mediation model is that student's motivation will mediate the effect of the intervention on student's achievement. In other words, student's motivation levels will impact how much they can benefit from the intervention. We call the student's motivation a mediator. The classical approach is traditionally common to estimate the mediation effect where the effect is simply a multiplication of the impact of the intervention on students' motivation and the impact of student's motivation on student's achievement. When working with the outcomes such as student's achievement, educational researchers may be interested in binary outcomes such as pass or fail, success or failure. The problem is when working with binary outcomes, the classical approach procedures to estimate the mediation effects causes some bias which may lead to inaccurate decisions such as applying ineffective teaching methods.

The causal inference approach is considered to be a better alternative. It's based on the theory of potential outcomes. Even though each student can only be observed in either treatment or control group, the potential outcome method can estimate the student's outcomes in both conditions. For instance, if a group of students is under the treatment condition, they will have the observed outcomes which are the outcomes after they receive the treatment and we can use the method to estimate their outcomes of what would have happened if they did not receive the treatment. It is similar if they're under the control condition. The mediation effect estimated by this approach is the average value of the difference between the students' outcomes in the two conditions depending on the mediator on each condition.

Two Monte Carlo Simulation studies were designed for this dissertation: one with a simple mediation model and the other one with a moderated mediation model with a treatment-moderator interaction. Both studies replicated 1000 datasets for each of the various experimental conditions. Specially, they included (a) proportion of outcome (.06, .07, .075, .08, .09, .1, .2, .3, .4, .5, .6, .7), (b) effect size (small, medium, large), and (c) sample size (350, 700, 1000). The datasets were generated by using the probit model because the outcome is binary. The analyses were run with two models: probit and logit. Results were evaluated by various criteria, including bias, standard error (*SE),* average standard error (A*SE*), mean square error *(MSE),* coverage probability, and power.

Results demonstrated that the causal inference approach produced a more accurate mediation effect than the classical approach in both outcome cases, both probit and logit models, and both mediation models. The first four evaluation criteria supported the demonstration. On the other hand, the last two criteria did not fully support the demonstration. For the coverage probability, the two approaches produced quite similar results. For power, in most of the

conditions, the two approaches produced mediation effects with power of 1. Generally, the causal inference approach still produced more accurate and reliable effects than the classical approach. Finally, study limitations, practical implications, and future research were discussed.

Keywords: mediation models, causal inference approach, classical approach, rare outcome cases, non-rare outcome cases.

Contents

List of Figures

List of Tables

Chapter 1. Introduction

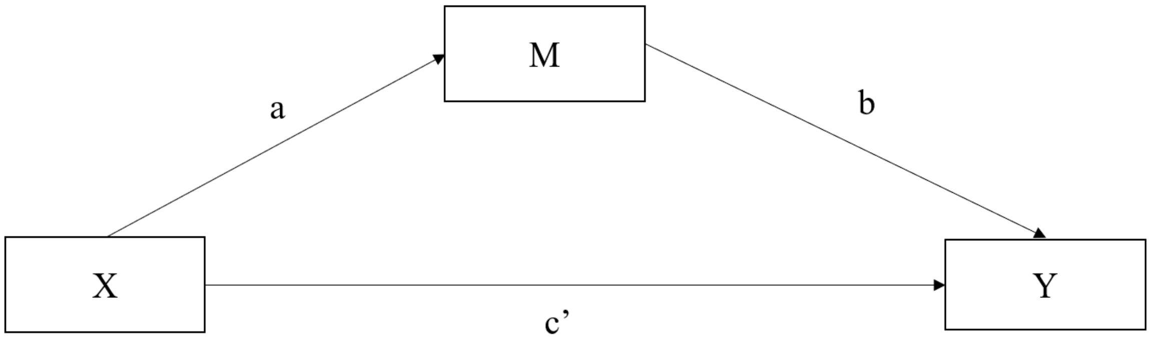
Mediation analysis is a useful technique for determining a mediation effect, which is the indirect effect of an independent variable on a dependent variable via a mediator. A mediation model is used to explain when and how a mediation effect occurs. Two approaches are commonly used when conducting mediation analysis: the classical approach and the causal inference approach. Researchers have been mostly using the classical approach. However, the way the classical approach analyzes and interprets mediation effects has some drawbacks, particularly when working with a binary outcome (Valeri & VanderWeele, 2013; VanderWeele & Vansteelandt, 2010). Following are brief presentations of mediation models, the classical approach, the causal inference approach, and the drawbacks of using the classical approach.

Mediation Models

Wright (1920) introduced indirect effects on variables via other variables. Then, Hyman (1955) and Lazarsfeld (1955) provided a series of statistical tests to determine the effect of a third variable on the relation between two variables. Later, Judd and Kenny (1981), James and Brett (1984), and Baron and Kenny (1986) translated these tests into what are known as mediation models.

Mediation models identify and explain the process that underlies an observed relation between an independent variable and a dependent variable through the use of a third variable known as a mediator. Rather than focusing only on a direct relation between independent and dependent variables, mediation models evaluate the relation between the independent variable and the mediator, as well as the relation between the mediator and the dependent variable. Figure 1 depicts a simple mediation model in which X is the independent variable, Y is the dependent variable, and M is the mediator. The a and b paths are direct paths from X to M and M to Y, respectively. The c' path is the path from X to Y conditioning on M. In addition, Figure 2 presents the c path, which is the direct path from X to Y.

Figure 1
Illustration of a Simple Mediation Model



Although X can be continuous or categorical, in this study I assume X to be a binary predictor for the purpose to interpret the treatment effect on an outcome Y with the existence of a mediator M. The treatment indicator X and the outcome Y in my study will be binary, while the mediator M will be continuous.

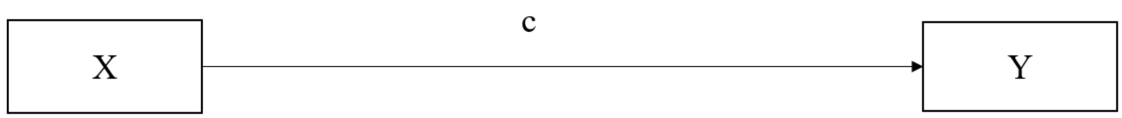
When working with binary outcomes, mediation analyses typically employ either the probit model or logit model. The probit model uses the cumulative normal distribution function, which is defined as

$$probit(P) = \Phi(P), \tag{1}$$

where Φ is the cumulative normal distribution function and P is the probability of outcome equal to 1. The logit model uses the cumulative logistic distribution function. It is the inverse of the link function, also known as log-odds, is defined as

$$logit(P) = ln\frac{P}{1-P}. \tag{2}$$

Figure 2
Illustration of the c Path



Classical Approach to Mediation Analysis

The classical approach to a simple mediation analysis was proposed by Judd and Kenny (1981) and Baron and Kenny (1986). To assess the mediation effect, this method employs a pair of regression equations. The first equation explains how the treatment indicator X affects the mediator M (i.e., $X \rightarrow M$). The mediator M is a linear function of the treatment indicator X, with the assumption that the relation between X and M is linear. That is,

$$M_i = \gamma_0 + \gamma_1 X_i + \epsilon_{M_i}, \tag{3}$$

where i is indexing for observations, X_i is the independent variable, M_i is the mediator, γ_0 is the M-intercept, γ_1 is the coefficient of the a path from X to M, respectively. In addition, ϵ_{M_i} is normally distributed residual with mean $\mathbf{0}$ and variances σ_m^2.

The second equation explains how the treatment indicator X influences the outcome Y (i.e., $X \rightarrow Y$) by conditioning on the mediator M. The outcome Y is a linear function of the treatment indicator X and the mediator M, with the assumption that the relation between X and M to Y is linear. That is,

$$Y_i = \beta_0 + \beta_1 M_i + \beta_2 X_i + \epsilon_{Y_i}, \tag{4}$$

where Y_i is the outcome, β_0 is the Y-intercept, β_1 and β_2 are the coefficients of b and c'

paths from M to Y and from X to Y, respectively. In addition, ϵ_{Y_i} is normally distributed residual with mean $\mathbf{0}$ and variance σ_y^2; σ_y^2 is 1 for probit and $\pi^2/3$ for logit.

As a result, the classical approach is inapplicable to nonlinear and interaction models. The combined model of Equation (3) and Equation (4) is as follows:

$$\begin{aligned} Y_i &= \beta_0 + \beta_2 X_i + \beta_1(\gamma_0 + \gamma_1 X_i + \epsilon_{M_i}) + \epsilon_{Y_i} \\ &= \beta_0 + \beta_2 X_i + \beta_1\gamma_0 + \beta_1\gamma_1 X_i + \beta_1\epsilon_{M_i} + \epsilon_{Y_i}. \end{aligned} \tag{5}$$

The mediation effect of X on Y via M is estimated by $\beta_1\gamma_1$, according to Equation (5). The direct effect of X on Y at a fixed level of M is estimated by β_2. This way of estimating the mediation effect and the direct effect is referred to as the classical approach.

When interpreting the cause of the outcome, Ato García et al. (2014) suggests that the classical approach must meet four assumptions in order to avoid bias in the estimation. The first assumption is linearity and non-interaction, implying that the classical approach is inapplicable to models with interactions or nonlinear terms. This assumption is explained above because the classical approach employs a system of two linear regression equations. The second assumption is that the treatment indicator X must come before the mediator M, and the mediator M must come before the outcome Y. Because we are interested in the causality of the relationships between X and M, M and Y, and X and Y, this assumption is required. Because this is a difficult assumption to meet with cross-sectional data, researchers (e.g., Ato García et al., 2014) suggested using mediation analyses with the classical approach only for longitudinal data. The third assumption is that there will be no errors in measuring the treatment indicator X and mediator M. If the variables are not measured with high reliability, the coefficients will be biased, as will the estimation of the mediation effect. The fourth assumption is that no confounder variables exist in the path from the treatment indicator X to the mediator M (i.e., $X \rightarrow M$) and the path from the mediator M to the outcome Y (i.e., $M \rightarrow Y$). Confounder variables influence the relationships between X and M as well as M and Y. Because we want to test the causality of M based solely on X and the causality of Y based solely on X and M, the classical approach requires no confounder variables. Some researchers (e.g., Ato García et al., 2014) believe that measuring all potential confounders and controlling their effects is the most effective way to prevent the problem of confounder variables.

To summarize, the classical approach to mediation analysis is straightforward. In a simple mediation model, the mediation effect is simply calculated by multiplying the two path coefficients, $\beta_1\gamma_1$. The four assumptions of the classical approach, however, are difficult to meet. As a result, it is reasonable to expect that the estimated effects will be biased.

Causal Inference Approach

Holland (1986) developed causal inference based on Jerzy Neyman's theory of potential outcomes (Rubin, 1990). Under each treatment state, the key assumption of potential outcomes is that each individual in the population of interest has two potential outcomes—observed potential outcome, which is the actual outcome when a condition is met, and unobserved potential outcome, which is the outcome of what would have happened if the condition was not met—under each treatment state. Despite the fact that each individual can only be observed in one treatment state at any given time, the potential outcome method attempts to estimate the expected values of unobserved outcomes.

Morgan and Winship (2015) provided one hypothetical scenario for potential outcomes: having a college degree rather than just a high school diploma will have a causal effect on subsequent earnings. Adults with only a high school diploma have theoretical what-if earnings under the state "have a college degree," while adults with a college degree have theoretical what-if earnings under the state "have only a high school diploma." These what-if scenarios are counterfactual in the sense that they exist in theory but are not observed in practice. The potential outcomes aid in answering causal questions such as

"Does a college degree increase a person's likelihood of earning a higher salary?" or "What effect does a college degree have on wages later in life?"

An example for using the potential outcome method in mediation analyses is to determine the mediation effect of an intervention on students' achievement through students' motivation. Students' motivation is the mediator in this case. For instance, a group of students is in the treatment group, they will have observed outcomes which are actual outcomes after they receive the treatment. The method will estimate their unobserved outcomes which are the outcomes of what would have happened if they did not receive the treatment. If the group is in the control condition, the situation is similar.

The causal inference approach to mediation analysis is the method of estimating an average causal mediation effect by using potential outcomes. The mediation effect is conceptualized as the expected value in the difference of the outcomes between the treatment condition and the control condition when conditioning on the mediator in the causal inference approach. This concept will be covered in depth in Chapter 2.

Statement of the Problems

Some issues arise when the classical approach to mediation analyses is applied to a model with a binary outcome (VanderWeele & Vansteelandt, 2010). First, the classical approach will produce a biased mediation effect when using the logit model with binary outcomes. The causal inference approach employs a risk-ratio scale to estimate a mediation effect using logit model. The risk-ratio is the ratio of the probability of an outcome in a treatment group to the probability of an outcome in a control group (Sistrom & Garvan, 2004). The classical approach, on the other hand, employs an odds-ratio scale for both probit and logit model. The odds-ratio, which is the ratio of the odds of an event occurring in the treatment group to the odds of it occurring in the control group (Sistrom & Garvan, 2004), is a measure of association between a treatment indicator and an outcome. In the case of a non-rare outcome in the logit model, where the response equals 1 occurs in more than 10% of all responses, a mediation effect estimated on the odds-ratio scale is not approximately equal to a mediation effect estimated on the risk-ratio scale. As a result, the estimated mediation effect is biased.

Second, when the outcome is binary, the classical approach is less suitable for estimating mediation effects in the presence of an interaction between the treatment indicator and a mediator or a covariate. According to MacKinnon et al. (2020), the classical approach can only be used to properly estimate mediation effects for mediation models with a continuous mediator and a continuous outcome when the interaction is present. However, when the outcome is binary, the classical approach's mediation effect will not be equal to the causal inference approach's mediation effect estimate (VanderWeele & Vansteelandt, 2010). In other words, using the classical approach to estimate mediation effects when there is a treatment-mediator or treatment-covariate interaction and a binary outcome will result in some bias.

These issues will be discussed in more detail in Chapter 2. Despite these issues, the classical approach to mediation analyses is still widely used in applied research, even when the outcome variable is not continuous. Therefore, the purpose of this study will be to show that the causal inference approach is better than the classical approach when conducting a mediation analysis with a binary outcome.

Chapter 2. Literature Review

The previous chapter provided a short history of mediation analysis as well as the core principles of the classical approach and the causal inference approach to mediation analysis. Issues that appeared while using the classical approach in mediation analysis with binary outcomes were addressed. This chapter will introduce mediation and moderated mediation models with continuous and binary outcomes. In addition, an explanation of why the causal inference approach to mediation and moderated mediation analyses is important for binary outcomes will be presented.

Mediation Models and Moderated Mediation Models

When working with mediation and moderated mediation analyses, there are two main types of outcomes: continuous and categorical. The majority of published research papers use mediation models and moderated mediation models with continuous outcomes. Therefore, this section will start with continuous outcomes.

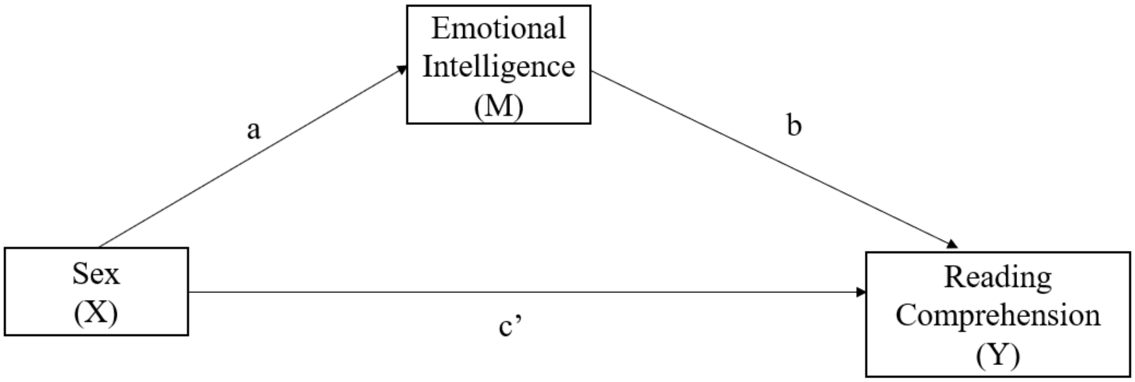
Continuous Outcomes

Mediation Models. They depict the relation between a treatment indicator and an outcome through the use of a mediator. Chapter 1 provided a brief history of mediation models. Figure 3 shows an example of how a mediation analysis is used in educational research. The diagram shows a simple mediation model in which *sex* is the binary predictor, *emotional intelligence* is the mediator, and *reading comprehension* is the outcome. The model is based on the work of Jimanez-Parez et al. (2021). The classical approach was used by the researchers to estimate the mediation effect. The effect of *sex* on *reading comprehension* is mediated by *emotional intelligence* in this case.

The causal process between the treatment indicator X and the outcome Y via a mediator is one of main reasons for investigators to explore third variable effects (James & Brett, 1984). Two causal paths relate the treatment indicator X and the outcome Y in

Figure 3

Example of A Mediation Analysis in Education Research. Adopted from "Sustainable Education Emotional Intelligence and Mother–Child Reading Competencies within Multiple Mediation Models" by E. J. Perez, M. V. Jana, R. Gutierrez-Fresneda, P. Garcia-Guirao, Sustainability, 2021, 13, 1803. Copyright 2021 by Multidisciplinary Digital Publishing Institute.



mediation analyses. The first path is directly from the treatment indicator X to the outcome Y (direct effect). That is

$$Y = \beta_{01} + \beta_{11}X + \epsilon_{Y_1}, \tag{6}$$

where β_{01} is an intercept of the regression with Y as an outcome without M as a predictor, β_{11} is the coefficient of the c path from X to Y, and ϵ_{Y_1} is a normally distributed residual. The second path from the treatment indicator X to the outcome Y through the mediator M (mediation effect) shown in Equation (4) where the relation between M and X is shown in Equation (3).

Baron and Kenny (1986) proposed a four-step technique for determining mediation effects. First, the coefficient β_{11} in Equation (6) is required to be statistically significant. Second, the coefficient γ_1 in Equation (3) is required to be statistically significant. Third, the coefficient β_1 in Equation (4) is required to be statistically significant when both the treatment indicator X and mediator M are the predictors of the outcome Y. Fourth, the coefficient in the relation between the treatment indicator X and the outcome Y (the first step) must be larger (in absolute value) than the coefficient in the relation of the treatment indicator X and the outcome Y through the mediator M. That is to say, β_{11} in Equation (6) must be greater than β_1 in Equation (4).

Moderated Mediation Models. They are extensions of mediation models that include an additional covariate known as a moderator and its interaction with the treatment indicator and/or the mediator. James and Brett (1984) were the first ones to introduce these concepts to the scientific community, such as the joint mediator-moderator models. The term *moderated mediation* was coined by the researchers to indicate that the mediation effect is dependent on the level of a moderator.

X is a treatment indicator, Y is a dependent variable, M is a mediator, and Z is a moderator, as shown in Figure 4. Furthermore, a_1 and b are direct paths from X to M and M to Y, c_1' is the path from X to Y conditioning on M, a_2 and c_2' are paths from Z to M and Z to Y, and a_3 and c_3' are paths from XZ to M and XZ to Y, respectively. The effects shown as the c_2' and c_3' paths are also known as the moderated mediation effects, which are mediation effects of the treatment on the outcome via the moderator. A moderated mediation analysis is used to determine these effects. A moderated mediation model used to explain how and when a specific effect occurs (Frone, 1999). A moderated mediation effect occurs when the strength of a mediation effect is affected by the level of a variable. Put another way, when mediation relations are contingent on the level of a moderator, a moderated mediation effect occurs (Preacher et al., 2007). A moderator can influence the magnitude of a mediation effect in a variety of ways. Preacher et al. (2007) summarizes five common moderated mediation models (see Appendix A). Figure 4 is the second of the five models, known as moderated mediation Model 2.

An example of moderated mediation Model 2 in educational research is presented in Figure 5. This example was provided by Arslan and Coşkun (2021). They used the classical approach to analyze the moderated mediation. The moderated mediation model is used in this study to see if the level of $mindfulness$ modifies the mediation role of $self\text{-}forgiveness$ on the effect of $social\,exclusion$ on $internet\,addiction$ in college students.

Figure 4

Model 2 represented as path diagram. Adopted from "Addressing Moderated Mediation Hypotheses: Theory, Methods, and Prescriptions." By K. J. Preacher, D. D. Rucker, and A. F. Hayes, Multivariate Behavioral Research, 2007, 42(1), p. 185-227. Copyright 2007 by Lawrence Erlbaum Associates, Inc.

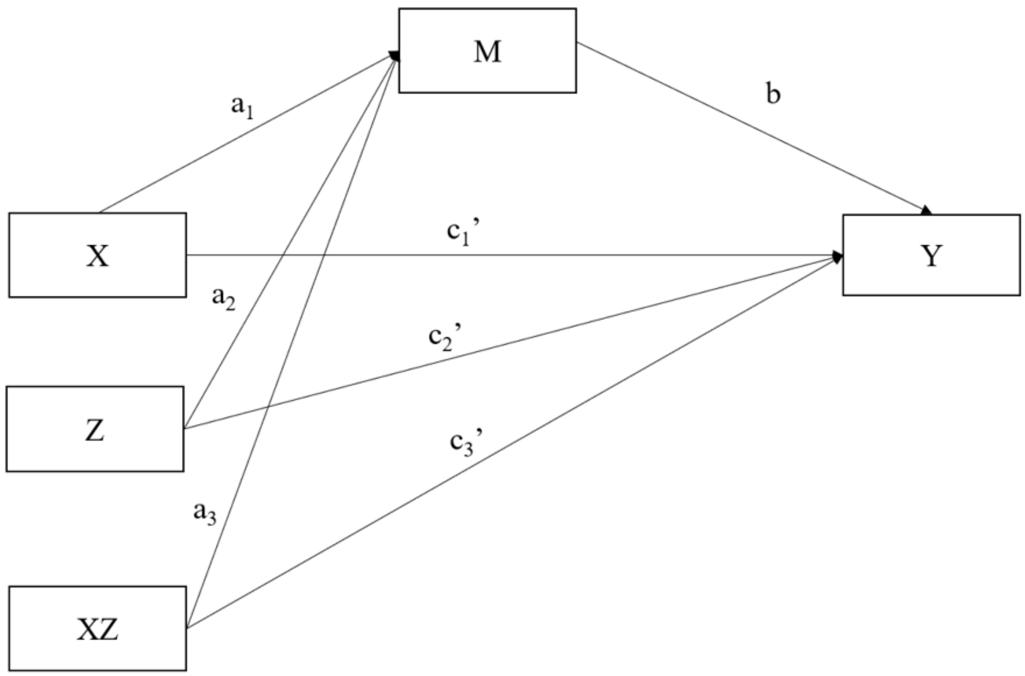
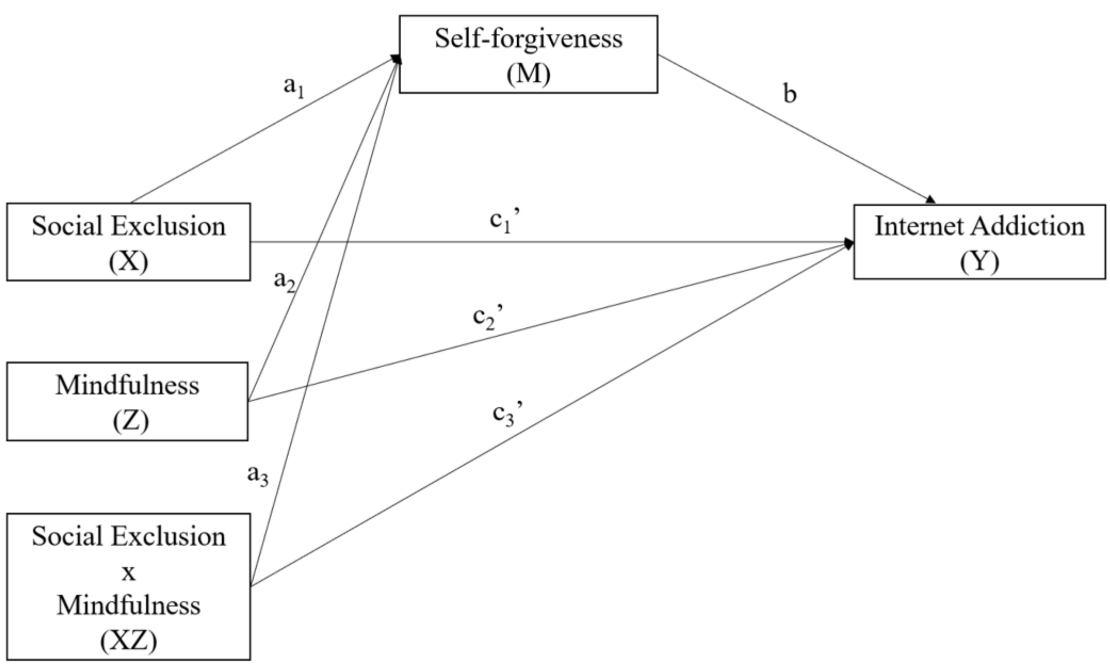


Figure 5

Second Example for Education Research in Moderated Mediation Analysis. Adopted from "Social Exclusion, Self-Forgiveness, Mindfulness, and Internet Addiction in College Students: a Moderated Mediation Approach" by G. Arslan & M. Coskun, International Journal of Mental Health and Addiction, 2021. Copyright 2021 by Springer Science Business Media, LLC.



To determine moderated mediation effects of moderated mediation models with continuous mediator and continuous outcome, three-step procedure is applied: (1) determine the mediation effect, (2) determine the moderation effect, and (3) determine the moderated mediation effect of the mediator M on the relation between the treatment indicator X and outcome Y through the moderator Z. Since the mediator M is continuous, the moderated mediation process will determine the high and low values (1 SD above and below) of the mediator. Most studies on moderated mediation models with continuous outcome and continuous mediator used SPSS Macro PROCESS (Hayes, 2012), Mplus MODMED (L. K. Muthén & Muthén, 2010), SmartPls (Ringle et al., 2015), Stata (StataCorp, 2007), or a combined method of IBM, SPSS, and AMOS (Arbuckle, 2017) to run moderated mediation analyses.

Binary Outcomes

The second type of outcomes used with mediation and moderated mediation models is categorical. Binary outcomes are part of categorical outcomes. For the purpose of exploring the utility of the causal inference approach in binary outcomes, the paper will focus on binary outcomes.

Mediation Analysis. When the outcomes in mediation analyses are binary, for example, students pass or fail a class, Equations (4) and (6) must change from the linear regression to either the probit or logit regression. Under the condition of binary outcomes, there are two cases: continuous mediator and binary mediator. For the purpose of this study, the paper only focuses on continuous mediator. In this case, Equation (3) will be with the linear regression. According to Muthén et al. (2015), the Equation (5) will have the conditional expectation

$$E(Y_i^*|X_i) = \beta_0 + \beta_1\gamma_0 + \beta_1\gamma_1 X_i + \beta_2 X_i \tag{7}$$

and conditional variance

$$V(Y_i^*|X_i) = V(\beta_1 \epsilon_{M_i} + \epsilon_{Y_i}) = \beta_1^2 \sigma_m^2 + c. \tag{8}$$

For the probit model, Equation (8) implies that Y^* conditioned on X has a normal distribution because it involves the sum of two normally distributed residuals ϵ_M and ϵ_Y. Therefore, the probability can be obtained using the standard normal distribution function Φ for the probit regression,

$$\begin{aligned} P(Y = 1|X) = P(Y^* > 0|X) = \Phi[\frac{E(Y^*|X)}{\sqrt{V(Y^*|X)}}] \\ = \Phi[\frac{\beta_0 + \beta_1\gamma_0 + \beta_1\gamma_1 X + \beta_2 X}{\beta_1^2 \sigma_m^2 + 1}], \end{aligned} \tag{9}$$

where β_0, β_1, β_2, γ_0, γ_1, and σ_m^2 are defined above.

If using the logit model instead of the probit model, Equation (8) implies that Y^* conditioned on X has a distribution that is a combination of the logit residual ϵ_Y and the normally distributed residual ϵ_M. Therefore, the effect will not be expressed explicitly in the probability scale as in the probit regression. Instead, it is obtained in the scale of log-odds-ratio, referred to as logit, through numerical integration.

Moderated Mediation Analysis. According to Muthén et al. (2015), the set of two regression equations for the moderated mediation Model 2 where M is continuous and Y is binary is given as

$$M = \gamma_0 + \gamma_1 X + \gamma_2 Z + \gamma_3 XZ + \epsilon_m \tag{10}$$

and

$$Y^* = \beta_0 + \beta_1 M + \beta_2 X + \beta_3 Z + \beta_4 XZ + \epsilon_y. \tag{11}$$

The combined model of Equation (10) and Equation (11) is:

$$Y^* = \beta_0 + \beta_1\gamma_0 + (\beta_2 + \beta_1\gamma_1)X + (\beta_1\gamma_2 + \beta_3)Z + (\beta_1\gamma_3 + \beta_4)XZ + \beta_1\epsilon_m + \epsilon_y, \tag{12}$$

where γ_0 and β_0 are the intercepts for the regressions with M and Y^* as an outcome, respectively, γ_1, γ_2, γ_3 are coefficients of a_1, a_2, a_3 paths, β_1, β_2, β_3, β_4 are coefficients of b, c'_1, c'_2, c'_3 paths, ϵ_m and ϵ_y are normally distributed residuals, and $V(\epsilon_y)$ =1.

For the probit model, according to Preacher et al. (2007), the conditional expectation and variance follows as

$$\begin{aligned} E(Y^*|M = m, X = x, Z = z) &= P(Y = 1|M = m, X = x, Z = z) \\ &= \Phi[probit(Y^*)] \end{aligned} \tag{13}$$

and

$$V(X) = (\beta_1 + \beta_3 Z)^2\sigma_m^2 + 1, \tag{14}$$

where $probit(Y^*) = \beta_0 + \beta_1 M + \beta_2 X + \beta_3 Z + \beta_4 XZ$. It can also be written as

$$probit(X_1, X_0) = \beta_0 + \beta_2 X_1 + \beta_3 Z + \beta_4 X_1 Z + \beta_1(\gamma_0 + \gamma_1 X_0 + \gamma_2 Z + \gamma_3 X_0 Z), \tag{15}$$

where X_1 and X_0 are the treatment indicator at the treatment and control conditions, respectively. Furthermore, σ_m^2 is the error variance for the regression with M as an outcome, β_0, β_1, β_2, β_3, β_4, γ_0, γ_1, γ_2, and γ_3 are defined above.

Similar to the simple mediation model, if using the logit model instead of the probit model, Equation (14) implies that Y^* conditioned on X has a distribution that is a combination of the logit residual ϵ_Y and the normally distributed residual ϵ_M. Therefore, the effect is obtained in the scale of log-odds-ratio through numerical integration and will not be expressed explicitly.

Total Natural Indirect Effects (TNIE). It is defined when the treatment indicator X is at treatment condition and the mediator M changes from control condition to treatment condition. That is,

$$TNIE = E[Y(X = 1, M = M_1)] - E[Y(X = 1, M = M_0)], \tag{16}$$

where $Y(X = 1, M = M_1)$ and $Y(X = 1, M = M_0)$ are the values of the outcome when X is at the treatment condition and M is at the treatment condition and control condition, respectively.

Pure Natural Indirect Effects (PNIE). It is defined when X is at the control condition and mediator changes from the control condition to the treatment condition. That is,

$$PNIE = E[Y(X = 0, M = M_1)] - E[Y(X = 0, M = M_0)], \tag{17}$$

where $Y(X = 0, M = M_1)$ and $Y(X = 0, M = M_0)$ are the values of the outcome when X is at the control condition and M is at the treatment condition and control condition, respectively.

Binary Outcomes vs. Continuous Outcomes

Mediation analyses and moderated mediation analyses with binary outcomes need to be explained separately because of two reasons. First, $TNIE$ and $PNIE$ are the same if the outcome and mediator are continuous. That is,

$$PNIE = TNIE = E[Y(X = x, M = M_0)] - E[Y(X = x, M = M_1)], \tag{18}$$

where $Y(X = x, M = M_0)$ and $Y(X = x, M = M_1)$ are the values of the outcome when the continuous X is at a fixed value x, and the mediator M is at the control condition and treatment condition, respectively. For binary variables, $PNIE$ and $TNIE$ are different (see Equations (16) and (17)).

Second, as mentioned in Chapter 1, we use different models in mediation and moderated mediation analyses with binary outcomes to estimate unbiased moderated mediation effects, which are probit or logit models. As probit and logit models function quite differently, mediation analyses and moderated mediation analyses with binary outcomes need to be considered carefully.

Causal Inference vs. Classical Approach

The two approaches, the classical approach and causal inference approach, to mediation analyses were briefly introduced in Chapter 1. The classical approach is simple while the causal inference approach is much more complicated. Therefore, this section will provide more details about the causal inference approach.

Causal Inference Approach

The causal inference approach to mediation analyses is based on the potential outcomes method, which assumes each individual has two potential outcomes, observed and unobserved, in each treatment state. The underlying motivation for the potential outcomes is to estimate the causal effects in mediation and moderated mediation analyses as an average effect.

Following Imai et al. (2011); Muthén (2011); Pearl (2012); Valeri and VanderWeele (2013), I assume a randomized binary treatment indicator X_i, where X_i=1 for the treatment group and X_i=0 for the control group, to be applied for participant i $(i = 1, \ldots, N)$. Then, the causal effect of the treatment will be $Y_i(1) - Y_i(0)$, where $Y_i(1)$ and $Y_i(0)$ are the outcome values of participant i when the participant is assigned to the treatment group and the control group, respectively. Since each participant can only be observed once, either $Y_i(1)$ or $Y_i(0)$, the causal effect is undefined at the individual level. However, the average effect of the treatment is identifiable. The effect is defined as $E[Y_i(1) - Y_i(0)]$.

Now let $M_i(1)$ and $M_i(0)$ be the values of the mediator obtained when the participant i under the treatment condition and control condition, respectively. Then, $Y[X_i = 1, M = M_i(1)]$ will be the value of the outcome obtained when both the treatment indicator X and the mediator M of participant i are under the treatment condition. Similarly, $Y[X_i = 0, M = M_i(0)]$ is the value of the outcome when both the treatment indicator X and the mediator M of participant i are under the control condition. According to Ato García et al. (2014), the average total effect (ATE) is defined by the causal inference approach as

$$ATE = E[Y_i(X = 1, M = M_i(1)) - Y_i(X = 0, M = M_i(0))]. \tag{19}$$

Here, ATE is average difference of unobserved outcomes under different conditions. That is to say, the effect is the expected value of the difference between the value of the outcome when the treatment indicator and the mediator are under the treatment condition and the value of the outcome when the treatment indicator and the mediator are under the control condition. Note that these expected values are a mixture of observed and unobserved outcomes. ATE consists of two components: the average causal mediation effect ($ACME$) and the average direct effect (ADE). $ACME$ is the average of the mediation effect of the treatment indicator X at level x on the outcome Y through the mediator M changing from treatment condition $X = 1$ to control condition $X = 0$. To rephrase it, $ACME$ shows the average of the difference of the outcome value under the treatment condition from the outcome value under the control condition. In equation, this effect is written as

$$ACME = E[Y_i(X = x, M = M_i(1)) - Y_i(X = x, M = M_i(0))], \tag{20}$$

where x takes a value either 1 or 0, indicating treatment or control condition. On the other hand, ADE is the average of the direct effects of X on Y by controlling M at level x when the treatment indicator changes between treatment condition and control condition,

$$ADE = E[Y_i(X = 1, M = M_i(x)) - Y_i(X = 0, M = M_i(x))], \tag{21}$$

where x is defined above.

According to Ato García et al. (2014), Equations (20) and (21) require the assumption of sequential ignorability. The assumption of sequential ignorability includes two parts. First, if one or more observed covariates are given, the treatment indicator will be ignorable. In other words, the treatment indicator is independent of all the potential values of the mediator and outcome. Second, if the treatment and one or more observed covariates are given, the mediator will be ignorable. Put another way, the mediator is independent of all the potential values of the outcome. Imai et al. (2010) explained three reasons why we need this assumption. First, it suggests the possibility of constructing a general method of estimating the average treatment effect for outcome and mediating variables of any type and using any parametric or nonparametric models. Second, it implies that we may estimate causal mediation effects while imposing weaker assumptions about the correct functional form or distribution of the observed data. Third, nonparametric identification analysis reveals the key role of the sequential ignorability assumption irrespective of the statistical models used by researchers.

The assumption of sequential ignorability also implies four other assumptions : (1) no confounder in the path from X to Y, (2) no confounder in the path from M to Y, (3) no confounder in the path from X to M, and (4) the treatment cannot be the cause of any confounder of the path from M to Y if it exists. As defined in Chapter 1, confounder variables are variables that affect the relation between X and M, M and Y, and X and Y. We need to control for confounder variables because we want to test the causality of M based solely on X and the causality of Y based solely on X and M. Moreover, when working with a mediation analysis, the causal inference approach requires to have the temporal precedence of cause assumption: the treatment indicator X must precede the mediator M, and the mediator M also must precede the outcome variable Y. The reason is that we need to have the causality from X to M, M to Y, and X to Y. Thus, X needs to happen before M and M needs to happen before Y.

Similarities of Causal Inference and Classical Approaches

In order to obtain the causal inference interpretation of the mediation effect when working with mediation analyses, both classical and causal inference approaches have the assumptions of no confounders and the temporal precedence of cause. The fourth assumption in the classical approach requires no external variables in the relations between X and M and between M and Y. Similarly, the sequential ignorability assumption of the causal inference approach requires the same thing. That is to say, both approaches require that the estimated regression equations are adjusted for the confounders of all paths in mediation models (MacKinnon, 2008; VanderWeele & Vansteelandt, 2010). Moreover, when working with mediation and moderated mediation models, both classical and causal inference approaches assume the temporal precedence of cause for the treatment indicator, the mediator, and the outcome (MacKinnon, 2008; Pearl, 2012). Put another way, the treatment indicator is assumed to happen before the mediator and the mediator is assumed to happen before the outcome.

Differences Between Causal Inference and Classical Approaches

Besides the two similarities described above, the classical approach and the causal inference approach have two main differences. These are two disadvantages of the classical approach when working with mediation and moderated mediation analyses with binary outcomes. First, as argued in Chapter 1, the classical approach will produce a biased mediation effect when using the logit model with non-rare outcomes. A mediation effect estimated using the classical approach in the odds-ratio scale is not approximately equal to a mediation effect estimated using the causal inference approach in the risk-ratio scale. The risk ratio (RR) scale for a binary outcome is

$$RR = \frac{P(Y=1|X=1)/[P(Y=1|X=1)+P(Y=0|X=1)]}{P(Y=1|X=0)/[P(Y=1|X=0)+P(Y=0|X=0)]} \tag{22}$$

and the odds ratio (OR) scale for a binary outcome is

$$OR = \frac{odds(Y=1)}{odds(Y=0)} = \frac{P(Y=1|X=1)/P(Y=0|X=1)}{P(Y=1|X=0)/P(Y=0|X=0)}. \tag{23}$$

In rare outcome cases, defined where the response equals 1 occurs in 10% or lower of the total responses (VanderWeele & Vansteelandt, 2010), $P(Y=1|X=1)$ and $P(Y=1|X=0)$ will be quite small. When those two terms are small, the two sums are approximately the same as the second terms. So, OR is approximately equal to the RR. However, when the outcome is non-rare, $P(Y=1|X=1)$ and $P(Y=1|X=0)$ will not be small. Thus, OR is not equal to the RR. Moreover, the causal inference approach was indicated to produce an unbiased mediation effect (Murphy et al., 2014). As a result, the classical approach will cause bias in estimating mediation effects if using OR scale in the logit model with non-rare outcome cases (Valeri & VanderWeele, 2013). In addition, the classical approach cannot use the risk-ratio scale because on risk-ratio scale, the mediation effect is estimated by the average of the difference of the values of observed outcome from unobserved outcome (VanderWeele & Vansteelandt, 2010). Put another way, to obtain the mediation effect using the risk-ratio scale, we need to use potential outcomes method from the causal inference approach.

Second, the classical approach cannot automatically incorporate with the interaction between the treatment indicator and a mediator or a covariate when the outcome is binary. The classical approach has limited guidance on how to estimate direct and mediation effects in mediation analyses under the presence of an interaction (Judd & Kenny, 1981; MacKinnon, 2008). Recently, MacKinnon et al. (2020) has presented a study that showed the classical approach can be used to estimate effects for mediation models with a continuous mediator and a continuous outcome. By recoding the treatment indicator and using group-mean centering on the mediator, the mediation effects estimated by the classical approach can be equivalent to the mediation effects estimated by the causal inference approach. However, when the outcome is binary and an interaction is present, the effects estimated by the classical approach cannot be equal to the effects estimated by the causal inference approach (VanderWeele & Vansteelandt, 2010). Hence, there will be bias when using the classical approach to estimate effects of mediation analyses with the presence of an interaction and a binary outcome.

Considering these problems with the classical approach, the causal inference has advantages. First, the causal inference approach estimates unbiased mediation effects regardless of rare or non-rare binary outcome cases. That is to say, the causal inference approach applies RR scale when using the logit model for both rare and non-rare outcome cases. Second, the causal inference approach provides definitions and estimators of direct and mediation effects that incorporate an interaction between a treatment indicator and a mediator or a covariate even when the outcome is binary (Pearl, 2001; Robins & Greenland, 1992). However, researchers have called for the importance of investigating the interaction when using the causal inference approach (Pearl, 2001; VanderWeele & Vansteelandt, 2010). The underlying method for estimating the effects in the causal inference approach is average effect, which is problematic when the treatment-mediator or treatment-covariate interaction is present. The average effect method ignores the important information on the direct and mediation effects at specific values of the treatment indicator and mediator or covariate, which does not provide complete insight in the causal mechanism (Pearl, 2001). Even though the causal inference cannot capture the effects fully when an interaction is present, the approach is still a better choice compared to the classical approach for mediation analyses with binary outcomes.

Published Research in Classical Approach vs. Causal Inference Approach

The problems of producing biased mediation effects when using the classical approach with binary outcomes have been indicated for quite a while. Several research papers have been published on comparing the classical approach and the causal inference approach in mediation analyses and have suggested the causal inference approach as an alternative for the classical approach. Ato García et al. (2014) summarized the two approaches with their assumptions and ran a simulation study in a simple mediation analysis with a continuous outcome to test whether violating the assumptions of omitting latent confounders would affect the mediation effects. The researchers used simulation data with four simulation factors: sample size, correlation between residuals, direct effect, and indirect effect. Sample size was specified to 200. The direct effect was set to 0.5 and the indirect effect was set to 0.09. The correlation between residuals were set to 0, 0.25, and 0.50. They used 1000 replications in each condition. The results indicated that the existence of latent confounders caused overestimating mediation effects. Based on the results, they recommended complementing the classical approach with the causal inference approach since the causal inference approach can be generalized to variables of different kinds and to many more complicated scenarios.

MacKinnon et al. (2020) compared the classical approach and causal inference approach in two models with a continuous mediator and a continuous outcome; the first model is simple mediation analysis and the second model is mediation analysis with a treatment-mediator interaction. The second model is the moderated mediation Model 1, where the treatment indicator is also the moderator. The researchers used simulation data with two simulation factors: sample size and effect size. Sample size was specified to 50, 100, 200, 500, and 1000. Effect size was specified to be zero, small (i.e., 2% of the variance in the outcome), medium (i.e., 13% of the variance in the outcome), and large (i.e., 26% of the variance in the outcome)(Cohen, 1988). Corresponding to the effect size, parameters were specified: a path from X to M (0, 0.14, 0.39, 0.59), b path from M to Y (0, 0.14, 0.39, 0.59), h path from X to Y (0, 0.14, 0.36, 0.51). In sum, they used 1,280 conditions with 500 replications for each condition. In addition, they ran simulations with a negative value of h (i.e., -39) to test the *power* between the two approaches in detecting mediation effects. Their findings indicated without the interaction term, mediation effects estimated by the two approaches were the same; with the interaction term, however, the mediation effects estimated by the causal inference approach in both treatment and control groups were clearly different than the classical approach. According to their findings, the authors recommended that researchers widely apply the causal inference approach in mediation analyses.

Following the work of MacKinnon et al. (2020), Rijnhart et al. (2021) compared the classical approach and causal inference approach in moderated mediation Model 1 with a continuous mediator and a binary outcome, specifically in rare outcome cases. They used a real-life data example, analytical comparisons, and a simulation study. The researchers used both simulation data and real-life data for the comparison. For the simulation data, they used five simulation factors: effect size, sample size, coefficient for d_1 path from XM to M, coefficient for d_2 path from XM to Y, and residual variance. Effect size was specified to be zero, small, medium, and large (Cohen, 1988). Sample size was specified to be 50, 100, 200, 500, and 1000. The d_1 and d_2 path coefficients were set to be 0 or 0.39. The residual variance were considered as 1, 4, and 9. Moreover, the thresholds for dichotomizing the continuous treatment indicator and the outcome were at the median in the variables, respectively. They concluded that the two approaches produced similar mediation effects, but different direct effects and total effects. Based on the findings, the researchers suggested that the classical approach should only be used to estimate the mediation effects and direct effects of mediation models with binary outcomes when the aim is to determine the direct effect conditional on specific mediator values. They also emphasized that the causal inference approach is the preferred approach for mediation analyses as its average causal direct and mediation effect estimates can be used to reveal causal mechanisms.

Chapter 3. Method

In theory, the causal inference approach is a better choice than the classical approach. The causal inference approach has advantages in working with mediation models with binary outcomes and the presence of a treatment-mediator or treatment-covariate interaction. Several methodological studies compared the classical approach and causal inference approach in mediation analysis. My dissertation builds on their work by focusing on two studies that compare the causal inference approach to the classical approach: one on a simple mediation analysis and the other on a moderated mediation Model 2 analysis. Both studies will use binary outcomes in both rare and non-rare cases. In two significant ways, my dissertation differs from previous published research. First, it employs two approaches in moderated mediation Model 2. Second, it exclusively considers binary outcomes, such as rare and non-rare cases. Table 1 shows a summary of the comparison between my dissertation and the other three papers.

The data for both studies will be generated by the probit model using Monte Carlo Simulation on Mplus via the MplusAutomation package (Hallquist & Wiley, 2018) in RStudio (Allaire, 2009). The generated data will next be analyzed using the probit and logit models. As mentioned in Chapter 1, when utilizing the logit model with binary outcomes, the classical approach will yield a biased mediation effect; the logit model will be assessed to validate this statement. Furthermore, since the data were generated using the probit model, $TNIE$ estimated using the probit model are expected to be gold quantities. The probit model will be evaluated to explore this hypothesis.

	García et al. (2014)	Mackinnon et al. (2020)	Rijnhart et al. (2021)	My Dissertation
Simple Mediation	X	X		X
Moderated Mediation		Model 1	Model 1	Model 2
Continuous Outcomes	X	X		
Rare Outcomes			X	X
Non-rare Outcomes				X

Table 1
Comparison Between Research Papers and My Dissertation

Study 1: Simple Mediation Model

Model

Study 1 was under a simple mediation model without any treatment-mediator or treatment-covariate interaction. The model was graphically shown in Figure 1 in Chapter 1. The model has three variables: a binary treatment indicator X, a continuous mediator M, and a binary outcome Y. There are also three direct paths: a, b, and c' paths from X to M, M to Y, and X to Y conditioning on M, respectively. As described in Chapter 1, this model has two regression equations. Equation (3) presents the regression with M as an outcome and one slope coefficient, which is the a path coefficient (γ_1). Equation (4) presents the regression with Y as an outcome and two slope coefficients. One of them is the b path coefficient (β_1), and the other is the c' path coefficient (β_2).

Data Generation

The data generation process will be based on a probit model. The mediator M and the continuous latent response variable Y^* will be generated based on Equations (3) and (5). The data generation process needs to specify population values of β_0, β_1, β_2, γ_0, γ_1, and σ_m^2. The threshold is also β_0 and will be determined by one of the simulation factors—the proportion of $Y = 1$. These population parameters will be specified in the Simulation Design. The number of replications will be 1000, which is believed to be sufficient for this study.

Simulation Design

In the proposed simulation study, there will be three simulation factors: (1) proportion of $Y = 1$, (2) effect size (of the mediation effect), and (3) sample size. Specifically, there will be 12 levels from the first factor, 3 levels from the second factor, and 3 levels from the third factor. Therefore, 12 proportion values $\times$ 3 effect sizes $\times$ 3 sample sizes will be 108 conditions.

Proportion of $\mathbf{Y = 1}$. This is the proportion of the binary outcome variable $Y = 1$. The proportion will be specified as .06, .07, .075, .08, .09, .1 for examining the rare outcome cases and .2, .3, .4, .5, .6, .7 for non-rare outcome cases. The values are chosen because as defined, the proportion of 10% or lower is considered as rare outcome cases. Therefore, the proportion of $Y = 1$ has 12 levels: 6 levels for rare cases and 6 levels for non-rare cases.

Effect size (ν). According to Lachowicz et al. (2017), the effect size of the mediation effect for mediation analyses is defined as

$$\nu = \gamma_1^2 \beta_1^2, \tag{24}$$

where γ_1 and β_1 are coefficients of the direct path from X to M and M to Y. Essentially, it is a squared value of the mediation effect by the classical approach. As a result, ν is standardized, a type of variance-explained measure, and a monotonic function in absolute value of the classical mediation effect. Moreover, it does not depend on sample size. Therefore, even though ν is derived from the classical mediation effect, it can be applied to the causal mediation effect without causing any bias.

This factor will depend on two coefficients γ_1 and β_1. For the purpose of comparing among quantities, γ_1 and β_1 will be specified to be small (i.e., 0.14 or 2% of the variance in the outcome), medium (i.e., 0.39 or 13% of the variance in the outcome), and large (i.e., 0.59 or 26% of the variance in the outcome) (Cohen, 1988). According to Equation (24), the product of γ_1^2 and β_1^2 will define the effect size. Thus, there will be 3 levels for effect size.

Sample size (n). It is the number of observations. Sample size will be assigned with a set of three values: small ($n = 350$), medium ($n = 700$), and large ($n = 1000$) to see how the effects will change accordingly. The other coefficients γ_2, γ_0 and σ_m^2 will be assigned a fixed value of 0.9, 1, and 0.84, respectively. The threshold for dichotomizing X will be 0.4, with 40% of the treatment condition and 60% of the control condition in the observed data.

Evaluation of Results

The mediation effects will be computed using the causal inference technique and the classical approach in both probit and logit models. For the causal inference approach, two mediation effects will be estimated using both models: $TNIE$ on the odds-ratio scale using the probit model and the logit model. The classical approach will calculate two mediation effects: the traditional indirect effect (TIE) on the odds-ratio scale using the probit and logit models. Another four mediation effects will be estimated using the population parameter values that correspond to these four mediation effects. Then I'll compute the difference between the four estimated mediation effects and the four population mediation effects. Furthermore, I will directly compare two pairs of different values of $TNIE$ and TIE from population values to see which approach is better in recovering population mediation effects. According to Muthén et al. (2015), the formulas for determining the mediation effects will be provided below.

TNIE in odds-ratio scale (probit model). It will be calculated using the population model parameters for the data generating probit mediation model with odds-ratio (OR). This quantity will also be estimated by fitting the probit mediation model. It is defined as follows

$$TNIE(OR) = \frac{\Phi[probit(1,1)]/(1-\Phi[probit(1,1)])}{\Phi[probit(1,0)]/(1-\Phi[probit(1,0)])}$$
$$= \frac{\frac{\Phi[\beta_0+\beta_2+\beta_1\gamma_0+\beta_1\gamma_1]}{\sqrt{\beta_1^2\sigma_m^2+1}}/[1-\frac{\Phi[\beta_0+\beta_2+\beta_1\gamma_0+\beta_1\gamma_1]}{\sqrt{\beta_1^2\sigma_m^2+1}}]}{\frac{\Phi[\beta_0+\beta_2+\beta_1\gamma_0]}{\sqrt{\beta_1^2\sigma_m^2+1}}/[1-\frac{\Phi[\beta_0+\beta_2+\beta_1\gamma_0]}{\sqrt{\beta_1^2\sigma_m^2+1}}]}, \tag{25}$$

where β_0, β_1, β_2, γ_0, γ_1, γ_2, and σ_m^2 are are defined above.

TIE in odds-ratio scale (probit model). It will be computed using the population model parameters for the data generating probit mediation model. It is defined as

$$TIE(OR) = \frac{\frac{\Phi(\beta_0+\beta_1\gamma_1+\beta_2)}{1-\Phi(\beta_0+\beta_1\gamma_1+\beta_2)}}{\frac{\Phi(\beta_0+\beta_2)}{1-\Phi(\beta_0+\beta_2)}}, \tag{26}$$

where β_1, and γ_1 are computed using the population model parameters for the data generating probit mediation model. Also, this quantity will be estimated by fitting the probit mediation model.

TNIE in odds-ratio scale (logit model). With the logit model, $TNIE$ in the risk-ratio scale is used when the binary outcome Y is non-rare. However, for the purpose of comparing between $TNIE$ and TIE, $TNIE$ in an odds-ratio scale will be computed for both outcome cases instead of an odds-ratio scale for rare cases and a risk-ratio scale for non-rare cases. According to VanderWeele and Vansteelandt (2010), the formula will be

$$TNIE(OR) \approx \exp^{\beta_0+\beta_2 X+\beta_1 M}, \tag{27}$$

where β_0, β_1, β_2 are defined above. This quantity will be estimated as part of fitting the logit mediation model.

TIE in odds-ratio scale (logit model). With the logit model, TIE in odds-ratio scale is used with the assumption that Y corresponds to a rare outcome. It is the exponential of mediation effect from X to Y via M. According to VanderWeele and Vansteelandt (2010), the formula is

$$TIE(OR) \approx \exp^{\beta_1 \gamma_1} . \tag{28}$$

This quantity will be estimated as part of fitting the logit mediation model.

To summarize, these mediation effect formulas will be utilized to calculate the four mediation effects estimated by the two approaches as well as the four population mediation effects. The differences between these four pairs of mediation effects are then evaluated using six evaluation criteria: (1) bias, (2) SE, (3) ASE, (4) MSE, (5) *coverage*, and (6) *power*. Each evaluation criterion will be assessed in relation to the three simulation factors outlined before. Following are the definitions and formulas for the six evaluation criteria.

Bias. It is the magnitude of a statistic to overestimate or underestimate a parameter. That is, *bias* of an parameter θ is the difference between the parameter's true value and the expected value of the parameter being estimated.

$$bias = mean(\hat{\theta}) - \theta, \tag{29}$$

where $\hat{\theta}$ is estimated values of the parameter θ, θ is the population parameter.

In this study, *bias* is determined in a different way than usual. Because the mediation effects are estimated on a probability scale for the probit model and a log-odds-ratio scale for the logit model, they must be calculated on a common scale for the two models to be comparable. As a result, *bias* is calculated on an odds-ratio scale. Since two odds-ratio values should be compared by their ratio, rather than their difference, the *bias* is calculated as a ratio between the estimated value in odds-ratio scale and the population value in odds-ratio scale. It is defined as

$$bias(OR) = \frac{\text{odds-ratio}(\hat{\theta})}{\text{odds-ratio}(\theta)}. \tag{30}$$

This method of computing the *bias* made the bias values comparable regardless of the population values, just like a relative bias, because the *bias* indicates how large or small the estimated parameter value compared to the population value by its proportion. This is in fact very similar to the idea of the relative bias. For example, when the *bias* is near to 1, the results are considered good on the odds-ratio scale because the estimated mediation effect is the same as the population mediation effects in that scenario.

Standard Error (SE). It is the standard deviation of the sampling distribution of a target parameter. Accordingly, the SE of a parameter θ is defined as

$$SE = \sqrt{\frac{\sum_{i=1}^{r}(\hat{\theta}_i - \bar{\hat{\theta}})^2}{r}}, \tag{31}$$

where r is the number of replications, $\hat{\theta}_i$ is the estimated value of the parameter θ for the i^{th} replication, $\bar{\hat{\theta}}$ is the expected value of the estimated value of the parameter θ.

Mean of Estimated Standard Errors (ASE). It is the average of the estimated standard errors across replications. It is defined as

$$ASE = \frac{\sum_{i=1}^{r}\hat{SE}_i}{r}, \tag{32}$$

where $\hat{SE}_i$ is the estimated standard error of the parameter θ at i^{th} replication, r is defined above.

Mean Square Error (MSE). It is an overall measure of estimation error. According to Rice (2007),

$$MSE = \frac{1}{r}\sum_{i=1}^{r}(\hat{\theta}_i - \theta)^2, \tag{33}$$

where $\hat{\theta}_i$, θ, and r are defined above.

Coverage Probability. It is the proportion of the time when the confidence interval (CI) contains the population parameter, over many replications. In other words, *coverage probability* is the probability of the population parameter in between the lower bound and upper bound of the CI of the parameter. When CI works well, which means 95% certain contains the values of the parameter, the *coverage probability* should be close to 0.95.

Power. It is the probability that a test correctly rejects a false null hypothesis. It is defined as the probability that we reject H_0 when it is false (Murphy et al., 2014), where

$$power = 1 - \beta = P(Reject\ H_0|H_0\ is\ false). \tag{34}$$

In many applications, 0.80 is considered to be sufficiently high.

$TNIE$ and TIE will be estimated on an odds-ratio scale based on these evaluation criteria. The goal is to determine which approach, causal inference or classical, is superior in recovering population mediation effects. Specifically, I will examine which approach provides a *bias* that is closer to 1, a lower SE, a lower ASE, a lower MSE, a higher *coverage*, and a higher *power*.

Study 2: Moderated Mediation Model 2

Model

Study 2 extends the simple mediation model in Study 1 to a moderated mediation model including an interaction between a treatment indicator and a covariate. The model is moderated mediation Model 2 in Figure 4, where the a_1 path from the treatment indicator X to the mediator M is moderated by the moderator Z and the treatment-moderator interaction XZ. There are four variables in this model: the binary treatment indicator X, continuous mediator M, binary moderator Z, and binary outcome Y. The letters a_1, a_2, a_3, and b are direct paths from X to M, Z to M, XZ to M, and M to Y, c'_1, c'_2, and c'_3 are paths from X to Y, Z to Y, and XZ to Y conditioning on M, respectively.

Data Generation

The data generation process will be based on a probit model. The mediator M and continuous latent response variable Y^* will be generated based on the Equations (14) and (15). The data generation process needs β_0, β_1, β_2, β_3, β_4, γ_0, γ_1, γ_2, γ_3, and σ_m^2. The threshold is also β_0 and will be determined by one of the simulation factors—the proportion of $Y = 1$. These population parameters will be specified in the Simulation Design. Number of replications will be 1000, which is believed to be sufficient for this study.

Simulation Design

The four simulation factors will be considered: (1) proportion of $Y = 1$, (2) effect size (of the mediation effect), (3) sample size, and (4) moderation effect. Specifically, there will be 12 levels from the first factor, 3 levels from the second factor, 3 levels from the third factor, and 2 levels for the fourth factor. Therefore, there will be 12 proportion values $\times$ 3 effect sizes $\times$ 3 sample sizes $\times$ 2 moderation effects = 216 conditions. The first three factors will be specified exactly the same as Study 1. The fourth factor, the moderator Z, will be dichotomous with values of 0 for no moderation effect and 1 for having moderation effect. The coefficients β_2, β_3, β_4, γ_0, γ_2, γ_3, and σ_m^2 will be assigned to fixed values.

Evaluation of Results

The moderated mediation effects will be computed using the causal inference approach and the classical approach by both probit and logit models. For the causal inference approach, two moderated mediation effects will be computed: $TNIE$ on the odds-ratio scale using the probit model and the logit model. The classical approach will compute two moderated mediation effects: TIE on the odd-ratio scale using the probit model and the logit model. In addition to these four moderated mediation effects, another four moderated mediation effects will be computed using the population parameter values. Then I'll compare the four pairs of moderated mediation effects: one from each of the two approaches, and the other from population values. Furthermore, I will directly compare $TNIE$ and TIE in the odds-ratio scale estimated using the probit model and the logit model to see which approach produces more accurate moderated mediation effects. According to VanderWeele and Vansteelandt (2010) and Muthén et al. (2015), the formulas for determining the moderated mediation effects will be provided below.

TNIE in odds-ratio scale (probit model). It will be calculated using the population model parameters for the data generating probit moderated mediation model with odds-ratio. This quantity will also be estimated by fitting the probit moderated mediation model. It is defined as

$$TNIE(OR) = \frac{\Phi[probit(1,1)]/(1-\Phi[probit(1,1)])}{\Phi[probit(1,0)]/(1-\Phi[probit(1,0)])} \tag{35}$$

$$= \frac{\Phi[\frac{\beta_0+\beta_1\gamma_0+\beta_2+\beta_1\gamma_1+(\beta_1\gamma_2+\beta_1\gamma_3+\beta_3+\beta_4)Z}{\sqrt{\beta_1^2\sigma_m^2+1}}]/[1-\Phi(\frac{\beta_0+\beta_1\gamma_0+\beta_2+\beta_1\gamma_1+(\beta_1\gamma_2+\beta_1\gamma_3+\beta_3+\beta_4)Z}{\sqrt{\beta_1^2\sigma_m^2+1}})]}{\Phi[\frac{\beta_0+\beta_1\gamma_0+\beta_2+(\beta_1\gamma_2+\beta_3+\beta_4)Z}{\sqrt{\beta_1^2\sigma_m^2+1}}]/(1-\Phi(\frac{\beta_0+\beta_1\gamma_0+\beta_2+(\beta_1\gamma_2+\beta_3+\beta_4)Z}{\sqrt{\beta_1^2\sigma_m^2+1}})]}, \tag{36}$$

where β_0, β_1, β_2, β_3, β_4, γ_0, γ_1, γ_2, γ_3, σ_m^2, and Z are defined above.

TIE in odds-ratio scale (probit model). It will be computed using the population model parameters for the data generating probit mediation model. It is defined as

$$TIE(OR) = \frac{\frac{\Phi(\beta_0+\beta_2+\beta_1\gamma_1+(\beta_1\gamma_2+\beta_1\gamma_3+\beta_3+\beta_4)Z)}{1-\Phi(\beta_0+\beta_2+\beta_1\gamma_1+(\beta_1\gamma_2+\beta_1\gamma_3+\beta_3+\beta_4)Z)}}{\frac{\Phi(\beta_0+\beta_2+(\beta_1\gamma_2+\beta_3+\beta_4)Z)}{1-\Phi((\beta_0+\beta_2+(\beta_1\gamma_2+\beta_3+\beta_4)Z))}}, \tag{37}$$

where β_1, and γ_1 are computed using the population model parameters for the data generating probit mediation model. Also, this quantity will be estimated by fitting the probit mediation model.

TNIE in odds-ratio scale (logit model). With logit model, $TNIE$ in the risk-ratio scale is used when the binary outcome Y is non-rare. However, for the purpose of comparing between $TNIE$ and TIE, $TNIE$ in odds-ratio scale will be computed for both outcome cases. Adjusting the formula provided by VanderWeele and Vansteelandt (2010) for Model 1 by adding the moderator Z and replacing the interaction term XM for XZ, the formula for Model 2 will be

$$TNIE(OR) \approx \exp^{\beta_0+\beta_2 X+\beta_1 M+\beta_3 Z+\beta_4 XZ}, \tag{38}$$

where β_0, β_1, β_2, β_3, and β_4 are defined above. This quantity will be estimated as part of fitting the logit mediation model.

TIE in odds-ratio scale (logit model). It is $TIE(OR) = \exp^{\beta_1(\gamma_1+\gamma_3 Z)}$. Three coefficients β_1, γ_1, and γ_3 are obtained by fitting the logit regression.

In sum, these moderated mediation effect formulas will be used to compute four moderated mediation effects from the two approaches and the four population moderated mediation effects from the population values. Then, the differences between these four pairs of moderated mediation effects will be estimated based on six evaluation criteria: (1) $bias$, (2) SE, (3) ASE, (4) MSE, (5) $coverage\,probability$, and (6) $power$. Each of these evaluation criteria will be evaluated with respect to the four simulation factors mentioned earlier. The six evaluation criteria will be calculated exactly the same as shown in Study 1.

Chapter 4. Results

Study 1: Simple Mediation Model

Probit Model

Overall, in the probit model analysis, $TNIE$ was more accurate than TIE in recovering the population values for the mediation effect in both rare and non-rare outcome cases. For each evaluation criterion, the difference between $TNIE$ and TIE was small for non-rare outcome cases. However, in rare outcome cases, the gap was much larger, and $TNIE$ clearly outperformed TIE.

Proportion Y = 1. The proportion of $Y = 1$ had 12 levels: six for cases with rare outcomes and six for cases with non-rare outcomes. Six evaluation criteria were used to evaluate the mediation effects, $TNIE$ and TIE, with respect to the proportion of $Y = 1$ in both rare and non-rare outcome cases.

As stated in Chapter 3, *bias* was calculated using an odds-ratio scale to compare $TNIE$ and TIE. On an odds-ratio scale, *bias* was the ratio of the odds of a mediation effect to the odds of a population value. As a result, the closer a mediation effect was to recovering a population value, the closer the *bias* was to 1. To put it another way, when the *bias* for a mediation effect equals 1, it indicates that the quantity is unbiased. When the proportion of rare outcomes increased, the *bias* for $TNIE$ and TIE became closer to 1 and the gap between them shrank. The gap was positive because the *bias* for $TNIE$ was closer to 1 than the *bias* for TIE. This was evident in the conditions where the sample size was 350 and the effect size was large (for the numerical information, see Table 2). Furthermore, as the proportion of $Y = 1$ approached .08, the *bias* for the two quantities increased faster , and then slowly after that. Figure 6 depicts this graphically.

This change did not exist in non-rare outcome cases. When the proportion of $Y = 1$ increased from 0.2 to 0.4 under the same conditions with a large effect size and a sample size of 350, the *bias* for the two quantities increased and the gap between them decreased.

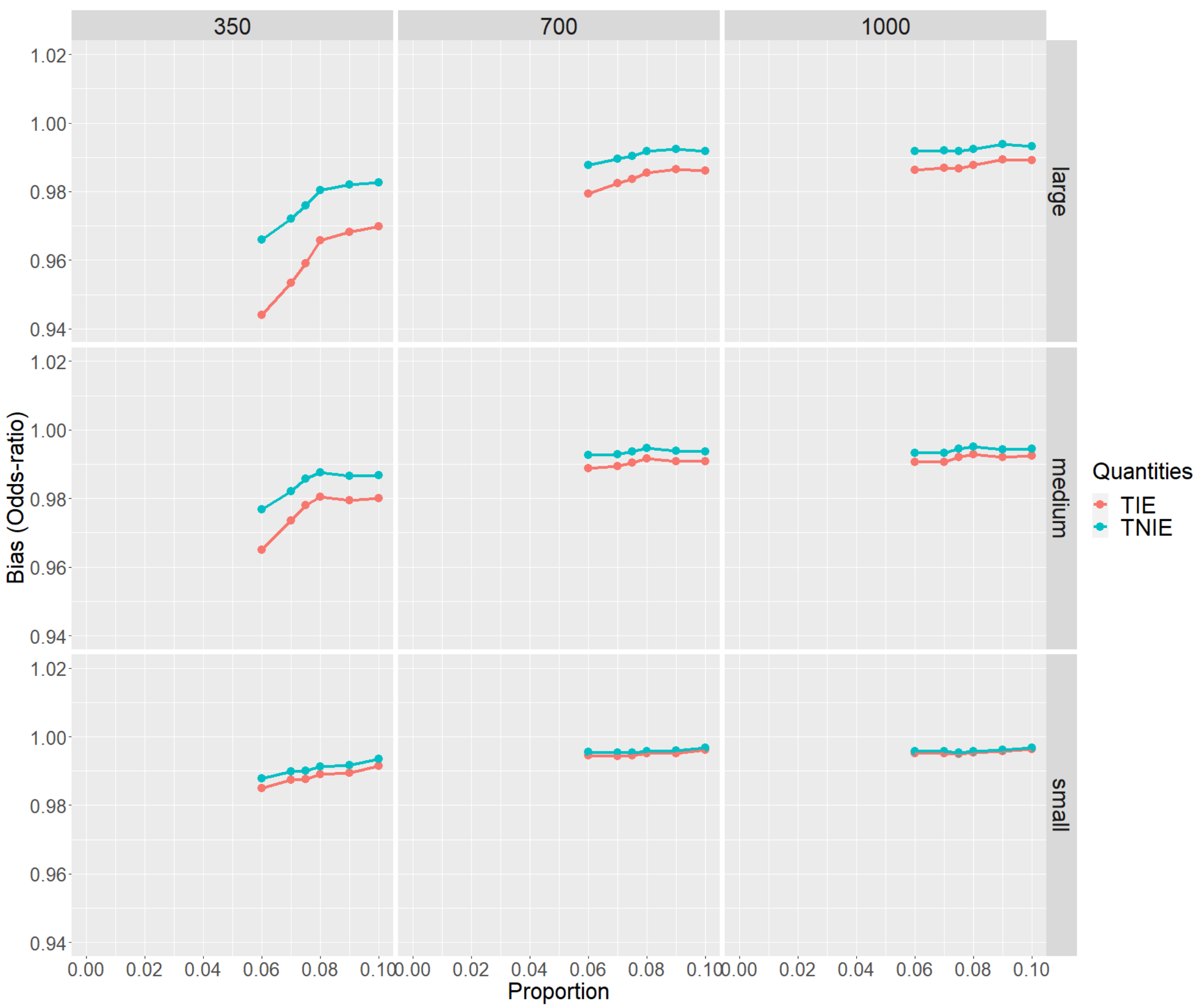
Effect Size	Sample Size	Proportions	Bias for TNIE	Bias for TIE	gap
large	350	0.06	0.9556	0.9256	0.03
		0.07	0.9658	0.94	0.0258
		0.075	0.972	0.95	0.022
		0.08	0.9714	0.9503	0.0211
		0.09	0.9784	0.961	0.0174
		0.1	0.9818	0.9661	0.0157

Table 2

The Changes in Bias under Different Proportions for Rare Outcome Cases in Probit

Figure 6

Bias over TIE and TNIE for Rare Outcomes in Probit for Simple Mediation Model



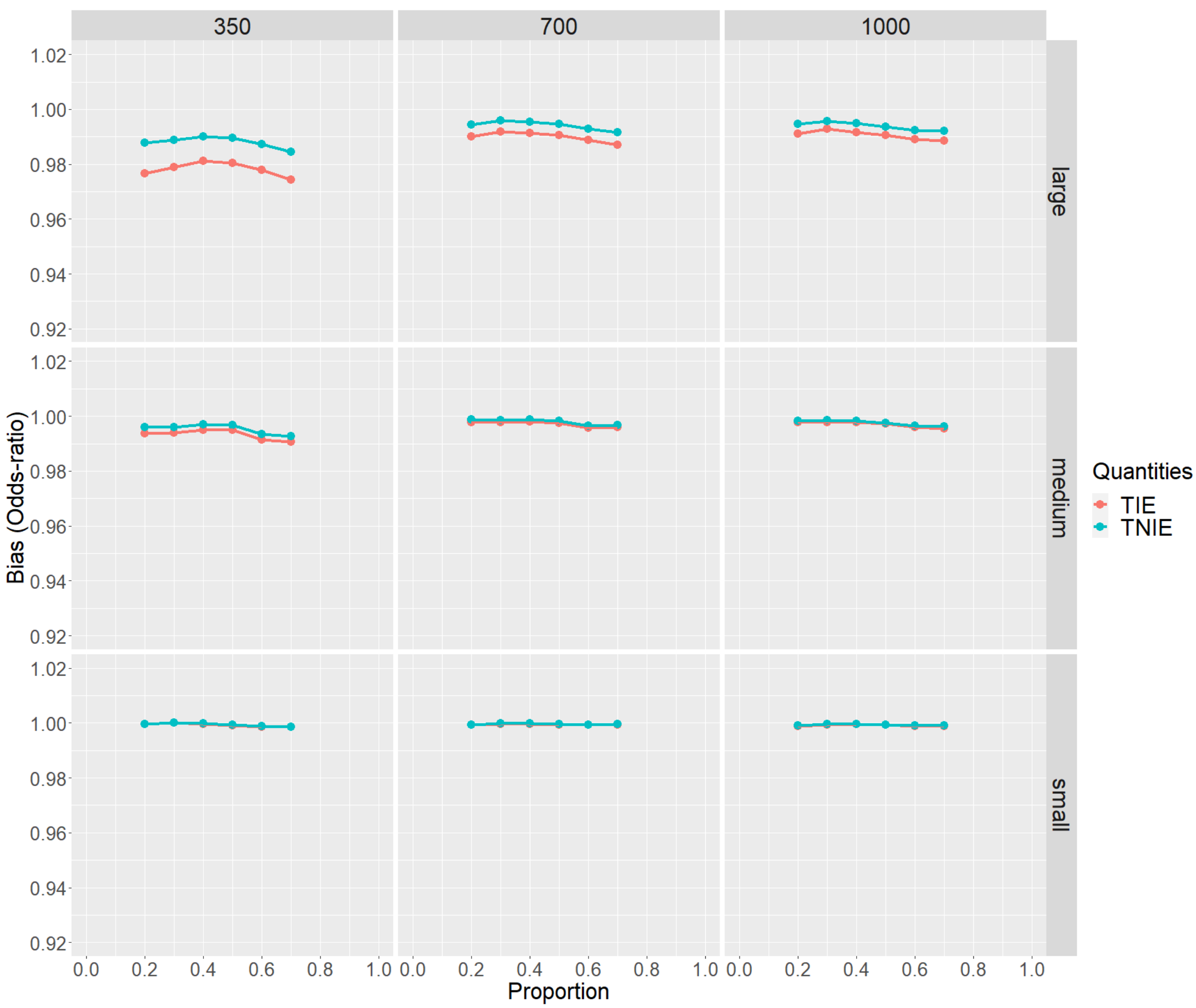
Effect Size	Sample Size	Proportions	Bias for TNIE	Bias for TIE	gap
large	350	0.2	0.9877	0.9765	0.0112
		0.3	0.9888	0.9788	0.01
		0.4	0.9902	0.9811	0.0091
		0.5	0.9897	0.9804	0.0093
		0.6	0.9873	0.9779	0.0094
		0.7	0.9844	0.9743	0.0101

Table 3

The Changes in Bias under Different Proportions for Non-rare Outcome Cases in Probit

Figure 7

Bias over TIE and TNIE for Non-rare Outcomes in Probit for Simple Mediation Model



Effect Size	Sample Size	Proportions	SE for TNIE	SE for TIE	gap
large	350	0.06	0.4594	0.6497	-0.1903
		0.07	0.4244	0.6065	-0.1821
		0.075	0.3916	0.538	-0.1464
		0.08	0.3929	0.5347	-0.1418
		0.09	0.3217	0.4042	-0.0825
		0.1	0.3119	0.3906	-0.0787

Table 4

The Changes in SE under Different Proportions for Rare Outcome Cases in Probit

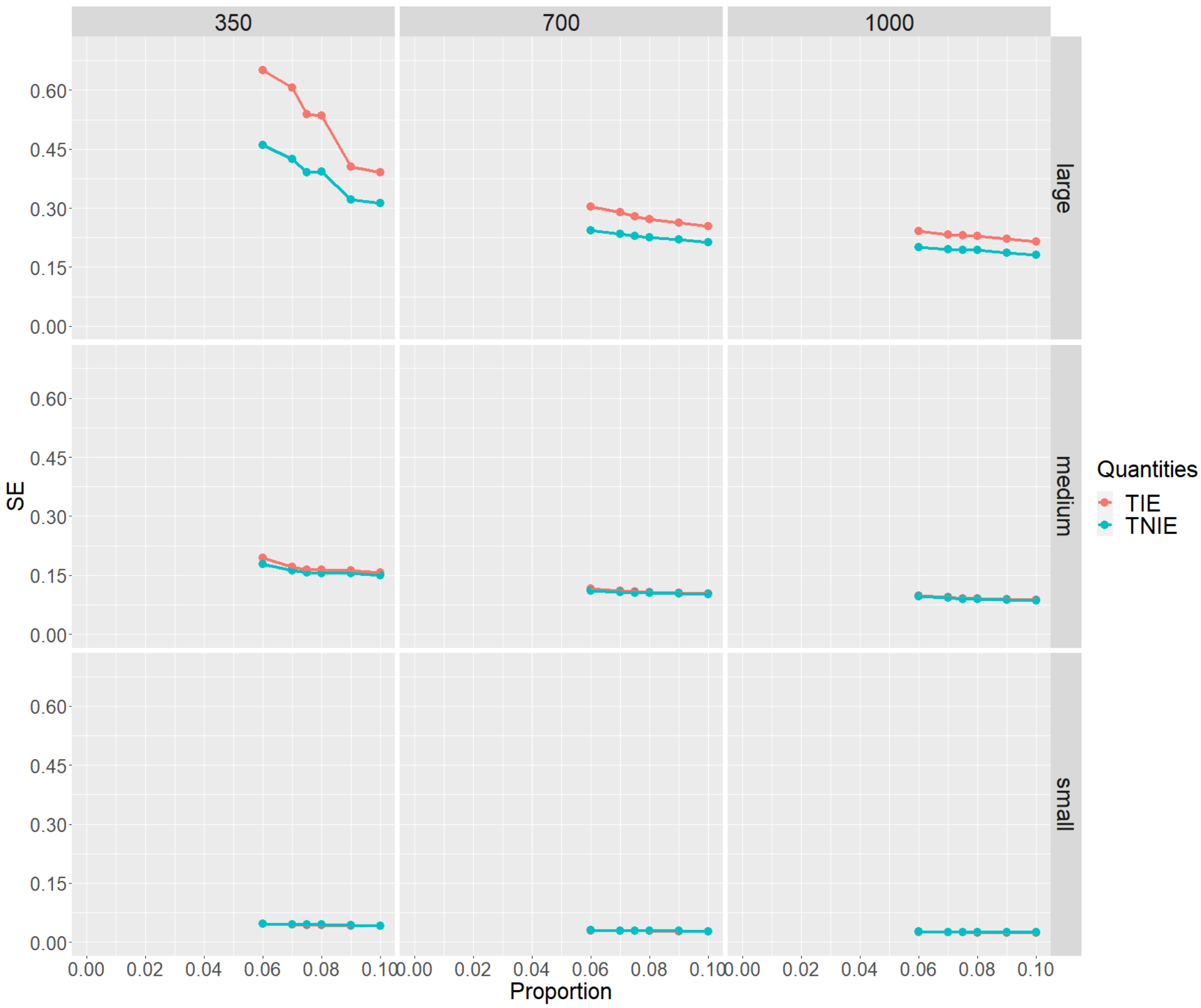
As the proportion increased to 0.7, the *bias* for the two quantities shifted away from 1 and the gap widened. This pattern is detailed in Table 3. The *bias* for $TNIE$ is closer to 1 than for the *bias* for TIE, as in rare outcome cases (f or a graphical representation, see Figure 7). In summary, as the proportion of $Y = 1$ increased, the *bias* for $TNIE$ and TIE became more consistent with 1 for rare outcome cases but not for non-rare cases. Furthermore, in both outcome cases, the *bias* for $TNIE$ was always closer to 1 than *bias* for TIE.

SE indicated how far the mediation effect deviated from the mean of mediation effects across 1000 replications. As a result, the smaller the SE, the more accurate the mediation effect. My findings revealed that the SE followed similar patterns to the *bias*, but in a different direction. The SE for the two quantities decreased as the proportion of $Y = 1$ increased, as did the gap between them in rare outcome cases. This is evident when the sample size is 350 and the effect size is large. Across all proportions of $Y = 1$, the SE for $TNIE$ was always less than the SE for TIE. The gap was all negative, as shown in Table 4. The pattern is depicted graphically in Figure 8.

In non-rare outcome cases, this pattern was not as obvious. Under the same conditions, with a sample size of 350 and a large effect size, increasing the proportion of $Y = 1$ from 0.2 to 0.4 reduced the SE for the two quantities and narrowed the gap. The SE for the two quantities continued to decrease as the proportion approached 0.5, but the gap widened. When the proportion reached 0.6, the SE for the two quantities continued to

Figure 8

SE over TIE and TNIE for Rare Outcomes in Probit for Simple Mediation Model



decrease, and the gap also shrank. Finally, as the proportion approached 0.7, the SE for the two quantities increased, as did the gap. Figure 9 depicts this graphically for all conditions. In general, the gaps in rare and non-rare outcome cases were all negative. This meant that the SE for $TNIE$ was always less than the SE for TIE across all conditions in both outcome cases.

Chapter 3 mentioned the formula and definition of ASE. The ASE was calculated primarily by averaging the SEs for mediation effects across 1000 replications. As with SE, the smaller the ASE, the better the mediation effect. The results for ASE were similar to those for SE. The ASE for $TNIE$ was consistently lower than the ASE for TIE. Tables

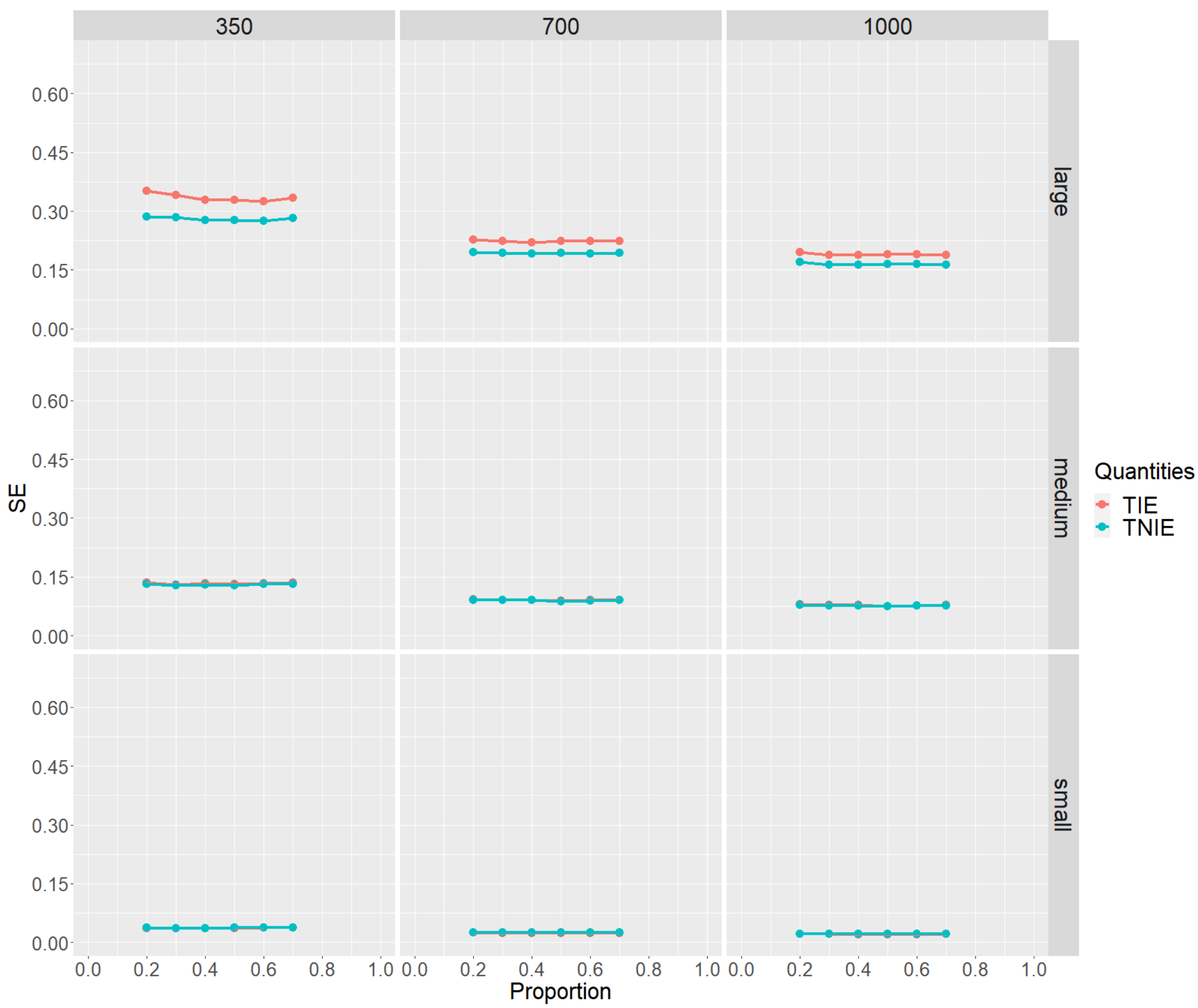
Effect Size	Sample Size	Proportions	SE for TNIE	SE for TIE	gap
large	350	0.2	0.2862	0.3517	-0.0655
		0.3	0.2833	0.3407	-0.0574
		0.4	0.2775	0.3279	-0.0504
		0.5	0.2771	0.329	-0.0519
		0.6	0.2745	0.3251	-0.0506
		0.7	0.2816	0.3335	-0.0519

Table 5

The Changes in SE under Different Proportions for Non-rare Outcome Cases in Probit

Figure 9

SE over TIE and TNIE for Non-Rare Outcomes in Probit for Simple Mediation Model



Effect Size	Sample Size	Proportions	ASE for TNIE	ASE for TIE	gap
large	350	0.06	0.5744	0.8015	-0.2271
		0.07	0.4343	0.6059	-0.1716
		0.075	0.3804	0.5085	-0.1281
		0.08	0.3764	0.5022	-0.1258
		0.09	0.319	0.3917	-0.0727
		0.1	0.312	0.3799	-0.0679

Table 6

The Changes in ASE under Different Proportions for Rare Outcome Cases in Probit

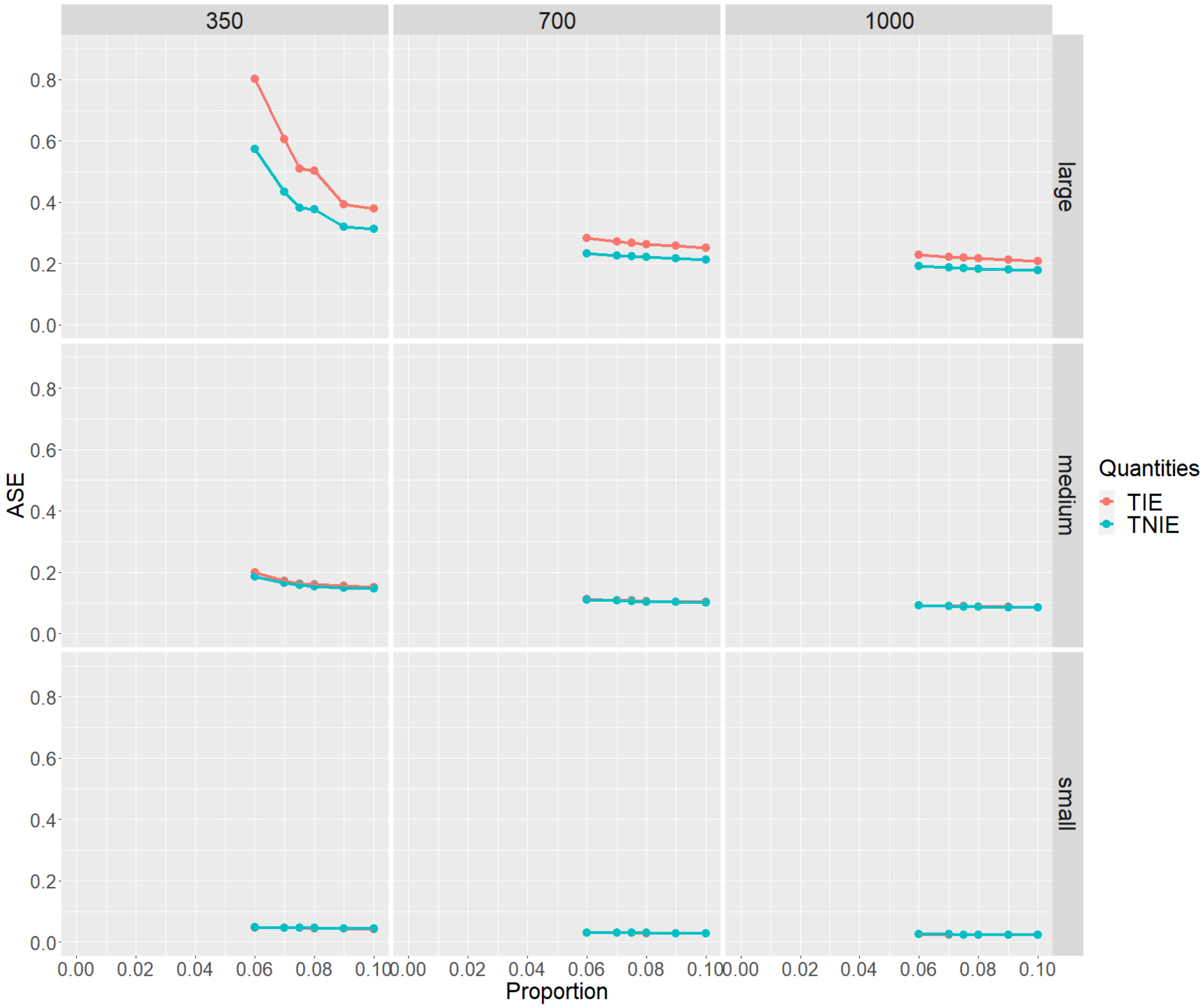
6 and 7 provide numerical examples for rare and non-rare outcome cases with a large effect size and a sample size of 350. Figures 10 and 11 depict the patterns of rare and non-rare cases, respectively.

The MSE is a measure of how close the estimate is to the population value over 1000 replications. Thus, the smaller the MSE, the more accurate the mediation effect. My findings revealed that the MSE followed similar patterns to the *bias*, but in a different direction. The MSE for the two quantities decreased as the proportion increased, as did the gap between them in rare outcome cases. When the sample size is 350 and the effect size is large, this is clearly visible. Across all proportions of $Y = 1$, the MSE for $TNIE$ was consistently less than that of TIE. The gap was all negative, as shown in Tables 8 and 9. The pattern is depicted graphically in Figures 12 and 13.

$Coverage$ *probability* represented the proportion of time when the population value was in the 95% confidence interval over 1000 replications. Thus, if the *coverage* for a mediation effect was equal to or greater than .95, it would indicate that the mediation effect was unbiased. The results did not show any clear patterns for *coverage* as the proportion of $Y = 1$ increased in rare and non-rare cases. For example, when the sample size was 350 and the effect size was large, the proportion of $Y = 1$ increased from 0.06 to 0.075, the *coverage* for the two quantities increased and the gap between them increased as well. Then, as the proportion increased to 0.09, the *coverage* for the two quantities decreased and did the gap between them. When the proportion reached 0.1, the *coverage*

Figure 10

ASE over TIE and TNIE for Rare Outcomes in Probit for Simple Mediation Model

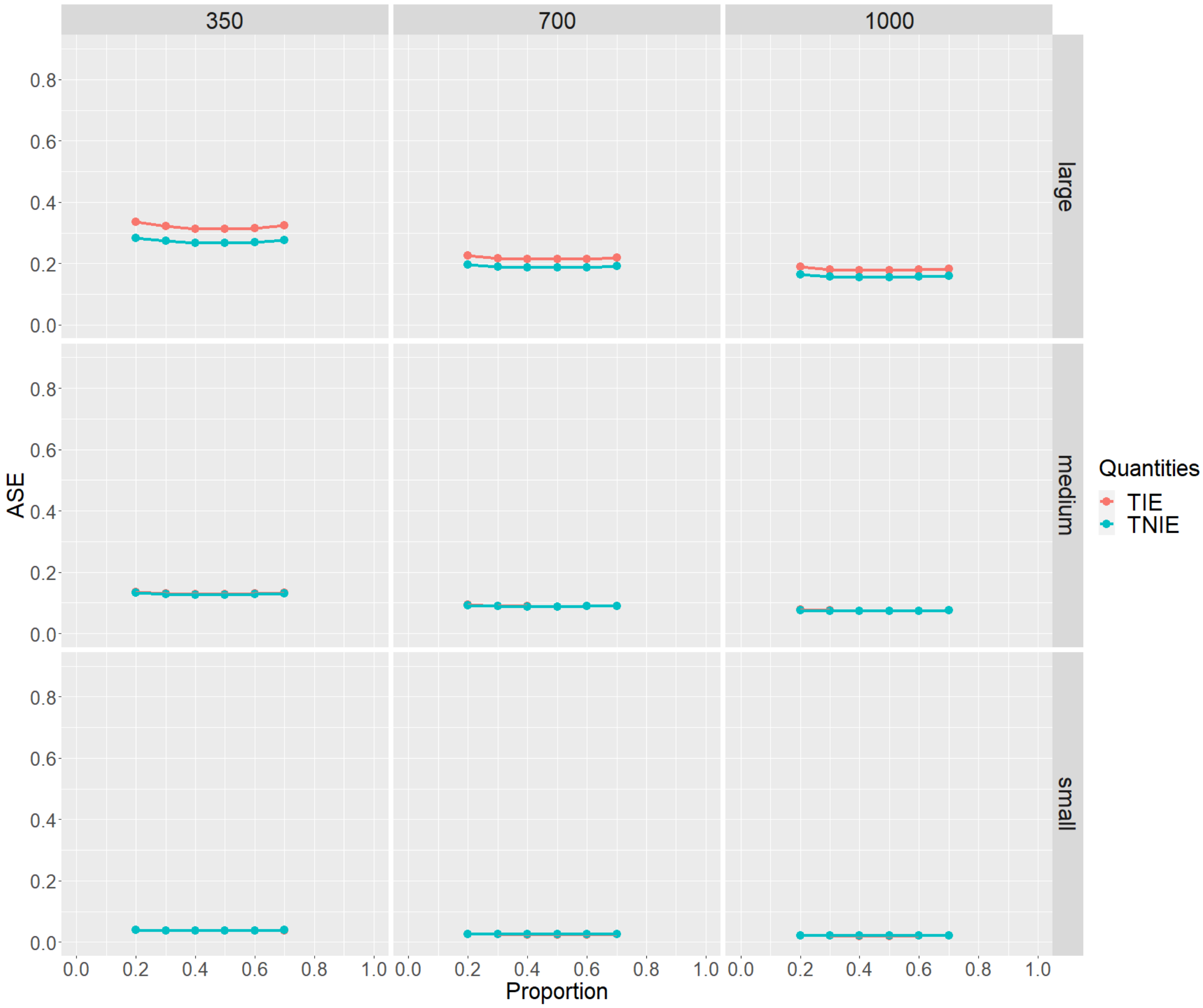


Effect Size	Sample Size	Proportions	ASE for TNIE	ASE for TIE	gap
large	350	0.06	0.2833	0.3357	-0.0524
		0.07	0.2732	0.3208	-0.0476
		0.075	0.2681	0.3129	-0.0448
		0.08	0.2672	0.3118	-0.0446
		0.09	0.2702	0.3156	-0.0454
		0.1	0.2765	0.3245	-0.048

Table 7

The Changes in ASE under Different Proportions for Non-rare Outcome Cases in Probit

Figure 11
ASE over TIE and TNIE for Rare Outcomes in Probit for Simple Mediation Model

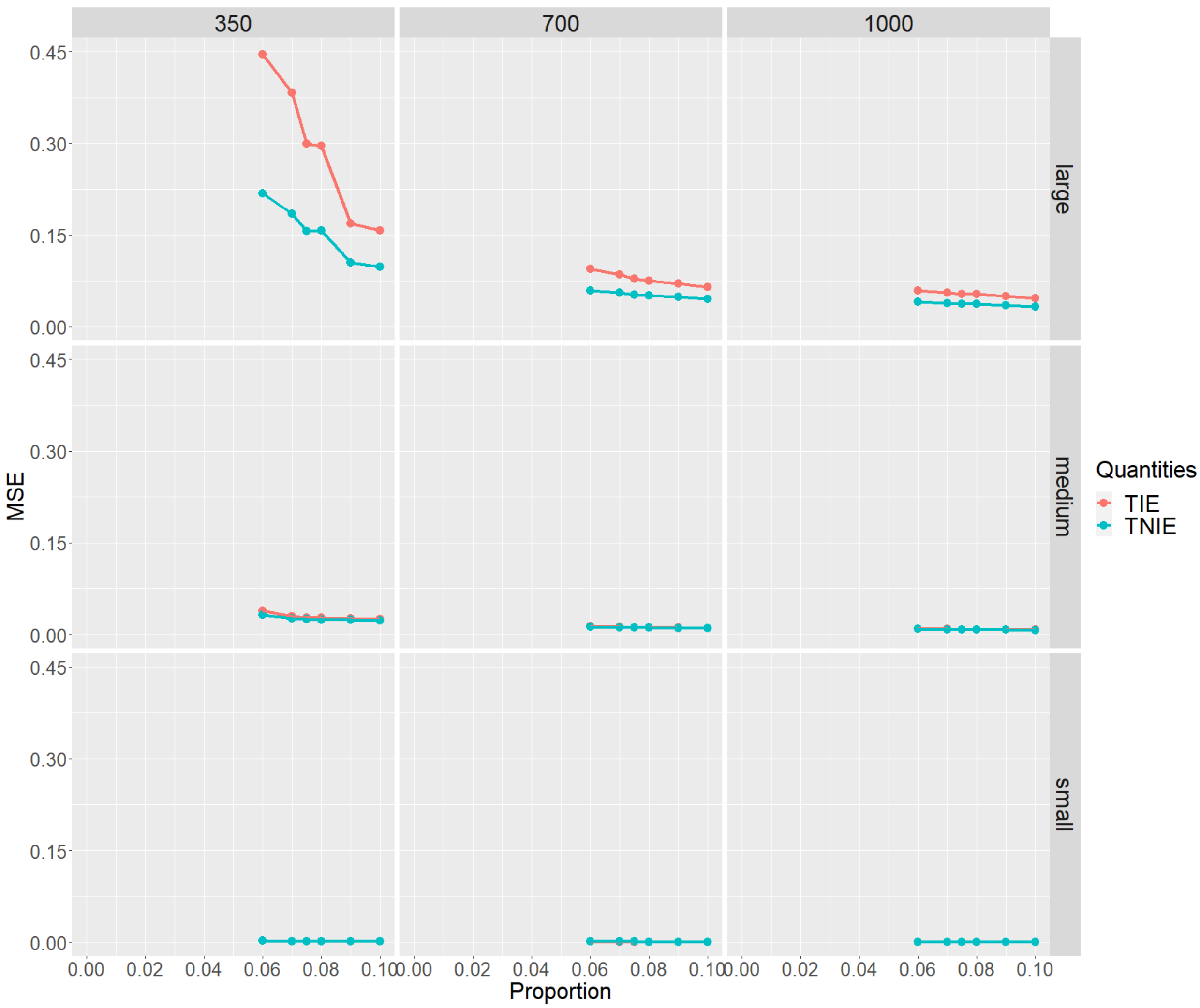


Effect Size	Sample Size	Proportions	MSE for TNIE	MSE for TIE	gap
large	350	0.06	0.2185	0.4452	-0.2267
		0.07	0.1844	0.3823	-0.1979
		0.075	0.1561	0.2993	-0.1432
		0.08	0.1572	0.2956	-0.1384
		0.09	0.1051	0.1692	-0.0641
		0.1	0.0984	0.157	-0.0586

Table 8
The Changes in MSE under Different Proportions for Rare Outcome Cases in Probit

Figure 12

MSE over TIE and TNIE for Rare Outcomes in Probit for Simple Mediation Model



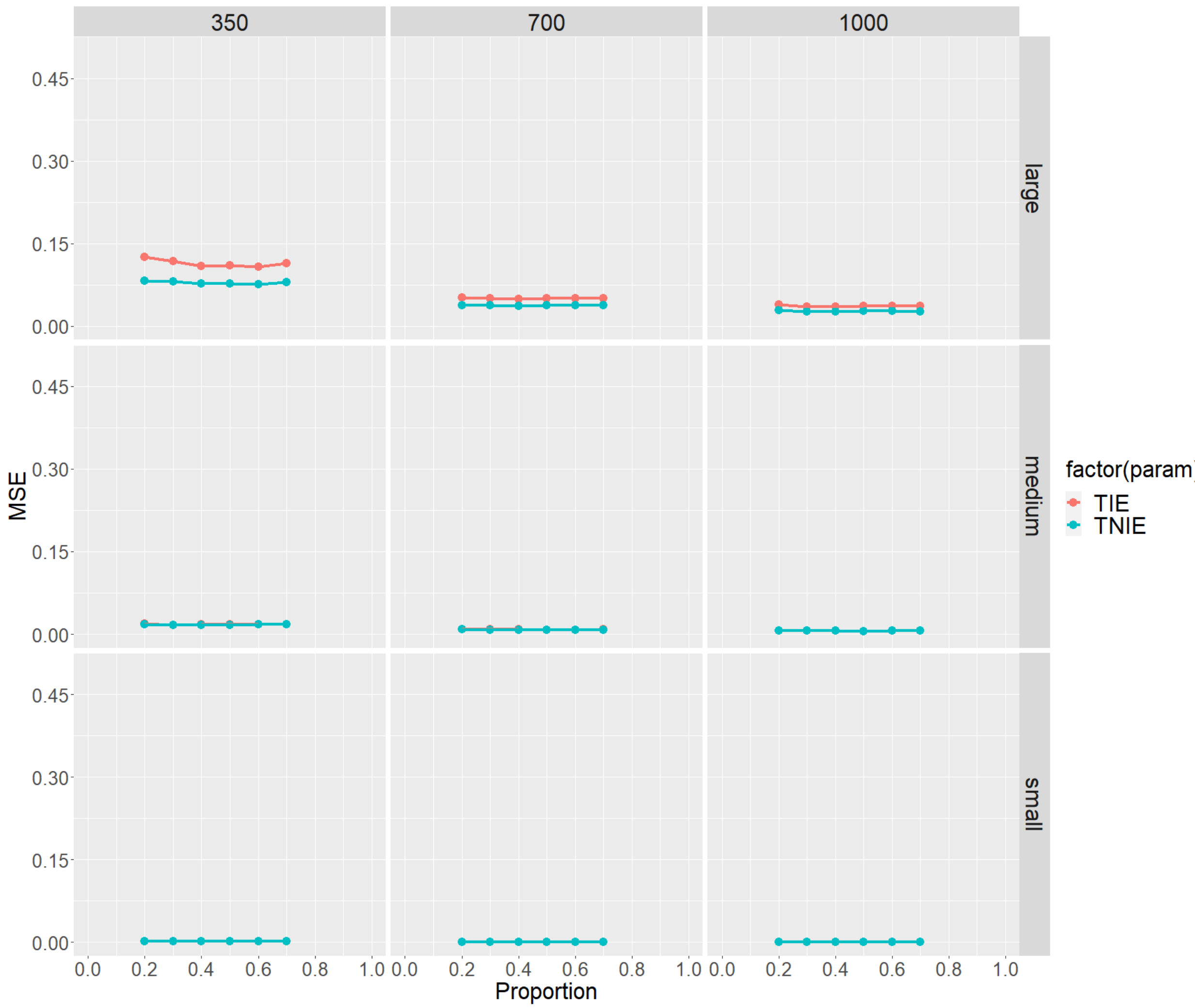
Effect Size	Sample Size	Proportions	MSE for TNIE	MSE for TIE	gap
large	350	0.06	0.0824	0.1257	-0.0433
		0.07	0.0806	0.1177	-0.0371
		0.075	0.0773	0.1088	-0.0315
		0.08	0.0771	0.1096	-0.0325
		0.09	0.0759	0.1074	-0.0315
		0.1	0.0801	0.1136	-0.0335

Table 9

The Changes in MSE under Different Proportions for Non-rare Outcome Cases in Probit

Figure 13

MSE over TIE and TNIE for Non-rare Outcomes in Probit for Simple Mediation Model



for the two quantities increased again and the gap between them continued to narrow. This pattern is detailed in Table 10. Figure 14 shows this graphically as well. The difference was positive but very small because $TNIE$ had slightly more coverage than TIE across all values of the proportion of $Y = 1$.

Non-rare outcome cases clearly showed no pattern in the difference between the two quantities in *coverage* when compared to rare outcome cases. When the proportion of $Y = 1$ increased from 0.2 to 0.3 under the same conditions with a large effect size and a sample size of 350, the *coverage* for the two quantities decreased but the gap increased. The gap was negative but small because $TNIE$ has slightly less *coverage* than TIE.

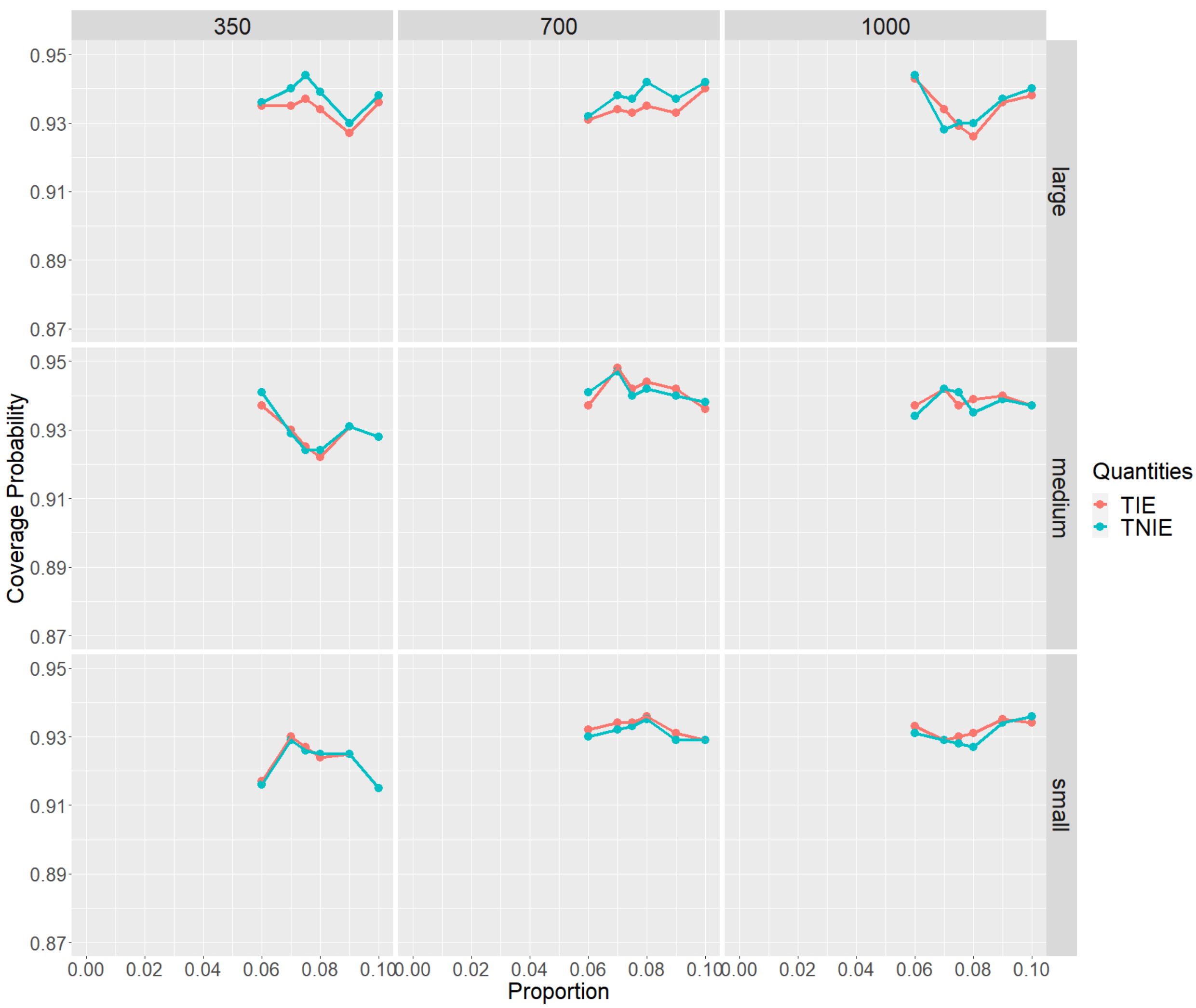
Effect Size	Sample Size	Proportions	Coverage for TNIE	Coverage for TIE	gap
large	350	0.06	0.935	0.932	0.003
		0.07	0.934	0.931	0.003
		0.075	0.939	0.931	0.008
		0.08	0.942	0.934	0.008
		0.09	0.936	0.937	-0.001
		0.1	0.942	0.94	0.002

Table 10

The Changes in Coverage under Different Proportions for Rare Outcome Cases in Probit

Figure 14

Coverage Probability over TIE and TNIE for Rare Outcomes in Probit for Simple Mediation Model



Effect Size	Sample Size	Proportions	Coverage for TNIE	Coverage for TIE	gap
large	350	0.2	0.939	0.941	-0.002
		0.3	0.922	0.927	-0.005
		0.4	0.925	0.915	0.01
		0.5	0.929	0.93	-0.001
		0.6	0.94	0.934	0.006
		0.7	0.939	0.935	0.004

Table 11

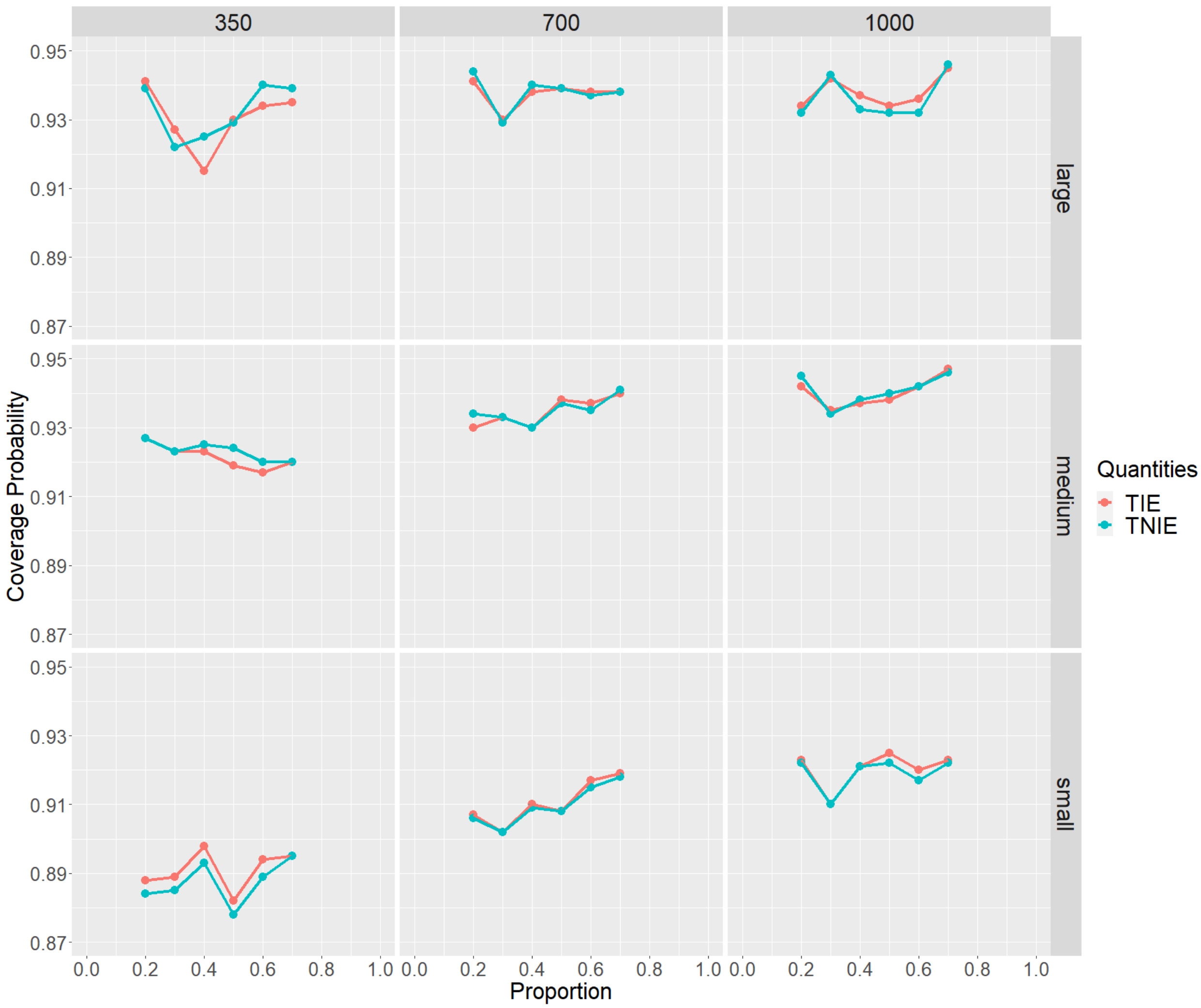
The Changes in Coverage under Different Proportions for Non-rare Outcome Cases in Probit

When the proportion was raised to 0.4, the *coverage* for $TNIE$ increased while the *coverage* for TIE decreased. Hence, the gap between them widened and became positive, because $TNIE$ has greater *coverage* than TIE. The proportion was then increased to 0.5, the *coverage* for the two quantities increased, and the gap became positive but much smaller. This meant that $TNIE$ had slightly less coverage than TIE. As the proportion approached 0.6, the *coverage* for the two quantities grew, and the gap widened and became positive. This demonstrated that $TNIE$ had greater *coverage* than TIE once more. Finally, when the proportion reached 0.7, the *coverage* for $TNIE$ dropped while the *coverage* for TIE dropped further. As a result, the gap narrowed but remained positive. This meant that $TNIE$'s *coverage* was still greater than TIE's. This pattern is depicted graphically in Figure 15 and shown numerically in Table 11.

The proportion of the time the statistical results were significant was used to calculate the $power$. When the *power* for a mediation effect is .8, it indicates that the mediation effect is unbiased. The findings of Study 1 revealed that the *power* for $TNIE$ and TIE increased as the proportion of $Y = 1$ increased in rare outcome cases. This is evident when the sample size is 350 and the effect size is large. When the proportion was 0.06, the *power* for $TNIE$ was slightly greater than the *power* for TIE, so the difference was small and positive. When the proportion was increased to 0.7, the *power* of the two quantities increased until they were equal. Therefore, the distance between them has

Figure 15

Coverage Probability over TIE and TNIE for Non-rare Outcomes in Probit for Simple Mediation Model



shrunk to zero. As the proportion increased to 0.8, the *power* of the two quantities reached 1 and the difference remained zero. Following that, regardless of the proportion's increase, the *power* for the two quantities remains 1 and the gap remains 0. This is illustrated numerically in Table 12 and graphically in Figure 14.

In non-rare cases, however, the *power* remained constant at 1 for both $TNIE$ and TIE as the proportion increased. This is illustrated graphically in Figure 17 and numerically in Table 13.

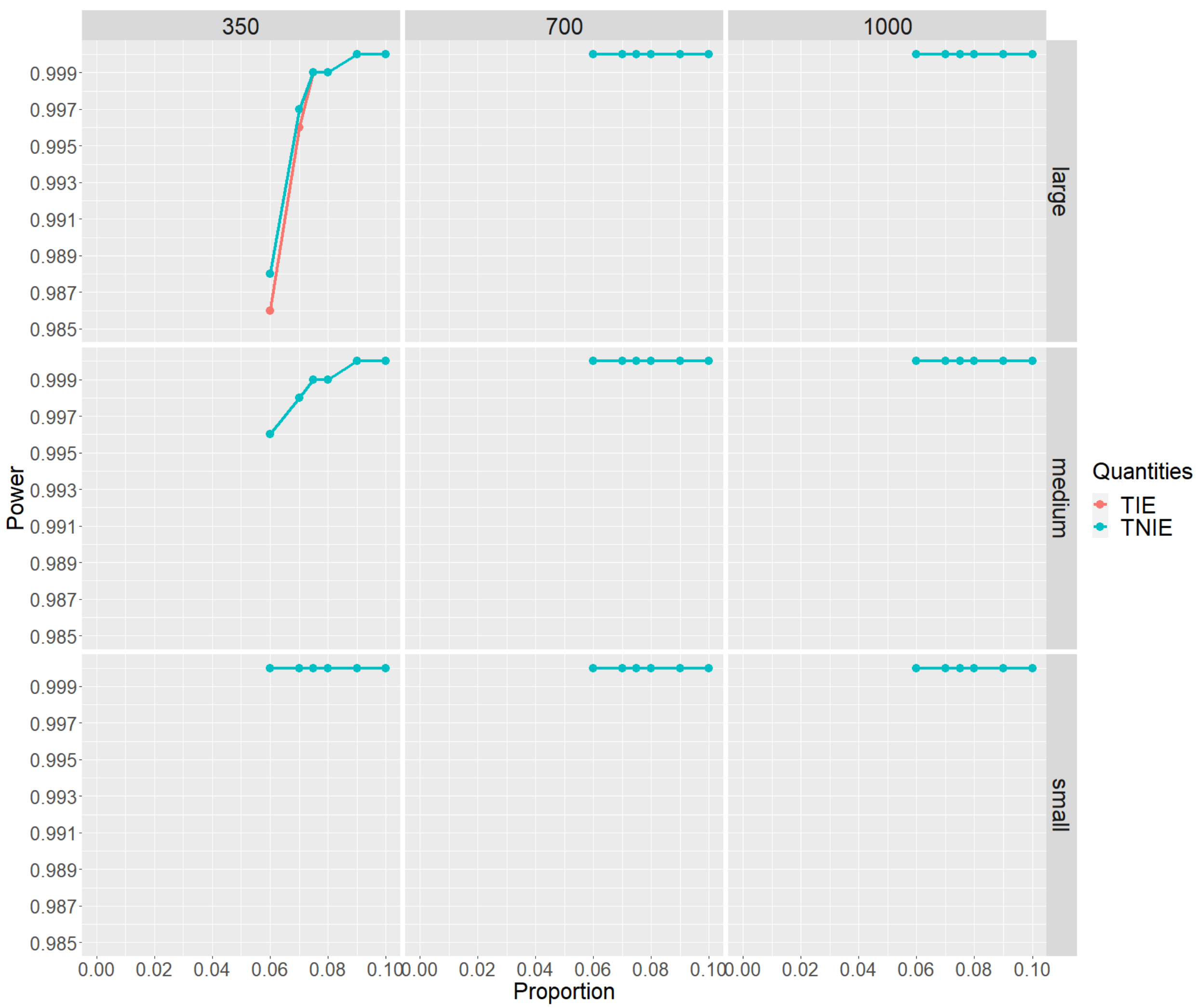
Effect Size	Sample Size	Proportions	Power for TNIE	Power for TIE	gap
large	350	0.06	0.988	0.986	0.002
		0.07	0.997	0.996	0.001
		0.075	0.999	0.999	0
		0.08	0.999	0.999	0
		0.09	1	1	0
		0.1	1	1	0

Table 12

The Changes in Power under Different Proportions for Rare Outcome Cases in Probit

Figure 16

Power over TIE and TNIE for Rare Outcomes in Probit for Simple Mediation Model



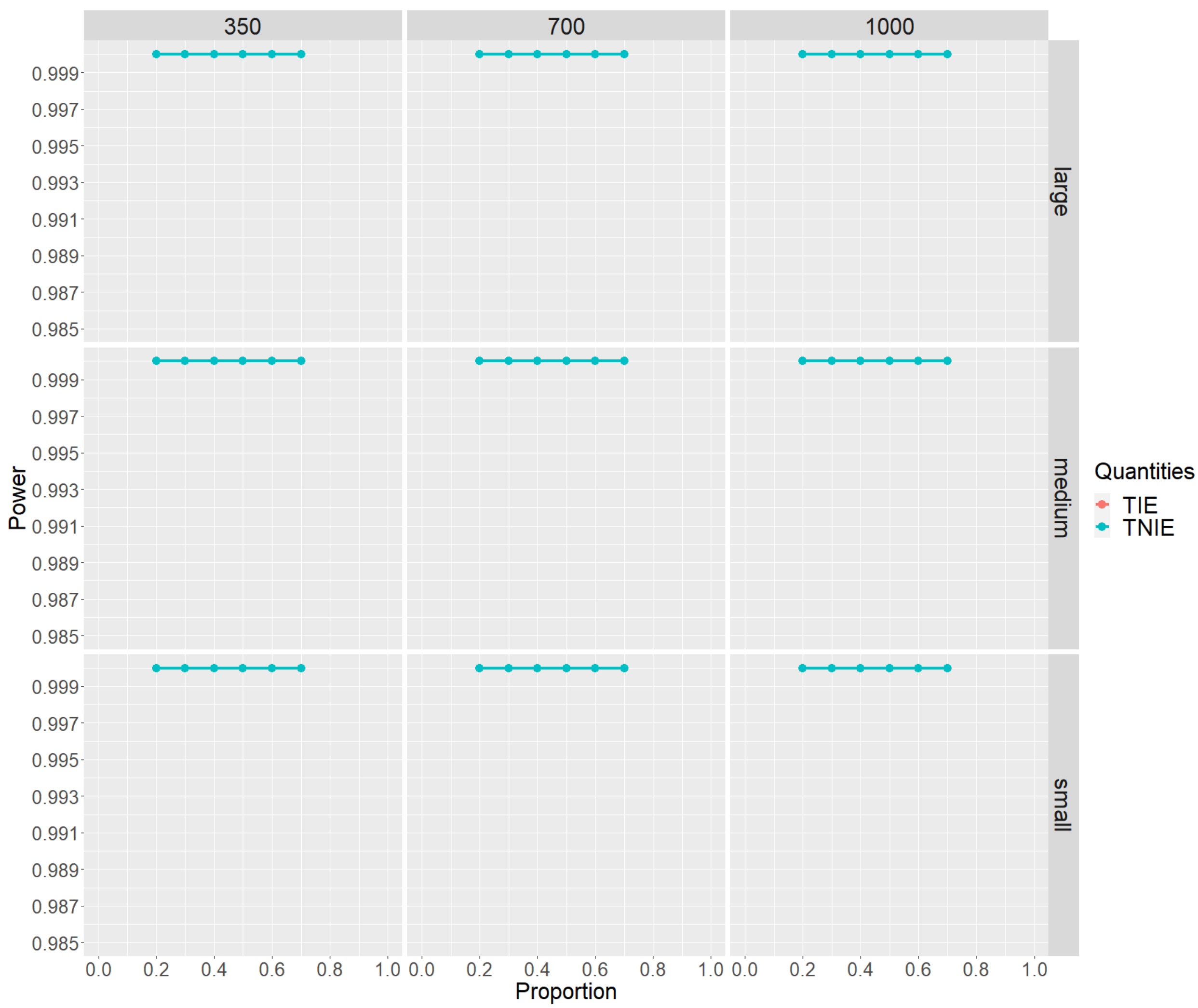
Effect Size	Sample Size	Proportions	Power for TNIE	Power for TIE	gap
large	350	0.2	1	1	0
		0.3	1	1	0
		0.4	1	1	0
		0.5	1	1	0
		0.6	1	1	0
		0.7	1	1	0

Table 13

The Changes in Power under Different Proportions for Non-rare Outcome Cases in Probit

Figure 17

Power over TIE and TNIE for Non-rare Outcomes in Probit for Simple Mediation Model



Proportion	Sample Size	Effect Size	TNIE	TIE	gap
0.06	350	small	0.9983	0.9982	1e-04
		medium	0.9841	0.9785	0.0056
		large	0.9556	0.9256	0.03

Table 14

The Changes in Bias under Different Effect Sizes for Rare Outcome Cases in Probit

Effect Size. There were three levels of effect size: small, medium, and large. When the effect size was increased, the *bias* gaps between $TNIE$ and TIE widened and the *bias* for each quantity moved further away from 1. This pattern was clearly visible in rare outcome cases when the proportion was 0.06 and the sample size was 350. The difference between the two values was positive. As a result, the *bias* for $TNIE$ was consistently smaller than *bias* for TIE as the effect size increased (for more information, see Table 14). Figure 7 shows this graphically as well.

For non-rare outcome cases, the pattern was similar but the gap between the two quantities was quite smaller than in the rare cases. Moreover, the *bias* for the two quantities in non-rare cases was closer to 1 than in rare cases. Table 15 and Figure 6 show this pattern numerically and graphically. In sum, $TNIE$ was less biased than TIE in both cases; however, the difference was shown more clearly in rare outcome cases than non-rare cases.

When the effect size increased, the gaps in SE between two quantities and SE for each quantity widened. In rare outcome cases, this is clearly visible when the proportion is 0.06 and the sample size is 350 (for more information, see Table 16 and Figure 8). The difference in SE was negative because the SE for $TNIE$ was lower than the SE for TIE. Thus, as the effect size increased, the SE for $TNIE$ was consistently smaller than the SE for TIE.

The SE for $TNIE$ and TIE was smaller in non-rare outcome cases and the gap between was also smaller (see Table 17 and Figure 9 for numerical and graphical

Proportion	Sample Size	Effect Size	TNIE	TIE	gap
0.2	350	small	0.9997	0.9996	1e-04
		medium	0.9961	0.9937	0.0024
		large	0.9877	0.9765	0.0112

Table 15

The Changes in Bias under Different Effect Size for Non-rare Outcome Cases in Probit

representations). Generally, SE for $TNIE$ was smaller than that for TIE and the gap was more visible in rare outcomes cases.

For both outcome cases, the results for ASE were similar to those for SE. Tables 18 and 19 show numerical examples of rare and non-rare outcome cases with a large effect size and a sample size of 350. Figures 10 and 11 show the ASE pattern over the changes in proportion across all conditions in rare and non-rare outcome cases.

MSE followed the same patterns as ASE and SE as the proportion of $Y = 1$ increased. This is shown in Tables 40 and 41 when the sample size is 350 and the effect size is large. Figures 21 and 22 graphically depict the pattern across all conditions. Overall, the SE, ASE, and MSE for $TNIE$ were consistently lower than those for TIE in both outcome cases across all conditions.

When the effect size changed, there was no clear pattern in the *coverage probability* for the two quantities or the gaps between them. Table 22 shows an example of a rare outcome case with a proportion of 0.06 and a sample size of 350. When the effect size was small, *bias* for $TNIE$ was slightly less than *bias* for TIE, resulting in a negative gap between them. When the effect size was medium, the *coverage* for the two quantities increased and the *coverage* for $TNIE$ increased, resulting in a positive gap. Then, as the effect size increased, the *coverage* for the two quantities decreased while the *coverage* for $TNIE$ increased; thus, the gap remained positive but shrank. Table 23 shows an illustrated example of a non-rare outcome case. When the effect size was small, the *coverage* for $TNIE$ was slightly lower than the *coverage* for TIE, resulting in a negative

Proportion	Sample Size	Effect Size	TNIE	TIE	gap
0.06	350	small	0.0471	0.0462	9e-04
		medium	0.1783	0.1944	-0.0161
		large	0.4594	0.6497	-0.1903

Table 16

The Changes in SE under Different Effect Size for Rare Outcome Cases in Probit

Proportion	Sample Size	Effect Size	TNIE	TIE	gap
0.2	350	small	0.0375	0.0366	9e-04
		medium	0.1324	0.136	-0.0036
		large	0.2862	0.3517	-0.0655

Table 17

The Changes in SE under Different Effect Size for Non-rare Outcome Cases in Probit

Proportion	Sample Size	Effect Size	TNIE	TIE	gap
0.2	350	small	0.0394	0.0384	0.001
		medium	0.1326	0.1351	-0.0025
		large	0.2833	0.3357	-0.0524

Table 18

The Changes in ASE under Different Effect Size for Rare Outcome Cases in Probit

Proportion	Sample Size	Effect Size	TNIE	TIE	gap
0.2	350	small	0.0394	0.0384	0.001
		medium	0.1326	0.1351	-0.0025
		large	0.2833	0.3357	-0.0524

Table 19

The Changes in ASE under Different Effect Size for Non-rare Outcome Cases in Probit

Proportion	Sample Size	Effect Size	TNIE	TIE	gap
0.2	350	small	0.0014	0.0013	1e-04
		medium	0.0176	0.0185	-9e-04
		large	0.0824	0.1257	-0.0433

Table 20

The Changes in MSE under Different Effect Size for Rare Outcome Cases in Probit

Proportion	Sample Size	Effect Size	TNIE	TIE	gap
0.2	350	small	0.0014	0.0013	1e-04
		medium	0.0176	0.0185	-9e-04
		large	0.0824	0.1257	-0.0433

Table 21

The Changes in MSE under Different Effect Size for Non-rare Outcome Cases in Probit

but small gap. As the effect size increased to medium, the *coverage* for the two quantities increased and became equal, resulting in a gap of 0. Then, as the effect size increased, the *coverage* for the two quantities increased, while the *coverage* for $TINE$ decreased slightly, resulting in a negative but small gap. These patterns are depicted graphically in Figures 14 and 15. To summarize, as the effect size increased, the *coverage* for $TNIE$ fluctuated between greater and lesser than the *coverage* for TIE, with no particular pattern.

In rare outcome cases, the effect size had an effect on the *power* only when the sample size was 350 and the proportion was less than 0.08. In those cases, as the effect size increased, the *power* for $TNIE$ and TIE decreased, and the *power* gap between the two increased. In all other cases, the *power* of the two quantities was 1 and remained constant. Figure 16 depicts this graphically. Table 24 shows one example under the conditions of 350 sample size and 0.06 proportion. When the effect size was small, the *power* for the two quantities was equal, and thus the gap was 0. When the effect size was medium, the *power* for the two quantities decreased and remained equal, resulting in a gap of 0. Then, as the effect size increased, the *power* for the two quantities decreased further, while the *power* for $TNIE$ increased slightly; thus, the gap was positive but small. In non-rare outcome cases, the *power* for the two quantities remained at 1 for all three sample size conditions. This pattern is depicted graphically in Figure 17.

Sample Size. As mentioned in Chapter 3, the sample size was divided into three levels: 350, 700, and 1000. The *bias* for $TNIE$ and TIE became closer to 1 as the sample size increased, and the gap between them shrank. This pattern was observed in both

Proportion	Sample Size	Effect Size	TNIE	TIE	gap
0.06	350	small	0.906	0.913	-0.007
		medium	0.929	0.927	0.002
		large	0.935	0.932	0.003

Table 22

The Changes in Coverage under Different Effect Sizes for Rare Outcome Cases in Probit

Proportion	Sample Size	Effect Size	TNIE	TIE	gap
0.2	350	small	0.884	0.888	-0.004
		medium	0.927	0.927	0
		large	0.939	0.941	-0.002

Table 23

The Changes in Coverage under Different Effect Size for Non-rare Outcome Cases in Probit

Proportion	Sample Size	Effect Size	TNIE	TIE	gap
0.06	350	small	1	1	0
		medium	0.996	0.996	0
		large	0.988	0.986	0.002

Table 24

The Changes in Power under Different Effect Size for Rare Outcome Cases in Probit

Proportion	Sample Size	Effect Size	TNIE	TIE	gap
0.2	350	small	1	1	0
		medium	1	1	0
		large	1	1	0

Table 25

The Changes in Power under Different Effect Size for Non-rare Outcome Cases in Probit

Proportion	Effect Size	Sample Size	TNIE	TIE	gap
0.06	large	350	0.9556	0.9256	0.03
		700	0.9856	0.9751	0.0105
		1000	0.9905	0.9838	0.0067

Table 26

The Changes in Bias under Different Sample Sizes for Rare Outcome Cases in Probit

outcomes. Table 26 numerically presents the pattern in rare outcome cases with large effect size and a proportion of 0.06. Under similar conditions, table 27 appears, but with a proportion of 0.2. In summary, when the sample size increased, $TNIE$ produced *bias* closer to 1 than TIE, and this was more visible in rare outcome cases.

When the sample size was increased, the difference between two quantities in SE and the SE for each quantity became smaller. This pattern can be seen clearly in rare outcome cases with a proportion of 0.2 and a large effect size (see Table 28 for numerical detail). Table 29 illustrates the pattern numerically for non-rare outcome cases.

The *coverage probability* for the two quantities and the gaps between them did not show any clear patterns when the sample size was changed. Table 30 shows one numerical example of a rare outcome case. When the sample size was 350, the *coverage* for $TNIE$ was slightly higher than that of TIE, indicating that the gap was positive. The *coverage* for $TNIE$ and TIE decreased as the sample size increased to 700, and the values for $TNIE$ remained slightly higher than those for TIE; thus, the gap remained the same. When the sample size was increased to 1000, the *coverage* for the two quantities increased, but the gap remained constant.

Table 31 shows an example of a non-rare outcome case. When the sample size was 350, the *coverage* for $TNIE$ was slightly lower than that of TIE, resulting in a negative but small gap. As the sample size increased to 700, the *coverage* for $TNIE$ and TIE increased, and the values for $TNIE$ increased more than those for TIE, resulting in a positive gap. When the sample size reached 1000, the *coverage* for the two quantities

Proportion	Effect Size	Sample Size	TNIE	TIE	gap
0.2	large	350	0.9877	0.9765	0.0112
		700	0.9944	0.99	0.0044
		1000	0.9946	0.9912	0.0034

Table 27
The Changes in Bias under Different Sample Sizes for Non-rare Outcome Cases in Probit

Proportion	Effect Size	Sample Size	TNIE	TIE	gap
0.06	large	350	0.4594	0.6497	-0.1903
		700	0.2426	0.3035	-0.0609
		1000	0.2005	0.2414	-0.0409

Table 28
The Changes in SE under Different Sample Sizes for Rare Outcome Cases in Probit

Proportion	Effect Size	Sample Size	TNIE	TIE	gap
0.2	large	350	0.2862	0.3517	-0.0655
		700	0.1958	0.2266	-0.0308
		1000	0.1702	0.196	-0.0258

Table 29
The Changes in SE under Different Sample Sizes for Non-rare Outcome Cases in Probit

Proportion	Effect Size	Sample Size	TNIE	TIE	gap
0.06	large	350	0.935	0.932	0.003
		700	0.933	0.94	-0.007
		1000	0.938	0.94	-0.002

Table 30
The Changes in Coverage under Different Sample Size for Rare Outcome Cases in Probit

Proportion	Effect Size	Sample Size	TNIE	TIE	gap
0.2	large	350	0.939	0.941	-0.002
		700	0.944	0.941	0.003
		1000	0.932	0.934	-0.002

Table 31

The Changes in Coverage under Different Sample Sizes for Non-rare Outcome Cases in Probit

decreased, but the gap became negative because the value for $TNIE$ became less than the value for TIE. Figures 14 and 15 depict these patterns graphically.

When the sample size increased from 350 to 700 in rare outcome cases, the *power* for $TNIE$ and TIE increased, as did the *power* gap between the two quantities. When the sample size increased to 700, the *power* for the two quantities reached 1 and stayed there. This is illustrated numerically in Table 32 and graphically in Figure 16.

In non-rare outcome cases, the *power* for the two quantities remained constant at 1 for all three sample size conditions. This pattern is depicted graphically in Figure 17.

Logit Model

Overall, in the logit model analysis, $TNIE$ was more accurate than TIE in recovering population values for the mediation effect in both rare and non-rare outcome cases across the first four evaluation criteria. $TNIE$, on the other hand, had slightly less *coverage* than TIE. In both outcomes cases, *power* for $TNIE$ and TIE remained at 1 for all conditions. Figure 18 represents this graphically.

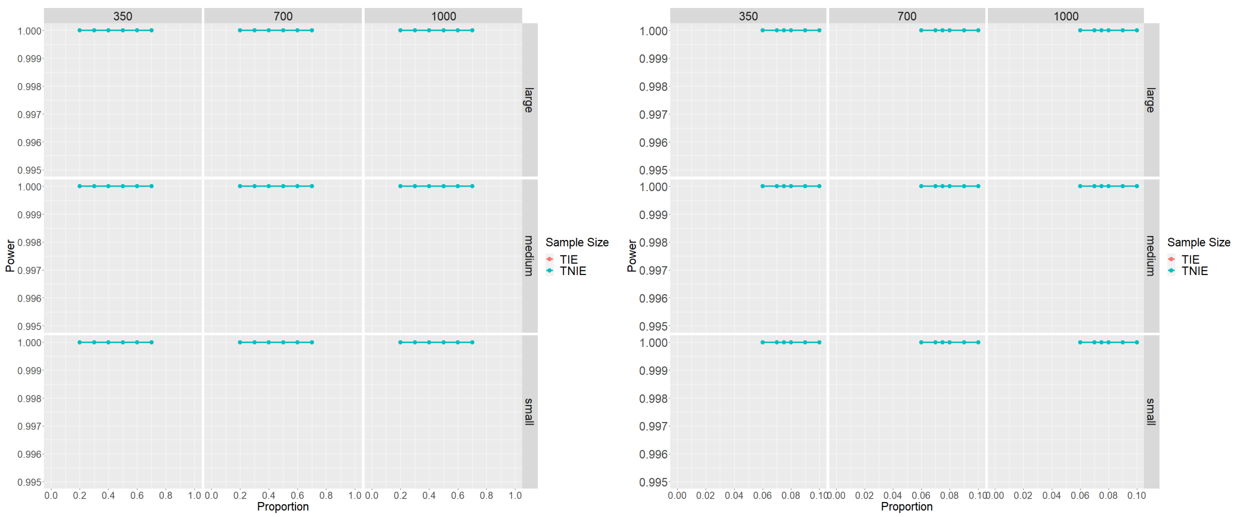
Proportion	Effect Size	Sample Size	TNIE	TIE	gap
0.06	large	350	0.988	0.986	0.002
		700	1	1	0
		1000	1	1	0

Table 32

The Changes in Power under Different Sample Sizes for Rare Outcome Cases in Probit

Figure 18

Power over TIE and TNIE for Nonrare and Rare Outcomes in Simple Mediation Model in Logit



Proportion of Y = 1. As mentioned in Chapter 3, the proportion of $Y = 1$ in the logit model also had 12 levels: six for cases with rare outcomes and six for cases with non-rare outcomes. Six evaluation criteria were used to assess the mediation effects, $TNIE$ and TIE, in relation to the proportion of $Y = 1$ cases in both rare and non-rare outcome cases.

When the proportion of $Y = 1$ in rare outcomes increased, the *bias* for $TNIE$ and TIE became closer to 1 and the gap between them shrank. The gap was positive. That is, the *bias* for $TNIE$ was closer to 1 than it was for TIE. This was evident in the conditions where the sample size was 350 and the effect size was large (for more information, see Table 34). Figure 19 graphically depicts the pattern in all conditions.

Non-rare outcome cases did not show this change. When the proportion of $Y = 1$

Proportion	Effect Size	Sample Size	Power for TNIE	Power for TIE	gap
0.2	large	350	1	1	0
		700	1	1	0
		1000	1	1	0

Table 33

The Changes in Power under Different Sample Sizes for Non-rare Outcome Cases in Probit

was 0.2 under the same conditions with a large effect size and a sample size of 350, the *bias* for $TNIE$ was closer to 1 than that for the *bias* for TIE. Thus, the difference between them was positive. When the proportion of $Y = 1$ increased from 0.2 to 0.5, the *bias* for the two quantities increased, and the gap between them shrank but remained positive. As the proportion increased to 0.6, the *bias* for the two quantities shifted further away from 1, but the gap remained small and positive. When the proportion reached 0.7, the *bias* for the two quantities shifted further away from 1, while the gap between them shrank but remained positive. This pattern is detailed in Table 35. The *bias* for $TNIE$ was consistently closer to 1 than the *bias* for TIE, as in rare outcome cases (for a graphical representation, see Figure 20). In brief overview, as the proportion of $Y = 1$ increased, the *bias* for $TNIE$ and TIE became closer to 1 in the rare outcome cases but not in the non-rare cases. Furthermore, in both outcome cases, the *bias* for $TNIE$ was always closer to 1 than the *bias* for TIE.

The SE displayed patterns similar to the *bias*, but in a different direction. The SE for the two quantities decreased as the proportion increased, as did the gap between them in rare outcome cases. This is evident under the conditions of a sample size of 350 and a large effect size. Across all proportions of $Y = 1$, the SE for $TNIE$ was always less than the SE for TIE. The difference between them was all negative, as shown in Table 36. The pattern is depicted graphically in Figure 21.

This pattern did not emerge clearly in cases with non-rare outcomes.Under the conditions of a sample size of 700 and a large effect size. The proportion of $Y = 1$ increased

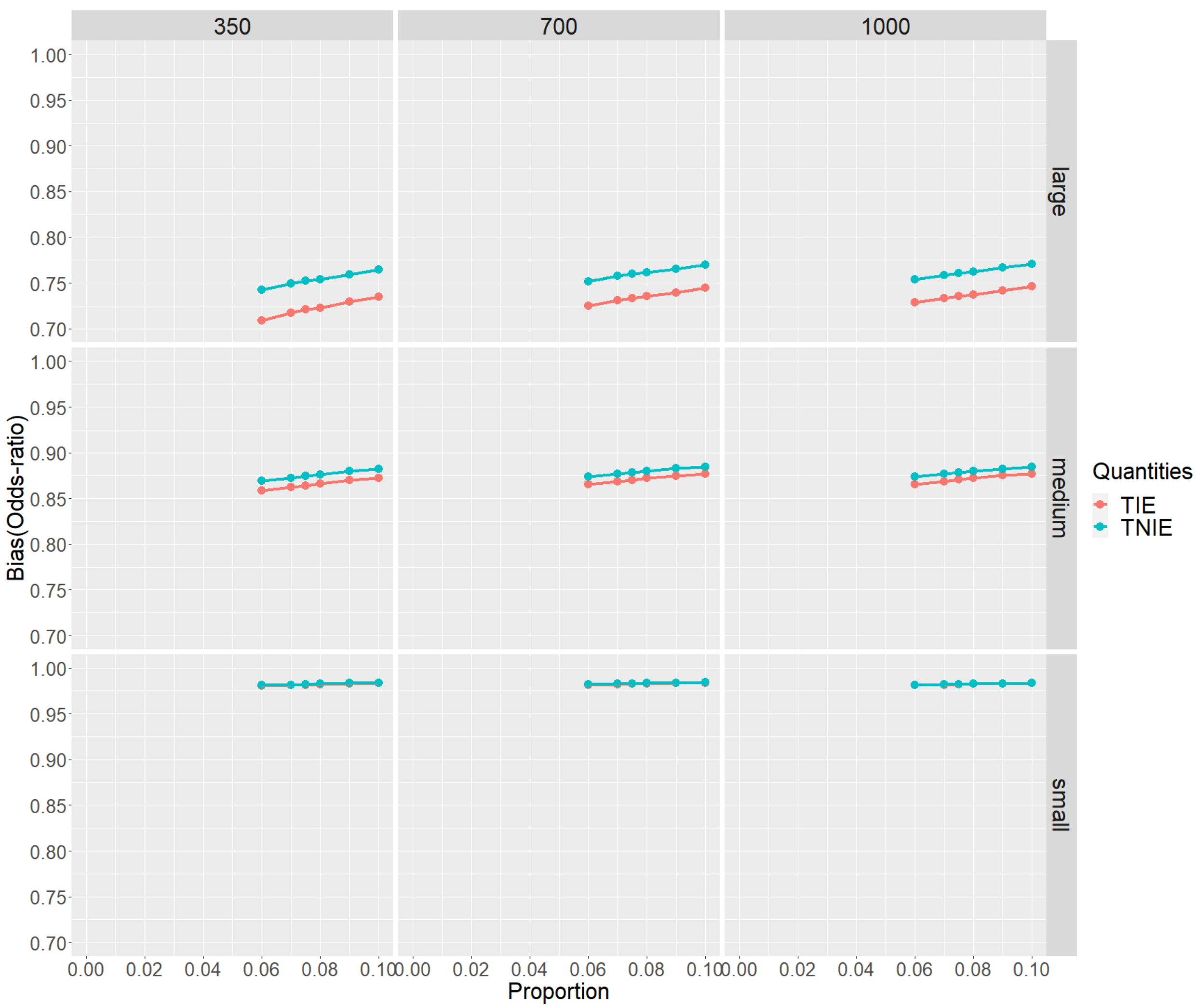
Effect Size	Sample Size	Proportions	Bias for TNIE	Bias for TIE	gap
large	350	0.06	0.7427	0.7091	0.0336
		0.07	0.7491	0.7171	0.032
		0.075	0.7526	0.7209	0.0317
		0.08	0.754	0.7228	0.0312
		0.09	0.7596	0.7292	0.0304
		0.1	0.7644	0.7348	0.0296

Table 34

The Change in Bias under Different Proportion for Rare Outcome Cases in Logit

Figure 19

Bias over TIE and TNIE for Rare Outcomes in Logit for Simple Mediation Model



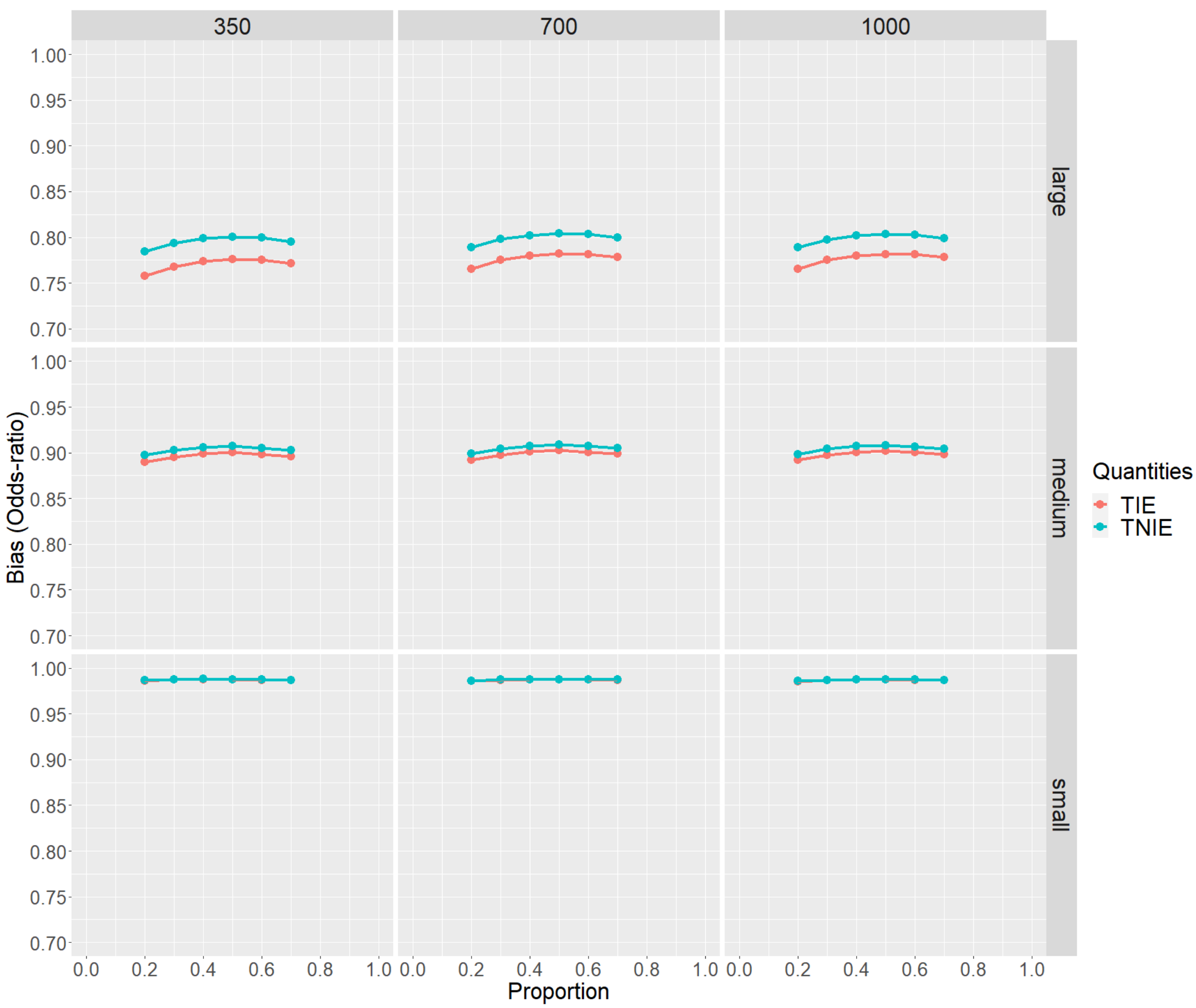
Effect Size	Sample Size	Proportions	Bias for TNIE	Bias for TIE	gap
large	350	0.2	0.7847	0.7581	0.0266
		0.3	0.7934	0.7678	0.0256
		0.4	0.7987	0.7738	0.0249
		0.5	0.8006	0.776	0.0246
		0.6	0.7996	0.7756	0.024
		0.7	0.7953	0.7715	0.0238

Table 35

The Change in Bias under Different Proportion for Non-rare Outcome Cases in Logit

Figure 20

Bias over TIE and TNIE for Non-rare Outcomes in logit for Simple Mediation Model



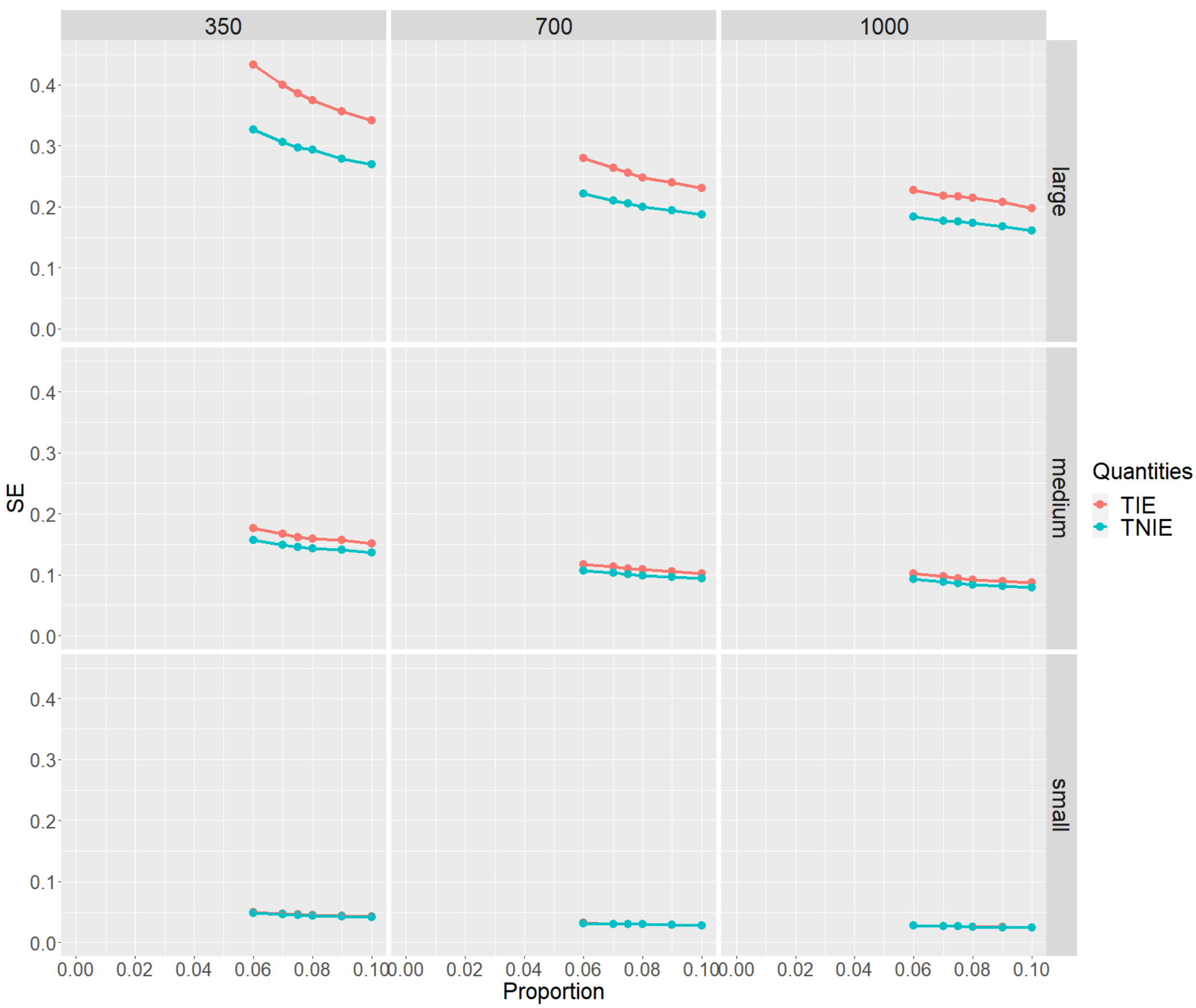
Effect Size	Sample Size	Proportions	SE for TNIE	SE for TIE	gap
large	350	0.06	0.3269	0.4332	-0.1063
		0.07	0.306	0.3995	-0.0935
		0.075	0.2973	0.3858	-0.0885
		0.08	0.293	0.3746	-0.0816
		0.09	0.2792	0.3569	-0.0777
		0.1	0.27	0.342	-0.072

Table 36

The Change in SE under Different Proportion for Rare Outcome Cases in Logit

Figure 21

SE over TIE and TNIE for Rare Outcomes in Logit for Simple Mediation Model



Effect Size	Sample Size	Proportions	SE for TNIE	SE for TIE	gap
large	700	0.2	0.1651	0.1979	-0.0328
		0.3	0.1606	0.1922	-0.0316
		0.4	0.1564	0.1862	-0.0298
		0.5	0.1574	0.1875	-0.0301
		0.6	0.1563	0.1861	-0.0298
		0.7	0.1567	0.1854	-0.0287

Table 37

The Changes in SE under Different Proportion for Non-rare Outcome Cases in Logit

from 0.2 to 0.4, the SE for the two quantities decreased, and the gap shrunk. As the proportion approached 0.5, the SE for the two quantities increased, as did the difference between them. The pattern changed once more when the proportion reached 0.6, with the SE for the two quantities and the gap decreasing. Finally, when the proportion reached 0.7, the SE for the two quantities and the difference between them began to shrink. This pattern is numerically represented in Table 37. Figure 22 depicts this graphically.

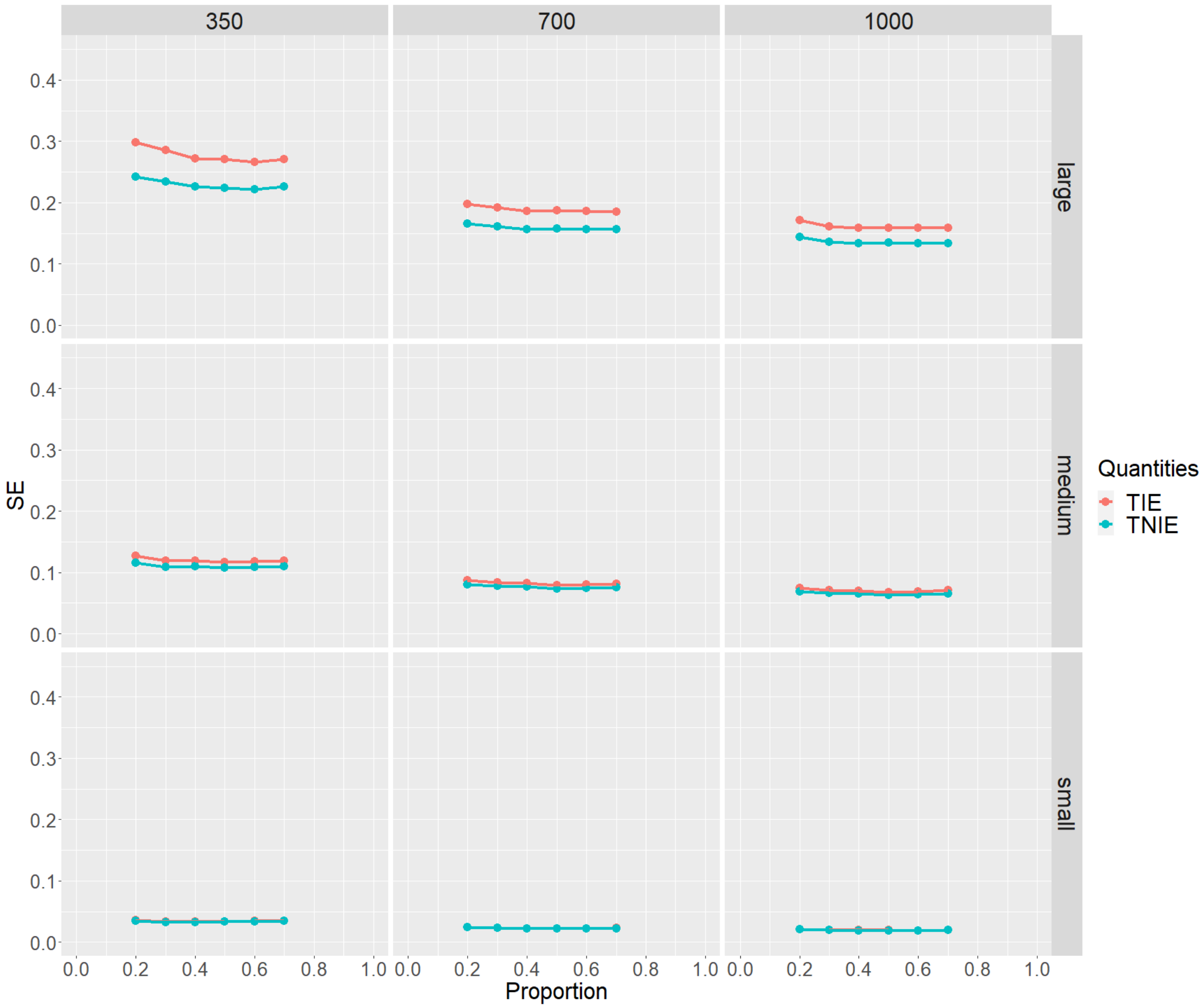
The ASE results followed a similar pattern to the SE results for both rare and non-rare outcome cases. Table 38 presents numerically one example under conditions with a large effect size and a sample size of 350, whereas Figure 23 graphically presents the pattern across all conditions. Similarly, the pattern for non-rare outcome cases is presented in Table 39 and Figure 24.

MSE followed the same patterns as ASE and SE as the proportion of $Y = 1$ increased. This is shown in Tables 40 and 41 when the sample size is 350 and the effect size is large. The pattern is depicted graphically in Figures 21 and 22. Overall, the SE, ASE, and MSE for $TNIE$ were consistently lower than those for TIE in both outcome cases across all conditions.

The results did not show any clear patterns for *coverage* as the proportion of $Y = 1$ in rare and non-rare cases increased. For example, when the sample size was 350 and the effect size was large, the proportion of $Y = 1$ increased from 0.06 to 0.07, the *coverage* for the two quantities decreased and the gap between them decreased. Then, as the proportion

Figure 22

SE over TIE and TNIE for Non-Rare Outcomes in logit for Simple Mediation Model



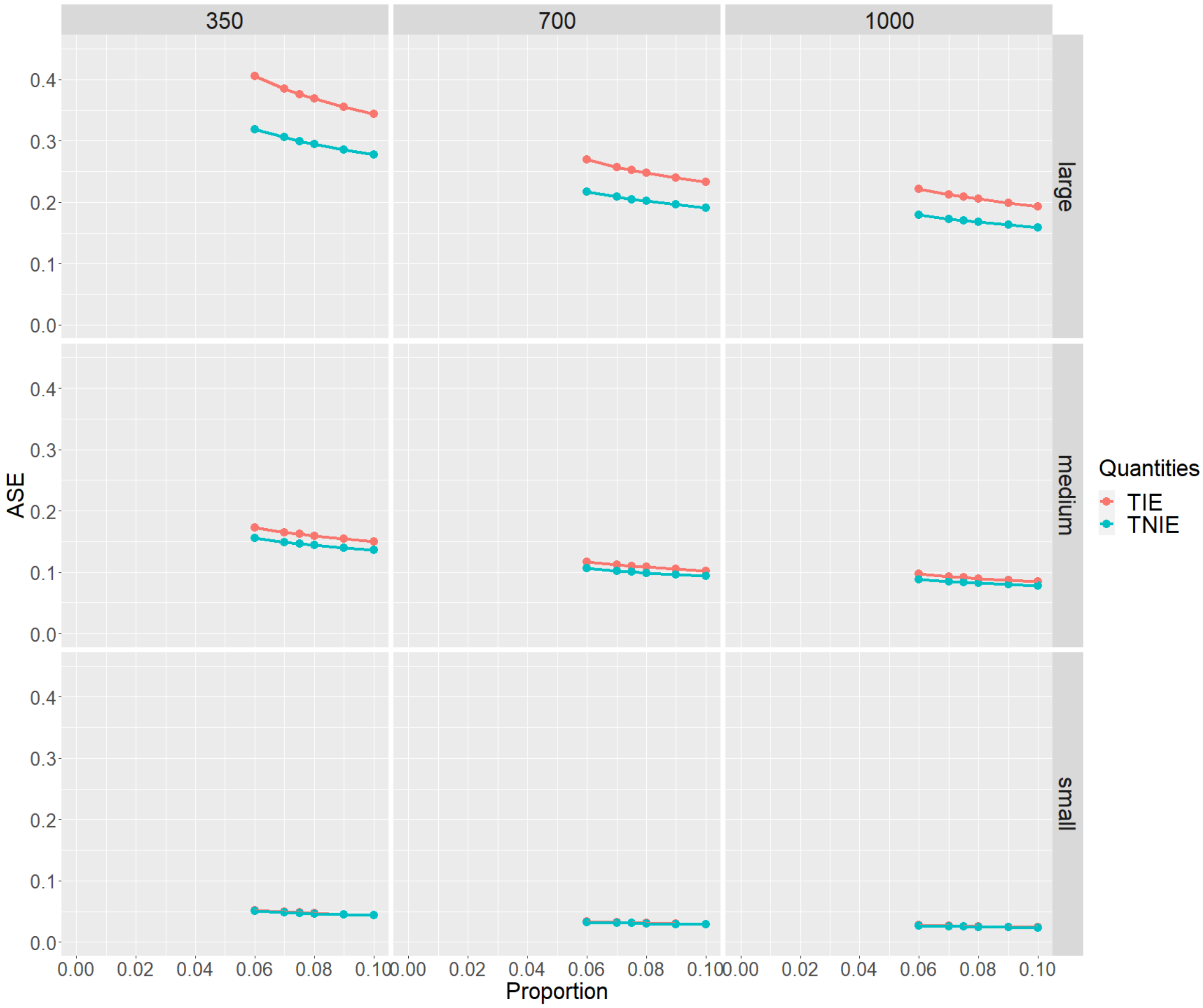
Effect Size	Sample Size	Proportions	ASE for TNIE	ASE for TIE	gap
large	350	0.06	0.3192	0.4058	-0.0866
		0.07	0.3059	0.385	-0.0791
		0.075	0.2993	0.3759	-0.0766
		0.08	0.2949	0.3692	-0.0743
		0.09	0.2851	0.3552	-0.0701
		0.1	0.2771	0.3435	-0.0664

Table 38

The Changes in ASE under Different Proportion in Rare Outcome Cases for Simple Mediation Model in Logit

Figure 23

ASE over TIE and TNIE for Rare Outcomes in logit for Simple Mediation Model

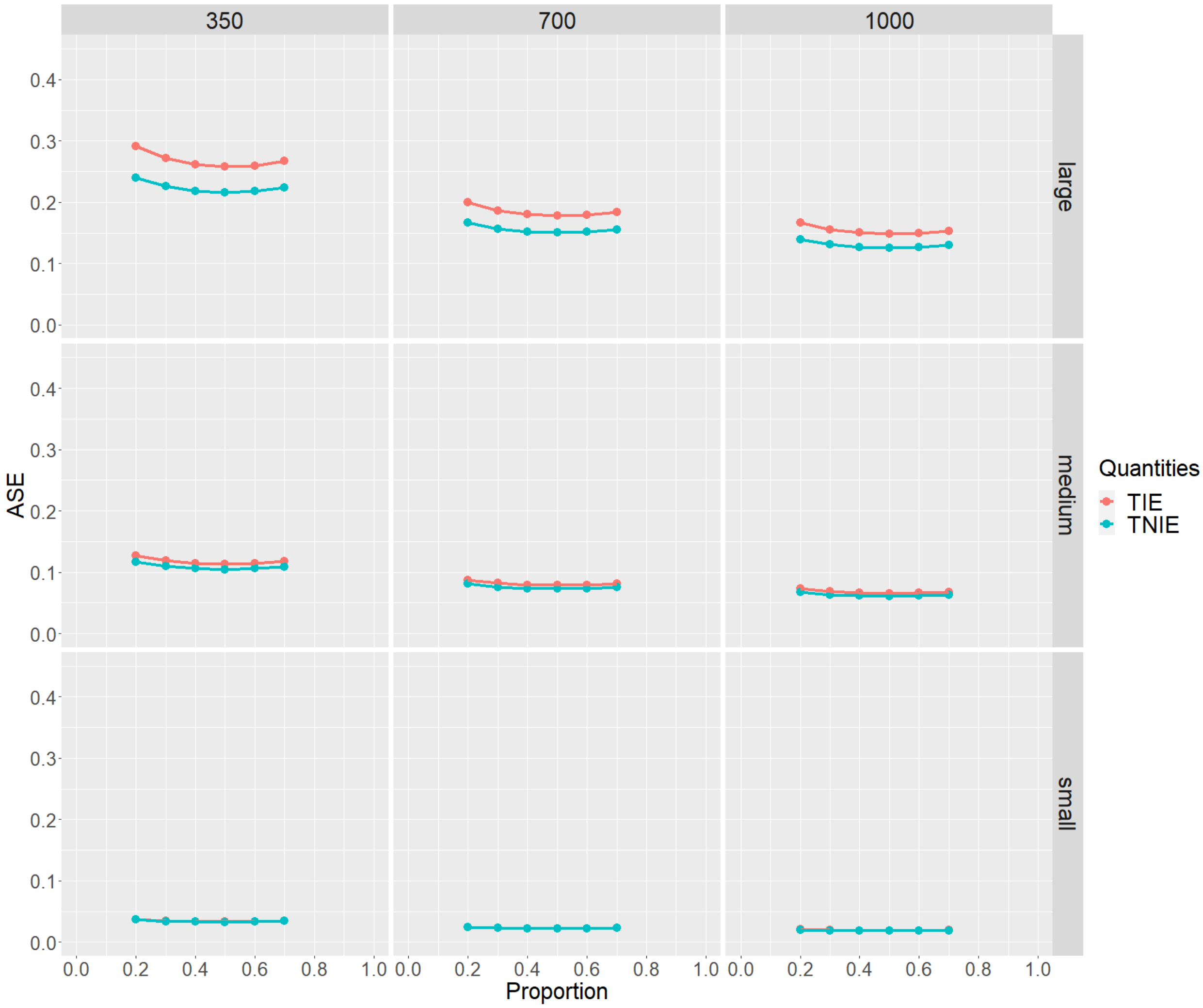


Effect Size	Sample Size	Proportions	ASE for TNIE	ASE for TIE	gap
large	350	0.06	0.2401	0.2911	-0.051
		0.07	0.2258	0.2715	-0.0457
		0.075	0.2184	0.2613	-0.0429
		0.08	0.216	0.2578	-0.0418
		0.09	0.2178	0.2592	-0.0414
		0.1	0.2241	0.2669	-0.0428

Table 39

The Changes in ASE under Different Proportion for Non-rare Outcome Cases in Logit

Figure 24
ASE over TIE and TNIE for Rare Outcomes in Logit for Simple Mediation Model

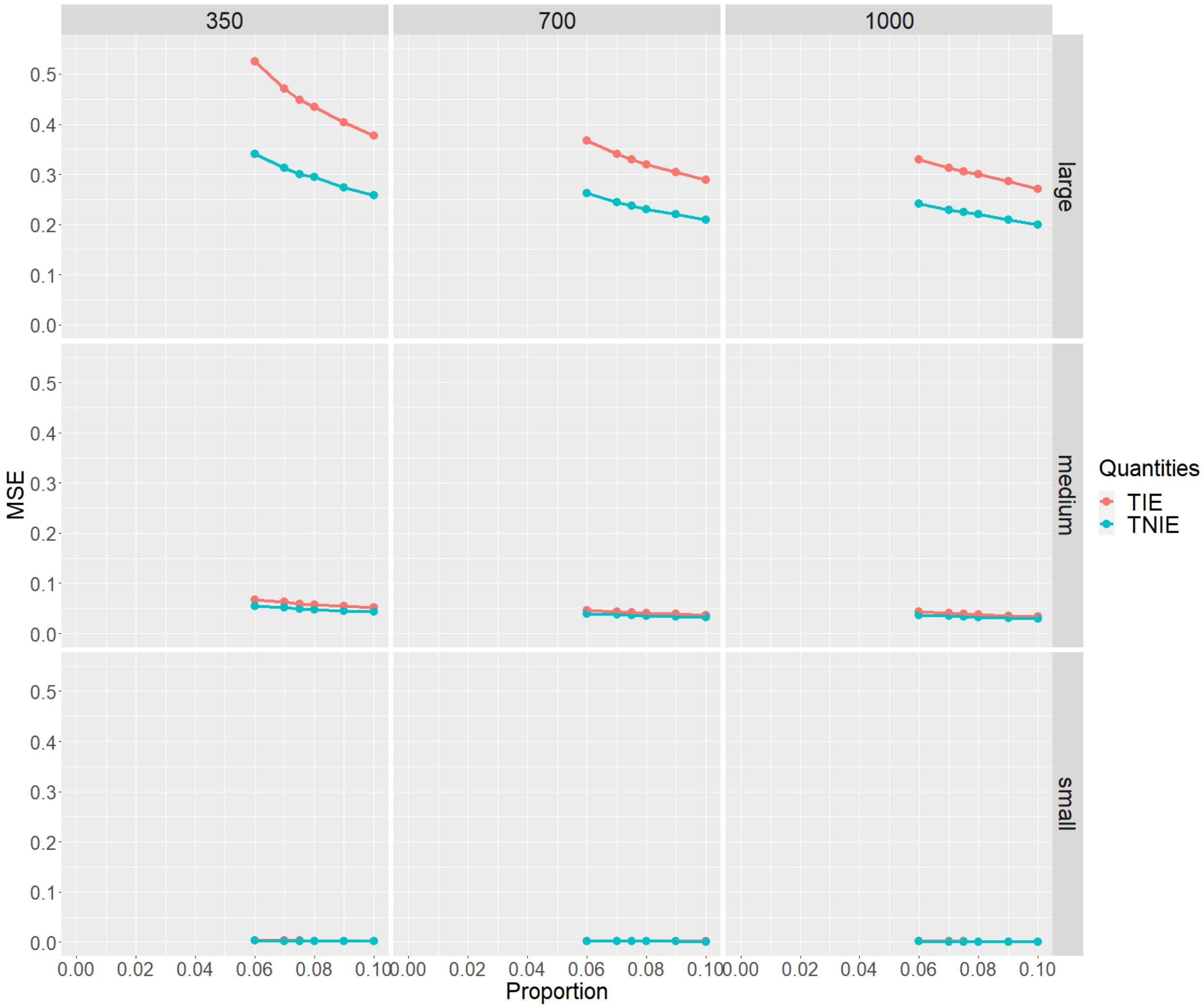


Effect Size	Sample Size	Proportions	MSE for TNIE	MSE for TIE	gap
large	350	0.06	0.3413	0.5245	-0.1832
		0.07	0.3129	0.4711	-0.1582
		0.075	0.2996	0.4488	-0.1492
		0.08	0.2939	0.4347	-0.1408
		0.09	0.2737	0.4034	-0.1297
		0.1	0.2586	0.3776	-0.119

Table 40
The Changes in MSE under Different Proportion for Rare Outcome Cases in Logit

Figure 25

MSE over TIE and TNIE for Rare Outcomes in Logit for Simple Mediation Model



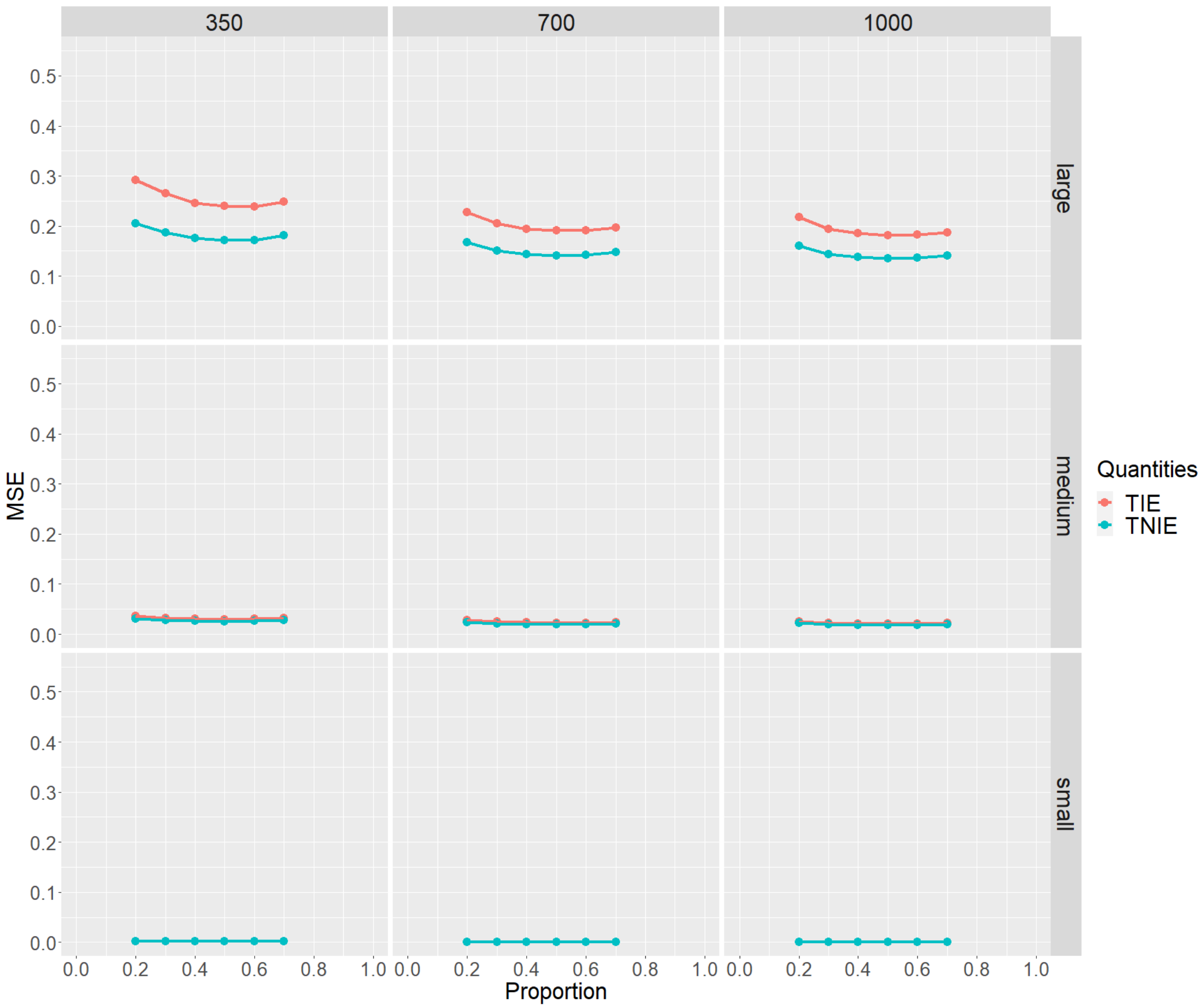
Effect Size	Sample Size	Proportions	MSE for TNIE	MSE for TIE	gap
large	350	0.06	0.2056	0.2923	-0.0867
		0.07	0.187	0.2645	-0.0775
		0.075	0.1753	0.2452	-0.0699
		0.08	0.1716	0.24	-0.0684
		0.09	0.1719	0.238	-0.0661
		0.1	0.1807	0.2485	-0.0678

Table 41

The Change for MSE under Different Proportion for Non-rare Outcome Cases in Logit

Figure 26

MSE over TIE and TNIE for Non-rare Outcomes in Logit for Simple Mediation Model



increased to 0.09, the *coverage* for $TNIE$ increased while the *coverage* for TIE decreased. Subsequently, the gap between them shrank. When the proportion reached 0.1, the *coverage* for $TNIE$ increased again while the *coverage* for TIE decreased, but the gap between them narrowed further. This pattern is detailed in Table 42. Figure 27 depicts this graphically for all conditions. The gap was negative. That is, the *coverage* for $TNIE$ was less than the *coverage* for TIE for all values of the proportion of $Y = 1$.

There was no pattern in the gap between the two quantities in *coverage* in non-rare outcome cases. When the proportion of $Y = 1$ increased from 0.2 to 0.3 under the same conditions as the rare outcome cases with a large effect size and a sample size of 350, the

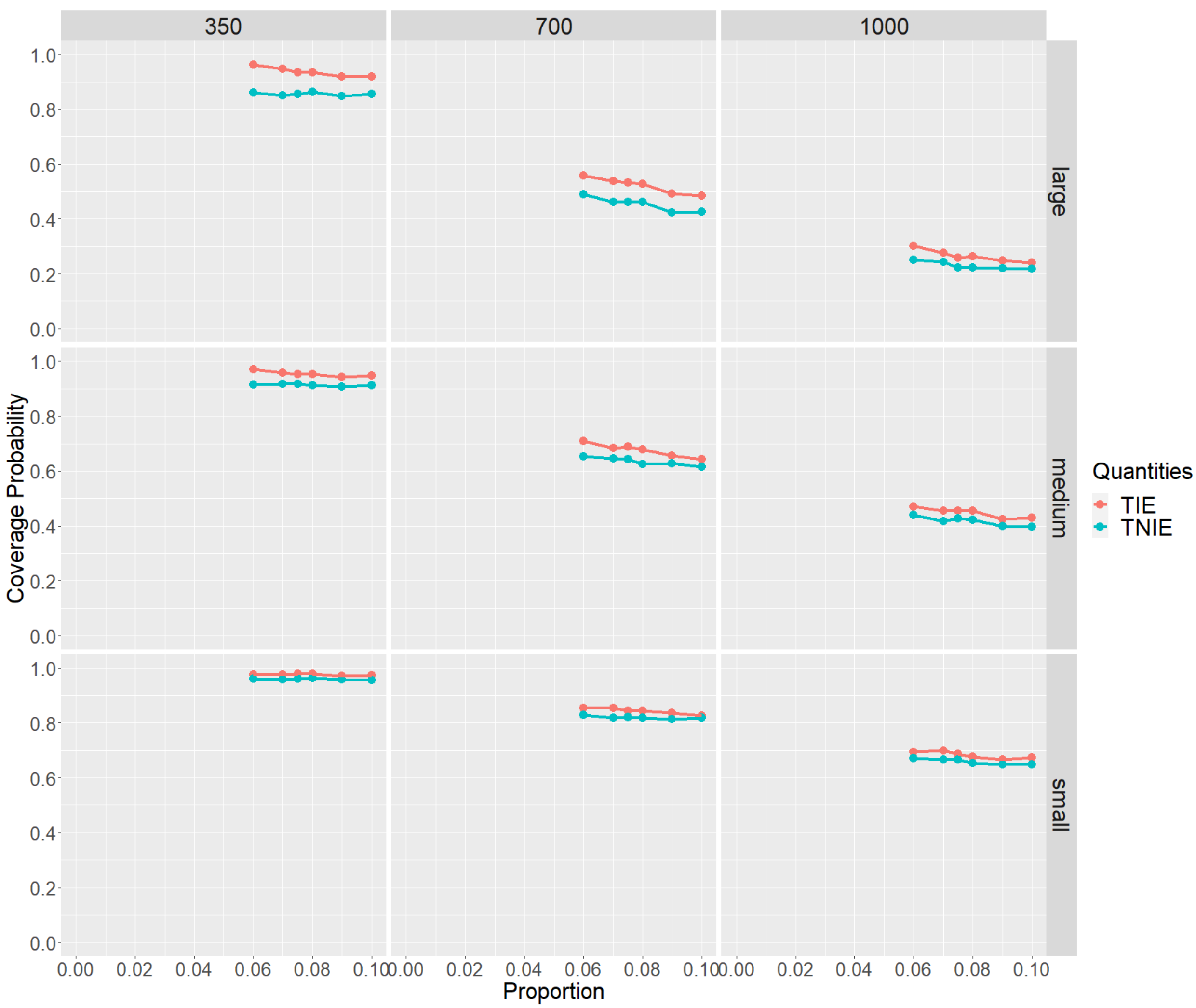
Effect Size	Sample Size	Proportions	Coverage for TNIE	Coverage for TIE	gap
large	350	0.06	0.842	0.96	-0.118
		0.07	0.835	0.936	-0.101
		0.075	0.83	0.936	-0.106
		0.08	0.824	0.923	-0.099
		0.09	0.82	0.921	-0.101
		0.1	0.826	0.917	-0.091

Table 42

The Changes in Coverage under Different Proportion for Rare Outcome Cases in Logit

Figure 27

Coverage logability over TIE and TNIE for Rare Outcomes in Logit for Simple Mediation Model



Effect Size	Sample Size	Proportions	Coverage for TNIE	Coverage for TIE	gap
large	350	0.2	0.785	0.84	-0.055
		0.3	0.76	0.82	-0.06
		0.4	0.752	0.811	-0.059
		0.5	0.756	0.806	-0.05
		0.6	0.763	0.812	-0.049
		0.7	0.76	0.824	-0.064

Table 43

The Changes in Coverage under Different Proportion for Non-rare Outcome Cases in Logit

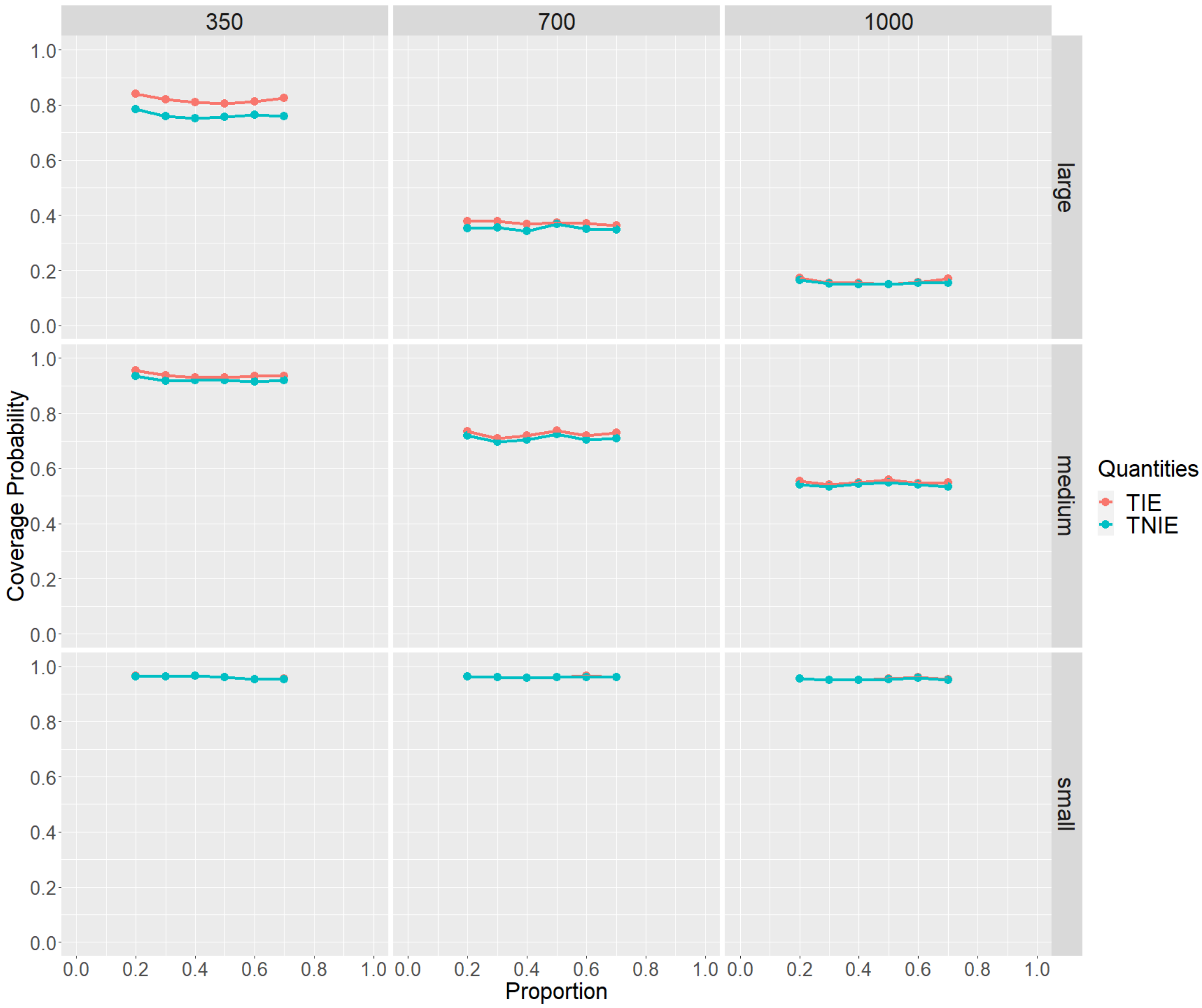
coverage for the two quantities decreased but the gap increased. When the proportion raised to 0.4, the *coverage* for $TNIE$ decreased while the *coverage* for TIE increased. Thus, the gap between them has shrunk slightly. The proportion then increased to 0.5, and the *coverage* for $TNIE$ increased while the *coverage* for TIE decreased, as did the gap. The *coverage* for the two quantities increased as the proportion approached 0.6, but the gap continued decreasing. Finally, once the proportion reached 0.7, the *coverage* for $TNIE$ decreased while the *coverage* for TIE increased. As a result, the gap widened. All of the gaps were positive. This meant that the *coverage* for $TNIE$ was less than the *coverage* for TIE across all values for the proportion of $Y = 1$. For all conditions, this pattern is shown in Table 43 and graphically in Figure 28.

Effect Size. When the effect size increased, gaps in the *bias* between $TNIE$ and TIE widened and the *bias* for each quantity moved further away from 1. This pattern was clearly visible in rare outcome cases when the proportion was 0.06 and the sample size was 350. All of the differences between the two quantities were positive. That is, the *bias* for $TNIE$ was greater than the *bias* for TIE at all levels of effect size (for more information, see Table 44). Figure 19 shows this graphically as well.

The pattern was similar for non-rare outcome cases, but the difference between the two quantities was smaller than in rare cases. Furthermore, in non-rare cases, the *bias* for the two quantities was closer to 1, and the gaps were smaller than in rare cases. This pattern is shown numerically in Table 45 for conditions with a sample size of 350 and a

Figure 28

Coverage logability over TIE and TNIE for Non-rare Outcomes in Logit for Simple Mediation Model



Proportion	Sample Size	Effect Size	Bias for TNIE	Bias for TIE	gap
0.06	350	small	0.9812	0.9806	6e-04
		medium	0.8693	0.8586	0.0107
		large	0.7427	0.7091	0.0336

Table 44

The Changes in Bias under Different Effect Size for Rare Outcome Cases in Logit

Proportion	Sample Size	Effect Size	Bias for TNIE	Bias for TIE	gap
0.2	350	small	0.9865	0.9861	4e-04
		medium	0.8974	0.8894	0.008
		large	0.7847	0.7581	0.0266

Table 45

The Changes in Bias under Different Effect Size for Non-rare Outcome Cases in Logit

proportion of 0.2, and graphically in Figure 20 for all conditions. In conclusion, $TNIE$ was less biased than TIE in both cases, with the gap being more visible in rare outcome cases than non-rare cases.

When the effect size increased, the gaps in SE between two quantities and SE for each quantity widened. In rare outcome cases, this is clearly visible under the proportion of 0.06 and a sample size of 350 (for numerical and graphical details, see Table 46 and Figure 21). The gaps in the SE were all negative. As the effect size increased, the SE for $TNIE$ became smaller than that for TIE.

The SE for $TNIE$ and TIE was smaller in non-rare outcome cases, as was the gap between (for numerical and graphical representations, see Table 47 and Figure 22). In general, SE for $TNIE$ was consistently lower than that for TIE, and the gap was more noticeable in rare outcome cases.

In both outcome cases, the ASE for the two quantities followed the same pattern as the SE. Tables 48 and 49 show numerical results with a sample size of 350 and a proportion of.06 for a rare outcom case and.2 for a non-rare case. Figures 23 and 24 show the pattern in all conditions for graphical representations.

In both outcome cases, the same pattern was seen for MSE for the two quantities. Figures 25 and 26 graphically depict the pattern in all conditions. Tables 50 and 51 show examples under conditions with large sample size and a proportion of.06 and.2 for rare and non-rare outcome cases, respectively.

When the effect size changed, the $coverage\, probability$ for the two quantities shrank

Proportion	Sample Size	Effect Size	SE for TNIE	SE for TIE	gap
0.06	350	small	0.0482	0.0493	-0.0011
		medium	0.1564	0.1757	-0.0193
		large	0.3269	0.4332	-0.1063

Table 46

The Changes in SE under Different Effect Size for Rare Outcome Cases in Logit

Proportion	Sample Size	Effect Size	SE for TNIE	SE for TIE	gap
0.2	350	small	0.0351	0.0357	-6e-04
		medium	0.116	0.127	-0.011
		large	0.2418	0.2978	-0.056

Table 47

The Changes in SE under Different Effect Sizes for Non-rare Outcome Cases in Logit

Proportion	Sample Size	Effect Size	ASE for TNIE	ASE for TIE	gap
0.06	350	small	0.0504	0.0515	-0.0011
		medium	0.1553	0.1726	-0.0173
		large	0.3192	0.4058	-0.0866

Table 48

The Changes in ASE under Different Effect Sizes for Rare Outcome Cases in Logit

Proportion	Sample Size	Effect Size	ASE for TNIE	ASE for TIE	gap
0.2	350	small	0.0367	0.0373	-6e-04
		medium	0.1163	0.1265	-0.0102
		large	0.2401	0.2911	-0.051

Table 49

The Changes in ASE under Different Effect Sizes for Non-rare Outcome Cases in Logit

Proportion	Sample Size	Effect Size	MSE for TNIE	MSE for TIE	gap
0.06	350	small	0.0027	0.0028	-1e-04
		medium	0.055	0.0675	-0.0125
		large	0.3413	0.5245	-0.1832

Table 50

The Changes in MSE under Different Effect Sizes for Rare Outcome Cases in Logit

Proportion	Sample Size	Effect Size	MSE for TNIE	MSE for TIE	gap
0.2	350	small	0.0014	0.0015	-1e-04
		medium	0.0312	0.037	-0.0058
		large	0.2056	0.2923	-0.0867

Table 51

The Changes in MSE under Different Effect Sizes for Non-rare Outcome Cases in Logit

and the gap between them widened. This pattern was evident in both outcomes. Table 52 shows an example of a rare outcome case with a proportion of 0.06 and a sample size of 350. Table 53 shows an illustrated example of a non-rare outcome case. These patterns are depicted graphically in Figures 27 and 28. Furthermore, because the gaps were negative across the two tables, it indicated that as the effect size increased, the *coverage* for $TNIE$ was less than the *coverage* for TIE.

Sample Size. In the rare outcome cases, the *bias* for $TNIE$ and TIE became closer to 1 as the sample size increased, and the gap between them shrank. Figure 19 depicts this pattern graphically for all conditions in rare outcome cases. Table 54 numerically presents the pattern under conditions with a large effect size and a proportion of 0.06.

This change did not appear clearly in all conditions for non-rare outcome cases (see Figure 20 for a graphical representation)). For example, when the sample size increased from 350 to 700 under conditions with a large effect size and a proportion of 0.4, the *bias* for $TNIE$ and TIE became closer to 1 and the gap between them shrank. Then, as the sample size increased to 1000, the *bias* for $TNIE$ and TIE moved further away from 1, but the gap between them still shrank. This pattern is numerically represented in Table 55. In general, as the sample size increased for both outcome cases, the *bias* for $TNIE$ got closer to 1 than the *bias* for TIE.

When the sample size increased, the SE for the two quantities and the gap between them shrank. Figures 21 and 22 depict this pattern graphically. Table 56 illustrates this

Proportion	Sample Size	Effect Size	Coverage for TNIE	Coverage for TIE	gap
0.06	350	small	0.982	0.982	0
		medium	0.948	0.977	-0.029
		large	0.842	0.96	-0.118

Table 52

The Changes in Coverage under Different Effect Sizes for Rare Outcome Cases in Logit

Proportion	Sample Size	Effect Size	Coverage for TNIE	Coverage for TIE	gap
0.2	350	small	0.964	0.966	-0.002
		medium	0.935	0.955	-0.02
		large	0.785	0.84	-0.055

Table 53

The Changes in Coverage under Different Effect Sizes for Non-rare Outcome Cases in Logit

Proportion	Effect Size	Sample Size	Bias for TNIE	Bias for TIE	gap
0.06	large	350	0.7427	0.7091	0.0336
		700	0.7518	0.7247	0.0271
		1000	0.7542	0.7287	0.0255

Table 54

The Changes in Bias under Different Sample Sizes for Rare Outcome Cases in Logit

Proportion	Effect Size	Sample Size	Bias for TNIE	Bias for TIE	gap
0.4	large	350	0.7987	0.7738	0.0249
		700	0.8023	0.7797	0.0226
		1000	0.8016	0.7795	0.0221

Table 55

The Changes in Bias under Different Sample Sizes for Non-rare Outcome Cases in Logit

Proportion	Effect Size	Sample Size	SE for TNIE	SE for TIE	gap
0.06	large	350	0.3269	0.4332	-0.1063
		700	0.2213	0.28	-0.0587
		1000	0.1837	0.2275	-0.0438

Table 56

The Changes in SE under Different Sample Sizes for Rare Outcome Cases in Logit

pattern in the rare outcome cases with a proportion of 0.06 and a large effect size. The pattern was similar in the non-rare outcome cases, but the gap was smaller. The conditions when the effect size was large and the proportion was 0.2 are numerically illustrated in Table 57. In total, the gaps between the two quantities were all negative. That is, the SE for $TNIE$ was consistently lower than the SE for TIE across all levels of the sample size in both outcome cases.

The ASE for $TNIE$ and TIE decreased as the sample size increased, as did the gap between them. Figures 23 and 24 graphically depict this pattern across all conditions. Tables 58 and 59 numerically emphasize the pattern under conditions with a large effect size and proportions of 0.06 for a rare outcome case and 0.2 for a non-rare case, respectively. In both tables, the gaps were negative. This indicated that the ASE for $TNIE$ was lower than the ASE for TIE under all levels of the sample size.

A similar pattern was shown for MSE. Figures 25 and 26 graphically depict this pattern across all conditions, whereas Tables 60 and 61 emphasize it numerically under conditions with a large effect size and a proportion of. 06 for a rare outcome case, and .02 for a non-rare outcome case.

The $coverage\, probabilities$ for the two quantities decreased as the sample size increased, as did the gap between them in both outcome cases. Furthermore, with a large effect size and a sample size of 1000, the $coverage$ for both quantities became extremely small (<.2). This pattern is numerically delineated in Tables 62 and 63. This pattern can also be seen across all conditions in Figures 27 and 28. Overall, $TNIE$ coverage was less

Proportion	Effect Size	Sample Size	TNIE	TIE	gap
0.2	large	350	0.2418	0.2978	-0.056
		700	0.1651	0.1979	-0.0328
		1000	0.1436	0.1714	-0.0278

Table 57

The Changes in SE under Different Sample Sizes for Non-rare Outcome Cases in Logit

Proportion	Effect Size	Sample Size	ASE for TNIE	ASE for TIE	gap
0.06	large	350	0.3192	0.4058	-0.0866
		700	0.2171	0.2691	-0.052
		1000	0.1795	0.2212	-0.0417

Table 58

The Changes in ASE under Different Sample Sizes for Rare Outcome Cases in Logit

Proportion	Effect Size	Sample Size	ASE for TNIE	ASE for TIE	gap
0.2	large	350	0.2401	0.2911	-0.051
		700	0.1664	0.1993	-0.0329
		1000	0.1388	0.1661	-0.0273

Table 59

The Changes in ASE under Different Sample Sizes for Non-rare Outcome Cases in Logit

Proportion	Effect Size	Sample Size	MSE for TNIE	MSE for TIE	gap
0.06	large	350	0.3413	0.5245	-0.1832
		700	0.2621	0.3673	-0.1052
		1000	0.2414	0.3292	-0.0878

Table 60

The Changes in MSE under Different Sample Sizes for Rare Outcome Cases in Logit

Proportion	Effect Size	Sample Size	MSE for TNIE	MSE for TIE	gap
0.2	large	350	0.2056	0.2923	-0.0867
		700	0.167	0.227	-0.06
		1000	0.161	0.217	-0.056

Table 61

The Changes in MSE under Different Sample Sizes for Non-rare Outcome Cases in Logit

Proportion	Effect Size	Sample Size	Coverage for TNIE	Coverage for TIE	gap
0.06	large	350	0.842	0.96	-0.118
		700	0.422	0.52	-0.098
		1000	0.198	0.241	-0.043

Table 62

The Changes in Coverage under Different Sample Sizes for Rare Outcome Cases in Logit

than TIE coverage, as evidenced by negative gaps.

Study 2: Moderated Mediation Model 2

Similar to the findings of Study 1, the results of Study 2 showed that the causal inference approach estimates mediation effects more accurately than the classical approach in both rare and non-rare outcome cases, as well as in probit and logit models. Furthermore, for the probit model, the causal inference approach was clearly superior to the classical approach in non-rare outcome cases, whereas for the logit model, the causal inference approach was clearly superior to the classical approach in rare outcome cases.

Probit Model

Overall, in the probit model analysis, $TNIE$ was more accurate than TIE in recovering the population values of the mediation effect in both rare and non-rare outcome cases. The gap between $TNIE$ and TIE was much smaller in rare outcome cases than in non-rare outcome cases.

Proportion of Y = 1. When the proportion increased in rare outcome cases with medium and a large effect sizes, the *bias* for $TNIE$ and TIE got closer to 1 and the gap between them got smaller. The gap was positive because the *bias* for $TNIE$ was closer to 1 than the *bias* for TIE. This was evident in the conditions where the sample size was 350 and the effect size was large (for more information, see Table 64). Figure 29 depicts this graphically. Furthermore, under small effect size conditions, the *bias* for the two quantities was very close to 1. As a result, there was no discernible pattern in the changes

Proportion	Effect Size	Sample Size	Coverage for TNIE	Coverage for TIE	gap
0.2	large	350	0.785	0.84	-0.055
		700	0.352	0.378	-0.026
		1000	0.165	0.173	-0.008

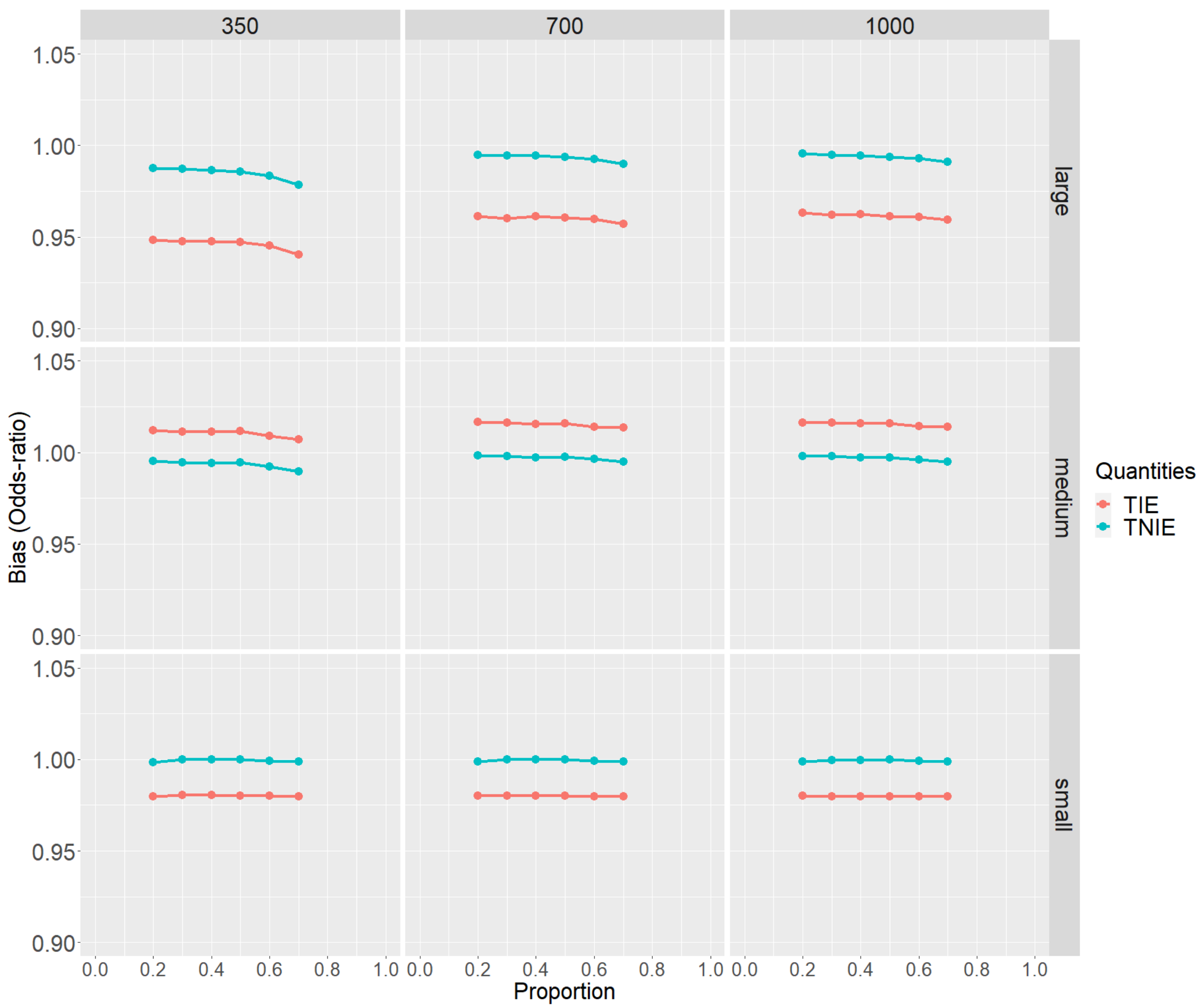
Table 63

The Changes in Coverage under Different Sample Sizes for Non-rare Outcome Cases in Logit

in *bias* as the proportion increased (for numerical details, see Table 65). Despite the fact that some of the differences between the two quantities were negative, the values were very small. In those conditions, the *bias* for the two quantities was nearly identical.

Figure 30

Bias over TIE and TNIE in Non-rare Outcome Cases in Probit with Moderation Effect



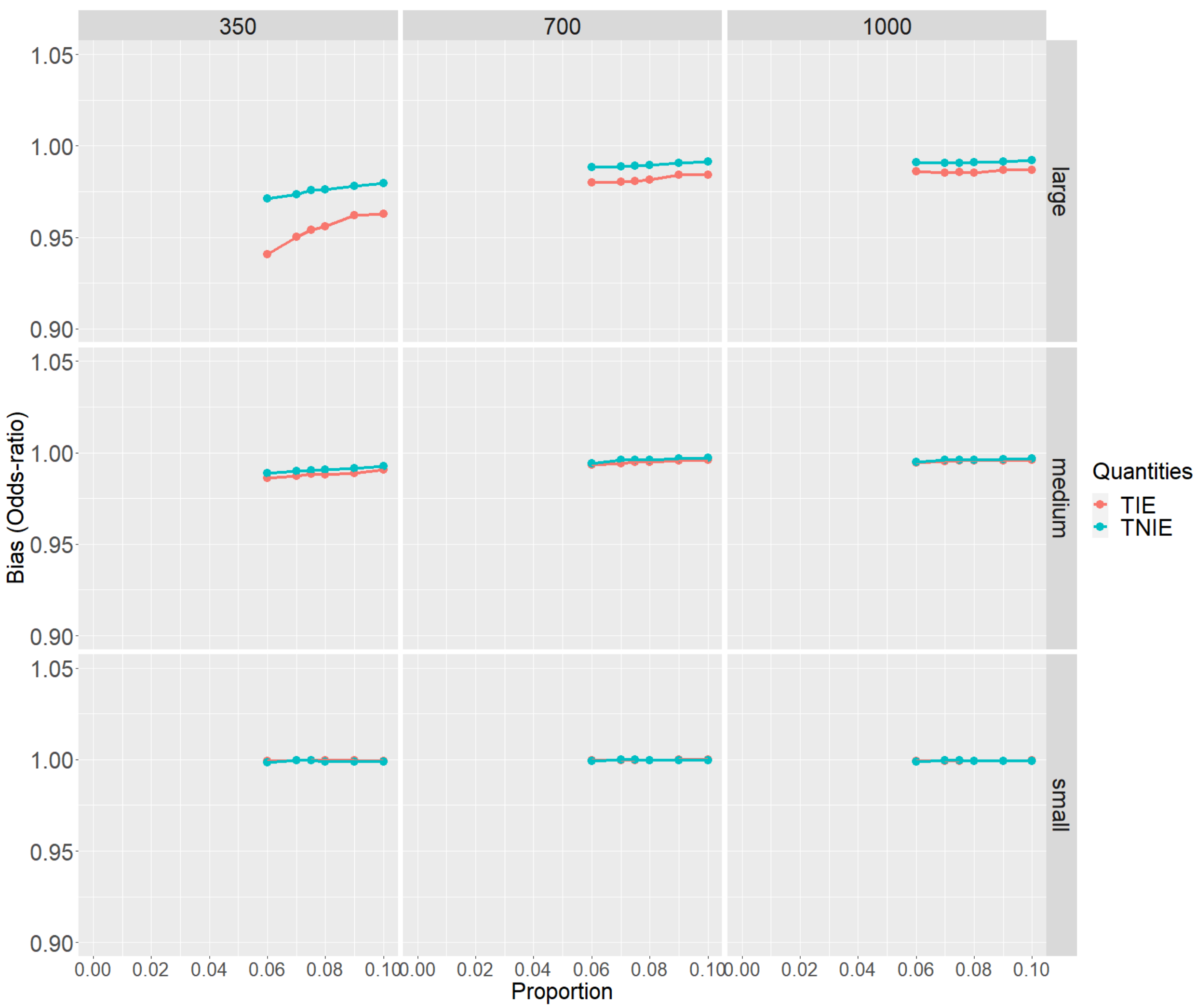
Effect Size	Sample Size	Proportions	Bias for TNIE	Bias for TIE	gap
large	350	0.06	0.9712	0.9406	0.0306
		0.07	0.9736	0.9503	0.0233
		0.075	0.9758	0.954	0.0218
		0.08	0.976	0.9561	0.0199
		0.09	0.978	0.9622	0.0158
		0.1	0.9795	0.9627	0.0168

Table 64

The Changes in Bias under Different Proportions for Rare Outcome Cases in Probit with Moderation Effect

Figure 29

Bias over TIE and TNIE for Rare Outcome Cases in Probit with Moderation Effect



Effect Size	Sample Size	Proportions	Bias for TNIE	Bias for TIE	gap
small	350	0.06	0.9983	0.9993	-0.001
		0.07	0.9996	0.9997	-1e-04
		0.075	0.9994	0.9996	-2e-04
		0.08	0.999	0.9994	-4e-04
		0.09	0.999	0.9994	-4e-04
		0.1	0.9987	0.9993	-6e-04
	700	0.06	0.9991	0.9995	-4e-04
		0.07	0.9998	0.9995	3e-04
		0.075	0.9998	0.9997	1e-04
		0.08	0.9997	0.9997	0
		0.09	0.9996	0.9998	-2e-04
		0.1	0.9994	0.9998	-4e-04
	1000	0.06	0.9991	0.9995	-4e-04
		0.07	0.9998	0.9995	3e-04
		0.075	0.9998	0.9997	1e-04
		0.08	0.9997	0.9997	0
		0.09	0.9996	0.9998	-2e-04
		0.1	0.9994	0.9998	-4e-04

Table 65

The Changes in Bias under Small Effect Size across Proportions for Rare Outcome Cases in Probit with Moderation Effect

Effect Size	Sample Size	Proportions	Bias for TNIE	Bias for TIE	gap
large	350	0.2	0.9877	0.9484	0.0393
		0.3	0.9873	0.9477	0.0396
		0.4	0.9864	0.9476	0.0388
		0.5	0.9856	0.9473	0.0383
		0.6	0.9834	0.9453	0.0381
		0.7	0.9783	0.9403	0.038

Table 66

The Changes in Bias under Different Proportions for Non-rare Outcome Cases in Probit with Moderation Effect

Effect Size	Sample Size	Proportions	SE for TNIE	SE for TIE	gap
large	350	0.06	0.2712	0.3943	-0.1231
		0.07	0.2608	0.3659	-0.1051
		0.075	0.2536	0.3524	-0.0988
		0.08	0.2503	0.3344	-0.0841
		0.09	0.2419	0.3096	-0.0677
		0.1	0.2335	0.3105	-0.077

Table 67

The Changes in SE under Different Proportions for Rare Outcome Cases in Probit with Moderation Effect

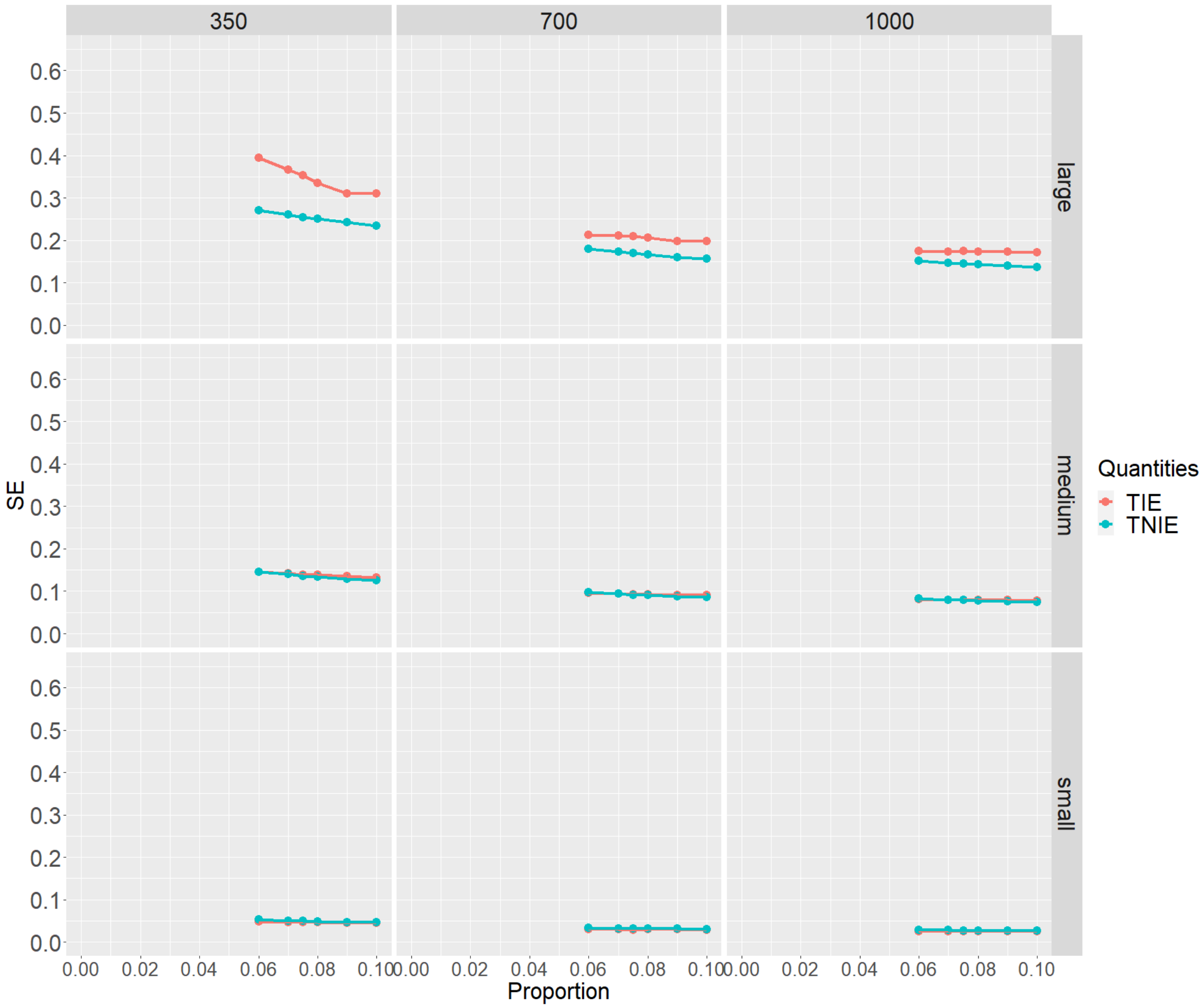
In the non-rare outcome cases, as the proportion of $Y = 1$ increased, the *bias* for the two quantities shifted away from 1 and the gap remained relatively constant. This pattern was detailed in Table 66. Similarly to rare outcome cases, the *bias* for $TNIE$ was closer to 1 than the *bias* for TIE at all proportion levels (for a graphical representation, see Figure 30). In summary, as the proportion of $Y = 1$ increased, the *bias* for $TNIE$ and TIE increased in the rare outcome cases but decreased in the non-rare cases. Furthermore, while the *bias* for $TNIE$ was nearly identical to or got closer to 1 than the *bias* for TIE in both outcome cases, the difference between the two quantities was much larger in non-rare outcome cases.

The results revealed that the SE followed similar patterns to the *bias*, but in a different direction. The SE for the two quantities decreased as the proportion increased, as did the gap between them in rare outcome cases. Across all proportions of $Y = 1$, the SE for $TNIE$ was always less than the SE for TIE. The gaps were all negative, as shown in Table 67. The pattern was depicted graphically in Figure 31.

This pattern did not emerge clearly in cases with the non-rare outcomes. The numerical conditions with a sample size of 350 and large effect size are presented in Table 68. The SE for $TNIE$ increased as the proportion of $Y = 1$ increased, whereas the SE for TIE did not show a clear pattern of change. Figure 32 shows this graphically as well.

Figure 31

SE over TIE and TNIE for Rare Outcome Cases in Probit with Moderation Effect



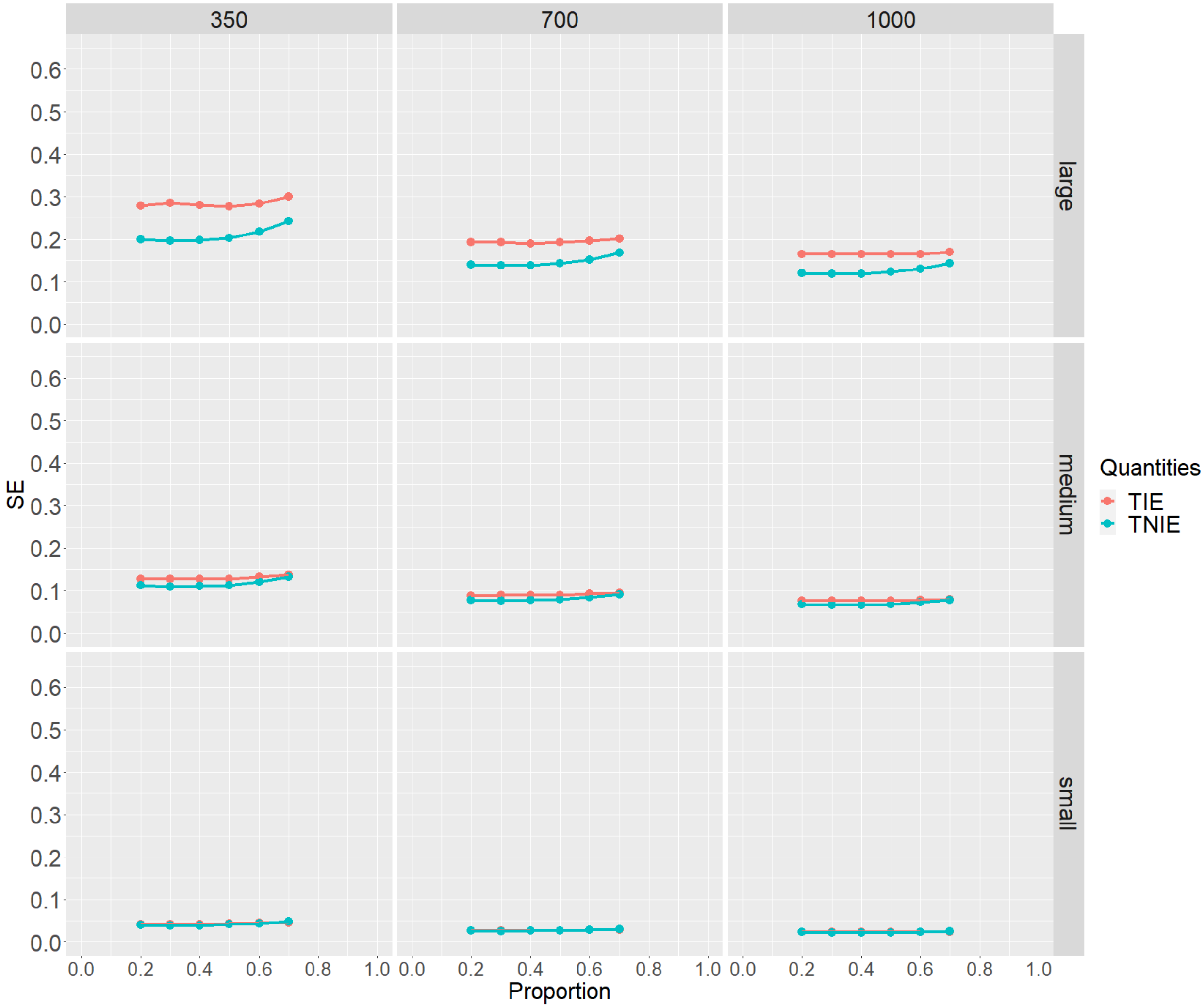
Effect Size	Sample Size	Proportions	SE for TNIE	SE for TIE	gap
large	350	0.2	0.1996	0.279	-0.0794
		0.3	0.1967	0.2845	-0.0878
		0.4	0.1973	0.28	-0.0827
		0.5	0.2027	0.2778	-0.0751
		0.6	0.2169	0.2842	-0.0673
		0.7	0.243	0.2995	-0.0565

Table 68

The Changes in SE under Different Proportions in Non-rare Outcome Cases in Probit with Moderation Effect

Figure 32

SE over TIE and TNIE in Non-Rare Outcome Cases in Probit with Moderation Effect



Under conditions with a sample size of 350, there was no clear pattern for ASE as the proportion of $Y = 1$ increased in the rare outcome cases. In other conditions, as the proportion of $Y = 1$ increased, the ASE for the two quantities decreased, and the gap between them increased. Furthermore, the ASE for $TNIE$ was lower than the ASE for TIE at all levels of the proportion. Table 69 numerically depicts the pattern under conditions with large effect size and a sample size of 700. The pattern is depicted graphically in Figure 33.

Under all levels of the proportion of $Y = 1$, the pattern of the ASE was clearer in

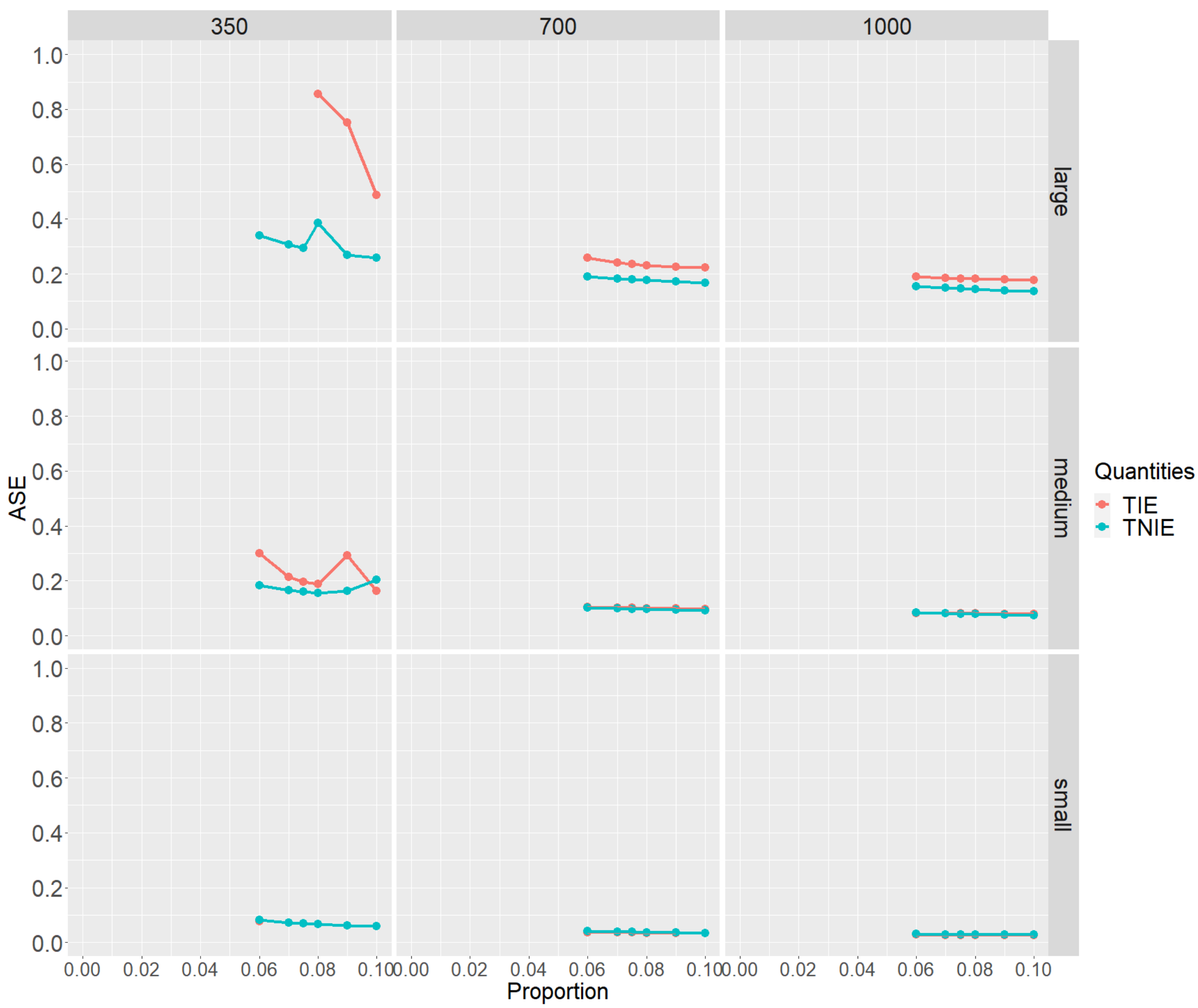
Effect Size	Sample Size	Proportions	ASE for TNIE	ASE for TIE	gap
large	700	0.06	0.191	0.2574	-0.0664
		0.07	0.1833	0.2407	-0.0574
		0.075	0.1796	0.236	-0.0564
		0.08	0.1766	0.2314	-0.0548
		0.09	0.171	0.225	-0.054
		0.1	0.1667	0.222	-0.0553

Table 69

The Changes in ASE under Different Proportions for Rare Outcome Cases in Probit with Moderation Effect

Figure 33

ASE over TIE and TNIE for Rare Outcome Cases in Probit with Moderation Effect



Effect Size	Sample Size	Proportions	ASE for TNIE	ASE for TIE	gap
large	350	0.2	0.2173	0.3305	-0.1132
		0.3	0.2075	0.3183	-0.1108
		0.4	0.2086	0.3148	-0.1062
		0.5	0.2188	0.3197	-0.1009
		0.6	0.2529	0.3696	-0.1167
		0.7	0.3786	0.6197	-0.2411

Table 70

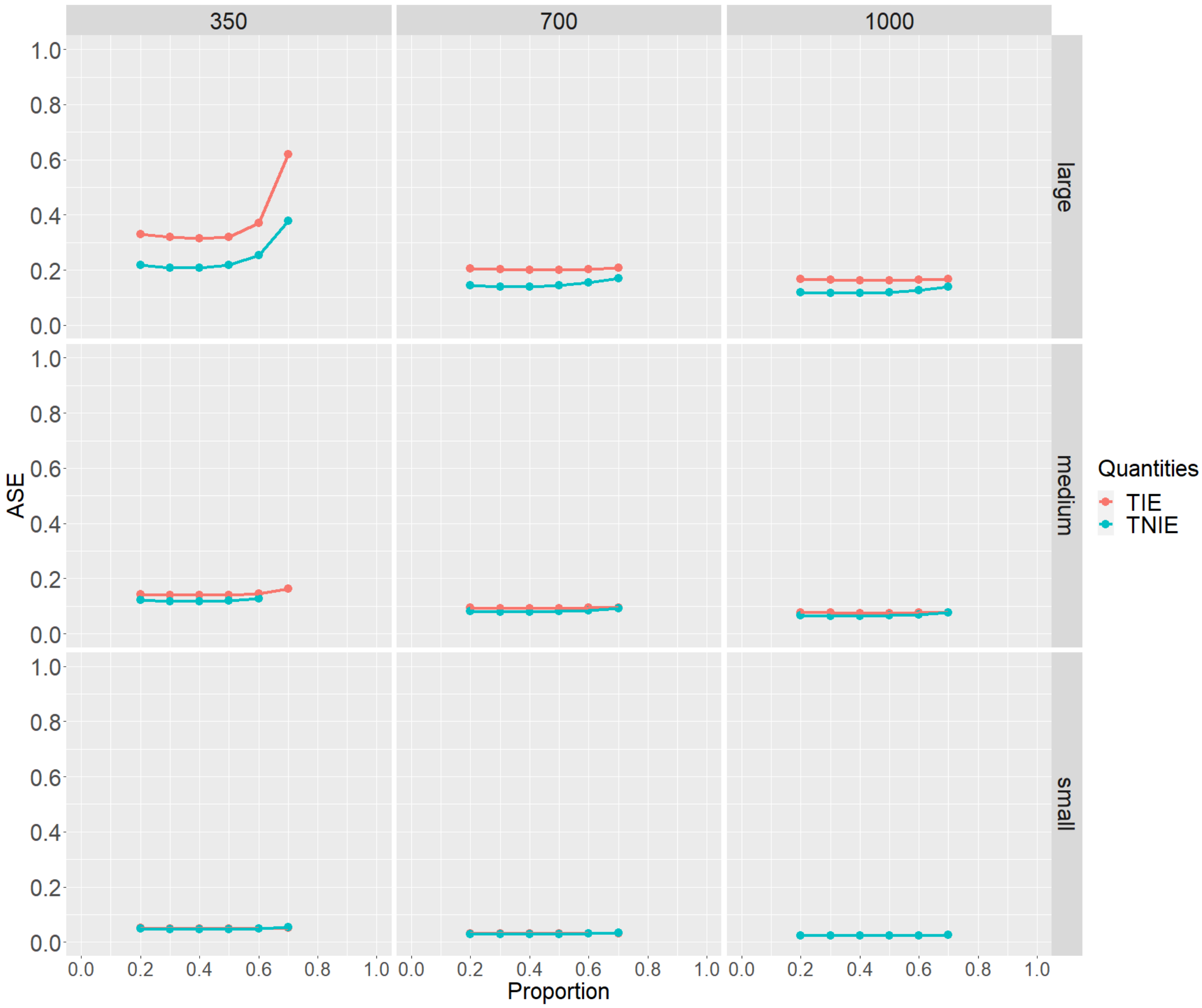
The Changes in ASE under Different Proportions in Non-rare Outcome Cases in Probit with Moderation Effect

the non-rare outcome cases. The ASE for $TNIE$ and TIE decreased as the proportion of $Y = 1$ increased, and the ASE for $TNIE$ was consistently smaller than the ASE for TIE. The results are presented numerically and graphically in Table 70 and Figure 34.

In both outcome cases, the MSE results were similar to the SE results. The MSE for the two quantities decreased, and the MSE for $TNIE$ was consistently less than the MSE for TIE across all proportions of $Y = 1$. The gaps were all negative, as shown in Tables 71 and 72. The pattern is depicted graphically in Figures 35 and 36.

The results revealed no clear patterns for *coverage* as the proportion increased in both rare and non-rare cases. Tables 73 and 74 numerically depict the ambiguous pattern under conditions with a sample size of 350 and a large effect size. This is illustrated graphically in Figures 37 and 38. In the rare outcome cases, $TNIE$ coverage was less than TIE coverage, but the gaps were very small. The gaps were larger in non-rare outcome cases, especially when the *coverage* for $TNIE$ was greater than the *coverage* for TIE. Furthermore, the *coverage* for $TNIE$ was high (above .94).

Figure 34
ASE over TIE and TNIE for Rare Outcome Cases in Probit with Moderation Effect

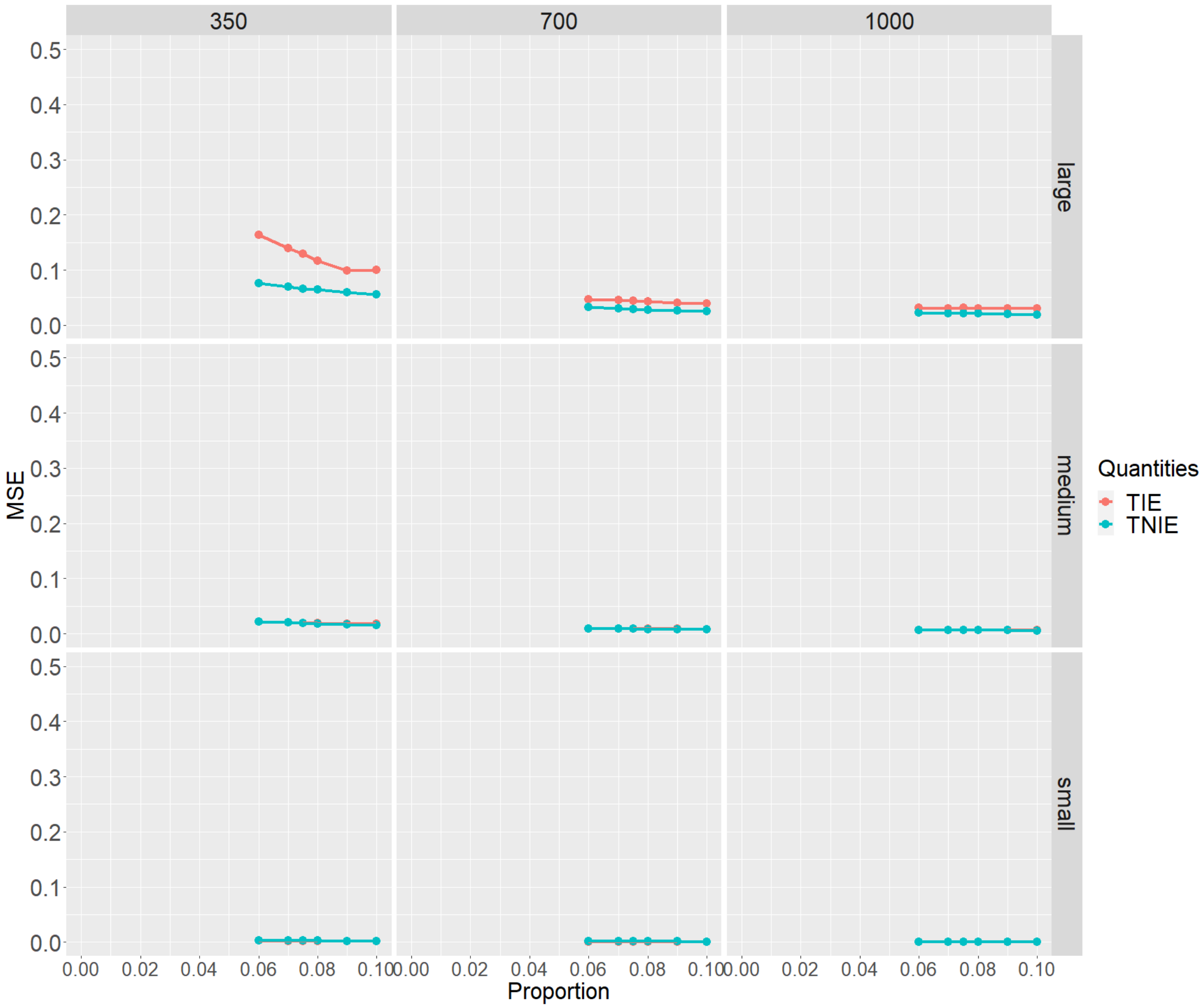


Effect Size	Sample Size	Proportions	MSE for TNIE	MSE for TIE	gap
large	350	0.06	0.0753	0.1642	-0.0889
		0.07	0.0695	0.1399	-0.0704
		0.075	0.0655	0.1293	-0.0638
		0.08	0.0639	0.1164	-0.0525
		0.09	0.0595	0.0992	-0.0397
		0.1	0.0554	0.0997	-0.0443

Table 71
The Changes in MSE under Different Proportions for Rare Outcome Cases in Probit with Moderation Effect

Figure 35

MSE over TIE and TNIE for Rare Outcome Cases in Probit with Moderation Effect



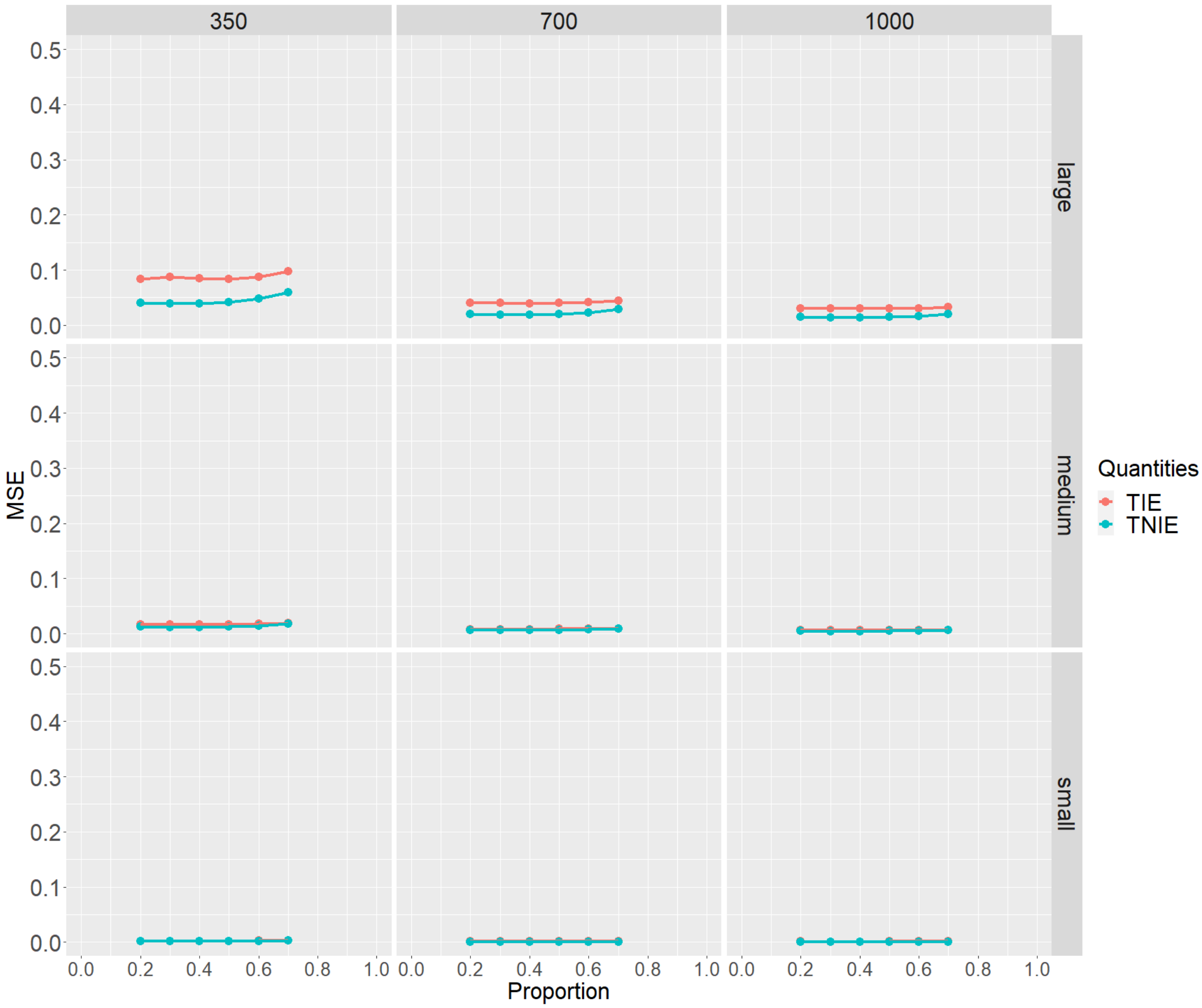
Effect Size	Sample Size	Proportions	MSE for TNIE	MSE for TIE	gap
large	350	0.06	0.0401	0.084	-0.0439
		0.07	0.039	0.0872	-0.0482
		0.075	0.0393	0.0847	-0.0454
		0.08	0.0414	0.0835	-0.0421
		0.09	0.0476	0.0876	-0.04
		0.1	0.06	0.098	-0.038

Table 72

The Changes in MSE under Different Proportions for Non-rare Outcome Cases in Probit with Moderation Effect

Figure 36

MSE over TIE and TNIE in Non-rare Outcome Cases in Probit with Moderation Effect



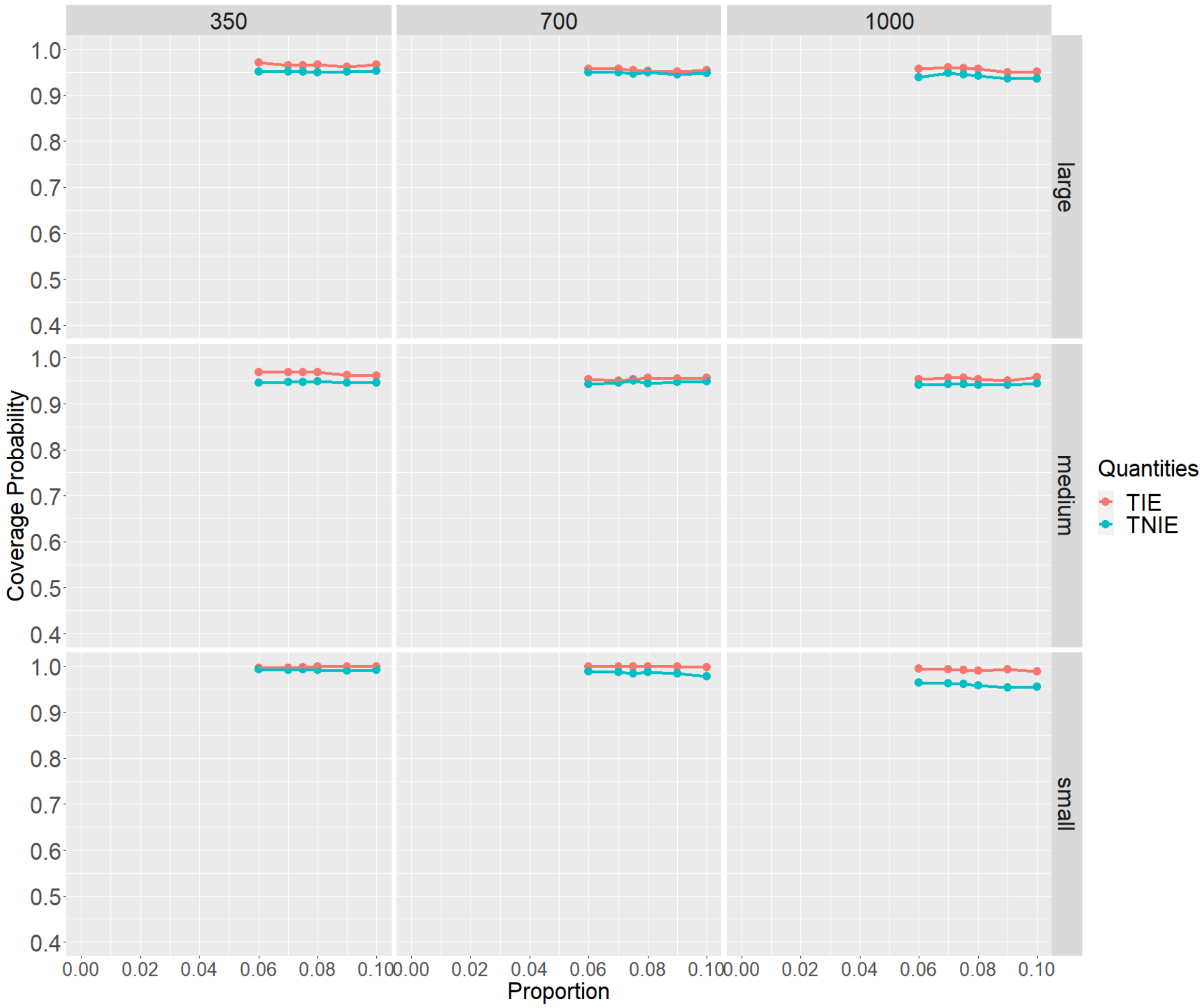
Effect Size	Sample Size	Proportions	Coverage for TNIE	Coverage for TIE	gap
large	350	0.06	0.951	0.971	-0.02
		0.07	0.952	0.965	-0.013
		0.075	0.951	0.965	-0.014
		0.08	0.95	0.967	-0.017
		0.09	0.952	0.962	-0.01
		0.1	0.953	0.966	-0.013

Table 73

The Changes in Coverage under Different Proportions for Rare Outcome Cases in Probit with Moderation Effect

Figure 37

Coverage Probability over TIE and TNIE for Rare Outcome Cases in Probit with Moderation Effect



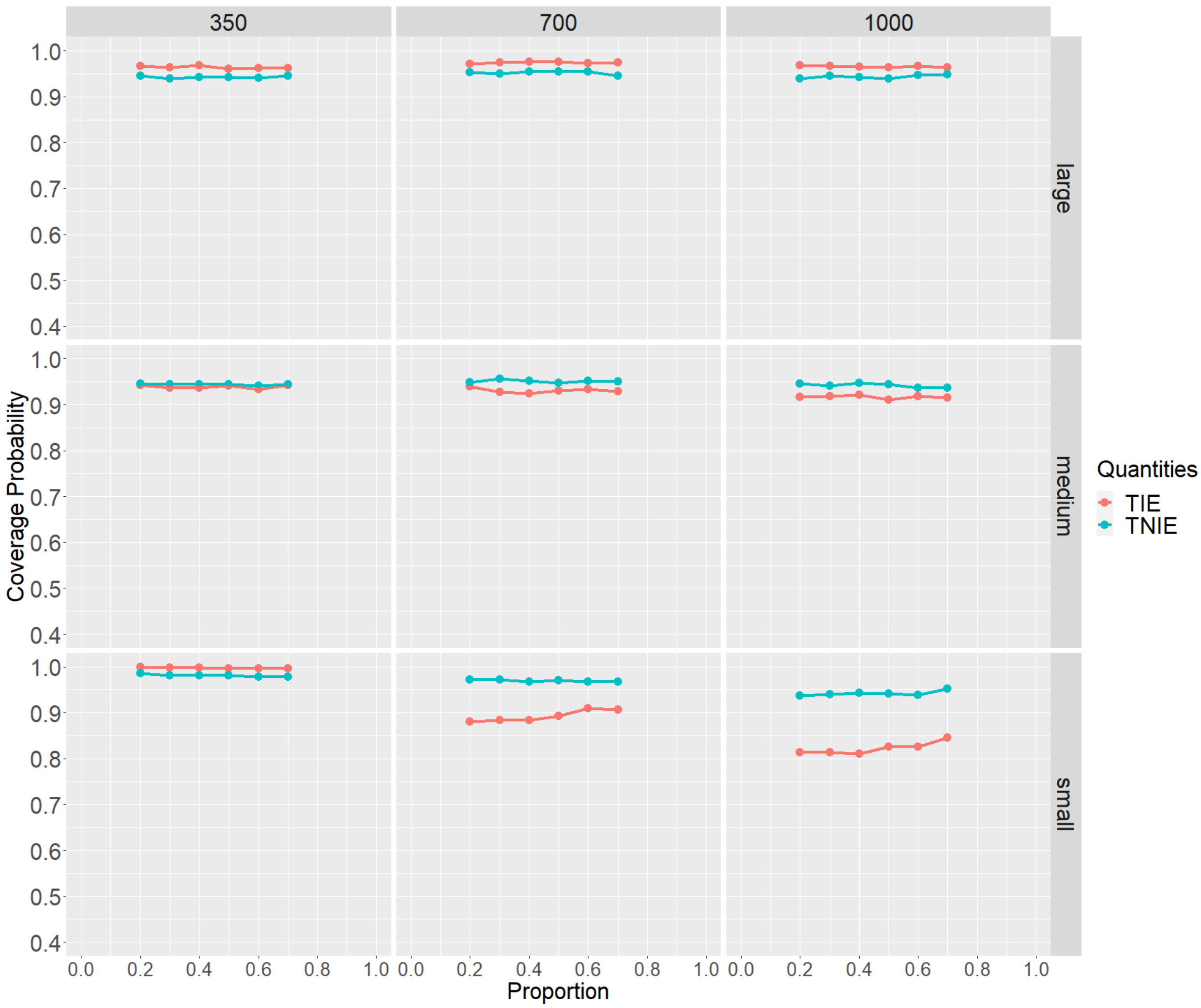
Effect Size	Sample Size	Proportions	Coverage for TNIE	Coverage for TIE	gap
large	350	0.2	0.946	0.967	-0.021
		0.3	0.94	0.964	-0.024
		0.4	0.942	0.968	-0.026
		0.5	0.943	0.961	-0.018
		0.6	0.941	0.962	-0.021
		0.7	0.945	0.962	-0.017

Table 74

The Changes in Coverage under Different Proportions in Non-rare Outcome Cases in Probit with Moderation Effect

Figure 38

Coverage Probability over TIE and TNIE in Non-rare Outcome Cases in Probit with Moderation Effect



In both outcome cases, the *power* for $TNIE$ remained constant across changes in the proportion of $Y = 1$. The *power* for TIE increased as the proportion of $Y = 1$ increased in rare outcome cases. Table 75 numerically depicts this pattern under conditions with a large effect size and a sample size of 350. In the non-rare outcome cases, on the other hand, the *power* for TIE was equal to 1 when the proportion of $Y = 1$ was 0.2 to 0.4, then decreased as the proportion of $Y = 1$ increased (see Table 76 for a numerical representation). These patterns are also depicted graphically in Figures 37 and 38. Overall, the *power* for $TNIE$ was greater than the *power* for TIE if the *power* for TIE was not 1; otherwise, the *power* for the two quantities was the same.

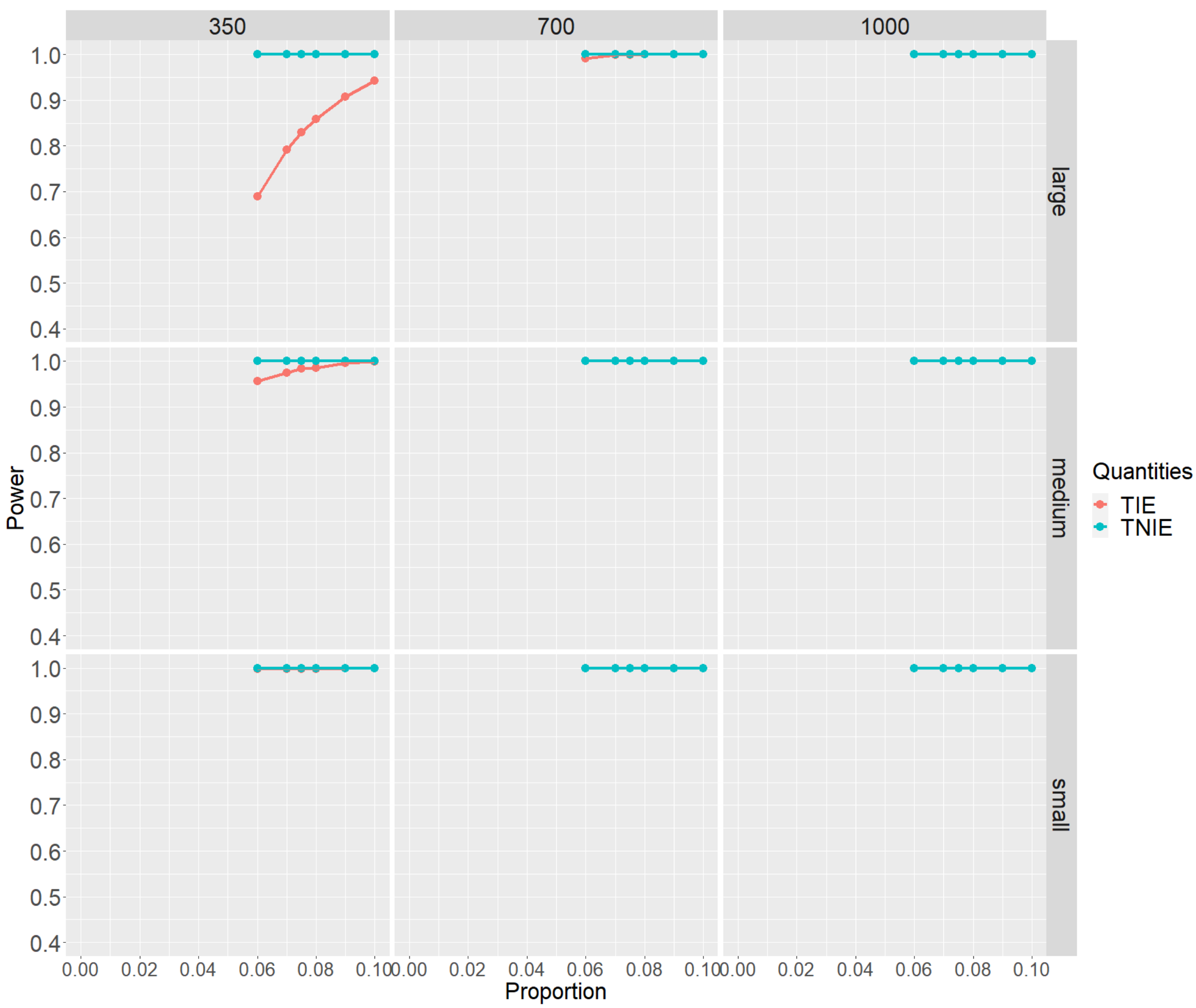
Effect Size	Sample Size	Proportions	Power for TNIE	Power for TIE	gap
large	350	0.06	1	0.689	0.311
		0.07	1	0.791	0.209
		0.075	1	0.83	0.17
		0.08	1	0.859	0.141
		0.09	1	0.907	0.093
		0.1	1	0.942	0.058

Table 75

The Changes in Power under Different Proportions for Rare Outcome Cases in Probit with Moderation Effect

Figure 39

Power over TIE and TNIE for Rare Outcomes in Probit in Probit with Moderation Effect



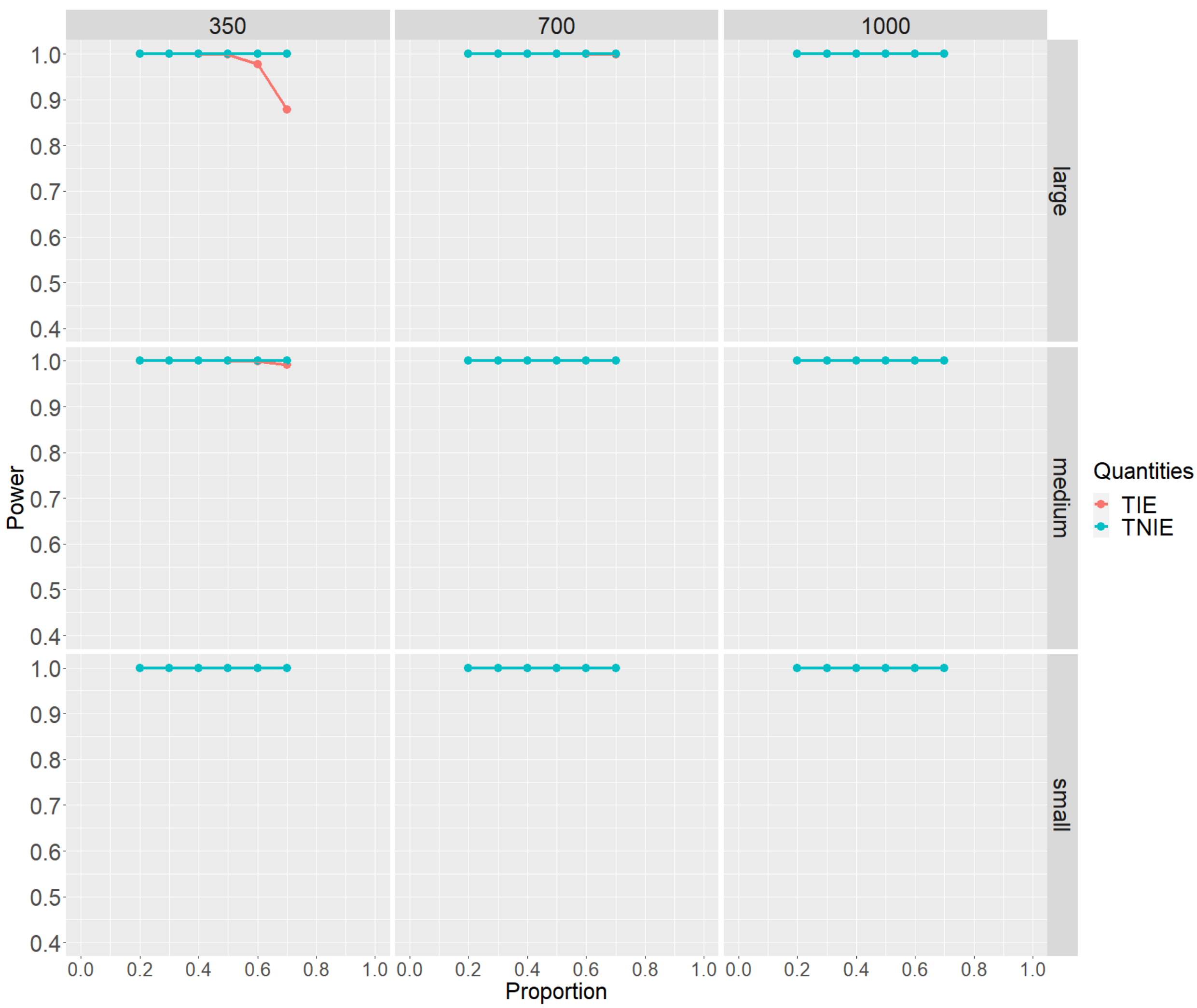
Effect Size	Sample Size	Proportions	Power for TNIE	Power for TIE	gap
large	350	0.2	1	1	0
		0.3	1	1	0
		0.4	1	1	0
		0.5	1	0.998	0.002
		0.6	1	0.978	0.022
		0.7	1	0.879	0.121

Table 76

The Changes in Power under Different Proportions in Non-rare Outcome Cases in Probit with Moderation Effect

Figure 40

Power over TIE and TNIE for Non-rare Outcomes in Probit in Probit with Moderation Effect



Proportion	Sample Size	Effect Size	TNIE	TIE	gap
0.075	700	small	0.9998	0.9997	1e-04
		medium	0.996	0.9949	0.0011
		large	0.989	0.9809	0.0081

Table 77

The Changes in Bias under Different Effect Size for Rare Outcome Cases in Probit with Moderation Effect

Effect Size. When the effect size increased, the *bias* gaps between $TNIE$ and TIE widened and the *bias* for each quantity moved further away from 1. In rare outcome cases, this pattern was clearly visible. The difference between the two quantities was positive. As a result, the *bias* for $TNIE$ was consistently smaller than the *bias* for TIE as the effect size increased (for more information, see Table 77). Figure 29 shows this graphically as well.

The *bias* for $TNIE$ was closer to 1 in the non-rare outcome cases, and the gaps were much larger than in the rare cases. The pattern of the gaps between the two quantities in *bias*, on the other hand, was not as clear as in the rare outcome cases. The gap narrowed slightly as the effect size increased from small to medium; when the effect size reached large, the gap widened significantly. This pattern was shown numerically in Table 78 and graphically in Figure 30. In summary, $TNIE$ had smaller bias than TIE in both cases, and it was more noticeable in non-rare outcome cases than in rare cases.

When the effect size was small and medium, the differences in SE between $TNIE$ and TIE were quite small. The gaps were more visible in both outcome cases given the large effect size (see Tables 79 and 80, and Figures 31 and 32 for numerical and graphical representations). In general, the SE for $TNIE$ was lower than that for TIE, and the difference was more visible in non-rare outcome cases.

Aside from conditions with a sample size of 350 in rare outcome cases, the ASE followed a similar pattern to the SE. The ASE did not converge well under those

Proportion	Sample Size	Effect Size	TNIE	TIE	gap
0.2	700	small	0.999	0.98	0.019
		medium	0.9984	1.0165	-0.0181
		large	0.995	0.9611	0.0339

Table 78

The Changes in Bias under Different Effect Size in Non-rare Outcome Cases in Probit with Moderation Effect

conditions. The pattern for MSE was similar to the pattern for SE across changes in the effect size. However, there was little difference between the rare and non-rare outcome cases. These patterns are depicted graphically in Figures 33, 34, 35, and 36.

When the effect size changed, there was no clear pattern in the *coverage probability* for the two quantities or the gaps between them. Tables 81 and 82 show the numerical changes in *coverage* in rare and non-rare outcome cases. In most cases, the differences between the two quantities were negative but relatively small. These patterns are depicted graphically in Figures 37 and 38. To summarize, as the effect size increased, the *coverage* for $TNIE$ fluctuated between greater and lesser than the *coverage* for TIE, with no discernible pattern.

In rare outcome cases, the *power* for $TNIE$ was 1 at all effect size levels. The effect size had an effect on the *power* for TIE only when the sample size was between 350 and 700. In those cases, as the effect size increased, the *power* for TIE decreased and the *power* gap between the two quantities increased. In all other cases, the *power* for TIE was 1 and stayed that way. This is depicted graphically in Figures 39 and 40. Tables 83 and 84 numerically demonstrate this pattern with a sample size of 350 and a proportion of 0.06 for a rare outcome and 0.6 for a non-rare outcome. When the effect size was small, the *power* for TIE was very close to 1, indicating that the gap was positive but small. When the effect size was medium, the *power* for TIE decreased, resulting in a wider gap. Then, as the effect size increased to large, the *power* for TIE decreased, resulting in a much larger gap.

Proportion	Sample Size	Effect Size	TNIE	TIE	gap
0.06	350	small	0.0525	0.0481	0.0044
		medium	0.145	0.1452	-2e-04
		large	0.2712	0.3943	-0.1231

Table 79

The Changes in SE under Different Effect Size in Rare Outcome Cases in Probit with Moderation Effect

Proportion	Sample Size	Effect Size	TNIE	TIE	gap
0.2	350	small	0.0402	0.0414	-0.0012
		medium	0.1122	0.1276	-0.0154
		large	0.1996	0.279	-0.0794

Table 80

The Changes in SE under Different Effect Size in Non-rare Outcome Cases in Probit with Moderation Effect

Proportion	Sample Size	Effect Size	Coverage for TNIE	Coverage for TIE	gap
0.06	350	small	0.993	0.997	-0.004
		medium	0.946	0.969	-0.023
		large	0.951	0.971	-0.02

Table 81

The Changes in Coverage under Different Effect Size for Rare Outcome Cases in Probit with Moderation Effect

Proportion	Sample Size	Effect Size	Coverage for TNIE	Coverage for TIE	gap
0.2	350	small	0.986	0.999	-0.013
		medium	0.946	0.943	0.003
		large	0.946	0.967	-0.021

Table 82

The Changes in Coverage under Different Effect Size in Non-rare Outcome Cases in Probit with Moderation Effect

Proportion	Sample Size	Effect Size	Power for TNIE	Power for TIE	gap
0.06	350	small	1	0.998	0.002
		medium	1	0.956	0.044
		large	1	0.689	0.311

Table 83

The Changes in Power under Different Effect Size for Rare Outcome Cases in Probit

Sample Size. The *bias* for $TNIE$ and TIE became closer to 1 as the sample size increased, and the gap between them shrank. This pattern was clearly visible in both outcome cases when the effect size was large. The pattern is numerically presented in Tables 85 and 86 under conditions with a large effect size and a proportion of 0.06 for rare cases and 0.2 for non-rare cases. In summary, $TNIE$ produced *bias* that was closer to 1 than TIE in both cases as the sample size increased, but it was more visible in conditions with a large effect size.

When the sample size was increased, the difference between two quantities in SE and the SE for each quantity became smaller. This pattern is clearly visible when the effect size is large (for numerical details, see Tables 87 and 88). This pattern is numerically represented in Figures 31 and 32. In general, as sample size increased, the gap in the SE between $TNIE$ and TIE became less and less clear.

The ASE and MSE results for the two quantities followed the same pattern as the SE. This is illustrated numerically in Tables 89, 90, 91, and 92 and graphically in Figure 33, 34, 35, and 36.

When the sample size was changed, the *coverage probability* for the two quantities did not show any patterns, whereas the gaps between them only showed a clear pattern when the effect size was large. In those cases, as the sample size increased, the gaps in *coverage* widened and became negative. As the sample size increased, the *coverage* for $TNIE$ became smaller than the *coverage* for TIE. This pattern is numerically shown in Tables 93 and 94.

Proportion	Sample Size	Effect Size	Power for TNIE	Power for TIE	gap
0.8	350	small	1	1	0
		medium	1	0.999	0.001
		large	1	0.978	0.022

Table 84

The Changes in Power under Different Effect Size in Non-rare Outcome Cases in Probit with Moderation Effect

Proportion	Effect Size	Sample Size	Bias for TNIE	Bias for TIE	gap
0.06	large	350	0.9712	0.9406	0.0306
		700	0.9883	0.98	0.0083
		1000	0.991	0.986	0.005

Table 85

The Changes in Bias under Different Sample Size for Rare Outcome Cases in Probit with Moderation Effect

Proportion	Effect Size	Sample Size	Bias for TNIE	Bias for TIE	gap
0.2	large	350	0.9877	0.9484	0.0393
		700	0.995	0.9611	0.0339
		1000	0.9955	0.963	0.0325

Table 86

The Changes in Bias under Different Sample Size in Non-rare Outcome Cases in Probit with Moderation Effect

Proportion	Effect Size	Sample Size	SE for TNIE	SE for TIE	gap
0.06	large	350	0.2712	0.3943	-0.1231
		700	0.1793	0.2133	-0.034
		1000	0.1518	0.1749	-0.0231

Table 87

The Changes in SE under Different Sample Size for Rare Outcome Cases in Probit with Moderation Effect

Proportion	Effect Size	Sample Size	SE for TNIE	SE for TIE	gap
0.2	large	350	0.1996	0.279	-0.0794
		700	0.1406	0.1925	-0.0519
		1000	0.1204	0.1642	-0.0438

Table 88

The Changes in SE under Different Sample Size in Non-rare Outcome Cases in Probit with Moderation Effect

Proportion	Effect Size	Sample Size	ASE for TNIE	ASE for TIE	gap
0.06	large	350	0.3387	77.5323	-77.1936
		700	0.191	0.2574	-0.0664
		1000	0.1541	0.1889	-0.0348

Table 89

The Changes in ASE under Different Sample Size for Rare Outcome Cases in Probit with Moderation Effect

Proportion	Effect Size	Sample Size	ASE for TNIE	ASE for TIE	gap
0.2	large	350	0.2173	0.3305	-0.1132
		700	0.1448	0.2051	-0.0603
		1000	0.1191	0.166	-0.0469

Table 90

The Changes in ASE under Different Sample Size in Non-rare Outcome Cases in Probit with Moderation Effect

Proportion	Effect Size	Sample Size	MSE for TNIE	MSE for TIE	gap
0.06	large	350	0.0753	0.1642	-0.0889
		700	0.0324	0.0464	-0.014
		1000	0.0232	0.031	-0.0078

Table 91

The Changes in MSE under Different Sample Size for Rare Outcome Cases in Probit with Moderation Effect

Proportion	Effect Size	Sample Size	MSE for TNIE	MSE for TIE	gap
0.2	large	350	0.0401	0.084	-0.0439
		700	0.0198	0.0404	-0.0206
		1000	0.0145	0.03	-0.0155

Table 92

The Changes in MSE under Different Sample Size in Non-rare Outcome Cases in Probit with Moderation Effect

Proportion	Effect Size	Sample Size	Coverage for TNIE	Coverage for TIE	gap
0.06	large	350	0.951	0.971	-0.02
		700	0.95	0.958	-0.008
		1000	0.939	0.958	-0.019

Table 93

The Changes in Coverage under Different Sample Size for Rare Outcome Cases in Probit with Moderation Effect

Proportion	Effect Size	Sample Size	Coverage for TNIE	Coverage for TIE	gap
0.2	large	350	0.946	0.967	-0.021
		700	0.953	0.971	-0.018
		1000	0.94	0.968	-0.028

Table 94

The Changes in Coverage under Different Sample Size in Non-rare Outcome Cases in Probit with Moderation Effect

Proportion	Effect Size	Sample Size	Power for TNIE	Power for TIE	gap
0.06	large	350	1	0.689	0.311
		700	1	0.991	0.009
		1000	1	1	0

Table 95

The Changes in Power under Different Sample Size for Rare Outcome Cases in Probit with Moderation Effect

Proportion	Effect Size	Sample Size	Power for TNIE	Power for TIE	gap
0.2	large	350	1	1	0
		700	1	1	0
		1000	1	1	0

Table 96

The Changes in Power under Different Sample Size in Non-rare Outcome Cases in Probit with Moderation Effect

When the sample size was increased from 350 to 700 in both outcome cases, the *power* for TIE increased and the gap in the *power* between the two quantities decreased. When the sample size was increased to 1000, the *power* for TIE was 1 and the gap was 0. This is depicted graphically in Figures 39 and 40.

Moderation Effect. This factor was only used in the moderated mediation model. The goal was to see if there was a significant difference in producing an accurate mediation effect between the moderated mediation model with a low moderation value (no moderation effect) and the moderated mediation model with a high moderation value (a moderation effect). In rare outcome cases, with a moderation effect, the *bias* for $TNIE$ and TIE was closer to 1, and the gap between them was larger than without a moderation effect. Tables 64 and 97 numerically present this pattern under conditions with a larger effect size and a sample size of 350. Figures 29 and 41 show graphical representations of this pattern where the title indicates whether the graph with or without moderation effect. The figures cannot show the gaps for the conditions with a small effect size because they are too small. The numerical results in these conditions are shown in Tables 65 and 98.

With the presence of a moderation effect, the *bias* for $TNIE$ and TIE was slightly closer to 1 and the gap between them was smaller in the non-rare outcome cases. Tables 66 and 99 numerically present this pattern under conditions with a small effect size. Figures 30 and 42 show this in graphical form.

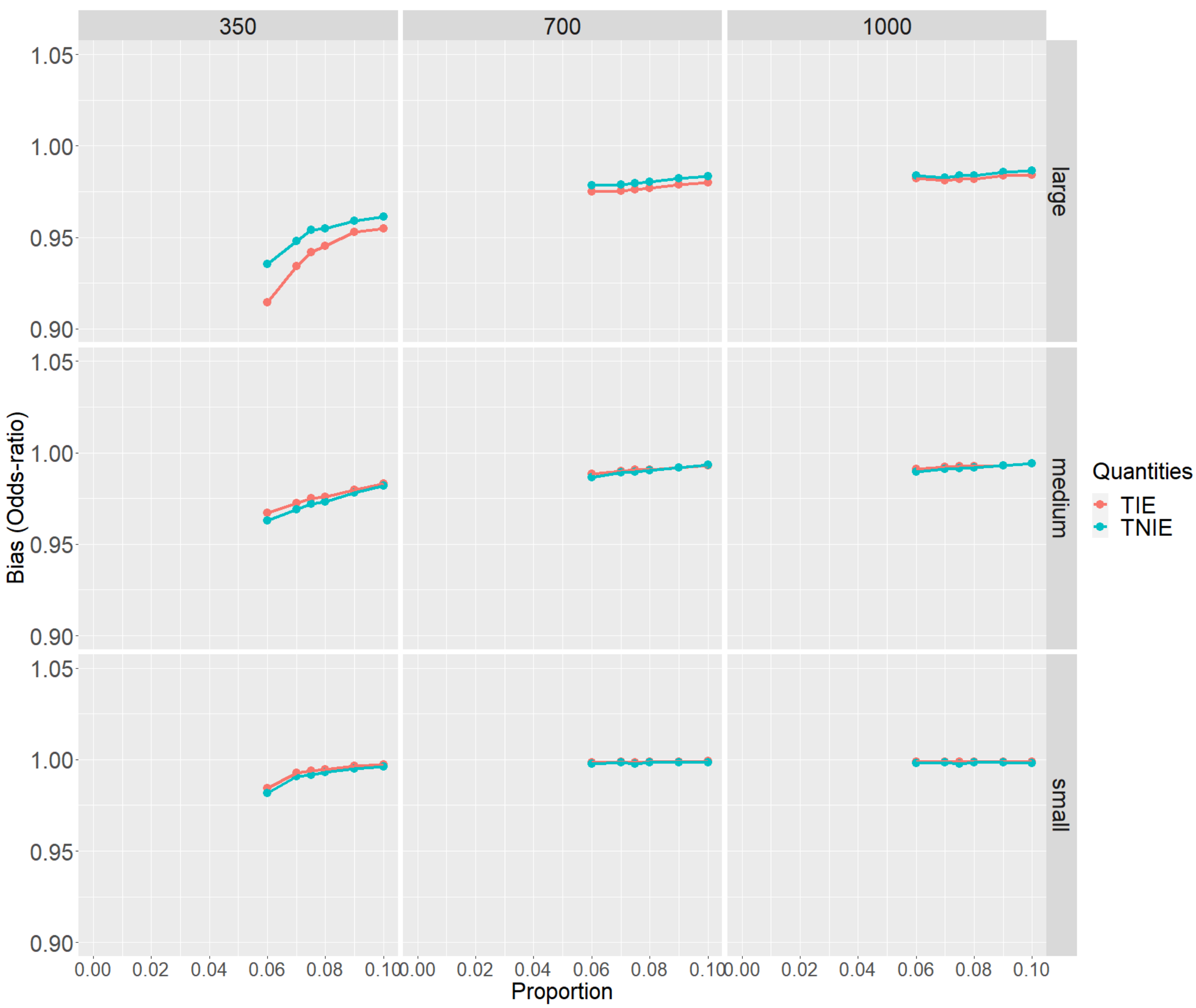
Effect Size	Sample Size	Proportions	Bias for TNIE	Bias for TIE	gap
large	350	0.06	0.9353	0.9143	0.021
		0.07	0.9481	0.9343	0.0138
		0.075	0.9539	0.9419	0.012
		0.08	0.9549	0.9454	0.0095
		0.09	0.959	0.9529	0.0061
		0.1	0.9612	0.9548	0.0064

Table 97

The Changes in Bias under Different Proportions for Rare Outcome Cases in Probit Without Moderated Effect

Figure 41

Bias over TIE and TNIE for Rare Outcomes in Probit Without Moderation Effect



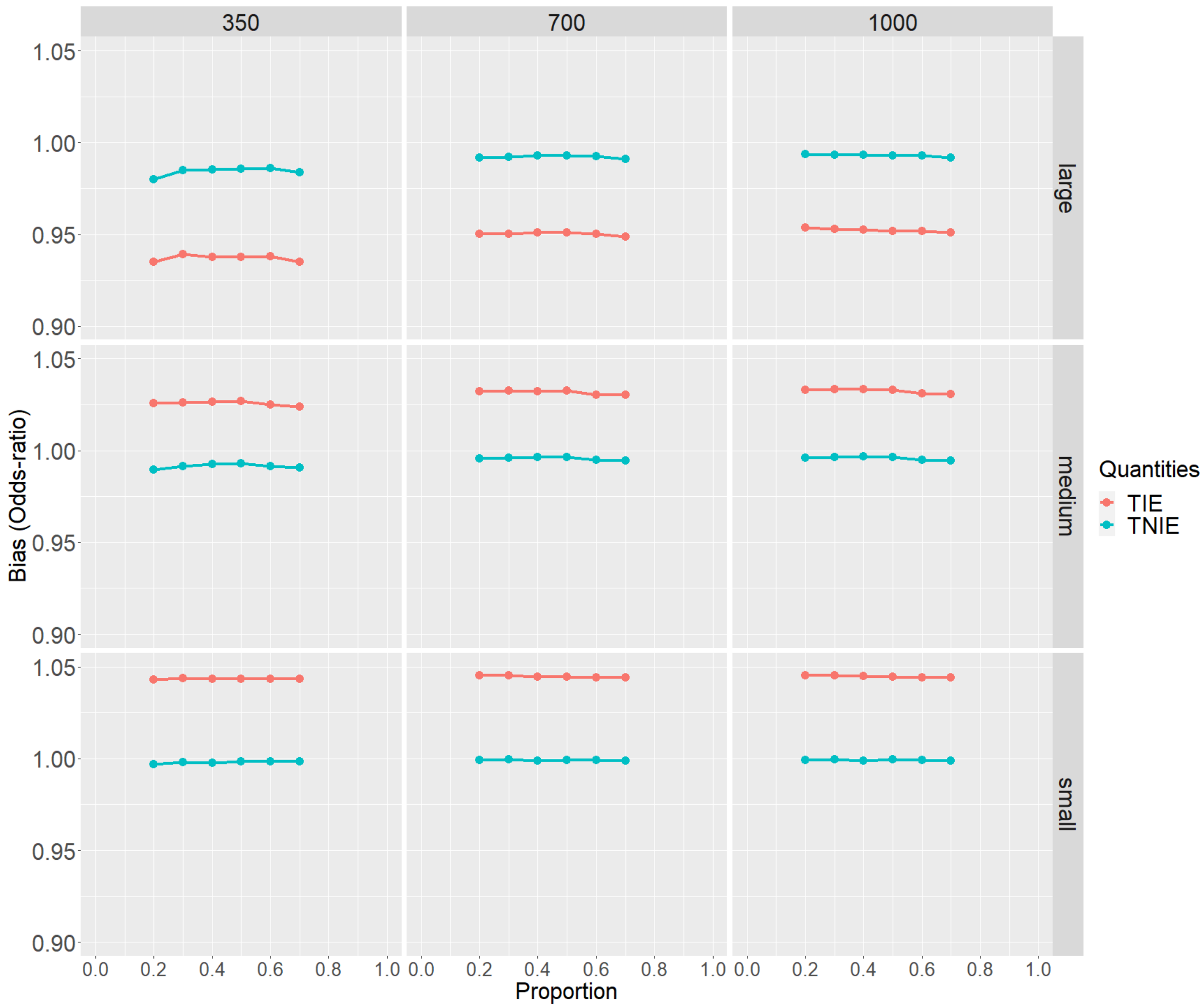
Effect Size	Sample Size	Proportions	Bias for TNIE	Bias for TIE	gap
small	350	0.06	0.9815	0.9844	-0.0029
		0.07	0.991	0.9929	-0.0019
		0.075	0.9915	0.9939	-0.0024
		0.08	0.993	0.9945	-0.0015
		0.09	0.995	0.9964	-0.0014
		0.1	0.996	0.9972	-0.0012
	700	0.06	0.9977	0.9985	-8e-04
		0.07	0.9984	0.9987	-3e-04
		0.075	0.9976	0.9985	-9e-04
		0.08	0.9984	0.9987	-3e-04
		0.09	0.9985	0.9989	-4e-04
		0.1	0.9985	0.999	-5e-04
	1000	0.06	0.9977	0.9985	-8e-04
		0.07	0.9984	0.9987	-3e-04
		0.075	0.9976	0.9985	-9e-04
		0.08	0.9984	0.9987	-3e-04
		0.09	0.9985	0.9989	-4e-04
		0.1	0.9985	0.999	-5e-04

Table 98

The Changes in Bias across Proportions for Rare Outcome Cases in Probit Without Moderation Effect

Figure 42

Bias over TIE and TNIE for Non-rare Outcomes in Probit Without Moderation Effect



In both outcome cases, the *bias* for $TNIE$ was closer to 1 than the *bias* for TIE in terms of having a moderation effect or not having a moderation effect. Furthermore, in the presence of a moderation effect, the gap between the two quantities in *bias* was larger in the rare outcome cases but smaller in the non-rare outcome cases.

Under conditions with a moderation effect, the SE for the two quantities was smaller in rare outcome cases, but the gap between the two quantities was slightly larger. Furthermore, in the absence of a moderation effect, $TNIE$ had a higher SE than TIE. The patterns of the SE and the gaps between the two quantities were very similar in

Effect Size	Sample Size	Proportions	Bias for TNIE	Bias for TIE	gap
small	350	0.2	0.9967	1.043	-0.0463
		0.3	0.9982	1.0436	-0.0454
		0.4	0.9977	1.0435	-0.0458
		0.5	0.9985	1.0435	-0.045
		0.6	0.9985	1.0432	-0.0447
		0.7	0.9984	1.0435	-0.0451
	700	0.2	0.9993	1.0453	-0.046
		0.3	0.9997	1.0452	-0.0455
		0.4	0.9987	1.0447	-0.046
		0.5	0.9993	1.0446	-0.0453
		0.6	0.999	1.044	-0.045
		0.7	0.9989	1.0443	-0.0454
	1000	0.2	0.9993	1.0453	-0.046
		0.3	0.9997	1.0452	-0.0455
		0.4	0.9987	1.0447	-0.046
		0.5	0.9993	1.0446	-0.0453
		0.6	0.999	1.044	-0.045
		0.7	0.9989	1.0443	-0.0454

Table 99

The Changes in Bias across Proportions for Non-rare Outcome Cases in Probit Without Moderation Effect

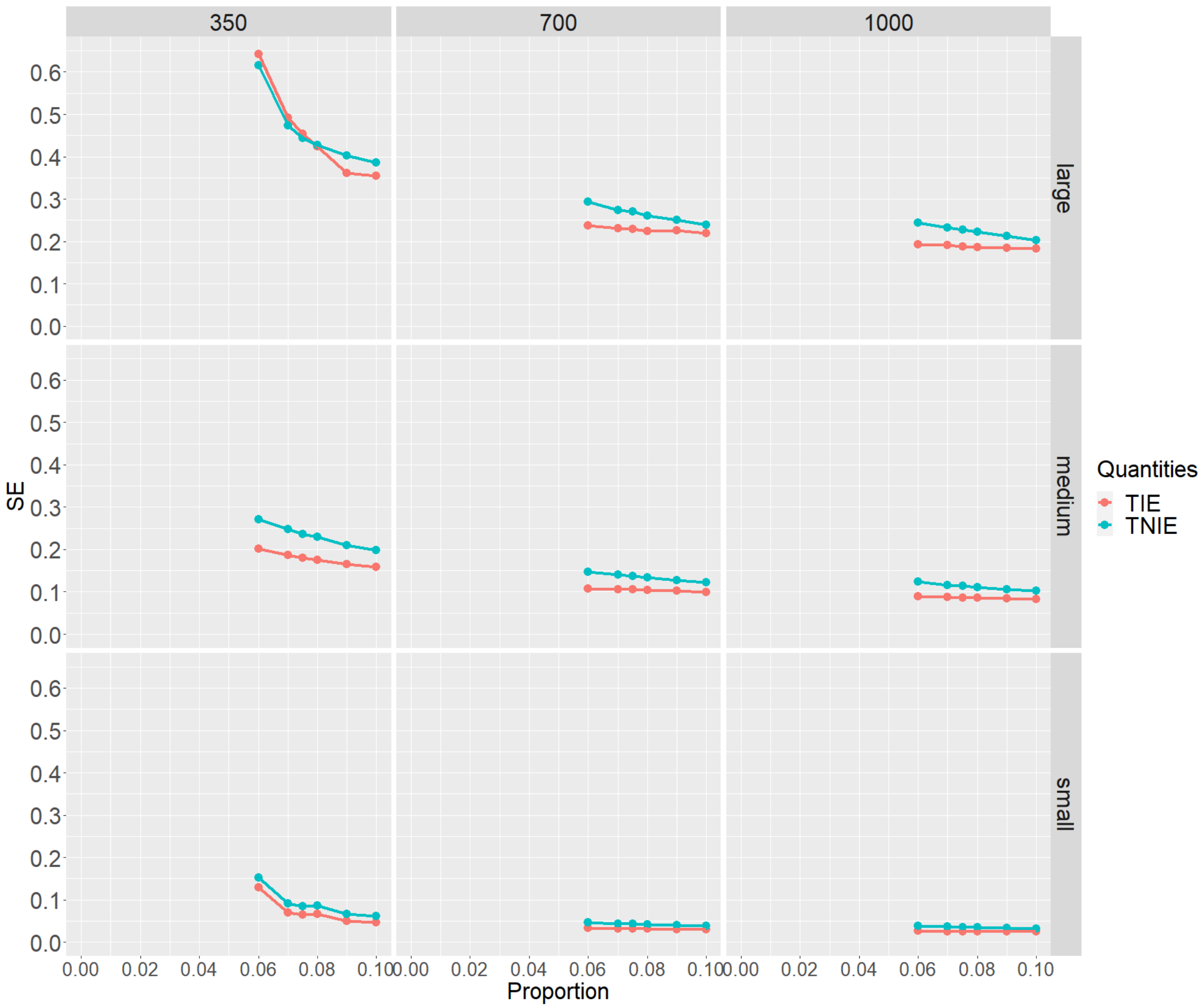
Effect Size	Sample Size	Proportions	SE for TNIE	SE for TIE	gap
small	350	0.06	0.1526	0.13	0.0226
		0.07	0.0909	0.0702	0.0207
		0.075	0.0853	0.0651	0.0202
		0.08	0.0858	0.0663	0.0195
		0.09	0.066	0.0497	0.0163
		0.1	0.0621	0.0475	0.0146
	700	0.06	0.0463	0.0329	0.0134
		0.07	0.0431	0.0313	0.0118
		0.075	0.0428	0.0314	0.0114
		0.08	0.0419	0.0311	0.0108
		0.09	0.0403	0.0305	0.0098
		0.1	0.0388	0.0301	0.0087
	1000	0.06	0.0379	0.0267	0.0112
		0.07	0.036	0.026	0.01
		0.075	0.0352	0.0258	0.0094
		0.08	0.0344	0.0254	0.009
		0.09	0.033	0.0249	0.0081
		0.1	0.0319	0.0246	0.0073

Table 100

The Changes in SE across Proportions for Rare Outcome Cases in Probit Without Moderation Effect

Figure 43

SE over TIE and TNIE for Rare Outcomes in Probit Without Moderation Effect

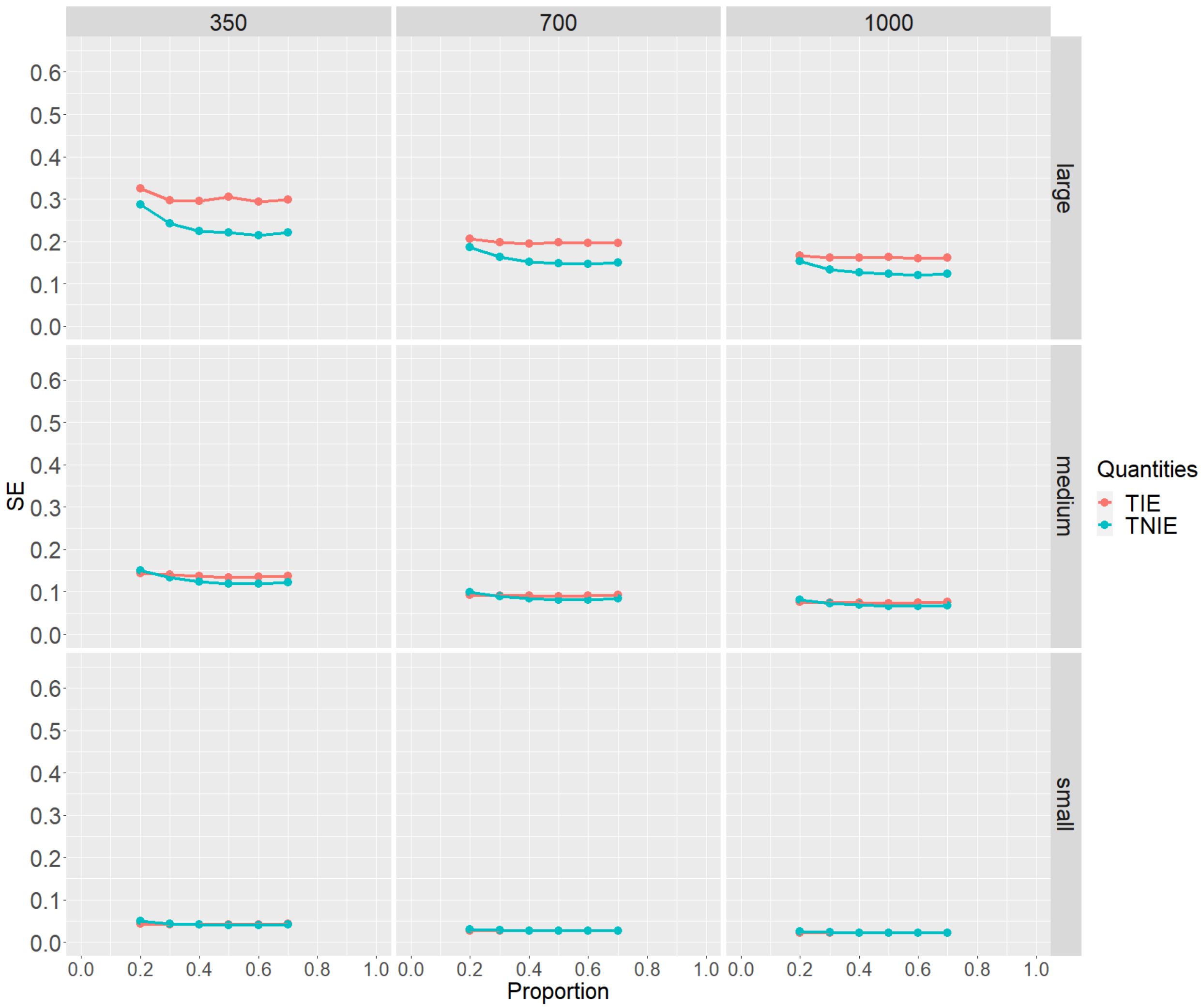


non-rare outcome cases. However, when the moderation effect was present, the SE was slightly smaller. Tables 67 and 100 numerically show this pattern for rare outcome cases, whereas Tables 68 and 101 show non-rare outcome cases under small effect size conditions. This pattern can be seen graphically in Figures 31 and 43 for rare outcome cases, and Figures 32 and 44 for non-rare outcome cases.

Without the presence of a moderation effect, the gap in the ASE between the two quantities was much clearer, and the ASE for $TNIE$ was slightly larger than the ASE for TIE in rare outcome cases, as shown graphically in Figures 31 and Figure 43. In non-rare outcome cases, the ASE for moderated effects was quite similar at both levels. The

Figure 44

SE over TIE and TNIE for Non-rare Outcomes in Probit Without Moderation Effect



pattern is numerically represented in Figures 32 and 44. Because the conditions with a small effect size had such a small gap between the two quantities, the lines representing the two quantities in ASE were overlapping. Tables 69 and 102 and Tables 70 and 103 present numerical values in rare and non-rare outcome cases, respectively.

Under conditions with a medium effect size, the MSE for the two quantities showed a clearer gap in rare outcome cases without a moderation effect. Figures 35 and 47 illustrate this pattern graphically. The pattern was quite similar between the levels of the moderation effect for non-rare outcome cases. Figures 36 and 48 present graphically for cases with non-rare outcomes. The gaps between the two quantities in MSE were either

Effect Size	Sample Size	Proportions	SE for TNIE	SE for TIE	gap
small	350	0.2	0.0502	0.0438	0.0064
		0.3	0.0432	0.041	0.0022
		0.4	0.0419	0.0417	2e-04
		0.5	0.0408	0.0417	-9e-04
		0.6	0.0407	0.0419	-0.0012
		0.7	0.0417	0.0427	-0.001
	700	0.2	0.031	0.0272	0.0038
		0.3	0.0282	0.0266	0.0016
		0.4	0.0273	0.027	3e-04
		0.5	0.0266	0.027	-4e-04
		0.6	0.0268	0.0273	-5e-04
		0.7	0.0275	0.0277	-2e-04
	1000	0.2	0.0252	0.0221	0.0031
		0.3	0.0231	0.0217	0.0014
		0.4	0.0219	0.0216	3e-04
		0.5	0.0212	0.0215	-3e-04
		0.6	0.0216	0.022	-4e-04
		0.7	0.0224	0.0225	-1e-04

Table 101

The Changes in SE across Proportions for Non-rare Outcome Cases in Probit Without Moderation Effect

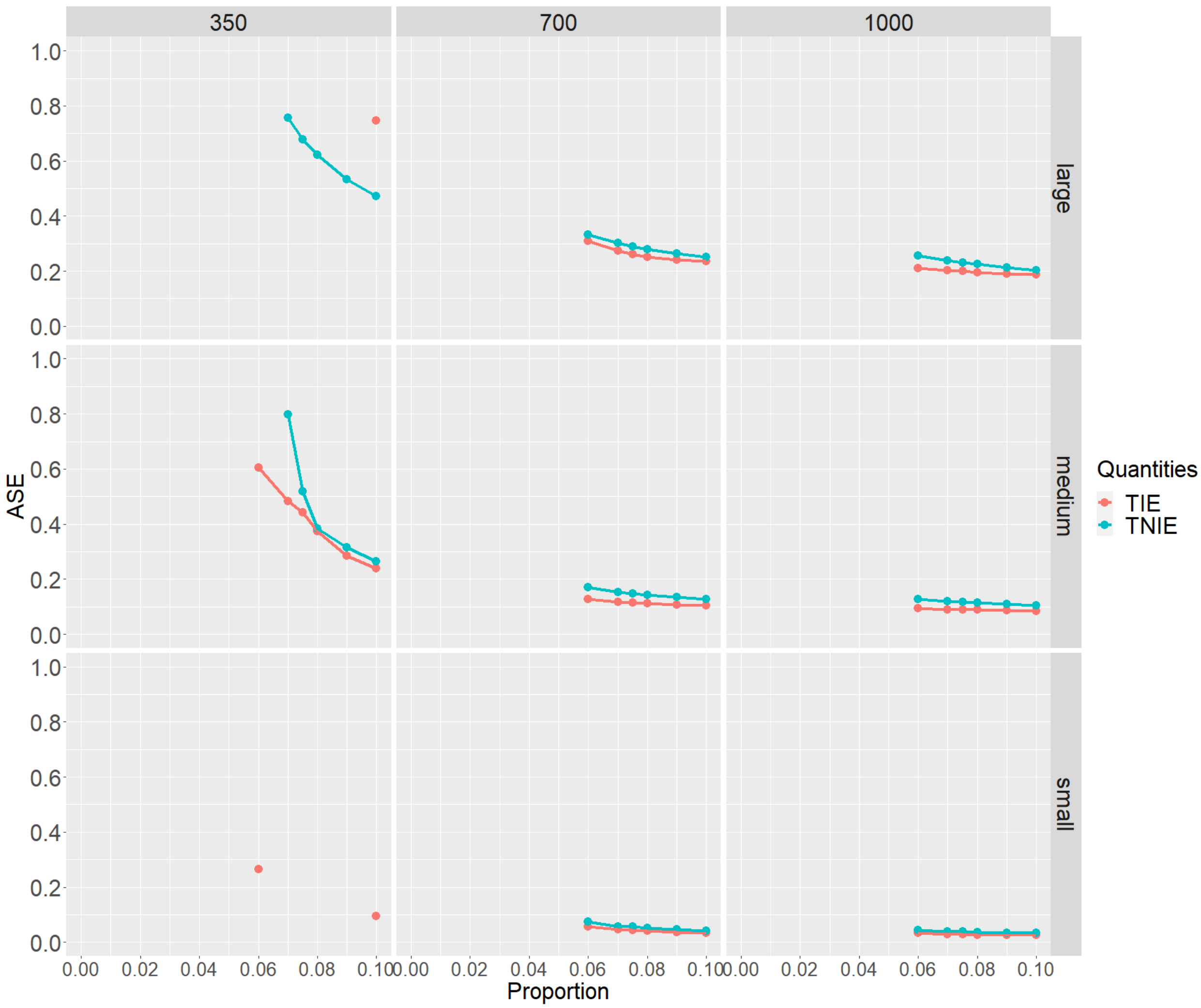
Effect Size	Sample Size	Proportions	ASE for TNIE	ASE for TIE	gap
large	350	0.06	NaN	0.2654	NaN
		0.07	NaN	126.8263	NaN
		0.075	NaN	94.7017	NaN
		0.08	NaN	38.1459	NaN
		0.09	NaN	28.6141	NaN
		0.1	NaN	0.0956	NaN
	700	0.06	0.0754	0.0573	0.0181
		0.07	0.057	0.047	0.01
		0.075	0.0562	0.0437	0.0125
		0.08	0.0513	0.0405	0.0108
		0.09	0.0454	0.0356	0.0098
		0.1	0.0426	0.0335	0.0091
	1000	0.06	0.0446	0.0338	0.0108
		0.07	0.0387	0.0289	0.0098
		0.075	0.0376	0.0282	0.0094
		0.08	0.0362	0.0272	0.009
		0.09	0.0347	0.0263	0.0084
		0.1	0.0335	0.0259	0.0076

Table 102

The Changes in ASE across Proportions for Rare Outcome Cases in Probit Without Moderation Effect

Figure 45

ASE over TIE and TNIE for Rare Outcomes in Probit Without Moderation Effect



very small or 0 for the conditions with a small effect size. This meant that the MSE for $TNIE$ and TIE were nearly identical. Tables 71 and 104, and Tables 72 and 105 present this numerically for rare and non-rare outcome cases.

In rare outcome cases, the $coverage\,probabilities$ for $TNIE$ and TIE were quite similar between the two levels of the moderation effect. Without the presence of a moderation effect, the gap in the $coverage\,probabilities$ between $TNIE$ and TIE was much larger in the non-rare outcome cases under conditions with a large effect size. This pattern is depicted graphically in Figures 37 and 49, as well as Figures 38 and 50. In general, the $coverage$ for $TNIE$ was slightly lower than the $coverage$ for TIE under large

Effect Size	Sample Size	Proportions	ASE for TNIE	ASE for TIE	gap
small	350	0.06	0.0548	0.0487	0.0061
		0.07	0.0478	0.0458	0.002
		0.075	0.0449	0.0451	-2e-04
		0.08	0.044	0.0453	-0.0013
		0.09	0.0441	0.0459	-0.0018
		0.1	0.0459	0.0472	-0.0013
	700	0.06	0.0331	0.0291	0.004
		0.07	0.0297	0.0282	0.0015
		0.075	0.0282	0.028	2e-04
		0.08	0.0276	0.0281	-5e-04
		0.09	0.0278	0.0285	-7e-04
		0.1	0.0288	0.0292	-4e-04
	1000	0.06	0.0265	0.0233	0.0032
		0.07	0.0239	0.0227	0.0012
		0.075	0.0228	0.0226	2e-04
		0.08	0.0223	0.0226	-3e-04
		0.09	0.0224	0.0229	-5e-04
		0.1	0.0231	0.0234	-3e-04

Table 103

The Changes in ASE across Proportions for Non-rare Outcome Cases in Probit Without Moderation Effect

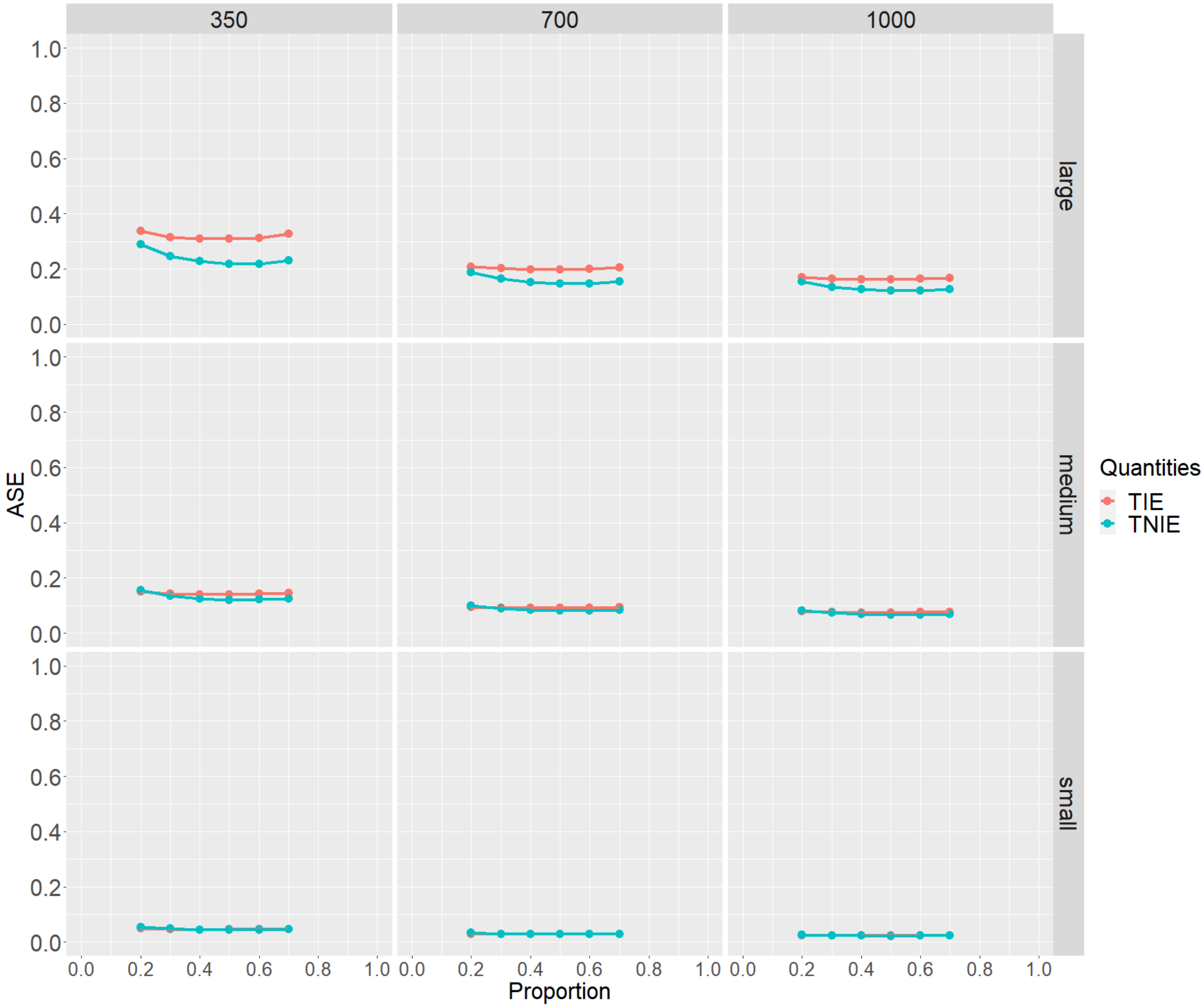
Effect Size	Sample Size	Proportions	MSE for TNIE	MSE for TIE	gap
small	350	0.06	0.0236	0.0172	0.0064
		0.07	0.0083	0.005	0.0033
		0.075	0.0073	0.0043	0.003
		0.08	0.0074	0.0044	0.003
		0.09	0.0044	0.0025	0.0019
		0.1	0.0039	0.0023	0.0016
	700	0.06	0.0021	0.0011	0.001
		0.07	0.0019	0.001	9e-04
		0.075	0.0018	0.001	8e-04
		0.08	0.0018	0.001	8e-04
		0.09	0.0016	9e-04	7e-04
		0.1	0.0015	9e-04	6e-04
	1000	0.06	0.0014	7e-04	7e-04
		0.07	0.0013	7e-04	6e-04
		0.075	0.0012	7e-04	5e-04
		0.08	0.0012	6e-04	6e-04
		0.09	0.0011	6e-04	5e-04
		0.1	0.001	6e-04	4e-04

Table 104

The Changes in MSE across Proportions for Rare Outcome Cases in Probit Without Moderation Effect

Figure 46

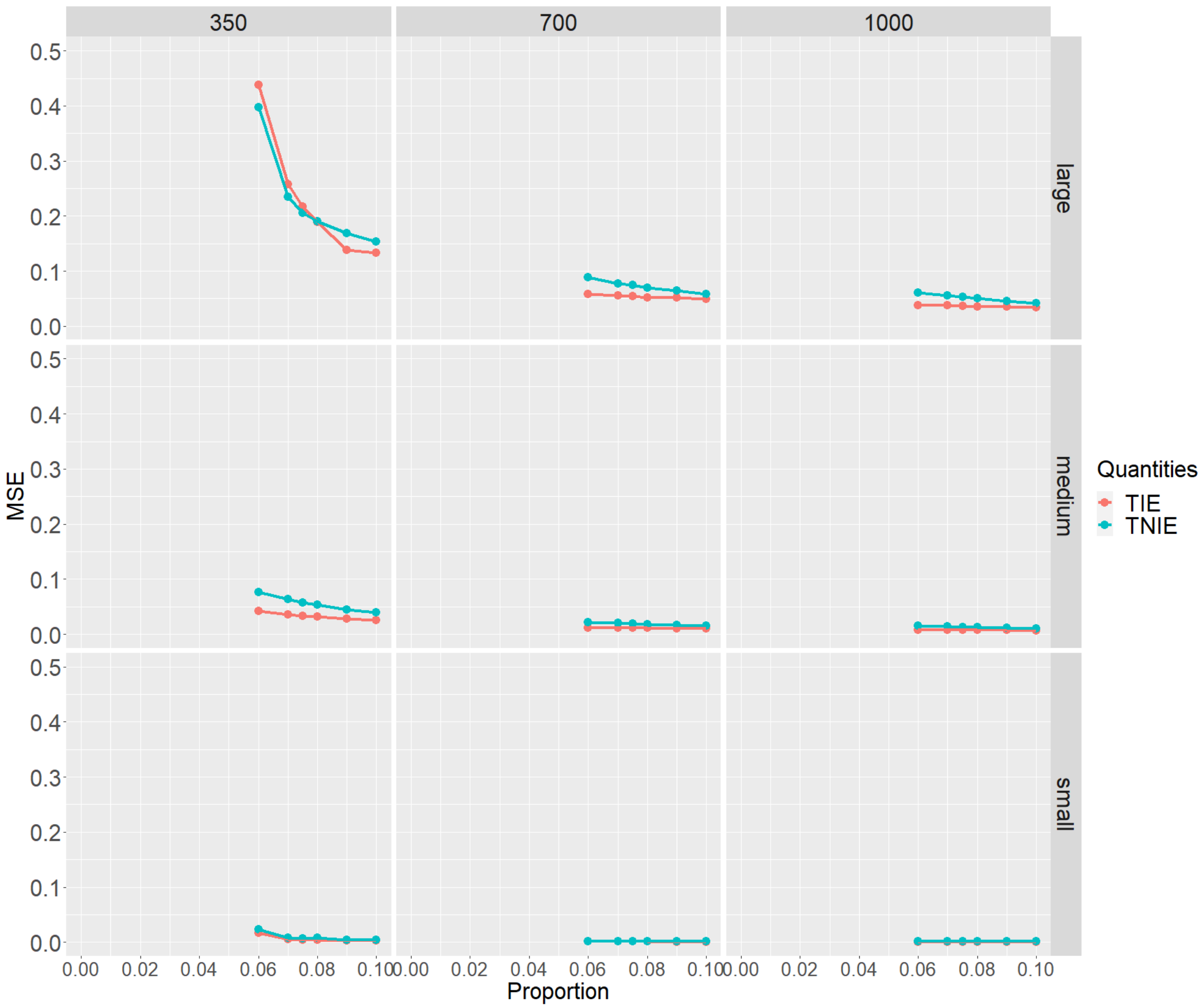
ASE over TIE and TNIE for Non-rare Outcomes in Probit Without Moderation Effect



effect size conditions and much higher than the *coverage* for TIE under small and medium effect size conditions.

The gaps in the *power* were larger in rare outcome cases when no moderation effect was present and a sample size of 350 was used. The *power* for $TNIE$ remained constant at both levels of the moderation effect, whereas the *power* for TIE was reduced. In all other cases, the *power* of the two quantities remained constant at 1. The *power* for the two quantities was 1 in non-rare outcome cases without the presence of a moderation effect. This pattern is depicted graphically in Figures 39 and 51, as well as Figures 40 and 52.

Figure 47
MSE over TIE and TNIE for Rare Outcomes in Probit Without Moderation Effect

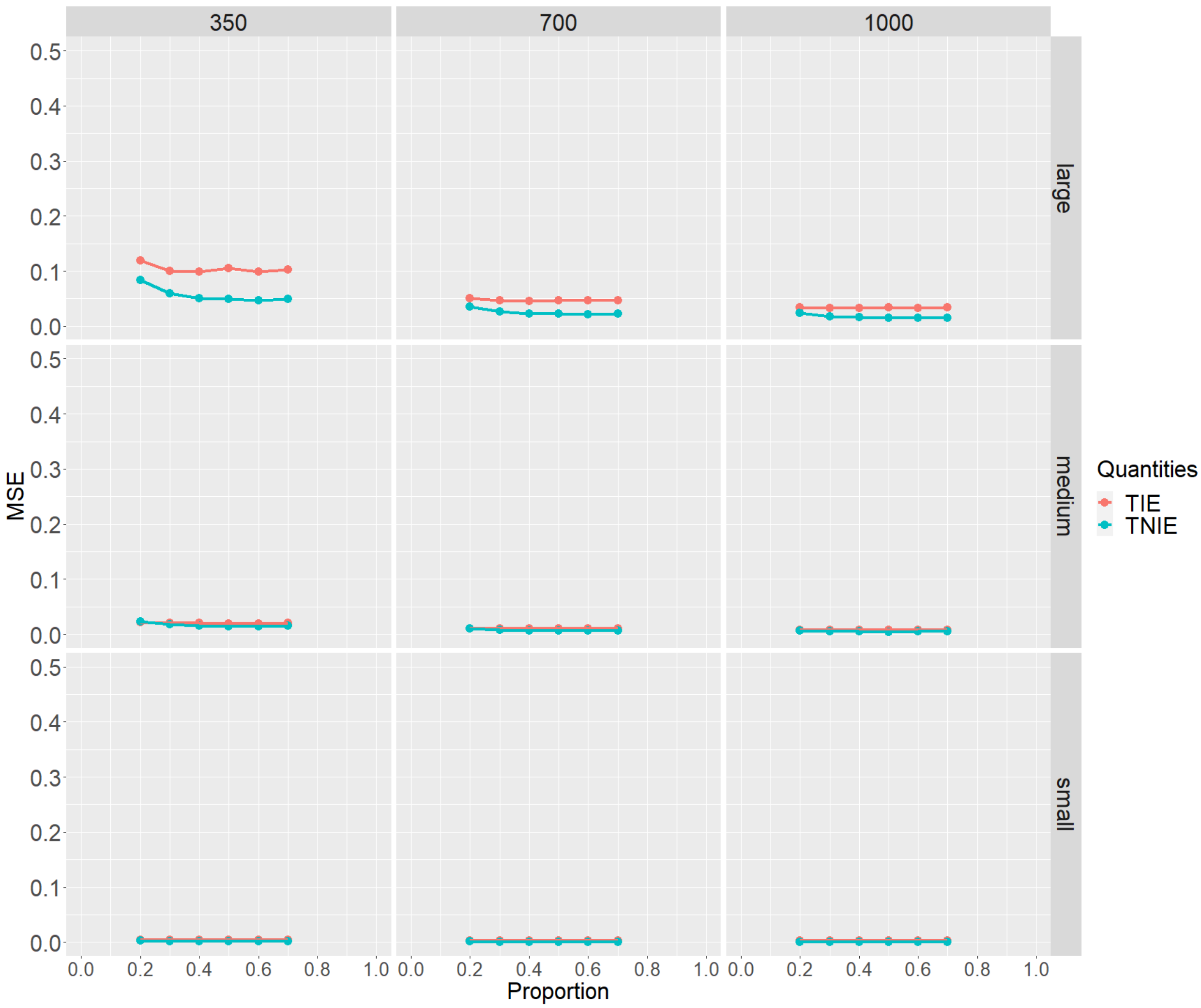


Logit Model

Overall, in the logit model analysis, $TNIE$ was more accurate than TIE in recovering the population values of the mediation effect in both rare and non-rare outcome cases for the first four evaluation criteria. The results of TIE were greater than those of $TNIE$ for *coverage probability*. For *power*, $TNIE$ outperformed TIE only when the sample size was 350, and the results of the two quantities were identical in all other conditions. Furthermore, the difference between $TNIE$ and TIE was smaller in non-rare cases than in rare cases.

Figure 48

MSE over TIE and TNIE for Non-rare Outcomes in Probit Without Moderation Effect



Proportion of Y = 1. When the proportion of $Y = 1$ increased in rare outcome cases, the *bias* for $TNIE$ and TIE got closer to 1 while the gap changed its pattern twice. In more detail, as the proportion increased from 0.06 to 0.08, the gap shrank; as the proportion approached 0.09, the gap widened; and finally, as the proportion reached 0.1, the gap shrank again. The gap was positive because the *bias* for $TNIE$ was closer to 1 than the *bias* for TIE. This was visible under conditions with large and medium effect sizes. The conditions with a large effect size and a sample size of 350 are numerically presented in table 106. Figure 53 depicts this graphically for all conditions. The difference between the two quantities was too small to be captured in the figure when the effect size

Effect Size	Sample Size	Proportions	MSE for TNIE	MSE for TIE	gap
small	350	0.2	0.0025	0.0039	-0.0014
		0.3	0.0019	0.0037	-0.0018
		0.4	0.0018	0.0038	-0.002
		0.5	0.0017	0.0038	-0.0021
		0.6	0.0017	0.0037	-0.002
		0.7	0.0017	0.0038	-0.0021
	700	0.2	0.001	0.0029	-0.0019
		0.3	8e-04	0.0029	-0.0021
		0.4	7e-04	0.0029	-0.0022
		0.5	7e-04	0.0029	-0.0022
		0.6	7e-04	0.0028	-0.0021
		0.7	8e-04	0.0029	-0.0021
	1000	0.2	6e-04	0.0027	-0.0021
		0.3	5e-04	0.0026	-0.0021
		0.4	5e-04	0.0026	-0.0021
		0.5	5e-04	0.0026	-0.0021
		0.6	5e-04	0.0026	-0.0021
		0.7	5e-04	0.0026	-0.0021

Table 105

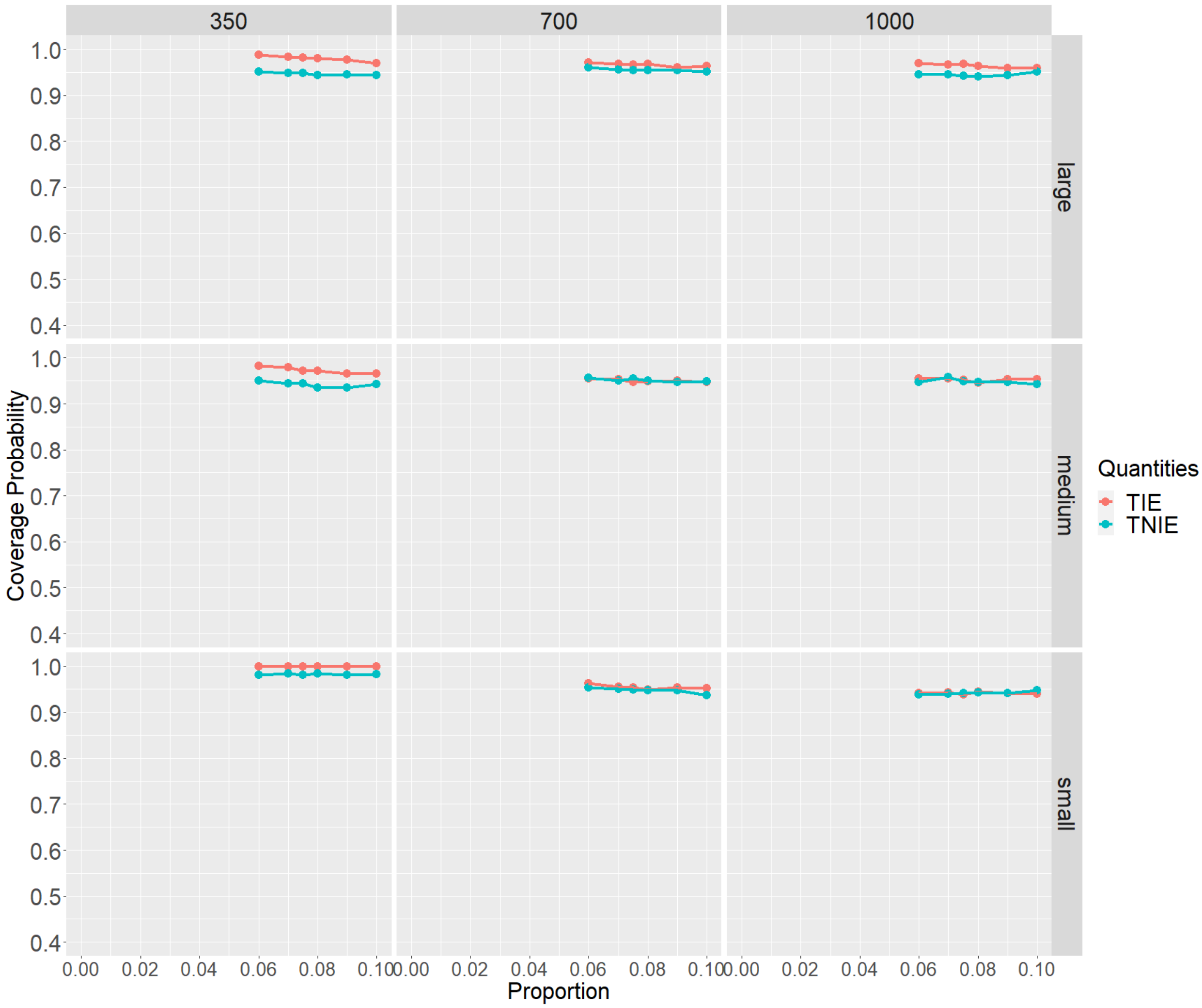
The Changes in MSE under Different Proportions for Non-rare Outcome Cases in Probit Without Moderation Effect

was small. Thus , I present numerically in Table 107. The *bias* for both quantities was greater than 1, and the gap was negative. This implied that the *bias* for $TNIE$ was closer to 1 than it was for TIE.

In non-rare outcome cases, there was no clear pattern for the *bias*. The *bias* for the two quantities increased as the proportion of $Y = 1$ increased from 0.2 to 0.4. When the proportion was increased to 0.5, the *bias* for $TNIE$ moved further away from one, while the *bias* for TIE remained close to one. The *bias* for the two quantities grew further away from 1 as the proportion increased to 0.7. As the proportion increased, the gap shrank. The gaps, like the rare outcome cases, were all positive. That is, the *bias* for $TNIE$ was closer to 1 than it was for TIE. The conditions with a large effect size and a sample size of 350 are numerically presented in Table 108. Figure 54 graphically depicts the pattern for all conditions. The difference between the two quantities was too small to capture under conditions with a small effect size. As a result, in Table 109, I numerically present the

Figure 49

Coverage over TIE and TNIE for Rare Outcomes in Probit Without Moderation Effect



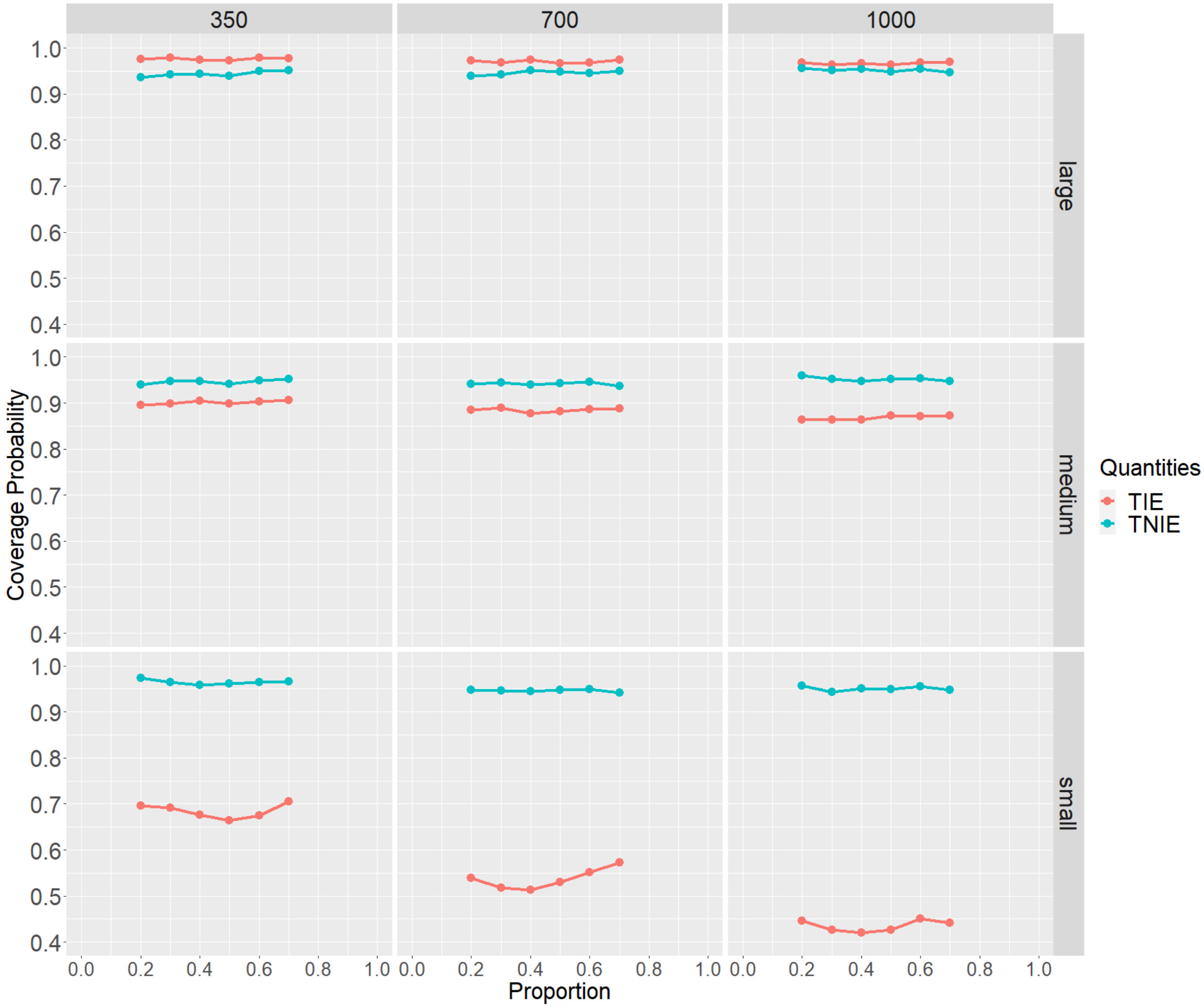
Effect Size	Sample Size	Proportions	Bias for TNIE	Bias for TIE	gap
large	350	0.06	0.8201	0.7734	0.0467
		0.07	0.8266	0.7797	0.0469
		0.075	0.8312	0.7841	0.0471
		0.08	0.8334	0.7857	0.0477
		0.09	0.8392	0.7922	0.047
		0.1	0.8441	0.7962	0.0479

Table 106

The Changes in Bias under Different Proportions for Rare Outcome Cases in Logit with Moderation Effect

Figure 50

Coverage over TIE and TNIE for Non-rare Outcomes in Probit Without Moderation Effect



pattern under those conditions. In these conditions, the *bias* for the two quantities was greater than 1, and the gap was negative. This indicated that the *bias* for $TNIE$ was closer to 1 than it was for TIE. In brief summary, as the proportion of $Y = 1$ increased, the *bias* for $TNIE$ and TIE became closer to 1 for the rare outcome cases but not for the non-rare cases. Furthermore, in both outcome cases, the *bias* for $TNIE$ got closer to 1 than the *bias* for TIE across all conditions.

Effect Size	Sample Size	Proportions	Bias for TNIE	Bias for TIE	gap
small	350	0.06	1.007	1.0072	-2e-04
		0.07	1.0074	1.0077	-3e-04
		0.075	1.0071	1.0074	-3e-04
		0.08	1.0068	1.0071	-3e-04
		0.09	1.0067	1.007	-3e-04
		0.1	1.0065	1.0067	-2e-04
	700	0.06	1.008	1.0082	-2e-04
		0.07	1.0077	1.0079	-2e-04
		0.075	1.0077	1.0079	-2e-04
		0.08	1.0076	1.0078	-2e-04
		0.09	1.0075	1.0077	-2e-04
		0.1	1.0073	1.0075	-2e-04
	1000	0.06	1.008	1.0082	-2e-04
		0.07	1.0077	1.0079	-2e-04
		0.075	1.0077	1.0079	-2e-04
		0.08	1.0076	1.0078	-2e-04
		0.09	1.0075	1.0077	-2e-04
		0.1	1.0073	1.0075	-2e-04

Table 107

The Changes in Bias under Small Effect Size across Proportions for Rare Outcome Cases in Logit with Moderation Effect

Effect Size	Sample Size	Proportions	Bias for TNIE	Bias for TIE	gap
large	350	0.2	0.8729	0.8267	0.0462
		0.3	0.8811	0.8365	0.0446
		0.4	0.8828	0.8406	0.0422
		0.5	0.881	0.8417	0.0393
		0.6	0.874	0.8386	0.0354
		0.7	0.8602	0.8282	0.032

Table 108

The Changes in Bias under Different Proportions for Non-rare Outcome Cases in Logit with Moderation Effect

Figure 51

Power over TIE and TNIE for Rare Outcomes in Probit Without Moderation Effect

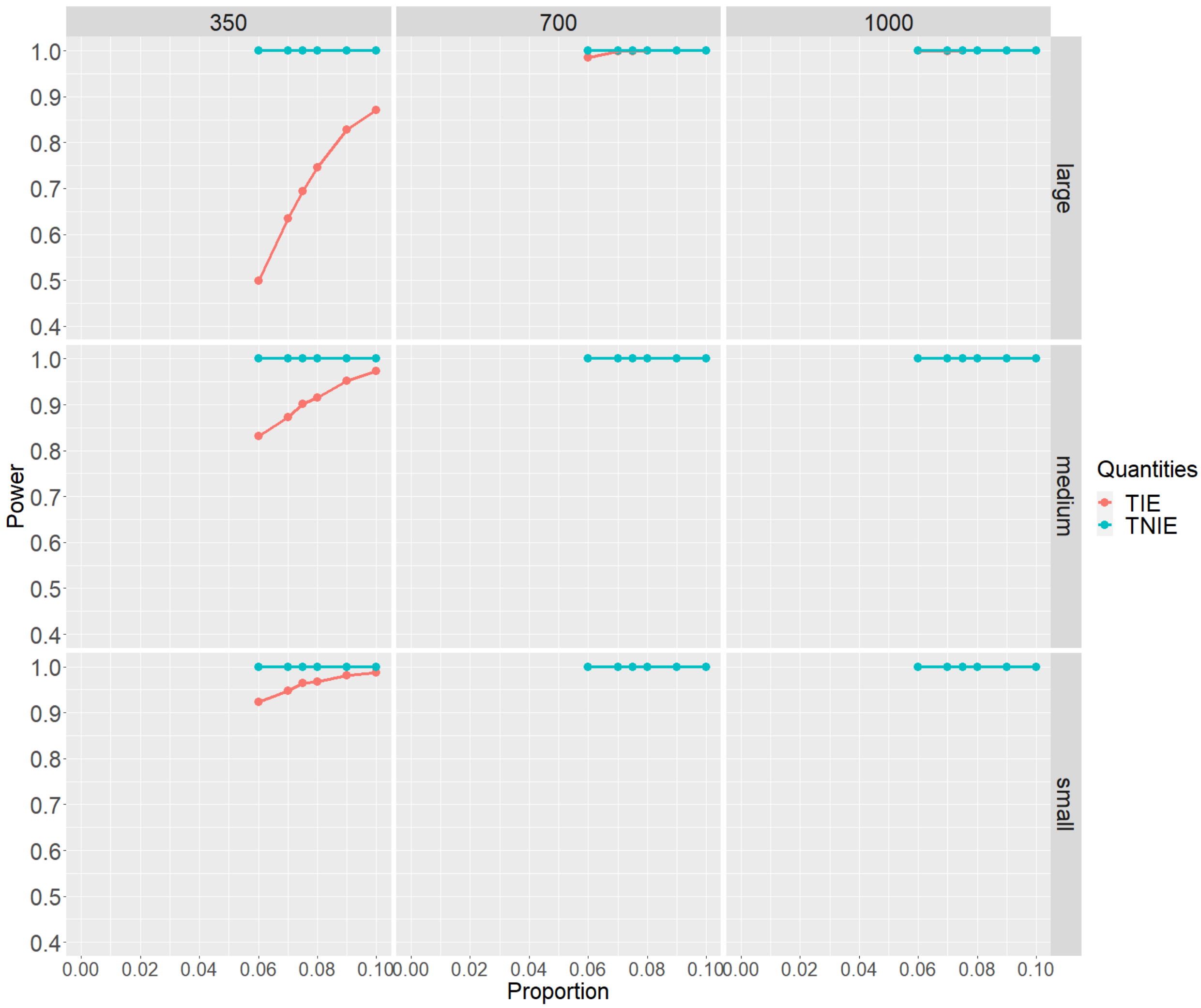


Figure 52

Power over TIE and TNIE for Non-rare Outcomes in Probit Without Moderation Effect

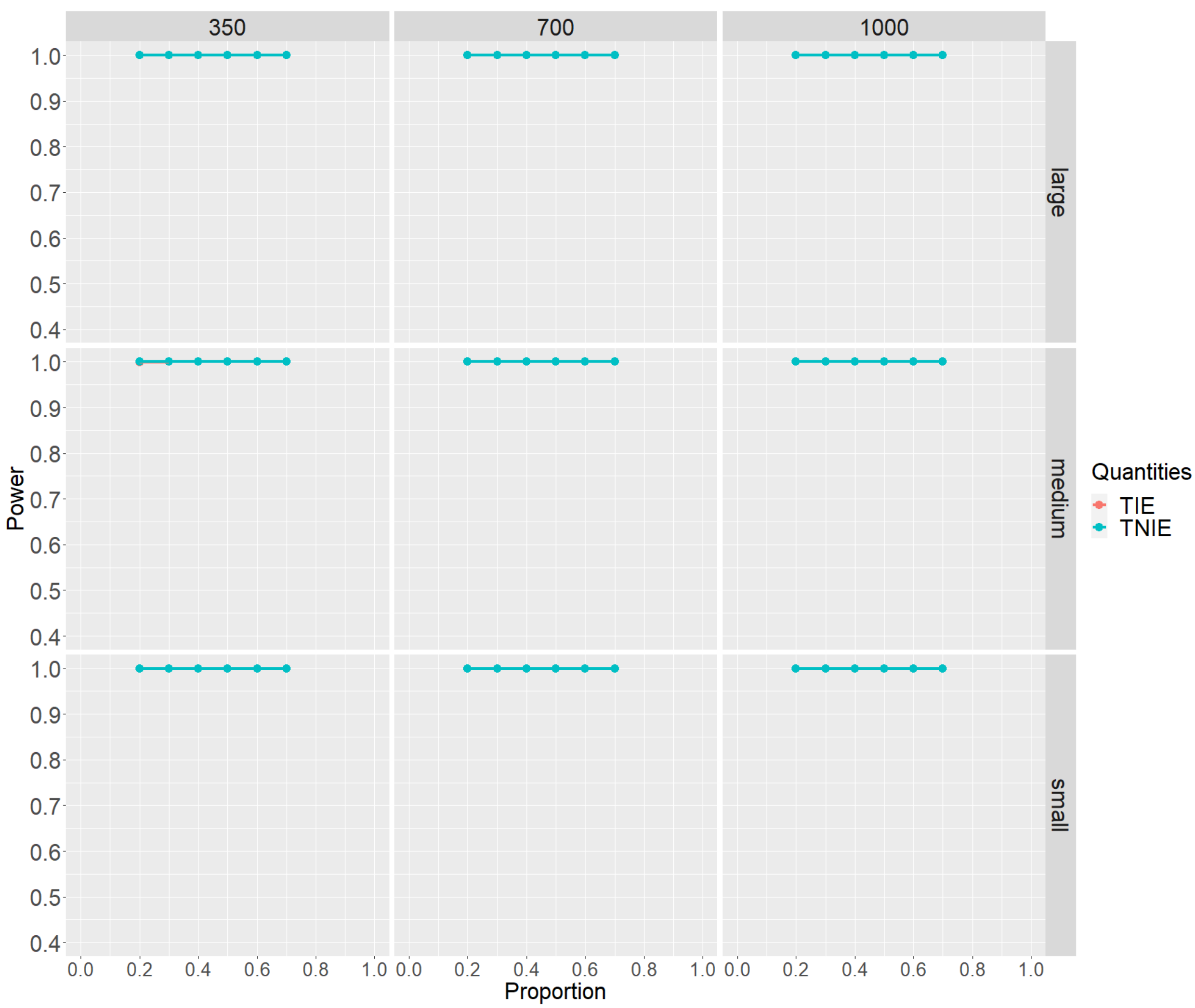
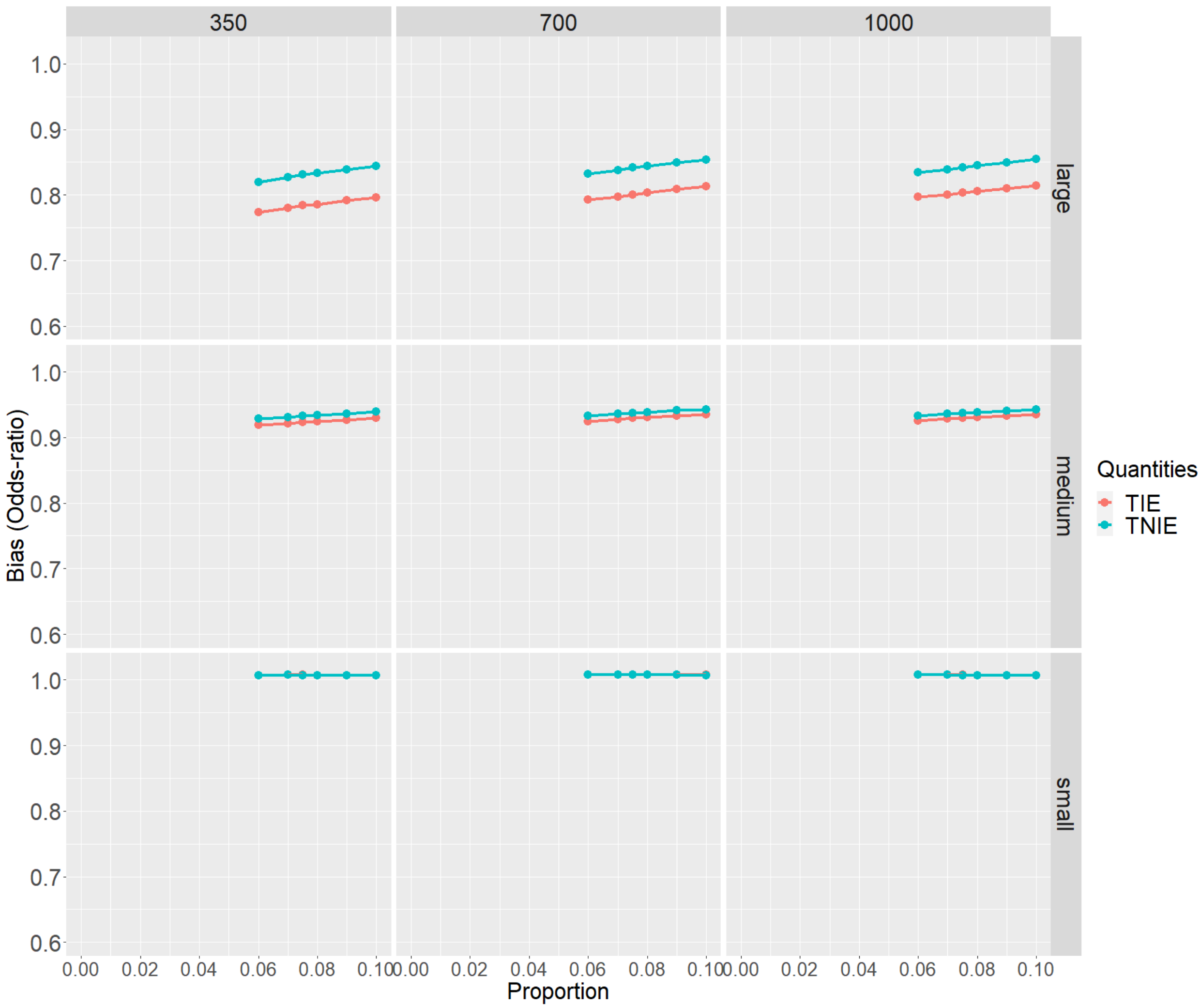


Figure 53

Bias over TIE and TNIE for Rare Outcomes in Logit with Moderation Effect



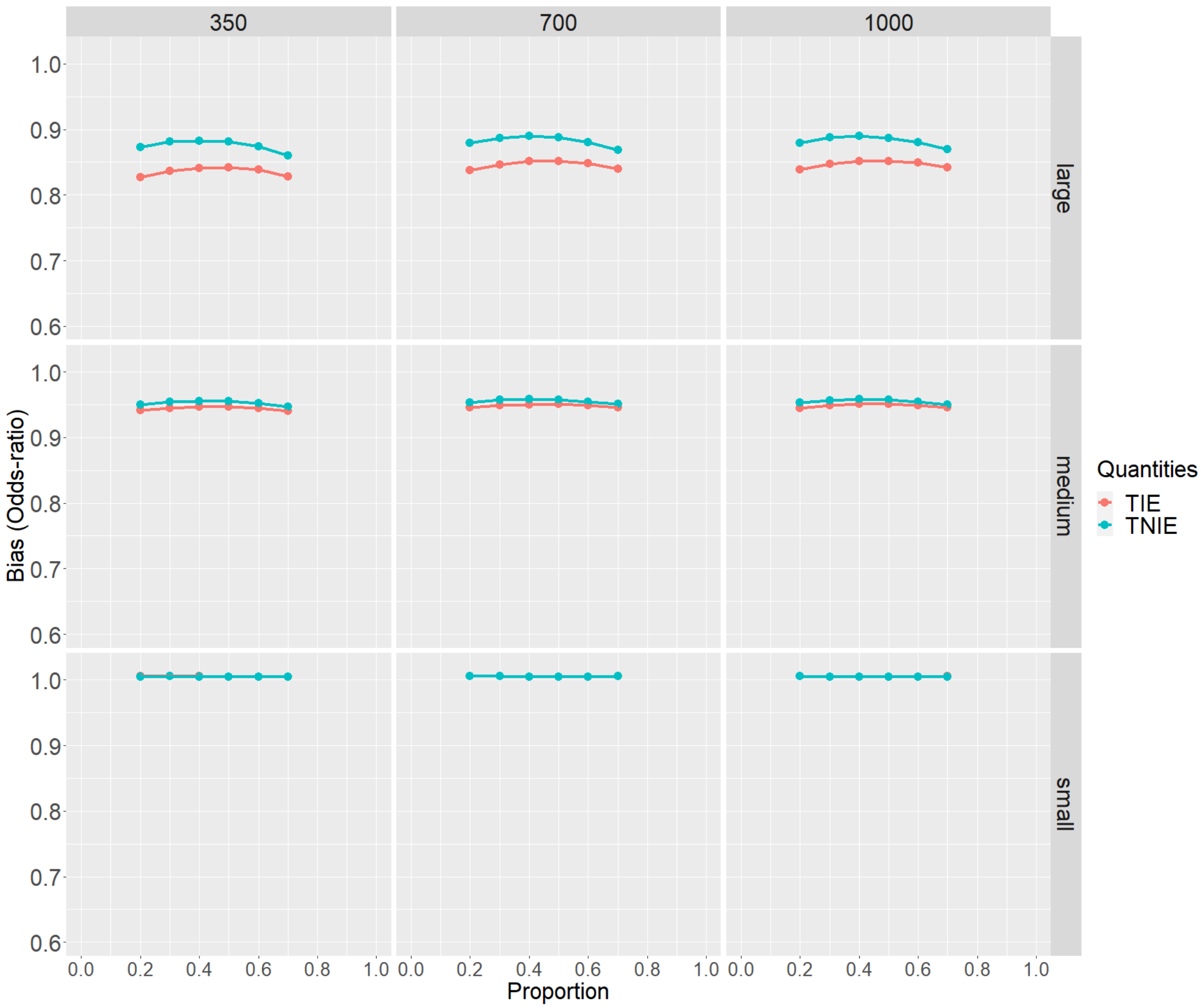
Effect Size	Sample Size	Proportions	Bias for TNIE	Bias for TIE	gap
small	350	0.2	1.0052	1.0055	-3e-04
		0.3	1.0056	1.0058	-2e-04
		0.4	1.0052	1.0055	-3e-04
		0.5	1.005	1.0052	-2e-04
		0.6	1.005	1.0052	-2e-04
		0.7	1.0051	1.0052	-1e-04
	700	0.2	1.0057	1.0059	-2e-04
		0.3	1.0053	1.0055	-2e-04
		0.4	1.005	1.0052	-2e-04
		0.5	1.0049	1.005	-1e-04
		0.6	1.005	1.0051	-1e-04
		0.7	1.0053	1.0054	-1e-04
	1000	0.2	1.0057	1.0059	-2e-04
		0.3	1.0053	1.0055	-2e-04
		0.4	1.005	1.0052	-2e-04
		0.5	1.0049	1.005	-1e-04
		0.6	1.005	1.0051	-1e-04
		0.7	1.0053	1.0054	-1e-04

Table 109

The Changes in Bias under under Small Effect Size across Proportions for Non-rare Outcome Cases in Logit with Moderation Effect

Figure 54

Bias over TIE and TNIE for Non-rare Outcomes in Logit with Moderation Effect



The SE showed similar patterns to the *bias*, but in the opposite direction. In the rare cases, as the proportion increased, the SE for the two quantities decreased while the gap between them increased. When the sample size was 350 and the effect size was large, this was clearly visible. Across all proportions of $Y = 1$, the SE for $TNIE$ was always less than that for TIE. The gaps were all negative, as shown in Table 110. Figure 55 graphically represented the pattern for all conditions. Because the difference between the two quantities was too small to see on the graph under the conditions of a small effect size, I present it numerically in Table 111.

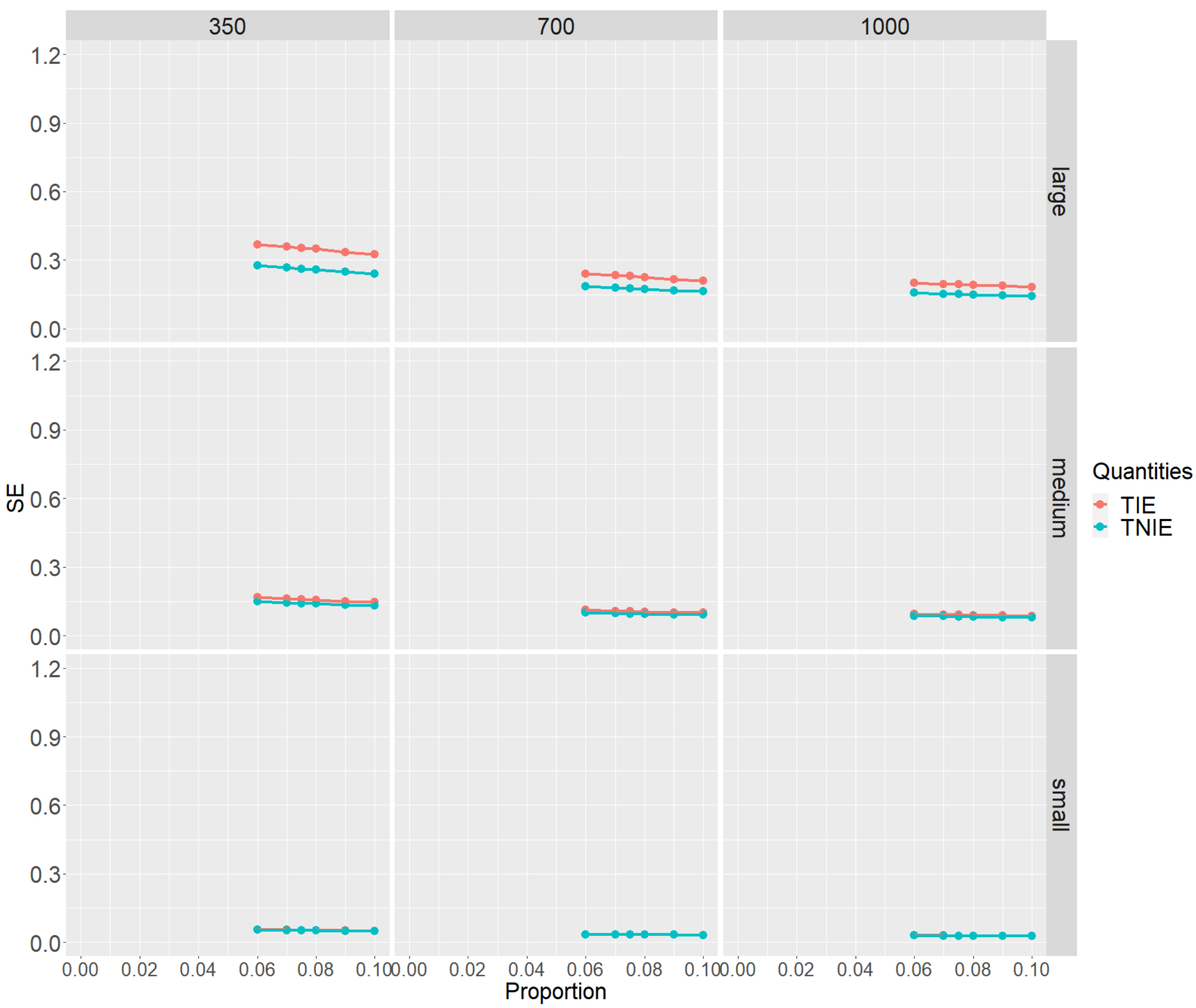
Effect Size	Sample Size	Proportions	SE for TNIE	SE for TIE	gap
large	350	0.06	0.2772	0.3673	-0.0901
		0.07	0.2672	0.3584	-0.0912
		0.075	0.2608	0.3515	-0.0907
		0.08	0.2579	0.3488	-0.0909
		0.09	0.2495	0.3349	-0.0854
		0.1	0.2414	0.3259	-0.0845

Table 110

The Changes in SE under Different Proportions for Rare Outcome Cases in Logit with Moderation Effect

Figure 55

SE over TIE and TNIE for Rare Outcomes in Logit with Moderation Effect



Effect Size	Sample Size	Proportions	SE for TNIE	SE for TIE	gap
small	350	0.06	0.0543	0.0559	-0.0016
		0.07	0.0528	0.0543	-0.0015
		0.075	0.0522	0.0538	-0.0016
		0.08	0.0512	0.0527	-0.0015
		0.09	0.0498	0.0513	-0.0015
		0.1	0.0486	0.05	-0.0014
	700	0.06	0.0348	0.0353	-5e-04
		0.07	0.0338	0.0343	-5e-04
		0.075	0.0333	0.0339	-6e-04
		0.08	0.0332	0.0338	-6e-04
		0.09	0.0332	0.0338	-6e-04
		0.1	0.0321	0.0327	-6e-04
	1000	0.06	0.0304	0.0308	-4e-04
		0.07	0.0295	0.0299	-4e-04
		0.075	0.0291	0.0295	-4e-04
		0.08	0.0291	0.0295	-4e-04
		0.09	0.0286	0.029	-4e-04
		0.1	0.028	0.0285	-5e-04

Table 111

The Changes in SE under Small Effect Size across Proportions for Rare Outcome Cases in Logit with Moderation Effect

Cases with non-rare outcomes followed a different pattern. The SE for the two quantities decreased as the proportion for $Y = 1$ increased from 0.2 to 0.4. The SE for the two quantities increased as the proportion approached 0.5. When the proportion was increased to 0.7, the SE for the two quantities increased further. Table 112 shows this pattern with a sample size of 350 and a large effect size. Figure 56 depicts this graphically for all conditions. Because the difference between the two quantities was too small with such a small effect size, I presented it numerically in Table 113. The gaps were negative across all conditions in both outcome cases. This implied that the SE for $TNIE$ was always less than the SE for TIE.

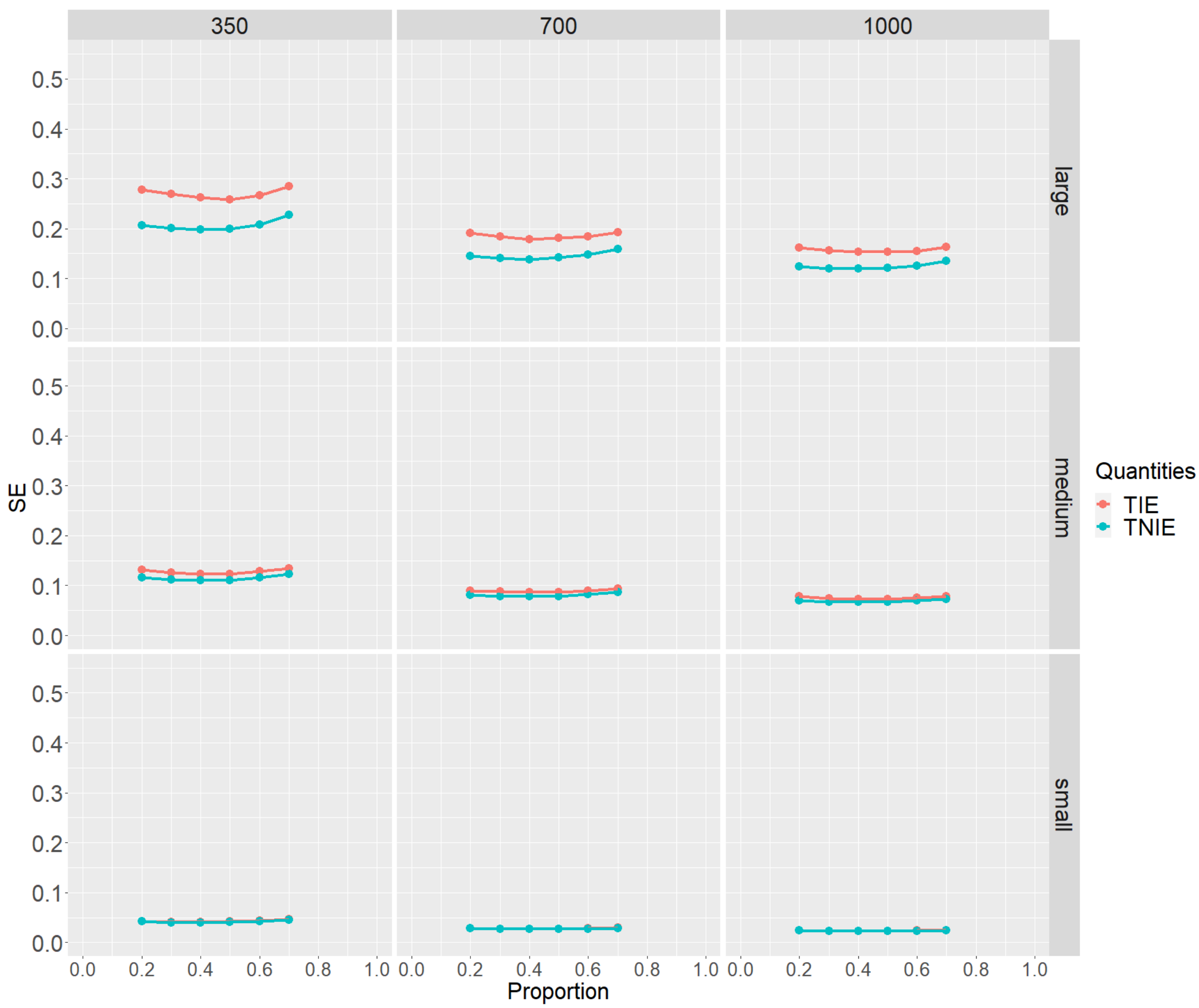
Effect Size	Sample Size	Proportions	SE for TNIE	SE for TIE	gap
large	350	0.2	0.2062	0.2784	-0.0722
		0.3	0.2005	0.2695	-0.069
		0.4	0.1979	0.2621	-0.0642
		0.5	0.1993	0.2586	-0.0593
		0.6	0.2085	0.266	-0.0575
		0.7	0.2274	0.2846	-0.0572

Table 112

The Changes in SE under Different Proportions for Non-rare Outcome Cases in Logit with Moderation Effect

Figure 56

SE over TIE and TNIE for Non-Rare Outcomes in Logit with Moderation Effect



In general, the SE for $TNIE$ was lower than the SE for TIE as the proportion of

Effect Size	Sample Size	Proportions	SE for TNIE	SE for TIE	gap
small	350	0.2	0.0418	0.0429	-0.0011
		0.3	0.04	0.041	-0.001
		0.4	0.0394	0.0405	-0.0011
		0.5	0.0408	0.0419	-0.0011
		0.6	0.0427	0.0437	-0.001
		0.7	0.0452	0.0461	-9e-04
	700	0.2	0.0278	0.0283	-5e-04
		0.3	0.0266	0.027	-4e-04
		0.4	0.0265	0.027	-5e-04
		0.5	0.0264	0.0268	-4e-04
		0.6	0.0276	0.0281	-5e-04
		0.7	0.0288	0.0292	-4e-04
	1000	0.2	0.0239	0.0243	-4e-04
		0.3	0.0227	0.0231	-4e-04
		0.4	0.0224	0.0227	-3e-04
		0.5	0.0225	0.0228	-3e-04
		0.6	0.0233	0.0236	-3e-04
		0.7	0.0235	0.0238	-3e-04

Table 113

The Changes in SE under Small Effect Size across Proportions for Non-rare Outcome Cases in Logit with Moderation Effect

$Y = 1$ increased in both outcome cases. Furthermore, in rare outcome cases, the difference between the two quantities was slightly larger.

The results for ASE were comparable to those for SE. Table 114 shows the pattern numerically in rare outcome cases with a small effect size. The pattern is depicted graphically in Figure 57. Table 115 presents numerically conditions with a small effect size for non-rare outcome cases, and Figure 58 presents the results graphically across all conditions.

MSE followed the same pattern as SE and ASE. Figures 59 and 60 depict rare and non-rare outcome cases graphically. As indicated by the negative gaps, the MSE for $TNIE$ was consistently lower than that for TIE across all conditions. Furthermore, under conditions with a small effect size, the MSE for the two quantities was nearly identical (for numerical representations, see Tables 116 and 117).

When the proportion of $Y = 1$ increased, the results for *coverage probability* did not

Effect Size	Sample Size	Proportions	ASE for TNIE	ASE for TIE	gap
large	350	0.06	0.0732	0.0764	-0.0032
		0.07	0.0691	0.0721	-0.003
		0.075	0.0677	0.0705	-0.0028
		0.08	0.066	0.0687	-0.0027
		0.09	0.0634	0.066	-0.0026
		0.1	0.0612	0.0636	-0.0024
	700	0.06	0.0416	0.0426	-0.001
		0.07	0.0399	0.0409	-0.001
		0.075	0.0393	0.0403	-0.001
		0.08	0.0386	0.0396	-0.001
		0.09	0.0376	0.0386	-0.001
		0.1	0.0366	0.0376	-0.001
	1000	0.06	0.0325	0.0332	-7e-04
		0.07	0.0313	0.0319	-6e-04
		0.075	0.0307	0.0314	-7e-04
		0.08	0.0303	0.0309	-6e-04
		0.09	0.0295	0.0302	-7e-04
		0.1	0.0289	0.0295	-6e-04

Table 114

The Changes in ASE under Small Effect Size across Proportions for Rare Outcome Cases in Logit with Moderation Effect

Effect Size	Sample Size	Proportions	ASE for TNIE	ASE for TIE	gap
small	350	0.06	0.0514	0.0533	-0.0019
		0.07	0.0476	0.0493	-0.0017
		0.075	0.0464	0.0479	-0.0015
		0.08	0.0466	0.048	-0.0014
		0.09	0.0475	0.0488	-0.0013
		0.1	0.0499	0.0511	-0.0012
	700	0.06	0.0312	0.0319	-7e-04
		0.07	0.0293	0.03	-7e-04
		0.075	0.0288	0.0295	-7e-04
		0.08	0.0289	0.0295	-6e-04
		0.09	0.0297	0.0303	-6e-04
		0.1	0.0309	0.0315	-6e-04
	1000	0.06	0.0248	0.0253	-5e-04
		0.07	0.0235	0.0239	-4e-04
		0.075	0.0231	0.0236	-5e-04
		0.08	0.0232	0.0236	-4e-04
		0.09	0.0237	0.0241	-4e-04
		0.1	0.0247	0.025	-3e-04

Table 115

The Changes in ASE under Small Effect Size across Proportions for Non-rare Outcome Cases in Logit with Moderation Effect

Effect Size	Sample Size	Proportions	MSE for TNIE	MSE for TIE	gap
small	350	0.06	0.003	0.0032	-2e-04
		0.07	0.0028	0.003	-2e-04
		0.075	0.0028	0.0029	-1e-04
		0.08	0.0027	0.0028	-1e-04
		0.09	0.0025	0.0027	-2e-04
		0.1	0.0024	0.0025	-1e-04
	700	0.06	0.0013	0.0013	0
		0.07	0.0012	0.0012	0
		0.075	0.0012	0.0012	0
		0.08	0.0012	0.0012	0
		0.09	0.0011	0.0012	-1e-04
		0.1	0.0011	0.0011	0
	1000	0.06	0.001	0.001	0
		0.07	9e-04	9e-04	0
		0.075	9e-04	9e-04	0
		0.08	9e-04	9e-04	0
		0.09	9e-04	9e-04	0
		0.1	8e-04	9e-04	-1e-04

Table 116

The Changes in MSE under Small Effect Size across Proportions for Rare Outcome Cases in Logit with Moderation Effect

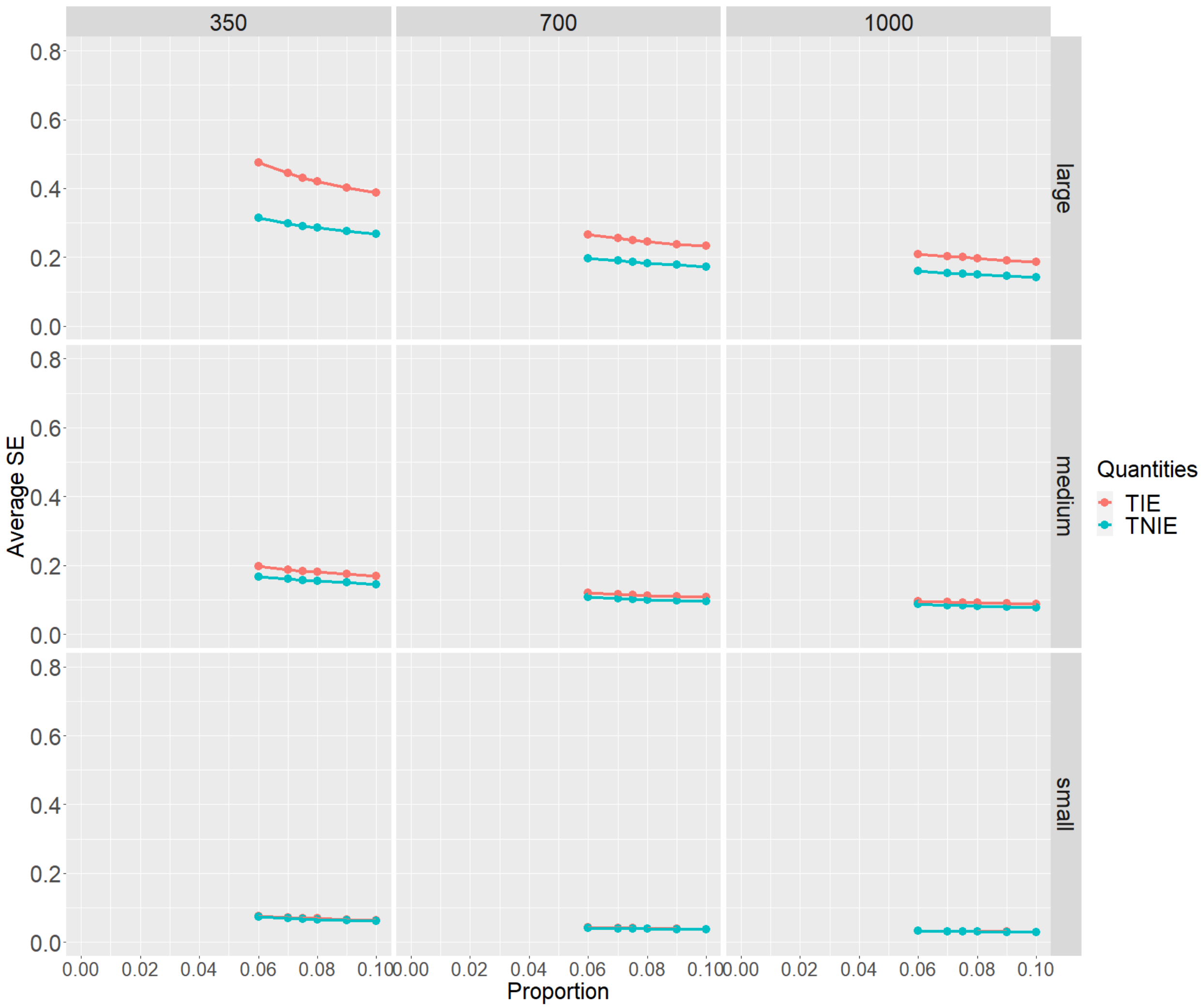
Effect Size	Sample Size	Proportions	MSE for TNIE	MSE for TIE	gap
small	350	0.2	0.0018	0.0019	-1e-04
		0.3	0.0016	0.0017	-1e-04
		0.4	0.0016	0.0017	-1e-04
		0.5	0.0017	0.0018	-1e-04
		0.6	0.0018	0.0019	-1e-04
		0.7	0.0021	0.0021	0
	700	0.2	8e-04	8e-04	0
		0.3	7e-04	8e-04	-1e-04
		0.4	7e-04	7e-04	0
		0.5	7e-04	7e-04	0
		0.6	8e-04	8e-04	0
		0.7	9e-04	9e-04	0
	1000	0.2	6e-04	6e-04	0
		0.3	5e-04	6e-04	-1e-04
		0.4	5e-04	5e-04	0
		0.5	5e-04	5e-04	0
		0.6	6e-04	6e-04	0
		0.7	6e-04	6e-04	0

Table 117

The Changes in MSE under Different Proportions for Non-rare Outcome Cases in Logit with Moderation Effect

Figure 57

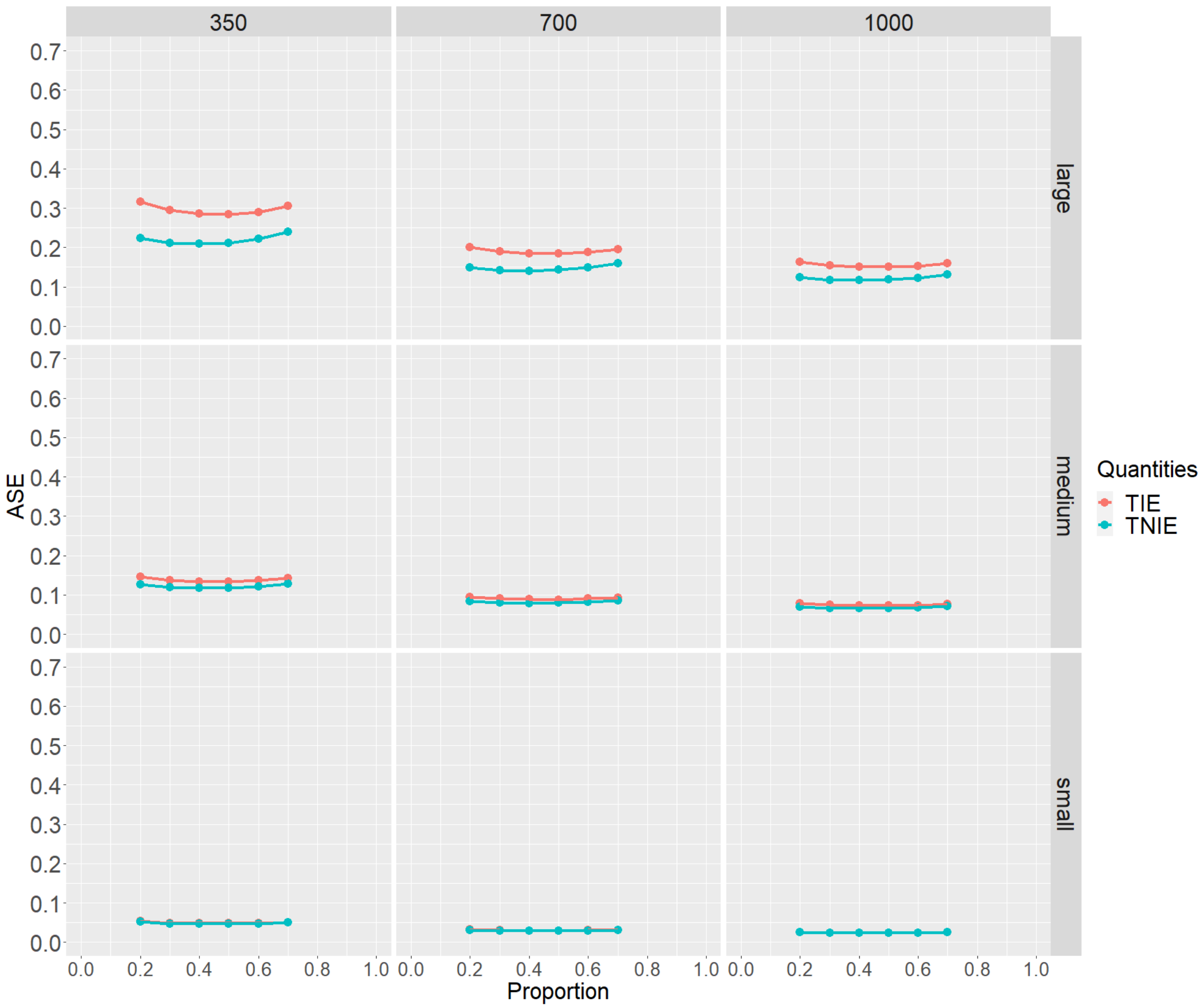
ASE over TIE and TNIE for Rare Outcomes in Logit with Moderation Effect



show any clear patterns. For example, in rare outcome cases with a sample size of 350 and a small effect size, the proportion for $Y = 1$ increased from 0.06 to 0.07, the *coverage* for the two quantities increased, and the gap between them remained constant. When the proportion raised to 0.075, the *coverage* for $TNIE$ decreased while the *coverage* for TIE increased, and the gap between them widened. As the proportion approached 0.8, the *coverage* for $TNIE$ increased while the *coverage* for TIE remained constant and the gap between them shrank. Then, as the proportion increased to 0.9, the *coverage* for $TNIE$ decreased while the *coverage* for TIE remained constant, increasing the gap between them. When the proportion reached 0.1, the *coverage* for $TNIE$ decreased further, while

Figure 58

ASE over TIE and TNIE for Rare Outcomes in Logit with Moderation Effect

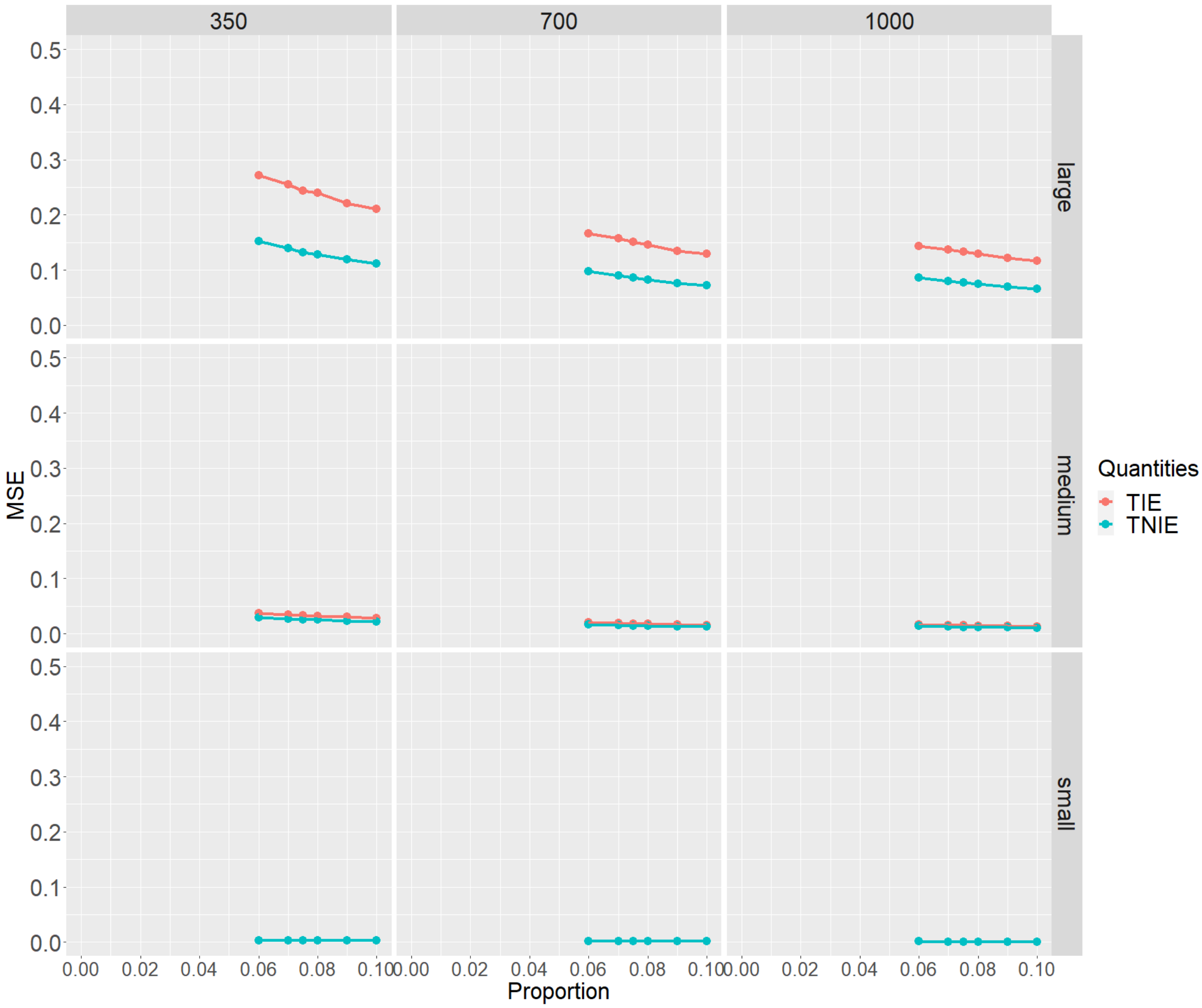


the *coverage* for TIE increased, and the gap between them widened. This pattern is numerically depicted in table 118. Figure 61 shows this graphically across all conditions. The gaps were positive, but they were quite small. This indicated that $TNIE$ had slightly less *coverage* than TIE across all proportion of $Y = 1$.

The gap between the two quantities in *coverage* showed a clearer pattern in the non-rare outcome cases than in the rare outcome cases. When the proportion of $Y = 1$ increased from 0.2 to 0.3 under conditions with a small effect size and a sample size of 350, the *coverage* for $TNIE$ decreased while the *coverage* for TIE remained the same and the gap increased. The *coverage* for $TNIE$ and TIE decreased as the proportion increased to

Figure 59

MSE over TIE and TNIE for Rare Outcomes in Logit with Moderation Effect



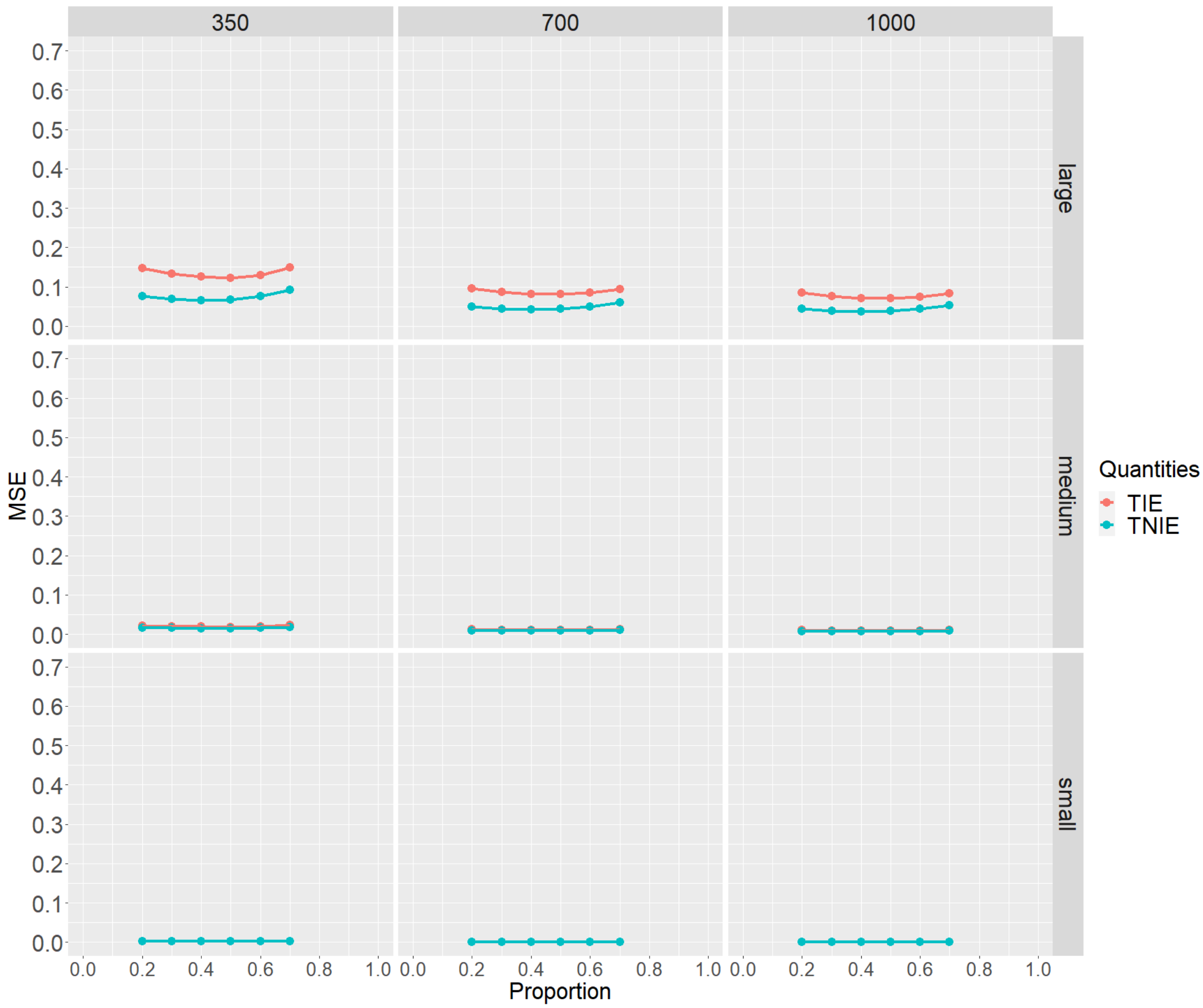
Effect Size	Sample Size	Proportions	Coverage for TNIE	Coverage for TIE	gap
large	350	0.06	0.992	0.996	-0.004
		0.07	0.993	0.997	-0.004
		0.075	0.992	0.998	-0.006
		0.08	0.993	0.998	-0.005
		0.09	0.991	0.998	-0.007
		0.1	0.989	0.999	-0.01

Table 118

The Changes in Coverage under Different Proportions for Rare Outcome Cases in Logit with Moderation Effect

Figure 60

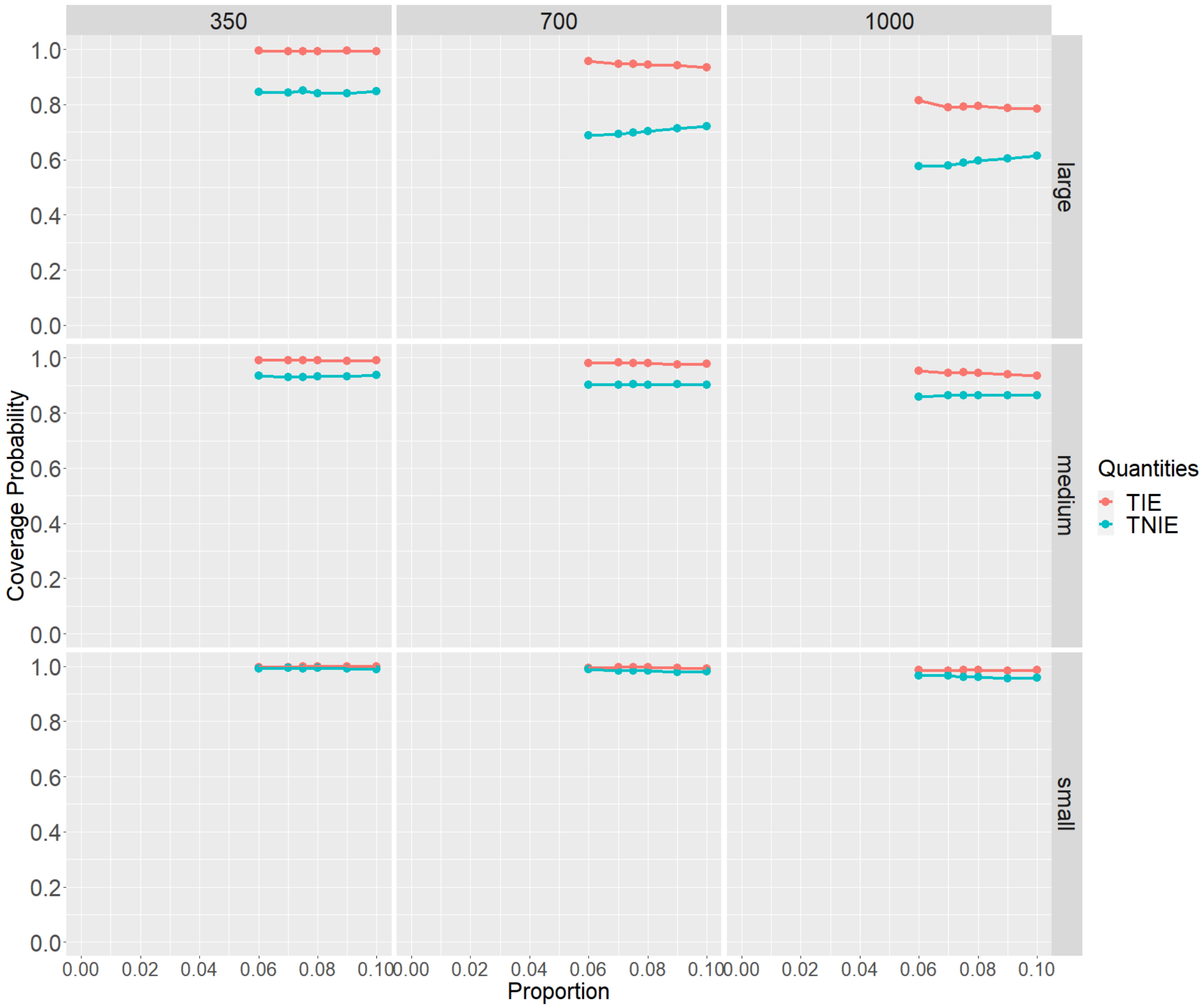
MSE over TIE and TNIE for Non-rare Outcomes in Logit with Moderation Effect



0.4, and the gap shrank. The proportion then increased to 0.5, the *coverage* for $TNIE$ decreased further, while the *coverage* for TIE remained constant, and the gap remained constant. The *coverage* for the two quantities decreased as the proportion approached 0.6, as did the gap. Finally, once the proportion reached 0.7, the *coverage* for $TNIE$ and TIE remained constant, as did the gap. For proportion, the gap was negative at all levels. This demonstrated that the *coverage* for $TNIE$ was slightly less than that for TIE. The pattern is shown numerically in Table 119, and graphically in Figure 62 across all conditions. The gap was negative but small in both outcome cases. That is, the *coverage* for $TNIE$ is slightly less than the *coverage* for TIE.

Figure 61

Coverage Probability over TIE and TNIE for Rare Outcomes in Logit with Moderation Effect



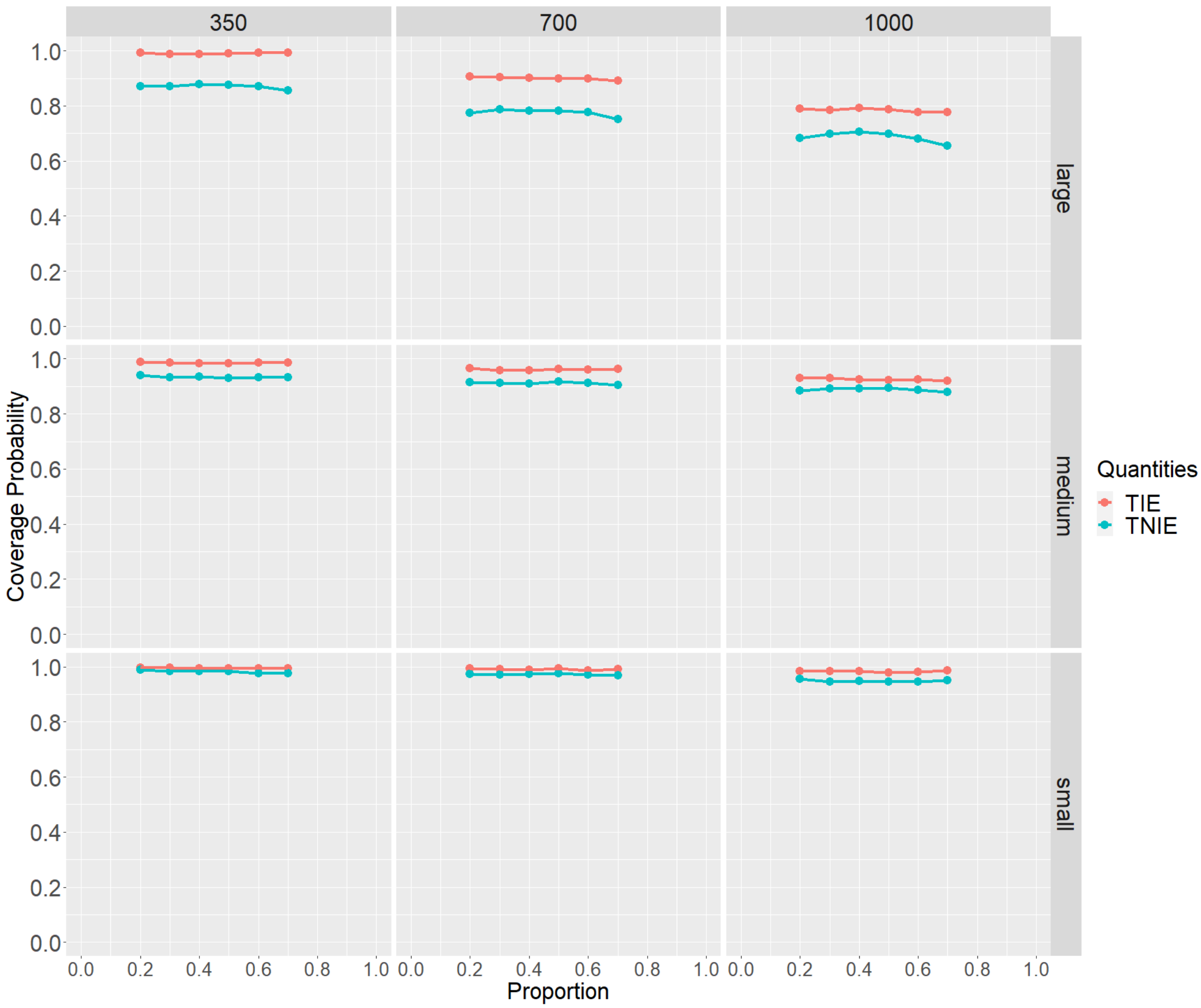
Effect Size	Sample Size	Proportions	Coverage for TNIE	Coverage for TIE	gap
large	350	0.2	0.988	0.997	-0.009
		0.3	0.985	0.997	-0.012
		0.4	0.984	0.994	-0.01
		0.5	0.984	0.994	-0.01
		0.6	0.977	0.993	-0.016
		0.7	0.977	0.993	-0.016

Table 119

The Changes in Coverage under Different Proportions for Non-rare Outcome Cases in Logit with Moderation Effect

Figure 62

Coverage Probability over TIE and TNIE for Non-rare Outcomes in Logit with Moderation Effect



In both outcome cases, the *power* for $TNIE$ and TIE was very high (above .95) across all conditions. For rare outcome cases, the *power* for $TNIE$ was 1 for all conditions and the *power* for TIE was all 1 except for conditions with a large effect size and a sample size of 350 (for a graphical representation, see Figure 63). As the proportion increased, the *power* for TIE increased, and the difference in *power* between the two quantities decreased. This is illustrated numerically in Table 120.

In non-rare cases, on the other hand, the *power* remained 1 for both $TNIE$ and TIE as the proportion increased. This can be seen numerically in Table 121 and

Effect Size	Sample Size	Proportions	Power for TNIE	Power for TIE	gap
large	350	0.06	1	0.981	0.019
		0.07	1	0.992	0.008
		0.075	1	0.994	0.006
		0.08	1	0.997	0.003
		0.09	1	0.998	0.002
		0.1	1	0.999	0.001

Table 120
The Changes in Power under Different Proportions for Rare Outcome Cases in Logit with Moderation Effect

graphically in Figure 64.

Effect Size. When the effect size increased, the gaps in the *bias* between $TNIE$ and TIE widened and the *bias* for each quantity moved further away from 1. Furthermore, when the effect size was small, the gap was negative but very small, and the *bias* for the two quantities was greater than 1; this indicated that the *bias* for $TNIE$ was slightly closer to 1 than the *bias* for TIE. Because the difference was so small (.0004), we could conclude that the *bias* for $TNIE$ and TIE was nearly identical. When the effect size was medium, the gap was positive, indicating that the *bias* for $TNIE$ was smaller than the *bias* for TIE. As the effect size increased, the gap widened and remained positive, indicating that the *bias* for $TNIE$ was clearly smaller than the *bias* for TIE. This pattern can be seen in both outcome cases. Tables 122 and 123 numerically present this pattern under conditions with a sample size of 350 and a proportion of 0.06 for rare outcome cases and 0.2 for non-rare outcome cases. Figures 53 and 54 show this graphically for all conditions.

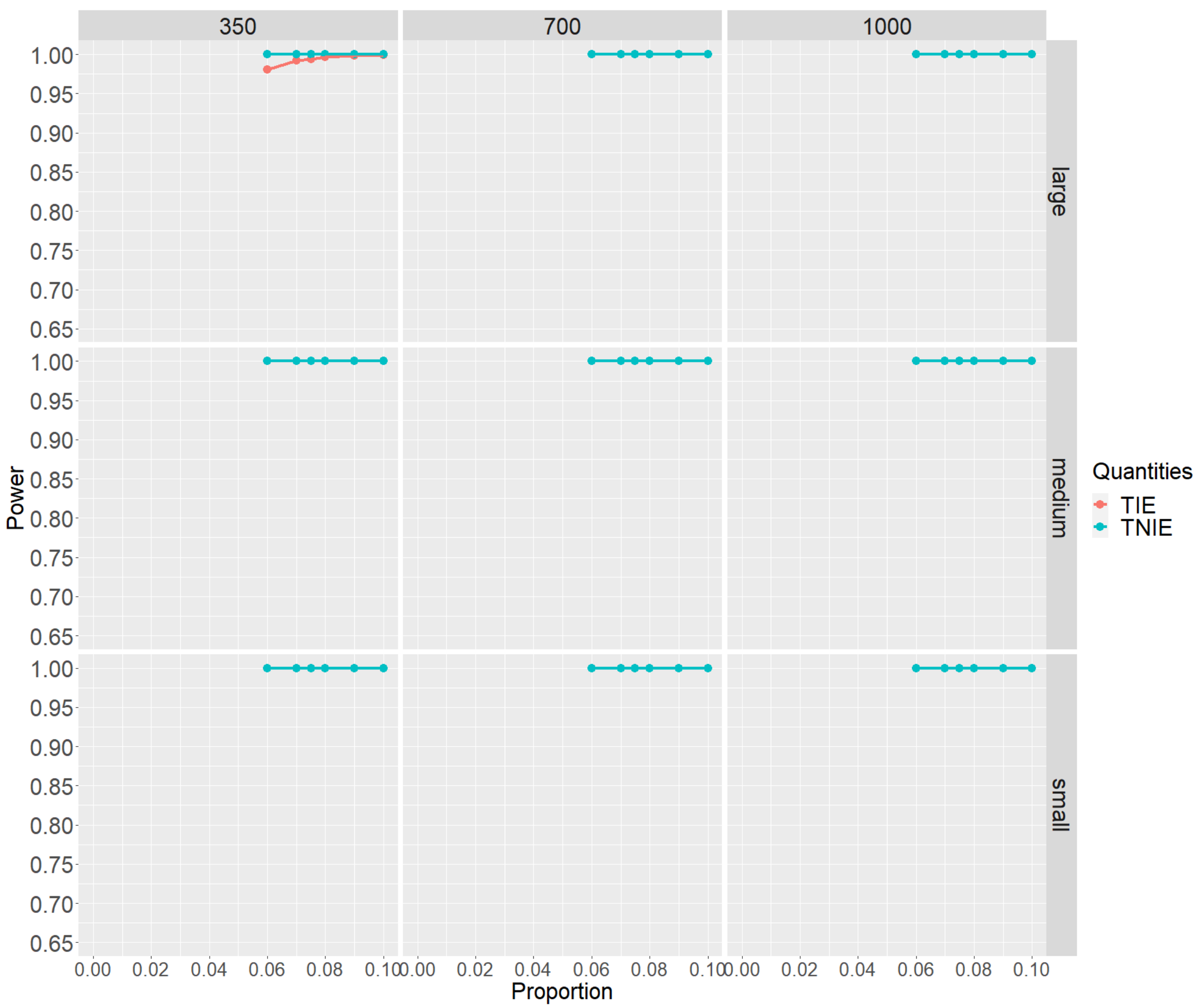
Effect Size	Sample Size	Proportions	Power for TNIE	Power for TIE	gap
large	350	0.2	1	1	0
		0.3	1	1	0
		0.4	1	1	0
		0.5	1	1	0
		0.6	1	1	0
		0.7	1	1	0

Table 121

The Changes in Power under Different Proportions for Non-rare Outcome Cases in Logit with Moderation Effect

Figure 63

Power over TIE and TNIE for Rare Outcomes in Logit with Moderation Effect



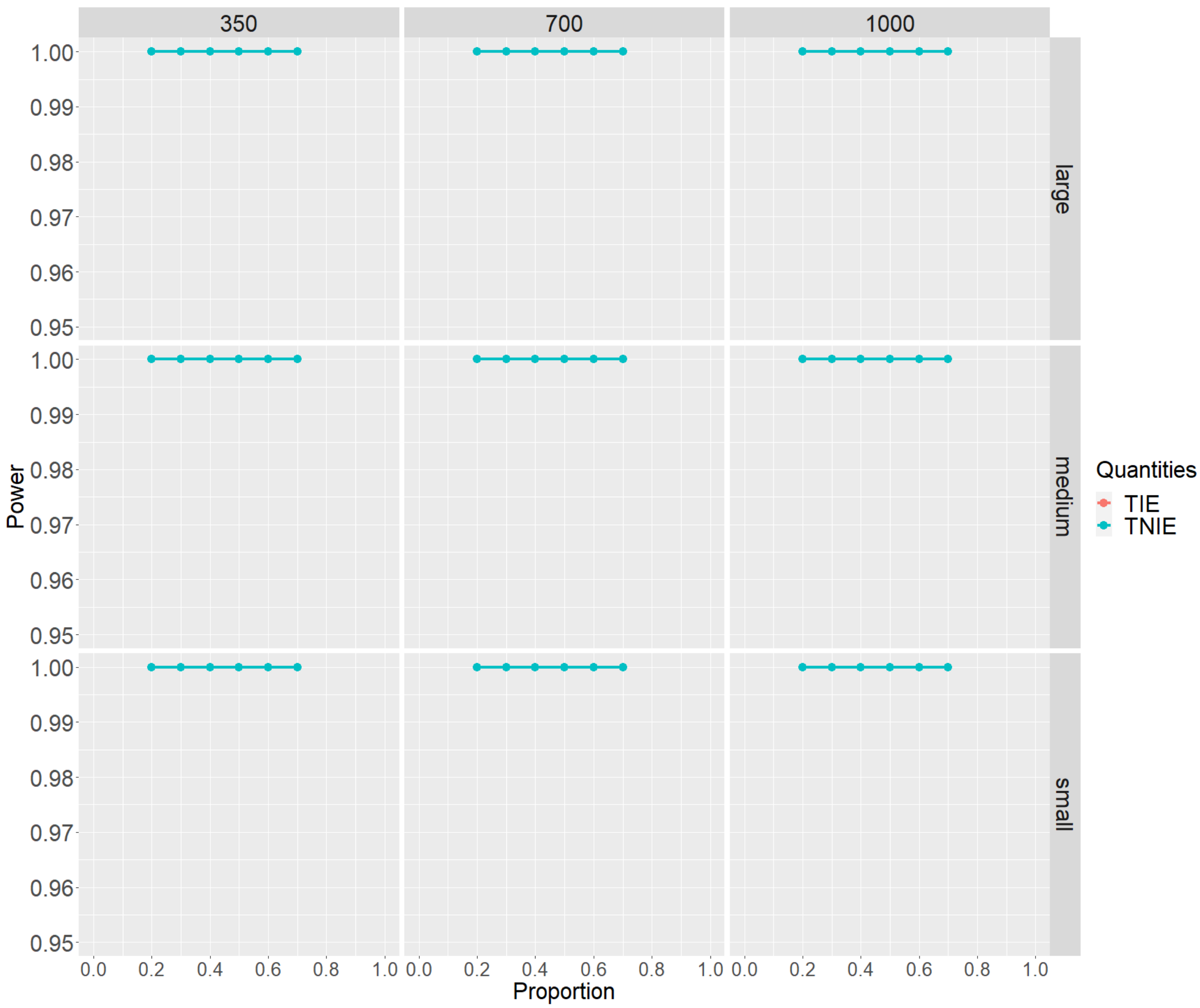
Proportion	Sample Size	Effect Size	Bias for TNIE	Bias for TIE	gap
0.06	350	small	1.007	1.0072	-2e-04
		medium	0.9282	0.9188	0.0094
		large	0.8201	0.7734	0.0467

Table 122

The Changes in Bias under Different Effect Size for Rare Outcome Cases in Logit with Moderation Effect

Figure 64

Power over TIE and TNIE for Non-rare Outcomes in Logit with Moderation Effect



When the effect size increased, the gaps in SE between two quantities and the SE for each quantity widened. More specifically, when the effect size was small, the SE for

Proportion	Sample Size	Effect Size	Bias for TNIE	Bias for TIE	gap
0.2	350	small	1.0052	1.0055	-3e-04
		medium	0.9503	0.941	0.0093
		large	0.8729	0.8267	0.0462

Table 123

The Changes in Bias under Different Effect Size in Non-rare Outcome Cases in Logit with Moderation Effect

$TNIE$ was slightly smaller than the SE for TIE so the gap was negative but small; when the effect size was medium, the SE was smaller than TIE because the gap grew larger but remained negative; and when the effect size was large, the SE for $TNIE$ was clearly smaller than the SE for TIE since the gap grew larger but remained negative. Under the conditions of a sample size of 350 and a proportion of 0.06 for a rare outcome case and 0.2 for a non-rare outcome case, this pattern can be seen in Tables 124 and 125. Figures 55 and 56 show graphical representations of all conditions.

The ASE results for the two quantities were the same as the SE results. As the effect size increased, the ASE for $TNIE$ became clearly smaller than the ASE for TIE. Tables 126 and 126 present this using a sample size of 350 and a proportion of 0.06 for a rare outcome case and 0.2 for a non-rare outcome case. This is depicted graphically for all conditions in Figures 57 and 58.

Similarly, the MSE for $TNIE$ was clearly smaller than the MSE for TIE across the effect size changes. This pattern is shown numerically in Tables 128 and 129, and graphically for all conditions in Figures 59 and 60.

The *coverage probability* for the two quantities decreased as the effect size increased, while the gap between them increased. More closely, when the effect size was small, the gap between them was negative but small, implying that $TNIE$ had slightly less *coverage* than TIE. When the effect size was medium, the gap grew slightly larger but remained negative, indicating that the *coverage* for $TNIE$ was less than that for TIE. When the

Proportion	Sample Size	Effect Size	SE for TNIE	SE for TIE	gap
0.06	350	small	0.0543	0.0559	-0.0016
		medium	0.1495	0.1674	-0.0179
		large	0.2772	0.3673	-0.0901

Table 124

The Changes in SE under Different Effect Size for Rare Outcome Cases in Logit with Moderation Effect

Proportion	Sample Size	Effect Size	SE for TNIE	SE for TIE	gap
0.2	350	small	0.0418	0.0429	-0.0011
		medium	0.1163	0.131	-0.0147
		large	0.2062	0.2784	-0.0722

Table 125

The Changes in SE under Different Effect Size in Non-rare Outcome Cases in Logit with Moderation Effect

Proportion	Sample Size	Effect Size	ASE for TNIE	ASE for TIE	gap
0.06	350	small	0.0732	0.0764	-0.0032
		medium	0.1674	0.1972	-0.0298
		large	0.3145	0.4747	-0.1602

Table 126

The Changes in ASE under Different Effect Size in Non-rare Outcome Cases in Logit with Moderation Effect

Proportion	Sample Size	Effect Size	ASE for TNIE	ASE for TIE	gap
0.2	350	small	0.0514	0.0533	-0.0019
		medium	0.1256	0.1451	-0.0195
		large	0.2239	0.3167	-0.0928

Table 127

The Changes in ASE under Different Effect Size in Non-rare Outcome Cases in Logit with Moderation Effect

Proportion	Sample Size	Effect Size	MSE for TNIE	MSE for TIE	gap
0.06	350	small	0.003	0.0032	-2e-04
		medium	0.0292	0.0371	-0.0079
		large	0.1517	0.271	-0.1193

Table 128

The Changes in MSE under Different Effect Size for Rare Outcome Cases in Logit with Moderation Effect

effect size was large, the gap widened, implying that the *coverage* for $TNIE$ was clearly less than the *coverage* for TIE. This pattern was visible equally in both outcome cases. Table 130 shows an example of a rare outcome case with a proportion of 0.06 and a sample size of 350. Table 131 shows an illustrated example of a non-rare outcome case of 0.2 and a sample size of 350. Figures 61 and 62 graphically depict these patterns for all conditions.

Only when the sample size was 350 did the effect size had an effect on the *power* for the two quantities in rare outcome cases. When the effect size was small or medium, the *power* for $TNIE$ and TIE was 1 and the gap was zero; when the effect size was large, the *power* for $TNIE$ remained 1 while TIE shrank and the gap widened. This implied that $TNIE$ had a higher *power* than TIE only when the effect size was large. In all other cases, the *power* of the two quantities was 1 and remained constant. This is illustrated numerically in Table 132 and graphically in Figure 63.

In non-rare outcome cases, the *power* for the two quantities remained constant at 1 for all three effect size conditions. This pattern is depicted graphically in Figure 64.

Sample Size. The *bias* for $TNIE$ and TIE became closer to 1 as the sample size increased, and the gap between them shrank. Furthermore, the *bias* for $TNIE$ was clearly smaller than the *bias* for TIE at all three sample size levels. This pattern can be seen clearly in both outcome cases under medium and large effect size. Tables 133 and 134 numerically present the pattern under conditions with a large effect size and a proportion of 0.06 for a rare outcome case and 0.2 for a non-rare outcome case. Figures 53 and 54

Proportion	Sample Size	Effect Size	MSE for TNIE	MSE for TIE	gap
0.2	350	small	0.0018	0.0019	-1e-04
		medium	0.0167	0.0217	-0.005
		large	0.0752	0.1472	-0.072

Table 129

The Changes in MSE under Different Effect Size in Non-rare Outcome Cases in Logit with Moderation Effect

Proportion	Sample Size	Effect Size	Coverage for TNIE	Coverage for TIE	gap
0.06	350	small	0.992	0.996	-0.004
		medium	0.934	0.992	-0.058
		large	0.846	0.996	-0.15

Table 130

The Changes in Coverage under Different Effect Size for Rare Outcome Cases in Logit with Moderation Effect

Proportion	Sample Size	Effect Size	Coverage for TNIE	Coverage for TIE	gap
0.2	350	small	0.988	0.997	-0.009
		medium	0.94	0.989	-0.049
		large	0.871	0.993	-0.122

Table 131

The Changes in Coverage under Different Effect Size in Non-rare Outcome Cases in Logit with Moderation Effect

Proportion	Sample Size	Effect Size	Power for TNIE	Power for TIE	gap
0.06	350	small	1	1	0
		medium	1	1	0
		large	1	0.981	0.019

Table 132

The Changes in Power under Different Effect Size for Rare Outcome Cases in Logit with Moderation Effect

Proportion	Effect Size	Sample Size	Bias for TNIE	Bias for TIE	gap
0.06	large	350	0.8201	0.7734	0.0467
		700	0.8326	0.7928	0.0398
		1000	0.8343	0.7967	0.0376

Table 133

The Changes in Bias under Different Sample Size for Rare Outcome Cases in Logit with Moderation Effect

graphically depict this pattern for all conditions.

When the sample size increased, the difference between two quantities in SE and the SE for each quantity became smaller. The gap was negative, indicating that the SE for $TNIE$ was less than the SE for TIE. This pattern is illustrated in Tables 135 and 136 under conditions with a large effect size and a proportion of 0.06 for a rare outcome case and 0.2 for a non-rare outcome case. This pattern is also depicted graphically in Figures 55 and 56.

The ASE and MSE results for the two quantities followed the same pattern as the SE. This is illustrated numerically in Tables 137, 138, 139, and 140, and graphically in Figures 57, 58, 59, and 60.

The *coverage probability* for the two quantities decreased as the sample size increased, but the gap between them increased rather than decreased. This pattern is numerically illustrated in Tables 141 and 142 under conditions with a large effect size and a proportion of 0.06 for rare outcome cases and 0.2 for non-rare outcome cases. Figures 61 and 62 show clear graphical representations for all conditions. Across all sample sizes, the *coverage* for $TNIE$ was lower than that for TIE. Furthermore, when the sample size was 1000, the *coverage* was quite low (below .85).

When the sample size was 350, the *power* for $TNIE$ was 1 and the *power* for TIE was slightly lower. As a result, the difference between the two quantities was positive. The *power* for $TNIE$ remained 1 when the sample size was increased to 700, while the *power*

Proportion	Effect Size	Sample Size	Bias for TNIE	Bias for TIE	gap
0.2	large	350	0.8729	0.8267	0.0462
		700	0.8795	0.8374	0.0421
		1000	0.8798	0.8389	0.0409

Table 134

The Changes in Bias under Different Sample Size in Non-rare Outcome Cases in Logit with Moderation Effect

Proportion	Effect Size	Sample Size	SE for TNIE	SE for TIE	gap
0.06	large	350	0.2772	0.3673	-0.0901
		700	0.1865	0.2401	-0.0536
		1000	0.1586	0.2007	-0.0421

Table 135

The Changes in SE under Different Sample Size for Rare Outcome Cases in Logit with Moderation Effect

Proportion	Effect Size	Sample Size	SE for TNIE	SE for TIE	gap
0.2	large	350	0.2062	0.2784	-0.0722
		700	0.1451	0.1905	-0.0454
		1000	0.1245	0.1618	-0.0373

Table 136

The Changes in SE under Different Sample Size in Non-rare Outcome Cases in Logit with Moderation Effect

Proportion	Effect Size	Sample Size	ASE for TNIE	ASE for TIE	gap
0.06	large	350	0.3145	0.4747	-0.1602
		700	0.1976	0.2649	-0.0673
		1000	0.1602	0.2095	-0.0493

Table 137

The Changes in ASE under Different Sample Size for Rare Outcome Cases in Logit with Moderation Effect

Proportion	Effect Size	Sample Size	ASE for TNIE	ASE for TIE	gap
0.2	large	350	0.2239	0.3167	-0.0928
		700	0.1497	0.2009	-0.0512
		1000	0.1232	0.1633	-0.0401

Table 138

The Changes in ASE under Different Sample Size in Non-rare Outcome Cases in Logit with Moderation Effect

Proportion	Effect Size	Sample Size	MSE for TNIE	MSE for TIE	gap
0.06	large	350	0.1517	0.271	-0.1193
		700	0.0976	0.166	-0.0684
		1000	0.0865	0.1436	-0.0571

Table 139

The Changes in MSE under Different Sample Size for Rare Outcome Cases in Logit with Moderation Effect

Proportion	Effect Size	Sample Size	MSE for TNIE	MSE for TIE	gap
0.2	large	350	0.0752	0.1472	-0.072
		700	0.05	0.0961	-0.0461
		1000	0.0443	0.0847	-0.0404

Table 140

The Changes in MSE under Different Sample Size in Non-rare Outcome Cases in Logit with Moderation Effect

Proportion	Effect Size	Sample Size	Coverage for TNIE	Coverage for TIE	gap
0.06	large	350	0.846	0.996	-0.15
		700	0.689	0.956	-0.267
		1000	0.577	0.815	-0.238

Table 141

The Changes in Coverage under Different Sample Size for Rare Outcome Cases in Logit with Moderation Effect

Proportion	Effect Size	Sample Size	Coverage for TNIE	Coverage for TIE	gap
0.2	large	350	0.871	0.993	-0.122
		700	0.774	0.906	-0.132
		1000	0.683	0.789	-0.106

Table 142

The Changes in Coverage under Different Sample Size in Non-rare Outcome Cases in Logit with Moderation Effect

for TIE increased to 1. Thus , the gap was zero. When the sample size reached 1000, the *power* for the two quantities was 1 and the gap was 0. This is shown numerically in Table 143 and graphically for all conditions in Figure 63.

In the non-rare outcome cases, the *power* for the two quantities remained constant at 1 for all three sample size levels. This pattern was depicted graphically in Figure 64.

Moderation Effect. In rare outcome cases, with a moderation effect, the *bias* for $TNIE$ and TIE was closer to 1, and the gap between them was larger than without a moderation effect. Tables 106 and 145 numerically present this pattern under conditions with a larger effect size and a sample size of 350. Figures 53 and 65 show this in graphical forms. The figures cannot show the gaps for the conditions with small effect sizes because they are too small. The numerical results in these conditions are shown in Tables 107 and 146.

Similarly to rare outcome cases, with a moderation effect, the *bias* for $TNIE$ and TIE was closer to 1 and the gap between them was larger than in non-rare cases without a moderation effect. Furthermore, as the proportion of $Y = 1$ increased, the gap between the two quantities in *bias* in conditions without a moderation effect moved in the opposite direction as in conditions with a moderation effect. To put it simply, when there was a moderation effect, the gap widened. Tables 109 and 147 numerically present this pattern under conditions with a small effect size. Figures 54 and 66 show this in graphical representations.

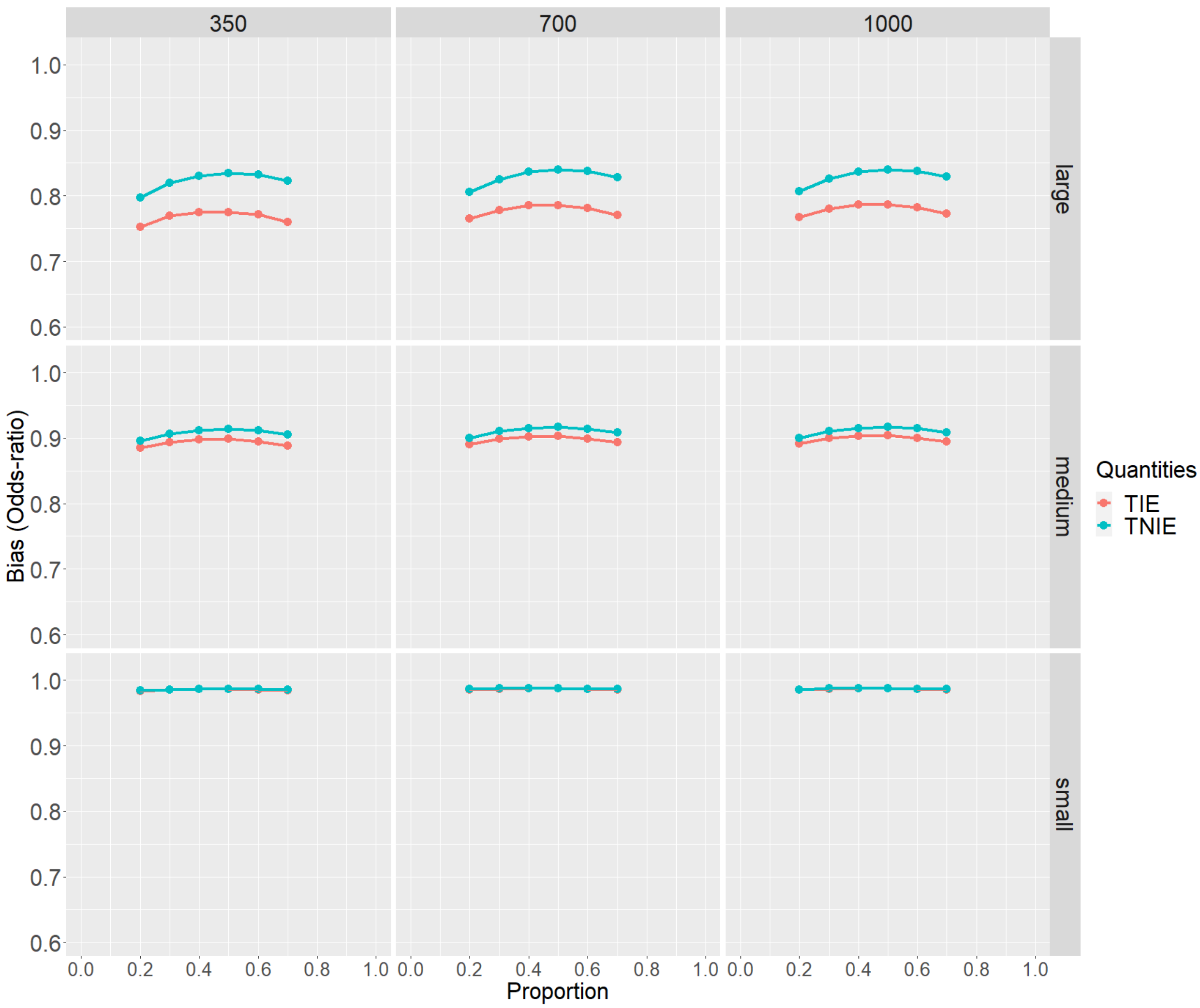
Proportion	Effect Size	Sample Size	Power for TNIE	Power for TIE	gap
0.06	large	350	1	0.981	0.019
		700	1	1	0
		1000	1	1	0

Table 143

The Changes in Power under Different Sample Size for Rare Outcome Cases in Logit with Moderation Effect

Figure 66

Bias over TIE and TNIE for Non-rare Outcomes in Logit Without Moderation Effect



In both outcome cases, the *bias* for $TNIE$ was closer to 1 than the *bias* for TIE in terms of having a moderation effect or not having a moderation effect. Furthermore, the

Proportion	Effect Size	Sample Size	Power for TNIE	Power for TIE	gap
0.2	large	350	1	1	0
		700	1	1	0
		1000	1	1	0

Table 144

The Changes in Power under Different Sample Size in Non-rare Outcome Cases in Logit with Moderation Effect

Effect Size	Sample Size	Proportions	Bias for TNIE	Bias for TIE	gap
large	350	0.06	0.7144	0.6831	0.0313
		0.07	0.724	0.691	0.033
		0.075	0.7296	0.6969	0.0327
		0.08	0.7334	0.6998	0.0336
		0.09	0.7423	0.7078	0.0345
		0.1	0.7488	0.7127	0.0361

Table 145

The Changes in Bias under Different Proportions for Rare Outcome Cases in Logit Without Moderated Effect

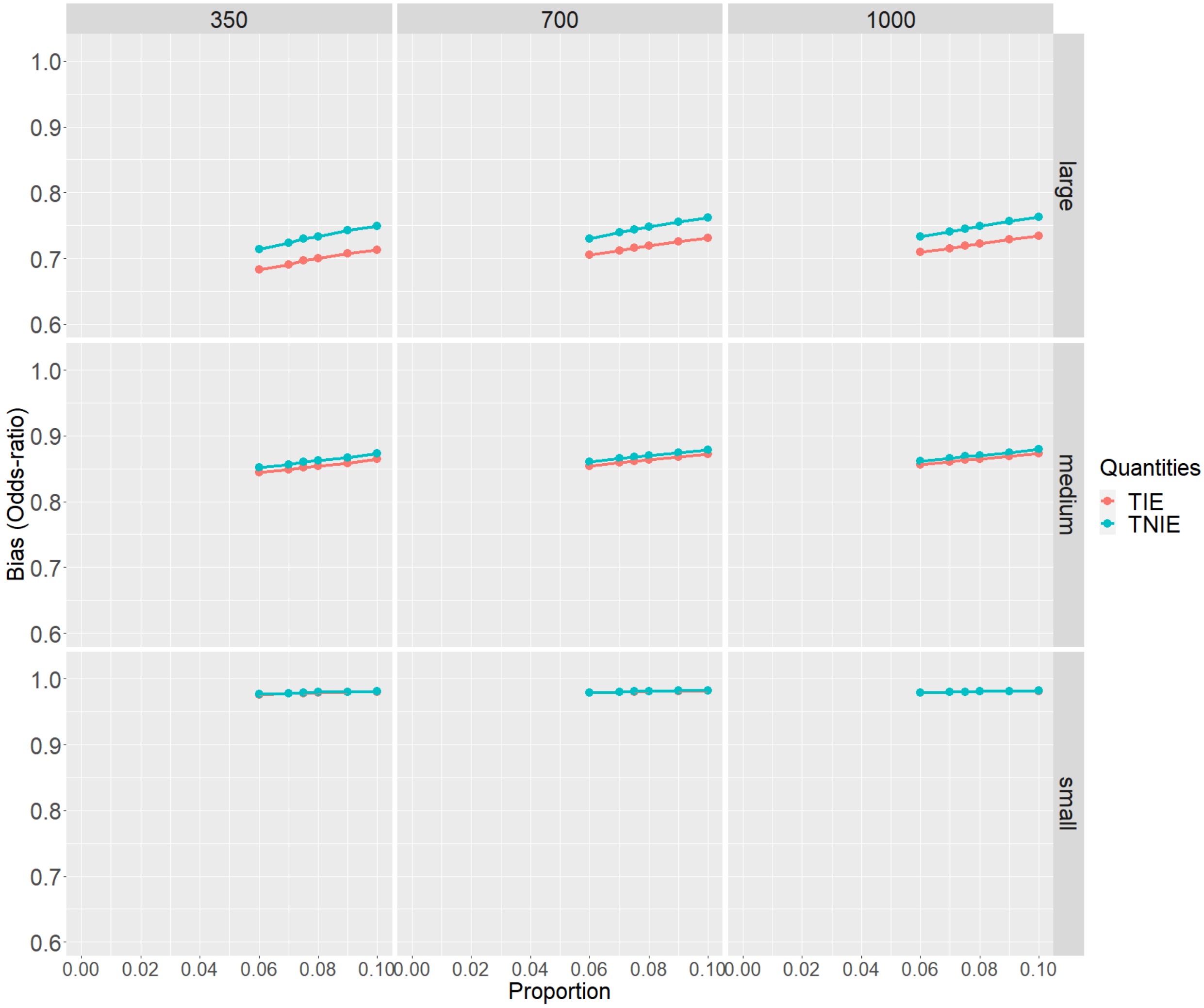
Effect Size	Sample Size	Proportions	Bias for TNIE	Bias for TIE	gap
small	350	0.06	0.9769	0.9765	4e-04
		0.07	0.9782	0.9779	3e-04
		0.075	0.9788	0.9783	5e-04
		0.08	0.9797	0.9794	3e-04
		0.09	0.9804	0.98	4e-04
		0.1	0.9811	0.9807	4e-04
	700	0.06	0.9795	0.9794	1e-04
		0.07	0.9807	0.9805	2e-04
		0.075	0.9808	0.9806	2e-04
		0.08	0.9812	0.9811	1e-04
		0.09	0.9818	0.9815	3e-04
		0.1	0.9824	0.9822	2e-04
	1000	0.06	0.9795	0.9794	1e-04
		0.07	0.9807	0.9805	2e-04
		0.075	0.9808	0.9806	2e-04
		0.08	0.9812	0.9811	1e-04
		0.09	0.9818	0.9815	3e-04
		0.1	0.9824	0.9822	2e-04

Table 146

The Changes in Bias across Proportions for Rare Outcome Cases in Logit Without Moderation Effect

Figure 65

Bias over TIE and TNIE for Rare Outcomes in Logit Without Moderation Effect



gap in *bias* between the two quantities for two levels of the moderation effect moved in the same direction in the rare outcome cases but in the opposite direction in the non-rare outcome cases.

The SE for $TNIE$ and TIE at the two levels of the moderation effect was very close to the *bias*. Under conditions with a moderation effect, the SE for the two quantities was smaller. Furthermore, under both levels of the moderated effects, $TNIE$ had a smaller SE than TIE in both outcome cases. In non-rare outcome cases, the SE and the gaps between the two quantities were slightly smaller. Tables 111 and 148 numerically show this pattern for rare outcome cases, whereas Tables 113 and 149 show non-rare outcome cases

Effect Size	Sample Size	Proportions	Bias for TNIE	Bias for TIE	gap
small	350	0.2	0.984	0.9835	5e-04
		0.3	0.986	0.9855	5e-04
		0.4	0.9867	0.9861	6e-04
		0.5	0.9868	0.9862	6e-04
		0.6	0.9863	0.9855	8e-04
		0.7	0.9857	0.9849	8e-04
	700	0.2	0.9862	0.9859	3e-04
		0.3	0.9873	0.987	3e-04
		0.4	0.9876	0.9871	5e-04
		0.5	0.9876	0.9871	5e-04
		0.6	0.9868	0.9864	4e-04
		0.7	0.9863	0.9857	6e-04
	1000	0.2	0.9862	0.9859	3e-04
		0.3	0.9873	0.987	3e-04
		0.4	0.9876	0.9871	5e-04
		0.5	0.9876	0.9871	5e-04
		0.6	0.9868	0.9864	4e-04
		0.7	0.9863	0.9857	6e-04

Table 147

The Changes in Bias across Proportions for Non-rare Outcome Cases in Logit Without Moderation Effect

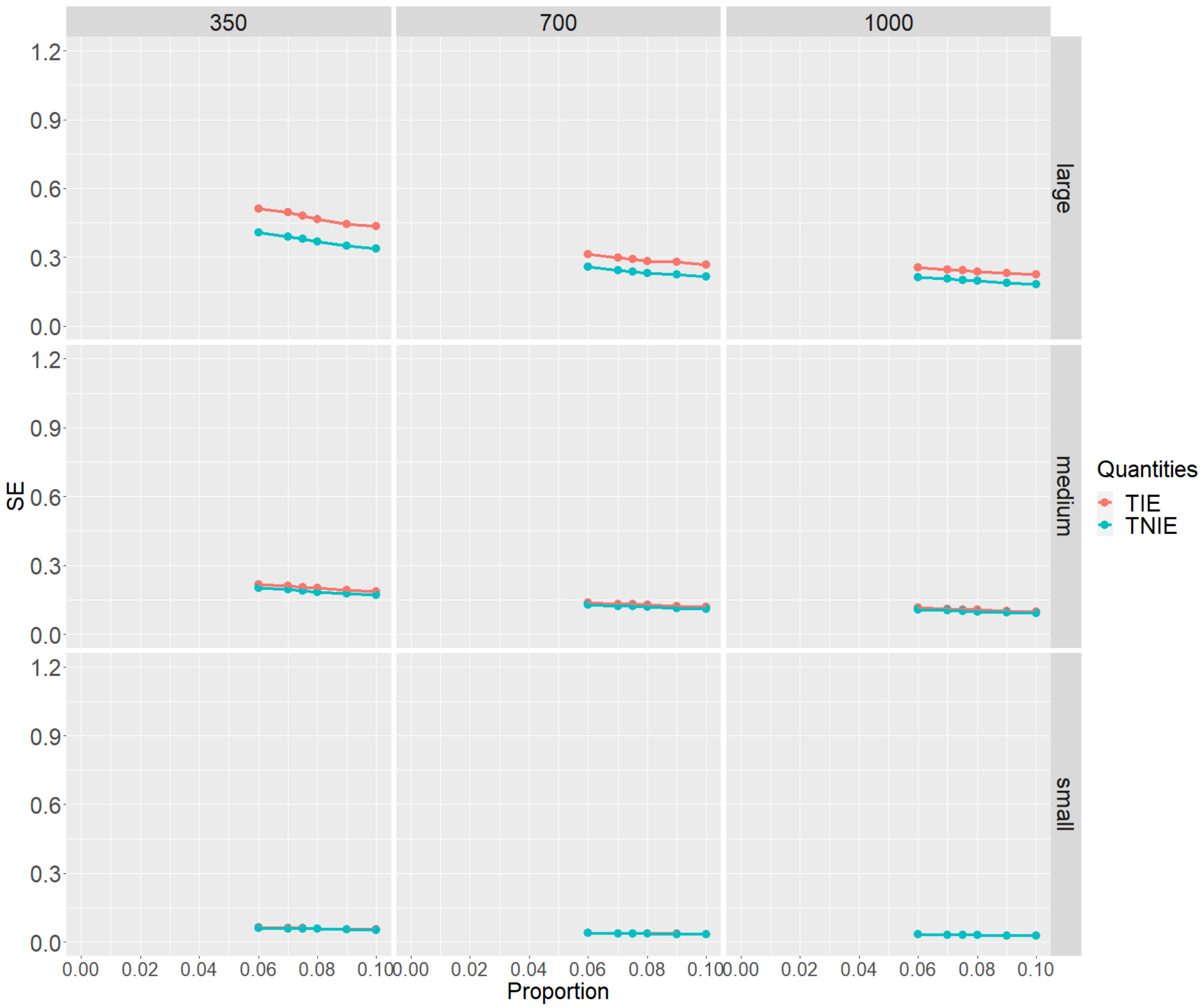
Effect Size	Sample Size	Proportions	SE for TNIE	SE for TIE	gap
small	350	0.06	0.0631	0.064	-9e-04
		0.07	0.0603	0.0613	-0.001
		0.075	0.0598	0.0607	-9e-04
		0.08	0.0585	0.0595	-0.001
		0.09	0.0559	0.0568	-9e-04
		0.1	0.0541	0.0549	-8e-04
	700	0.06	0.04	0.0404	-4e-04
		0.07	0.0374	0.0378	-4e-04
		0.075	0.0372	0.0375	-3e-04
		0.08	0.0367	0.0371	-4e-04
		0.09	0.0357	0.0361	-4e-04
		0.1	0.0349	0.0353	-4e-04
	1000	0.06	0.033	0.0333	-3e-04
		0.07	0.0315	0.0318	-3e-04
		0.075	0.031	0.0313	-3e-04
		0.08	0.0303	0.0305	-2e-04
		0.09	0.0293	0.0296	-3e-04
		0.1	0.0286	0.0289	-3e-04

Table 148

The Changes in SE across Proportions for Rare Outcome Cases in Logit Without Moderation Effect

Figure 67

SE over TIE and TNIE for Rare Outcomes in Logit Without Moderation Effect



under small effect size conditions. This pattern can be seen graphically in Figures 55 and 67 for rare outcome cases, and Figures 56 and 68 for non-rare outcome cases.

The ASE results followed the same pattern as the SE results. This is depicted graphically in Figures 57 and 69 for cases with rare outcomes, and Figures 58 and 70 for cases with non-rare outcomes. Because the conditions with a small effect size had such a small gap between the two quantities, the lines representing the two quantities in ASE were overlapping. Tables 114 and 150 and 115 and 151 present numerical values for rare and non-rare outcome cases, respectively.

Effect Size	Sample Size	Proportions	SE for TNIE	SE for TIE	gap
small	350	0.2	0.0463	0.0472	-9e-04
		0.3	0.0414	0.0424	-0.001
		0.4	0.0412	0.0424	-0.0012
		0.5	0.0409	0.0422	-0.0013
		0.6	0.0414	0.0428	-0.0014
		0.7	0.0432	0.0448	-0.0016
	700	0.2	0.029	0.0294	-4e-04
		0.3	0.0271	0.0276	-5e-04
		0.4	0.0269	0.0275	-6e-04
		0.5	0.0267	0.0273	-6e-04
		0.6	0.0273	0.028	-7e-04
		0.7	0.0284	0.0292	-8e-04
	1000	0.2	0.0236	0.0239	-3e-04
		0.3	0.0222	0.0226	-4e-04
		0.4	0.0215	0.0219	-4e-04
		0.5	0.0213	0.0217	-4e-04
		0.6	0.022	0.0225	-5e-04
		0.7	0.0231	0.0236	-5e-04

Table 149

The Changes in SE across Proportions for Non-rare Outcome Cases in Logit Without Moderation Effect

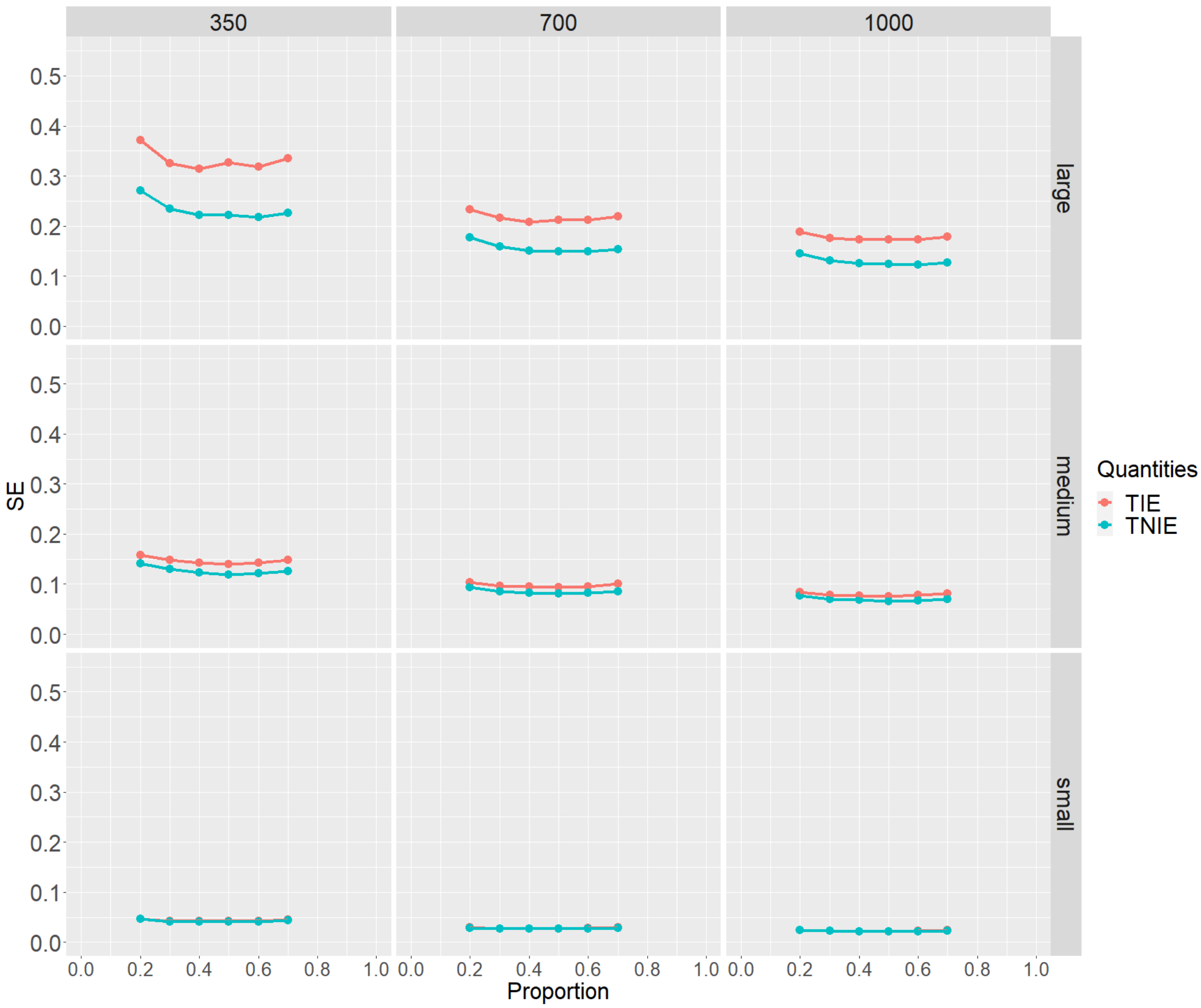
Effect Size	Sample Size	Proportions	ASE for TNIE	ASE for TIE	gap
large	350	0.06	0.0754	0.0771	-0.0017
		0.07	0.0707	0.0723	-0.0016
		0.075	0.0689	0.0705	-0.0016
		0.08	0.067	0.0685	-0.0015
		0.09	0.0641	0.0656	-0.0015
		0.1	0.0616	0.063	-0.0014
	700	0.06	0.0438	0.0443	-5e-04
		0.07	0.0416	0.0422	-6e-04
		0.075	0.0409	0.0414	-5e-04
		0.08	0.04	0.0406	-6e-04
		0.09	0.0387	0.0393	-6e-04
		0.1	0.0375	0.038	-5e-04
	1000	0.06	0.0348	0.0351	-3e-04
		0.07	0.0332	0.0335	-3e-04
		0.075	0.0325	0.0328	-3e-04
		0.08	0.0319	0.0322	-3e-04
		0.09	0.0309	0.0312	-3e-04
		0.1	0.03	0.0304	-4e-04

Table 150

The Changes in ASE across Proportions for Rare Outcome Cases in Logit Without Moderation Effect

Figure 68

SE over TIE and TNIE for Non-rare Outcomes in Logit Without Moderation Effect



The MSE for the two quantities was clearly lower with a moderation effect than without one. Figures 59 and 71 depict this for rare outcome cases, and Figures 60 and 72 depict this for non-rare outcome cases. The gaps between the two quantities in MSE were either very small or zero for the conditions with a small effect size. This indicated that the MSE for $TNIE$ and TIE were nearly identical. Tables 116 and 152, as well as Tables 117 and 153, show this numerically for rare and non-rare outcome cases.

The *coverage probabilities* for $TNIE$ and TIE under no moderation effect conditions were much smaller, and the gap between them was larger than under moderation effect conditions. This pattern can be found in all conditions and in both

Effect Size	Sample Size	Proportions	ASE for TNIE	ASE for TIE	gap
small	350	0.06	0.0503	0.0517	-0.0014
		0.07	0.0459	0.0473	-0.0014
		0.075	0.0442	0.0458	-0.0016
		0.08	0.0441	0.0458	-0.0017
		0.09	0.0449	0.0468	-0.0019
		0.1	0.0473	0.0494	-0.0021
	700	0.06	0.0309	0.0314	-5e-04
		0.07	0.0286	0.0292	-6e-04
		0.075	0.0277	0.0284	-7e-04
		0.08	0.0277	0.0284	-7e-04
		0.09	0.0283	0.0292	-9e-04
		0.1	0.0297	0.0306	-9e-04
	1000	0.06	0.0248	0.0252	-4e-04
		0.07	0.023	0.0235	-5e-04
		0.075	0.0224	0.0229	-5e-04
		0.08	0.0224	0.0229	-5e-04
		0.09	0.0228	0.0234	-6e-04
		0.1	0.0239	0.0245	-6e-04

Table 151

The Changes in ASE across Proportions for Non-rare Outcome Cases in Logit Without Moderation Effect

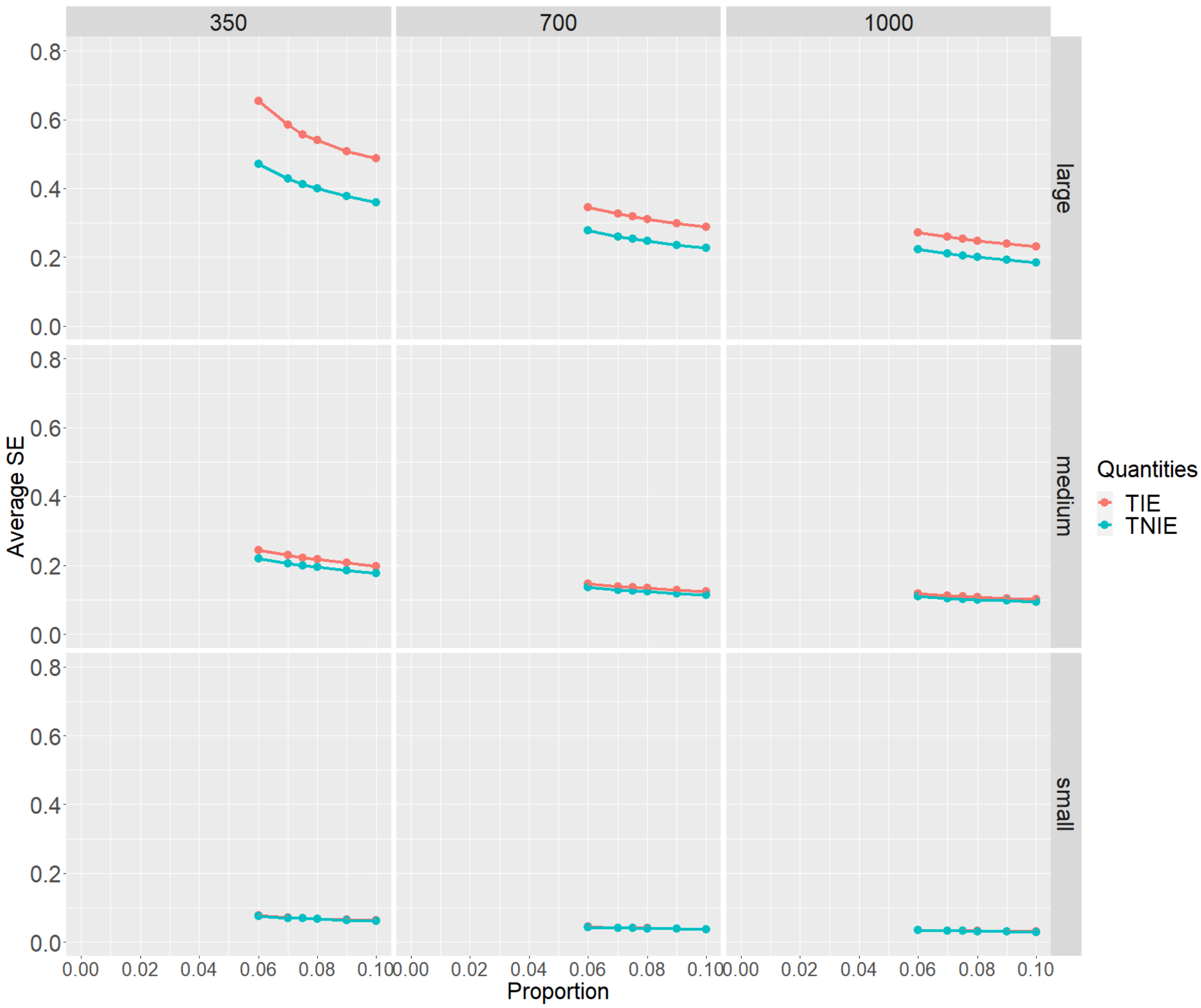
Effect Size	Sample Size	Proportions	MSE for TNIE	MSE for TIE	gap
small	350	0.06	0.0046	0.0047	-1e-04
		0.07	0.0042	0.0043	-1e-04
		0.075	0.0041	0.0042	-1e-04
		0.08	0.0039	0.004	-1e-04
		0.09	0.0036	0.0037	-1e-04
		0.1	0.0033	0.0034	-1e-04
	700	0.06	0.0021	0.0021	0
		0.07	0.0018	0.0019	-1e-04
		0.075	0.0018	0.0018	0
		0.08	0.0017	0.0018	-1e-04
		0.09	0.0016	0.0017	-1e-04
		0.1	0.0016	0.0016	0
	1000	0.06	0.0016	0.0016	0
		0.07	0.0014	0.0015	-1e-04
		0.075	0.0014	0.0014	0
		0.08	0.0013	0.0013	0
		0.09	0.0012	0.0013	-1e-04
		0.1	0.0012	0.0012	0

Table 152

The Changes in MSE across Proportions for Rare Outcome Cases in Logit Without Moderation Effect

Figure 69

ASE over TIE and TNIE for Rare Outcomes in Logit Without Moderation Effect

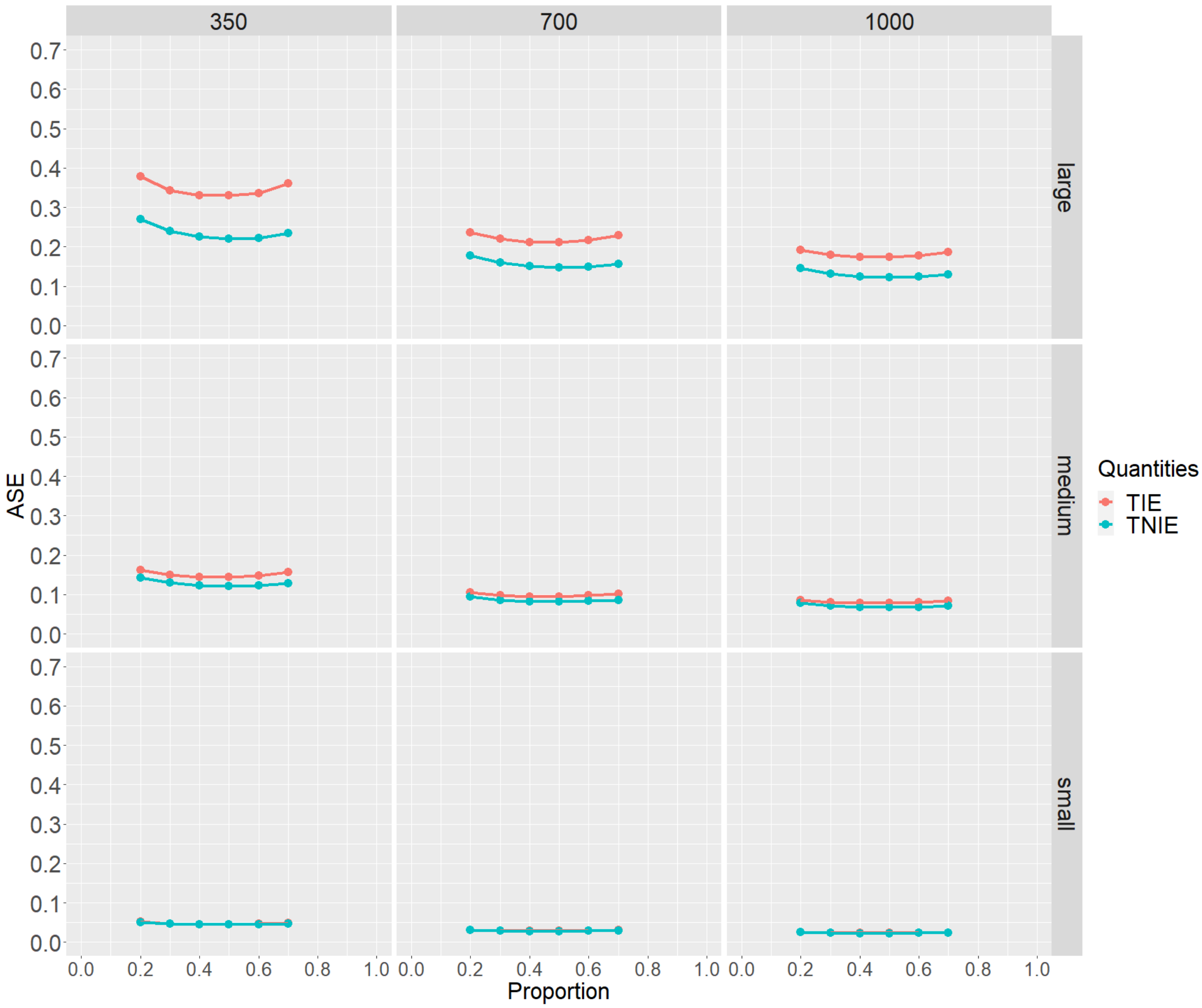


outcome cases. This pattern is depicted graphically in Figures 61 and 73, as well as Figures 62 and 74. Across both levels of the moderation effect, the *coverage* for $TNIE$ was significantly lower than the *coverage* for TIE.

The results of *power* were far superior to those of *coverage probability*. Under the two levels for the moderation effect, the *power* for $TNIE$ and TIE was quite similar in both outcome cases. In rare outcome cases, with medium and large effect sizes and a sample size of 350, the *power* for $TNIE$ remained constant at 1, whereas the *power* for TIE increased as the proportion of $Y = 1$ increased. As a result, the gap between them was positive and narrowed as the proportion increased. The *power* for TIE without a

Figure 70

ASE over TIE and TNIE for Non-rare Outcomes in Logit Without Moderation Effect



moderation effect was lower than the *power* for TIE with a moderation effect. In all other cases, the *power* of the two quantities remained constant at 1. Under all conditions, the *power* for the two quantities in non-rare outcome cases was 1. This pattern is depicted graphically in Figures 63 and 75 and Figures 64 and 76.

Figure 71

MSE over TIE and TNIE for Rare Outcomes in Logit Without Moderation Effect

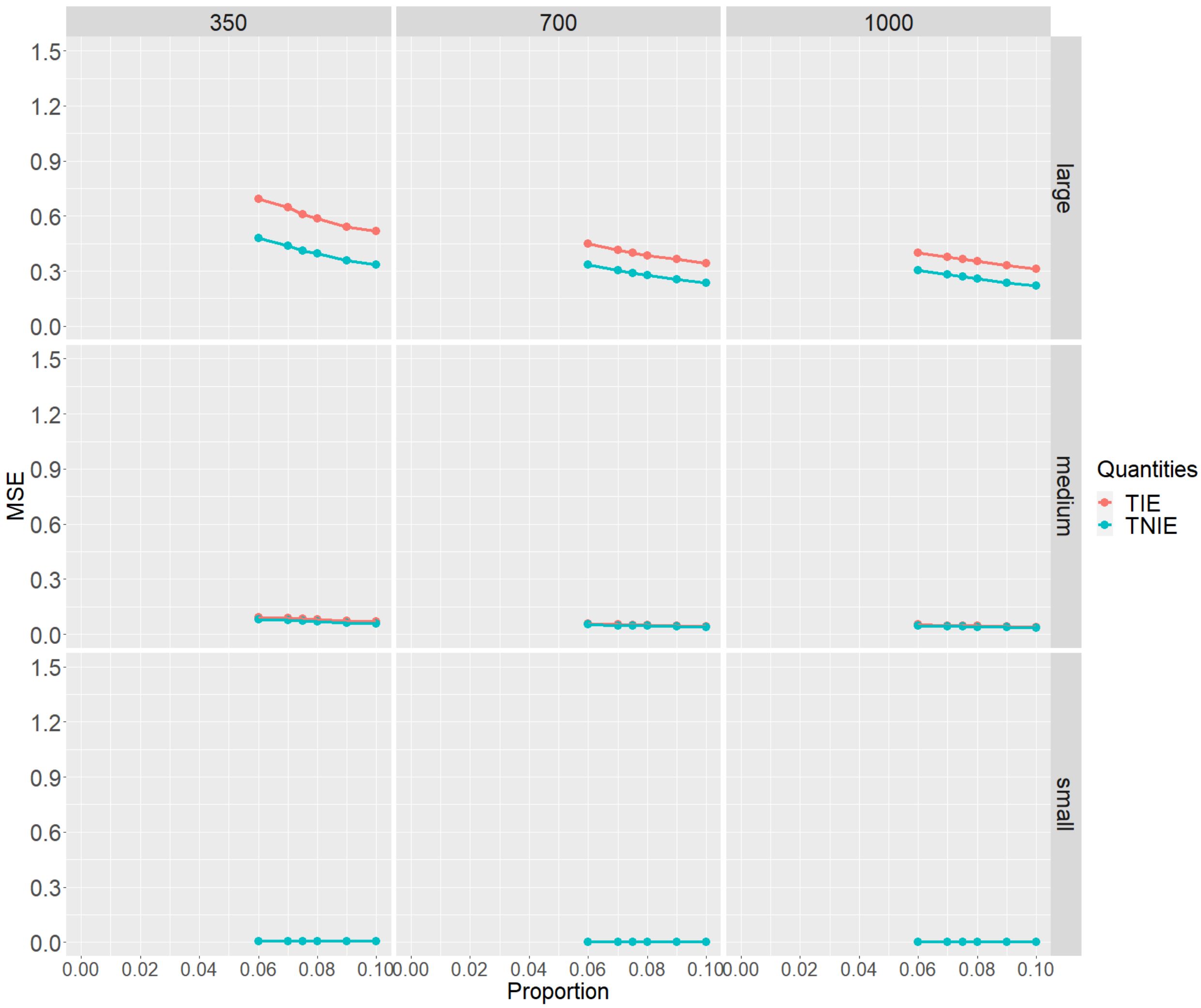
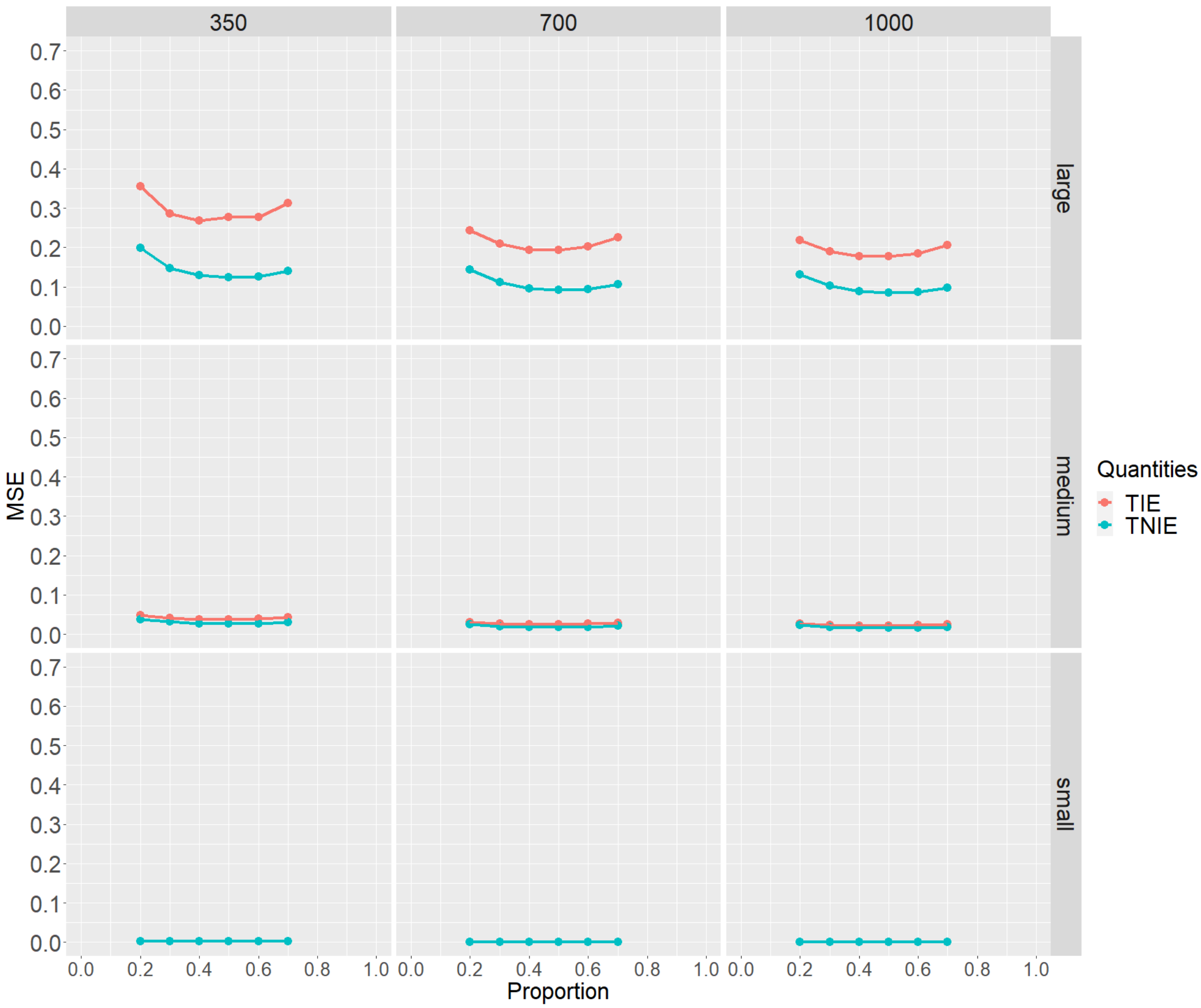


Figure 72

MSE over TIE and TNIE for Non-rare Outcomes in Logit Without Moderation Effect



Effect Size	Sample Size	Proportions	MSE for TNIE	MSE for TIE	gap
small	350	0.2	0.0024	0.0025	-1e-04
		0.3	0.0019	0.002	-1e-04
		0.4	0.0019	0.002	-1e-04
		0.5	0.0019	0.002	-1e-04
		0.6	0.0019	0.0021	-2e-04
		0.7	0.0021	0.0023	-2e-04
	700	0.2	0.0011	0.0011	0
		0.3	9e-04	9e-04	0
		0.4	9e-04	9e-04	0
		0.5	9e-04	9e-04	0
		0.6	9e-04	0.001	-1e-04
		0.7	0.001	0.0011	-1e-04
	1000	0.2	8e-04	8e-04	0
		0.3	7e-04	7e-04	0
		0.4	6e-04	7e-04	-1e-04
		0.5	6e-04	7e-04	-1e-04
		0.6	7e-04	7e-04	0
		0.7	7e-04	8e-04	-1e-04

Table 153

The Changes in MSE under Different Proportions for Non-rare Outcome Cases in Logit Without Moderation Effect

Figure 73

Coverage over TIE and TNIE for Rare Outcomes in Logit Without Moderation Effect

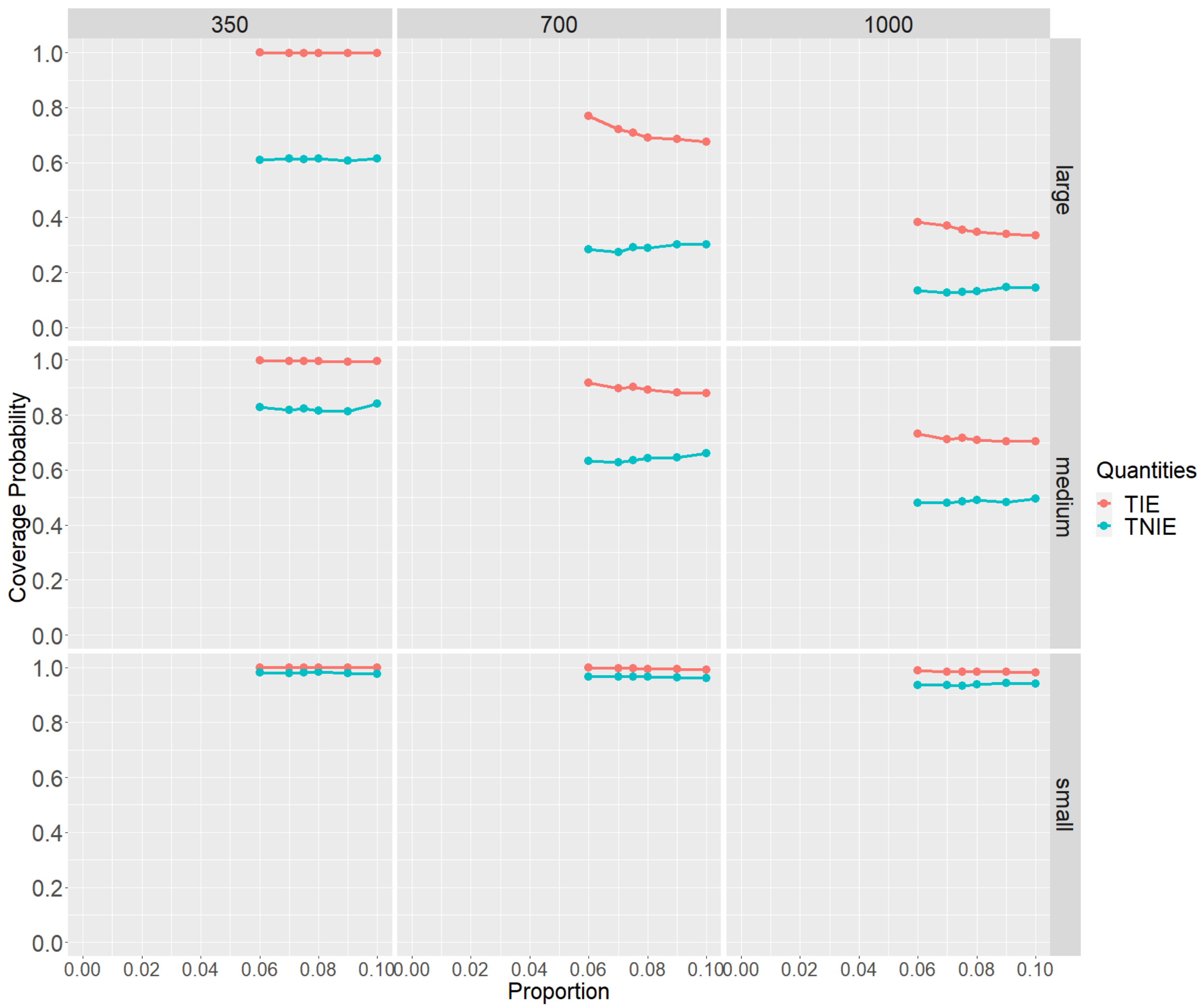


Figure 74

Coverage over TIE and TNIE for Non-rare Outcomes in Logit Without Moderation Effect

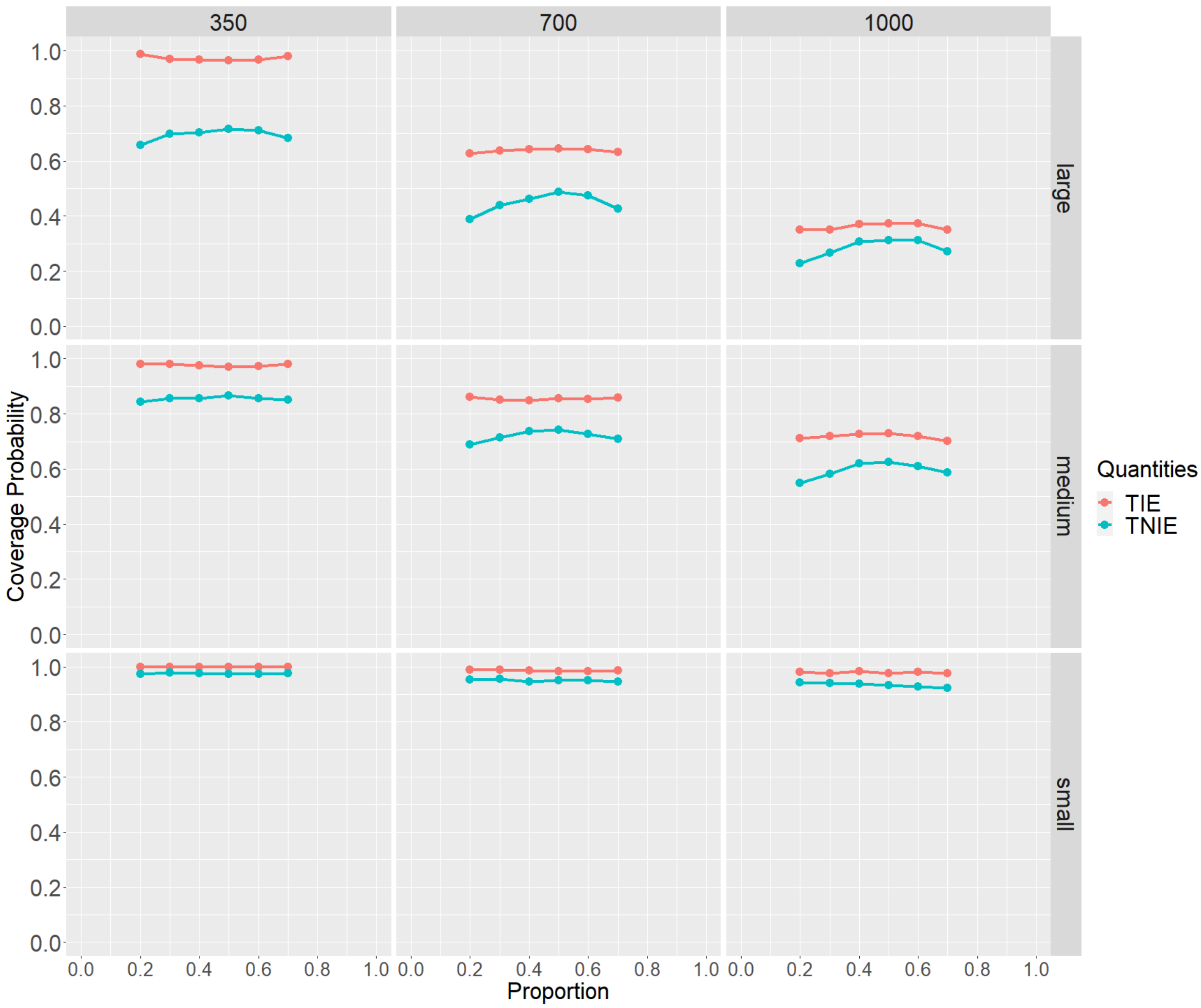


Figure 75

Power over TIE and TNIE for Rare Outcomes in Logit Without Moderation Effect

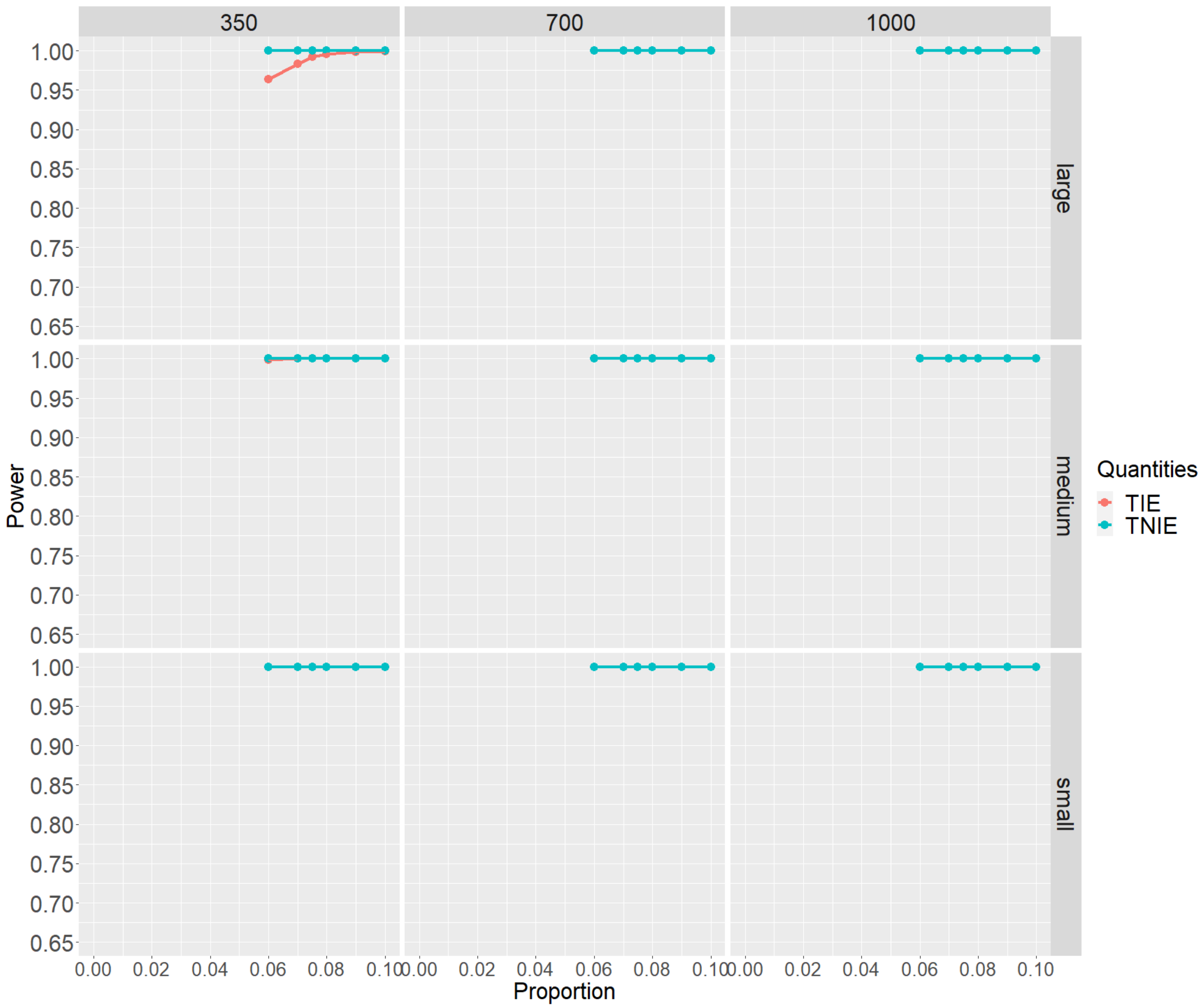
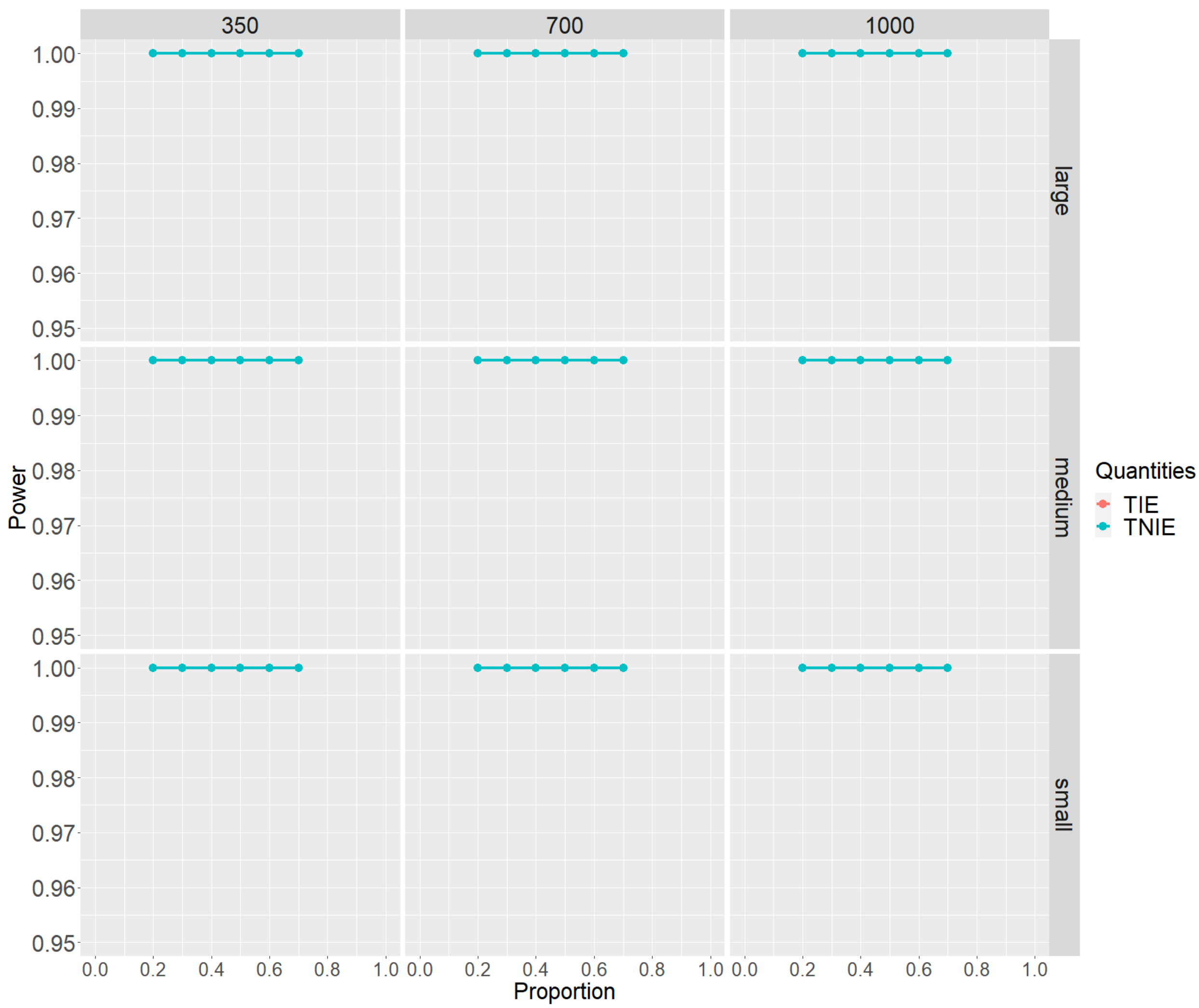


Figure 76

Power over TIE and TNIE for Non-rare Outcomes in Logit Without Moderation Effect



Chapter 5. Discussion & Implication

The results from Chapter 4 will be interpreted in this chapter, as will the discrepancies between my hypothesis and the results. Furthermore, the significance of my study will be recognized, as well as how my dissertation will contribute to the field's literature. Furthermore, several limitations will be highlighted, and future research will be conducted.

Discussion

Study 1: Simple Mediation Model

Overall, the results did not fully support my original hypotheses. In the probit and logit models, the causal inference approach estimated mediation effects more accurately than the classical approach in both rare and non-rare outcome cases. Furthermore, the mediation effects from the probit model outperformed those from the logit model. These indications corroborate my hypotheses. The gap between the two effects, however, was more visible in the rare outcome cases than I anticipated. The specifics are shown below.

Probit Model. The first four evaluation criteria results supported the hypothesis that $TNIE$ was more accurate than TIE. The last two evaluation criteria, however, did not fully support my hypotheses. This is true for all three simulation factors.

Proportion Y=1. The evaluation criterion *bias* on the odds-ratio scale was used to determine the accuracy of the mediation effect by looking at how close to 1 the *bias* was. The closer the *bias* was to 1, the more accurate the mediation effect. In all conditions, the *bias* for $TNIE$ was closer to 1 than for the *bias* for TIE. This indicated that $TNIE$ was more accurate than TIE across all the proportions of $Y = 1$ in both outcome cases and was more visible in the rare outcome cases. This contradicts my hypothesis. I expected the *bias* difference between $TNIE$ and TIE to be more visible in the non-rare outcome cases than in the rare outcome cases.

SE, ASE, and MSE, on the other hand, were used to determine how deviant the mediation effect was from the population value. Subsequently, the smaller the criteria, the better the results. In both outcome cases, $TNIE$ was consistently smaller than TIE in these evaluation criteria across all levels of the proportion of $Y = 1$. This is similar to my hypotheses. However, I anticipated that the gap in these criteria would not be visible in the rare outcome cases, whereas the results were visible in both outcomes.

$Coverage\,probability$ was used to assess the reliability of the effects. Thus, the *coverage probability* of an accurate effect should be close to or greater than .95. In the rare outcome cases, *coverage* for both $TNIE$ and TIE was very close to .95, with a very small difference between them. When the effect size was small in the non-rare outcome cases, some *coverage probabilities* for the two quantities were less than .90. Except for conditions with a small effect size, the results of $TNIE$ and TIE were equally reliable in rare and non-rare outcome cases. My hypothesis does not correspond to the results. I predicted that the *coverage* of the two quantities would be .95 or higher in all conditions, and that $TNIE$ would be better than TIE in non-rare outcome cases.

The precision of the results was measured using *power*. As a result, if the *power* was .80 or higher, the mediation effect would be considered good. The *power* for $TNIE$ and TIE was greater than .99 in both outcome cases. This indicated that the $TNIE$ and TIE results were precise in all conditions of proportion of $Y = 1$. This result matches my hypothesis, but not entirely. In non-rare outcome cases, I expected the *power* for $TNIE$ to be greater than the *power* for TIE.

Effect Size. With increasing effect size, the *bias* for $TNIE$ became closer to 1 than the *bias* for TIE, and the gap between them widened in both outcome cases. The difference between the two quantities was greater in the rare outcome cases than in the non-rare cases. This indicated that as the effect size increased, it became clear that $TNIE$ was more accurate than TIE, particularly in the rare outcome cases. In more detail, when the effect size was small, the *bias* for $TNIE$ was slightly smaller than the *bias* for TIE;

when the effect size was medium, the *bias* for $TNIE$ was smaller than the *bias* for TIE; and when the effect size was large, the *bias* for $TNIE$ was clearly smaller than the *bias* for TIE, as seen in both cases. Except for the visibility of the rare and non-rare outcome cases, these results match my hypotheses.

Similarly to the *bias*, the SE, ASE, and MSE for $TNIE$ outperformed TIE in both outcome cases across all effect size levels. These findings support my hypotheses.

In the cases of rare outcomes, the *coverage probabilities* for $TNIE$ and TIE were very close to .95. The effect size change had little effect on the changes in *coverage probabilities*. When the effect size was small in non-rare outcome cases, some *coverage probabilities* for the two quantities were below .90; when the effect size was medium, the *coverage* for the two quantities increased and became nearly identical; when the effect size was large, the *coverage* for $TNIE$ was slightly better than the *coverage* for TIE. Furthermore, the results indicated that $TNIE$ and TIE in both rare and non-rare outcome cases were reliable, with the exception of conditions with a small effect size. These findings contradict my hypotheses. In non-rare outcome cases, I expected $TNIE$ to have better *coverage* than TIE across all levels of effect size.

In the non-rare outcome cases, the *power* for the two quantities was equal to 1 and was unaffected by the change in effect size. In the rare outcome cases, when the effect size was small, the two *power* were identical; when the effect size was medium, the two *power* were smaller but still identical; and when the effect size was large, the two *power* shrank and the *power* for $TNIE$ was slightly better than the *power* for TIE. My hypotheses were not supported by the results. In summary, when the effect size was small, $TNIE$ recovered the population value slightly better than TIE; when the effect size is medium, $TNIE$ recovered better than TIE; and when the effect size is large, the gap between the two quantities was much larger. That is, $TNIE$ was clearly superior to TIE. Across three effect size conditions, the gap between the two quantities was larger in the rare outcome cases than in the non-rare outcome cases. This pattern was clearly presented in the first four evaluation criteria. The values for the last two criteria, *coverage probability* and *power*, were quite close to each other for $TNIE$ and TIE. Because the final two criteria evaluate the reliability and precision of the results, it makes sense that the two values were good and close to each other.

Sample Size. When the sample size was increased, the *bias* for $TNIE$ became closer to 1 than the *bias* for TIE, and the difference between them shrank in both outcome cases. The gap was larger in the rare outcome cases than in the non-rare cases. This implied that as the sample size grew, it became less clear that the *bias* for $TNIE$ was smaller than the *bias* for TIE, particularly in non-rare outcome cases. In more detail, when the sample size was 350, the *bias* for $TNIE$ was clearly smaller than the *bias* for TIE; when the sample size was 700, the *bias* for $TNIE$ was smaller than the *bias* for TIE; and when the sample size was 1000, the *bias* for $TNIE$ was slightly smaller than the *bias* for TIE, but this was not presented clearly in both cases because the gaps were too small. The pattern affected by the sample size change was similar to my hypotheses, but the visibility of rare and non-rare outcome cases was the opposite of what I expected.

The SE, ASE, and MSE for $TNIE$ were better than those for TIE in rare outcome cases than non-rare outcome cases, but they became less clearly as the sample size increased. When the sample size reached 1000, these criteria of the two quantities became closer and almost identical. The pattern confirmed my hypotheses, but the visibility of the rare and non-rare outcome cases contradicted them.

This demonstrated that the results of $TNIE$ and TIE in both rare and non-rare outcome cases were reliable. My hypotheses were not completely supported by the results. I expected $TNIE$ coverage to be better than TIE coverage, and the gaps between them to be more visible in non-rare outcome cases.

In the non-rare outcome cases, the *powers* for the two quantities were equal to

1 and were unaffected by the change in sample size. When the sample size was 350, the two *powers* were around .99, and $TNIE$ was slightly better than TIE; when the sample size was 700, the *powers* for both quantities reached 1; and when the sample size was 1000, the two *powers* were identical and remained at 1. The results for the rare outcome cases match my hypotheses, whereas I expected the results for $TNIE$ to be more accurate than the results for TIE in the non-rare cases.

Logit Model. Sum up, the logit model's values for the first five evaluation criteria for $TNIE$ and TIE were not as good as the probit model's. This is explained by the fact that the probit model was used to simulate data sets. The causal inference approach produced more accurate mediation effects in the logit model than the classical approach. This was demonstrated by the findings that $TNIE$ outperformed TIE in recovering population values in both rare and non-rare outcome cases across all evaluation criteria except *coverage probability*, and *power*. In terms of *power*, both quantities have a consistent *power* of 1 under all conditions. This indicated that the $TNIE$ and TIE results were precise.

Proportion Y=1. When the proportion of $Y = 1$ increased, the *bias* for $TNIE$ became closer to 1 than for TIE, and the gap between them shrank slightly for rare outcome cases. However, regardless of the proportion of $Y = 1$ increments, the *bias* for $TNIE$ consistently got closer to 1 than the *bias* for TIE. This indicated that $TNIE$ recovered the population mediation effects better than TIE in both outcome cases. The pattern aligned with my hypotheses.

When the proportion of $Y = 1$ increased in the rare outcome cases, the SE for $TNIE$ was better than the SE for TIE in recovering the population values, but the difference between them shrank. The SE for the two quantities got closer to each other as the proportion of $Y = 1$ increased, but $TNIE$ is still better than TIE. The pattern did not appear clearly in the non-rare outcome cases. Regardless of the changes in the proportion of $Y = 1$, the SE for $TNIE$ was better than the SE for TIE. In summary, for the criterion

SE, $TNIE$ outperforms TIE in both cases; the difference is slightly larger in rare outcome cases than in non-rare outcome cases. Furthermore, the larger the gap between $TNIE$ and TIE in SE, the smaller the proportion of $Y = 1$ in the rare outcome cases. ASE and MSE produced similar results to SE. The pattern of the SE, ASE, and MSE matched my hypotheses, but the visibility between the rare and non-rare outcome cases did not.

In both outcome cases, there was no clear pattern in the *coverage probabilities* for the two quantities. In both outcome cases, $TNIE$ had a lower *coverage probability* than TIE. The difference between the two quantities was greater in the rare outcome cases than in the non-rare outcome cases.Regardless of the proportion values, the majority of *coverage probabilities* for the two quantities were less than .90. This indicated that the $TNIE$ and TIE results were not very reliable. The results did not match my hypothese.

For the first four criteria, the results are generally consistent with my hypotheses. As the proportion of $Y = 1$ increased, the difference between the values of $TNIE$ and TIE became smaller. This was evident in the rare outcome cases but not in the non-rare cases, despite the fact that the $TNIE$ values were better than the TIE values across all conditions in the first four evaluation criteria. This implies that $TNIE$ is more accurate than TIE in getting closer to population values for *bias*, SE, ASE, and MSE. In both outcome cases, however, the *coverage* for TIE was greater than that of $TNIE$. In the rare outcome cases, as the proportion of $Y = 1$ increased, the gap in *coverage* between the two quantities shrank. The gap did not change significantly in the non-rare cases. Despite the fact that the results suggested that TIE was more reliable than $TNIE$, the differences were not significant. As a result, there is no reason to be concerned about the outcomes of $TNIE$.

Effect Size. With increasing effect size, the *bias* for $TNIE$ became closer to 1 than the *bias* for TIE, and the gap between them widened in both outcome cases. The gap was larger in the rare outcome cases than in the non-rare cases. This meant that as the effect size increased, it became clear that *bias* for $TNIE$ was better than the *bias* for TIE, especially in the rare outcome cases. In more detail, when the effect size was small, the *bias* for $TNIE$ was slightly better than the *bias* for TIE; when the effect size was medium, the *bias* for $TNIE$ was better than the *bias* for TIE; and when the effect size was large, the *bias* for $TNIE$ was clearly better than the *bias* for TIE, as demonstrated in both cases. My hypotheses are confirmed by the results besides the visibility of the two outcome cases.

Similar to the *bias* results, the SE, ASE, and MSE for $TNIE$ were clearly better than those for TIE in rare outcome cases than in non-rare outcome cases. This indication also partly corresponds to my hypotheses. I anticipated that the gaps were clearer in the non-rare outcome cases.

In the rare outcome cases, *coverage probabilities* for both $TNIE$ and TIE decreased, while the gap between them increased as the effect size increased. Some *coverage probabilities* for the two quantities were below .90 when the effect size was small; when the effect size was medium, the *coverage* for the two quantities decreased and the *coverage* for $TNIE$ was slightly worse than the *coverage* for TIE; when the effect size was large, the *coverage* for $TNIE$ was slightly better than the *coverage* for TIE. The gaps between the two quantities were much smaller in non-rare outcome cases. The change in effect size did not follow the same pattern under the same conditions with a sample size of 1000. This suggested that, with the exception of conditions with a small effect size, the results of $TNIE$ and TIE in rare and non-rare outcome cases were reliable in some conditions. My hypotheses were not supported by the results. I expected that the results would be reliable in all conditions and that non-rare outcome cases would outperform rare outcome cases.

In summary, when the effect size was small, $TNIE$ recovered the population effect values slightly better than TIE; when the effect size was medium, $TNIE$ recovered the population effect values better than TIE; and when the effect size was large, the gap between the two quantities was much larger. To put it simply, $TNIE$ was clearly superior to TIE. Across three effect size conditions, the gap between the two quantities was larger in the rare outcome cases than in the non-rare outcome cases. The first four evaluation criteria clearly demonstrated this pattern. The *coverage probability* and *power* for $TNIE$ and TIE were very close for the last two criteria. Because the last two criteria evaluate the results' reliability and precision, the fact that the two values were good and close to each other implied that we could trust the results.

Sample Size. When the sample size was increased, the *bias* for $TNIE$ became closer to 1 than the *bias* for TIE, and the difference between them shrank in both outcome cases. The gap was larger in the rare outcome cases than in the non-rare cases. This meant that as the sample size grew, it became less clear that the *bias* for $TNIE$ was better than the *bias* for TIE, especially in non-rare outcome cases. In more detail, when the sample size was 350, the *bias* for $TNIE$ was clearly better than the *bias* for TIE; when the sample size was 700, the *bias* for $TNIE$ was better than the *bias* for TIE; and when the sample size was 1000, the *bias* for $TNIE$ was slightly better than the *bias* for TIE, but this could not be seen clearly in both cases. My hypotheses were supported by the pattern, but not by the visibility of the rare and non-rare outcome cases.

Similarly to *bias*, the SE, ASE, and MSE for $TNIE$ were clearly better than those for TIE in rare outcome cases than non-rare outcome cases. When the sample size was increased, the SEs for the two quantities became closer and almost identical. My hypotheses are confirmed by the results, but not by the visibility of the two outcome cases.

The *coverage probabilities* for both $TNIE$ and TIE decreased as the sample size increased, as did the gap between them. When the sample size was 350, the *coverage* for the two quantities was close to.95. Furthermore, the *coverage* for $TNIE$ was always less than that of TIE. This means that the results for $TNIE$ and TIE in both rare and non-rare outcome cases were only reliable with a sample size of 350. My hypotheses do not account for this.

In both outcome cases, the *powers* for the two quantities were equal to one in non-rare outcome cases and were unaffected by the change in sample size. The *power* of $TNIE$ was one in rare outcome cases, while the *power* of TIE increased to one as the sample size increased. This indicated that the $TNIE$ and TIE results were accurate in all conditions. My hypotheses are confirmed by the results.

Except for the visibility of rare and non-rare outcome cases, the results for the sample size factor are generally consistent with my hypotheses for the first four criteria. Further elaborated, when the sample size was 350, the criteria for $TNIE$ were clearly better than the criteria for TIE; when the sample size was 700, the criteria for $TNIE$ were better than the criteria for TIE; and when the sample size was 1000, the criteria for $TNIE$ were slightly better than or nearly identical to the criteria for TIE. Furthermore, the results showed that the gap between the two quantities was larger in the rare outcome cases than in the non-rare outcome cases, contrary to my hypotheses. However, the *coverage probability* for $TNIE$ was lower than that of TIE, while the *power* for the two quantities was the same.

In the first four evaluation criteria for Study 1, the results are aligned with my hypothesis for non-rare outcome cases but not for rare outcome cases. I predicted that the results for $TNIE$ and TIE would be nearly identical in rare cases, with $TNIE$ outperforming TIE in non-rare cases. Based on the results, when the proportion of $Y = 1$ changed, $TNIE$ was more accurate than TIE in both cases. Because my hypotheses were based on the results of Rijnhart et al. (2021), a number of factors contributed to the discrepancy between my hypotheses and the results. First, the researchers only looked at rare outcomes. My hypotheses were not tested in cases with non-rare outcomes. Second, the model used in the paper included an interaction term between the independent variable and the mediator, whereas my model did not. The additional term can significantly improve the impact of $TNIE$ on estimating mediation effects. This explains why $TNIE$ is identical to TIE in their paper when it comes to recovering population results.

Study 2: Moderated Mediation Model 2

Overall, the results match my hypothesis that the causal inference approach produced a more accurate mediation effect than the classical approach. Moreover, the results from the probit model was exactly as I anticipated that the gap between the two approaches were more visible in the non-rare outcome cases.

Probit Model. According to the results of the probit model analysis, $TNIE$ performed more accurately than TIE in recovering the population values. In all simulation factors, the results of *bias* and *coverage probability* were more visible in the non-rare outcome cases, whereas the results of other evaluation criteria were more noticeable in the rare outcome cases.

Proportion Y=1. When the proportion of $Y = 1$ increased, the *bias* for $TNIE$ approached 1 in both rare and non-rare outcome cases. This suggested that $TNIE$ was more accurate than TIE in recovering population mediation effects. Furthermore, the disparities between the two quantities were more noticeable in non-rare outcome cases. This is consistent with my hypotheses.

When the proportion of $Y = 1$ was increased for SE and MSE, the results for $TNIE$ were better than the results for TIE in recovering the population values. The gap between the two quantities narrowed as the proportion of $Y = 1$ increased, but $TNIE$ is still better than TIE in both outcome cases. This is exactly what I was expecting. Even though the results for ASE did not converge well in the rare outcome cases for a sample size of 350, they still showed that $TNIE$ outperformed TIE.

The *coverage probability* results match my hypotheses quite well. Aside from the fact in rare outcome cases and a large effect size in non-rare outcome cases, the *coverage* for $TNIE$ was slightly lower than that of TIE. Even though $TNIE$ had less coverage than TIE in those cases, the difference was very small, especially in the rare outcome cases. Furthermore, the *coverage probabilities* for the two quantities were high (above .94

for $TNIE$ and .8 for TIE) across all proportion levels. This indicated that the results were quite consistent.

Except when the proportion of $Y = 1$ was 0.06 or 0.07, the *power* for TIE was good (above .80) in both outcome cases. Furthermore, in both outcome cases, the *power* for $TNIE$ remained constant at 1 across all conditions, whereas the *power* for TIE was 1 in some conditions. Despite the fact that the results do not support my hypothesis that as the proportion of $Y = 1$ increased, the *power* for $TNIE$ was consistently greater than the *power* for TIE, the results still indicated that $TNIE$ recovered the population effects better than TIE.

In general, the results closely match my hypotheses as the proportion of $Y = 1$ increased. The difference between $TNIE$ and TIE values became smaller as the proportion of $Y = 1$ increased, but $TNIE$ values were still better than TIE across all conditions in the first four evaluation criteria. This implied that $TNIE$ was more accurate than TIE for *bias*, SE, ASE, and MSE. The *coverage* for TIE, on the other hand, was sometimes better and sometimes worse than that of $TNIE$. Even though the results suggested that TIE was more reliable than $TNIE$, the differences were small. As a result, my hypotheses that $TNIE$ was a better choice are still supported.

Effect Size. With increasing effect size, the *bias* for $TNIE$ became closer to 1 than the *bias* for TIE, and the gap between them widened in both outcome cases. This meant that as the effect size increased, it became clear that *bias* for $TNIE$ was smaller than the *bias* for TIE, especially in the rare outcome cases. More specifically, when the effect size was small, the *bias* for $TNIE$ was slightly smaller in the rare outcome cases and smaller in the non-rare cases than the *bias* for TIE; when the effect size was medium, the *bias* for $TNIE$ was smaller in rare outcome cases and non-rare outcome cases than the *bias* for TIE; and when the effect size was large, the *bias* for $TNIE$ was smaller in rare outcome cases and clearly smaller in non-rare outcome cases than the *bias* for TIE. My hypotheses are confirmed by the results.

The results for the SE, ASE, and MSE showed that $TNIE$ was clearly more accurate than TIE in recovering population values in rare outcome cases than in non-rare outcome cases. Further elaborated, when the effect size was small, $TNIE$ in these criteria was slightly smaller than TIE or nearly identical to TIE (i.e., MSE); when the effect size was medium, $TNIE$ in these criteria was smaller than TIE; and when the effect size was large, $TNIE$ in these criteria was clearly smaller than TIE. These outcomes are exactly what I expected.

The *coverage probabilities* results do not match my hypotheses. In rare outcome cases, regardless of effect size changes, the *coverage probability* for $TNIE$ was slightly smaller than the *coverage probability* for TIE. When the effect size was small, the *coverage probability* for $TNIE$ was slightly smaller under a sample size of 350 and clearly larger under other sample sizes than the *coverage probability* for TIE; when the effect size was medium, the *coverage probability* for $TNIE$ was larger than the *coverage probability* for TIE; when the effect size was large, the *coverage probability* for $TNIE$ was smaller than the *coverage probability* for TIE.

When the effect size was small, the *power* for the two quantities remained at 1; when the effect size was medium, the *power* for $TNIE$ was greater than the *power* for TIE only when the sample size was 350; and when the effect size was large, the *power* for $TNIE$ was greater than the *power* for TIE only when the sample size was 350 or 700; otherwise, the two quantities had the same *power* of 1. The findings confirm my hypotheses, as I expected the *power* for the two quantities to be closer to 1. I also predicted that as the effect size increased, $TNIE$ would consistently outperform TIE in terms of *power*. The results do not support this. However, $TNIE$ remained at the highest *power* across all conditions.

In summary, when the effect size was small, the *bias*, SE, ASE, and MSE for $TNIE$ were slightly better than those for TIE; when the effect size was medium, the criteria values for $TNIE$ were better than those for TIE; and when the effect size was large, the gap between the two quantities was much larger. To put it simply, $TNIE$ was clearly superior to TIE. In terms of *coverage probability*, even though $TNIE$ was less than TIE in some conditions, the difference was very small. The gap was wide when the *coverage probability* for $TNIE$ was greater than that for TIE. The *power* for $TNIE$ and TIE were close in most of the conditions. Because the last two criteria assess the results' reliability and precision, the fact that the two values were good and close to each other implied that we can trust the results.

Sample Size. When the sample size increased, the *bias* for $TNIE$ became closer to 1 than the *bias* for TIE, and the difference between them shrank in both outcome cases. This meant that as the sample size grew larger, it became less clear that the *bias* for $TNIE$ was smaller than the *bias* for TIE. More specifically, when the sample size was 350, the *bias* for $TNIE$ was clearly smaller than the *bias* for TIE; when the sample size was 700, the *bias* for $TNIE$ was smaller than the *bias* for TIE; and when the sample size was 1000, the *bias* for $TNIE$ was slightly smaller than the *bias* for TIE. This pattern can be seen more clearly in rare outcome cases than in non-rare outcome cases, despite the fact that the difference in *bias* between the two quantities was much larger in non-rare outcome cases. My hypotheses are not fully supported by the results. I expected the change in *bias* to be more noticeable in non-rare outcome cases.

The SE, ASE, and MSE results revealed that $TNIE$ performed more accurately than TIE in recovering population values. The values were slightly higher in the rare outcome cases than in the non-rare outcome cases. When the sample size increased, the criteria for the two quantities became more similar and nearly identical. This corresponded to my hypotheses for non-rare outcome cases. For rare outcome cases, I expected the criteria values for $TNIE$ and TIE to be slightly identical when the sample size was 350; almost identical when the sample size was 700; and identical when the sample size was 1000.

The *coverage probabilities* for both $TNIE$ and TIE decreased as the sample size increased, and the gap between them widened. This was clearly visible in both outcome cases with a small effect size. In the rare outcome cases, w hen the sample size was 350, the *coverage* for $TNIE$ was slightly lower than the *coverage* for TIE; as the sample size increased to 700, the *coverage* for $TNIE$ was lower than that of TIE; and finally, when the sample size reached 1000, the *coverage* for $TNIE$ was lower than the *coverage* for TIE. In the non-rare outcome cases, when the sample size was 350, the *coverage* for $TNIE$ was slightly lower than the *coverage* for TIE; when the sample size increased to 700, the *coverage* for $TNIE$ was higher than the *coverage* for TIE; and when the sample size reached 1000, the *coverage* for $TNIE$ was clearly higher than the *coverage* for TIE. This contradicts my hypotheses. I expected the difference between the two quantities to narrow as the sample size grew larger.

When the sample size was 350, the *power* for $TNIE$ was greater than the *power* for TIE; when the sample size was 700, the *power* for $TNIE$ was greater than the *power* for TIE only when the effect size was large; otherwise, the *power* for the two quantities was 1; and when the sample size was 1000, the *power* for $TNIE$ and TIE remained constant at 1. These results do not fully support my hypotheses.

Except for the visibility of rare and non-rare outcome cases, the results for the sample size factor were generally consistent with my hypotheses for the first four criteria. In more detail, when the sample size was 350, $TNIE$ were clearly better than TIE in recovering population effect values; when the sample size was 700, the $TNIE$ were better than TIE; and when the sample size was 1000, the $TNIE$ were slightly better than or nearly identical to TIE. Furthermore, the results showed that the gap between the two quantities was more visible in non-rare outcome cases for *bias* and in rare outcome cases for other criteria, whereas I expected the results to be more visible in non-rare outcome cases for all criteria. In terms of *coverage probability*, $TNIE$ was less than TIE in rare outcome cases and, under certain conditions in non-rare outcome cases. The *power* for $TNIE$, on the other hand, was either greater than or equal to the *power* for TIE. Despite the fact that the results do not fully support my hypotheses, they do show that $TNIE$ is more accurate than TIE.

Moderation Effect. When the moderation effect was present, the *bias* for the two quantities was closer to 1. In the rare outcome cases, the *bias* for $TNIE$ was clearly closer to 1 than the *bias* for TIE under a large effect size in both levels of the moderation effect, whereas it was slightly farther away from 1 than or nearly identical to the *bias* for TIE under small and medium effect sizes without a moderation effect. In non-rare outcome cases, the *bias* for $TNIE$ was closer to 1 than the *bias* for TIE across all conditions. My hypotheses are confirmed by the results.

The outcomes for SE, ASE, and MSE were fairly similar. The criteria values of the two quantities were smaller in the presence of the moderation effect, and the criteria values for $TNIE$ were also smaller than those for TIE. In the absence of the moderation effect, however, the criteria values for $TNIE$ were slightly higher than those for TIE in rare outcome cases. My hypotheses are not supported by the results.

Without the moderation effect, the *coverage* for the two quantities was reduced and the gap between them widened. The differences were more noticeable in non-rare outcome cases than in rare outcome cases. My hypotheses are validated by this result. Despite this, $TNIE$ had less *coverage* than TIE at both levels of the moderation effect. This contradicts my hypotheses.

In both outcome cases, $TNIE$ had a power of 1 under two levels of moderation effects. In the rare outcome cases, on the other hand, TIE had less power in the absence of a moderation effect. Without the presence of a moderation effect, TIE had a power of 1 in all non-rare outcome cases. Furthermore, the *power* for TIE was less than 0.8 in both moderation effects with a large effect size and a sample size of 350. My hypotheses were not supported by the results.

In both outcome cases, the presence of a moderation effect improved the results for the moderation effect factor. Only in *bias* do the results match my hypotheses.

Logit Model. Overall, the logit model's values for the first five evaluation criteria for $TNIE$ and TIE were not as good as the probit model's. This is explained by the fact that the probit model was used to simulate data sets. The causal inference approach produced more accurate mediation effects in the logit model than the classical approach. This was demonstrated by the findings that $TNIE$ outperformed TIE in recovering population values in both rare and non-rare outcome cases across all evaluation criteria except *coverage probability*, and *power*. Both quantities have a high *power* across all conditions. This indicated that the $TNIE$ and TIE results were precise.

Proportion of Y=1. As the proportion of $Y = 1$ increased, the *bias* for $TNIE$ approached 1 in both rare and non-rare outcome cases. This suggested that $TNIE$ was more accurate than TIE in recovering population mediation effects. Furthermore, the gaps between the two quantities were nearly identical in both outcome cases. This contradicts my hypotheses. In the rare outcome cases, I expected the *bias* for $TNIE$ to be similar to that of TIE.

When the proportion of $Y = 1$ increased for SE, ASE, and MSE, the results for $TNIE$ were better than those for TIE in recovering the population values. The gap between the two quantities narrowed as the proportion of $Y = 1$ increased, but $TNIE$ is still better than TIE in both outcome cases. Furthermore, the disparity was more pronounced in rare outcome cases versus non-rare outcome cases. This is the polar opposite of what I expected.

Except for the part where the *coverage* for $TNIE$ was slightly smaller than the *coverage* for TIE, the results for *coverage probability* match quite well with my hypotheses. Even though the *coverage* for $TNIE$ was less than that of TIE, the difference was very small, especially in the rare outcome cases. Furthermore, across all levels of the proportion, the majority of *coverage, probabilities* of the two quantities were less than.85.

This indicates that the $TNIE$ and TIE results were not very reliable. My hypotheses were not supported by the results.

In both outcome cases, the *power* for the two quantities was quite high (above .95). Furthermore, in non-rare outcome cases, the *power* for $TNIE$ and TIE remained constant at 1. For rare outcome cases, the *power* for $TNIE$ remained constant at one, whereas the *power* for TIE remained constant only when the sample size was 700 or 1000. This contradicts my hypotheses.

Aside from the *coverage*, the results in the visibility of the two outcome cases do not match my hypotheses as the proportion of $Y = 1$ increased. As the proportion of $Y = 1$ increased, the difference between the values of $TNIE$ became smaller than that of TIE. This was evident in the rare outcome cases but not in the non-rare cases, despite the fact that the $TNIE$ values were better than the TIE values across all conditions in the first four evaluation criteria. In other words, the results of *bias*, SE, ASE, and MSE indicated that $TNIE$ was more accurate than TIE in recovering the population values. In both outcome cases, however, the *coverage* for TIE was greater than the *coverage* for $TNIE$. In the rare outcome cases, as the proportion of $Y = 1$ increased, the *coverage* gap between the two quantities shrank. The gap did not change significantly in the non-rare cases. Despite the fact that the results suggested that TIE was more reliable than $TNIE$, the differences were not significant. As a result, there is no reason to be concerned about the outcomes of $TNIE$.

Effect Size. When the effect size was increased, the *bias* for $TNIE$ became closer to 1 than the *bias* for TIE, and the gap between them widened in both outcome cases. This indicated that as the effect size increased, it became increasingly clear that the $TNIE$ was less bias than TIE, particularly in the rare outcome cases. In more detail, when the effect size was small, the *bias* for $TNIE$ was slightly smaller than the *bias* for TIE; when the effect size was medium, the *bias* for $TNIE$ was smaller than the *bias* for TIE; and when the effect size was large, the *bias* for $TNIE$ was clearly smaller than the *bias* for TIE, as demonstrated in both cases. My hypotheses are supported by the results.

The results for the SE, ASE, and MSE showed that $TNIE$ was clearly more accurate than TIE in recovering population values in rare outcome cases than in non-rare outcome cases. Precisely, when the effect size was small, $TNIE$ in these criteria was slightly smaller than TIE or nearly identical to TIE (i.e., MSE); when the effect size was medium, $TNIE$ in these criteria was smaller than TIE; and when the effect size was large, $TNIE$ in these criteria was clearly smaller than TIE. These outcomes were exactly what I expected.

The $coverage\,probabilities$ results contradicted my hypotheses completely. When the effect size was small, the $coverage, probabilities$ for $TNIE$ was slightly smaller than the *coverage* for TIE; when the effect size was medium, the *coverage* for the two quantities decreased, and the *coverage* for $TNIE$ was smaller than the *coverage* for TIE; and when the effect size was large, the *coverage* for $TNIE$ was clearly smaller than the *coverage* for TIE. This pattern was evident in both outcomes.

In the last criterion, as the effect size was small and medium in the rare outcome cases, the *power* for the two quantities remained at 1. When the effect size was large, the *power* for $TNIE$ was greater than the *power* for TIE only when the sample size was 350; otherwise, the two quantities had the same *power* of 1. The *power* for $TNIE$ and TIE in non-rare outcome cases was 1 for all conditions. The results partially confirm my hypotheses, as I expected the *power* for the two quantities to be closer to 1. I also predicted that as the effect size increased, the *power* for $TNIE$ would be closer to 1 than the *power* for TIE. That does not correspond to the results.

In summary, when the effect size was small, the *bias*, SE, ASE, and MSE for $TNIE$ recovered the population effect values slightly more accurate than those for TIE; when the effect size was medium, $TNIE$ recovered the population effect values more accurate than TIE; and when the effect size was large, the gap between the two quantities was much larger. That is to say, $TNIE$ was clearly smaller than TIE in those values. Across three effect size conditions, the gap between the two quantities was larger in the rare outcome cases than in the non-rare outcome cases. The *coverage probability*, and *power* for $TNIE$ and TIE were very close. Because the last two criteria assess the reliability and precision of the results, the fact that the two values were good and close to each other implied that we could trust the results.

Sample Size. When the sample size was increased, the *bias* for $TNIE$ became closer to 1 than the *bias* for TIE, and the difference between them shrank in both outcome cases. This meant that as the sample size grew larger, it became less clear that the *bias* for $TNIE$ was smaller than the *bias* for TIE. In more detail, when the sample size was 350, the *bias* for $TNIE$ was clearly smaller than the *bias* for TIE; when the sample size was 700, the *bias* for $TNIE$ was smaller than the *bias* for TIE; and when the sample size was 1000, the *bias* for $TNIE$ was slightly smaller than the *bias* for TIE. This pattern is visible in both outcome cases. This perfectly matched my hypotheses.

The results of SE, ASE, and MSE indicated that $TNIE$ recovered population values better than TIE. The values were slightly higher in the rare outcome cases than in the non-rare outcome cases. When the sample size was increased, the SEs for the two quantities became closer and almost identical. This corresponded to my hypotheses for non-rare outcome cases. For rare outcome cases, I expected the criteria values for $TNIE$ and TIE to be slightly identical when the sample size was 350; almost identical when the sample size was 700; and identical when the sample size was 1000.

The *coverage probabilities* for both $TNIE$ and TIE decreased as the sample size increased, but the gap between them grew larger before narrowing slightly. More closely, when the sample size was 350, the *coverage* for $TNIE$ was lower than the *coverage* for TIE; when the sample size increased to 700, the *coverage* for $TNIE$ was clearly lower than the *coverage* for TIE; and finally, when the sample size reached 1000, the *coverage* for $TNIE$ was lower than the *coverage* for TIE. My hypotheses were not supported by the results. I anticipated that $TNIE$ had higher *coverage* than TIE.

When the sample size was 350 for the rare outcome cases, the *power* for $TNIE$ was greater than the *power* for TIE only when the effect size was large; otherwise, the *power* for the two quantities was 1. The *power* for $TNIE$ and TIE remained constant when the sample size was increased to 700 and 1000, respectively. The *powers* for the two quantities were equal to 1 in the non-rare outcome cases and were unaffected by the change in sample size. These findings contradict my hypotheses.

With the exception of the visibility for rare and non-rare outcome cases, the results for the sample size factor were generally consistent with my hypotheses for the first four criteria. In more detail, when the sample size was 350, the $TNIE$ criteria were clearly better than the TIE criteria in recovering population effect values; when the sample size was 700, the $TNIE$ criteria were better than the TIE criteria; and when the sample size was 1000, the $TNIE$ criteria were slightly better than or nearly identical to the TIE criteria. Furthermore, the results showed that the gap between the two quantities was similar in both outcome cases, whereas I expected the results in non-rare outcome cases to be more visible. The *coverage probability* for $TNIE$ was lower than the *coverage probability* for TIE, and the *coverage, probabilities* was quite low. The *power* for the two quantities, on the other hand, was nearly identical. Even though the results do not match my hypotheses, the fact that the *power* was greater than.95 indicated that the $TNIE$ and TIE results were precise in all conditions.

Moderation Effect. When the moderation effect was present, the *bias* for the two quantities was closer to 1. Furthermore, in both levels of the moderation effect, the *bias* for $TNIE$ was clearly closer to 1 than the *bias* for TIE. This pattern was slightly more visible in the cases with non-rare outcomes. My hypotheses are verified by the results.

The outcomes of SE, ASE, and MSE were comparable. The criteria values for the two quantities were smaller in the presence of the moderation effect, and the values for

$TNIE$ were also smaller than TIE. Furthermore, the values were slightly lower in non-rare outcome cases. I correctly predicted the different outcomes in the two levels of the moderation effect. My hypotheses, however, are diametrically opposed to the visibility of the outcome cases. I expected the cases with rare outcomes to be less visible.

Without the moderation effect, the *coverage* for the two quantities was much lower, and the gap between them was much wider. The gaps were much larger in cases with rare outcomes than in cases with non-rare outcomes. For the visibility of the two outcome cases, the results do not match my hypotheses.

In the two levels of the moderation effect, the *power* for the two quantities was very similar. The *power* for TIE was slightly higher when a moderation effect was present, whereas the *power* for $TNIE$ was 1 under all conditions. This result corroborates my hypothesis. There was no discernible difference between rare and non-rare outcome cases. This result contradicts my hypothesis. I expected that the outcomes of non-rare outcome cases would be more visible than those of rare outcome cases.

In both outcome cases, the presence of a moderation effect improved the results for the moderation effect factor. Furthermore, in both levels of the moderation effect, $TNIE$ outperformed TIE in the first four evaluation criteria. The results for TIE were better and more clear for the *coverage probability* without a moderation effect. Because I used Mplus results directly, the calculation may be inaccurate. The results for $TNIE$ and TIE for *power* were very high and very close to each other.

To summarize, the findings of Study 2 closely match my hypotheses, with the exception of the distinction between rare and non-rare outcome cases. Because my visibility hypotheses were based on the findings of Rijnhart et al. (2021), where the researchers only looked at the rare outcome cases. My study's findings indicated that $TNIE$ were superior to TIE, whereas Rijnhart et al. (2021) claimed that the two quantities were identical. The main reason for the disparity between my results and theirs is that we did not use the same model; mine is moderated mediation Model 2 with treatment-moderator interaction, while theirs is moderated mediation Model 1 with treatment-mediation interaction.

Implications

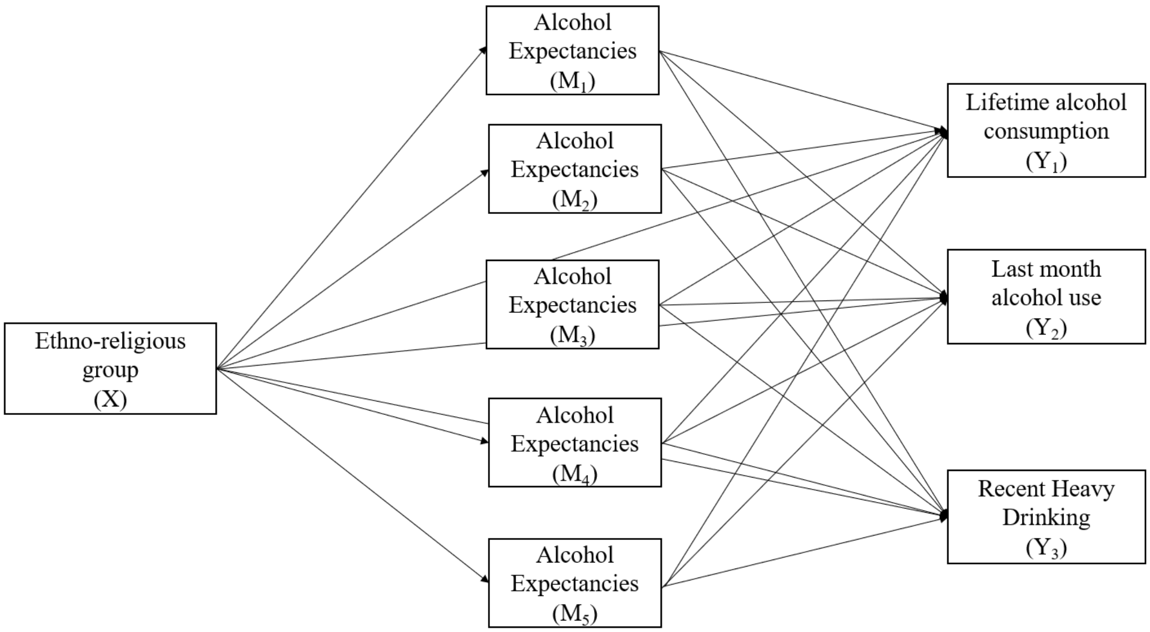
The causal inference approach outperforms the classical approach in estimating accurate mediation effects, according to the results and discussions. The findings would support the use of the causal inference approach with mediation analyses when the outcomes are binary. For example, when it is believed that the effect is mediated by students' interest in learning, researchers should use the causal inference approach to test whether an intervention on an education program will improve students' academic achievement, either pass or fail.

The classical approach is a common method for calculating mediation effects. It is widely used in many topics that have binary outcomes, such as alcohol addiction in university students, the effect of students' mental health on academic achievement, and the effect of parental involvement on students' initial alcohol consumption. Because the classical approach has been applied to a wide range of topics, there may be significant effects in demonstrating that the causal inference approach produces more accurate results than the classical approach. Sznitman et al. (2015), for example, presented a study in which the outcomes and mediators are binary. The graphical representation of the model used in the study is shown in Figure 77. The study's goal was to use a mediation model to test the relationship between ethno-religious groups (treatment indicator) and alcohol uses (outcomes) among university students via life expectancies (mediators). The researchers used the traditional method to obtain those effects and the bias-corrected bootstrap resampling method to compute confidence intervals for the mediation effect.

The findings revealed that Israeli Arab students were more likely to abstain from alcohol than Israeli Jewish students, but those Israeli Arab students who did drink were at a higher risk of heavy drinking. Their findings suggested that Ethno-religious differences in

Figure 77

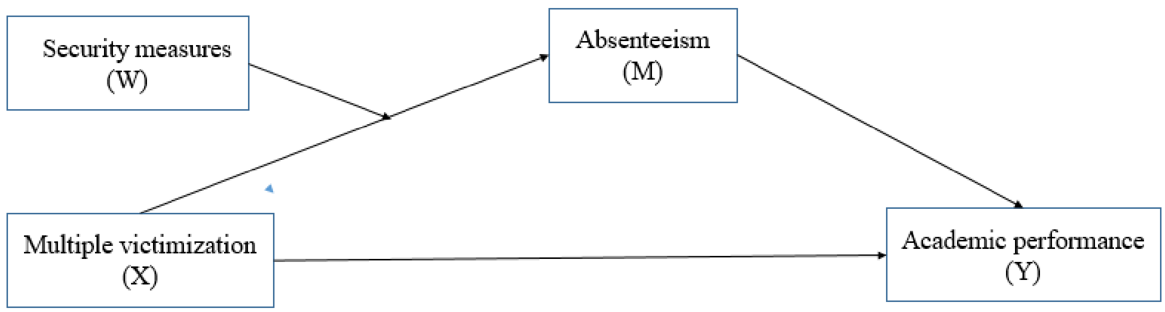
Example for Mediation Analysis with Binary outcomes and Binary Mediators. Adopted from "Examining differences in drinking patterns among Jewish and Arab university students in Israel" by S. R. Sznitman, S. Bord, W. Elias, A. Gesser-Edelsburg, Y. Shiftan, O. Baron-Epel, Ethnicity and Health, 2015, 20(6), p. 594–610. Copyright 2015 by National Library of Medicine.



heavy drinking among Israeli Arabs and Jews were moderated by drinking expectations. This aided public health interventions aimed at better understanding Ethno-religious group differences in harmful drinking. Because the researchers used the classical approach to estimate the mediation effects, we must consider whether the results are accurate. If the results were incorrect, based on the findings of this study that the classical approach produces inaccurate effects when working with binary outcomes, this suggests that the understanding of group differences in harmful drinking may be invalid. As a result, an ineffective intervention is designed. Focusing on the wrong group may mean ignoring those who are at risk of harmful drinking and in need of assistance.

Another example is Gardella et al. (2016)'s paper. The researchers presented a study of moderated mediation Model 2 with a binary mediator and a categorical outcome. The graphical representation of the model used in the study is shown in Figure 78. The researchers employed a moderated mediation analysis to examine the relationship between multiple victimization (treatment indicator) and academic performance (outcome), which was mediated by school absenteeism (mediator), and the relationship between multiple victimization and school absenteeism was moderated by the presence of school security measures (moderator). They used the classical approach to estimate the moderated mediation effect.

Figure 78
Example for Moderated Mediation Analysis with Categorical Outcome and Binary Mediation. Adopted from "Academic Consequences of Multiple Victimization and the Role of School Security Measures" by J. H. Gardella, E. E. Tanner-Smith, B. W. Fisher, American Journal of of Community Psychology, 2016, 58, p. 36-46. Copyright 2016 by Community Research and Action.

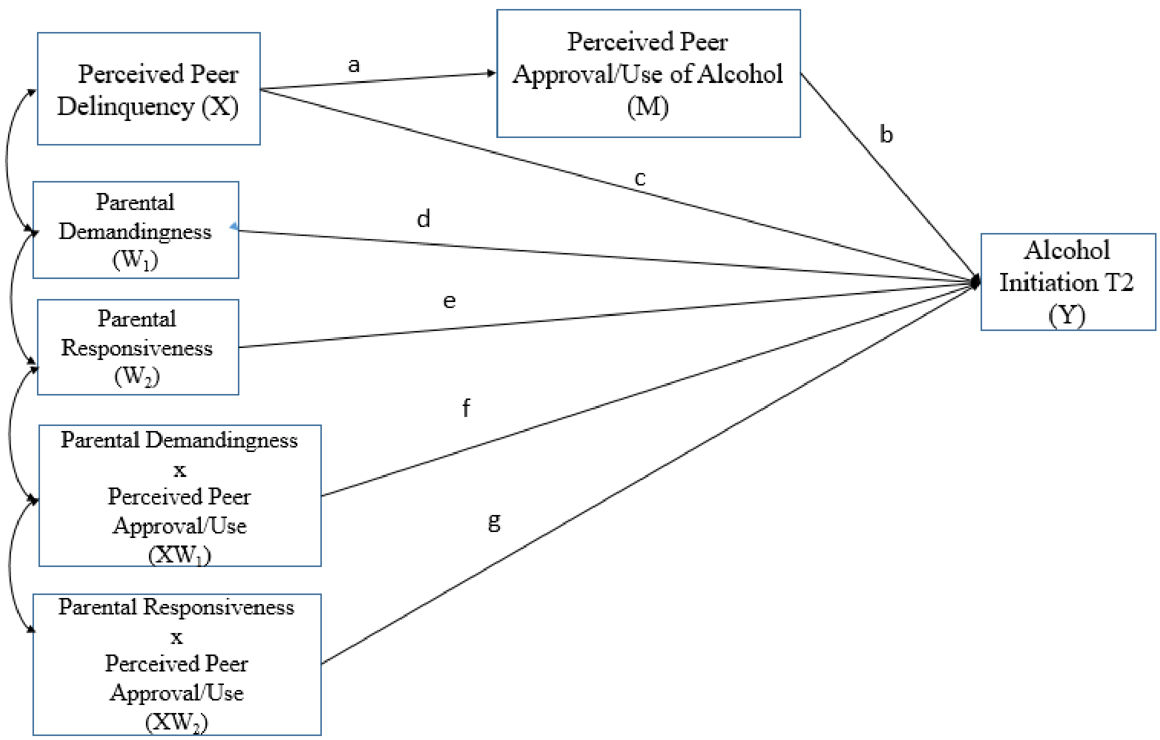


The findings indicated that absenteeism moderated the relationship between multiple victimization and academic performance, and that both metal detectors and security guards moderated the relationship between multiple victimization and absenteeism. In other words, victims of multiple victimization reported higher absenteeism in schools with security guards or metal detectors. The researchers advised practitioners to exercise caution when implementing security measures in schools because some security measures had been linked to increased absenteeism and poor academic performance in some students. Because this study also used the classical approach, the findings may need to be revisited. If the results were inaccurate, schools may need to consider implementing additional security measures to protect students.

The third example is about peer pressure and parental involvement in students' alcohol use. Trucco et al. (2011) presented a study of moderated mediation Model 1 with a binary outcome and a continuous mediator. The graphical representation of the model used in this study is shown in Figure 79. The study's goal was to test the relationship between perceived peer delinquency (treatment indicator) and initiation of alcohol use (outcome) via perceived peer approval/use of alcohol (mediator) under different levels of parental demandingness or parental responsiveness (moderator). The researchers used the classical approach to estimate the moderated mediation effect.

Figure 79

Example for Moderated Mediation Analysis with Binary Outcomes and Continuous Mediations. Adopted from "Vulnerability to peer influence: A moderated mediation study of early adolescent alcohol use initiation by E. M. Trucco, C. R. Colder, W. F. Wieczorek. Psychology of Addictive Behaviors, 2011, 36(7), p. 729–736. Copyright 2017 by 2011 by America Psychological Association



The study's findings revealed that high levels of peer delinquency predicted the initiation of alcohol use through perceived peer approval and use of alcohol; however, parental warmth or parental control was not related to the initiation of alcohol use. According to the researcher, parental warmth and control may not be enough to protect an adolescent from the influence of deviant peers. Because they used the classical approach to estimate moderated mediation effects when the outcome is binary, the results may need to be reexamined. If the results were inaccurate, as suggested by the classical approach producing biased results when working with binary outcomes, the decision on whether parents should increase their warmth or control to prevent adolescents from initiating alcohol use may need to be reconsidered.

These examples highlight the importance of revisiting studies that used the classical approach in mediation analysis with binary outcomes. The inaccuracy of the classical approach's results may cause problems in decision making. Furthermore, this study strongly encourages researchers to experiment with new statistical analytics techniques in order to obtain more robust results. Robust outcomes will lead to accurate and precise decisions, which will aid in academic achievement and the prevention of actions or harmful habits that have caused mental and physical illness.

Furthermore, my research fills a gap in the field's literature by examining the differences between the causal inference approach and the classical approach. Other studies concentrated on continuous and rare outcome cases with moderated mediation Model 1, in which the treatment indicator and mediator interact. My study included non-rare outcome cases as well as rare outcome cases in moderated mediation Model 2, where the treatment indicator and moderator interact. I was able to determine that the mediation effects of the causal inference approach and the classical approach were not equal by expanding the research in other conditions. To put it another way, the causal inference approach produced more accurate mediation effects than the classical approach. This demonstrated the efficacy of the causal inference approach and the importance of using it to estimate accurate mediation effects in mediation analyses with binary outcomes. In addition, the effect will assist researchers and practitioners in making critical and accurate decisions such as changing the teaching curricula, implementing interventions, or implementing school safety measures.

Finally, this study will add to the literature by informing applied education researchers about the applications of the two approaches in mediation and moderated mediation analyses with binary outcomes, as well as clearly demonstrating that the causal inference approach provided a more accurate estimate of a mediation effect. The classical approach was never more effective than the causal-inference approach. More specifically, while it worked well, the classical approach performed only as well as the causal inference approach. As a result, the causal inference approach is recommended wherever feasible. The causal inference approach, on the other hand, is not always practicable. When there are two or more mediators in an indirect path in mediation models, the causal inference approach cannot be applied because Mplus cannot utilize the causal inference approach. Despite the fact that this is a software constraint, I am unaware of any alternative software that can handle it in a convenient manner.

Limitations & Future Research

First and foremost, the sample sizes are quite large. The vast majority of educational research does not employ such a large sample size. However, the results were inconsistent for smaller sample sizes. So, no conclusions can be drawn from the analysis (see Appendix B for these graphs). Second, Mplus was used to calculate the results of the evaluation criteria other than *bias* in this study. The results were generated across all 1000 replications at the same time. Another option is to manually generate the evaluation criteria using the formulas given in the Method Chapter after collecting the data from each replication. I chose Mplus because it is a useful and powerful tool. Third, I calculated mediation effects on an odds-ratio scale for both approaches in order to compare $TNIE$ and TIE in both probit and logit models. As indicated in the literature review, the causal inference approach for the logit model utilizes a risk-ratio scale rather than an odds-ratio scale. As a consequence, the mediation effects estimated by the two approaches in the logit model were substantially closer to each other than in the probit model.

For future research, a real data analysis should be considered as a follow-up study. By analyzing a real data set by both classical and causal inference approaches, it will provide an opportunity to reflect the results from the current study by evaluating the discrepancy between the results from the two approaches. In addition, having a real data analysis is a bridge to connect methodology research to practical applications.

Furthermore, based on the literature review, the causal inference approach is estimated on a risk-ratio scale when using the logit model. The risk-ratio scale is not available for the classical approach since it requires to use the potential outcome method. Thus, the odds-ratio scale was chosen since that is the common scale for all quantities in the study. A study on calculating TNIE on the risk-ratio scale to compare with the TNIE on the odds-ratio scale from the present study might be examined for future research.

References

Allaire, J. J. (2009). *About RStudio*. https://rstudio.com/about/.

Arbuckle, J. K. (2017). *IBM AMOS 25 user's guide*.

Arslan, G., & Coşkun, M. (2021). Social Exclusion, Self-Forgiveness, Mindfulness, and Internet Addiction in College Students: A Moderated Mediation Approach. *International Journal of Mental Health and Addiction*. https://doi.org/10.1007/s11469-021-00506-1

Ato García, M., Vallejo Seco, G., & Ato Lozano, E. (2014). Classical and causal inference approaches to statistical mediation analysis. *Psicothema*, *26.2*, 252–259. https://doi.org/10.7334/psicothema2013.314

Baron, R. M., & Kenny, D. A. (1986). The moderator–mediator variable distinction in social psychological research: Conceptual, strategic, and statistical considerations [Journal Article]. *Journal of Personality and Social Psychology*, *51*(6), 1173–1182. https://doi.org/10.1037/0022-3514.51.6.1173

Cohen, J. (1988). *Statistical power analysis for the behavioral sciences*. Hillsdale: Erlbaum.

Frone, M. R. (1999). Work stress and alcohol use [Journal Article]. *Alcohol Research & Health: The Journal of The National Institute on Alcohol Abuse and Alcoholism*, *23*(4), 284–291.

Gardella, J. H., Tanner-Smith, E. E., & Fisher, B. W. (2016). Academic Consequences of Multiple Victimization and the Role of School Security Measures. *American Journal of Community Psychology*, *58*(1-2), 36–46. https://doi.org/10.1002/ajcp.12075

Hallquist, M. N., & Wiley, J. F. (2018). MplusAutomation: An R package for facilitating large-scale latent variable analyses in Mplus. *Structural Equation Modeling*, 621–638. https://doi.org/10.1080/10705511.2017.1402334

Hayes, A. F. (2012). *PROCESS: A versatile computational tool for observed variable* *mediation, moderation, and conditional process modeling[white paper].* http://www.afhayes.com/%0Apublic/process2012.pdf

Holland, P. W. (1986). Statistics and Causal Inference. *Journal of the American Statistical Association*, *81*(396), 945–960. https://doi.org/10.2307/2289064

Hyman, H. H. (1955). *Survey Design and Analysis: Principles, Cases and Procedures.* Glencoe IL: Free Press.

Imai, K., Keele, L., Tingley, D., & Yamamoto, T. (2011). Unpacking the Black Box of Causality: Learning About Causal Mechanisms from Experimental and Observational Studies. *American Political Science Review*, *105*. https://doi.org/10.1017/S0003055411000414

Imai, K., Luke, J. K., & Tingley, D. (2010). A general approach to causal mediation analysis. *Psychological Methods*, *15*, 309–334. https://doi.org/10.1037/a0020761

James, L. R., & Brett, J. M. (1984). Mediators, moderators, and tests for mediations [Journal Article]. *Journal of Applied Psychology*, *69*(2), 307–321.

Jimanez-Parez, E., Vicente-Yagae Jara, M. I., Gutierrez Fresneda, R., & Garcia Guirao, P. (2021). Sustainable Education, Emotional Intelligence and MotherChild Reading Competencies within Multiple Mediation Models. In *Sustainability* (Vol. 13). https://doi.org/10.3390/su13041803

Judd, C. M., & Kenny, D. A. (1981). Process Analysis: Estimating Mediation in Treatment Evaluations. *Evaluation Review*, *5*(5), 602–619. https://doi.org/10.1177/0193841X8100500502

Lachowicz, M., Preacher, K., & Kelley, K. (2017). A Novel Measure of Effect Size for Mediation Analysis. *Psychological Methods*, *23*. https://doi.org/10.1037/met0000165

Lazarsfeld, P. F. (1955). Interpretation of statistical relations as a research operation. *The Language of Social Research*, 115–125.

MacKinnon, D. P. (2008). *Introduction to Statistical Mediation Analysis* (1 edition). Routledge.

MacKinnon, D. P., Valente, M. J., & Gonzalez, O. (2020). The Correspondence Between Causal and Traditional Mediation Analysis: The Link Is the Mediator by Treatment Interaction. *Prevention Science*, *21*(2), 147–157. https://doi.org/10.1007/s11121-019-01076-4

Morgan, S. L., & Winship, C. (2015). *Counterfactuals and Causal Inference*. Cambridge University Press.

Murphy, K. R., Myors, W., & Brett, A. (2014). *Statistical Power Analysis: A Simple and General Model for Traditional and Modern Hypothesis Tests*.

Muthén. (2011). *Applications of Causally Defined Direct and Indirect Effects in Mediation Analysis using SEM in Mplus*.

Muthén, L. K., & Muthén, B. O. (2010). *Mplus user's guide* (6th ed.).

Muthén, Muthén, L. K., & Asparouhov, T. (2015). Causal effects in mediation modeling: An introduction with applications to latent variables. In *Regression and mediation analysis using Mplus* (pp. 307–379). Muthén & Muthén.

Pearl, J. (2001). Direct and indirect effects. *Proceedings of the Seventeenth Conference on Uncertainty in Artificial Intelligence*, 411–420.

Pearl, J. (2012). The Causal Mediation Formula: A Guide to the Assessment of Pathways and Mechanisms. *Prevention Science*, *13*(4), 426–436. https://doi.org/10.1007/s11121-011-0270-1

Preacher, K. J., Rucker, D. D., & Hayes, A. F. (2007). Addressing moderated mediation hypotheses: Theory, methods, and prescriptions [Journal Article]. *Multivariate Behav Res*, *42*(1), 185–227. https://doi.org/10.1080/00273170701341316

Rice, J. A. (2007). *Mathematical statistics and data analysis* (3rd ed).

Thomson/Brooks/Cole.

Rijnhart, J. J. M., Valente, M. J., MacKinnon, D. P., Twisk, J. W. R., & Heymans, M. W. (2021). The Use of Traditional and Causal Estimators for Mediation Models with a Binary Outcome and Exposure-Mediator Interaction. *Structural Equation Modeling: A Multidisciplinary Journal*, *28*(3), 345–355. https://doi.org/10.1080/10705511.2020.1811709

Ringle, C. M., Wende, S., & Becker, J. M. (2015). *SmartPLS 3*. http://www.smartpls.com

Robins, J. M., & Greenland, S. (1992). Identifiability and exchangeability for direct and indirect effects. *Epidemiology (Cambridge, Mass.)*, *3*(2), 143–155. https://doi.org/10.1097/00001648-199203000-00013

Rubin, D. B. (1990). [On the Application of Probability Theory to Agricultural Experiments. Essay on Principles. Section 9.] Comment: Neyman (1923) and Causal Inference in Experiments and Observational Studies. *Statistical Science*, *5*(4), 472–480. http://www.jstor.org/stable/2245383

Sistrom, C. L., & Garvan, C. W. (2004). Proportions, odds, and risk. *Radiology*, *230(1)*. https://doi.org/10.1148/radiol.2301031028

StataCorp. (2007). *Stata stistical software: Release 10*.

Sznitman, S. R., Bord, S., Elias, W., Gesser-Edelsburg, A., Shiftan, Y., & Baron-Epel, O. (2015). Examining differences in drinking patterns among Jewish and Arab university students in Israel. *Ethnicity & Health*, *20*(6), 594–610. https://doi.org/10.1080/13557858.2014.961411

Trucco, E. M., Colder, C. R., & Wieczorek, W. F. (2011). Vulnerability to peer influence: A moderated mediation study of early adolescent alcohol use initiation [Journal Article]. *Addictive Behaviors*, *36*(7), 729–736. https://doi.org/10.1016/j.addbeh.2011.02.008

Valeri, L., & VanderWeele, T. J. (2013). Mediation analysis allowing for exposure-mediator interactions and causal interpretation: Theoretical assumptions and implementation with SAS and SPSS macros. *Psychological Methods*, *18*(2), 137–150. https://doi.org/10.1037/a0031034

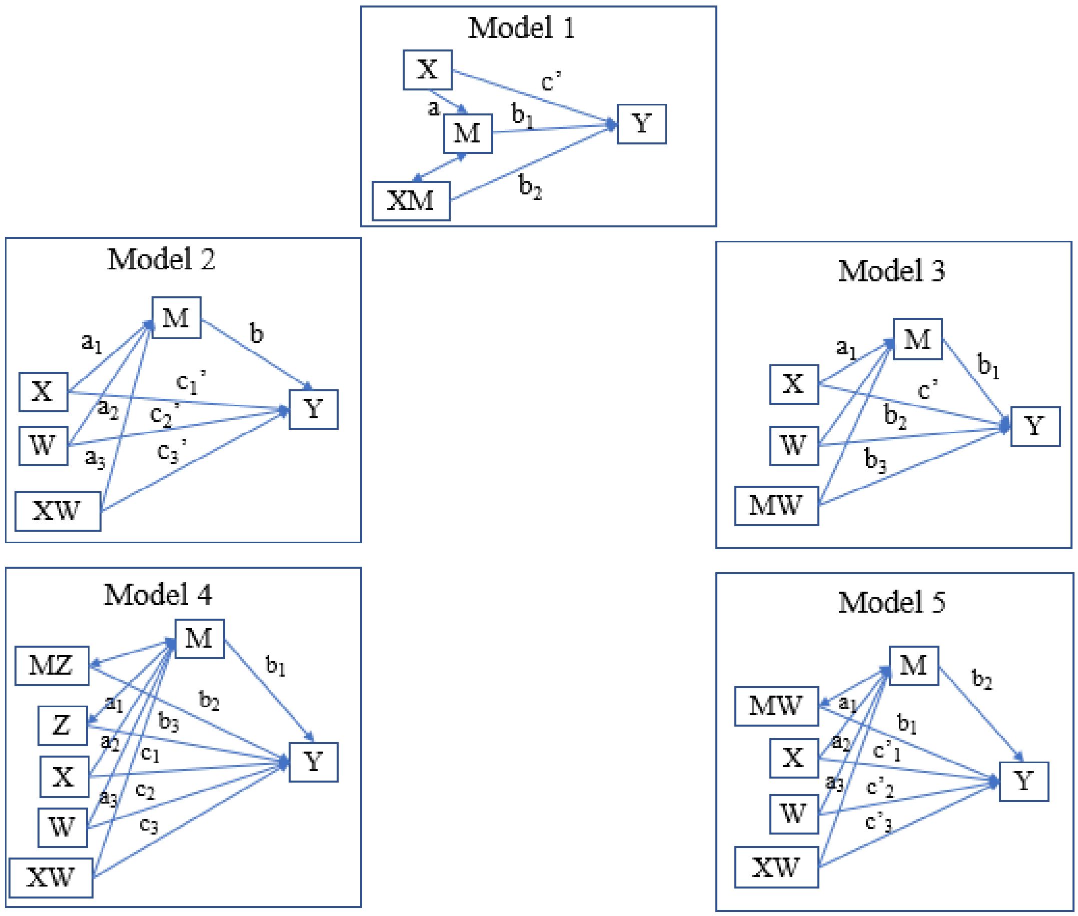
VanderWeele, & Vansteelandt, S. (2010). Odds Ratios for Mediation Analysis for a Dichotomous Outcome. *American Journal of Epidemiology*, *172*(12), 1339–1348. https://doi.org/10.1093/aje/kwq332

Wright, S. (1920). The Relative Importance of Heredity and Environment in Determining the Piebald Pattern of Guinea-Pigs. *Proceedings of the National Academy of Sciences of the United States of America*, *6*(6), 320–332. https://doi.org/10.1073/pnas.6.6.320

Appendix A

Supplimental Graphs

Figure A1
Graphs for Five Common Moderated Mediation Models



Appendix B

Graphs for Inconsistent Results

Figure B1
SE over TIE and TNIE for Rare Outcomes in Probit for Simple Mediation Model

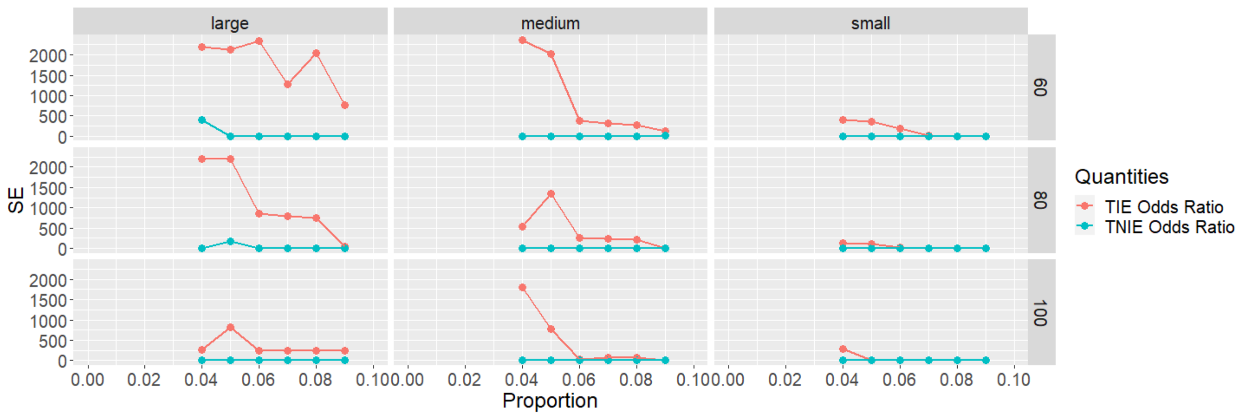


Figure B2
SE over TIE and TNIE for Non-Rare Outcomes in Probit for Simple Mediation Model

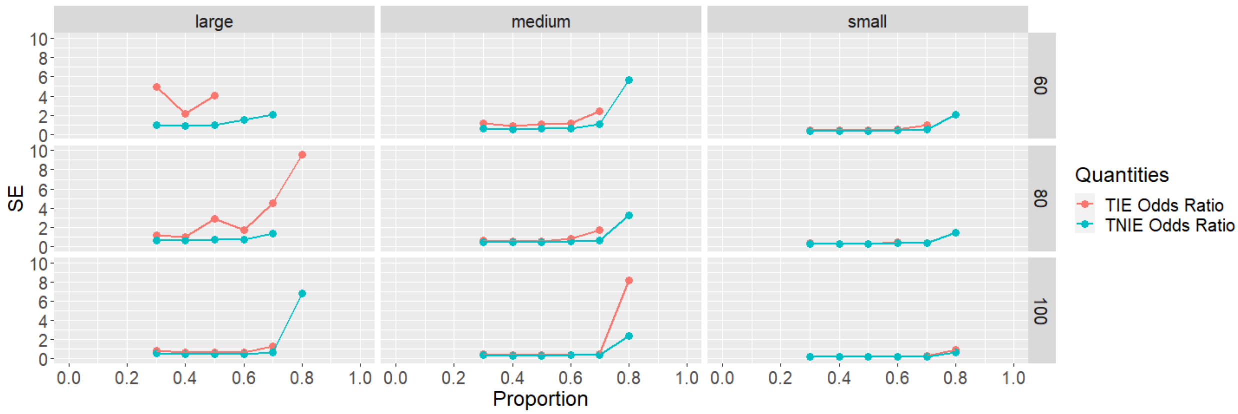


Figure B3
ASE over TIE and TNIE for Rare Outcomes in Probit for Simple Mediation Model

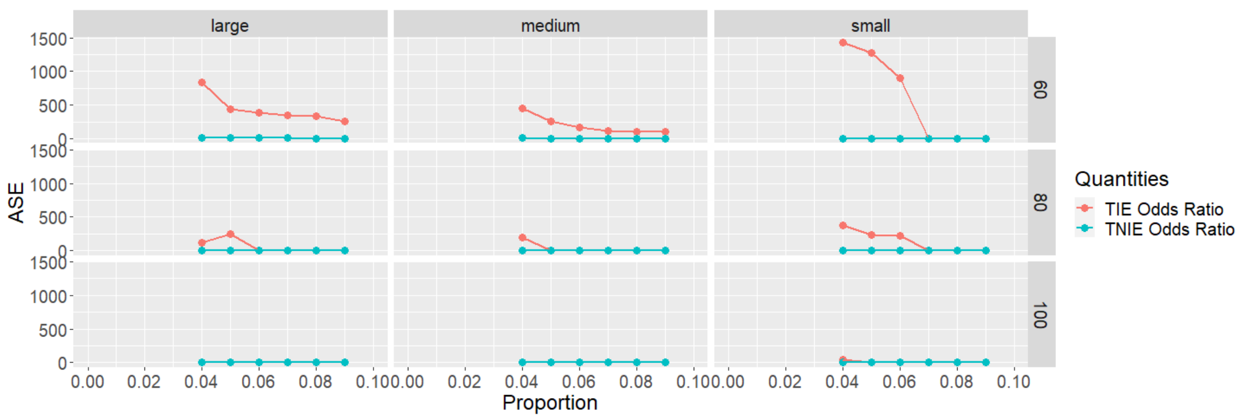


Figure B4
ASE over TIE and TNIE for Non-Rare Outcomes in Probit for Simple Mediation Model

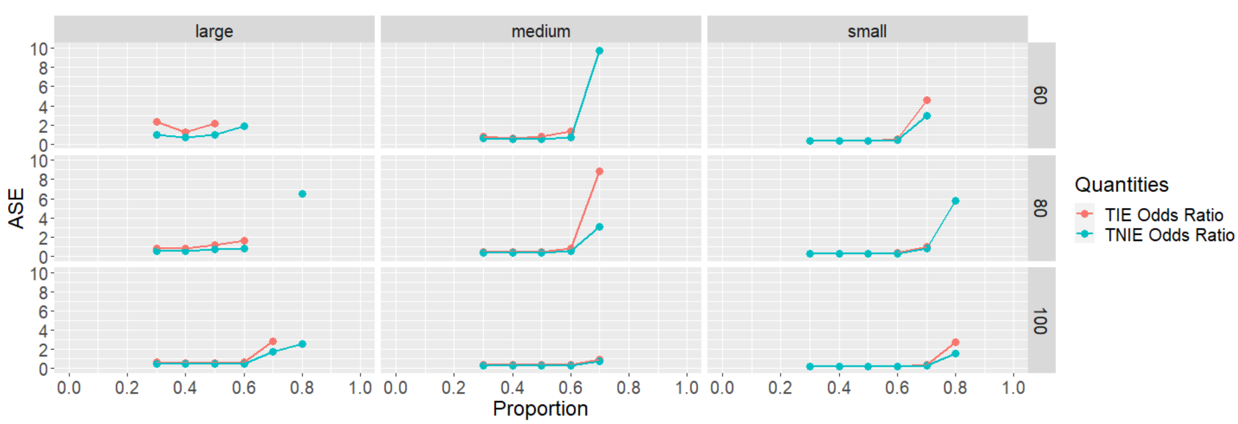


Figure B5

MSE over TIE and TNIE for Rare Outcomes in Probit for Simple Mediation Model

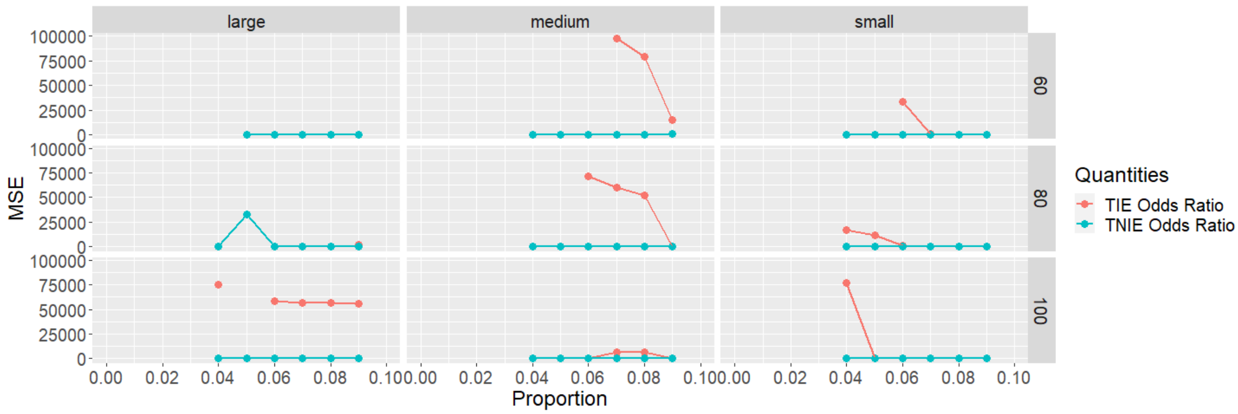


Figure B6

MSE over TIE and TNIE for Non-Rare Outcomes in Probit for Simple Mediation Model

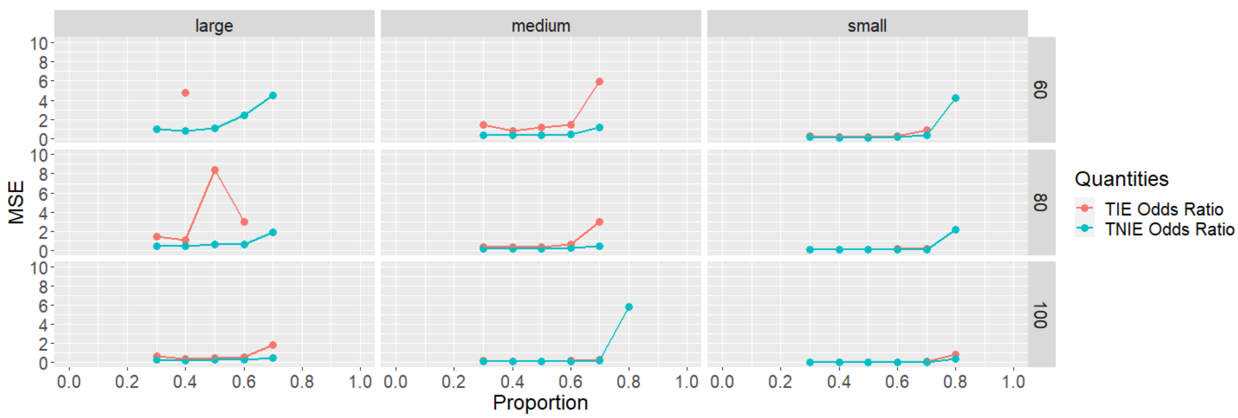


Figure B7

SE over TIE and TNIE for Rare Outcomes in Logit for Simple Mediation Model

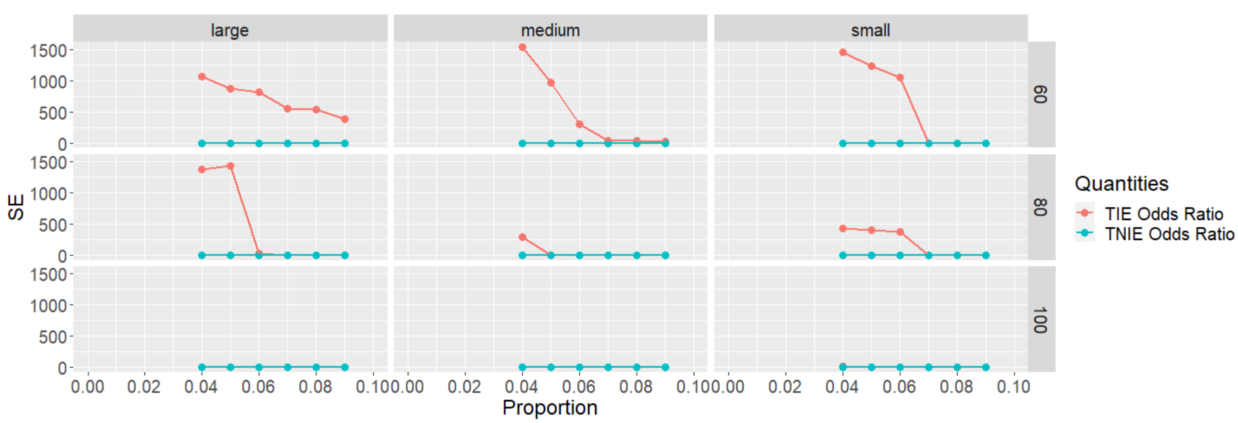


Figure B8

SE over TIE and TNIE for Non-Rare Outcomes in Logit for Simple Mediation Model

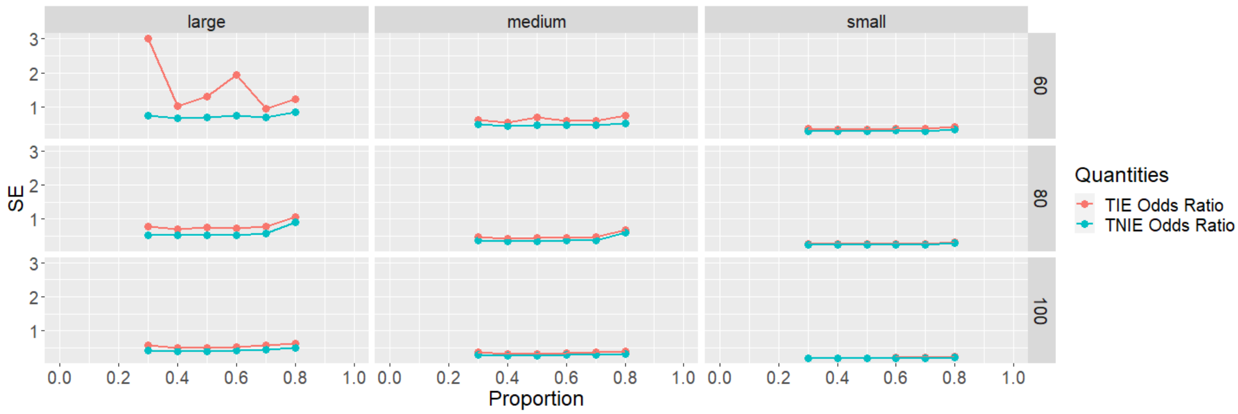


Figure B9

ASE over TIE and TNIE for Rare Outcomes in Logit for Simple Mediation Model

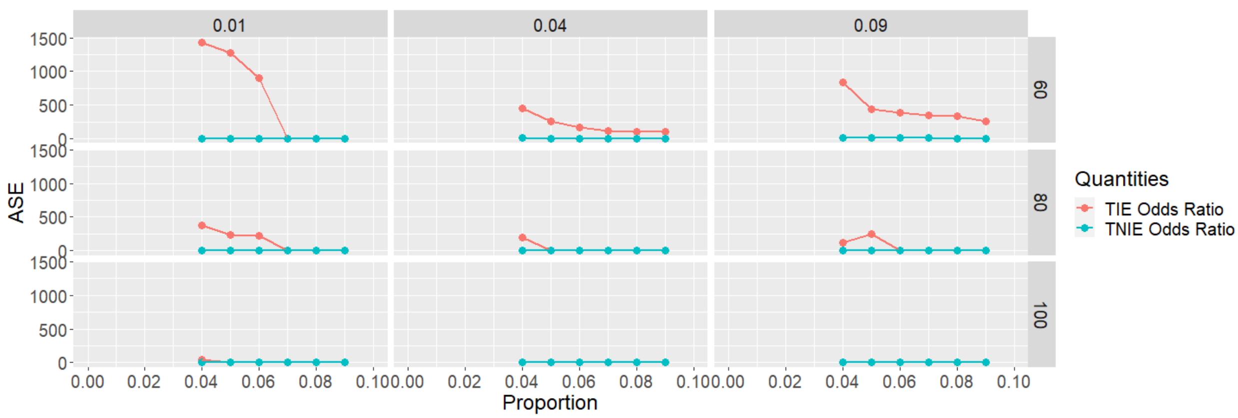


Figure B10

MSE over TIE and TNIE for Rare Outcomes in Logit for Simple Mediation Model