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SUBSPACE AVERAGING OF AUDITORY EVOKED POTENTIALS

Approved by:

Prof. Carlos Davila
Associate Professor of Electrical Engineering

Prof. Dinesh Rajan
Professor of Electrical Engineering

Prof. Dario Villarreal
Assistant Professor of Electrical Engineering

SUBSPACE AVERAGING OF AUDITORY EVOKED POTENTIALS

A Thesis Presented to the Graduate Faculty of

Lyle School of Engineering

Southern Methodist University

in

Partial Fulfillment of the Requirements

for the degree of

Master of Science in Electrical Engineering

with a

Major in Electrical Engineering

by

David Xiaoliang Wang

B.E., Electrical Engineering, Beijing Union University, Beijing

May 19, 2018

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Wang, Xiaoliang B.S., Electrical Engineering, Beijing Union University, Beijing, 2016

Subspace Averaging of Auditory Evoked Potentials

Advisor: Professor Carlos E. Davila

Master of Science conferred May 19, 2018

Thesis completed April 15, 2018

The auditory evoked potential (AEP) is an electric potential generated in the brain in response to auditory stimuli. It has clinical importance in the detection of newborn infant hearing loss. The signal to noise ratio (SNR) of the AEP is low, so signal averaging is typically employed to estimate it. Often, thousands of trials must be averaged before a sufficiently high SNR estimate is obtained.

In this research, we have developed a new AEP averaging method called subspace averaging. The subspace averaging method projects onto the signal subspace: the span of the principal eigenvectors of the signal correlation matrix. The signal subspace has low dimensionality and captures the key features of the signal. We compare the signal-to noise ratio (SNR) estimates of the conventional averaging method and the subspace averaging method. The subspace average has lower noise power and therefore has a higher SNR compared to the conventional average. Further, we have demonstrated that a discrete wavelet transform (DWT) filter bank can be employed to further boost the performance of subspace averaging.

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ACKNOWLEDGMENTS

This work could not have been accomplished without the help and support of Prof. Carlos Davila, along with his many initial thoughts on the auditory evoked potential. I would like to thank my thesis committee members, Prof. Davila, Prof. Rajan and Prof. Villarreal for their supervision and kind comments on this thesis. Also, I would like to thank Prof. Epstein and Dr. Silva who contributed to the open source AEP database at Northeastern University.

I. INTRODUCTION

The auditory evoked potential (AEP) is generated in the brain in response to an auditory stimulus and the AEP has very low signal-to-noise ratio (usually lower than 0 dB), and it is very likely to be interfered by external factors such as measuring equipment, low-frequency noise, body movements, etc. The methods to improve the SNR of AEP have been developed for decades due to its clinical importance. Major SNR improvement methods can be categorized as filter-based method [1, 2], averaging-based [3], as well as new techniques such as machine learning and brain-computer interface (BCI) [4, 5, 6].

Nevertheless, averaging is the most commonly used method that efficiently improves the SNR of AEP trials. This research develops a new robust AEP averaging method that not only achieves higher SNR ensemble averages but also requires fewer AEP single trials, compared with conventional averaging.

1.1 Auditory Evoked Potentials

The evoked potential (EP) is an electrical potential generated by human or animal nervous systems when an external stimulus is presented. EPs are named after the type of external sensory stimulus such as visual auditory evoked potential (VEP), auditory evoked potential (AEP), somatosensory evoked potential (SEP), etc. This research focuses on the auditory evoked potential, it is produced by external ‘sound’ events to the brain. The research of AEP signals has meaningful clinical importance such as hearing loss detection in newborn infants and aids for cognition problems [7].

The AEP is recorded by the electrodes on the subject's scalp while stimuli are transmitted through audiometric earphones, the signal is then processed by filters and amplifiers [8]. Tone-burst and 'click' are two types of stimuli commonly used when recording AEPs. 'Click' is a square wave signal with a short duration of 50-200 microseconds sent to the subject's earphones. The spectrum of a 'click' stimulus is wider than a tone burst and the energy is concentrated to 2-4 kHz; the tone-burst is a frequency-specific (1kHz or 4kHz) pure tone sound signal that has a certain rise, fall, and duration. [9, 10]

Before analyzing AEP signals, it is necessary to consider several important properties of these signals. First, AEP signals consist of multiple dominant frequencies: an AEP single trial contains an auditory brainstem response (ABR) in the first few milliseconds, which has various components [11]. Besides, AEP signals are non-stationary and have various peaks and wave morphologies; AEP waveforms can vary across subjects [12]. More importantly, the SNR of the AEP is very low for several reasons: the amplitudes of the AEP are low, making it easily to be interfered by external noise such as 60 Hz noise and the noise caused by body movements; AEP signals are recorded with the presence of ongoing EEG activities which are considered as additive (backgrounding) noise [11].

1.2 Conventional Averaging

Averaging is the most common used method to average out background noise to improve the SNR of AEP [13, 14]. Usually, a full-length AEP record consists of thousands of single-trials. To apply averaging of an AEP record, it is necessary to divide the full-length record into time-aligned individual single trials. An AEP single trial can be modeled as:

$$x_i = s_i + z_i \quad (1.1)$$

where s_i is the AEP signal and z_i is the background EEG. Assume the signal components across trials are identical $s_i = s$, and the noise components are zero-mean and uncorrelated across trials. Let σ_s^2 and σ_z^2 denotes the power of single-trial signal and noise, respectively. Then the SNR of an AEP single trial is:

$$SNR = \frac{\sigma_s^2}{\sigma_z^2} \quad (1.2)$$

If there are m trials in total and each trial has a length of n , an AEP trial matrix can be written as:

$$A_{m \times n} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} \quad (1.3)$$

The conventional average calculates the mathematical mean of all trials, it is given by:

$$\begin{aligned} \bar{x} &= \frac{1}{m} \sum_{i=1}^m x_i = \frac{1}{m} \sum_{i=1}^m s_i + \frac{1}{m} \sum_{i=1}^m z_i \\ &= \bar{s} + \bar{z} \end{aligned} \quad (1.4)$$

Note signal components are identical across trial so that $\bar{s} = s_i = s$. After averaging, the signal power remains the same and noise power is reduced by a factor of m . The signal to noise ratio of the ensemble average can be written as:

$$\begin{aligned}
SNR_{avg} &= \frac{\frac{1}{n} \sigma_s^2}{\frac{1}{n} \sigma_z^2} = \frac{E[ss^T]}{E[\bar{z}\bar{z}^T]} = \frac{E[ss^T]}{E\left[\frac{1}{m^2} \sum_{i,j=1}^m z_i z_j^T\right]} \\
&= \frac{\sigma_s^2}{\frac{1}{m^2} \sum_{i=1}^m \sigma_z^2} = \frac{\sigma_s^2}{\frac{1}{m} \sigma_z^2} \\
&= mSNR
\end{aligned} \tag{1.5}$$

Compared to a single trial, the signal to noise ratio of ensemble average is increased by a factor of m . Hence, the conventional average method has been utilized for SNR improvement of various types of signals. However, in some cases, the ideal assumption is violated such as the phase or amplitude of the signal changes among trials, the performance of conventional averaging will be influenced. Conventional averaging requires thousands of trials to yield an ensemble average with a satisfying SNR. If more than one average is needed, the AEP recording process has to be lengthy.

In this thesis, a new averaging method named ‘subspace averaging’ has been developed for AEP signals to obtain higher SNR averages. Also, a wavelet filter bank has been introduced to boost the performance of subspace averaging. Related experiments focused on both synthetic and actual AEP signals have demonstrated the high performance of the method. Finally, the author discusses the key factors of subspace averaging and tradeoffs of using a filter bank.

II. SUBSPACE AVERAGING

The method is accomplished by the orthogonal projection of the AEP matrix onto a low dimensional subspace, which is determined by the principle eigenvectors of the signal correlation matrix [15]. In the previous chapter, an AEP single trial has been defined as $x_i = s_i + z_i$, $i = 1, 2, 3, \dots, m$, where s is signal component and z_i is noise caused by background ongoing EEG activity [13, 16]. It is worthwhile to point out the noise component z_i is uncorrelated and zero-mean across trials whereas signal component s_i varies from trial to trial, $s_i \approx s$. We can assume that the signal components s_i exist in a low dimensional signal subspace S , $s_i \in S$; Whereas the noise exists in entire linear space ($z_i \in \mathbb{R}^n$). Hence, the SNR of a trial can be increased by calculating the projection of the trial onto the signal subspace.

2.1 Correlation Matrix and Subspace

The determination of the signal subspace is the key step to perform subspace averaging, given a row vector $x = s + z$ of a length n , the signal subspace can be inferred by the autocorrelation matrix of x .

$$R_{xx} = E[x^T x] = SPS^T + \sigma_z^2 I \quad (2.1)$$

where S is the matrix whose columns are principle eigenvectors, P is a positive definite matrix formed by a random vector w that $E[w^T w] = P$, and I is an identical matrix of dimension $m \times m$. Hence, the autocorrelation matrix of vector x suggests that the signal component s can be expressed as $s = Sw$.

The signal subspace can be estimated by finding the principle eigenvectors of the actual AEP sample correlation matrix, which is given by:

$$\hat{R} = A^T A \quad (2.2)$$

The eigenvectors and eigenvalues of correlation matrix can be determined by:

$$\hat{R}v_k = \lambda_k v_k, k = 1, 2, \dots, n, \lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n \quad (2.3)$$

The so-called principle eigenvectors corresponding to r largest eigenvalues of $\hat{R}$ are the eigenvectors corresponding to the non-zero eigenvalues of SPS^T , i.e., $v_1, v_2, \dots, v_r$. The span of all these r eigenvectors is the signal subspace consisting of [17]. These principal eigenvectors $v_1, v_2, \dots, v_r$ are linearly independent and form a basis of the signal subspace of which has a dimension of r .

2.2 Projection and Averaging

Figure 1 Projection theorem illumination

When an AEP trial x_i is multiplied by linear combination of linearly independent principle vectors $v_1, v_2, \dots, v_r$:

$$\hat{s}_i = \sum_{j=1}^r \langle v_j, x_i \rangle v_j, i = 1, 2, 3, \dots, m \quad (2.4)$$

The resulting vector $\hat{s}_i$ is called the orthogonal projection of AEP trial x_i onto the signal subspace. Also, such linear combination can be written in matrix form, let Q_s be the matrix of principal eigenvectors:

$$Q_s = [v_1, v_2, \dots, v_r] \quad (2.5)$$

Then, a projection matrix can be obtained by:

$$P_s = Q_s(Q_s^T Q_s)^{-1} Q_s^T = Q_s Q_s^T \quad (2.6)$$

The matrix P_s is a projection matrix that maps the AEP vectors (single trial) to the span of principle eigenvectors [18]. The projection is a linear combination of these principle eigenvectors $v_1, v_2, v_3, \dots, v_r$. Therefore, if an AEP matrix (or a single trial) is multiplied by the projection matrix, the orthogonal projection of the AEP matrix (or a single trial) onto the signal subspace is obtained. Let B be the matrix of projected trials [19]:

$$B = A Q_s Q_s^T = [\hat{s}_1^T, \hat{s}_2^T, \hat{s}_3^T, \dots, \hat{s}_m^T]^T \quad (2.7)$$

The projected trials matrix has the same dimension as the AEP matrix A . The projection matrix is idempotent, so the signal power of a projected trial is:

$$\sigma_{s_avg}^2 = \frac{1}{n} E[s_i^T P_s s_i] = \frac{1}{n} s_i^T s_i = \sigma_s^2 \quad (2.8)$$

The noise power of a projected trial is:

$$\sigma_{z_avg}^2 = \frac{1}{n} E[z_i^T P_s z_i] = \frac{1}{n} \sigma_z^2 \text{tr}\{P_s\} = \frac{r}{n} \sigma_z^2 \quad (2.9)$$

Where $\text{tr}\{Ps\}$ is the trace of projection matrix, equal to r . The noise power is reduced by a factor of r/n . Generally, $n \gg r$ so the SNR of projection of a single trial onto signal subspace is significantly increased. The SNR is given by:

$$SNR_{SA_single} = \frac{\sigma_{s_avg}^2}{\sigma_{z_avg}^2} = \frac{n \sigma_s^2}{r \sigma_z^2} \quad (2.10)$$

Meanwhile, averaging all these projected trials improves the SNR even further. The ensemble average can be obtained by computing the mean of projected trials:

$$\bar{B} = \frac{1}{m} \sum_{i=1}^m \hat{s}_i \quad (2.11)$$

Therefore, as m projected trials in average and their noise power being reduced again, the SNR of the resulting ensemble average is now:

$$\begin{aligned} SNR_{SA} &= m SNR_{SA_single} \\ &= m \frac{n \sigma_s^2}{r \sigma_z^2} \end{aligned} \quad (2.12)$$

2.3 Signal to Noise Ratio Estimator

It is challenging to directly calculate the SNR of actual AEP trials due to the unknown signal components. Alternatively, the noise power and signal power can be estimated separately, then a SNR estimator can be accomplished for measuring the performance of the method.

Firstly, a column wise variance of resulting ensemble averages can be calculated, such a variance vector gives the power of noise residue of ensemble averages. To see how the variance vector performs as a noise power estimator, we have to take a look at column wise variance of multiple AEP trials. Under the same assumption, an AEP matrix can be re-written as:

$$A = \begin{bmatrix} x_1(n) \\ x_2(n) \\ \vdots \\ x_m(n) \end{bmatrix} = \begin{bmatrix} s(n) + z_1(n) \\ s(n) + z_2(n) \\ \vdots \\ s(n) + z_m(n) \end{bmatrix} \quad (2.13)$$

Then, a column-wise variance vector of these trials can be calculated by:

$$\begin{aligned} \text{Var}[x_i(n)] &= E[(x_i(n) - \bar{x}(n))^2] \\ &= E[(z_i(n) - \bar{z}(n))^2] \\ &= E[z_i(n)^2] = \sigma_z^2(n) \end{aligned} \quad (2.14)$$

Therefore, such a variance vector only gives the power of the noise component of the signal since the signal components are canceled out. In practice, the expectations in (3.14) (3.16) and (3.17) are obtained using sample means across the rows to the corresponding matrices.

For ensemble averages, the variance vector calculation can be applied to the matrix whose rows are conventional averages.

$$A_{CA} = \begin{bmatrix} \bar{x}_1(n) \\ \bar{x}_2(n) \\ \vdots \\ \bar{x}_p(n) \end{bmatrix} = \begin{bmatrix} s(n) + \bar{z}_1(n) \\ s(n) + \bar{z}_2(n) \\ \vdots \\ s(n) + \bar{z}_p(n) \end{bmatrix} \quad (2.15)$$

where each row is the averaged of q single trials. The variance vector of these ensembles is:

$$\begin{aligned} \text{Var}[\bar{x}_i(n)] &= E[(\bar{x}_i(n) - x_{GM}(n))^2] \\ &= E[(\bar{z}_i(n) - z_{GM}(n))^2] \\ &= E[\bar{z}_i(n)^2] \\ &= \frac{1}{q} \sigma_z^2(n) \end{aligned} \quad (2.16)$$

where $x_{GM}(n)$ is the grand mean of all trials and $z_{GM}(n)$ is the grand mean of all noise components.

We have used the fact that the noise component of each ensemble average $\bar{z}_i(n)$ is not equal to

zero whereas the grand mean of noise component $z_{GM}(n) \approx 0$. The column-wise variance vector implies that the ensemble averages' noise power is decreased by a factor of q . Similarly, the variance vector of subspace averaging is given by:

$$\begin{aligned}
 Var[\bar{x}_{i_{ss}}(n)] &= E[(\bar{x}_{i_{ss}}(n) - x_{GM}(n))^2] \\
 &= E[(\bar{z}_{i_{ss}}(n) - z_{GM}(n))^2] \\
 &= E[\bar{z}_{i_{ss}}(n)^2] \\
 &= \frac{r}{qn} \sigma_z^2
 \end{aligned} \tag{2.17}$$

where q is the number of trials in each average, n is the trial length, and the subspace is spanned by r eigenvectors. The reason the noise power is reduced in proportion to the trial length is that the noise component z_i tends to be equally distributed among the basis vectors in signal subspace [15]. Therefore, less noise power projects onto subspace. The noise power of each method is estimated by taking the mean of the variance vector:

$$\begin{aligned}
 \hat{\sigma}_z^2 &= \frac{1}{n} \sum_{i=1}^n Var[\bar{x}_i(i)] \\
 \hat{\sigma}_{z_{ss}}^2 &= \frac{1}{n} \sum_{i=1}^n Var[\bar{x}_{i_{ss}}(i)]
 \end{aligned} \tag{2.18}$$

Meanwhile, the grand mean of AEP trials can be used for signal power estimation. The grand mean converges to the signal component as the number of trails goes to infinity. In practice, as many trials as possible have to be used for a fair grand mean, the signal power is estimated by calculating the square of the Euclidean vector norm of the grand mean:

$$\hat{\sigma}_s^2 = \frac{\|GM\|^2}{n} \tag{2.19}$$

Hence, the SNR estimator is obtained:

$$\widehat{SNR} = \frac{\hat{\sigma}_s^2}{\hat{\sigma}_z^2} \quad (2.20)$$

This SNR estimator is used for both synthetic signals and actual AEP data in the experiments to measure the performance of subspace averaging and its filter bank applications.

III. DISCRETE WAVELET TRANSFORM FILTER BANK

A filter bank is a set of band-pass filters that splits the input signal into multiple sub-bands. This feature makes the filter bank an ideal tool for AEP signal analysis since the signal has more than one frequency component. The wavelet filter bank for evoked potential signal analysis is well-established and some literature has demonstrated the benefit of wavelet transforms for AEP and ABR. [20, 21, 22, 23, 24, 25]. The discrete-wavelet-transform (DWT) is the most commonly used wavelet transform algorithm for AEP analysis because of its efficiency and orthogonality (or bi-orthogonality) [26, 20, 25].

The use of the DWT to improve the SNR of AEP estimates is as far as we know, novel. To see why a filter bank improves the performance of the subspace averaging, we propose a signal model that has multiple frequencies. If the signal is divided into multiple sub-bands, each sub-band has fewer principle eigenvectors, then the noise power of each sub band is reduced. After the reconstruction, the noise power of the output is less than that of the input signal. Hence, the projected trials of each sub-band have higher SNR, and the ensemble average after the summation has even further increased SNR. Based on experiments, the output ensemble average of the DWT filter bank with subspace averaging showed an improved SNR. This chapter provides several filter bank configuration options and discusses the key features.

3.1 Single Level DWT Filter Bank

The single level filter bank splits signals in two sub bands to be down-sampled, decomposed, up-sampled and then reconstructed. The first two filters are called decomposition

filters and the last two are called reconstruction filters. Thus, the first consideration of a filter bank is the appropriate design of decomposition and reconstruction filters. Plenty of literature has discussed the wavelet transform [27, 28], its applications [29, 30], and specifically its applications to biomedical signals such as EEG, EPs and etc. [25, 21]. Also, a Matlab-based wavelet toolbox is available to help researchers design the wavelet filters [31]. Therefore, the chapter is not aimed at mathematical design of wavelet filters, but offers structural configurations and discusses the performance of subspace averaging. The discussion will cover the theoretical SNR improvement of a multi-frequency signal model, as well as some design considerations for actual AEP signals.

The single level DWT subspace consists of five steps from start to finish. Firstly, decomposition filters produce two sets of coefficients: a low pass decomposition filter produces approximation coefficients cA_I and a high pass decomposition filter produces detail coefficients cD_I . Then, down-sampling each set of coefficients by two, reduces the length of coefficients to $n/2$ (or $(n+1)/2$ if m is an odd number). Thirdly, subspace averaging method is performed on two sets of coefficients, cA_I' and cD_I' are the projections of coefficients cA_I and cD_I onto the corresponding subspaces, respectively. Finally, the two matrices cA_I' and cD_I' are up-sampled, reconstructed and eventually summed up, forming a new projected trials matrix B' .

Figure 2 Single level DWT filter bank block diagram

The Daubechies 19th order wavelet filters [27] are selected as the decomposition and reconstruction filters, Lower-order filters do not provide sufficient attenuation in the stop band. In most cases, higher order filters are preferred for poor SNR trials [26, 32].

Figure 3 Coefficients of 19 order Daubechies filter

To discuss how a filter bank with subspace averaging works, we first propose a signal model with two frequencies, let s' be the a two-frequency signal:

$$s' = \alpha_1 \cos(\omega_1 n + \phi_1) + \alpha_2 \cos(\omega_2 n + \phi_2) \quad (3.1)$$

where ω_1 and ω_2 are frequencies, ϕ_i is uniformly distributed on $[0 \ 2\pi]$. For each frequency component, the signal subspace is spanned by two eigenvectors [17]. Hence the signal has 4 principle eigenvectors in total, $r=4$. Also, a model trial has a length of n and there are m trials in the average. Let the power of ω_1 be $\sigma_{s_1}^2$ and the power of ω_2 be $\sigma_{s_2}^2$, then $\sigma_s^2 = \sigma_{s_1}^2 + \sigma_{s_2}^2$. As discussed in the previous chapter, the SNR of subspace average is: $m \frac{n \sigma_s^2}{4 \sigma_z^2}$.

When the signal is divided into two sub-bands, the signal frequencies may be in the same sub-band, or, the frequencies are in the different sub-bands. If two frequencies are in the same sub-band, the signal power of the sub band remains the same whereas the noise power is reduced by 2 ($\frac{1}{2} \sigma_z^2$), due to the filters, then the subspace projection process reduces the noise power to $\frac{4}{n} \times \frac{1}{2} \sigma_z^2$. Meanwhile, the r of the other sub band with no signal component is equal to 0, which is a zero-out operation. Hence, after reconstruction, the SNR of the output average is now:

$$m \frac{\sigma_{s_1}^2 + \sigma_{s_2}^2}{\frac{4}{n} \times \frac{1}{2} \sigma_z^2} = m \frac{n \sigma_s^2}{2 \sigma_z^2}. \quad (3.2)$$

For the other situation that two frequencies are in the different sub bands, each sub band has one frequency ($r=2$) and a noise power of $\frac{1}{2} \sigma_z^2$. Then, subspace projection process reduces the noise power of each sub band to $\frac{2}{n} \times \frac{1}{2} \sigma_z^2$. Hence, the SNR of the output average is:

$$m \frac{\sigma_{s_1}^2 + \sigma_{s_2}^2}{\frac{2}{n} \times \frac{1}{2} \sigma_z^2 + \frac{2}{n} \times \frac{1}{2} \sigma_z^2} = m \frac{n \sigma_s^2}{2 \sigma_z^2} \quad (3.3)$$

Therefore, compared to the SNR of subspace average, the SNR of the single-level filter bank output is increased by 2 no matter where signal components exist. If the signal has multiple

frequencies, the single level filter bank is not capable of providing sufficient sub bands. In practice, when applying a single-level filter bank to actual AEP signals, as shown in figure 4, the signal components only exist in low frequency sub band. To maximally increase the SNR of actual AEP signals, a multi-level DWT filter bank is required.

Figure 4 Single-level DWT coefficients of an AEP single-trial (DB 19 filters)

3.2 Packet Tree Multi-level DWT Filter Bank

One of the approaches for multi-level wavelet filter banks is packet tree (a.k.a. binary tree) decomposition that splits the signal into several sub-bands with equal frequency bandwidth. Packet tree can be considered a multiple single-level filter bank applied to each sub-band from the previous level. In this process, a k -level packet tree filter bank yields 2^k sub-bands, an AEP trial of

length n is decomposed to coefficients of length $n/2^k$ prior to subspace averaging, accordingly, the number of principle eigenvectors of each sub-band is reduced.

Figure 5 Packet tree multi-level DWT filter bank block diagram

The number of eigenvectors is determined by the signals in the sub-band. For illustrative purposes, we use the previous model that has eight frequency components. Given a two-level packet tree filter bank, the number of sub-bands is 4, and the noise power of each sub band is $\frac{1}{4}\sigma_z^2$. If the two signal frequencies are in two sub band, the projection process reduces the noise power to $\frac{2}{n} \times \frac{1}{4}\sigma_z^2$ while the sub bands without signal component are zero-out. Hence, the SNR of the filter bank output is then:

$$SNR_{DWT_{PT2}} = m \frac{\sigma_{s_1}^2 + \sigma_{s_2}^2}{\frac{2}{n} \times \frac{1}{4}\sigma_z^2 + \frac{2}{n} \times \frac{1}{4}\sigma_z^2} = mn \frac{\sigma_s^2}{\sigma_z^2} \quad (3.4)$$

Similarly, the SNR of other situations can be calculated accordingly by the noise power of effective sub bands. Compared to a single-level filter bank, the multi-level packet tree filter bank increases the SNR of ensemble average under the same assumption. In practice, the actual AEP frequency components are concentrated in the low frequency band. By inspecting the coefficients

of the three-level packet tree filter bank, more sub-bands are useful than that of a single level filter bank. As shown in figure 6, the level three coefficients $cA_{4,1}$ $cD_{4,1}$ $cA_{4,2}$ and $cD_{4,2}$ are further decomposed by the coefficients cA_1 of the single level filter bank whereas $cA_{4,3}$ $cD_{4,3}$ $cA_{4,4}$ and $cD_{4,4}$ are decomposed by coefficients cD_1 , which has barely no signals. Even through the SNR of actual AEP signals by single level filter bank does not change too much compare to no filter bank, a multi-level packet tree filter bank does increase the SNR since frequency components are divided into narrower band, each sub-band then has fewer frequency component, therefore fewer principle eigenvectors.

Figure 6 Three-level DWT packet tree coefficients of an AEP single-trial (DB19 filters)

3.3 Constant Q Multi-level DWT Filter Bank

Another strategy for multi-level wavelet filter banks is the constant Q decomposition where the higher level sub-bands only spilt low frequency sub-bands from the previous level and the high frequency sub-bands of each level go to subspace averaging without further decomposition. Therefore, desired low frequency AEP components have more detail for subspace averaging and high frequency sub-bands do not consume more computing resources.

Figure 7 Constant Q multi-level DWT filter bank block diagram

A k -level Constant Q filter bank yields $k+1$ sub-bands, an AEP trial of length n is decomposed to sub-bands with coefficients of length $n/2^k, n/2^{k-1}, n/2^{k-2}, \dots, n/2$ prior to subspace averaging. The number of principle eigenvectors is reduced, depending on the frequency components of each sub-band. For instance, if a two-level constant Q filter bank is applied on the s' model, the noise power of the sub band from lowest to highest frequency are $\frac{1}{4}\sigma_z^2, \frac{1}{4}\sigma_z^2$ and $\frac{1}{2}\sigma_z^2$; if two frequencies are in two low frequency sub bands. The SNR of the output is then:

$$SNR_{DWT_{Q2}} = m \frac{\sigma_{s_1}^2 + \sigma_{s_2}^2}{\frac{2}{n} \times \frac{1}{4}\sigma_z^2 + \frac{2}{n} \times \frac{1}{4}\sigma_z^2} = mn \frac{\sigma_s^2}{\sigma_z^2} \quad (3.5)$$

Meanwhile, the signal components can exist in other sub bands, accordingly, the SNR changes. This suggests a spectral inspection of actual AEP data prior to filter bank design. In fact,

the signal components of actual AEP data are considered to exist in low frequency sub band. Thus, even though a constant Q filter bank does not increase the SNR, it avoids further decomposition and saves computational recourse, compare to a packet tree filer bank of same level.

Figure 8 Three-level DWT constant Q coefficients of an AEP single trial (DB19 filters)

As shown in figure 8, the approximation coefficients cA_3 and detail coefficients cD_3 of the constant Q filter bank are the same as the coefficients $cA_{4,1}$ and $cD_{4,1}$ of the packet tree filter bank respectively, whereas coefficients cD_2 is the reconstruction of coefficients $cA_{4,2}$ and $cD_{4,2}$, and coefficients cD_3 is the reconstruction of $cA_{4,3}$ $cD_{4,3}$ $cA_{4,4}$ and $cD_{4,4}$.

IV. EXPERIMENTS AND RESULTS

In the thesis, series of experiments were held to testify the high performance of subspace averaging and filter bank methods. Both synthetic signals and actual AEP signals were used. The synthetic noisy signal consists of multiple sinusoids and, then the actual AEP data was used for comparing the performance of each method.

4.1 Simulation

The synthetic signal was generated as:

$$x[n] = \sum_{i=1}^3 \alpha_i \cos(\omega_i(n-1)T + \phi_i) + z[n] \quad (4.1)$$

where $\omega_1 = 12\pi$, $\omega_2 = 24\pi$, $\omega_3 = 48\pi$, $\alpha_1 = 1$, $\alpha_2 = 0.75$, $\alpha_3 = 0.35$ and ϕ_i is randomly picked from $[0, 2\pi]$, to let the phases of the signals vary from trial to trial. The sampling interval is $T=1/400$, trial length is $n=300$, and there were 200 trials in the average. The noise component $z[n]$ was Gaussian noise and the SNR of each trial was approximately -13dB.

Before applying subspace averaging, an eigenvalue inspection of the signal components is necessary. One frequency should have two non-zero eigenvalues corresponding to two principle eigenvectors. In this case, the synthetic three-frequency signal component should have six non-zero eigenvalues that denotes six principle eigenvectors.

Figure 9 Synthetic signal eigenvalue inspection.

Then, both conventional averaging and subspace averaging was implemented for the 200-trial matrix (dimension 200×300). As shown in Figure 11, compared to synthetic signal $x_i = s_i + z_i$, the projection of the signal onto signal subspace $\hat{s}_i$ was very close to the signal component s and the noise power has been significantly reduced. The noise power of projections can be calculated by $\text{var}(\hat{s}_i - s)$. The simulation also attempted the different setups. Note that the $r=2$ was accomplished by only introducing one signal frequency, the initial SNR of -21 dB was by having a noise variance of 100. The results can be seen in Table 1. Meanwhile, ten independently generated matrices with $m=200$ $n=300$, $r=6$ and noise variance of 20 were averaged. The mean values of the variance vectors of trials for conventional ensemble averages and subspace ensemble averages are 20.16, 0.0984 and 1.27×10^{-3} , respectively.

Figure 10 Comparison of synthetic trial, single projection and signal component.

Figure 11 Comparison of conventional average, subspace average and signal component.

No.	m	n	r	Initial SNR	Con. Avg. SNR	Improve-ment	Subs. Avg. SNR	Improve-ment	Theoretical Improvement
1	200	300	6	-13 dB	8.92 dB	21.92 dB	24.83 dB	37.83 dB	40.00 dB
2	100	300	6	-13 dB	5.91 dB	18.91 dB	19.35 dB	32.35 dB	36.98 dB
3	200	150	6	-13 dB	8.66 dB	21.66 dB	20.79 dB	33.79 dB	36.98 dB
4	200	150	2	-13 dB	9.01 dB	22.01 dB	27.55 dB	40.55 dB	41.76 dB
5	200	300	6	-21 dB	0.21 dB	21.21 dB	14.91 dB	35.90 dB	40.00 dB

Table 1 Simulation comparison of conventional averaging and subspace averaging. The last column indicates the theoretical SNR improvement of the subspace average: $\frac{mn}{r}$ (in dB).

Further, another four sets of synthetic signals are generated to demonstrate how a filter bank improves the performance of the subspace average. In this experiment, the signal frequencies ω_1 and ω_2 existed in various sub bands, and ω_3 were removed. Besides, the length of each trial was set as 300 and there were 200 trials in total. As shown in Table 2, when two frequencies were close to each other (e.g. No.6), a multi-level filter bank was capable of improving the SNR, because a multi-level filter bank is more frequency specific: the sub-bands with signal components are narrower, then the noise power is lower whereas other ‘empty’ sub-bands are zero-out. On the other hand, if two frequencies were far from each other (e.g. No.9), a two-level constant Q filter bank was less useful for SNR improvement, though the SNR could be improved by a packet tree filter bank. Also, the simulations results were very close to the theoretical SNR of the filter bank (table 3), as discussed in chapter 3.

No.	ω_1	ω_2	Initial SNR	Con. Avg. SNR	Subs. Avg. SNR	1-level filter bank	2-level constant Q
6	$[0 \ 1/4\pi]$	$[0 \ 1/4\pi]$	-13 dB	9.87 dB	24.91 dB	27.38 dB	29.53 dB
7	$[0 \ 1/4\pi]$	$[1/4\pi \ 1/2\pi]$	-13 dB	9.13 dB	23.52 dB	26.21 dB	28.98 dB
8	$[0 \ 1/4\pi]$	$[1/2\pi \ \pi]$	-13 dB	9.31 dB	24.17 dB	27.11 dB	28.27 dB
9	$[1/4\pi \ 1/2\pi]$	$[1/2\pi \ \pi]$	-13 dB	9.22 dB	24.55 dB	27.38 dB	27.74 dB

Table 2 Simulation comparison of subspace averaging and multiple constant Q filter banks.

No.	Initial SNR	Theoretical Subs. Avg. SNR	Theoretical 1-level filter bank	Theoretical 2-level constant Q
6	-13 dB	28.76 dB	31.77 dB	34.78 dB
7	-13 dB	28.76 dB	31.77 dB	34.78 dB
8	-13 dB	28.76 dB	31.77 dB	33.01 dB
9	-13 dB	28.76 dB	31.77 dB	33.01 dB

Table 3 Theoretical SNR of experiments No.6 to No.9.

In the case of clearly known signal components, the maximal SNR of the subspace average is obtained by the multi-level filter bank that slips each signal frequency into an individual sub-band and keeps the signal sub-band as narrow as possible (reduces noise power as much as possible), then performs subspace projections of signal sub-band while filtering out ‘empty’ sub-bands. However, the signal components of actual AEP data are unknown, some necessary pre-processing and inspections have to be held prior to the subspace averaging and filter bank design.

4.2 Actual AEP Data and Pre-processing

In this experiment, the actual AEP data was open-source and provided by Michael J. Epstein and Ikaro Silva, who recorded AEPs at Northeastern University. The database is accessible on the PhysioNet website and it was a part of a study of EP loudness growth [33]. the AEP signals were recorded along with otoacoustic emission (OAE) and researcher provided the necessary annotations of each record to extract the AEP from mixed signal and split each single trial.

The signal recoding details are found in [33]. Each full-length AEP signal record lasted around forty-five seconds consisting over two million samples which provided over one thousand trials of length $n=2002$. Therefore, an AEP record can be divided into multiple time-aligned single trials in AEP matrix of dimension $m \times 2002$ to be further processed.

Figure 12 AEP Single-trial example: time domain observation of trial No.1.

Decimation:

The actual AEP data was sampled at 48000 samples/sec, which is considered too high due to the fact that the data was filtered by a 30 Hz to 3000 Hz bandpass filter. Therefore, the AEP trials are decimated by 4 to reduce the sample rate from 48 KHz to 12 KHz, resulting in that each trial has a length of 500. Thus, the dimension of the AEP matrix is then $m \times 500$.

Notch filter:

High energy 60Hz noise and its higher order harmonics were detected in the AEP data; this might be caused by poor grounding of power-line or be re-introduced by A/D converter or audio equipment when recording. Therefore, a set of notch filters were designed for 60 Hz noise removal: A notch filter with a stop frequency of 60Hz (a.k.a. anti-hum filter) can be simply achieved by

setting a low frequency of 59 Hz, a middle frequency of 60 Hz and a high frequency of 61 Hz. Similarly, the higher order harmonics of 60 Hz can be removed by implementing multiple notch filters with frequencies that correspond to the harmonics.

Figure 13 Frequency domain observation of the full-length AEP record (0 Hz to 1500Hz low frequency area).

Mean Euclidean distance for outlier removal:

As mentioned earlier, AEP signals are very likely to be influenced by body or scalp movements of the subject which causes abnormal waveforms (extremely high peaks) [34]. It is easier to remove them from the matrix before averaging, otherwise, these high peaks (positive or negative) may corrupt the averaging.

Figure 14 Abnormal high peaks of the full-length AEP record

The mean Euclidean distance (MED) method is very similar to the variance vector discussed in previous chapters, but it calculates the row-wise MED between two trials and returns it in a column vector. The MED can be written as:

$$MED_i = \sqrt{\frac{1}{m} \sum_{n=1}^m (x_i(n) - x_1(n))^2} \quad (4.2)$$

The procedure calculates the mean Euclidean distance (MED) between the first AEP trial x_1 and the rest of the trials. The first trial x_1 is usually a non-influenced trial because most trials are not affected; most MEDs are fluctuating within a stable range. If MEDs exceed this range, they are simply removed from the AEP matrix. On the other hand, if the first trial is influenced, the normal MED range goes higher but still stable so that the abnormal trials still stand out.

Figure 15 Mean Euclidean distance between the first trial x_1 and rest of the trials

Improved subspace average

As discussed in chapter 2, the signal subspace can be estimated by the sample correlation matrix. However, the SNR of actual AEP single trial is so low that the noise can influence the eigenvalues of the correlation matrix due to the fact that ongoing EEG noise are not uncorrelated from trial to trial. Ideally, the correlation matrix R is only made up of signal components. To minimize the influence on the signal subspace due to the noise components, an autocorrelation matrix can be calculated by:

$$\hat{R} = \sum_{i=1}^{m-a} \sum_{j=i+a}^m x_i^T x_j \quad (4.3)$$

$$\hat{R}_{xx} = \hat{R}^T + \hat{R} \quad (4.4)$$

where a is chosen so that the noise component is assumed to be uncorrelated after a trials. To see the contribution of noise components to $\hat{R}_{xx}$, the eigenvalues of $\hat{R}_{xx}$ can be inspected. As shown in figure 16, the noise component contributes less to autocorrelation matrices computed via (4.3) than to the sample correlation matrix computed via (2.2). Therefore, the autocorrelation matrix gives a better signal subspace estimate. Note that when the autocorrelation matrix is calculated by every trial or every two trials ($a=1$ or 2), the signal components are reduced. In this case, the autocorrelation matrix provides reasonable SNR improvement when a is from 3 to 8.

Figure 16 Eigenvalue inspection of sample correlation matrix and autocorrelation matrix with various a . (Normalized by setting the largest eigenvalue to 1)

4.3 Experiments

The experiments were focused on a preprocessed AEP matrix of dimension 900×500 . To compare the SNR of results, the AEP matrix was split into 9 groups, each of which had 100 trials. Then, each method was implemented on these 9 groups and yielded 9 ensemble averages. Further, as discussed in chapter 2, the SNR estimates of ensemble averages were calculated.

Firstly, the SNR estimates of the actual AEP single-trial, the subspace single-projection and the improved subspace single-projection, which was based on the autocorrelation matrix via (4.4), were calculated. As eigenvalues inspection shows, the largest ten eigenvalues can be considered effective. Hence, the number of eigenvectors for each subspace averaging was set to 8 ($r=8$). As shown in figure 17, compared to the signal trial, the SNR estimate of a single projection is increased, and the SNR estimate of a single projection by the signal subspace of the autocorrelation matrix is increased even further. Compared to conventional average, the subspace averages and improved subspace averages have higher SNR estimate.

Figure 17 Trial-scale SNR estimates comparison (left) and average-scale SNR estimates comparison (right).

Secondly, multiple filter banks were also applied on actual AEP data groups to test how a filter bank elevates the performance of subspace averaging. In this experiment, we compared the performance of the conventional averaging, subspace averaging, a two-level constant Q filter bank, a tree-level constant Q filter bank, and a four-level constant Q filter bank. Besides, each subspace method was tested with various r and SNR estimates were calculated accordingly. The reason for neglecting a multi-level packet tree filter bank is, that actual AEP signal components are mainly located in low frequency sub-band as discussed in chapter 3. Note that the signal subspaces were estimated by corresponding autocorrelation matrices as the previous experiment demonstrated.

Meanwhile, when distributing r to filter bank in the experiment, we followed the principle that no eigenvector was given to the frequency band $[\frac{1}{2}\pi \pi]$. The actual eigenvector distribution to each filter bank can be seen in table 3. For example, [6 2 0] of two-level filter bank represents that the low frequency sub band has 6 eigenvectors, the mid-frequency sub-band has 2 eigenvectors and high frequency sub band has 0 eigenvector. As shown in table 3, the resulting SNR estimates can be compared horizontally to demonstrate how a filter bank improves the SNR, or be compared vertically to see how r affects the result.

r	Con. Avg. $\overline{SNR}$	Subs. Avg. $\overline{SNR}$	2-level constant Q $\overline{SNR}$	3-level constant Q $\overline{SNR}$	4-level constant Q $\overline{SNR}$
8	-4.21 dB	-1.17 dB	1.18 dB [6 2 0]	1.26 dB [6 2 0 0]	1.56 dB [6 2 0 0 0]
6	-4.21 dB	-0.54 dB	1.93 dB [4 2 0]	2.05 dB [4 2 0 0]	2.26 dB [4 2 0 0 0]
4	-4.21 dB	2.32 dB	4.06 dB [2 2 0]	4.12 dB [2 2 0 0]	4.23 dB [2 2 0 0 0]

Table 4 SNR estimates comparison of averages.

Compared to conventional averaging, all ensemble averages via the subspace method had higher SNR estimates. The four-level constant Q filter bank yielded averages with highest SNR

estimates. Nevertheless, the eigenvectors distributions can vary, depending on which sub bands the signal components are considered to exist. As shown in table 4, some other eigenvectors distributions were also tested.

$r=8$	[8 0 0 0 0]	[6 2 0 0 0]	[4 4 0 0 0]	[4 2 2 0 0]	[2 2 2 2 0]
4-level constant Q $\widehat{SNR}$	1.05 dB	1.56 dB	2.21 dB	2.19 dB	4.09 dB

Table 5. SNR estimates comparison of 4-level constant Q filter banks with various eigenvectors distributions.

The experiment also inspected the periodogram power spectral density (PSD) estimate of the resulting ensemble averages. Compared to conventional averaging, none of subspace averaging methods resulted in frequency component loss at low frequencies where the signal was thought to exist.

Figure 18 Periodogram PSD estimate comparison of ensemble averages. (low frequency area: 1 Hz to 1 KHz)

4.4 Discussion

The number of principle eigenvectors r is the main factor when applying subspace averaging to AEP signals since r determines the signal subspace dimension. Unlike the simulation, the signal component of an AEP trial is unknown so r cannot be calculated theoretically. Further, if a filter bank is utilized, it is important to determine which sub-bands should be given eigenvectors whereas ‘empty’ sub bands can be ignored. The best way to determine r is inspecting the eigenvalues. If r is too large, AEP trials can be projected to noise; if r is too small, some signal components may be neglected.

The level of a DWT filter bank is another consideration when utilizing a filter bank. For each type of the filter bank, if the signal components can be divided into narrow sub bands while empty sub bands are ignored, the SNR of the output can be improved. Combined with the number of principle eigenvector r , the filter bank design should comprehensively consider the number of levels and the eigenvalue distribution. Ideally, each signal component in an individual sub-band should have two eigenvectors. In practice, the eigenvalue inspection of the autocorrelation matrix and the spectral inspection of actual AEP data are necessary.

Figure 19 Ensemble averages comparison of conventional averages, subspace averages and four-level constant Q tiler bank with subspace averaging. Initial eigenvectors, $r=8$, were distributed as $[2\ 2\ 2\ 2\ 0]$ for four-level constant Q filter banks.

V. CONCLUSION

In this thesis, a new AEP signal averaging method called ‘subspace averaging’ and its filter bank application have been proposed. The experiments successfully showed that subspace averaging is capable of yielding AEP ensemble averages with higher SNR. Also, two types of multi-level discrete wavelet filter bank were used to further to boost the performance of subspace averaging. Compared to conventional averaging, the subspace ensemble averages have less noise power and therefore, have enhanced SNR; the subspace average with a DWT filter bank yields much higher SNR ensemble averages.

Further, the principles of subspace averaging and related filter bank applications have been discussed. Researchers should primarily balance three tradeoffs: the number of principle eigenvectors, the number of AEP trials in averaging, and the number of levels of the filter bank.

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