Essays in Advertising, Regulation and Consumer Naivety

Aastha Gupta
aagupta@smu.edu

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ESSAYS IN ADVERTISING, REGULATION
AND CONSUMER NAIVETY

Approved by:

Prof. Santanu Roy
Professor of Economics

Prof. Bo Chen
Associate Professor of Economics

Prof. James Lake
Associate Professor of Economics

Prof. R. Canan Savaskan-Ebert
Associate Professor of ITOM
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AND CONSUMER NAIVETY

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by
Aastha Gupta
B.A., Economics, University of Delhi
M.A., Economics, University of Delhi
M.A., Economics, Southern Methodist University

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I am also thankful to my parents for their support in my journey and giving me the freedom to choose my own path.
This dissertation consists of three essays that focus on the theoretical analysis of regulation of firm’s communication regarding the quality of its product and the impact of such regulation on market outcome.

The first essay, “Advertising Through Influencers and Disclosure Regulation”, focuses on the recent FTC regulations which require mandatory disclosure of all paid advertising content through social media influencers. This chapter investigates the impact of this disclosure policy on market outcomes when the influencer has the expertise to evaluate product quality and influence the beliefs of both potential buyers as well as the firm. In markets where the influencers do not care much about their followers or lack sufficient expertise and in markets with high uncertainty about product quality, the disclosure regulation changes the outcome from hidden paid advertising to unbiased independent reviews; this improves not only the consumer and social welfare but also the expected profit of the firm. Further, when the firm has private information about the distribution of its product quality, the disclosure policy facilitates signaling of this private information; paid (independent) review is posted when quality is more likely to be low (high).

The second essay, “Consumer Naivety and Price Signaling”, focuses on naive consumers who cannot judge the quality of the product through prices because of cognitive limitations. In a
static signaling model, I analyze the pricing behavior of a monopolist, selling product of uncertain quality, in the presence of such naive consumers. In the high-quality state, the presence of high proportion of naive consumers reduces the price of the product and hence improves overall welfare of the society. On the other hand, in the low-quality state, it increases the price of the product (depending on the valuations) thereby reducing social surplus. Allowing for disclosure as an alternative to communicate quality, the high-quality monopolist has no incentive to disclose in the presence of high proportion of naive consumers. This provides explanation for the infrequent voluntary disclosure by some industries.

The third essay, “Deceptive Advertising, Regulation and Naive Consumers”, analyzes markets where buyers have incomplete information about product quality, consumer sophistication increases the case for strong regulation of deceptive advertising by firms. In a model where a fraction of buyers are naive (i.e., cannot update beliefs based on market signals and believe all advertising claims) and prior beliefs of the buyers about product quality are optimistic, results show that the socially optimal level of penalty is (a) higher than the penalty required to merely avoid deception by firms and (b) increasing in the proportion of sophisticated buyers. The optimal penalty for false advertising not only discourages deception but also reduces prices by eliminating signaling distortion. Moreover, a low level of penalty is worse than no penalty from a social welfare standpoint.
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This is dedicated to my mother, Dr. Pavan Gupta.
Firms have private information about the various attributes and quality of their product which can be useful to the consumers for making an optimal purchase decision. Firms can communicate their private information through various channels which are important ways for buyers to learn about the product and other options in the market. However, there is a possibility that the consumers can be deceived by such communication. This dissertation is an attempt to investigate the impact of regulatory measures directed towards the protection of consumers, including the behaviorally naive consumers, from any misleading information from the firms.

A firm can disclose the true quality of its product through credible and verifiable mechanisms, such as third-party certifications, labelling etc. However, empirical evidence suggests that voluntary disclosure from the firms is infrequent and inadequate. Instead, firms may indirectly communicate quality or important features of their product. One way is through price signals: firm producing higher quality product charges a higher price.

Alternatively, firms may directly communicate product information to the consumers through advertisements. There are traditional ways of advertising through billboards, television, radio etc. With the advent of internet, advertising format has changed: digital advertising has become popular and has allowed firms to reach a large consumer base. As advertising is a way to communicate and persuade consumers to purchase the product, a firm has an incentive to make false claims through advertisements. Over time, consumers have started to ignore traditional advertisements and use ad-blockers for digital advertisements.
Due to this, native advertisements have gained importance in promoting products whereby the advertisements are hidden within a general text, such as blogs, news articles etc. One such form of native advertisements is promotions through social media influencers. Influencers are experts who review different products, inform consumers about the product quality and help them in their purchase decision. They are promoting products through creating electronic word of mouth which has a strong impact on their local follower network. Firm has an incentive to intervene in the review process by making a payment offer to influencer to generate a positive review of the product. Such paid reviews are hidden from the consumers who believe these reviews to be independent and genuine.

Regulation’s main role is to protect consumers from any form of deception by the firms. Regulatory authorities, such as the FTC in the USA, penalize any firm found guilty of false advertising. How does this regulation change with the presence of behavioral consumers? FTC has also clearly stated that the regulation holds for new forms of advertisement as well. Additionally, it is required that the influencers clearly disclose any private relationship with the seller. Is this regulation effective with new and innovative ways of advertisements? This dissertation is a collection of three essays which aims to answer these questions. The ultimate goal is to assess the impact on social welfare.

In the first essay, Advertising through Influencers and Disclosure Regulation, I focus on the emerging phenomenon of influencer advertising wherein the firms secretly promote their products through product reviews by the influencers. The influencers try out products and post the reviews on either their own blogs or social media accounts such as Facebook, Instagram etc. The followers of the influencer, who are the potential consumers for the firms, take into account these product reviews in their purchase decision without observing the relationship between the influencers and the firms. The firm has an incentive to influence the review process through payments that are contingent on favorable reviews as it can allow the firm to charge a higher price and earn higher profit. This implies that there is a possibility to generate biased or inflated reviews, which the consumers perceive them as
honest opinions of the influencer. In response, FTC requires mandatory disclosure of all paid advertising content so that buyers can differentiate between paid and independent product reviews. This chapter investigates the impact of this disclosure policy on market outcome and social welfare.

The influencer faces a key trade off when thinking about accepting payment from a firm for their review. On one hand, the influencer values the payment because this is their only income source from the review. On the other hand, the influencer knows that doing a paid review will ultimately lead to them producing a less accurate review that is biased towards the firm’s product being high quality. This will happen at least probabilistically, otherwise the firm would not get any benefit from paying the influencer for their review. Because influencers value the information they provide to their followers, the payoff of the influencer falls with a paid review. This trade-off ties together the equilibrium payment by the firm to the influencer and the extent of bias in the review provided by the influencer.

Surprisingly, the disclosure requirement is ineffective if the influencer has a high level of expertise, highly values information provision to their followers, or has a high probability of being inherently honest. A high degree of imperfect information pushes in this direction too through highly uncertain product quality. This explains the empirical findings of persistence of paid reviews even after they are required to be disclosed. Further, when the disclosure regulation is effective in improving welfare (i.e. markets where the influencers do not care much about their followers, lack sufficient expertise or operate under high uncertainty about product quality), the market outcome changes from hidden paid advertising to unbiased independent reviews. Surprisingly, this also increases the expected profit of firms so that firms have an incentive to lobby for such regulation. I extend my model to include a case where the firm has private information about the distribution of product quality. In this scenario, the disclosure policy facilitates signaling of this private information. This shows that consumers' pessimism about the quality of the products promoted through paid reviews may be well-founded in certain markets.
In the second essay, *Consumer Naivety and Price Signaling*, the focus is on signaling quality through prices. The monopolist may produce a high-quality product or a low-quality product. Only he knows the actual quality of the product. Consumers are unaware about the product quality and can be of two types. They can either be sophisticated or naive. Sophisticated consumers understand product quality signals and therefore, are able to make wise decisions. On the other hand, naive consumers are unable to interpret signals from the firm and make a purchase decision based on their prior beliefs about the product quality. I investigate the impact of the presence of consumer naivety on the pricing strategies of the monopolist. The findings show that the presence of naive consumers creates a positive externality in a high-quality state. The monopolist has no incentive to signal quality through higher prices as the naive consumers would not be able to incorporate it in their decision. The social surplus increases through reduced prices and eliminating monopoly distortion. However, naive consumers create a negative externality in a low quality state. The low-type monopolist has an incentive to charge a price which is higher than the valuation to take advantage of the consumer naivety. Therefore, the social surplus falls through increased prices and negative consumer surplus. In case the firm has an option to voluntarily disclose the quality, a high quality firm would rather not in the presence of naive consumers. This provides an explanation for infrequent voluntary disclosure in many industries.

The final essay, *Deceptive Advertising, Regulation and Naive Consumers*, focuses on signaling quality through prices and advertisements (traditional or digital). Similar to the second essay, the monopolist is the only one who is aware about the actual quality of the product. The consumers can be sophisticated or naive. However, I use an alternative definition of consumer naivety. When naive consumers observe a price signal, they disregard it and when they observe an advertising signal, they believe it at face value. In other words, they are naive and gullible. Such consumers will be deceived in reality. In this chapter, I formulate an optimal regulation that prevents deception of naive consumers and also maximizes the social welfare. Contrary to what one may expect, consumer sophistication (rather than
naivety) increases the case for strong regulation of deceptive advertising by firms. When
naive consumers are in a high proportion in the market, the optimal regulation is required
to only prevent deception which increases the overall consumer surplus. As the proportion
of sophisticated consumers increase in the market, the high-type monopolist signals qual-
ity to such consumers through higher prices. This creates a significant signaling distortion.
Here, the optimal regulation requires not only to prevent deception but also to eliminate any
signaling distortion. This leads to a stronger regulation as the proportion of sophisticated
consumers increase. A stronger regulation increases the credibility of the advertising signal
which leads to lower prices and maximum social welfare. An additional striking result is
that weak regulation is worse than no regulation from a social welfare standpoint.
Chapter 2

ADVERTISING THROUGH INFLUENCERS AND DISCLOSURE REGULATION

2.1 Introduction

In the current world of social media and blogging, the advertising of products has taken on a new, more creative form. Influencers are the “local” celebrities who post reviews of various products on social media platforms (such as Facebook, Twitter, Instagram) or their own blogs. Their followers rely on these reviews as sources of information about the product. Influencer advertising has gained prominence over other forms of advertising for various reasons. Each influencer has his/her own local network of followers (consumers), which leads to the diffusion of information.\(^1\) It is also found that electronic word of mouth leads to a direct increase in sales.\(^2\) Further, there is significant empirical evidence that social media platforms can strongly influence the beliefs and perceptions of their followers.\(^3\)

However, there is a widespread concern that the influencers may be affiliated with the sellers of the product, in which case the influencer may post a biased or an inflated review in exchange for payment. In that case, these reviews are a form of native advertisement; the followers may not observe the fact that it is a paid promotion.\(^4\) Such advertisements are also deceptive, as illustrated by recent cases prosecuted by the Federal Trade Commission (FTC). For example, an online company, Teami LLC, reportedly paid influencers to inflate their views about the weight loss characteristics of its herbal tea without any clinical evidence.\(^5\)

\(^1\)Kiss & Bichler (2008) and Hamami (2019)
\(^2\)Godes & Mayzlin (2009); Rosario et al. (2016)
\(^3\)Del Vicario et al. (2016); Alatas et al. (2019); Diehl et al. (2019); Muller & Schwarz (2020)
In response, regulatory bodies in several countries have made it mandatory for influencers to disclose if they have been paid to review a product. The FTC, the Competition Bureau in Canada, and the European Advertising Standards Alliance require a clear and prominent disclosure from the influencer of any “material” relationship with the seller. This paper addresses the effect of this kind of regulation on the market outcome; in particular, it characterizes the nature of the market environments in which such disclosure policy is likely to have a positive impact as well as to characterize the economic environments where it is likely to be ineffective.

I formulate a stylized model where buyers are aligned with social influencers according to their tastes and preferences. The influencer is an expert who can judge which products will appeal to his followers. His followers are the buyers who have the same tastes and preferences. I assume that the influencer has an established relationship with his followers. The followers are the potential buyers for the seller. The seller does not know the true quality of his product. However, he would like to know how his product matches with the preferences of the followers. Therefore, the influencer’s review is a potential source of valuable information on product quality not just for buyers but also for the seller.

The seller has an incentive to influence the review process through payments that are contingent on favorable reviews as it can potentially allow the seller to charge a higher price and earn a higher profit. In particular, if buyers cannot observe that the review is paid for and believe that the influencer is independent, the seller has a very high incentive to enter into a paid arrangement and deceive the buyers. It is this incentive that creates the basic rationale for this regulation.

I allow for both honest influencers who always provide unbiased reviews as well as strategic influencers who are willing to inflate their reviews in return for monetary compensation.

Without any regulation, consumers update their beliefs after observing a review and the

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corresponding price charged by the seller. When a disclosure policy is enforced, it requires that the influencer inform the consumers whether it is a paid promotion by the seller. Thus, consumers are able to update their beliefs on the basis of the review type as well as the review.

I find that the disclosure policy is ineffective if the influencer has a high level of expertise, cares a lot about the welfare of his followers, or has a high probability of being honest. On the other hand, the regulation is effective if the influencers do not care much about their followers or lack sufficient expertise; it is also highly effective in markets with high uncertainty about product quality. In these settings, the disclosure regulation improves consumer and social welfare.

Surprisingly, in such economic environments, disclosure regulation also increases the expected profit of the seller. In the absence of regulation, if buyers anticipate an independent review, the seller cannot help but secretly influence the review. When a disclosure policy is implemented, the affiliation is observable to the consumers and a paid review actually hurts the seller. As the seller can charge a higher price for an independent review due to its credibility, it is in the interest of the seller to lobby for such regulation.

When the firm has private information about the distribution of its product quality, the disclosure policy facilitates the signaling of this private information. In such markets, the impact of disclosure policy is twofold — it prevents the possibility of sending out biased product reviews by better firms and helps buyers sort out firms that have inferior products by observing the fact that they sponsored paid reviews.

From the point of view of positive economics, the results in the paper help us understand the kind of markets where disclosure regulation is likely to be most effective in leading to an independent (unsponsored) review. It also helps us to understand the settings in which the regulation is superfluous and therefore, any cost incurred by the society in the implementation of the regulation would be a deadweight loss. The results of this paper also explain the strong empirical evidence of the persistence of paid reviews even when
the disclosure policy is in place.\textsuperscript{7} Finally, the fact that disclosure regulation allows better firms to signal their private information about quality by choosing independent rather than paid review provides a foundation for observed consumer pessimism about products that are publicized through paid reviews.

The fact that the disclosure policy is ineffective when the expertise level of the influencer is high is somewhat surprising. Consumers value the opinion of an influencer with a high level of expertise; his review will have a considerable impact on the consumers’ beliefs. Such an influencer’s review can generate a high level of economic value for the followers and therefore, the seller will have to pay a high level of compensation to introduce bias in the review. As a result, a paid review outcome cannot be sustained and the market outcome is one with independent review even if there is no disclosure regulation.

There is a small literature that has looked at influencer advertising. Pei & Mayzlin (2021) model the relationship between the firm and the influencer with the focus on finding the optimal affiliation level under disclosure policy. Their results show that the level of affiliation between the firm and the influencer depends on the consumer’s prior beliefs and awareness about the product. However, in their framework, the prices are fixed and do not change with the influencer’s review. As a result, a clear comparison of the outcomes with and without the disclosure policy cannot be obtained. In contrast, endogenous price in this paper allows us for a richer characterization of the equilibrium and allows us to obtain a clear comparison between the outcomes with and without disclosure policy. Janssen & Williams (2021) focus on the effect of recommendations from the influencers on the search process of consumers who are deciding on purchasing one out of many products available. In my paper, I analyze the effect of influencer’s recommendations under quality considerations and without any search involved.

Mitchell (2020) presents a dynamic model which focuses on how the influencer maintains a long-term relationship with his followers and shows that the dynamic effect of disclosure

\textsuperscript{7}See Ershov & Mitchell (2020)
policy acts as a tax and may not have the desired effect. In contrast, my paper focuses on the interplay of the influencer’s relationship with both the firm and the followers. As influencer maintains both relationships simultaneously, the effect of disclosure policy can go either way. For markets with multiple influencers, Fainmesser & Galeotti (2020) show how a search technology, which matches the followers with those influencers who provide the highest utility, provides a better social outcome than a disclosure requirement. In contrast, my paper shows how disclosure can be effective under some circumstances and why regulation may be superfluous otherwise.

This paper is also related to Inderst & Ottaviani (2012). In their paper, firms pay commissions and compete to influence an intermediary who is an expert and can advise the consumer to buy their product; the consumers know that the intermediary has a commercial relationship with the firms. The business stealing effect of influencing the influencer’s recommendation is absent in the monopoly setting in my paper. Instead, it focuses on the effect of disclosure of whether or not there is any commercial relationship between the firm and the influencer (without any disclosure of the actual payments).  

The economic effects highlighted in this paper are also somewhat related to the product certification literature, where the certification intermediary can potentially misreport the true quality for certain compensation. For instance, see Choi & Mukherjee (2020) for a situation where the firm decides on whether to certify the product through the intermediary knowing the true quality of its product.

There is also a large literature on the manipulation of consumer reviews and ratings by the firm to improve the beliefs of future buyers. For example, Mayzlin (2006), Burguet et al. (2015) and Aköz et al. (2020). There is no parallel to the analysis of the impact of disclosure policy in these papers. Moreover, the reviewers do not derive any payoff from informing the consumers about the product quality. Finally, my paper is related to the well-established

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8Also, in their frameworks, the intermediary does not internalize the effect of his recommendation on the price paid by the buyers (and, therefore, their surplus). In fact, firms set product prices before observing the recommendation of the intermediary.
quality disclosure literature. For example, Daughety & Reinganum (2008 a, b), Rayo & Segal (2010), Janssen & Roy (2015).

The paper is organized as follows: Section 2 presents the model. Section 2.3 solves the influencer’s and firm’s problems and fully characterizes the equilibrium outcome. It also analyzes the impact of disclosure policy. Section 2.4 considers an extended version of the model. It solves for the market outcome and analyzes the impact of disclosure policy. Section 2.5 concludes. The detailed proofs are contained in the Appendix A.

2.2 Benchmark Model

Consider a product market with a seller, an influencer and many potential buyers of the product. The seller invests in the production of a new product or in a new technology for product improvement. The investment can be successful with a probability $\eta$ or can be a failure with a probability $(1 - \eta)$. This is common knowledge. If investment is successful, the product is of high-quality and if it fails, the product is of low-quality. The seller does not observe the outcome of his investment.

There is a unit mass of buyers, each with a unit demand for the product. All buyers are rational and sophisticated. They have a common valuation $v$ for the low-quality product. For the high-quality product, valuations are uniformly distributed between $v$ and $1 + v : U[v, 1 + v]$. Like sellers, buyers also do not observe the product quality.

There is a single influencer who may review the quality of the product. If he does, he incurs a cost $c_I$ to receive an imperfect private signal about the product quality, $s \in \{0, 1\}$. $s = 1$ signals high-quality and $s = 0$ signals low-quality. The signal is accurate with probability $\gamma$, i.e., $Pr(s = 1|H) = Pr(s = 0|L) = \gamma$ where $\gamma > 0.5$ and represents the expertise of the influencer. If $\gamma < 0.5$, the influencer is not competent enough to judge the true product quality. Therefore, in equilibrium, seller would not approach with an advertising offer and buyers would not follow the influencer. After observing the signal, the influencer who reviews the product, decides on the message to be sent, $m \in \{0, 1\}$. $m = 1$ indicates
that the influencer observed a high-quality signal and \( m = 0 \) indicates that the influencer observed a low-quality signal. Influencer can be of two types: honest or strategic. With probability \( \mu \), influencer is an honest type. An honest influencer is a commitment type who always posts an honest review, i.e., \( m = s \). A strategic influencer is a non-commitment type who may post an honest review or a biased review, i.e. chooses \( m \) that maximizes his expected payoff. Note that the seller and the buyers do not observe the private signal of the influencer; they only observe the message sent by the influencer.

The seller may offer an opportunity for paid advertisement to the influencer wherein the influencer posts a positive message in exchange for some payment \( \tau(m) \). The seller has the ability to commit to this contingent payment. If influencer accepts such an offer, then he sends a message which is influenced by his secret affiliation with the seller. Such a message is a paid review (PR). In a paid review equilibrium, only \( m = 1 \) is on the equilibrium path and \( \beta(1) = 1 \). However, \( m = 0 \) is off the equilibrium path and \( \beta(0) = 0 \). If influencer rejects the offer or was not made an offer, he may send a message without any affiliation with the seller. Such a message is an independent review (IR). In an independent review equilibrium, both messages can arise on the equilibrium path and \( \beta(1) = \beta(0) = 1 \). The seller’s offer is not observed by the buyers, nor the influencer’s decision to accept or reject the offer.

Depending on the type of the influencer and his expertise level, the message may not indicate the true state of the quality. When buyers observe a message, they believe that it may be an independent review or a paid review. Let \( \beta(m) \) be the probability that buyers assign to the influencer being paid for the review after receiving a message \( m \) and \( (1 - \beta(m)) \) to the reviewer being independent. The buyers update their beliefs about product quality to \( \hat{\eta} \) using Bayes rule depending on their beliefs about the equilibrium strategies of the seller and the influencer. Influencer’s message informs buyers about the quality of the product and helps them with their purchase decision. This implies that the review generates an economic value for each buyer. This is denoted by \( EVM - \) economic value of the message – and is calculated as the change in the ex-post utility of the consumers due to the review. It is the
ex post value created for buyers whose purchase decision would have been different if there was no review at all.

It is assumed that buyers are not sophisticated enough to see the price change that would have occurred if there was no review. Thus, the change in ex post value created by the review is evaluated at the same price. Once quality is realized after purchase, for buyers who would have bought at this price without the message there is no difference in surplus and so they associate no value with the review. The same holds for buyers who do not buy and would not have bought without the message. For each possible realized product quality, the review makes a difference only to buyers that buy or do not buy because of the review. $\alpha \in [0, 1]$ is the weight an influencer places on the welfare of the buyers. The expected EVM generated by sending a message $m$ after observing a signal $s$ depends on the buyers’ beliefs $\beta(m)$.

The timeline of the game is as follows:

(i) Nature determines the product quality, drawn from an independent distribution which assigns probability $\eta$ to the high-quality. It is not observed by any player.

(ii) Nature determines the influencer type, drawn from an independent distribution which assigns probability $\mu$ to the honest type. Only the influencer observes his type.

(iii) Seller decides whether to offer a contingent payment contract to the influencer or not. This is not observed by the buyers.

(iv) The influencer decides on whether to accept or reject the seller’s offer. This is not observed by the buyers.

(v) The influencer decides on whether to review the product. If he reviews, he incurs a cost $c_I$ and sends a message $m$.

(vi) Based on the influencer’s decision, seller sets a price, $p$.

(vii) After observing the message sent by the influencer and the price set by the seller, buyers update their beliefs and make a purchase decision.

(viii) Product quality is finally realized after consumption.

Finally, the above setup changes slightly when a disclosure policy is in effect. Such a
policy requires the influencer to disclose if the message was sent in affiliation with the seller. I denote the affiliation level by $\theta \in \{0, 1\}$. If $\theta = 1$, the influencer is affiliated with the seller. If $\theta = 0$, the influencer has no affiliation with the seller. This changes the posterior beliefs of the buyers, which is now dependent on the type of review and the review itself. Rest of the setup remains the same.

### 2.2.1 Demand Curve and Calculation of EVM

Before solving for the equilibrium outcome, this section derives the demand curve and calculates the EVM. Given the prior beliefs of the consumers and the valuation structure described above, maximizing the expected utility function generates a linear demand curve as shown in Figure 2.1. For any price $p$, $\left(1 + \frac{v - p}{\eta}\right)$ fraction of consumers wish to purchase the product. When an influencer reviews the product, the consumers observe the message and the demand curve changes depending on the consumers’ beliefs about the type of review they are anticipating in equilibrium.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{demand_curve.png}
\caption{Demand Curve under no message}
\end{figure}
I derive the change in the demand curve due to the influencer’s review of the product and the resulting economic value generated. EVM is the change in the ex-post utility of the consumers after a review is posted. It is measured by the difference between the consumer surplus with review and consumer surplus without review.

Consider the case where the consumers believe to be in an independent review equilibrium. Both positive and negative review outcomes are possible. A positive review is posted if the influencer receives a high signal about the product quality. When consumers observe a positive message, they update their beliefs positively about the product quality as they trust an independent review. The consumers now believe that the product is of high quality with a higher probability than their prior beliefs. This leads to an increase in their willingness to pay for the product. Therefore, the demand curve rotates upward. If influencer deviates to no review, the demand generated at the given price is lower. EVM generated from a positive review, \( m = 1 \), is shown in Figure 2.2.

![Figure 2.2: Calculation of EVM under \( m = 1 \) in an independent review equilibrium](image)

As described in Section 2, the influencer’s signal may or may not be accurate. With
probability $\gamma$, the signal is accurate. This implies that the product is of high-quality and the review is correct. Therefore, the review helped the consumers to make better purchase decision. This results in an increase in consumer surplus and thereby, positive EVM. With probability $(1 - \gamma)$, the influencer’s signal is inaccurate. This implies that the product is of low-quality and the review is incorrect. Moreover, the consumers paid a high price for a low-quality product which is valued at $v$, generating a negative consumer surplus. Therefore, this results in a negative EVM.

EVM generated from a negative review, $m = 0$ is shown in Figure 2.3. A negative review is posted if the influencer receives a low signal about the product quality. Upon observing a negative message, the consumers update their beliefs. The consumers now believe that the product is of high quality with a lower probability than their prior beliefs. This leads to a decrease in their willingness to pay for the product. Therefore, the demand curve rotates downward. If influencer deviates to no review, the demand generated at the given price is higher.
With probability $\gamma$, the influencer’s signal is accurate. This implies that the product is of low-quality and the review is correct. Therefore, the review helped the consumers to make better purchase decision: some consumers decided to not purchase the product after the review and the rest paid a lower price for it. This results in an increase in consumer surplus and thereby, positive EVM. With probability $(1 - \gamma)$, the influencer’s signal is inaccurate. This implies that the product is of high-quality and the review is incorrect. The review reduced the demand which means that after the review, less number of consumers were able to benefit from purchasing a high-quality product at a low price. Therefore, this results in a negative EVM. The expected EVM from an independent review is a weighted average of both the possible review outcomes where weights depend on the accuracy of the signal received.

Next, consider the case where the consumers believe to be in a paid review equilibrium. The only possible outcome is a positive review. In this scenario, an honest influencer will continue to post a positive review only when he receives a high signal. The EVM generated remains the same as mentioned above. However, a strategic influencer will post a positive review even after receiving a low private signal about product quality. The actual signal received by the influencer is $s = 0$ but the message sent is $m = 1$. The consumers observe a positive review but do not observe the influencer type. They believe that the message is coming from an honest influencer with probability $\mu$ and from a strategic influence with probability $(1 - \mu)$. If $\mu = 0$, there is no change in the beliefs of the consumers as they would not value any message coming from a strategic influencer. The demand curve does not change and EVM is zero. However, if $\mu > 0$, there is a possibility that the message is posted by an honest influencer which leads to an increase in the consumers’ beliefs about the product being high quality. Therefore, the demand curve rotates upward.

To understand the EVM generated in this particular state, focus on Figure 2.4. With probability $\gamma$, the signal is accurate and the product is of low-quality. Because of the biased review posted by the influencer, more consumers purchase the low-quality product at a high
price compared to no-review case. This results in a negative EVM. With probability \((1 - \gamma)\), the signal is inaccurate and the product is of high-quality. This implies that the review is correct and the consumers gained by making better purchase decisions. This results in a positive EVM. A detailed mathematical calculation of the EVM is given in the Appendix, as part of the proof for equilibrium outcome.

\[\text{Figure 2.4: Calculation of EVM under } m = 1 \text{ in a paid review equilibrium}\]

2.3 Market Outcome

In this section, I characterize the equilibrium outcome with and without the disclosure policy. I provide an informal description of the equilibrium outcome for the various ranges of the parameter space. A formal description of the equilibrium outcome is given in the Appendix. Further, I derive the range of parameters for which the disclosure policy is effective and range for which disclosure policy is ineffective.
2.3.1 Equilibrium without Disclosure Policy

In a scenario with no regulation of any kind, the consumer is not aware whether it is a paid or an independent review. In equilibrium, they have beliefs about what type of a review is anticipated, defined by $\beta(m)$. Based on $\beta(m)$, they update their beliefs about product quality with respect to the review and the influencer type. In case of a positive review, consumers update their beliefs on the basis of the influencer type. With probability $\mu$, the influencer is honest type who reports his private signal truthfully and thus, can be trusted. With probability $(1 - \mu)$, the influencer is strategic type who posts his private signal truthfully under an independent review but may bias his message under a paid review. In case of a negative review, consumers believe the review completely as the seller would never pay for a negative review. Therefore, consumers trust a negative review but may not trust a positive review if they anticipate a paid affiliation between the seller and the influencer.

The influencer decides whether to review the product and whether to accept the seller’s offer, if any. At the time the influencer makes the decision about the type of review to post, he is not certain about the exact EVM he will generate. Therefore, he takes into account the expected EVM that can be generated. Both the influencer types generate the same expected EVM under the independent review. For an honest influencer, the paid and independent reviews generate the same expected EVM as he always reports his signal truthfully. For a strategic influencer, paid review generates a lower expected EVM than the independent review because of generating negative economic value for buyers in exchange of some payment. Therefore, the influencer’s decision depends on the comparison of his expected payoff under each review type, which in turn is dependent on the seller’s offer.

The seller’s price and advertisement strategy depends on influencer’s expertise level, how much he cares about the welfare of the buyers and beliefs about the influencer type. I elaborate on this by defining two cases based on the level of $\alpha$.

**Case I:** Low $\alpha$, i.e., the influencer does not care much for the welfare of the buyers. Therefore, he does not generate an independent review. He either posts a paid review or no
review at all.

Given we are in the scenario where only a paid review can be posted, consumers perceive any review as paid. The seller accordingly makes the pricing decision. If a negative message is posted, the influencer must be honest who received a low quality signal. Therefore, the consumers trust the review and update their beliefs. The demand curve rotates downward as seen in Figure 2.3. At the new demand curve, price maximizing the seller’s profit is given by:

\[ p = \frac{n(m = 0) + v}{2} \]

A positive message can be posted if (i) the influencer received a high signal or (ii) the influencer is strategic, received a low signal and posted an inflated review. When consumer observes a positive message, he takes into account both the possibilities and updates his beliefs about product quality accordingly. If \( \mu = 0 \), then the consumers believe the influencer to be strategic and they disregard any positive message from him. They continue to hold their prior beliefs about product quality and the demand curve is unchanged. If \( \mu > 0 \), there is some positive belief that the influencer is honest and the positive message is based on the true private signal. Therefore, the consumers positively updates their beliefs about product quality and the demand curve rotates upward. At the new demand curve, price maximizing the seller’s profit is given by:

\[ p = \frac{n(m = 1) + v}{2} \]

In case no review is posted, there is no change in the consumers’ beliefs and the demand curve. The price charged by the seller is based on the prior beliefs:

\[ p = \frac{n + v}{2} \]

If the seller makes an advertisement offer, he decides on the payment amount, \( \tau(m) \). As
the cost is high enough to prevent the influencer to post a review, the seller can compensate the influencer for only the difference and make him indifferent between paid review and no review. Therefore, the amount by which the cost is higher than the expected EVM from a paid review is sufficient compensation. However, the strategic influencer generates a lower expected EVM under paid review than an honest influencer. This implies that the minimum payment for which strategic influencer accepts the advertisement offer is higher than that for an honest influencer. As seller is not aware about the influencer type, he must offer the following payment to the influencer so that any influencer type accepts as well as his profit is maximized:

$$\tau (m) = \begin{cases} c_I - \alpha [\mathbb{E}(V)] & \text{if } m = 1 \\ 0 & \text{if } m = 0 \end{cases}$$

Given the optimal price charged by the seller and the optimal payment offered by him, a comparison of his expected profits earned from a paid review and no review can be made. The comparison yields the following condition: seller will make an offer to the influencer iff $\bar{\alpha}_1(\eta, \gamma, \mu) \leq \alpha$. Hence, a paid review is posted for all $\alpha \geq \bar{\alpha}_1(\eta, \gamma, \mu)$ and no review is posted for all $\alpha \leq \bar{\alpha}_1(\eta, \gamma, \mu)$. $\alpha$ represents how much the influencer cares about the welfare of his followers. A higher $\alpha$ implies a higher satisfaction or moral benefit from helping the followers with their purchase decision and increasing their welfare. As the only way he can post a review is by accepting the seller’s advertisement offer, a paid review is desirable. If $\alpha$ is low, the influencer does not care much for his followers and would need a higher compensation to post a review. Such compensation amount would not be profitable for the seller, who does not make an offer in equilibrium.

$\bar{\alpha}_1(\eta, \gamma, \mu)$ changes with respect to the prior beliefs of the consumers, expertise level of the influencers and the influencer type. I now look into the effect of a change in each of these parameters on the threshold and the equilibrium outcome. The effect of $\eta$ on $\bar{\alpha}_1(\eta, \gamma, \mu)$ is shown in Figure 8. When $\eta$ is very low (close to zero), the general perception of the product
being high quality is very low. It is almost certain that the product is low quality and even a paid review would not have a desired effect on the beliefs of the consumers. Therefore, seller does not make an advertisement offer to the influencer and no review is posted in equilibrium. As $\eta$ increases, the uncertainty of the product quality increases and $\tilde{\alpha}_1(\eta, \gamma, \mu)$ decreases. This implies that there is a possibility for the seller to influence the beliefs of the consumers.

Two possibilities arise. First, if $\alpha$ is high, influencer cares enough for his followers to post a review. Also, given the payment scheme, a high $\alpha$ along with a low $\eta$ would decrease the payment offer made by the seller. Therefore, a paid review is posted in equilibrium. Second, if $\alpha$ is low, influencer does not care much about his followers’ welfare. Therefore, he does not care much to help his followers to make an informed decision. Seller would have to pay a higher amount for influencer to accept. Such a payment is not profit-maximizing and hence, no offer is made. No review is posted in equilibrium and consumers make a decision based on their priors. For the mid-values of $\eta$, the uncertainty about the product quality is the highest and a paid review is posted in equilibrium. As $\eta$ increases beyond this point, there is an increasingly strong perception that the product is of high quality. Again, the outcome depends on the value of $\alpha$. As long as $\bar{I}(\eta, \gamma, \mu) \leq \alpha$, a paid review is posted and otherwise,
no review is posted. When $\eta$ is very high (close to one), the consumers are almost certain that the product is of high quality. Given the general distribution of the quality is high, the seller does not need to intervene by making an advertisement offer and no review is posted. As before, at the extreme values of $\eta$, the review process does not have much effect on the consumers’ beliefs. It is the intermediate range that has the strong impact.

$\gamma \in \left[\frac{1}{2}, 1\right]$ represents the expertise of the influencer: how well he can judge the quality of the product upon trial. Its effect on the equilibrium outcome is shown in Figure 9. A higher expertise level means a higher probability that the influencer’s private signal is correct, leading to a lower expected EVM from a paid review. If the level of expertise is towards the lower bound, the expected EVM is high reducing the payment to be made by the seller. Therefore, the seller would like to make an advertisement offer and a paid review is posted in equilibrium.

As $\gamma$ increases, $\bar{a}_1(\eta, \gamma, \mu)$ increases. Two possibilities arise. First, if $\alpha$ is high, influencer cares enough for his followers to post a review. Also, given the payment scheme, a high $\alpha$ would decrease the payment offer made by the seller. Therefore, a paid review is posted in equilibrium. Second, if $\alpha$ is low, influencer does not care much about his followers’ welfare.
Therefore, he does not care much to help his followers to make an informed decision. Seller would have to pay a higher amount for influencer to accept. Such a payment is not profit-maximizing and hence, no offer is made. No review is posted in equilibrium and consumers make a decision based on their priors. When $\gamma = 1$, the influencer has the highest level of expertise. In this case, a paid review generates zero expected EVM implying that there is no feasible advertisement offer that the seller can make and thus, no review is posted in equilibrium.

$\mu$ represents the probability that the influencer is an honest type. Its effect on the equilibrium outcome is shown in Figure 10. If $\mu = 0$, the influencer is strategic type and willing to post a biased review if he receives a low signal. Thus, probability of posting a positive message is equal to one. However, as the consumers are aware that it is a paid review, they disregard the review coming from a strategic influencer. Therefore, it is not profitable for the seller to make any advertisement offer to the influencer and no review is posted in equilibrium.

As $\mu$ increases, the probability that the influencer is honest type increases. While updating their beliefs, the consumers take into consideration that the message could be from an
honest influencer. Therefore, demand curve rotates upward and there is an increase in the price charged by the seller. Hence, the seller makes an offer to the influencer but only if \( \alpha \) is high. For a low \( \alpha \), the payment is too high for the advertisement to be profitable. When \( \mu = 1 \), the influencer is an honest type. The seller would not make an advertisement offer to the honest influencer as he would not change his review under any circumstance. Therefore, no review is posted in equilibrium.

**Case II:** High \( \alpha \), i.e., the influencer cares for the welfare of the buyers such that a positive payoff is generated for the influencer when he posts an independent review. This implies that the influencer is willing to and can post an independent review in the absence of a paid advertisement offer from the seller or if the offer made by the seller is not appealing. Therefore, the influencer either posts an independent review or a paid review and the consumers always observe a message.

The seller chooses an optimal pricing strategy depending on the review type and the message sent. A negative message can be posted under two circumstances. First, an independent review where influencer received a negative signal and second, a paid review where the influencer was honest and received a negative signal. The consumers always trust the negative message irrespective of the review type. Therefore, they update their beliefs such that the the demand curve rotates downward as seen in Figure 2.3. At the new demand curve, price maximizing the seller’s profit is given by:

\[
p = \frac{\bar{\eta}(m = 0) + v}{2}
\]

If the influencer chooses to post an independent review and sends a positive message, the seller optimally would like to signal to the consumers that it is an honest review as consumers cannot see the review type. One way for seller to signal is to charge a price corresponding to a scenario where consumers know that it is an independent review. Consumers would completely trust such a positive message and update their beliefs, with posterior probability for high quality higher than in a paid review case. This implies that the demand curve
is steeper for the independent review case. Therefore, the profit-maximizing price charged under independent review is higher. Thus, the seller signals the review type via the price corresponding to an independent review. Consumers observe a high price along with a positive message and infer the review as independent. Alternatively, if the influencer chooses to post a positive message under a paid review, the seller can fool consumers by charging a high price corresponding to an independent review. Therefore, the price charged by the seller in case of a positive message is given by:

\[ p = \frac{\bar{\eta}(m = 1) + v}{2} \]

If the seller makes an advertisement offer, he decides on the payment amount, \( \tau(m) \). For an honest influencer, the expected EVM is same for both the types of reviews which means that he will accept seller’s advertisement offer even without any payment. However, for strategic influencer, the seller should make a payment that at least compensates for the loss in the expected EVM to accept the advertisement offer. This also makes him indifferent between the two types of reviews. Just like the consumers, the seller cannot observe the influencer type. Thus, he must offer the following payment so that any influencer type accepts:

\[ \tau(m) = \begin{cases} 
\alpha [\mathbb{E}(V_{IR}) - \mathbb{E}(V_{PR})] & \text{if } m = 1 \\
0 & \text{if } m = 0
\end{cases} \]

Given the optimal price charged by the seller and the optimal payment offered by him, a comparison of his expected profits earned from either type of the review can be made. The comparison yields the following condition: seller will make an offer to the influencer iff \( \alpha_2(\eta, \gamma, \mu) \geq \alpha \). Hence, a paid review is posted for all \( \alpha \leq \alpha_2(\eta, \gamma, \mu) \) and an independent review is posted for all \( \alpha \geq \alpha_2(\eta, \gamma, \mu) \). \( \alpha \) represents how much the influencer cares about the welfare of his followers. A higher \( \alpha \) implies a higher satisfaction or moral benefit from
increasing the welfare of the followers. Therefore, the result implies that more the influencer cares about his followers’ welfare, more he would be willing to post an independent review. Providing an honest opinion increases the welfare of the consumers and thereby, the payoff of the influencers.

The above threshold on $\alpha$ changes with the given parameters. I now look into the effect of change in these parameters on the threshold and the equilibrium outcome. Prior beliefs about the product quality can influence the type of review that will be posted. The effect of $\eta$ on $\tilde{\alpha}_2 (\eta, \gamma, \mu)$ is shown in Figure 5. When $\eta$ is very low (close to zero), the general perception of the product being high quality is very low and $\tilde{\alpha}_2 (\eta, \gamma, \mu) < 0$. It is almost certain that the product is low quality and even a paid review would not have a desired effect on the beliefs of the consumers. Therefore, seller does not make an advertisement offer to the influencer and an independent review is posted in equilibrium. As $\eta$ increases, the uncertainty of the product quality increases and $\tilde{\alpha}_2 (\eta, \gamma, \mu)$ increases above zero. This implies that there is a possibility for the seller to influence the beliefs of the consumers. Two possibilities arise. First, if $\alpha$ is low, influencer does not care much about his followers’ welfare. Therefore, the influencer will accept seller’s offer to post a paid review. Second, if
α is high, influencer cares enough for his followers that he would only post an independent review. The seller cannot optimally change the influencer’s mind as a high α along with a low η would require him to increase the payment offer to the influencer. Such a payment cannot be profit-maximizing for the seller and therefore, he does not make an advertisement offer in equilibrium. For the mid-values of η, the uncertainty about the product quality is the highest and only a paid review is posted in equilibrium.

As η increases beyond this point, there is an increasingly strong perception that the product is of high quality. Again, the outcome depends on the value of α. As long as \( \bar{\alpha}_2(\eta, \gamma, \mu) \geq \alpha \), a paid review is posted and when \( \bar{\alpha}_2(\eta, \gamma, \mu) \leq \alpha \), an independent review is posted. When η is very high (close to one), the consumers are almost certain that the product is of high quality. Given the general distribution of the quality is high, the seller does not need to interfere with the review process and does not make an advertisement offer. An independent review is posted in equilibrium. Hence, at the extreme values of η, the review process does not have much effect on the consumers’ beliefs. It is the intermediate range that has the strong impact.

\[ \gamma \in \left[ \frac{1}{2}, 1 \right] \] represents the expertise of the influencer: how well he can judge the quality of the product upon trial. Its effect on the equilibrium outcome is shown in Figure 6.
the level of expertise is towards the lower bound, the seller would like to interfere in the review process by making an advertisement offer. This avoids any misreporting from the influencer’s part under an independent review. As $\gamma$ increases, $\tilde{\alpha}_2(\eta, \gamma, \mu)$ falls. This implies that a higher level of expertise corresponds to a broader range of $\alpha$ where independent review is posted. A higher expertise level means a higher probability that the influencer’s private signal is correct, leading to a higher expected EVM from an independent review and a lower expected EVM from a paid review.

Two possibilities arise. First, if $\alpha$ is low, influencer does not care much about their followers’ welfare. Therefore, even though the expected EVM is much lower, the influencer will accept seller’s offer to post a paid review. Second, if $\alpha$ is high, influencer cares enough for his followers that he would only post an independent review. The seller cannot optimally change the influencer’s mind as a high $\alpha$ combined with a high expertise level would require him to increase the payment offer to the influencer. Such a payment cannot be profit-maximizing for the seller and therefore, he does not make an advertisement offer in equilibrium. When $\gamma = 1$, the influencer has the highest level of expertise and $\tilde{\alpha}_2(\eta, \gamma, \mu) = 0$. In this case, a paid review generates zero expected EVM implying that there is no feasible advertisement offer that the seller can make and thus, an independent review is posted in equilibrium.

$\mu$ represents the probability that the influencer is an honest type. Its effect on the equilibrium outcome is shown in Figure 7. If $\mu = 0$, the influencer is strategic type and willing to post a biased review if he receives a low signal. Thus, probability of posting a positive message is equal to one. As the seller can fool consumers into thinking that it is an independent review through price signaling, a paid review is posted in equilibrium. As $\mu$ increases, the probability that the influencer is honest type increases. This implies that there is a fall in the expected profit of the seller from a paid review because by making the same payment, the probability of a positive message is now less than one. This results in a fall in $\tilde{\alpha}_2(\eta, \gamma, \mu)$ leading to a smaller range of $\alpha$ which is suitable for a paid review outcome. When $\mu = 1$, the influencer is an honest type. The seller would not make an advertisement offer.
offer to the honest influencer as he would not change his review under any circumstance. The probability of posting a positive message is same irrespective of the review type. Therefore, an independent review is posted in equilibrium.

The results are summarized in the following proposition. A formal proof is contained in the Appendix.

**Proposition 2.1** A unique perfect Bayesian equilibrium exists. It is characterized as follows:

(a) No review is posted if $\alpha \leq \bar{\alpha}_1 (\eta, \gamma, \mu)$. The seller does not make an offer to the influencer. A fraction $\left( \frac{\eta + v}{2\eta} \right)$ of buyers purchase the product at a price $p = \frac{\eta + v}{2}$.

(b) A paid review is posted if $\bar{\alpha}_1 (\eta, \gamma, \mu) \leq \alpha \leq \bar{\alpha}_2 (\eta, \gamma, \mu)$. The influencer accepts the seller’s offer of $\tau (m = 1) = \alpha [\mathbb{E} (V_{1R}) - \mathbb{E} (V_{PR})]$. A fraction $\left( \frac{\bar{\eta}(m) + v}{2\bar{\eta}(m)} \right)$ of buyers purchase the product at a price $p = \frac{\bar{\eta}(m) + v}{2}$.

(c) An independent review is posted if $\bar{\alpha}_2 (\eta, \gamma, \mu) \leq \alpha$. The seller does not make an offer to the influencer. A fraction $\left( \frac{\bar{\eta}(m) + v}{2\bar{\eta}(m)} \right)$ of buyers purchase the product at a price $p = \frac{\bar{\eta}(m) + v}{2}$.
2.3.2 Equilibrium with Disclosure Policy

When a disclosure policy is imposed, the influencer is required to disclose if the review was paid for by the seller. That is, if it is a paid review represented by \( \theta = 1 \) or if it is an independent review represented by \( \theta = 0 \).

The consumers are able to observe the message along with the type of review. They update their beliefs differently for a paid review and for an independent review. In case of a negative message, consumers believe the review completely as the seller would never pay for a negative review. In case of a positive review through independent review, consumers trust the review completely as it is honest. In case of a positive message through a paid review, consumers update their beliefs on the basis of the influencer type. With probability \( \mu \), the influencer is honest type who reports his private signal truthfully and thus, can be trusted. With probability \( (1 - \mu) \), the influencer is strategic type who posts a high signal truthfully but lies upon receiving a low signal.

If a negative message is posted, the consumers trust the review and update their beliefs. The demand curve rotates downward as seen in Figure 2.3. At the new demand curve, the price maximizing the seller’s profit is given by:

\[
p = \frac{\bar{\eta}(m = 0) + v}{2}
\]

If a positive message is posted under an independent review, the consumers trust it and update their beliefs. The demand curve rotates upward, as seen in Figure 3. At the new demand curve, the price maximizing the seller’s profit is given by:

\[
p = \frac{\bar{\eta}(m = 1, \theta = 0) + v}{2}
\]

Under a paid review, a positive message can be posted if (i) the influencer received a high signal or (ii) the influencer is strategic, received a low signal and posted an inflated review. When the consumer observes a positive message, he takes into account both the possibilities.
and updates his beliefs about the product quality accordingly. As there is a possibility that the message comes from an honest influencer, the consumer positively updates his beliefs. The demand curve rotates upward but the amount of shift is smaller than under an independent review. At the new demand curve, price maximizing the seller’s profit is given by:

\[ p = \frac{\bar{\eta}(m = 1, \theta = 1) + v}{2} \]

which is less than the price under an independent review. In case no review is posted, there is no change in the consumers’ beliefs and the demand curve. The price charged by the seller is based on the prior beliefs:

\[ p = \frac{\eta + v}{2} \]

The influencer makes the decision to post an independent review if it is feasible for him, i.e., if he generates a high enough economic value from the independent review. He can still choose to post a paid review depending on the seller’s offer and the expected EVM generated in either case. Therefore, the influencer’s decision depends on the comparison of his expected payoff under each review type, which in turn is dependent on the seller’s offer. The seller’s advertisement strategy depends on the influencer’s decision of whether to post an independent review. I elaborate on this by defining two cases which are again dependent on the level of \( \alpha \).

**Case I:** Low \( \alpha \). This part of the result remains the same as without the disclosure policy. As only a paid review can be posted, disclosure policy does not have any role to play in this case.

**Case II:** High \( \alpha \). Both the influencer types generate the same expected EVM under the independent review. For an honest influencer, the paid review generates lower expected EVM because the price charged is lower along with a lower shift in the demand curve. For
a strategic influencer, paid review generates a lower expected EVM than the independent review because of (i) lower price and lower shift in the demand curve and (ii) generating negative economic value for buyers in exchange of some payment. Therefore, the seller should make a payment that at least compensates the strategic influencer for the loss in the expected EVM to accept the advertisement offer. As seller is not aware about the influencer type, he must offer the following payment to the influencer so that any influencer type accepts as well as his profit is maximized:

$$\tau(m) = \begin{cases} 
\alpha \left[ \mathbb{E}(V_{IR}) - \mathbb{E}(V_{PR}) \right] & \text{if } m = 1 \\
0 & \text{if } m = 0
\end{cases}$$

Given the optimal price charged by the seller and the optimal payment offered by him, a comparison of his expected profits earned from either type of the review can be made. I find that expected profit from an independent review is always higher than from a paid review. Because of the disclosure policy, the review type is visible to the consumers. The seller can no longer fool the consumers into thinking that the review is independent. Define $\check{\alpha}(\eta, \gamma, \mu)$ which distinguishes low $\alpha$ from a high $\alpha$. It determines $\alpha$ at which the influencer cares enough for the welfare of the buyers to be indifferent between an independent review and a paid review.

The following proposition summarizes the results. A formal proof is given in the Appendix.

**Proposition 2.2** A unique perfect Bayesian equilibrium exists. It is characterized as follows:

(a) No review is posted if $\alpha \leq \check{\alpha}_1(\eta, \gamma, \mu)$. The seller does not make an offer to the influencer. A fraction $\left( \frac{n+v}{2n} \right)$ of buyers purchase the product at a price $p = \frac{n+v}{2}$.

(b) A paid review is posted if $\check{\alpha}_1(\eta, \gamma, \mu) \leq \alpha \leq \check{\alpha}(\eta, \gamma, \mu)$. The influencer accepts the seller’s offer of $\tau(m = 1) = \alpha \left[ \mathbb{E}(V_{IR}) - \mathbb{E}(V_{PR}) \right]$. A fraction $\left( \frac{n(m)+v}{2n(m)} \right)$ of buyers purchase
the product at a price \[ p = \frac{\bar{g}(m) + v}{2}. \]

(c) An independent review is posted if \( \bar{\alpha}(\eta, \gamma, \mu) \leq \alpha \). The seller does not make an offer to the influencer. A fraction \( \left( \frac{\bar{g}(m) + v}{2\bar{g}(m)} \right) \) of buyers purchase the product at a price \( p = \frac{\bar{g}(m) + v}{2} \).

### 2.3.2 Effect of Disclosure Policy

Depending on the parameter values, the effect of disclosure policy can either be positive or none at all. Let’s focus on the case where \( \alpha \) is high. If \( \bar{\alpha}(\eta, \gamma, \mu) \leq \alpha \leq \bar{\alpha}_2(\eta, \gamma, \mu) \), a paid review is posted. When a disclosure policy is implemented, the outcome changes to that of an independent review. This shows a strong positive effect of disclosure on the equilibrium outcome. An independent review is honest, the influencer reports his private signal truthfully. Disclosure policy works as a credible mechanism to inform the consumers about the type of review. It has a strong effect on the consumers’ beliefs. Moreover, with disclosure, the seller cannot wrongfully signal the review type through prices. If \( \bar{\alpha}_2(\eta, \gamma, \mu) \leq \alpha \), an independent review is posted irrespective of the disclosure policy. Therefore, there is no effect of disclosure on the review process. The market outcome remains the same.

Our earlier analysis about the change in \( \bar{\alpha}_2(\eta, \gamma, \mu) \) due to the changes in the underlying factors can be helpful for in determination of the effectiveness of the disclosure policy. The disclosure policy is highly effective for four main reasons. First, if the influencer does not care much about the consumer’s welfare, i.e., the influencer is mainly focusing on making money out of the review process. Second, if the prior beliefs about quality distribution is uncertain, i.e., \( \eta \) is in the intermediate range. Third, the expertise level of the influencer is low. The influencer is not able to discern the quality of the product correctly. Lastly, the influencer is a strategic type. He would be willing to post a biased review in exchange for money. In each case, the consumer is the most affected as he is relying on the review process, which is inefficient due to the above factors. Therefore, disclosure policy is helping consumers to know which review to trust on the basis of the type which increases the reliability of the
When $\alpha$ is low, the disclosure policy has no effect. The consumers are aware that they are in a world where only a paid review can be generated. Hence, the disclosure policy does not have much role to play here.

Expected social welfare in this market is calculated as the weighted average of the total surplus - consumer surplus plus profits of the firm - with the probability of sending out a positive or a negative message as the weights. It is expressed by:

$$E(SW) = Pr(m = 1) \cdot [CS(m = 1) + \pi(m = 1)] + Pr(m = 0) \cdot [CS(m = 0) + \pi(m = 0)]$$

Consumer surplus is the consumers’ utility after the review process and realization of the actual quality. The profits of the firm is the revenue generated from the review process, which is simply the price times the demand. I compare the change in the expected social welfare after the disclosure policy is implemented. For a high $\alpha$, the change in expected social welfare is shown in Figure 2.11. There is a clear increase when $\bar{\alpha}_2(\eta, \gamma, \mu) \geq \alpha$. For this particular range of $\alpha$, the seller charges a price corresponding to an independent review. However, without disclosure policy, the review posted is paid which generates a lower ex-post consumer surplus than under the independent review posted after disclosure policy is imposed.

On the other hand, the seller’s profit declines as now a negative review can also be posted by a strategic influencer. The increase in consumer surplus is higher than the decrease in the seller’s profits. The seller’s profit decline due to the increase in the probability of posting a negative review. However, the consumer surplus increases post-policy due to the improvement in the review by probability equal to one. As $\alpha$ goes beyond $\bar{\alpha}_2(\eta, \gamma, \mu)$, the disclosure policy has no bite and the social welfare is unchanged. The findings are summarized in the following proposition:
Proposition 2.3 The effect of disclosure policy on the social welfare is as follows:

(a) When $\alpha \leq \bar{\alpha}(\eta, \gamma, \mu)$, no effect on social welfare.

(b) When $\bar{\alpha}(\eta, \gamma, \mu) \leq \alpha$,
(i) social welfare improves if $\bar{\alpha}_2(\eta, \gamma, \mu) \geq \alpha$.
(ii) no effect if $\bar{\alpha}_2(\eta, \gamma, \mu) \leq \alpha$.

2.4 Extended Model

Now consider an extended version of the core model. Suppose there are two types of seller: good or bad. With probability $r$, the seller is a good type. The good seller produces a high-quality product with a higher probability than the bad seller:

$$\eta_g > \eta_b$$

where $\eta_g$ is the probability with which good seller produces high-quality product and $\eta_b$ is the probability with which bad seller produces high-quality product. Only seller observes
his type. So the buyers do not know the product quality as well as its distribution. The prior beliefs of the buyers are given by:

\[
Pr(H) = \eta = r\eta_g + (1 - r)\eta_b
\]  

(1)

The rest of the model setup remains unchanged. The solution concept is that of perfect Bayesian equilibrium in pure strategies satisfying the Intuitive criterion (Cho and Kreps, 1987).

2.4.1 Equilibrium without Disclosure Policy

When there are two types of sellers, a pooling equilibrium outcome is generated as defined by Proposition 2.1. Both the seller types choose the same optimal pricing and advertising strategy. The consumers update beliefs as for an average seller and which type of review they anticipate in equilibrium. The proof for the result follows the same argument as the base model without any regulation.

**Proposition 2.4** A unique pooling perfect Bayesian equilibrium satisfying the Intuitive criterion exists. It is characterized as follows:

(a) No review is posted if \( \alpha \leq \bar{\alpha}_1(\eta, \gamma, \mu) \). Either seller does not make an offer to the influencer. A fraction \( \frac{n + v}{2\eta} \) of buyers purchase the product at a price \( p = \frac{n + v}{2} \).

(b) A paid review is posted if \( \bar{\alpha}_1(\eta, \gamma, \mu) \leq \alpha \leq \bar{\alpha}_2(\eta, \gamma, \mu) \). Both seller types make an advertisement offer. The influencer accepts the seller’s offer of \( \tau(m = 1) = \alpha [\mathbb{E}(V_{1R}) - \mathbb{E}(V_{PR})] \). A fraction \( \frac{n_{(m)} + v}{2n_{(m)}} \) of buyers purchase the product at a price \( p = \frac{n_{(m)} + v}{2} \).

(c) An independent review is posted if \( \bar{\alpha}_2(\eta, \gamma, \mu) \leq \alpha \). Either seller does not make an offer to the influencer. A fraction \( \frac{n_{(m)} + v}{2n_{(m)}} \) of buyers purchase the product at a price \( p = \frac{n_{(m)} + v}{2} \).
2.4.2 Equilibrium with Disclosure Policy

When a disclosure policy is implemented, there are two equilibrium outcomes. First, a pooling equilibrium outcome is defined by Proposition 2.2. Both the seller types choose the same optimal pricing and advertising strategy. The consumers update beliefs as for an average seller and which type of review they anticipate in equilibrium. The proof for the result follows the same argument as the base model with disclosure policy. A formal proof is given in the Appendix.

Proposition 2.5 A unique pooling perfect Bayesian equilibrium satisfying the Intuitive criterion exists. It is characterized as follows:

(a) No review is posted if \( \alpha \leq \bar{\alpha}_1(\eta, \gamma, \mu) \). Either seller does not make an offer to the influencer. A fraction \( \left( \frac{\eta + v}{2n} \right) \) of buyers purchase the product at a price \( p = \frac{\eta + v}{2} \).

(b) A paid review is posted if \( \bar{\alpha}_1(\eta, \gamma, \mu) \leq \alpha \leq \bar{\alpha}(\eta, \gamma, \mu) \). Both seller types make an advertisement offer. The influencer accepts the seller's offer of \( \tau (m = 1) = \alpha [\mathbb{E}(V_{IR}) - \mathbb{E}(V_{PR})] \). A fraction \( \left( \frac{\eta(m) + v}{2n(m)} \right) \) of buyers purchase the product at a price \( p = \frac{\eta(m) + v}{2} \).

(c) An independent review is posted if \( \bar{\alpha}(\eta, \gamma, \mu) \leq \alpha \). Either seller does not make an offer to the influencer. A fraction \( \left( \frac{\eta(m) + v}{2n(m)} \right) \) of buyers purchase the product at a price \( p = \frac{\eta(m) + v}{2} \).

Second, a separating equilibrium outcome is defined in Proposition 2.6. This is particularly an interesting result. To understand the basis of this result, focus on the condition where influencer is indifferent between an independent review and a paid review: \( \alpha = \bar{\alpha}(\eta, \gamma, \mu) \). It varies for the type of the seller. If the seller is good type, he produces high quality product with a higher probability than the bad seller. This implies that if the seller type was known to the consumers, expected EVM generated under an independent review from a good seller will be higher than that from a bad seller. Therefore, \( \overline{\alpha}_g(\eta_g, \gamma) < \overline{\alpha}_b(\eta_b, \gamma) \). As it is already proven, for high \( \alpha \), an independent review is posted and for high cost, either a paid or no review is posted. Therefore, for all \( \alpha \leq \overline{\alpha}_g(\eta_g, \gamma) \), a paid or no review is posted and for all
\( \alpha \geq \alpha_g(\eta_g, \gamma) \), an independent review is posted. Similarly, for all \( \alpha \leq \alpha_b(\eta_b, \gamma) \), a paid or no review is posted and for all \( \alpha \geq \alpha_b(\eta_b, \gamma) \), an independent review is posted. For a range of \( \alpha \), the choice of review type is different for the seller types as shown in Figure 2.12. The good type chooses an independent review and the bad type chooses a paid or no review depending on the \( \alpha_1(\eta_b, \gamma, \mu) \). This can be sustained as neither type has an incentive to mimic the other type. The good seller type is able to signal its superior product quality distribution by separating himself from the bad seller type. If the bad seller were to imitate the good type seller, the influencer would be able to discover the truth during the review process. This reduces the expected payoff of the bad seller. Hence, a separating PBE is generated wherein the review type reveals the seller type and provides more information about product quality to the consumers to make better purchase decisions. A formal proof for the proposition is given in the Appendix.

![Figure 2.12: Separating PBE under Disclosure Policy](image)

**Proposition 2.6** When \( \alpha \in [\alpha_g(\eta_g, \gamma), \alpha_b(\eta_b, \gamma)] \), a unique separating perfect Bayesian equilibrium satisfying the intuitive criterion exists:

(a) The bad seller type

(i) makes an offer to the influencer of \( \tau (m = 1) = [c_I - \alpha (E(V_b))] \) if \( \alpha_1(\eta_b, \gamma, \mu) \leq \alpha \). A paid review is posted. A fraction \( \frac{\bar{\eta}(m_b, \theta = 1) + v}{2\bar{\eta}(m_b, \theta = 1)} \) of buyers purchase the product at a price \( p = \frac{\bar{\eta}(m_b, \theta = 1) + v}{2} \).

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(ii) does not make an offer to the influencer if $\bar{\alpha}_1 (\eta_b, \gamma, \mu) \geq \alpha$. No review is posted. A fraction $\left(\frac{n_b + v}{2b}\right)$ of buyers purchase the product at a price $p = \frac{n_b + v}{2}$.

(b) The good seller type does not make an offer to the influencer. An independent review is posted. A fraction $\left(\frac{n_2 (m_g, \theta = 0) + v}{2n_2 (m_g, \theta = 0)}\right)$ of buyers purchase the product at a price $p = \frac{n_2 (m_g, \theta = 0) + v}{2}$.

### 2.4.3 Effect of Disclosure Policy

Depending on the parameter values, the effect of disclosure policy can either be positive or none at all. Let’s focus on the case where $\alpha$ is high. If $\bar{\alpha}_2 (\eta, \gamma, \mu) \geq \alpha$, a paid review is posted. When a disclosure policy is implemented, the outcome changes to that of an independent review. This shows a strong positive effect of disclosure on the equilibrium outcome. An independent review is honest, the influencer reports his private signal truthfully. However, the consumers only infer the product quality for an average seller and not the quality distribution. For the intermediate range of influencer’s cost, there is an additional effect. Through separating equilibrium, the disclosure policy is able to inform consumers of the quality distribution through the type of review and the product quality through the review itself.

As before, the disclosure policy has no effect when $\bar{\alpha}_2 (\eta, \gamma, \mu) \leq \alpha$. The analysis of the role of different parameters in determining $\bar{\alpha}_2 (\eta, \gamma, \mu)$ and thereby, the effectiveness of the disclosure policy is same as the base model.

The expected social welfare is calculated in the same way as the base model. The disclosure policy has a two-fold effect. First, the improvement in the pooling equilibrium outcome leads to an increase in the expected social welfare. This is same as shown in Figure 2.11. Second, the emergence of a separating equilibrium outcome can lead to an even larger increase in the expected social welfare, compared to the first effect. This is shown in Figure 2.13.
If we are in the state with the good seller, an independent review is generated. The social welfare is higher than without disclosure policy because it is an independent review and the quality distribution is good. When we are in the state with the bad seller, a paid review is posted. The social welfare is higher than without policy case as the consumers are aware that the product quality distribution is bad and are able to make a more informed decision. However, when in the no-policy case, there is a shift from paid to independent review, the social welfare is higher than with-policy case. The bad seller makes an advertisement offer and a paid review is posted. The consumers are aware that it is a bad seller and might not even purchase the product depending on the message posted. The no-policy case is an honest review and more trusted than a paid review coming from a bad seller. The findings are summarized in the following proposition:

**Proposition 2.7** In a scenario where the quality distribution is unknown, the disclosure policy has the following effect on the expected social welfare:

(a) When $\alpha \leq \bar{\alpha}_b(\eta, \gamma, \mu)$, no effect on social welfare.

(b) When $\bar{\alpha}_b(\eta, \gamma, \mu) \leq \alpha \leq \bar{\alpha}_g(\eta_g, \gamma, \mu)$,

   (i) improves the expected social welfare if $Pr(H) = \eta_g$
(ii) deteriorates the expected social welfare if \( \Pr(H) = \eta_b \)

(c) has no effect if \( \tilde{\alpha} g (\eta_g, \gamma, \mu) \leq \alpha \).

### 2.5 Conclusion

Advertising through influencers is an emerging phenomenon and rapidly growing. This industry was valued at $8 billion in 2019 and is expected to increase to $15 billion by the end of 2022. Such native advertising deceives consumers by not only keeping relations with the seller private but also, in some instances, sending out biased reviews about product quality. My paper evaluates the effect of the FTC’s disclosure regulation on this particular advertising industry. The results highlight the economic environments where the effect of disclosure regulation is highly effective and where it is entirely ineffective. Therefore, the effect of disclosure regulation can go either way, depending on the market characteristics.

There are four main implications. First, an independent review can be sustained even without the regulation. Second, paid affiliations can continue to persist after the implementation of the policy. This supports the empirical evidence in Ershov and Mitchell (2020). In both cases, the regulation is ineffective. As the implementation of the policy is costly, it leads to a pure deadweight loss. Third, the disclosure policy can have a significant effect on eliminating the possibility of posting a biased review. This leads to an increase in the expected consumer surplus and expected profits of the seller. Therefore, the seller has an incentive to lobby for such regulation. Lastly, the disclosure also leads to a more informed separating outcome which further helps buyers in making better purchase decisions. This result provides the economic rationale behind the pessimism towards products that are promoted through influencers.

To the best of my knowledge, this is the first paper to outline a clear comparison of the impact of the disclosure versus non-disclosure of any affiliation of the influencer with the seller, while focusing on both sides of the strategic interaction: (i) between the buyers and the influencer and (ii) between the sellers and the influencer.
3.1 Introduction

In many markets, sellers have better information about the product than the buyers. In particular, information about the quality attributes of the product may not be available to the buyers. Past literature has emphasized that prices may be used by the seller to convey information about the product quality to the buyers. If, after observing the prices, the buyers update their beliefs about the product quality in a Bayesian fashion, then such a price signaling is effective. However, not all consumers possess such level of sophistication to extract these signals and connect them to the underlying private information of the sellers. Recent research in behavioral industrial organization provides evidence for the presence of “naive” consumers who face cognitive limitations and have limited reasoning abilities.\footnote{See DellaVigna (2009) and Grubb (2015) for an excellent literature review of research on behavioral industrial organization.} It is important to understand how the presence of such naive consumers affect the outcome in these markets. This paper is an attempt to understand this issue in a monopoly setting\footnote{I develop a model with single seller to show the pure effect of naive consumers on prices. Adding competition may not capture the actual impact of such consumers.} where seller has private information about the product quality and a section of the buyers do not update their beliefs based on the observed price.\footnote{If the monopolist chooses to voluntarily disclose its true quality instead, when there is a mechanism for credible disclosure available.} The basic model has been inspired by Bagwell and Riordan (1991). Their paper models a single seller, with private information about its quality, facing two types of consumers: one
type is informed and the other is uninformed about the true quality. The main result of their paper is that the high-quality type seller charges a high signaling price, in the presence of a high proportion of uninformed consumers, that leads to further loss of social surplus in addition to monopoly distortion. The profits of the monopolist and the overall social surplus are reduced compared to the full information monopoly outcome. Hence, the seller always has an incentive to voluntarily disclose the quality of its product if a low-cost disclosure mechanism is available.

My paper introduces a third type of consumer – the naive consumer. These consumers have prior beliefs about the quality of the product but do not update their beliefs after observing the price. This is because the naive consumers have limited reasoning abilities: they cannot process the price signals to connect it to the quality of the product. While there is always a separating equilibrium in the presence of high proportion of naive consumers, it reduces the price and to that extent, reduces the signaling distortion plus the monopoly deadweight loss when the monopolist is high-quality type. This results in a fall in the profits and market power of the monopolist. On the other hand, the low-quality type has an incentive to increase its price in the presence of high proportion of naive consumers because they cannot infer quality through prices and hence, are easier to be misled. Low-quality type completely abandons the other types of consumers which leads to social welfare loss.

Another interesting observation is that the incentive for the high-quality type to voluntarily disclose its quality under a credible mechanism reduces with an increase in the proportion of naive consumers in the market. In fact, when the naive consumers are dominant in the market, the high-quality type has no incentive to disclose quality. This result gives partial explanation for the infrequent disclosure of product quality by firms in some markets (see Fishman and Hagerty, 2003; Janssen and Roy, 2015).

The key mechanism driving these results is that the low-quality type finds it less attractive to imitate the high-quality type in the presence of high fraction of naive consumers because these consumers do not respond to price signals. For the same reason, high-quality type
experiences a loss in sales from the naive consumers if it charges a high signaling price. Therefore, the high-quality type does not need to have a higher gap in prices to separate itself from the low-quality type. This fall in market power improves social surplus.

There is an extensive literature on product quality signaling (see Ellingsen, 1997; Daughety and Reinganum, 2008a; Adriani and Deidda, 2009; Janssen and Roy, 2010). Daughety and Reinganum (2008a) is particularly of interest here. They introduce “naively-optimistic” consumers who always believe the product to be of high-quality. This increases the profits of both low-quality type and high-quality type and reduces the incentive of high-quality type to disclose. They also introduce “naively-pessimistic” consumers who always believe the product to be of low-quality, which reduces the signaling profits of the high-quality type but increases its incentive to disclose. Both these type of naive consumers have prior beliefs different from other consumers and do not update these beliefs on the basis of prices. On the other hand, I extend this framework to naive consumers who have strictly interior prior beliefs which are identical to the common prior beliefs of all the other consumers in the market.

Rest of the paper is organized as follows: Section 3.2 presents the model. Section 3.3 solves the firm’s problem. I solve for separating and pooling perfect Bayesian equilibria satisfying the intuitive criterion. This section also analyzes the results: it shows the effect on the price and profit functions of the monopolist. Also, I discuss the welfare implications of the results. Section 3.4 outlines the effect of voluntary disclosure. Section 3.5 concludes. Proofs are provided in the Appendix B.

3.2 Model

Consider a one-period product market with a monopolist firm and many potential buyers of the product. The product quality is uncertain and can either be of high (H) or low (L) quality. The monopolist knows the true quality of the product and produces it at a constant marginal cost, \( c_\tau \geq 0 \), which depends on the firm type \( \tau, \tau \in \{H, L\} \). The marginal cost of a
high-quality product is \( c_H > 0 \) and that of a low-quality product is \( c_L = 0 \). This production technology is common knowledge. The objective of the monopolist is to set a price \( p \) that maximizes its profits.

There is a unit mass of potential consumers of the product, each with a unit demand. All consumers believe that the quality is drawn from an independent distribution which assigns probability \( r \) to high-quality state, \( \tau = H \). They have a common reservation price, \( v > 0 \), for a low-quality product whereas for a high-quality product, the reservation prices are uniformly distributed between \( v \) and \( (1 + v) \), where \( v > c_H \). That is, the minimum additional utility generated by a unit of high-quality product exceeds its marginal production cost.

Consumers maximize their expected net surplus in order to make a purchase decision. The expected net surplus from buying the product is equal to the expected reservation price minus the price paid:

\[
EU = [rx + (1 - r)v] - p
\]

where \( x \in (v, 1 + v) \) and \( p \) is the price set by the monopolist. There are broadly two types of consumers defined in the market:

1. Informed consumers: They observe the true quality of the product before making the purchase decision. They simply assign \( r = 1 \) when they observe \( \tau = H \) and \( r = 0 \) when \( \tau = L \).

2. Uninformed consumers: They do not know the true quality of the product before making the purchase decision and continue to have common belief assigning probability \( r \) to high-quality state. However, they may or may not update their beliefs after observing the price. Because of different Bayesian updating, uninformed consumers are further divided into two categories: (a) Sophisticated consumers, who update their beliefs after observing the price set by the monopolist in a Bayesian fashion. (b) Naive consumers, who do not update their beliefs at all after observing the price.

Let \( z \) denote the fraction of informed consumers in the market. Out of the remaining
(1−z) consumers, let s denote the fraction of sophisticated consumers. Therefore, (1−z)s is the fraction of sophisticated consumers and (1−z)(1−s) is the fraction of naive consumers in the market.

Before the start of the game, all the agents know that the quality is drawn from an independent distribution which assigns probability r to product being high-quality. Given this common prior assumption, the timeline of the game is as follows: First, nature determines quality drawn from the independent distribution. The monopolist and the informed consumers observe the true quality of the product. Next, the monopolist sets the price p after which only the sophisticated consumers update their beliefs to \( \hat{r} \). Finally, all the consumers decide whether to purchase the product or not by maximizing their expected net surplus. This gives an informed demand curve in which \((1+v−p)\) fraction of the informed consumers buy when \( p \in [v, 1+v] \) and \( r = H \); a sophisticated demand curve in which \((1+(v−p)/\hat{r})\) fraction of the sophisticated consumers buy when \( p \in [v, \hat{r}+v] \) and the quality is believed to be high with probability \( \hat{r} \); and a naive demand curve in which \((1+(v−p)/r)\) fraction of the naive consumers buy when \( p \in [v, r+v] \) and the quality is believed to be high with probability r. The payoff of the monopolist is equal to its profit. The profit function is defined by \( \pi(\tau, r, \hat{r}, p) \): it depends on the type of the firm \( \tau \in H, L \), the beliefs of the naive consumers r, the posterior beliefs of the sophisticated consumers \( \hat{r} \) and the price set by the monopolist p. The payoff of each consumer is his/her ex-post utility.

The solution concept used is that of perfect Bayesian equilibrium where the out-of-equilibrium beliefs satisfy the intuitive criterion (Cho and Kreps, 1987) in every subgame.

### 3.3 Optimal Pricing Strategy

#### 3.3.1 Separating Equilibrium Outcome

A high-quality type’s profits are maximized at full-information monopoly price, \( P^F = \frac{1+v+cH}{2} \). This is the price it ideally wants to charge in a perfect information case. However,
in an imperfect information world, it deviates to a higher price to signal its true quality to sophisticated consumers and separate itself from the low-quality type, but only when there is a sufficient proportion of sophisticated consumers for it to be worthwhile. On the other hand, if there is a high proportion of naive consumers in the market, signaling quality does not work because of no impact on the purchase decision of the naive consumers. Instead, the high-quality type maximizes its profits given the demand functions of each type of consumer in the market:

\[
\max_p \left\{ (p - c_H)[(1 + v - p)z + (1 + v - p)(1 - z)s + (1 + \frac{v - p}{r})(1 - z)(1 - s)] \right\}
\]

which yields:

\[
P^M(s, z) = \left(\frac{v + c_H}{2}\right) + \frac{r}{2[1 - (1 - r)(z + (1 - z)s)]}
\]

One may argue that why cannot the high-quality type simply charge \(P^F\) if it cannot signal its quality to a large proportion of naive consumers? This is because by charging \(P^F\), the firm loses out on a lot of potential demand from naive consumers (and from some of the remaining consumers in the market) and hence receives lower profits. Observe that \(P^M(s, z)\) is increasing in \(s\) and \(z\) and is equal to \(P^F\) at \(z = 1\). Therefore, the high-quality type prefers to charge a price lower than its full-information monopoly when there are a large number of naive consumers in the market. On the other hand, a low-quality type can serve all the consumers in the market by simply charging \(p^L = v\) or serve only the naive consumers by charging \(p^L > v\). This is because in a separating equilibrium, the sophisticated consumers update their beliefs to \(\hat{r} = 0\) after observing \(p^L\) but the naive consumers do not and hence they are easier to fool. To determine the price charged for the naive consumers:

\[
\max_p \pi(L, r, 0, p) = p \left\{ (1 + \frac{v - p}{r})(1 - s)(1 - z) \right\}
\]
This gives:

\[ p^L = \frac{(r + v)}{2} \]

The pricing strategy of the low-quality type is to charge the above price when there is a large proportion of naive consumers in the market. Even though the monopolist is losing all the business from the sophisticated and informed consumers, it still makes a huge profit as the gains from serving only the naive consumers are a lot higher than the loss from not serving the rest of the consumers. But, when the proportion of rest of the consumers is dominant (given by equation (B.1)), the monopolist switches to \( v \) because losses outweigh the gains if it continues to charge higher than \( v \). However, this strategy is only valid under the assumption that \( r > v \). If \( r < v \), the low-quality type always charges \( p^L = v \) irrespective of the proportion of each type of consumer in the market and their beliefs. This is because, at this price, the monopolist serves all the consumers in the market and still earns the maximum possible profit. If low-quality type continues to charge \( p^L = \frac{(r+v)}{2} \) (which is less than \( v \) in this case), the informed and sophisticated consumers also purchase the product, taking away some of the monopolist’s surplus and making it no more a profit-maximizing price.

It is possible that the low-quality type has an incentive to mimic the price charged by the high-quality type. By doing so, the sophisticated consumers (and not just the naive consumers), after observing the high-quality price, are tricked into purchasing a low-quality product at high-quality price. The informed consumers still do not purchase the product as they know its true quality. This way the monopolist made both the naive as well as the sophisticated consumers to purchase its product at a price higher than the reservation price and ends up earning much higher profits. This is a possible scenario when the proportion of uninformed consumers is large enough in the market. To avoid such a situation to occur, the high-quality type separates itself from the low-quality type by charging a high enough price such that mimicry is no more profitable. If a low-quality type chooses to mimic a high signaling price, it loses a lot of demand from the uninformed consumers as they can no longer
afford it, resulting in lower profits than charging the reservation price.

To determine the optimal signaling price, I find a range of prices for which the low-quality type has an incentive to mimic the high-quality prices. When there is a high proportion of naive consumers in the market, this price range is represented by \( p \in [P(s), \overline{P}(s)] \) (given by equations (B.3) and (B.4)). At the upper and lower bounds, the low-quality type earns the same profit as charging \( p^L = \frac{(r+v)}{2} \). For prices within the range, the low-quality finds mimicry profitable. The high-quality type can charge any price outside this range to avoid mimicry. Prices below the lower bound are too low and hence, not profitable. The high-quality type can earn higher profit by letting the the low-quality type to mimic its prices. Prices equal to or above the upper bound are too high but still signals the quality to the sophisticated consumers. Therefore, the best response of the high-quality type is to charge a price equal to the upper bound because this way he can separate itself from the low-quality type by losing minimum possible profits.

When the proportion of naive consumers is low in the market, the price range for which the low-quality type has an incentive to mimic high-quality prices is represented by \( p \in [\underline{P}(s, z), \overline{P}(s, z)] \) (given by equations (B.5) and (B.6)). For the same reasoning as above, the best response of the high-quality type is to charge price equal to the upper bound. Therefore, when the proportion of uninformed consumers is high, the high-quality type chooses to signal its quality represented by the upper bounds. However, when the proportion of informed consumers is high, the high-quality type finds \( P^M(s, z) \) high enough to avoid mimicry because the low-quality type does not find it profitable to mimic for a small proportion of sophisticated consumers in the market. Hence, the low-quality type never deviates from its pricing strategy given by Lemma 5.

The last step in the construction of the separating equilibrium is to ensure that the high quality type has no incentive to deviate from such a pricing strategy. This is possible when it charges the high signaling price and may have an incentive to deviate to \( P^M(s, z) \). To show that this never occurs, I find a range of prices for which the high-quality type earns
higher profits than $P^M(s, z)$ under the case of high proportion of uninformed consumers in the market. This range is represented by $\widetilde{P}(s, z)$ – given by equation (B.7). Any price in this range is preferred over $P^M(s, z)$. Both the signaling prices – $\overline{P}(s)$ and $\overline{P}(s, z)$ – lie in this range. Hence, the high-quality type has no incentive to deviate from its pricing strategy.

![Diagram of pricing strategy](image)

**Figure 3.1: Pricing Strategy of high-quality type in separating equilibrium satisfying intuitive criterion**

The complete separating equilibrium satisfying the intuitive criterion is given in Figure 3.1 and is summarized in the following proposition. The formal proof of the proposition is given in the Appendix.

**Proposition 3.1.** Given (B.8) holds, the unique separating equilibrium satisfying intuitive criterion is as follows:

(i) If (B.1) holds and $r > v$, $p^L = \frac{r + v}{2}$ and $p^H = \max\{\overline{P}(s), P^M(s, z)\}$, where $P(s)$ is given by (B.3).

(ii) If (B.2) holds and $r > v$ or if $r < v$, $p^L = v$ and $p^H = \max\{\overline{P}(s, z), P^M(s, z)\}$, where $P(s, z)$ is given by (B.5).
3.3.1.1 Welfare Analysis

For the above equilibrium, I analyze the pricing strategy and the profit functions for both types of the monopolist firm, providing economic reasoning behind the equilibrium outcome. Next, I compute the social surplus for a welfare analysis of the equilibrium outcome. I restrict my attention to \( r > v \) case. The analysis is divided into two parts – one when the proportion of informed consumers is low and the other when the proportion of the informed consumers is high – to highlight how the strategies of each type of monopolist firm differs. Notice that the \( r < v \) case is similar to the part where the proportion of informed consumers is high and hence, the analysis of the results is similar for both these scenarios.

I. Low Proportion of Informed Consumers

Fixing \( z \) at a point in the range \( 0 \leq z \leq \left( \frac{r-v}{r+v} \right)^2 \), I plot the price and profit functions of the low-quality type and high-quality type under the separating equilibrium as a function of \( s \), as shown in Figures 3.2 and 3.3. \( s^L(z) \) is the point where the low-quality type shifts from charging \( r+v \) to \( v \), for a fixed value of \( z \) in 2 the range \( 0 \leq z \leq \left( \frac{r-v}{r+v} \right)^2 \). Similarly, \( s^H(z) \) is the point where the high-quality type shifts from charging \( P^M(s, z) \) to signaling price, for a fixed value of \( z \) in the range \( 0 \leq z \leq \left( \frac{r-v}{r+v} \right)^2 \). It is clear from the diagram that when \( z \) is small (that is, the proportion of sophisticated and naive consumers is large), the high-quality type charges a price strictly increasing in \( s \). When \( s \) is small (below \( s^H(z) \)), the proportion of naive consumers is dominant in the market. The price falls below the full-information monopoly price because with the low-quality type having no incentive to mimic the high-quality type, there is no point in signaling quality. Also, charging \( p^H \) negatively affects profits because the demand from naive consumers reduces by a huge number. Therefore, because of the presence of naive consumers in the market, the price of the high-quality product is lower for everyone. When \( s \) is large (above \( s^L(z) \) threshold), the proportion of sophisticated consumers is dominant in the market and the low-quality type has an incentive to mimic high-quality type because of which prices increase to very high level to signal quality. The
high-quality type’s profits increase initially with s because the price is moving up closer to the full-information monopoly price but the profits drop drastically after $s^L(\bar{z})$.

However, the price of the low-quality product has increased above the reservation price due to the presence of naive consumers. While more of the informed and sophisticated consumers are able to purchase the high-quality product, none of them are able to purchase the low-quality product. As for the low-quality type, the profits are highest when all the consumers in the market are naive but it starts to fall with increasing s as none of the sophisticated / informed consumers buy the low-quality product for higher than their reservation price. After switching to $p^L = v$, low-quality type’s profit is lowest but all consumers purchase its product. So, the presence of naive consumers reduces the profit of a high-quality type but increases the profit of a low-quality type.

There lies an intermediate case between $s^H(\bar{z})$ and $s^L(\bar{z})$ where the proportion of sophisticated consumers starts to get prominent in the market. The low-quality type is close to...
exhausting all the benefits from charging a price higher than the reservation price and hence, likely to mimic the high-quality type. But as the incentive to mimic is low (compared to when \( s > s^L(z) \)), the high-quality type has to increase its prices slightly to signal its quality.

Next, I plot the social surplus from selling low-quality and high-quality products under the separating equilibrium as a function of \( s \), shown in Figure 3.4. Initially, the social surplus from a low-quality product is strictly falling because of charging higher than reservation price but later with increasing \( s \), it becomes constant at a higher level as it switches to reservation price. The social surplus from a high-quality product is falling throughout with increasing \( s \). After \( s^L(z) \), the social surplus starts falling at an increasing rate because of high signaling price and goes below \( v \). This shows the huge negative effect of the presence of sophisticated consumers on the social welfare. However, due to the presence of the naive consumers, the social surplus from a high-quality product is increasing – a positive externality. But at the same time, these consumers create a negative externality in case of low-quality product by increasing the price and reducing the social surplus way below \( v \). Note that there are
two types of distortion associated with the pricing of high-quality product under separating equilibrium. One is the usual distortion arising from monopoly pricing (occurring when there is large proportion of informed consumers). Second is the distortion arising from signaling quality (occurring when there is a large proportion of sophisticated consumers). However, due to the presence of the naive consumers in the market, both types of distortions are reduced and social surplus increases. Therefore, the naive consumers create a positive externality for the rest of the consumers. On the other hand, for a low-quality product, the presence of naive consumers increases the price and reduces the social surplus, creating a negative externality for the rest of the consumers in the market.

![Figure 3.4: Social Surplus of low-quality and high-quality products under separating equilibrium when z is small](image)

**Proposition 3.2.**

(i) When $0 \leq z \leq (\frac{1-v}{1+v})^2$ and $\tau = H$,

(a) the price is increasing in $s$. When $s$ is small, price lies below $P^F$ and when $s$ is large, price lies entirely above $P^F$. 

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(b) the profit function is a non-monotonic discontinuous function of \( s \) and lie entirely below the full-information monopoly profits.

(c) the social surplus is a monotonically decreasing and discontinuous function of \( s \).

(ii) When \( 0 \leq z \leq \left( \frac{r-v}{r+v} \right)^2 \) and \( \tau = L \),

(a) the price is a step function with respect to \( s \). When \( s \) is small, price lies above \( v \) and when \( s \) is large, price is exactly \( v \).

(b) initially, the profits are falling with \( s \) but become constant as \( s \) increases further.

(c) the social surplus is a weakly decreasing function of \( s \). Initially, the social surplus is falling but becomes constant when \( s \) increases further.

II. High Proportion of Informed Consumers

Fixing \( z \) at a point in the range \( \left( \frac{r-v}{r+v} \right)^2 \leq z \leq 1 \), I plot the price and profit functions of both the firm types under the separating equilibrium as a function of \( s \), shown in Figures 3.5 and 3.6. Observe that the price of high-quality type is an increasing function of \( s \) in this case too. The high-quality type is simply charging its monopoly price everywhere and only gets to charge the full-information monopoly price when \( s = 1 \) because here \( z \) is already very large and even if all the remaining consumers are sophisticated, it is not worthwhile to signal quality through high prices. In this scenario too, the naive consumers have helped to reduce the price of the high-quality product for everyone in the market. The price of the low-quality type is unaffected by the presence of naive consumers in this case and simply charges \( v \) because it is scared to lose out on huge demand from the informed consumers.

Next, I plot the social surplus from selling low-quality and high-quality products under the separating equilibrium as a function of \( s \), shown in Figure 3.7. The social surplus from the low-quality product is constant throughout as it is charging one price throughout but is also the maximum possible welfare from the low-quality state. For the high-quality product, the social surplus is falling with increasing \( s \) because its price is increasing as \( s \) increases. With a higher proportion of informed consumers, the high-quality price converges
to full-information monopoly price reducing overall social welfare. Therefore, the presence of informed consumers creates a positive externality for everyone under low-quality state but creates a negative externality under high-quality state.

Figure 3.5: Price functions of low-quality and high-quality types under separating equilibrium when $z$ is large

Figure 3.6: Profit functions of low-quality and high-quality types under separating equilibrium when $z$ is large
Proposition 3.3.

(i) When \( \left( \frac{r - v}{r + v} \right)^2 \leq z \leq 1 \) and \( \tau = H \),

(a) the monopolist charges \( P^M(s, z) \) which approaches \( P^F \) as \( z \to 1 \).

(b) the profit function is monotonically increasing in \( s \), approaching the full-information monopoly profits as \( s \to 1 \).

(c) the social surplus is a monotonically decreasing function of \( s \).

(ii) When \( \left( \frac{r - u}{r + v} \right)^2 \leq z \leq 1 \) and \( \tau = L \),

(a) the monopolist charges reservation price, \( v \).

(b) the profits are constant at \( v \) throughout.

(c) the social surplus is a constant throughout the range of \( s \).

Figure 3.7: Social Surplus of low-quality and high-quality products under separating equilibrium when \( z \) is large
3.3.2 Pooling Equilibrium Outcome

A pooling equilibrium is possible when the proportion of informed consumers is low in the market. A low-quality type never pools with the high-quality type when the proportion of informed consumers is high because then all the informed consumers buy the high-quality product negatively affecting the low-quality type’s profits. For this equilibrium, both types of firms charge the same price, $p^H = p^L$ and the posterior belief of the sophisticated consumers becomes $r$ as they cannot interpret the quality through the same price charged by each type.

To calculate the pooling equilibrium, we need to find the range of prices for which the low-quality type is willing to pool with the high-quality type. That is, a range of prices for which the low-quality type earns at least as much profits as from the separating equilibrium outcome. When proportion of naive consumers is high in the market, the range of pooling prices is represented by $p \in [P_r(s), \overline{P}_r(s)]$ (given by equations (B.9) and (B.10)). This price range is depicted in Figure 3.8. It is clear that for any $0 \leq s \leq (r-v)^2$ (corresponding to high proportion of naive consumers), pooling equilibrium exists for the highlighted region.

Similarly, for a higher value of $s$, when there is a high proportion of sophisticated consumers in the market, the range of pooling prices is represented by $p \in [P_r(z), \overline{P}_r(z)]$ (given by equations (B.11) and (B.12)). This price range is depicted in Figure 3.9. It is clear that for any $0 \leq z \leq (r-v)^2$ and any value of $s$, the pooling equilibrium exists for the highlighted region. Combining the two pictures, we get the entire range of pooling equilibria as given in Figure 3.10.

Note that the high-quality type does not have any incentive to deviate from this pooling range. This is because for $(1-s)(1-z) \leq \frac{(4rv)}{(r+v)^2}$, even though the high-quality type is charging slightly lower price than $P^M$, there is an increase in demand which results in higher profits. This happens due to the fact that informed consumers already know the true quality and naive consumers do not update their beliefs. Both the types simply increase their demand when high-quality price falls. Similarly, for $(1-s)(1-z) \geq \frac{(4rv)}{(r+v)^2}$, the pooling range coincides with the signaling portion of the separating equilibria where the high-quality type is charging
a very high price and earning very low profits. Therefore, for the high-quality type, it is profitable to charge a lower price under pooling.

Thus, we have the following proposition for the pooling equilibria. The formal proof is given in the Appendix.
Proposition 3.4. For \( r > v \), any pooling equilibria satisfying the intuitive criterion is given by:

(i) when (B.1) holds and \( 0 \leq s \leq \left( \frac{r-v}{r+v} \right)^2 \), \( p_H = p_L = p \in [P_r(s), \bar{P}_r(s)] \)

(ii) when (B.2) holds and \( 0 \leq z \leq \left( \frac{r-v}{r+v} \right)^2 \), \( p_H = p_L = p \in [P_r(z), \bar{P}_r(z)] \)

3.4 Disclosure: Alternative to Signaling

At this point, it is important to discuss about an alternative option to signaling quality: disclosure. Throughout this section, I set the proportion of informed consumers at \( z \) which lies in the range \( 0 \leq z \leq \left( \frac{r-v}{r+v} \right)^2 \) because it does not make sense to disclose quality when there are high proportion of informed consumers in the market. As we can see from the above profit function of the high-quality type, signaling quality is extremely costly. Therefore, when the fraction of sophisticated consumers in the market is high, the high-quality type prefers to disclose its true quality as long as the cost of disclosure is less than the loss in profits from signaling. The range of disclosure cost that satisfies this condition is given by:
\[0 \leq D < (P^F - p) + [1 - (\bar{z} + s(1 - \bar{z})) \frac{(1 - r)}{r}][v + c_H - P^F - p](P^F - p)\]

where \(p\) is either \(P(s)\) or \(P(s, z)\). Notice that as \(s\) keeps on increasing, the high-quality type is charging signaling price which is increasing at an increasing rate. This leads to the cost range to expand as \(s\) increases. Therefore, the presence of sophisticated consumers expands the disclosure range giving the high-quality type a strong incentive to voluntarily disclose its quality through a credible mechanism. This takes us to the social cost of signaling which is not completely internalized by a high-quality type when deciding between disclosing and signaling quality. When there is a high proportion of sophisticated consumers in the market, the range of disclosure cost is wider when we incorporate the social cost and it is going to be beneficial for both the consumers and the firm. The social cost of signaling here refers to the huge loss in consumer surplus (of the current and the potential buyers) plus the loss in profits of the high-quality type. As \(s\) increases, disclosing becomes the best option for the high-quality type.

When the monopolist chooses to disclose its quality, the sophisticated consumers observe and understand the disclosure information. They update their beliefs and are transformed into the informed consumers of the market. However, the naive consumers face difficulty in processing such disclosure information and remain unaware about the true quality. Hence, following the disclosure, the market is left with informed and naive consumers only. If the proportion of sophisticated consumers was high initially, then disclosing quality is the best case scenario as the high-quality type now reduces its price to \(P^F(s, z)\) which increases the overall social welfare in the society. If the proportion of sophisticated consumers was low initially (below \(s^L(\bar{z})\)) threshold the naive consumers are dominant in the market and the monopolist has no choice but to continue to charge \(P^M(s, z)\). Disclosing quality in this situation implies that everything remains the same except that the monopolist’s profit as well as the social surplus reduce by the amount of disclosure cost. Therefore, the monopolist has no incentive to disclose its true quality in the presence of naive consumers in the market.
On the other hand, the low-quality type has no incentive to disclose its true quality, irrespective of the value of $s$. Even if it discloses, its pricing strategy remains unchanged but the profits and social surplus fall by the amount of disclosure cost. With this disclosure strategy of each type of the monopolist firm, if the consumers observe non-disclosure, they immediately relate it to the product being low-quality. However, this holds only for the sophisticated consumers and not for the naive consumers because they cannot process any such observation while making the purchase decision.

To sum up, the high-quality as well as the low-quality type strongly prefer not to disclose their quality in the presence of high proportion of naive consumers in the market. On the other hand, in the presence of high proportion of sophisticated consumers, only the high-quality type chooses to disclose but non-disclosure by the low-quality is enough signal for the sophisticated consumers to accurately update their beliefs.

3.5 Conclusion

Not all consumers are sophisticated enough to interpret price signals in markets where quality attribute of the product is unknown. This paper considers the presence of such naive consumers and their effect on the pricing strategy of a monopolist firm selling product of uncertain quality. Results show that the presence of naive consumers forms a negative externality in low-quality state by increasing the price of the low-quality product and thereby reducing overall social welfare. On the other hand, these consumers create a positive externality in the high-quality state by reducing the price of the high-quality product and thereby increasing overall social welfare. Moreover, the high-quality type and low-quality types do not find it profitable to disclose their true type in the presence of naive consumers in the market. It is, however, surprising that the increase in proportion of naive consumers reduces the incentive to disclose by the the high-quality type. This, to some extent, explains the non-disclosure of quality attributes in some industries. However, even if mandatory disclosure is imposed, the social welfare falls.
Chapter 4

DECEPTIVE ADVERTISING, REGULATION AND NAIVE CONSUMERS

4.1 Introduction

In a large number of markets, buyers are uninformed about the product quality attributes such as performance, satisfaction, durability, safety, reliability etc. Sellers take advantage of advertising and other methods of direct communication to inform buyers about various quality attributes of their products. However, sellers have an incentive to deceive buyers by communicating false or misleading information to boost sales. While sophisticated buyers can judge the veracity of this kind of information on the basis of market signals and their understanding of the incentives of the firms, naive buyers may not be able to make such decisions. In this paper, I examine the incentives for firms to engage in deceptive communication in the presence of naive buyers. Further, I argue that it is the presence of sophisticated buyers that actually increases the case for regulation of deception by firms.

There are a large number of instances where firms often engage in communications that falsely misrepresent their product’s attributes. For example, in separate instances during 2009, Sketchers, New Balance and Reebok falsely advertised that their toning shoes were superior to walking shoes because they helped in weight loss and strengthening muscles. These advertisements led to a peak in sales of toning shoes at $1 billion in 2010. Later, scientific tests found that toning shoes provided no additional benefit and to the contrary, may lead to injury. Similar instances can be found in the food industry, such as Dannon exaggerating health benefits of its Activia yogurt and DanActive dairy drink; Kellogg’s made deceptive claims in 2008 about its Frosted Mini-Wheats that it increases child’s attentiveness.
and again in 2009, claimed that its Rice Krispies cereal could improve a child’s immunity – both times clinical studies could not prove such claims. In the automobile industry, Volkswagen falsely claimed that their diesel cars were low-emission and environment-friendly by cheating on performance tests; Kia and Hyundai misled consumers by inflating the expected gas mileage of their cars during 2011-13 when gasoline prices were extremely high.\(^1\)

Various countries have designed regulations to impose standards on advertising content and to monitor and prevent deceptive advertising. In the US, the Federal Trade Commission (FTC) declares “all unfair or deceptive acts or practices in or affecting commerce” as unlawful\(^2\); in Canada, the Competition Bureau states “a person engages in reviewable conduct who, for the purpose of promoting, directly or indirectly, the supply or use of a product or for the purpose of promoting, directly or indirectly, any business interest, by any means whatever, makes a representation to the public that is false or misleading in a material respect.”\(^3\) They enforce these laws by making it difficult for firms to lie about their product’s characteristics. They do so by imposing penalties or some expected cost on firms engaging in deceptive advertising. Both the FTC and the Competition Bureau require offenders to pay a civil penalty as well as consumer damages.\(^4\) We refer to this broadly as information regulation.

One of the stated objectives of information regulation is to protect buyers. In fact, the

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\(^2\)Federal Trade Commission Act, 15 U.S.C. §§ 45(a) and 52
\(^3\)Competition Act (R.S.C., 1985, c. C-34) Section 74.01
\(^4\)FTC Act imposes a civil penalty of not more than $10,000 for each violation and “necessary payments to redress injury to consumers or other persons, partnerships, and corporations resulting from the rule violation or the unfair or deceptive act or practice, as the case may be.” Competition Act imposes monetary penalty of not more than $750,000 in the case of an individual and $10,000,000 in the case of a corporation, along with consumer damages. For instance, Skechers paid $40 million and Reebok paid $25 million in consumer damages to settle charges for unsupported claims about their toning shoes; Volkswagen paid $14.7 billion to settle allegations of cheating emission tests, out of which $10 billion was to compensate consumers, and remaining $4.7 billion to mitigate pollution; Dannon paid a total of $21 million to 39 US states to resolve investigations into its advertising of Activia and DanActive.
FTC defines consumer protection as its mission. In particular, prevention of deception of buyers is an important part of this mission of consumer protection. Of course, deception would be impossible if buyers are fully informed. As such, naturally, the case for regulation is in markets with incomplete information. If buyers are rational and sophisticated, they will be able to extract product information from the market signals or other actions of the seller and they can also make judgements about whether the messages sent by the firms are credible. Indeed, it has been shown in the literature that deception cannot arise in equilibrium if buyers are sophisticated and firms can signal information through prices or other means. In such a scenario, the effect of regulation will not be to prevent deception but rather influence the market outcome in other ways. In some cases, it can have a perverse effect on welfare and market outcomes (Janssen and Roy, 2020). The point of departure of this paper is to argue that in the real world, not all buyers can interpret the market signals and moreover, not all buyers have the sophistication to correctly understand the credibility of the information sent by the firms.

This paper models “naive” consumers who deviate from sophisticated behavior in two important ways: (i) inability to make rational inference from market signals and (ii) inability to understand the incentives of the firm to send out incorrect product information in carefully designed advertisements, that is, believe all advertising messages at face value irrespective of prices and hence, are gullible. Real-world markets are likely to be populated by naive as well as sophisticated consumers. Then the question arises that in markets with naive consumers present alongside sophisticated consumers, what is the role of information regulation? What is the effect of such regulation on the market outcome and social welfare?

5“The FTC is a bipartisan federal agency with a unique dual mission to protect consumers and promote competition.” - https://www.ftc.gov/about-ftc/what-we-do

6Recent papers such as Rhodes and Wilson (2018) and Piccolo et al (2015, 2017) have demonstrated the possibility of deception through a framework where the marginal cost of various product qualities are identical and all buyers are rational/sophisticated. They show that low-quality firm sends out the same advertising claim as its high-quality counterpart resulting in successful deception of rational buyers.

7See Schwarz et al (2016) for a summary of latest psychology literature on human gullibility and misinformation; Grubb (2015) for a summary of industrial organization literature with behavioral consumers (or rationally inattentive consumers); Avery et al (2013) for empirical evidence on how sophisticated consumers are less likely to be influenced by deceptive advertising.
The basic framework used to develop this economic argument is borrowed from the classic paper by Bagwell and Riordian (1991). I consider a monopoly where the seller has private information about the quality (high or low) of its product. The firm can signal its quality to potential consumers by a combination of a price and an advertising message. The advertising message can be false, in which case the firm incurs a fixed penalty. This fixed penalty captures the expected future cost imposed by the regulator on the firm. My paper augments the Bagwell and Riordian (1991) model by: (i) allowing for naive buyers, (ii) allowing firms to send an advertising message, which can be false and (iii) imposing a fixed penalty in case of false advertisement. The key focus is on how the penalty affects the market outcome.8

In this model, there is no pooling equilibrium as in the original Bagwell and Riordian (1991) framework. This implies that sophisticated consumers are never deceived in equilibrium; only naive consumers are deceived. Thus, when the proportion of sophisticated consumers is high, there is no incentive to deceive. I characterize a unique separating perfect Bayesian equilibrium. For a low level of penalty, the low type falsely advertises its product as high-quality to deceive naive consumers and, at the same time, the high type signals its true quality. Once penalty exceeds a certain level, there is no deception. However, the penalty continues to influence the signaling distortion, which is directly related to the fraction of sophisticated consumers in the market: as the penalty increases, the signaling distortion comes down. The signaling distortion is completely eliminated for a high level of penalty, and equilibrium reaches the full information outcome.

Though the consumers do not observe the quality, the regulatory bodies can observe quality ex-post. As the seller is risk-neutral, they care about the expected future penalty. I compute the minimum level of penalty, which can achieve a specific objective for two reasons. In practice, the nominal penalty imposed by regulatory bodies cannot be very high. If it is out of sync with the actual damage caused to buyers due to false advertising, this

8Examples cited earlier are a vindication of the statement that firms engage in false advertising or make false claims. Some examples may not fit the model but it makes sense to focus on these models where price signaling works and even then there is a possibility for deception.
penalty may be contested in a court of law. This naturally places an upper bound on the nominal level of penalty. Further, the expected future penalty depends on the probability of enforcement or detection of deception. Increasing this probability is costly for real-life regulators as monitoring cost is involved (such as cost to gather evidence). This is why it makes sense to focus on the minimum level of expected future penalty.\(^9\)

The desired level of penalty (strength of regulation) that the government needs to set depends upon the type of policy goal: whether to avoid deception or to maximize social welfare (optimal regulation). The minimum level of penalty needed to avoid deceptive advertising decreases in the proportion of sophisticated consumers. In fact, no regulation is needed if the proportion of sophisticated consumers is very high in the market because the low type cannot deceive sophisticated consumers and hence, no fear of deception. However, if the policy goal is to maximize social welfare, then a much higher level of penalty is required. Such a level of penalty leads to full information outcome and removes any signaling distortion present in the case of a high proportion of sophisticated consumers (Bagwell and Riordan, 1991). The resulting optimal penalty depends on the prior beliefs of the consumers about the product quality. If prior beliefs are optimistic, the optimal penalty level increases in the proportion of sophisticated consumers. This outcome contradicts the accepted notion that consumer naivety and protection of naive consumers from deception necessitates strong regulation. Instead, it is the increased buyer sophistication that requires stronger regulatory effort from a welfare perspective. If prior beliefs are pessimistic, the optimal penalty is set at a very high level overall to meet the goal. The low type indulges in deceptive advertising up to a high level of penalty to attract naive consumers so as to offset the low prior beliefs or low confidence in the product. In either scenario, increasing the penalty initially brings down social welfare. This implies that weak regulation with a low level of penalty can be detrimental to social welfare compared to no regulation. In such a case, deception still occurs so that the firm incurs the penalty.

\(^9\)See Polinsky and Shavell (2007).

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This paper is closely related to the renewed literature on false advertising. This literature focuses on the design of public policy on the prevention of false advertising in a framework where buyers do not infer information from prices. There are two papers that are of particular interest here. Rhodes and Wilson (2018) show how false advertising can emerge in a market with rational buyers and study the welfare effects of penalizing false advertising. Similarly but in a competitive setting, Piccolo et al. (2015) analyze the effect of false advertising on consumer surplus. In both the papers, as the cost of supplying the product does not depend on quality, a signaling equilibrium does not emerge. Additionally, they show how false advertising, under certain conditions, can lead to a positive impact on consumer surplus, making a case for a lenient regulatory action. In contrast, the cost structure in my paper allows for a signaling equilibrium to emerge so that prices are distorted because of signaling. Moreover, my framework allows for naive buyers to be present in the market alongside rational buyers. Note that Rhodes and Wilson (2018) extend their model to incorporate small cost asymmetry and show that false advertising continues to emerge under a pooling outcome. My paper is still different as, with any level of cost asymmetry, no false advertising occurs when all consumers are rational.

The existing literature on regulation of false advertising when firms also signal through prices has confined attention to markets where all buyers are sophisticated. Thus, Corts (2014) analyzes a particular case of false advertising wherein a seller can make speculative claims about the product quality. Janssen and Roy (2020) examine the welfare effects of regulation through the impact of policy penalizing false claims on the market outcome when no false advertising occurs in equilibrium. Similar to this strand of the literature, my paper addresses the question of regulation of false advertising for markets where price signaling is an alternative channel of communication of private information but focuses on the effect of consumer naivety.

The literature has also discussed the welfare effects of deceptive advertising when all buyers are gullible i.e., are assumed to be persuaded about product quality by advertising
messages of sellers. See among others Glaeser and Ujhelyi (2010) and Baumann and Rasch (2020). Within this literature, there are papers where a seller can spend more money to mislead consumers as a comparative strategy. Hattori and Higashida (2012) examine the strategic competition between firms when regulation makes it more costly to change the perceived quality and therefore, the extent of vertical differentiation. Hattori and Higashida (2015) show that it may not be ideal to regulate this kind of misleading advertising as it can reduce the inefficiencies of the market. In contrast to this strand of the literature, rational buyers in my paper infer product quality from advertising and prices using the equilibrium strategies of the seller. Moreover, the definition of naivety is a broader concept than gullibility.

Rest of the paper is organized as follows: Section 4.2 presents the model. Section 4.3 solves the firm’s problem and fully characterizes the equilibrium concept. Section 4 analyzes regulation under two cases: (i) to avoid deception and (ii) to maximize overall social welfare. Section 4.5 discusses extensions to the main analysis given in Sections 4.3 and 4.4. Section 4.6 concludes. Appendix C contains all the proofs.

4.2 Model

Consider a one-period product market with one firm and many potential buyers of the product. The product can either be of high-quality or low-quality, \( q = \{ H, L \} \). It is produced at a constant marginal cost, \( c_q \geq 0 \), which depends on the true quality of the product. The marginal cost of a high-quality product is \( c_H > 0 \) and of a low-quality product is \( c_L = 0 \) (without loss of generality). Nature determines the product quality for the firm. The product quality is only observed by the firm and is not revealed to the consumers. Each type of firm sets a price \( p \) and sends an advertising message, \( a \in \{ 0, 1 \} \). Message \( a = 0 \) implies that the firm did not advertise its product whereas \( a = 1 \) implies that the firm advertised its product as high-quality.
If a firm lies about its product quality, then it incurs an expected future fixed penalty equal to $d$. I will interpret $d$ as a regulatory parameter and it will capture the strength of the regulation, i.e., how stringent is the regulation. The firm maximizes expected profit net of any penalty it has to pay. We refer to this as expected net profit.

The potential consumers of the product are given by a unit mass, each with a unit demand. The consumers have homogeneous valuation, $v > 0$, for a low-quality product and heterogeneous valuation for a high-quality product, which is uniformly distributed between $v$ and $(1 + v)$. The high-quality market is active, i.e., $v > c_H$. It is common knowledge that consumers have prior belief that the product quality is high with probability $r$. Further, consumers are divided into two categories:

1. Sophisticated consumers observe the price and the advertising message set by each firm and update their beliefs in a Bayesian fashion depending on their beliefs about the equilibrium strategies of the firm.

2. Naive consumers have the same prior probability as the sophisticated consumers. However, upon observing an advertising message, $a = 1$, these consumers update their beliefs to product being high-quality, irrespective of the price. Their beliefs are unaffected by any other factor (price and/or $a = 0$). In particular, if they do not observe any advertising message, their belief is identical to the prior belief, independent of the price.

Let $s$ denote the fraction of sophisticated consumers and $(1 - s)$ the fraction of naive consumers in the market, where $s \in [0, 1]$. In addition, let $r'$ denote the posterior probability of the sophisticated consumers after they observe the price and the advertising message. Each consumer maximizes their expected valuation from buying based on the posterior probability minus the price set by the firm. We refer to this as expected net surplus.

The timeline of the game is as follows: First, Nature determines the quality of the firm, which is drawn from an independent distribution that assigns probability $r$ to high type. The firm observes its quality and simultaneously sets a price and chooses an advertising message. Finally, all the consumers decide whether to purchase the product or not. The firm’s payoff
is equal to expected net profit. The payoff of each consumer is equal to their expected net surplus.

The solution concept used is that of perfect Bayesian equilibrium where the out-of-equilibrium beliefs satisfy the intuitive criterion (Cho and Kreps, 1987) in every subgame. In equilibrium, consumers maximize their expected net surplus in order to make a purchase decision. This results in separate demand curves for each type of consumer, as shown in Figures 4.1 and 4.2. At price $P$ and $a = 1$, $(1 + \frac{P-v}{r})$ fraction of sophisticated consumers buy for any price $P \in [v, r' + v]$ and $(1 + v - P)$ fraction of naive consumers buy for any price $P \in [v, 1 + v]$. When $a = 0$, all the sophisticated consumers buy as long as $P \leq v$ and $(1 + \frac{P-v}{r})$ fraction of naive consumers buy for any $P \in [v, r + v]$.

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fraction of naive consumers buy for any $P \in [v, r + v]$.

Proposition 4.1 No pooling equilibrium exists that satisfies the intuitive criterion.

A pooling equilibrium is one where both the price and the advertising message sent by each type of firm are identical. Given this model, a pooling equilibrium where both types charge the same price and send the same advertising message is not possible for any value of $s$ and $d$. This result is in contrast with the findings of the previous literature (Rhodes and Wilson, 2018; Piccolo et al., 2015, 2017), which shows that false advertising occurs only under a pooling outcome. The polarity in results arises since previous literature rules out consumer naivety. Whereas in my paper, some consumers (sophisticated) always update their beliefs and some do not. Moreover, pooling is required on both the price and the message, which can never be the same for both types of firms even when all consumers are naive. A direct implication of this result is that sophisticated consumers can never be deceived in equilibrium. A formal proof is given in the appendix.

4.3 Equilibrium

A separating equilibrium defines pricing and advertising strategies of each type of firm, which reveals their true quality, at least to the sophisticated consumers. A fully separating equilibrium exists for all values of $s$ and $d$. A formal description of the equilibrium strategies for the various regions of the parameter space are specified in the appendix (Proposition A.1). As the equilibrium configuration of price and advertisement changes across the various ranges of parameter space, I would provide a more informal description of the equilibrium for the different cases in this section.

4.3.1 No Regulation (Benchmark Case)

In this subsection, I characterize the equilibrium for the benchmark case when there is no regulation, i.e., $d = 0$. This equilibrium is depicted in Figure 4.3.
The low type can serve all the consumers in the market by charging the reservation price, \( v \) and not advertising its product, \( a = 0 \). However, as naive consumers incorporate only the advertising message in their beliefs, the low type can increase its price by falsely advertising its product as of high-quality, especially when it does not have to incur any penalty. By deceiving naive consumers, the low type has to forgo the demand from sophisticated consumers who understand such deceptive behavior. Therefore, the low type faces a tradeoff: whether to serve all consumers at a low price or serve only the naive consumers at a high price through false advertising.

When \( s = 0 \), all consumers in the market are naive. As such, the low type falsely advertises its product by sending out \( a = 1 \) message and thereby charges a profit-maximizing price. As \( s \) increases slightly over zero, a small fraction of sophisticated consumers are introduced in the market. Sophisticated consumers will understand the deceptive strategies of the low type and update their beliefs accordingly. They would not be willing to pay more than the reservation price. However, the low type will continue with false advertising as the total gain from deceiving naive consumers will still be greater than the loss of demand from sophisticated consumers. As \( s \) increases further, the loss of demand from sophisticated consumers rises and the total gain from deceiving naive consumers falls. At \( s = \left(1 - \frac{v}{1 + v}\right)^2 \), the benefit from false advertising is equal to its cost. This is the point where the low type shifts to the reservation price. The fraction of sophisticated consumers is now so high that the low type can no longer afford to lose out on their demand and hence, switches to \( a = 0 \) to serve all the consumers in the market. Therefore, when \( s \geq \left(1 - \frac{v}{1 + v}\right)^2 \), sophisticated consumers are dominant in the market, which is why the low type charges a reservation price along with \( a = 0 \) message.

The high type always advertises its product and sends out \( a = 1 \) message. The high type’s strategy to signal quality depends on the type of consumer. For the naive consumers, advertising message is sufficient. However, for sophisticated consumers, the high type combines advertising with a high price to signal its true quality and separate itself from the low
The profits of the high type are maximized at the full-information monopoly price and thus, its ideal price. When naive consumers are dominant, the high type charges its ideal price as the advertising signal alone serves the purpose. There is no fear of imitation as it would not have any impact on naive consumers’ beliefs. As $s$ increases and enters the intermediate range – represented by $\left(\frac{cH}{1+v}\right)^2 \leq s \leq \left(\frac{1-v}{1+v}\right)^2$ – the high type needs to change its pricing strategy to serve the sophisticated consumers. While continuing to signal its quality through $a = 1$ to naive consumers, the high type additionally increases its price above the profit-maximizing level to signal its quality to sophisticated consumers. When sophisticated consumers become dominant in the market, the signaling price reaches the highest level.
4.3.2 Weak Regulation

In this subsection, I introduce regulation in the form of a fixed penalty, $d$. As $d$ increases, it affects the pricing and advertising signals of each type of firm in different ways. Accordingly, the penalty level is divided into four appropriate ranges. First, a weak regulation or a low-level penalty case is depicted in Figure 4.4. In particular, $d$ lies between 0 and $\bar{d}_1(s)$ where $\bar{d}_1(s)$ is given by

$$\bar{d}_1(s) = \frac{(1 - s)(1 - r)(r - v^2)}{4r}$$  \hspace{1cm} (1)

Introducing weak regulation does not have any major impact on the low type’s behavior. When both $s$ and $d$ are low, the incentive to deceive remains high because the benefit from false advertising is higher than the penalty level. Therefore, the low type deceives naive consumers and incurs the penalty. It continues to indulge in false advertising for intermediate values of $s$. However, once I reach $s = \frac{(1-v^2)^2-4d}{(1+v)^2}$, the low type switches to the reservation price along with $a = 0$. In the benchmark case, the switching point was where the gain from deceiving naive consumers (benefit) equals the loss of demand from sophisticated consumers (cost). Under the current case, the cost also includes the penalty, which shifts the point slightly to the left. Thus, the region of deception has narrowed down due to weak regulation.

For the high type, the strategy is the same as the benchmark case when naive consumers are dominant in the market. Advertising signal truthfully reveals the quality to naive consumers. However, high type’s pricing strategy changes for the intermediate and high proportion of sophisticated consumers. It employs a combination of price and advertising signals to reveal quality to sophisticated consumers. The penalty introduces some credibility to its advertising signal. Therefore, a lower signaling price along with $a = 1$ still has the same desired effect on the sophisticated consumers’ beliefs as the benchmark case. Further, no firm type has an incentive to mimic the other. The low type cannot possibly
deviate to the high type’s signaling price and pay the penalty as well. Even when $s$ is low, the low type does not want to deviate away from its profit-maximizing price. Similarly, the high type does not want to deviate to a lower price irrespective of the advertising message, especially after incurring a cost to produce its high-quality product.

**4.3.3 Intermediate Regulation**

Second, intermediate regulation or a moderate-level penalty is depicted in Figures 4.5 and 4.6. Here, $d$ lies between $\tilde{d}_1(s)$ and $\tilde{d}_2(s)$ where $\tilde{d}_1(s)$ is given by (1) and $\tilde{d}_2(s)$ is given by

$$\tilde{d}_2(s) = \left( \frac{1 - s(1 - r)}{4r} \right) \left[ \left( \frac{r}{1 - s(1 - r)} - v \right)^2 - c_H^2 \right]$$

(2)

The penalty has reached the range where the low type needs to decide whether to continue
to falsely advertise or not. This decision depends on the value of \( r \), the prior belief that the quality is high. If the prior is high, i.e. \( r > v \), the prior quality distribution is optimistic. The false advertising is not much helpful here. In fact, for any \( s \), the gain from false advertising is always lower than the loss of demand from sophisticated consumers plus the penalty. Therefore, the low type chooses \( a = 0 \) for all \( s \). However, this does not prevent it from taking advantage of the asymmetry in information as well as the naivety of the consumers to charge a price higher than the valuation. Because naive consumers make purchase decisions based on the expected valuation when they observe \( a = 0 \), the low type sets an average price. Thus, the low type can either serve all the consumers by charging the reservation price or serve only the naive consumers at a higher price.

\[
\frac{r(c_H^2 + 4d) - (1-r)(r-v^2)}{(r+v)^2} \leq \frac{r-v}{r+v}\]

\[
\frac{r^2}{(r+v)^2}.
\]

\( a = 1 \)

\( a = 0 \)

\( p^u \)

\( p^l \)

Figure 4.5: Separating Perfect Bayesian Equilibrium under Intermediate Regulation \((r > v)\)

When \( s \leq \frac{r(c_H^2 + 4d) - (1-r)(r-v^2)}{(r+v)^2} \), naive consumers are dominant in the market. The total surplus extracted from the naive consumers is much higher than the loss of demand from
sophisticated consumers. Therefore, the low type sets a price higher than valuation and sends \( a = 0 \) message. As \( s \) increases, the loss of demand from sophisticated consumers rises and the total surplus extracted from naive consumers falls. At \( s = \left( \frac{r-v}{r+v} \right)^2 \), total surplus equals total loss and thus, the low type’s strategy changes to cater to sophisticated consumers. As such, for \( s \geq \left( \frac{r-v}{r+v} \right)^2 \), the low type shifts to reservation price.

If prior belief is low, i.e. \( r < v \), the prior quality distribution is pessimistic. In this case, the low type must advertise to alter the beliefs of naive consumers. In the absence of any advertising, the price reduces to valuation level, lowering the profits and hence, not an equilibrium. However, false advertising is only profitable for low and intermediate \( s \). As the sophisticated consumers become dominant, the price needs to be reduced to valuation level as the low type cannot afford to lose the demand from sophisticated consumers. Therefore, the pricing strategy is the same as the weak regulation scenario with a smaller range of
deception.

The high type’s strategy remains the same as the benchmark case for low values of $s$. It is still optimal to signal through $a = 1$ because naive consumers believe only advertising signals irrespective of the intensity of regulation. The change in strategy occurs for intermediate and high values of $s$. A higher $d$ (i) increases the range of $s$ for which a full-information price is charged, (ii) reduces the signaling price by increasing the credibility of the advertising signal and (iii) reduces the incentives of the low type to mimic the high type and combined with a signaling price, makes it impossible to mimic without reducing profits.

4.3.4 Strong Regulation

Third, strong regulation or a high-level penalty is depicted in Figures 4.7 and 4.8. Here, $d$ lies between $\overline{d}_2(s)$ and $\overline{d}_2$ where $\overline{d}_2(s)$ is given by (2) and $\overline{d}_2$ is given by

$$\overline{d}_2 = \frac{[(1 - v)^2 - c_H^2]}{4}$$ (3)
The equilibrium outcome depends on the prior beliefs. With optimistic beliefs, the low type’s pricing and advertising signals do not change from the previous case of intermediate regulation. It serves only the naive consumers at a higher than valuation price for low and intermediate values of $s$ and all consumers at the reservation price for high values of $s$. For benchmark and weak regulation cases, low type’s strategy was dependent on the penalty level, $d$ as well as the proportion of sophisticated consumers, $s$. But once penalty enters the intermediate range, the low type has no incentive to deceive and $s$ becomes the only determining factor. The only impact of increasing $d$ on the low type’s strategy is that it reduces the incentive to mimic the high type.

On the other hand, high penalty has a positive impact on the high type’s strategy as it reduces the cost of signaling quality in two ways. First, the high type is able to charge its full-information price for a wider range of $s$. As mentioned in all the previous cases, when...

![Figure 4.8: Separating Perfect Bayesian Equilibrium under Strong Regulation ($r < v$)](image-url)
naive consumers are dominant in the market, the advertising signal is sufficient to reveal quality. Thus, the high type charges the full-information price. But now for intermediate values of $s$ as well, the high type reduces its price to the full-information level as there is no fear of imitation. If the low type mimics the high type, it would have to pay a high penalty, which is greater than the increase in demand. Therefore, the low type is better off losing out on increased demand and profits rather than paying a high penalty. Second, the high type sets a lower signaling price for high values of $s$ because of the increased credibility of the advertising message. The sophisticated consumers understand that such a claim can come only from the high type as the low type cannot afford to pay a high penalty in exchange for a small gain in profits.

With pessimistic beliefs, the low type’s strategy is same as the intermediate regulation case. The difference lies in the area of deception. When both $d$ and $s$ increase simultaneously, it is difficult to sustain deceptive behavior. The high type’s strategy also remains the same as the previous case but sets a lower signaling price with increasing penalty.

4.3.5 Substantial Regulation

Fourth, substantial regulation is depicted in Figures 4.9, 4.10(i) and 4.10(ii). Here, $d$ lies above $\overline{d}_2$ where $\overline{d}_2$ is given by (3).

When prior beliefs are optimistic, the low type’s strategy remains unchanged from the previous case of strong regulation. However, now penalty has entered a very high range, which makes it impossible to mimic the high type. Pessimistic beliefs still give rise to deceptive behavior from the low type, though the area of deception has narrowed down and only occurs when $s$ is low and $d \leq \overline{d}_1 (s)$, where $\overline{d}_1 (s)$ is defined as:

$$\overline{d}_1 (s) = \frac{(1 - v)^2 - s (1 + v)^2}{4}$$

This implies that when naive consumers are dominant in the market, the low type can earn higher profits by deceiving them than charging the reservation price. As the penalty
increases, deceptive advertising is no more profitable and affordable for all $s$. Therefore, the low type charges the reservation price, which gives the highest profit.

\[ \frac{(1-v^2)-4d}{(1+v)^2} \]

**Figure 4.9:** Separating Perfect Bayesian Equilibrium under Substantial Regulation

\[ \frac{1+v+c_H}{2} \]

\[ \frac{r+v}{2} \]

\[ v \]

\[ \frac{a=0}{a=0} \]

\[ \frac{a=1}{a=1} \]

\[ \frac{p_H}{p_H} \]

\[ \frac{p_L}{p_L} \]

**Figure 4.10(i):** Separating Perfect Bayesian Equilibrium under Substantial Regulation ($r < v$)
Irrespective of the prior beliefs, the high type’s price converges to the full-information price for all $s$ as there is no fear of imitation. The advertising message $a = 1$ is adequate to reveal the high type’s quality. As such, $d$ brings down the price of a high-quality product in two stages: first, strong regulation reduces the price for intermediate values of $s$ and then substantial regulation reduces the price for high values of $s$.

![Figure 4.10(ii): Separating Perfect Bayesian Equilibrium under Substantial Regulation ($r < v$)](image)

Hence, the above cases completely define the unique separating equilibrium satisfying the intuitive criterion. The results are summarized in the following propositions. A formal proof is given in the appendix.

**Proposition 4.2** In a separating equilibrium, the low type monopolist’s pricing and advertising strategies are as follows:

1. For low values of $s$,
   
   (a) Optimistic beliefs
   
   (i) deceives naive consumers, for $d \leq \tilde{d}_1(s)$. 

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(ii) charges higher than reservation price but no deception, otherwise.

(b) Pessimistic beliefs

(i) deceives naive consumers and charges higher than reservation price, for $d \leq \bar{d}_1(s)$.

(ii) charges reservation price, otherwise.

2. For intermediate values of $s$,

(a) Optimistic beliefs

(i) deceives naive consumers, for $d \leq \bar{d}_1(s)$.

(ii) charges higher than reservation price but no deception, otherwise.

(b) Pessimistic beliefs

(i) deceives naive consumers and charges higher than reservation price, for $d \leq \bar{d}_1(s)$.

(ii) charges reservation price, otherwise.

3. For high values of $s$, always charges reservation price.

Proposition 4.3 In a separating equilibrium, the high type monopolist’s pricing and advertising strategies are as follows:

1. For low values of $s$, always charges full-information price.

2. For intermediate values of $s$,

(a) Optimistic beliefs

(i) charges signaling price, for $d \leq \bar{d}_2(s)$.

(ii) charges full-information price, otherwise.

(b) Pessimistic beliefs

(i) charges signaling price, for $d \leq \bar{d}_2$.

(ii) charges full-information price, otherwise.

3. For high values of $s$,

(a) charges signaling price, for $d \leq \bar{d}_2$.

(b) charges full-information price, otherwise.
4.4 Social Welfare Analysis and Policy Implications

An optimal policy can have two types of goals: either to prevent false advertising/deception or to maximize overall social welfare. Under the separating equilibrium, each goal results in a different minimal required penalty. As the equilibrium strategy varies according to the proportion of sophisticated consumers, I base my policy analysis on $s$.

4.4.1 Penalty to Avoid Deception

First, consider the case when $r > v$. For $0 \leq s \leq \left( \frac{r-v}{r-v'} \right)^2$, the low type has an incentive to lie about its quality only below the $\bar{d}_1(s)$ threshold as defined in (1). Therefore, $\bar{d}_1(s)$ is the minimum penalty required to prevent deception up to $s = \left( \frac{r-v}{r-v'} \right)^2$. For $\left( \frac{r-v}{r-v'} \right)^2 \leq s \leq \left( \frac{1-v}{1+v} \right)^2$, the incentive to deceive begins to fall sharply. As the proportion of naive consumers decreases, the range of $d$ for which the low type can profitably deceive narrows down. Once I reach $s = \left( \frac{1-v}{1+v} \right)^2$, the proportion of naive consumers is so low that there is no net gain from false advertising. Therefore, for high values of $s$, there is no fear of deception, which implies that zero penalty should suffice.

Note that I do not need to set a high value for $d$ to avoid deception. Moreover, as $s$ increases, the minimum penalty falls implying that the presence of sophisticated consumers reduces the need to have such kind of regulation. The results are summarized in Figure 4.11.

When $r < v$, $\bar{d}_1(s)$ is the minimum penalty required to avoid deception. The low type only lies below the $\bar{d}_1(s)$ threshold. Beyond this point, there is no deceptive advertising as sophisticated consumers are dominant in the market. Figure 4.12 depicts this result.

**Proposition 4.4** To avoid deception,

(a) the penalty decreases with $s$

(b) high $s$ eliminates any regulatory measure.
Figure 4.11: Minimum Penalty to Avoid Deception
\((r > v)\)

\[ \bar{d}_v(s) \]

\[ \frac{(1-v)(r-v^2)}{4r} \]

\[ (1/v) \]

\[ (r-v) \]

\[ (1-v) \]

\[ 1 \]

Figure 4.12: Minimum Penalty to Avoid Deception
\((r < v)\)

\[ d_{1/2}(d) \]

\[ \frac{1+v}{2} \]

\[ \frac{1-v}{2} \]

\[ 1 \]
4.4.2 Optimal Penalty

For the second type of policy goal, I first calculate the social welfare in low and high-quality states under the separating equilibrium. The social welfare is the sum of consumer surplus and the monopolist’s profit. It is calculated by the difference in the valuation and the cost of producing a unit of the good times the total demand at a price $P$, which yields:

$$SW_L = \begin{cases} v[Q^D(P_L)] & \text{if } a = 0 \\ v[Q^D(P_L)] - d & \text{if } a = 1 \end{cases}$$

$$SW_H = [1 + v - P^H]\left[\frac{1 + v + P^H}{2} - c_H\right]$$

Next, I calculate the expected social welfare to determine the socially optimal level of penalty. Expected social welfare is the weighted average of social welfare in low- and high-quality states, where weights are equal to the prior beliefs:

$$\mathbb{E}(SW) = rSW^H + (1 - r)SW^L$$

The optimal penalty is defined as the minimum level of penalty that maximizes the expected social welfare. The following proposition states the properties of the optimal penalty function. An intuitive reasoning behind the calculation of the optimal penalty is given in the subsections 4.4.2.1 and 4.4.2.2. A formal proof is given in the appendix.

**Proposition 4.5** The optimal penalty:

1. is higher than the penalty required to avoid deception.
2. For low values of $s$,
   (a) prevents deception
   (b) is decreasing with $s$
3. For intermediate and high values of $s$,
(a) eliminates signaling distortion
(b) is increasing with $s$, when $r > v$
(c) is constant, when $r < v$.

4.4.2.1 Optimal Penalty when $r > v$

For low values of $s$, naive consumers are dominant in the market. It is defined as:

$$0 \leq s \leq \left(\frac{cH}{1 + v}\right)^2$$  \hspace{1cm} (5)

We know that in equilibrium, the low type takes advantage of naive consumers by either falsely advertising its product or charging a higher than valuation price. When $d \leq \tilde{d}_1(s)$, social welfare is low because the monopolist indulges in false advertising. False advertising is associated with loss of demand from sophisticated consumers as well as negative surplus of naive consumers. This low welfare level is further falling with $d$ because a part of the surplus is taken away as a penalty, which is equivalent to deadweight loss. As $d$ increases above $\tilde{d}_1(s)$, false advertising is no more profitable and thus, the monopolist charges a higher than valuation price. Therefore, social welfare shifts up and becomes constant.

On the other hand, in the high-quality state, the social welfare is constant for all $d$ as the monopolist always charges the full-information price. The advertising signal truthfully reveals its quality. As the high type’s strategy is constant, the pattern of expected social welfare function is solely dependent on the changes in the low type’s strategy. Below $\tilde{d}_1(s)$, the expected social welfare is low and decreasing with $d$ because the low type indulges in false advertising. Above $\tilde{d}_1(s)$, the expected social welfare shifts up and is a constant because the low type reduces its prices. There is no signaling distortion present, for any $d$, as the high type signals quality through advertising message alone. Therefore, expected social welfare is simply maximized by preventing deception at $d \geq \tilde{d}_1(s)$ and hence, $\tilde{d}_1(s)$ is the optimal penalty for low values of $s$. 

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For intermediate values of $s$, sophisticated and naive consumers are more or less in similar proportions in the market. It is defined as:

$$\left( \frac{c_H}{1+v} \right)^2 \leq s \leq \left( \frac{1-v}{1+v} \right)^2$$

(6)

In the low-quality state, the monopolist’s strategy changes from false advertising to charging a higher than valuation price to full-information outcome as $s$ increases within the above range. When $d \leq \bar{d}_1(s)$, the low type always deceives naive consumers, which results in a downward sloping social welfare. As $s$ increases within the intermediate range, $\bar{d}_1(s)$ decreases, thereby reducing the area of deception. This implies that $s$ reduces the incentive for the low type to deceive and hence, increases the social welfare. When $d \geq \bar{d}_1(s)$, social welfare jumps to a higher level as there is no deception. Therefore, the role of $d$ in a low-quality state is to prevent false advertising.

However, there is another factor that plays an important role in determining the low type’s strategy: the proportion of sophisticated consumers. For the first two subsets of $s$, the low type shifts to a higher than valuation price as it can still profitably extract surplus from naive consumers. But for the third subset of $s$, the sophisticated consumers are rising to become prominent in the market and hence, it shifts to the full-information outcome. Both cases lead to an increase in social welfare, which is maximized at the full-information level. Therefore, the role of $s$ is to maximize social welfare in a low-quality state.

In the high-quality state, there are two opposite forces working simultaneously. As noted in Section 3, in equilibrium, as $s$ enters the intermediate range, the monopolist begins to signal its quality through high prices. A resulting fall in profits (signaling distortion) is the cost paid by the monopolist to reveal its true quality. Hence, social welfare falls below the full-information outcome. Moreover, in equilibrium, higher $s$ implies a higher signaling price. Thus, as $s$ increases across different subsets of the intermediate range, signaling price increases, which reduces the social welfare further. At the same time, an increase in $d$ reduces the need to charge a high signaling price as the credibility of the advertising signal
increases. Therefore, signaling price falls with \( d \) leading to an increase in social welfare. Two conclusions are drawn: (i) signaling price rises and thus, social welfare falls with \( s \) and (ii) signaling price falls and thus, social welfare rises with \( d \).

Note that in a high-quality state, the role of \( d \) is to eliminate signaling distortion to reach the maximum level of social welfare. As higher \( s \) implies higher signaling distortion, it requires a higher \( d \) to maximize social welfare. Hence, as \( s \) increases within the intermediate range, the area of price signaling expands with respect to \( d \). Broadly, the intermediate range showcases the U-shaped effect of \( d \) on expected social welfare function. When \( d \) increases to \( \bar{d}_1(s) \), it first corrects the low type’s price and advertising signal. As \( d \) increases further up to \( \bar{d}_2(s) \), it then corrects the high type’s prices, removing entire signaling distortion thereby increasing the expected social welfare. Thus, penalty first prevents false advertising and then removes any underlying distortion. However, the optimal penalty is going to vary with \( s \).

For \( \left( \frac{cH}{1+v} \right)^2 \leq s \leq \frac{r(c_H^2+4d_2(s))-(1-r)(r-v^2)}{(r+v)^2} \), the expected social welfare is maximized at 
\[
d = \frac{s(r+v)^2+(1-r)(r-v^2)-rc_H^2}{4r},
\]
lying between \( \bar{d}_1(s) \) and \( \bar{d}_2(s) \). When \( s \) increases within this subset, the optimal penalty increases from \( \bar{d}_1(s) \) to \( \bar{d}_2(s) \). For \( \frac{r(c_H^2+4d_2(s))-(1-r)(r-v^2)}{(r+v)^2} \leq s \leq \left( \frac{r-v}{r+v} \right)^2 \), the expected social welfare is maximized at \( \bar{d}_2(s) \) and for \( \left( \frac{r-v}{r+v} \right)^2 \leq s \leq \left( \frac{1-v}{1+v} \right)^2 \), at \( \bar{d}_2 \). Therefore, as \( s \) increases, higher signaling distortion requires a higher optimal penalty to reach the full-information outcome.

For the high range of \( s \), sophisticated consumers are dominant in the market. It is defined as:
\[
\left( \frac{1-v}{1+v} \right)^2 \leq s \leq 1
\]

(7)

In the low-quality state, the social welfare is constant at the full-information level as the monopolist simply charges the reservation price to serve all the consumers in the market. On the other hand, in the high-quality state, the social welfare is a monotone function of \( d \). For \( d < \bar{d}_2 \), the monopolist signals its quality through high prices. I know that in equilibrium, this signaling price falls with \( d \). As the cost to signal quality falls, social welfare increases. For
\( d \geq \bar{d}_2 \), the price converges to the full-information outcome, thereby reaching the maximum level of social welfare.

As the low type’s strategy is constant, the pattern of expected social welfare function is solely dependent on the changes in the high type’s strategy. Even though deception is not possible, signaling distortion is still present for initial values of \( d \). Thus, the expected social welfare falls below the full-information level. As \( d \) increases, the need for price signaling falls and expected social welfare increases. At \( d = \bar{d}_2 \), the entire signaling distortion is eliminated, thereby maximizing expected social welfare. Therefore, \( \bar{d}_2 \) is the optimal penalty level for the high range of \( s \).

![Figure 4.13: Optimal Penalty](image)

To summarize, the optimal penalty is the same as the penalty to avoid deception for low values of \( s \). However, for intermediate and high values of \( s \), the optimal penalty rises and the minimum penalty to avoid deception falls. This emphasizes that the optimal penalty has a dual role of preventing deception and eliminating signaling distortion. The results are
summarized in Figure 4.13.

This takes us to an unconventional outcome from the analysis: overall, optimal penalty increases with $s$ or equivalently, decreases with the proportion of naive consumers. Generally, it would be expected for the naive consumers to strengthen the case for a higher penalty. But the above result implies the opposite: that the presence of naive consumers weakens the case for strong regulation. This occurs because the penalty is not only disincentivizing false advertising for the low type but also eliminating signaling distortion created by the high type’s prices. Under the case of a high proportion of sophisticated consumers, there is no deception but the signaling distortion is at its peak. Hence, it requires highest level of optimal penalty. Note that monopoly distortion will always be present.

### 4.4.2.2 Optimal Penalty when $r < v$

For the low range of $s$, naive consumers are dominant in the market. It is defined by equation (5). We know that in equilibrium, the low type takes advantage of naive consumers by falsely advertising its product and charging a higher than valuation price. When $d \leq \overline{d}_1(s)$, social welfare is low because the monopolist indulges in false advertising. False advertising is associated with loss of demand from sophisticated consumers as well as negative surplus of naive consumers. This low welfare level is further falling with $d$ because a part of the surplus is taken away as a penalty, which is equivalent to deadweight loss. As $d$ increases above $\overline{d}_1(s)$, false advertising is no more profitable and thus, the monopolist charges the valuation price. Therefore, both consumer surplus and social welfare shift up to become constant.

On the other hand, in the high-quality state, the social welfare is constant as the monopolist always charges the full-information price. The advertising signal truthfully reveals its quality. As the high type’s strategy is constant, the pattern of expected social welfare is solely dependent on the changes in the low type’s strategy. Below $\overline{d}_1(s)$, because the low type indulges in false advertising: (i) the consumer surplus is constant at a low level and (ii) the expected social welfare is low and decreasing with $d$. Above $\overline{d}_1(s)$, the expected consumer
surplus and hence, expected social welfare shift up to become constant because the low type reduces its prices. There is no signaling distortion present, for any $d$, as the high type signals quality through advertising message alone. Therefore, expected consumer surplus and expected social welfare are maximized at $d \geq \bar{d}_1(s)$ and hence, $\bar{d}_1(s)$ is the optimal penalty for low values of $s$.

The intermediate range of $s$ is defined by equation (6). The low type continues to falsely advertise and charge a higher than valuation price when $d \leq \bar{d}_1(s)$. The consumer surplus and social welfare are very low. As $d$ increases above $\bar{d}_1(s)$, the low type reduces its price to the valuation as the penalty and proportion of sophisticated consumers are increasing. The high type signals its quality for the initial levels of $d$. As $d$ increases above $\bar{d}_2$, the penalty is high enough to disincentivize the low type to mimic. Therefore, the high type switches to the full-information price. Therefore, the expected expected social welfare is maximized at $d = \bar{d}_2$, because it prevents false advertising as well as eliminates the signaling distortion. For the high range of $s$, sophisticated consumers are dominant in the market. It is defined
by equation (7). The analysis is exactly the same as $r > v$ case. Therefore, the expected social welfare is maximized at $d = \overline{d}_2$. The results are summarized in Figure 4.14.

4.5 Discussion

4.5.1 Optimal Penalty and Expected Consumer Surplus

The goal of regulatory authorities is to protect consumers. Therefore, it would be interesting to find the minimum level of penalty that maximizes expected consumer surplus. In this section, I argue that such minimum penalty is the same as the optimal penalty derived in Section 4.4.2.

Imagine the goal is to maximize expected consumer surplus. For low values of $s$, false advertising under a low-quality state is the only reason that reduces the expected consumer surplus. Under false advertising, the naive consumers pay a price higher than their valuation, which leads to a negative surplus in a low-quality state. Moreover, the sophisticated consumers do not even purchase the low-quality product. Minimum penalty, which can prevent false advertising, is $\overline{d}_1(s)$ under optimistic beliefs and $\overline{d}_1(s)$ under pessimistic beliefs. For intermediate values of $s$, false advertising under a low-quality state and signaling distortion under a high-quality state reduce the expected consumer surplus. Under a high-quality state, the monopolist signals its quality to the increasing proportion of sophisticated consumers by charging a price higher than the full-information case, which negatively impacts the consumer surplus. The minimum penalty, which eliminates false advertising as well as signaling distortion, is $\overline{d}_2(s)$ under optimistic beliefs and $\overline{d}_2(s)$ under pessimistic beliefs. For high values of $s$, signaling distortion under a high-quality state is the only reason that reduces the expected consumer surplus. The minimum penalty, which eliminates signaling distortion, is $\overline{d}_2$ for both types of beliefs. In short, the expected social welfare is maximized when false advertising and signaling distortion are eliminated. These two factors also maximize the expected consumer surplus at the same time. Hence, the resulting optimal penalty in either
case is the same.

The intuition behind this outcome is that in the high-quality state, increasing the penalty means lower signaling distortion, which reduces the price, moving closer to the full-information outcome. This leads to higher profits as well as higher consumer surplus. Therefore, after a certain level of penalty, when entire signaling distortion is eliminated, both profits and consumer surplus are maximized, leading to the maximum level of social welfare. In the low-quality state, increasing penalty reduces and eventually eliminates any possibility of deception. When penalty is high enough to prevent deception, there is an upward shift in consumer surplus as the low type switches to reservation price. The profits are unchanged because at the optimal level of penalty, the profit from deceptive advertising is equal to that from no advertising. This implies that the social welfare increases solely because of the increase in consumer surplus. A formal proof is given in the appendix.

**Proposition 4.6** The minimum level of penalty that maximizes expected consumer surplus is identical to the optimal penalty level, which maximizes expected social welfare.

### 4.5.2 Lobbying for Regulation

In this section, I calculate the profit functions of each type of monopolist under the separating equilibrium to evaluate their incentives to lobby for regulation against false advertising.

For low and intermediate values of $s$, the profit function of the low type monopolist is given by $\pi_L = P^L(1 + v - P^L)(1 - s) - d$ as it lies about its quality. In such a case, it will always lobby against regulation because it finds false advertising extremely profitable despite serving only the naive consumers. Lower the $s$, higher the profit from false advertising and hence, greater the incentive to lobby against regulation. For high values of $s$, the profit function is given by $\pi_L = v$. The low type does not lie about its quality with few naive consumers present in the market. Thus, it is indifferent about regulation for high range of $s$.

The profit function of the high type monopolist is $\pi_H = (P^H - c_H)(1 + v - P^H)$. For low
values of $s$, the high type is indifferent about regulation as it still earns monopoly profits. However, for intermediate and high values of $s$, the high type charges a high signaling price, which negatively affects its profits. As $s$ increases, the signaling price increases as well. Increasing the optimal penalty can reduce the signaling price. Therefore, the high type monopolist strongly lobbies for regulation. I know that higher $s$ requires a higher $d$ to eliminate signaling price. Hence, higher the $s$, stronger the intensity with which the high type lobbies for regulation.

The following proposition summarizes the results. A formal proof is given in the appendix.

**Proposition 4.7** Irrespective of the prior beliefs,

1. the high type:
   (a) has a strong incentive to lobby for regulation for intermediate and high $s$
   (b) is indifferent otherwise.

2. the low type:
   (a) has strong incentive to lobby against the regulation for low and intermediate $s$
   (b) is indifferent otherwise.

### 4.5.3 Intermediate Sophistication among Consumers

Till now, I have defined the naive consumers such that if an advertising message is observed, they believe the firm to be a high type and if no advertising message is observed, they make the purchase decision based on the prior. This implies that naive consumers do not use beliefs about equilibrium strategies of the monopolist to update their priors. Therefore, an interesting question arises about the outcome of the model when naive consumers update their beliefs partially according to the equilibrium strategies of the seller, i.e., exhibit an intermediate level of sophistication. However, they ignore the pricing strategy of the monopolist and only update their beliefs using the advertising signals. In this section, I evaluate the impact of naive consumers with such intermediate levels of sophistication on
Consider the case when all consumers are naive, i.e., $s = 0$. As the naive consumers do not understand price signals, the high type would always want to advertise its product to signal quality. In addition, it charges the full-information price to maximize its profit. The low type can either pool with the high type on the advertising message or it does not advertise its product. For a reasonable penalty, the low type would not miss an opportunity to pool and deceive the naive consumers. It can falsely advertise as high-quality and charge a high monopoly price. However, the low type shifts to no advertisement as penalty increases beyond the gains from false advertising. Therefore, depending on the level of penalty, there are two equilibrium outcomes possible. First, both types advertise their products for a low and intermediate level of penalty and charge monopoly prices. Second, for a high level of penalty, the high type advertises its product and the low type does not. A formal proof is given in the appendix.

**Proposition 4.8** When all consumers have intermediate sophistication,

(a) For $d \leq \tilde{d}_1(s = 0)$,

(i) the high type advertises and charges the full-information price

(ii) the low type advertises and charges higher than the reservation price.

(b) For $d \geq \tilde{d}_1(s = 0)$,

(i) high type advertises and charges the full-information price

(ii) low type does not advertise and charges the reservation price.

where $\tilde{d}_1(s = 0)$ is defined by (4).

The equilibrium outcome is the same as that under the pessimistic beliefs case of the original model. It differs from the optimistic beliefs case in the sense that the low type

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10 This section has been added as per the referee’s comment about the set of beliefs which define the naive consumers: if in an equilibrium, only the high type advertises its product, why the naive consumers do not relate no-advertisement to the low-quality product?
falsely advertises for a broader range of penalty. Now that the advertising message holds all the weightage, the low type would want to mimic the high type beyond the low levels of penalty to benefit off of the naive consumers. In order to avoid deception and maximize social welfare, the above equilibrium outcome suggests an optimal penalty of $d_1(s = 0)$. Compared to the original model, this penalty is identical to the case of pessimistic beliefs and is higher than the case of optimistic beliefs.

Now, the question arises as to how the above situation compares to $s = 1$, i.e., when all consumers are (fully-) sophisticated. As discussed in Bagwell and Riordan (1991) and Section 3, the low type charges the valuation price and does not falsely advertise when all consumers are sophisticated. On the other hand, the high type signals quality through high prices for low and intermediate levels of penalty and reduces to the full-information price for a high level of penalty. In order to eliminate signaling distortion and maximize social welfare, the optimal penalty should be $d_2$, as defined by (3). Comparing the optimal penalties at $s = 0$ and $s = 1$, notice that even though the penalties at both points are very high, the penalty falls by a small fraction at $s = 1$. This outcome is identical to the pessimistic belief case of the original model. Therefore, introducing intermediate sophistication does not lead to any qualitative new results.

### 4.5.4 Educating Naive Consumers

An alternative method of preventing false advertising is to educate naive consumers. As I increase the proportion of sophisticated consumers, signaling distortion increases. As a result, it is unclear whether it is desirable to have a higher proportion of sophisticated consumers. For instance, when all consumers are sophisticated, the low type has no incentive to send a false advertising message, but the high type has an incentive to signal its quality through high prices creating some signaling distortion. When all consumers are naive, the low type has an incentive to falsely advertise its product, which negatively impacts consumer surplus. While regulation can prevent false advertising as well as signaling distortion, education can
be effective only to combat false advertising. Therefore, if education is used as an instrument against false advertising, signaling distortion will always be present and this will reduce the social welfare.

4.6 Conclusion

False advertising needs our immediate attention, especially with the emergence of online retail, which has made deceiving consumers extremely easy. Moreover, not all consumers are sophisticated enough to understand the price and advertising signals and, in fact, believe everything at face value. My paper shows that the low type can successfully separate itself from the high type and still deceive some consumers. The results imply that a low level of penalty is adequate to avoid false advertising practices. However, a higher level of penalty ensures full information outcome, which maximizes social welfare. While regulation that penalizes false advertising can prevent the deception of naive buyers, stronger regulation may be needed if one looks at the effect of such a penalty on social welfare. Interestingly, buyer sophistication – and not naivety – strengthens the case for imposing a penalty on false advertising. Future work in this area can incorporate competition in the above framework to formulate optimal regulation and analyze its welfare implications. I conjecture that in a duopoly setting, the signaling distortion tends to increase market power (Janssen and Roy, 2010). However, consumer naivety is likely to reduce this market power. Broad qualitative conclusions of this paper are likely to continue to hold in the presence of competition. Note that competition does not necessarily eliminate signaling distortion. The signaling distortion with sophisticated consumers will be high and regulation can allow for credible disclosure, which can bring down the signaling distortion. This also reduces the optimal penalty level to obtain maximum social welfare.
APPENDIX A

APPENDIX TO CHAPTER 2

Proof of Proposition 2.1

The proof is divided into the following steps:

Step 1 – Buyers’ purchase decision

If an independent review is posted, then buyers update their beliefs based on the message sent through the review, irrespective of the influencer type. If \( m = 1 \),

\[
Pr(H|m = 1) = \tilde{\eta}(m = 1) = \frac{\gamma \eta}{\gamma \eta + (1 - \gamma)(1 - \eta)}
\]

If \( m = 0 \),

\[
Pr(H|m = 0) = \tilde{\eta}(m = 0) = \frac{(1 - \gamma)\eta}{(1 - \gamma)\eta + \gamma(1 - \eta)}
\]

If a paid review is posted, then the buyers’ updated beliefs are as follows:

\[
Pr(H|m = 1) = \tilde{\eta}(m = 1) = \frac{[\gamma + (1 - \gamma)(1 - \mu)]\eta}{[\gamma + (1 - \gamma)(1 - \mu)]\eta + [(1 - \gamma) + \gamma(1 - \mu)](1 - \eta)}
\]

However, under the “no-disclosure” case, buyers observe a review but are unaware if it is independent or paid. If a review is posted, then buyers update their beliefs based on the message sent through the review and the influencer type.

If \( m = 1 \),

\[
Pr(H|m = 1) = \tilde{\eta}(m = 1) = \frac{[\gamma + (1 - \gamma)(1 - \mu)]\eta}{[\gamma + (1 - \gamma)(1 - \mu)]\eta + [(1 - \gamma) + \gamma(1 - \mu)](1 - \eta)}
\]
Depending on the message sent and the seller type, buyers purchase the product if:

\[ EU = \bar{\eta}(m) \cdot v_H + (1 - \bar{\eta}(m)) \cdot v - p \geq 0 \]

\[ \Rightarrow EU = \bar{\eta}(m) \cdot v_H + (1 - \bar{\eta}(m)) \cdot v \geq p \]

Maximizing the expected utility function generates a linear demand curve: \( \left( 1 + \frac{v-p}{\bar{\eta}(m)} \right) \) fraction of consumers wish to buy at a price \( p \).

If no review is posted, then buyers make a purchase decision according to their prior beliefs. The buyers purchase the product if:

\[ EU = \eta \cdot v_H + (1 - \eta) \cdot v - p \geq 0 \]

\[ \Rightarrow \eta \cdot v_H + (1 - \eta) \cdot v \geq p \]

Maximizing the expected utility function generates a linear demand curve: \( \left( 1 + \frac{v-p}{\eta} \right) \) fraction of consumers wish to buy at a price \( p \).

**Step 2 – Seller’s price decision**

Given the demand curve, the seller faces the following profit function if a review is posted:

\[ \pi(p) = \left( 1 + \frac{v-p}{\bar{\eta}(m)} \right) p \]

It is maximized at \( p = \frac{\bar{\eta}(m)+v}{2} \) at which \( \frac{\bar{\eta}(m)+v}{2\bar{\eta}(m)} \) fraction of consumers buy. If no review is posted, the profit function is given as:

\[ \pi(p) = \left( 1 + \frac{v-p}{\eta} \right) p \]

It is maximized at \( p = \frac{\eta+v}{2} \) at which \( \frac{\eta+v}{2\eta} \) fraction of consumers buy.

**Step 3 – Calculation of EVM**

Economic value of the message (EVM) sent by the influencer measures the value added
to the buyers’ ex-post utility. In other words, how did the influencer’s message help the buyers in their decision making process. The EVM is measured by the increase in consumer surplus due to the message sent by the influencer.

Because an honest influencer always posts a truthful message, expected economic value of the message generated is same for paid as well as independent review. The expected economic value of the message is defined as:

$$E(EMR(m)) = Pr(m = 1) \cdot EM(m = 1)|_s + Pr(m = 0) \cdot EM(m = 0)|_s \quad (A.1)$$

The event $m = 1$ occurs when the honest influencer receives the signal $s = 1$. The influencer’s signal can be correct with probability $\gamma$, in which case it generates the following EVM:

$$EM(m = 1) = CS|_{m=1} - CS|_{m=\phi}$$

$$= \left(\frac{\bar{\eta}(m = 1) + v}{4}\right)\left(\frac{\bar{\eta}(m = 1) + v}{2\bar{\eta}(m = 1)}\right) - \left(\frac{2\eta - \bar{\eta}(m = 1) + v}{4}\right)\left(\frac{2\eta + v - \bar{\eta}(m = 1)}{2\eta}\right)$$

$$\Rightarrow \quad EM(m = 1) = \frac{(\bar{\eta}(m = 1) + v)^2}{8\bar{\eta}(m = 1)} - \frac{(2\eta - \bar{\eta}(m = 1) + v)^2}{8\eta}$$

Alternatively, the honest influencer’s signal can be incorrect with probability $(1 - \gamma)$, in which case it generates the following EVM:

$$EM(m = 1) = CS|_{m=1} - CS|_{m=\phi}$$

$$= \left(v - \frac{\bar{\eta}(m = 1) + v}{2}\right)\left(\frac{\bar{\eta}(m = 1) + v}{2\bar{\eta}(m = 1)}\right) - \left(v - \frac{(\bar{\eta}(m = 1) + v)}{2}\right)\left(1 + \frac{(v - \bar{\eta}(m = 1))}{2\eta}\right)$$

$$\Rightarrow \quad EM(m = 1) = \left(v - \frac{\bar{\eta}(m = 1)}{2}\right)\left[\frac{(\bar{\eta}(m = 1) - \eta)(\bar{\eta}(m = 1) - v)}{2\bar{\eta}(m = 1)\eta}\right]$$

$$\Rightarrow \quad EM(m = 1) = \frac{(\eta - \bar{\eta}(m = 1))(\bar{\eta}(m = 1) - v)^2}{4\bar{\eta}(m = 1)\eta}$$
The event \( m = 0 \) occurs when the honest influencer receives the signal \( s = 0 \). The influencer’s signal can be correct with probability \( \gamma \), in which case it generates the following EVM:

\[
EVM(m = 0) = CS|_{m=0} - CS|_{m=\phi} \\
= \left( v - \frac{\bar{\eta}(m = 0) + v}{2} \right) \left( \frac{\bar{\eta}(m = 0) + v}{2\bar{\eta}(m = 0)} \right) - \left( v - \frac{\bar{\eta}(m = 0) + v}{2} \right) \left( 1 + \frac{v - \bar{\eta}(m = 0))}{2\eta} \right)
\]

\[
\Rightarrow \quad EVM(m = 0) = \left( \frac{v - \bar{\eta}(m = 0)}{2} \right) \left[ \frac{(\bar{\eta}(m = 0) - \eta)(\bar{\eta}(m = 0) - v)}{2\bar{\eta}(m = 0)\eta} \right]
\]

\[
\Rightarrow \quad EVM(m = 0) = \frac{(\eta - \bar{\eta}(m = 0))(\bar{\eta}(m = 0) - v)^2}{4\bar{\eta}(m = 0)\eta}
\]

Alternatively, the honest influencer’s signal can be incorrect with probability \( (1 - \gamma) \), in which case it generates the following EVM:

\[
EVM(m = 0) = CS|_{m=0} - CS|_{m=\phi} \\
= \frac{1}{2} \left[ \left( \frac{\bar{\eta}(m = 0) + v}{2} \right) \left( \frac{\bar{\eta}(m = 0) + v}{2\bar{\eta}(m = 0)} \right) - \left( \frac{2\eta - \bar{\eta}(m = 0) + v}{2} \right) \left( \frac{2\eta + v - \bar{\eta}(m = 0)}{2\eta} \right) \right]
\]

\[
\Rightarrow \quad EVM(m = 0) = \frac{(\bar{\eta}(m = 0) + v)^2}{8\bar{\eta}(m = 0)} - \frac{(2\eta - \bar{\eta}(m = 0) + v)^2}{8\eta}
\]

Breaking down the expression for expected EVM in (A.1):

\[
Pr(m = 1) \cdot EVM(m = 1) = (\gamma \eta) \left[ \frac{(\bar{\eta}(m = 1) + v)^2}{8\bar{\eta}(m = 1)} - \frac{(2\eta - \bar{\eta}(m = 1) + v)^2}{8\eta} \right]
\]

\[
+ ((1 - \gamma)(1 - \eta) \left[ \frac{(\eta - \bar{\eta}(m = 1))(\bar{\eta}(m = 1) - v)^2}{4\bar{\eta}(m = 1)\eta} \right] \quad (A.2)
\]
\[ Pr(m = 0) \cdot EV M(m = 0) = ((1 - \gamma) \eta) \left[ \frac{(\bar{\eta}(m = 0) + v)^2}{8\bar{\eta}(m = 0)} - \frac{(2\eta - \bar{\eta}(m = 0) + v)^2}{8\eta} \right] \]

\[ + (\gamma(1 - \eta)) \left[ \frac{(\eta - \bar{\eta}(m = 0))(\bar{\eta}(m = 0) - v)^2}{4\bar{\eta}(m = 0)\eta} \right] \quad (A.3) \]

A strategic influencer always posts a truthful message under an independent review. Therefore, the expected EVM is same as (A.1). In case of a paid review, strategic influencer only posts a positive message. If the influencer receives a signal \( s = 1 \), then he is being truthful and the economic value generated is same as (A.2). However, if he receives a signal \( s = 0 \) and reports \( m = 1 \), then it is a biased review. Denote a biased review by \( m^\beta = 1 \). With probability \( \gamma \), his signal is correct which means the product is of low-quality and buyers are worse-off than in no-review case.

\[ EV M(m^\beta = 1) = CS|_{m^\beta=1} - CS|_{m=\phi} \]

\[ \Rightarrow EV M(m = 1) = \left( \frac{v - \bar{\eta}(m = 1)}{2} \right) \left[ \frac{(\bar{\eta}(m = 1) - \eta)(\bar{\eta}(m = 1) - v)}{2\bar{\eta}(m = 1)\eta} \right] \]

\[ \Rightarrow EV M(m = 1) = \frac{(\eta - \bar{\eta}(m = 1))(\bar{\eta}(m = 1) - v)^2}{4\bar{\eta}(m = 1)\eta} \]

With probability \( (1 - \gamma) \), the influencer’s signal is incorrect which means the product is of high-quality and buyers are better-off than in no-review case:

\[ EV M(m^\beta = 1) = CS|_{m^\beta=1} - CS|_{m=\phi} \]

\[ \Rightarrow EV M(m^\beta = 1) = \frac{(\bar{\eta}(m = 1) + v)^2}{8\bar{\eta}(m = 1)} - \frac{(2\eta - \bar{\eta}(m = 1) + v)^2}{8\eta} \]

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Therefore, the expected EVM generated under a paid review by a strategic influencer is expressed as:

\[ \mathbb{E}(EV_{PR}^{st}(m)|_s) = Pr(m = 1) \cdot EV_{PR}(m = 1)|_s + Pr(m^\beta = 1) \cdot EV_{PR}(m^\beta = 1)|_s \]

where \(Pr(m = 1) \cdot EV_{PR}(m = 1)|_s\) is given by equation (A.2) and

\[ Pr(m^\beta = 1) \cdot EV_{PR}(m^\beta = 1)|_s = ((1 - \gamma) \eta) \left[ \frac{(\bar{\eta}(m = 1) + v)^2}{8\bar{\eta}(m = 1)} - \frac{(2\eta - \bar{\eta}(m = 1) + v)^2}{8\eta} \right] \\
+ (\gamma(1 - \eta)) \left[ \frac{(\eta - \bar{\eta}(m = 1)) (\bar{\eta}(m = 1) - v)^2}{4\bar{\eta}(m = 1)\eta} \right] \quad (A.4) \]

**Step 4 – Influencer’s decision**

Influencer has to make two decisions:

1. If influencer is made an advertisement offer by the seller, should he accept or reject the offer? This depends on his expected payoff. If \(\tau(m) + \alpha [\mathbb{E}(EV_{PR}(m)|_s)] - c_I \geq \max \{0, \alpha [\mathbb{E}(EV_{IR}(m)|_s)] - c_I\}\), then accept seller’s offer. Otherwise, reject the offer.

   (2) If influencer rejects the offer or is not made any offer, should he post an independent review or not? This again depends on his expected payoff. If \(\alpha [\mathbb{E}(EV_{IR}(m)|_s)] - c_I \geq 0\), then post an independent review. Otherwise, do not post a review.

**Step 5 – Seller’s advertisement decision**

Seller has to decide whether or not to advertise through the influencer. If it advertises and influencer accepts, what should be \(\tau(m)\) that maximizes his profits?

(i) If \(\alpha [\mathbb{E}(EV_{IR}(m)|_s)] - c_I \geq 0\), then influencer posts an independent review in case no offer is made by the seller or offer is rejected by the influencer. Expected profit of the seller is expressed as:
Note that buyers only observe the message and not the affiliation with the seller. So, the posterior beliefs are same irrespective of the type of review. This also means that the price charged by the seller and the corresponding demand are same under independent as well as paid reviews. However, the probabilities with which a particular message is sent are different for independent and paid reviews. Probability of sending \( m = 1 \) is higher under paid review as strategic influencer will always post a positive review. Probability of sending \( m = 0 \) is lower under paid review as only honest influencer can send out a negative review.

The influencer’s payoff from posting a paid review (minus the payment) is:

\[
\alpha \mathbb{E} (EVM_{PR}(m)|s) - c_I
\]

Because the influencer will post an independent review in the absence of an offer, such a deviation generates following payoff for him:

\[
\alpha \mathbb{E} (EVM_{IR}(m)|s) - c_I
\]

For an honest influencer, the expected EVM is same in both cases which means that he will accept seller’s advertisement offer even without any payment. However, for strategic influencer, \( \mathbb{E} (EVM_{IR}(m)|s) > \mathbb{E} (EVM_{PR}^{st}(m)|s) \) because of generating negative economic value for buyers when he posts a biased paid review. This means that the seller should make a payment of at least \( \tau(m) = \alpha [\mathbb{E} (EVM_{IR}(m)|s) - \mathbb{E} (EVM_{PR}^{st}(m)|s)] \) for the strategic influencer to accept the advertisement offer. As seller is not aware about the influencer type, he must offer the following payment to the influencer so that any influencer type accepts as
well as his profit is maximized:

$$
\tau(m) = \begin{cases} 
\alpha \left[ \mathbb{E} (EV M_{IR}(m)|s) - \mathbb{E} (EV M_{PR}(m)|s) \right] & \text{if } m = 1 \\
0 & \text{if } m = 0
\end{cases}
$$

Next, we need to check, with the above payment, which expected profit is higher for
the seller. Comparing the expected profit expressions for the paid review and independent
review cases, seller makes an advertisement offer if and only if:

$$
\mathbb{E} \pi_{PR} \geq \mathbb{E} \pi_{IR}
\implies Pr(m^\beta = 1) \left( \frac{(\bar{\eta}(m = 1) + v)^2}{4\bar{\eta}(m = 1)} \right) - (Pr(m = 1) + Pr(m^\beta = 1)) \tau(m)
\geq (1 - \mu)Pr(m = 0) \left( \frac{(\bar{\eta}(m = 0) + v)^2}{4\bar{\eta}(m = 0)} \right) 
\implies \frac{Pr(m^\beta = 1) \left( \frac{(\bar{\eta}(m = 1) + v)^2}{4\bar{\eta}(m = 1)} \right) - (1 - \mu)Pr(m = 0) \left( \frac{(\bar{\eta}(m = 0) + v)^2}{4\bar{\eta}(m = 0)} \right)}{(Pr(m = 1) + Pr(m^\beta = 1)) [\mathbb{E} (EV M_{IR}(m)|s) - \mathbb{E} (EV M_{PR}(m)|s)]} = \bar{\alpha}_1 \geq \alpha \quad (A.5)
$$

Plugging in the values, we get the following expression for $\bar{\alpha}_1$:

$$
\bar{\alpha}_1(\eta, \gamma, \mu) = \frac{(1 - \mu)}{4(\gamma \eta + (1 - \gamma)(1 - \eta))} \left[ \eta(1 - v^2) - v^2(1 - \eta) \left( \frac{\gamma^2 + (1 - \gamma)^2}{\gamma(1 - \gamma)} \right) + \frac{(1 - \eta)}{\eta} \right] 
$$

where

$$
f(\eta, \gamma, \mu) = \frac{v^2(1 - \eta)(2\gamma - 1)}{4\eta(1 - \gamma)} + \frac{\gamma \eta(1 - \eta)^3 ((1 - \gamma)^3 - \gamma^3)}{4(\gamma \eta + (1 - \gamma)(1 - \eta))^2 (\gamma(1 - \eta) + (1 - \gamma)\eta)^2}
+ \frac{v \eta(1 - \eta)(\gamma^2 + (1 - \gamma)^2)}{8\gamma (\gamma \eta + (1 - \gamma)(1 - \eta)) (\gamma(1 - \eta) + (1 - \gamma)\eta)}
$$
This implies that seller will make an offer to the influencer if $\alpha_1(\eta, \gamma, \mu) \geq \alpha$. Hence, a paid review is posted for all $\alpha > \alpha$ and an independent review is posted for all $\alpha_1(\eta, \gamma, \mu) \leq \alpha$.

(ii) If $aE(EM_{IR}(m)|s) \leq c_I$, the influencer does not post any review in the absence of an offer from the seller. So, his payoff is zero. Seller's expected profit is given by:

$$
E\pi = \begin{cases} 
(Pr(m = 1) + Pr(m^g = 1)) \left[ \frac{(\eta(m=1)+v)^2}{4\eta(m=1)} - \tau(m) \right] + \mu Pr(m = 0) \left( \frac{(\eta(m=0)+v)^2}{4\eta(m=0)} \right) & \text{if } PR \\
\left( \frac{(\eta+v)^2}{4\eta} \right) & \text{if } IR
\end{cases}
$$

For the influencer to accept the offer, expected payoff from posting a paid review must be higher than from no review case:

$$
\tau(m) + a [E(EM_{PR}(m)|s)] - c_I \geq 0
$$

$$
\Rightarrow \tau(m) \geq c_I - a [E(EM_{PR}(m)|s)] \quad (A.6)
$$

Therefore, for any payment satisfying (A.6), influencer accepts seller’s advertisement offer. However, strategic influencer generates a lower expected EVM under paid review than an honest influencer. This implies that the minimum payment for which strategic influencer accepts is higher than for honest influencer. As seller is not aware about the influencer type, he must offer the following payment to the influencer so that any influencer type accepts as
well as his profit is maximized:

$$
\tau(m) = \begin{cases} 
c_I - \alpha \left[ \mathbb{E}(EV\text{M}^*_{PR}(m)|s) \right] & \text{if } m = 1 \\
0 & \text{if } m = 0 
\end{cases} \quad (A.7)
$$

Given the above payment, seller makes an advertisement offer if expected profit from paid review is higher than from no review:

$$
\mathbb{E}\pi_{PR} \geq \mathbb{E}\pi_{NR} \\
\Rightarrow \left( Pr(m = 1) + Pr(m^\beta = 1) \right) \left( \frac{(\bar{\eta}(m = 1) + v)^2}{4\bar{\eta}(m = 1)} - \tau(m) \right) \\
+ \mu Pr(m = 0) \left( \frac{(\bar{\eta}(m = 0) + v)^2}{4\bar{\eta}(m = 0)} \right) \geq \frac{(\eta + v)^2}{4\eta}
$$

After substituting for the value of $\tau(m)$, this generates the following condition:

$$
\Rightarrow \frac{(\eta + v)^2}{4\bar{\eta}} - \mu Pr(m = 0) \frac{(\bar{\eta}(m=0)+v)^2}{4\bar{\eta}(m=0)} \\
\left( Pr(m = 1) + Pr(m^\beta = 1) \right) \frac{c_I - \frac{(\bar{\eta}(m=1)+v)^2}{4\bar{\eta}(m=1)}}{\mathbb{E}(EV\text{M}^*_{PR}(m)|s)} + \mathbb{E}(EV\text{M}^*_{PR}(m)|s) = \bar{\alpha}_2 \leq \alpha \quad (A.8)
$$

After plugging in the values, $\bar{\alpha}_2$ can be expressed as:

$$
\bar{\alpha}_2(\eta, \gamma, \mu) = \frac{(\eta + v)^2}{4\eta} - \frac{\mu[\eta(1-\gamma)(1+v)+v(1-\gamma)]^2}{4\eta(1-\gamma)} \\
\mathbb{E}(EV\text{M}^*_{PR}(m)|s) \left[ \gamma\eta + (1-\gamma)(1-\eta) + (1-\mu)(\gamma(1-\eta) + (1-\gamma)\eta) \right]
$$

$$
+ \frac{c_I - \frac{[(\gamma(1-\gamma)(1-\mu))\eta(1+v)+v(1-\gamma)+\gamma(1-\mu)](1-\eta)]^2}{4\eta[\gamma(1-\gamma)(1-\mu)](1-\eta) + (1-\gamma)(1-\gamma)(1-\eta)} \\
\mathbb{E}(EV\text{M}^*_{PR}(m)|s)
$$

where

$$
\mathbb{E}(EV\text{M}^*_{PR}(m)|s) = \frac{(1-\eta)\left(\gamma(1+v)\eta + (1-\gamma)v(1-\eta)\right)}{8\gamma\eta[\gamma\eta + (1-\gamma)(1-\eta) + (1-\mu)(\gamma(1-\eta) + (1-\gamma)\eta)]} +
$$
If $\bar{\alpha}_2(\eta, \gamma, \mu) \leq \alpha$, the seller finds it profitable to make an offer to the influencer, which is accepted, for the payment defined in (A.7). A paid review is posted and a fraction of buyers purchase the product. If $\bar{\alpha}_2(\eta, \gamma, \mu) \geq \alpha$, the seller does not make an offer as there is no payment for which influencer can be compensated and seller’s expected profit can be maximized at the same time. No review is posted and buyers make a purchase decision based on their prior beliefs.

**Proof of Proposition 2.2**

The proof is divided into the following steps:

**Step 1 – Buyers’ purchase decision**

If an independent review is posted, then buyers update their beliefs based on the message sent through the review. If $m = 1$,

$$Pr(H|m = 1) = \bar{\eta}(m = 1) = \frac{\gamma m}{\gamma \eta + (1 - \gamma)(1 - \eta)}$$

If $m = 0$,

$$Pr(H|m = 0) = \bar{\eta}(m = 0) = \frac{(1 - \gamma)\eta}{(1 - \gamma)\eta + \gamma(1 - \eta)}$$

If a paid review is posted, then the buyers’ updated beliefs are as follows:

$$Pr(H|m = 1) = \bar{\eta}(m = 1) = \frac{\gamma (1 - \gamma)(1 - \mu)\eta}{\gamma (1 + \gamma)(1 - \mu)\eta + [(1 - \gamma) + \gamma(1 - \mu)](1 - \eta)}$$
Depending on the message sent and the seller type, buyers purchase the product if:

\[ EU = \eta(m, \theta) \cdot v_H + (1 - \eta(m, \theta)) \cdot v - p \geq 0\]

\[ \Rightarrow \quad EU = \eta(m, \theta) \cdot v_H + (1 - \eta(m, \theta)) \cdot v \geq p\]

Maximizing the expected utility function generates a linear demand curve: \( \left( 1 + \frac{v - p}{\eta(m, \theta)} \right) \)
fraction of consumers wish to buy at a price \( p \).

If no review is posted, then buyers make a purchase decision according to their prior beliefs. The buyers purchase the product if:

\[ EU = \eta v_H + (1 - \eta) v - p \geq 0\]

\[ \Rightarrow \quad \eta v_H + (1 - \eta) v \geq p\]

Maximizing the expected utility function generates a linear demand curve: \( \left( 1 + \frac{v - p}{\eta} \right) \)
fraction of consumers wish to buy at a price \( p \).

**Step 2 – Seller’s price decision**

Given the demand curve, the seller faces the following profit function if a review is posted:

\[ \pi(p) = \left( 1 + \frac{v - p}{\eta(m, \theta)} \right) p \]

It is maximized at \( p = \frac{\eta(m, \theta) + v}{2} \) at which \( \frac{\eta(m, \theta) + v}{2\eta(m, \theta)} \) fraction of consumers buy. If no review is posted, the profit function is given as:

\[ \pi(p) = \left( 1 + \frac{v - p}{\eta} \right) p \]

It is maximized at \( p = \frac{\eta + v}{2} \) at which \( \frac{\eta + v}{2\eta} \) fraction of consumers buy.

**Step 3 – Calculation of EVM**

As the price charged by the seller varies by the review type, expected economic value of
the message generated is different for paid and independent review. The expected economic value of the message is defined as:

\[ \mathbb{E}(EVM_{IR}(m)|s) = Pr(m = 1) \cdot EVM(m = 1)|s + Pr(m = 0) \cdot EVM(m = 0)|s \]  

(A.10)

For an independent review, \( \frac{\eta(m=1, \theta=0)+v}{2} \) is the price charged by the seller and both influencer types report the signal truthfully. The event \( m = 1 \) occurs when the influencer receives the signal \( s = 1 \). The influencer’s signal can be correct with probability \( \gamma \), in which case it generates the following EVM:

\[
EV M(m = 1) = CS|_{m=1} - CS|_{m=0} \\
EV M(m = 1) = \left( \frac{\bar{\eta}(m = 1, \theta = 0) + v}{4} \right) \left( \frac{\bar{\eta}(m = 1, \theta = 0) + v}{2\bar{\eta}(m = 1, \theta = 0)} \right) \\
- \left( \frac{2\eta - \bar{\eta}(m = 1, \theta = 0) + v}{4} \right) \left( \frac{2\eta + v - \bar{\eta}(m = 1, \theta = 0)}{2\eta} \right) \\
\Rightarrow EV M(m = 1) = \frac{(\bar{\eta}(m = 1, \theta = 0) + v)^2}{8\eta(m = 1, \theta = 0)} - \frac{(2\eta - \bar{\eta}(m = 1, \theta = 0) + v)^2}{8\eta}
\]

Alternatively, the influencer’s signal can be incorrect with probability \( (1 - \gamma) \), in which case it generates the following EVM:

\[
EV M(m = 1) = CS|_{m=1} - CS|_{m=0} \\
EV M(m = 1) = \left( v - \frac{(\bar{\eta}(m = 1, \theta = 0) + v)}{2} \right) \left( \frac{\bar{\eta}(m = 1, \theta = 0) + v}{2\eta(m = 1, \theta = 0)} \right) \\
- \left( v - \frac{(\bar{\eta}(m = 1, \theta = 0) + v)}{2} \right) \left( 1 + \frac{(v - \bar{\eta}(m = 1, \theta = 0))}{2\eta} \right) \\
\Rightarrow EV M(m = 1) = \left( \frac{v - \bar{\eta}(m = 1, \theta = 0)}{2} \right) \left[ \frac{(\bar{\eta}(m = 1, \theta = 0) - \eta) (\bar{\eta}(m = 1, \theta = 0) - v)}{2\eta(m = 1, \theta = 0)\eta} \right] \\
\Rightarrow EV M(m = 1) = \frac{(\eta - \bar{\eta}(m = 1, \theta = 0)) (\bar{\eta}(m = 1, \theta = 0) - v)^2}{4\eta(m = 1, \theta = 0)\eta}
\]
The event $m = 0$ occurs when the influencer receives the signal $s = 0$. The influencer’s signal can be correct with probability $\gamma$, in which case it generates the following EVM:

$$EVM(m = 0) = CS|_{m=0} - CS|_{m=\phi}$$

$$EVM(m = 0) = \left( v - \frac{(\bar{\eta}(m = 0) + v)}{2} \right) \left( \frac{\bar{\eta}(m = 0) + v}{2\bar{\eta}(m = 0)} \right)$$

$$- \left( v - \frac{(\bar{\eta}(m = 0) + v)}{2} \right) \left( 1 + \frac{(v - \bar{\eta}(m = 0))}{2\eta} \right)$$

$$\Rightarrow EVM(m = 0) = \left( v - \frac{\bar{\eta}(m = 0)}{2} \right) \left[ \frac{(\bar{\eta}(m = 0) - \eta)(\bar{\eta}(m = 0) - v)}{2\bar{\eta}(m = 0)\eta} \right]$$

$$\Rightarrow EVM(m = 0) = \frac{(\eta - \bar{\eta}(m = 0))(\bar{\eta}(m = 0) - v)^2}{4\bar{\eta}(m = 0)\eta}$$

Alternatively, the influencer’s signal can be incorrect with probability $(1 - \gamma)$, in which case it generates the following EVM:

$$EVM(m = 0) = CS|_{m=0} - CS|_{m=\phi}$$

$$EVM(m = 0) = \left( \frac{\bar{\eta}(m = 0) + v}{4} \right) \left( \frac{\bar{\eta}(m = 0) + v}{2\bar{\eta}(m = 0)} \right)$$

$$- \left( \frac{2\eta - \bar{\eta}(m = 0) + v}{4} \right) \left( \frac{2\eta + v - \bar{\eta}(m = 0)}{2\eta} \right)$$

$$\Rightarrow EVM(m = 0) = \frac{(\bar{\eta}(m = 0) + v)^2}{8\bar{\eta}(m = 0)} - \frac{(2\eta - \bar{\eta}(m = 0) + v)^2}{8\eta}$$

Breaking down the expression for expected EVM in (A.10):
In case of a paid review, an honest influencer continues to post a truthful message. The only difference is the price charged by the seller. Therefore, the economic value generated from a positive message changes to:

\[
Pr(m = 1) \cdot EV M(m = 1)|s = (\gamma \eta) \left[ \frac{(\bar{\eta}(m = 1, \theta = 0) + v)^2}{8\bar{\eta}(m = 1, \theta = 0)} - \frac{(2\eta - \bar{\eta}(m = 1, \theta = 0) + v)^2}{8\eta} \right] \\
+ ((1 - \gamma)(1 - \eta)) \left[ \frac{(\eta - \bar{\eta}(m = 1, \theta = 0)) (\bar{\eta}(m = 1, \theta = 0) - v)^2}{4\bar{\eta}(m = 1, \theta = 0)\eta} \right] \tag{A.11}
\]

\[
Pr(m = 0) \cdot EV M(m = 0)|s = ((1 - \gamma) \eta) \left[ \frac{(\bar{\eta}(m = 0) + v)^2}{8\bar{\eta}(m = 0)} - \frac{(2\eta - \bar{\eta}(m = 0) + v)^2}{8\eta} \right] \\
+ (\gamma(1 - \eta)) \left[ \frac{(\eta - \bar{\eta}(m = 0)) (\bar{\eta}(m = 0) - v)^2}{4\bar{\eta}(m = 0)\eta} \right] \tag{A.12}
\]

The economic value from a negative message is unchanged, as shown in (A.12). Therefore, the expected EVM generated from a paid review by an honest influencer is given as:

\[
E \left( EV M^h_{PR}(m)|s \right) = Pr(m = 1) \cdot EV M(m = 1)|s + Pr(m = 0) \cdot EV M(m = 0)|s
\]

where \( Pr(m = 1) \cdot EV M(m = 1)|s \) is given by equation (A.13) and \( Pr(m = 0) \cdot EV M(m = 0)|s \) is given by equation (A.12).

A strategic influencer only posts a positive message under a paid review. Therefore, if the influencer receives a signal \( s = 1 \), then he is being truthful and the economic value generated is same as (A.13). However, if he receives a signal \( s = 0 \) and reports \( m = 1 \), then
it is a biased review. With probability $\gamma$, his signal is correct which means the product is of low-quality and buyers are worse-off than in no-review case.

$$EV M(m^\beta = 1) = CS|_{m^\beta = 1} - CS|_{m = \phi}$$

$$\Rightarrow EV M(m = 1) = \left(\frac{v - \bar{\eta}(m = 1, \theta = 1)}{2}\right) \left[\frac{\eta(m = 1, \theta = 1) - \bar{\eta}(m = 1, \theta = 1) - v}{2\bar{\eta}(m = 1, \theta = 1)\eta}\right]$$

$$\Rightarrow EV M(m = 1) = \frac{(\eta - \bar{\eta}(m = 1, \theta = 1))(\bar{\eta}(m = 1, \theta = 1) - v)^2}{4\bar{\eta}(m = 1, \theta = 1)\eta}$$

With probability $(1 - \gamma)$, the influencer’s signal is incorrect which means the product is of high-quality and buyers are better-off than in no-review case:

$$EV M(m^\beta = 1) = CS|_{m^\beta = 1} - CS|_{m = \phi}$$

$$\Rightarrow EV M(m^\beta = 1) = \frac{(\bar{\eta}(m = 1, \theta = 1) + v)^2}{8\bar{\eta}(m = 1, \theta = 1)} - \frac{(2\eta - \bar{\eta}(m = 1, \theta = 1) + v)^2}{8\eta}$$

Therefore, the expected EVM generated under a paid review is expressed as:

$$\mathbb{E}(EV M_{PR}^st(m)|_s) = Pr(m = 1) \cdot EV M(m = 1)|_s + Pr(m^\beta = 1) \cdot EV M(m^\beta = 1)|_s$$

where $Pr(m = 1) \cdot EV M(m = 1)|_s$ is given by equation (A.13) and

$$Pr(m^\beta = 1) \cdot EV M(m^\beta = 1)|_s$$

is given by:

$$= (\gamma(1 - \eta)) \left[\frac{(\eta - \bar{\eta}(m = 1, \theta = 1))(\bar{\eta}(m = 1, \theta = 1) - v)^2}{4\bar{\eta}(m = 1, \theta = 1)\eta}\right]$$

$$+ ((1 - \gamma)\eta) \left[\frac{(\bar{\eta}(m = 1, \theta = 1) + v)^2}{8\bar{\eta}(m = 1, \theta = 1)} - \frac{(2\eta - \bar{\eta}(m = 1, \theta = 1) + v)^2}{8\eta}\right]$$

(A.14)

**Step 4 – Influencer’s decision**

Influencer has to make two decisions (same as the case with no disclosure policy):
(1) If influencer is made an advertisement offer by the seller, should he accept or reject the offer? This depends on his expected payoff. If $\tau(m) + \alpha [\mathbb{E}(EVM_{PR}(m)|s)] - c_I \geq max \{0, \alpha [\mathbb{E}(EVM_{IR}(m)|s)] - c_I\}$, then accept seller’s offer. Otherwise, reject the offer.

(2) If influencer rejects the offer or is not made any offer, should he post an independent review or not? This again depends on his expected payoff. If $\alpha [\mathbb{E}(EVM_{IR}(m)|s)] - c_I \geq 0$, then post an independent review. Otherwise, do not post a review.

Step 5 – Seller’s advertisement decision

Seller has to decide whether or not to advertise through the influencer. If it advertises and influencer accepts, what should be $\tau(m)$ that maximizes his profits?

(i) If $\alpha [\mathbb{E}(EVM_{IR}(m)|s)] \geq c_I$, then influencer posts an independent review in case no offer is made by the seller or offer is rejected by the influencer. Expected profit of the seller is expressed as:

$$
\mathbb{E}\pi = \begin{cases} 
(Pr(m = 1) + Pr(m^\beta = 1)) \left[ \frac{(\delta(m=1,\theta=0)+\nu)^2}{4\delta(m=1,\theta=0)} - \tau(m) \right] + \mu Pr(m = 0) \left( \frac{(\delta(m=0)+\nu)^2}{4\delta(m=0)} \right) & \text{if PR} \\
Pr(m = 1) \left( \frac{(\delta(m=1,\theta=0)+\nu)^2}{4\delta(m=1,\theta=0)} \right) + Pr(m = 0) \left( \frac{(\delta(m=0)+\nu)^2}{4\delta(m=0)} \right) & \text{if IR}
\end{cases}
$$

The influencer’s payoff from posting a paid review (minus the payment) is:

$$
\alpha \mathbb{E}(EVM_{PR}(m)|s) - c_I
$$

Because the influencer will post an independent review in the absence of an offer, such a deviation generates following payoff for him:

$$
\alpha \mathbb{E}(EVM_{IR}(m)|s) - c_I
$$

For an honest influencer, the difference in the EVM stems from the difference in price charged by the seller, i.e., $\mathbb{E}(EVM_{IR}(m)|s) > \mathbb{E}(EVM_{PR}^h(m)|s)$. However, for the strategic influencer, $\mathbb{E}(EVM_{IR}(m)|s) > \mathbb{E}(EVM_{PR}^t(m)|s)$ because of (i) difference in price charged
by the seller and (ii) generating negative economic value for buyers when he posts a biased paid review. This means that $\mathbb{E} (EV M^s_{PR}(m)|s) > \mathbb{E} (EV M^b_{PR}(m)|s)$. Therefore, the seller should make a payment of at least

$$\tau (m) = \alpha \left[ \mathbb{E} (EV M_{IR}(m)|s) - \mathbb{E} (EV M^s_{PR}(m)|s) \right]$$

for the strategic influencer to accept the advertisement offer. As seller is not aware about the influencer type, he must offer the following payment to the influencer so that any influencer type accepts as well as his profit is maximized:

$$\tau (m) = \begin{cases} 
\alpha \left[ \mathbb{E} (EV M_{IR}(m)|s) - \mathbb{E} (EV M^s_{PR}(m)|s) \right] & \text{if } m = 1 \\
0 & \text{if } m = 0
\end{cases}$$

Next, we need to check, with the above payment, which expected profit is higher for the seller. Comparing the expected profit expressions for the paid review and independent review cases:

$$\mathbb{E} \pi_{PR} - \mathbb{E} \pi_{IR} \Rightarrow Pr(m = 1) \left( \frac{(\bar{\eta}(m = 1, \theta = 1) + v)^2}{4\bar{\eta}(m = 1, \theta = 1)} - \frac{(\bar{\eta}(m = 1, \theta = 0) + v)^2}{4\bar{\eta}(m = 1, \theta = 0)} \right)$$

$$+ Pr(m^\beta = 1) \left( \frac{(\bar{\eta}(m = 1, \theta = 1) + v)^2}{4\bar{\eta}(m = 1, \theta = 1)} \right)$$

$$- (Pr(m = 1) + Pr(m^\beta = 1)) \tau (m) - (1 - \mu) Pr(m = 0) \left( \frac{(\bar{\eta}(m = 0) + v)^2}{4\bar{\eta}(m = 0)} \right) \geq 0$$

As $Pr(m = 1) \left( \frac{(\bar{\eta}(m=1,\theta=1)+v)^2}{4\bar{\eta}(m=1,\theta=1)} - \frac{(\bar{\eta}(m=1,\theta=0)+v)^2}{4\bar{\eta}(m=1,\theta=0)} \right) + Pr(m^\beta = 1) \left( \frac{(\bar{\eta}(m=1,\theta=1)+v)^2}{4\bar{\eta}(m=1,\theta=1)} \right) < 0$

$$\Rightarrow \mathbb{E} \pi_{PR} - \mathbb{E} \pi_{IR} < 0$$

Therefore, an independent review is posted and a fraction of buyers purchase the product.
(ii) If $\alpha \mathbb{E}(EV_{MIR}(m)|s) \leq c_I$, the influencer does not post any review in the absence of an offer from the seller. Thus, his payoff is zero. Seller’s expected profit is given by:

$$\mathbb{E}\pi = \begin{cases} 
(Pr(m = 1) + Pr(m = 1) \left[ \frac{\hat{\eta}(m=1, \theta=1) + \psi}{4\eta(m=1, \theta=1)} - \tau(m) \right] + \mu Pr(m = 0) \left( \frac{\hat{\eta}(m=0) + \psi}{4\eta(m=0)} \right) & \text{if } PR \\
\left( \frac{(\eta + \psi)^2}{4\eta} \right) & \text{if } IR 
\end{cases}$$

For the influencer to accept the offer, expected payoff from posting a paid review must be higher than from no review case:

$$\tau(m) + \alpha \left[ \mathbb{E}(EV_{MPR}(m)|s) \right] - c_I \geq 0$$

$$\Rightarrow \tau(m) \geq c_I - \alpha \left[ \mathbb{E}(EV_{MPR}(m)|s) \right] \quad (A.15)$$

Therefore, for any payment satisfying (A.15), influencer accepts seller’s advertisement offer. However, strategic influencer generates a lower expected EVM under paid review than an honest influencer. This implies that the minimum payment for which strategic influencer accepts is higher than for honest influencer. As seller is not aware about the influencer type, he must offer the following payment to the influencer so that any influencer type accepts as well as his profit is maximized:

$$\tau(m) = \begin{cases} 
c_I - \alpha \left[ \mathbb{E}(EV_{MPR}^{st}(m)|s) \right] & \text{if } m = 1 \\
0 & \text{if } m = 0 
\end{cases} \quad (A.16)$$

Given the above payment, seller makes an advertisement offer if expected profit from paid review is higher than from no review:

$$\mathbb{E}\pi_{PR} \geq \mathbb{E}\pi_{NR}$$
\[
(Pr(m = 1) + Pr(m^\beta = 1)) \left( \frac{(\bar{\eta}(m = 1, \theta = 1) + v)^2}{4\bar{\eta}(m = 1, \theta = 1)} - \tau(m) \right) \\
+ \mu Pr(m = 0) \left( \frac{(\bar{\eta}(m = 0) + v)^2}{4\bar{\eta}(m = 0)} \right) \geq \frac{(\eta + v)^2}{4\eta}
\]

After substituting for the value of \(\tau(m)\), this generates the following condition:

\[
\Rightarrow \frac{(\eta + v)^2 - \mu Pr(m = 0)\frac{(\bar{\eta}(m = 0) + v)^2}{4\bar{\eta}(m = 0)}}{(Pr(m = 1) + Pr(m^\beta = 1))} \mathbb{E}(EV_{PR}^s(m)_{|s}) + \frac{c_I - \frac{(\bar{\eta}(m = 1, \theta = 1) + v)^2}{4\bar{\eta}(m = 1, \theta = 1)}}{\mathbb{E}(EV_{PR}^s(m)_{|s})} = \bar{\alpha}_2 \leq \alpha \quad (A.17)
\]

After plugging in the values, \(\bar{\alpha}_2\) is expressed as:

\[
\bar{\alpha}_2 (\eta, \gamma, \mu) = \frac{(\eta + v)^2}{4\eta} - \frac{\mu[\eta(1-\gamma)(1+v)+v\gamma(1-\eta)]^2}{4\eta(1-\gamma)} \mathbb{E}(EV_{PR}^s(m)_{|s}) [\gamma\eta + (1-\gamma)(1-\eta) + (1-\mu) (\gamma(1-\eta) + (1-\gamma)\eta)]
\]

\[
+ \frac{c_I - \frac{[(\gamma+(1-\gamma)(1-\mu))\eta(1+v)+v((1-\gamma)+\gamma(1-\mu))(1-\eta)]^2}{4\gamma\eta((1-\gamma)+(1-\mu))(1-\gamma)\eta+(1-\gamma)+\gamma(1-\mu)(1-\eta))}}{\mathbb{E}(EV_{PR}^s(m)_{|s})}
\]

where \(\mathbb{E}(EV_{PR}^s(m)_{|s})\) is given by (A.9).

If \(\bar{\alpha}_2 (\eta, \gamma, \mu) \leq \alpha\), the seller finds it profitable to make an offer to the influencer, which is accepted, for the payment defined in (A.16). A paid review is posted and a fraction of buyers purchase the product. If \(\bar{\alpha}_2 (\eta, \gamma, \mu) \geq \alpha\), the seller does not make an offer as there is no payment for which influencer can be compensated and seller’s expected profit can be maximized at the same time. No review is posted and buyers make a purchase decision based on their prior beliefs.

**Proof of Proposition 2.4**

The proof follows the same steps as Proposition 2.1, except that prior beliefs are defined by (1) in the main text of Chapter 2.
Proof of Proposition 2.5

The proof follows the same steps as Proposition 2.2, except that prior beliefs are defined by (1) in the main text of Chapter 2.

Proof of Proposition 2.7

We prove the case where bad type makes an advertisement offer to the influencer and the good type does not. This separating equilibrium can be sustained if no type has an incentive to imitate the other. First, we check that the bad type has no incentive to mimic the good type.

The expected profit earned by the bad type from making an offer to the influencer is:

\[
\mathbb{E}_i^{PR} = \left( Pr(m_b = 1) + Pr(m_b^θ = 1) \right) \left( \frac{(\bar{\eta}(m_b = 1, \theta = 1) + v)^2}{4\bar{\eta}(m_b = 1, \theta = 1)} - \tau(m_b) \right) + \mu Pr(m_b = 0) \left( \frac{(\bar{\eta}(m_b = 0) + v)^2}{4\bar{\eta}(m_b = 0)} \right)
\]

The expected profit from deviating and imitating the good type is:

\[
\mathbb{E}_i^{IR} = Pr(m_b = 1) \frac{(\bar{\eta}(m_g = 1, \theta = 0) + v)^2}{4\bar{\eta}(m_g = 1, \theta = 0)} + Pr(m_b = 0) \frac{(\bar{\eta}(m_g = 0) + v)^2}{4\bar{\eta}(m_g = 0)}
\]

Deviation is not profitable iff:

\[
\mathbb{E}_i^{PR} \geq \mathbb{E}_i^{IR}
\]

\[
\left( Pr(m_b = 1) + Pr(m_b^θ = 1) \right) \left( \frac{(\bar{\eta}(m_b = 1, \theta = 1) + v)^2}{4\bar{\eta}(m_b = 1, \theta = 1)} - \tau(m_b) \right) + \mu Pr(m_b = 0) \left( \frac{(\bar{\eta}(m_b = 0) + v)^2}{4\bar{\eta}(m_b = 0)} \right) \geq Pr(m_b = 1) \frac{(\bar{\eta}(m_g = 1, \theta = 0) + v)^2}{4\bar{\eta}(m_g = 1, \theta = 0)} + Pr(m_b = 0) \frac{(\bar{\eta}(m_g = 0) + v)^2}{4\bar{\eta}(m_g = 0)}
\]
Simplifying the equation:

\[
Pr(m_b = 1) \left( \frac{(\bar{\eta}(m_g = 1, \theta = 0) + v)^2}{4\bar{\eta}(m_g = 1, \theta = 0)} \right) + Pr(m_b = 0) \left( \frac{(\bar{\eta}(m_g = 0) + v)^2}{4\bar{\eta}(m_g = 0)} - \mu \frac{(\bar{\eta}(m_g = 0) + v)^2}{4\bar{\eta}(m_g = 0)} \right)
\]

\[
\left( Pr(m_b = 1) + Pr(m_b^\beta = 1) \right) \mathbb{E} \left( EV M_{PR}^{b, st}(m) \right) + \frac{c_I}{\mathbb{E} \left( EV M_{PR}^{b, st}(m) \right)} = \bar{\alpha}_1 \leq \alpha
\]

After plugging in all the values, we get the following expression for \( \bar{\alpha}_1 \):

\[
\bar{\alpha}_1 (\eta, \gamma, \mu) = \frac{c_I}{\mathbb{E} \left( EV M_{PR}^{b, st}(m) \right)} - \frac{[(\gamma + (1-\gamma)(1-\mu))\eta_b(1+v)((1-\gamma)+\gamma(1-\mu))(1-\eta_b))^2}{4\eta_b((\gamma + (1-\gamma)(1-\mu))\eta_b+(1-\gamma)(1-\mu))(1-\eta_b))} \]

\[
+ \frac{((1-\gamma) + \gamma(1-\mu))(1-\eta_b) \left( \frac{[\eta_b(1-\gamma)(1+v)+v\gamma(1-\eta_b)]^2}{4\eta_b(1-\gamma)} - \mu[\eta_b(1-\gamma)(1+v)+v\gamma(1-\eta_b)]^2}{4\eta_b(1-\gamma)} \right)}{\mathbb{E} \left( EV M_{PR}^{b, st}(m) \right) \left[ \gamma\eta_b + (1-\gamma)(1-\eta_b) + (1-\mu) (\gamma(1-\eta_b) + (1-\gamma)\eta_b) \right]}
\]

where \( \mathbb{E} \left( EV M_{PR}^{b, st}(m) \right) \) is given by (A.9).

Note that \( \bar{\alpha}_1 (\eta, \gamma, \mu) \) lies close to \( \bar{\alpha}_b \). As long as \( \alpha \) is greater than \( \bar{\alpha}_1 (\eta, \gamma, \mu) \), bad type has no incentive to imitate good type.

Next, we need to check that the good type has no incentive to mimic the bad type. The expected profit earned by the good type from not advertising his product is:

\[
\mathbb{E}\pi_{IR} = Pr(m_g = 1) \left( \frac{(\bar{\eta}(m_g = 1, \theta = 0) + v)^2}{4\bar{\eta}(m_g = 1, \theta = 0)} \right) + Pr(m_g = 0) \left( \frac{(\bar{\eta}(m_g = 0) + v)^2}{4\bar{\eta}(m_g = 0)} \right)
\]

The expected profit from deviating to making an advertisement offer and imitating the bad type is:
\[ \mathbb{E}\pi_{PR} = \mu Pr(m_g = 0) \left( \frac{\bar{\eta}(m_b = 0) + v}{4\bar{\eta}(m_b = 0)} \right) + \]

\[ (Pr(m_g = 1) + Pr(m_g^\beta = 1)) \left( \frac{\bar{\eta}(m_b = 1, \theta = 1) + v}{4\bar{\eta}(m_b = 1, \theta = 1)} - \tau(m_b) \right) \]

Deviation is not profitable iff:

\[ \mathbb{E}\pi_{IR} \geq \mathbb{E}\pi_{PR} \]

\[ Pr(m_g = 1) \left( \frac{\bar{\eta}(m_g = 1, \theta = 0) + v}{4\bar{\eta}(m_g = 1, \theta = 0)} \right) + Pr(m_g = 0) \left( \frac{\bar{\eta}(m_g = 0) + v}{4\bar{\eta}(m_g = 0)} \right) \geq \]

\[ (Pr(m_g = 1) + Pr(m_g^\beta = 1)) \left( \frac{\bar{\eta}(m_b = 1, \theta = 1) + v}{4\bar{\eta}(m_b = 1, \theta = 1)} - \tau(m_b) \right) + \mu Pr(m_g = 0) \left( \frac{\bar{\eta}(m_b = 0) + v}{4\bar{\eta}(m_b = 0)} \right) \]

Simplifying the equation:

\[ \frac{Pr(m_g = 1) \left( \bar{\eta}(m_g = 1, \theta = 0) + v \right) + Pr(m_g = 0) \left( \bar{\eta}(m_g = 0) + v - \mu \bar{\eta}(m_g = 0) \right)}{(Pr(m_g = 1) + Pr(m_g^\beta = 1)) \mathbb{E} \left( EV M_{PR}^{b,st}(m) \right)_{s}} \]

\[ + \frac{\bar{c}_I \bar{\eta}(m_b = 1, \theta = 1) + v}{\mathbb{E} \left( EV M_{PR}^{b,st}(m) \right)_{s}} = \bar{\alpha}_2 \geq \alpha \]

After plugging in all the values, we get the following expression for \( \bar{\alpha}_2 \):
\[
\overline{\alpha}_2(\eta, \gamma, \mu) = \frac{c_I - \frac{[(\gamma + (1-\gamma)(1-\mu))\eta_b(1+v)+v(1-\gamma)+(1-\gamma)(1-\eta_b)]^2} {4\eta_b((\gamma + (1-\gamma)(1-\mu))\eta_b + (1-\gamma)(1-\eta_b))}} + \\
\mathbb{E}\left(\text{EV}_{M_{PR}^{b,st}}(m|s)\right)
\]

\[
\frac{((\gamma + (1-\gamma)(1-\mu))\eta_g)[(\gamma + (1-\gamma)(1-\mu))\eta_b(1+v)+v(1-\gamma)+(1-\gamma)(1-\eta_b)]^2} {4\eta_b((\gamma + (1-\gamma)(1-\mu))\eta_b + (1-\gamma)(1-\eta_b))} \\
\] 

\[
\mathbb{E}\left(\text{EV}_{M_{PR}^{b,st}}(m|s)\right)[\gamma \eta_g + (1-\gamma)(1-\eta_g) + (1-\mu)(\gamma(1-\eta_g) + (1-\gamma)\eta_g)]
\]

\[
+ \frac{((1-\gamma) + \gamma(1-\mu))(1-\eta_g)\left[\frac{[\eta_b(1-\gamma)(1+v)+v\gamma(1-\eta_b)]^2}{4\eta_b(1-\gamma)} - \frac{\mu[\eta_b(1-\gamma)(1+v)+v\gamma(1-\eta_b)]^2}{4\eta_b(1-\gamma)}\right]} {\mathbb{E}\left(\text{EV}_{M_{PR}^{b,st}}(m|s)\right)[\gamma \eta_g + (1-\gamma)(1-\eta_g) + (1-\mu)(\gamma(1-\eta_g) + (1-\gamma)\eta_g)]}
\]

where \(\mathbb{E}\left(\text{EV}_{M_{PR}^{b,st}}(m|s)\right)\) is given by (A.9).

Note that \(\overline{\alpha}_2(\eta, \gamma, \mu)\) lies close to \(\overline{\alpha}_g\). As long as \(\alpha\) is greater than \(\overline{\alpha}_2(\eta, \gamma, \mu)\), good type has no incentive to mimic the bad type. Hence, incentive compatibility constraints are satisfied.
APPENDIX TO CHAPTER 3

Proof of Proposition 3.1

Some parts of the proof have been outlined in the main text of Section 3.3. It remains to show the following:

(i) characterizing the optimal pricing strategy of the low-quality type under separating equilibrium.

(ii) characterizing the optimal pricing strategy of the high-quality type under separating equilibrium satisfying intuitive criterion.

(iii) mathematically showing why high-quality type does not deviate from such an equilibrium.

Starting with (i), the low-quality type’s pricing strategy has been partly mentioned in the main text. When \( r > v \), the pricing strategy of the low-quality type is to charge \( p^L = \frac{(r+v)}{2} \) when there is a large proportion of naive consumers in the market. For higher values of \( z \) and \( s \), it switches to charging \( v \). To determine the \((s, z)\) locus for which it charges \((r + v)\), solve

\[
\pi(L, r, 0, v) \leq \pi(L, r, 0, \frac{r + v}{2})
\]

that is, where the profit from charging \( \frac{r+v}{2} \) is higher than the profit from charging \( v \).

This gives the following condition:

\[
(1 - s)(1 - z) \leq \frac{(4rv)}{(r + v)^2} \quad (B.1)
\]
The monopolist switches to charging $v$ when:

$$(1 - s)(1 - z) \geq \frac{(4rv)}{(r + v)^2} \quad (B.2)$$

**Lemma 3.1.** The optimal pricing strategy of the low-quality type under the separating equilibrium is as follows:

(a) For $r > v$, the low-quality type charges $\frac{r + v}{2}$ if (B.1) holds and charges $v$ if (B.2) holds.

(b) For $r < v$, the low-quality type always charges $v$.

Given the above pricing strategy of the low-quality type, next step is to solve for (ii). As shown in the main text, the high-quality type prefers to charge $P^M(s, z)$, which approaches $P^F$ as $z \to 1$ when there is a large proportion of naive consumers in the market. Given such pricing, for some values of $s$, the low-quality type has an incentive to mimic the high-quality prices to fool the naive and sophisticated consumers. To ensure that the low-quality type does not mimic the high-quality prices, the high-quality type signals its quality to separate itself out from the low-quality type.

When $p^L = \frac{(r + v)}{2}$ (valid under (B.1)) and high-quality type is charging some price $p$, it must be that $\pi(L, 0, \frac{r + v}{2}) \geq \pi(L, 1, p)$ so that the low-quality type does not mimic the high-quality type. To find the range of prices where the low-quality type has an incentive to mimic, we need to solve for $\{P| \pi(L, 0, \frac{r + v}{2}) = \pi(L, 1, p)\}$. This gives us the following roots:

$$\overline{P}(s) = \frac{v}{2} + \frac{r}{2(1 - s + sr)} + \frac{\sqrt{(r + v(1 - s + sr))^2 - (1 - s + sr)(1 - s)(r + v)^2}}{2(1 - s + sr)} \quad (B.3)$$

$$\underline{P}(s) = \frac{v}{2} + \frac{r}{2(1 - s + sr)} - \frac{\sqrt{(r + v(1 - s + sr))^2 - (1 - s + sr)(1 - s)(r + v)^2}}{2(1 - s + sr)} \quad (B.4)$$

For $p = \frac{\underline{P}(s)}{\overline{P}(s)}$, the low-quality type is indifferent about mimicking the high-quality type. For any $p \in (\underline{P}(s), \overline{P}(s))$, the low-quality type earns a higher profit from mimicking the
high-quality type because the gains from mimicking and serving only naive and sophisticated consumers is higher than the loss of business from the informed consumers. To avoid such a situation, a high-quality type should charge either \( p^H \geq \overline{P}(s) \) or \( p^H \leq P(s) \) in a separating equilibrium for the region \((1 - s)(1 - z) \leq \frac{4ru}{(r + v)^2}\). A price below the lower bound reduces the monopolist’s profits by a huge amount. So, for this section of \((s, z)\) locus, the optimal best response of the high-quality type which survives the refinement under the intuitive criterion is to charge \( p^H = \max \overline{P}(s), P^M(s, z) \). That is, high-quality type should charge just high enough to prevent low-quality type from mimicking its price. By doing so, the high-quality type chooses the minimum possible price from the range that separates out the low-quality type and also signals its quality. Therefore, it chooses to signal its quality \([P^M(s, z) < \overline{P}(s, z)]\) for a small section of the \((s, z)\) locus satisfied by \((1 - s) < \left( \frac{(r + v(1 - s + sr))}{(1 - s + sr)(r + v)} \right)^2 \).

For the rest of this section, satisfied by \((1 - s)(1 - z) \leq \frac{4ru}{(r + v)^2}\) and \((1 - s) > \left( \frac{(r + v(1 - s + sr))}{(1 - s + sr)(r + v)} \right)^2 \), it chooses to charge \( p^H = P^M(s, z) \).

**Lemma 3.2.** If (B.1) holds, the optimal pricing strategy of the high-quality type under the separating equilibrium satisfying the intuitive criterion is given as:

(i) \( p^H = \overline{P}(s) \), if \((1 - s) < \left( \frac{(r + v(1 - s + sr))}{(1 - s + sr)(r + v)} \right)^2 \)

(ii) \( p^H = P^M(s, z) \), if \((1 - s) > \left( \frac{(r + v(1 - s + sr))}{(1 - s + sr)(r + v)} \right)^2 \)

When \( p^L = v \) and the high-quality type is charging some price \( p \), then we require \( \pi(L, 0, v) \geq \pi(L, 1, p) \) to ensure the low-quality type does not mimic the high-quality prices. Following the same procedure as the previous case, to find the range of prices where the low-quality type has an incentive to mimic, we need to solve for \( \{P|\pi(L, 0, v) = \pi(L, 1, p)\} \). This gives us the following roots:

\[
\overline{P}(s, z) = \frac{v}{2} + \frac{r}{2(1 - s + sr)} + \frac{\sqrt{(r + v(1 - s + sr))^2 - 4rv(1 - s + sr)^2}}{2(1 - s + sr)} (B.5)
\]
\[
P(s, z) = \frac{v}{2} + \frac{r}{2(1 - s + sr)} - \frac{\sqrt{(r + v(1 - s + sr))^2 - 4rv(1-s+sr)}}{2(1 - s + sr)} \quad (B.6)
\]

Notice that \(\overline{P}(s, z)\) becomes equal to \(P(s, z)\) at \(z = \left(\frac{(r-v(1-s+sr))}{(r+v(1-s+sr))}\right)^2\). So, mimicry is not possible for any \(z > \left(\frac{(r-v(1-s+sr))}{(r+v(1-s+sr))}\right)^2\). Hence, for any \(p \in (P(s, z), \overline{P}(s, z))\) and \(z < \left(\frac{(r-v(1-s+sr))}{(r+v(1-s+sr))}\right)^2\) the low-quality type prefers to mimic the high-quality type because even though now it serves only sophisticated and naive consumers, it still earns a higher profit – losing out on the demand from the informed consumers is more than compensated by the higher price. Therefore, for the region satisfied by \((1 - s)(1 - z) \geq \frac{(4rv)}{(r+v)^2}\) and \(z < \left(\frac{(r-v(1-s+sr))}{(r+v(1-s+sr))}\right)^2\), a high-quality type should charge either \(p^H \geq \overline{P}(s)\) or \(p^H \leq P(s)\) in a separating equilibrium. For the region satisfied by \((1 - s)(1 - z) \geq \frac{(4rv)}{(r+v)^2}\) and \(z > \left(\frac{(r-v(1-s+sr))}{(r+v(1-s+sr))}\right)^2\), the high-quality type should simply charge \(P^M(s, z)\) as there is no risk of mimicry. So, for this entire section of \((s, z)\) locus – satisfied by \((1 - s)(1 - z) \geq \frac{(4rv)}{(r+v)^2}\), the necessary condition for a separating equilibrium satisfying intuitive criterion is to charge \(p^H = max \overline{P}(s), P^M(s, z)\).

This condition holds for both \(r > v\) and \(r < v\) cases.

**Lemma 3.3.** If (B.2) holds or \(r < v\), the optimal pricing strategy of the high-quality type under the separating equilibrium satisfying the intuitive criterion is given as:

(i) \(p^H = \overline{P}(s, z)\), if \(z < \left(\frac{(r-v(1-s+sr))}{(r+v(1-s+sr))}\right)^2\)

(ii) \(pH = P^M(s, z)\), if \(z > \left(\frac{(r-v(1-s+sr))}{(r+v(1-s+sr))}\right)^2\)

One last step is to prove the existence of the separating equilibrium. For this, we need to ensure that the high-quality type does not deviate from \(\overline{P}(s)\) or \(\overline{P}(s, z)\) to \(P^M(s, z)\). Such a deviation is possible when the high-quality type is charging a very high signaling price in the presence of a high proportion of sophisticated consumers in the market. In such a situation, the monopolist is earning very low profits and may have an incentive to deviate to the profit-maximizing price, \(P^M(s, z)\). To find the “no-defect” root, we require
that \( \pi(H, 1, \overline{P}(s)) \geq \pi(H, 0, P^M(s, z)) \). Therefore, set \( \pi(H, 1, P) = \pi(H, 0, P^M(s, z)) \) which yields the following root:

\[
\bar{P}(s, z) = \frac{b + \sqrt{b^2 - 4a}}{2} \quad (B.7)
\]

where \( b = 1 + (v + c_H) \left( z + \frac{(1-z)(1-s+sr)}{r} \right) \) and

\[
a = \left( P^M(s, z) - c_H \right) \left[ s(1 - z) - (v - P^M(s, z)) \left( z + \frac{(1-z)(1-s)}{r} \right) \right] - P^M(s, z)
\]

The high-quality type does not deviate if and only if

\[
p^H \leq \bar{P}(s, z) \quad (B.8)
\]

Similarly, we require that \( \pi(H, 1, \overline{P}(s, z)) \geq \pi(H, 0, P^M(s, z)) \). Therefore, set \( \pi(H, 1, P) = \pi(H, 0, P^M(s, z)) \) which yields the same root as above. Both the signaling prices \( \overline{P}(s) \) and \( \overline{P}(s, z) \) lie below \( \bar{P}(s, z) \) throughout the \((s, z)\) locus. This implies that the high-quality type has no incentive to deviate from the above separating equilibrium.

**Proof of Proposition 3.4**

The proof requires to show the following:

(i) the range of prices for which the low-quality type is willing to pool with high-quality type under the case of high proportion of naive consumers in the market.

(ii) the range of prices for which the low-quality type is willing to pool with high-quality type under the case of high proportion of sophisticated consumers in the market.

When \( p^L = \frac{r + u}{2} \), the low-quality type is willing to pool with the high-quality type as long as \( \pi(L, r, p^H) \geq \pi(L, r, 0, \frac{r + u}{2}) \). That is, it earns a higher profits under the pooling case. We need a set of prices \( \{ p | \pi(L, r, p) = \pi(L, r, 0, \frac{r + u}{2}) \} \) for which the low-quality type is just willing to pool. This gives us the following roots:
\[ \overline{P}_r(s) = \frac{r + v}{2} \left( 1 + \sqrt{s} \right) \quad (B.9) \]

\[ \underline{P}_r(s) = \frac{r + v}{2} \left( 1 - \sqrt{s} \right) \quad (B.10) \]

The low-quality type is willing to pool for any \( p \in [\underline{P}_r(s), \overline{P}_r(s)] \). For \( s = 0 \), \( \underline{P}_r(s) = \overline{P}_r(s) = \frac{r + v}{2} \) so that no pooling exists. For \( s = \left( \frac{r - v}{r + v} \right)^2 \), \( \overline{P}_r(s) = r \) and \( \underline{P}_r(s) = v \). Therefore, this is only valid for \( r > v \) case.

When \( p^L = v \), then the low-quality type is willing to pool with the high-quality type as long as \( \pi(L, r, p^H) \geq \pi(L, r, 0, p^L) \). We need a set of prices \( p | \pi(L, r, p) = \pi(L, r, 0, v) \) for which the low-quality type is just willing to pool. This gives us the following roots:

\[ \overline{P}_r(z) = \frac{r + v}{2} + \frac{\sqrt{(1 - z)^2(r + v)^2 - 4rv(1 - z)}}{2(1 - z)} \quad (B.11) \]

\[ \underline{P}_r(z) = \frac{r + v}{2} - \frac{\sqrt{(1 - z)^2(r + v)^2 - 4rv(1 - z)}}{2(1 - z)} \quad (B.12) \]

The low-quality type is willing to pool for any \( p \in [\underline{P}_r(z), \overline{P}_r(z)] \). For \( z = 0 \), \( \overline{P}_r(z) = r \) and \( \underline{P}_r(z) = v \). Therefore, this is again valid only for \( r > v \) case. Combining both the cases, the pooling equilibrium exists for all \( 0 \leq z \leq \left( \frac{r - v}{r + v} \right)^2 \) and \( 0 < s \leq 1 \).
APPENDIX TO CHAPTER 4

Proof of Proposition 4.1

There are two parts of the proof depending on the choice of advertising message:

(i) If $a = 0$, the monopolist faces an average demand: $Q_D(P) = 1 + (\frac{v-P}{r})$. Given this demand and marginal cost $c_H$, $P^* = \frac{r+v-c_H}{2}$ is the profit-maximizing price for the high type monopolist which generates $\pi_H^* = (P^* - c_H)(1 + \frac{v-P^*}{r}) = \frac{(r+v-c_H)^2}{4r}$. Given this demand and marginal cost, $P^\nabla = r + v + c_H$ is the profit-maximizing price for the high type monopolist which generates $\pi_H^\nabla = (P^\nabla - c_H)(1 + \frac{v-P^\nabla}{r}) = \frac{(r+v-c_H)^2}{4r}$. Consider any pooling equilibrium where $P^H = P^L = P'$, $a = 0$ and buyers believe the monopolist to be high type with probability $r$. The high- and low type earn profits $\pi_H = (P' - c_H)(1 + \frac{v-P'}{r}) \leq \frac{(r+v-c_H)^2}{4r}$ and $\pi_L = (P')(1 + \frac{v-P'}{r})$. Suppose the sophisticated consumers believe that any firm deviating from such a strategy is a high type monopolist. If high type deviates to a higher price $P$ along with $a = 1$, its profit changes to $\pi_H(P) = (P - c_H)(1 + v - P)$ as both the sophisticated and naive consumers update their beliefs to $r = 1$. Construct a price $\hat{P}$ such that $\pi_H(\hat{P}) = (\hat{P} - c_H)(1 + v - \hat{P}) = \frac{(r+v-c_H)^2}{4r}$ which yields $\hat{P} = \frac{1+v+c_H}{2} + \sqrt{(1+v-c_H)^2 - \frac{(r+v-c_H)^2}{4r}}$. At $\hat{P}$, the low type earns $\pi_L(\hat{P}) = (\hat{P})(1 + v - \hat{P}) < (P')(1 + \frac{v-P'}{r})$, by the definition of pooling equilibrium. Therefore, any price $\hat{P} + \varepsilon$ generates $\pi_H(\hat{P} + \varepsilon) > \frac{(r+v-c_H)^2}{4r}$ and $\pi_L(\hat{P} + \varepsilon) < (P')(1 + \frac{v-P'}{r})$. Hence, the high type can profitably deviate to a higher price which low type would never mimic and the intuitive criterion fails.

(ii) If $a = 1$, the monopolist faces an average demand only from the sophisticated consumers: $Q_D(P) = 1 + (\frac{v-P}{r})(s + r(1-s))$. Given this demand and marginal cost $c_H$, $P^* = \frac{v+c_H}{2} + \frac{r}{2(s+r(1-s))}$ is the profit-maximizing price for the high type monopolist which generates $\pi_H^* = (P^* - c_H)(1 + \frac{v-P^*}{r}(s + r(1-s))) = \frac{(v-c_H)^2(s+r(1-s)) + \frac{r}{4(s+r(1-s))}}{4(s+r(1-s))}$. Consider any pooling equilibrium where $P^H = P^L = P'$, $a = 1$ and sophisticated buyers believe the monopolist to be high type with probability $r$. The high and low types earn profits:
Suppose the sophisticated consumers believe that any firm deviating from such a strategy is a high type monopolist. If high type deviates to a higher price \( P \), it profits change to \( \pi_H = (P - c_H)(1 + v - P) \) as the sophisticated consumers update their beliefs to \( r = 1 \). Construct a price \( \hat{P} \) such that \( \pi_H(\hat{P}) = (P - c_H)(1 + v - \hat{P}) = \pi_H^* \) which yields \( \hat{P} = \frac{1 + v + c_H}{2} + \sqrt{(\frac{1 + v - c_H}{2})^2 - \pi_H^*} \). At \( \hat{P} \), the low type earns \( \pi_L(\hat{P}) = (\hat{P})(1 + v - \hat{P}) < (P')(1 + \frac{v - P'}{r}(s + r(1 - s))) \), by the definition of pooling equilibrium. Therefore, any price \( \hat{P} + \varepsilon \) generates \( \pi_H(\hat{P} + \varepsilon) > \frac{(r + v - c_H)^2}{4r} \) and \( \pi_L(\hat{P} + \varepsilon) < (P')(1 + \frac{v - P'}{r}) \). Hence, the high type can profitably deviate to a higher price which low type would never mimic and the intuitive criterion fails.

**Proof of Propositions 4.2 and 4.3**

There are two parts of the proof: (i) when \( r > v \) and (ii) when \( r < v \)

(i) A low type firm can serve all the consumers in the market by simply charging \( v \). But when the naive consumers dominate the market, it has an opportunity to charge a higher-than-valuation price and lie about its quality given the penalty is low. Ideal price for the low type to charge in such a scenario would be the one that maximizes its profit:

\[
\max \pi_L(P_L) = P_L(1 + v - P_L)(1 - s) - d
\]

\[
\Rightarrow P_L = \frac{1 + v}{2}, a = 1
\]

However, if the penalty is high, the low type can still charge a high price even without lying about its quality because naive buyers do not understand price signals and therefore, maximize their expected utility. Ideal price in such a case is:

\[
\max \pi_L(P_L) = P_L \left( 1 + \frac{v - P_L}{r} \right)(1 - s)
\]
\[
\Rightarrow P_L = \frac{r + v}{2}, a = 0
\]

As \( r > v \), the above price is greater than \( v \), generating a higher profit to low type even without lying about its quality. Therefore, when the naive consumers are dominant in the market, the low type chooses to lie about its quality if and only if:

\[
\pi_L(P_L = \frac{1 + v}{2}, a = 1) \geq \pi_L(P_L = \frac{r + v}{2}, a = 0)
\]

\[
\Rightarrow d \leq \frac{(1 - s)(1 - r)(r - v^2)}{4r} = \bar{d}_1(s)
\]

As long as \( d \leq \bar{d}_1(s) \), low type charges \( P_L = \frac{1 + v}{2} \) and indulges in deception. For \( d \geq \bar{d}_1(s) \), low type still charges a higher price, \( P_L = \frac{r + v}{2} \), but does not lie about its quality due to an increased penalty. In both the scenarios, it only serves the naive consumers. However, when the proportion of sophisticated consumers increases in the market, the low type deviates to charge \( v \) because loss of demand from sophisticated consumers is more than the gain from naive consumers. If \( d \leq \bar{d}_1(s) \), such deviation occurs when:

\[
\pi_L(P_L = \frac{1 + v}{2}, a = 1) \leq \pi_L(P_L = v, a = 0)
\]

\[
\Rightarrow s \geq \frac{(1 - v)^2 - 4d}{(1 + v)^2}
\]

If \( d \geq \bar{d}_1(s) \):

\[
\pi_L(P_L = \frac{r + v}{2}, a = 1) \leq \pi_L(P_L = v, a = 0)
\]

\[
\Rightarrow s \geq \left(\frac{r - v}{r + v}\right)^2
\]
Lemma 4.1. If \( r > v \), the pricing and advertising strategy of the low type monopolist is summarized as follows:

1. For \( d \leq \bar{d}_1(s) \), \( P^L = \frac{1+v}{2}, a = 1 \) if \( s \leq \frac{(1-v)^2-4d}{(1+v)^2} \) and \( P^L = v, a = 0 \) if \( s \geq \frac{(1-v)^2-4d}{(1+v)^2} \).
2. For \( d \geq \bar{d}_1(s) \), \( P^L = \frac{r+v}{2}, a = 0 \) if \( s \leq \left(\frac{r-v}{r+v}\right)^2 \) and \( P^L = v, a = 0 \) if \( s \geq \left(\frac{r-v}{r+v}\right)^2 \).

The high type monopolist’s ideal price is \( P^H = \frac{1+v+c_H}{2} \), the full information price. This price along with the message \( a = 1 \) is the best response of the high type when naive consumers are dominant in the market (\( s \) is very low) because advertising message is enough for the naive consumers to believe that the product is of high-quality. However, when the proportion of sophisticated consumers increases in the market, the fear of mimicry from the low type increases as well. Therefore, with increasing \( s \), the high type should deviate to a higher signaling price to separate itself from the low type firm. It can do so by choosing a price, \( P \) that is not profitable for the low type to mimic. When \( d \leq \bar{d}_1(s) \) and \( 0 \leq s \leq \frac{(1-v)^2-4d}{(1+v)^2} \),

\[
\pi_L(P^L = \frac{1+v}{2}, a = 1) \geq P(1+v-P) - d
\]

\[\Rightarrow P \geq \bar{P}_{a=1}(s) = (1 + \sqrt{s}) \left(\frac{1+v}{2}\right)\]

or

\[P \leq \bar{P}_{a=1}(s) = (1 - \sqrt{s}) \left(\frac{1+v}{2}\right)\]

As \( \pi_H(\bar{P}_{a=1}(s)) \) > \( \pi_H(\bar{P}_{a=1}(s)) \), all the prices \( P \leq \bar{P}_{a=1}(s) \) are ruled out. Intuitive criterion suggests that the high type chooses the least-cost signaling price which is just enough to prevent low type to mimic high type’s prices. This is satisfied by \( P^H = \bar{P}_{a=1}(s) \) because \( \pi_H(\bar{P}_{a=1}(s)) \) is the highest for the price range \( P \geq \bar{P}_{a=1}(s) \) and sufficient to separate from the low type.

Therefore, the high type’s best response is to charge \( P^H = \max\left\{\frac{1+v+c_H}{2}, \bar{P}_{a=1}(s)\right\} \).
\[ \hat{P}^H = \frac{1 + v + c_H}{2} \] when:

\[ \frac{1 + v + c_H}{2} \geq \bar{P}_{a=1}(s) \Rightarrow s \leq \left( \frac{c_H}{1 + v} \right)^2 \]

Hence, \( \hat{P}^H = \bar{P}_{a=1}(s) \) when \( \left( \frac{c_H}{1 + v} \right)^2 \leq s \leq \frac{(1-v)^2 - 4d}{(1+v)^2} \). That is, the high type charges full information price when \( s \) is very low and switches to a higher signaling price, \( \bar{P}_{a=1}(s) \) for intermediate values of \( s \).

Similarly, when \( d \geq \bar{d}_1(s) \) and \( 0 \leq s \leq \left( \frac{r-v}{r+v} \right)^2 \),

\[ \pi_L(P^L = \frac{r + v}{2}, a = 0) \geq P(1 + v - P) - d \]

\[ \Rightarrow P \geq \bar{P}_{a=0}(s, d) = \frac{1 + v}{2} + \frac{\sqrt{(1 + v)^2 - \frac{(1-s)(r+v)^2}{r} - 4d}}{2} \]

or

\[ P \leq \bar{P}_{a=0}(s, d) = \frac{1 + v}{2} - \frac{\sqrt{(1 + v)^2 - \frac{(1-s)(r+v)^2}{r} - 4d}}{2} \]

Following the similar argument as above, the intuitive criterion suggests that the high type charges:

\[ \hat{P}^H = \max \left\{ \frac{1 + v + c_H}{2}, \bar{P}_{a=0}(s, d) \right\} \]

This generates a second threshold for \( d \):

\[ \frac{1 + v + c_H}{2} \geq \frac{1 + v}{2} + \frac{\sqrt{(1 + v)^2 - \frac{(1-s)(r+v)^2}{r} - 4d}}{2} \]

\[ \Rightarrow d \geq \left( \frac{1 - s(1 - r)}{4r} \right) \left[ \left( \frac{r}{1 - s(1 - r)} - v \right)^2 - c_H^2 \right] = \bar{d}_2(s) \]
Therefore, if $d \geq \bar{d}_2(s)$, the high type always charges the full information price for the entire range of $s$ given by $0 \leq s \leq (\frac{r-v}{r+v})^2$. However, if $d \leq \bar{d}_2(s)$, the high type charges $P^H = \frac{1+v+cH}{2}$ when $s \leq \frac{r(c_H^2+4d)-(1-r)(r-v^2)}{(r+v)^2}$ (generated by solving the $\bar{d}_2(s)$ expression for $s$). But when $\frac{r(c_H^2+4d)-(1-r)(r-v^2)}{(r+v)^2} \leq s \leq (\frac{r-v}{r+v})^2$, high type switches to signaling price: $P^H = \bar{P}_{a=0}(s,d)$.

When $d \leq \bar{d}_1(s)$ and $\frac{(1-v)^2-4d}{(1+v)^2} \leq s \leq 1$ or $d \geq \bar{d}_1(s)$ and $(\frac{r-v}{r+v})^2 \leq s \leq 1$, the sophisticated consumers are dominant in the market and the low type charges $v$. The high type signals its quality such that
\[
\pi_L(P^L = v, a = 0) \geq P(1 + v - P) - d
\]
\[
\Rightarrow P \geq \bar{P}(d) = \frac{1 + v}{2} + \frac{\sqrt{(1 - v)^2 - 4d}}{2}
\]
or
\[
P \leq \underline{P}(d) = \frac{1 + v}{2} - \frac{\sqrt{(1 - v)^2 - 4d}}{2}
\]
Again, intuitive criterion suggests that the high type’s best response is:
\[
P^H = \max \left\{ \frac{1+v+cH}{2}, \bar{P}(d) \right\}
\]
This generates a threshold which is a modified version of $\bar{d}_2(s)$:
\[
\frac{1 + v + c_H}{2} \geq \frac{1 + v}{2} + \frac{\sqrt{(1 - v)^2 - 4d}}{2}
\]
\[
\Rightarrow d \geq \frac{[(1 - v)^2 - c_H^2]}{4} = \bar{\bar{d}}_2
\]
Note that $\bar{d}_2(s) = \bar{\bar{d}}_2$ at $s = 1$. If $d \geq \bar{\bar{d}}_2$, the high type continues to charge the full information price, irrespective of $s$, because a high penalty is enough to prevent low type to
mimic high type’s price. If \( d \leq \bar{d}_2 \), high type switches to signaling price, \( \bar{P}(d) \) for the entire specified range of \( s \).

**Lemma 4.2.** If \( r > v \), the pricing and advertising strategy of the high type monopolist is as follows:

1. For \( d \leq \tilde{d}_1(s) \),
   
   \[
   (a) \quad P^H = \frac{1+v+cu}{2}, \quad a = 1 \quad \text{if} \quad 0 \leq s \leq \left( \frac{cH}{1+v} \right)^2.
   
   \]
   
   (b) \( P^H = \bar{P}_{a=1}(s) \), \( a = 1 \) if \( \left( \frac{cH}{1+v} \right)^2 \leq s \leq \frac{(1-v)^2-4d}{(1+v)^2} \).
   
   (c) \( P^H = \bar{P}(d) \), \( a = 1 \) if \( \frac{(1-v)^2-4d}{(1+v)^2} \leq s \leq 1 \).

2. For \( \tilde{d}_1(s) \leq d \leq \tilde{d}_2(s) \),
   
   \[
   (a) \quad P^H = \frac{1+v+cu}{2}, \quad a = 1 \quad \text{if} \quad 0 \leq s \leq \frac{r(cH^2+4d)-(1-r)(r-v^2)}{(r+v)^2}.
   
   \]
   
   (b) \( P^H = \bar{P}_{a=0}(s,d) \), \( a = 1 \) if \( \frac{r(cH^2+4d)-(1-r)(r-v^2)}{(r+v)^2} \leq s \leq \left( \frac{r-v}{r+v} \right)^2 \).
   
   (c) \( P^H = \bar{P}(d) \), \( a = 1 \) if \( \left( \frac{r-v}{r+v} \right)^2 \leq s \leq 1 \).

3. For \( \tilde{d}_2(s) \leq d \leq \bar{d}_2 \),
   
   \[
   (a) \quad P^H = \frac{1+v+cu}{2}, \quad a = 1 \quad \text{if} \quad 0 \leq s \leq \left( \frac{r-v}{r+v} \right)^2.
   
   \]
   
   (b) \( P^H = \bar{P}(d) \), \( a = 1 \) if \( \left( \frac{r-v}{r+v} \right)^2 \leq s \leq 1 \).

4. For \( d \geq \bar{d}_2 \), \( P^H = \frac{1+v+cu}{2}, \quad a = 1 \) for all \( s \).

Lemmas 4.1 and 4.2 together complete the first part of the proof. Hence, I have the following proposition, supported by Figure C.1.

**Proposition C.1.** If \( r > v \), the unique separating equilibrium satisfying the intuitive criterion is defined as:

1. When \( d \leq \tilde{d}_1(s) \),
   
   \[
   (a) \quad P^H = \frac{1+v+cu}{2}, \quad a = 1 \quad \text{and} \quad P^L = \frac{1+v}{2}, \quad a = 1 \quad \text{if} \quad 0 \leq s \leq \left( \frac{cH}{1+v} \right)^2.
   
   \]
   
   (b) \( P^H = \bar{P}_{a=1}(s) \), \( a = 1 \) and \( P^L = \frac{1+v}{2}, \quad a = 1 \) if \( \left( \frac{cH}{1+v} \right)^2 \leq s \leq \frac{(1-v)^2-4d}{(1+v)^2} \).
   
   (c) \( P^H = \bar{P}(s,d) \), \( a = 1 \) and \( P^L = v, \quad a = 0 \) if \( \frac{(1-v)^2-4d}{(1+v)^2} \leq s \leq 1 \).

2. When \( \tilde{d}_1(s) \leq d \leq \tilde{d}_2(s) \),
   
   \[
   (a) \quad P^H = \frac{1+v+cu}{2}, \quad a = 1 \quad \text{and} \quad P^L = \frac{r+v}{2}, \quad a = 0 \quad \text{if} \quad 0 \leq s \leq \frac{r(cH^2+4d)-(1-r)(r-v^2)}{(r+v)^2}.
   
   \]

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(b) \( P^H = P_{a=0}(s, d), a = 1 \) and \( P^L = \frac{r+v}{2}, a = 0 \) if \( \frac{r(v^2+4d)-(1-r)(r-v^2)}{(r+v)^2} \leq s \leq \left(\frac{r-v}{r+v}\right)^2 \).

(c) \( P^H = \overline{P}(s, d), a = 1 \) and \( P^L = v, a = 0 \) if \( \left(\frac{r-v}{r+v}\right)^2 \leq s \leq 1 \).

3. When \( \tilde{d}_2(s) \leq d \leq \frac{v}{2} \),

(a) \( P^H = \frac{1+v+cH}{2}, a = 1 \) and \( P^L = \frac{r+v}{2}, a = 0 \) if \( 0 \leq s \leq \left(\frac{r-v}{r+v}\right)^2 \).

(b) \( P^H = \overline{P}(s, d), a = 1 \) and \( P^L = v, a = 0 \) if \( \left(\frac{r-v}{r+v}\right)^2 \leq s \leq 1 \).

4. When \( d \geq \frac{v}{2} \),

(a) \( P^H = \frac{1+v+cH}{2}, a = 1 \) and \( P^L = \frac{r+v}{2}, a = 0 \) if \( 0 \leq s \leq \left(\frac{r-v}{r+v}\right)^2 \).

(b) \( P^H = \frac{1+v+cH}{2}, a = 1 \) and \( P^L = v, a = 0 \) if \( \left(\frac{r-v}{r+v}\right)^2 \leq s \leq 1 \).

Figure C.1: Separating Perfect Bayesian Equilibrium

\((r > v)\)
(ii) When \( r < v \), the low type still has an advantage to falsely advertise its product to the naive consumers. Therefore, when the naive consumers dominate the market,

\[
\max \pi_L(P^L) = P^L(1 + v - P^L)(1 - s) - d
\]

\[
\Rightarrow P^L = \frac{1 + v}{2}, a = 1
\]

However, when the proportion of sophisticated consumers increases in the market, the low type deviates to charge \( v \) because loss of demand from sophisticated consumers is more than the gain from naive consumers. Such a deviation occurs when:

\[
\pi_L(P^L = \frac{1 + v}{2}, a = 1) \leq \pi_L(P^L = v, a = 0)
\]

\[
\Rightarrow s \geq \frac{(1 - v)^2 - 4d}{(1 + v)^2}
\]

or

\[
\Rightarrow d \geq \frac{(1 - v)^2 - s(1 + v)^2}{4} = d_1(s)
\]

Hence, I have the following lemma:

**Lemma 4.3.** If \( r < v \), the pricing and advertising strategy of the low type monopolist is summarized as follows:

1. For \( d \leq d_1(s) \), \( P^L = \frac{1 + v}{2}, a = 1 \)
2. For \( d \geq d_1(s) \), \( P^L = v, a = 0 \).

The high type monopolist’s ideal price is \( P^H = \frac{1 + v + c_H}{2} \), the full information price. This price along with the message \( a = 1 \) is the best response of the high type when naive consumers are dominant in the market (\( s \) is very low) because advertising message is enough for the
naive consumers to believe that the product is of high-quality. However, when the proportion of sophisticated consumers increases in the market, the fear of mimicry from the low type increases as well. Therefore, with increasing $s$, the high type should deviate to a higher signaling price to separate itself from the low type firm. It can do so by choosing a price, $P$ that is not profitable for the low type to mimic. When $d \leq \overline{d}_1(s)$,

$$\pi_L(P^L = \frac{1+v}{2}, a = 1) \geq P(1 + v - P) - d$$

$$\Rightarrow P \geq \bar{P}_{a=1}(s) = (1 + \sqrt{s}) \left( \frac{1+v}{2} \right)$$

or

$$P \leq \bar{P}_{a=1}(s) = (1 - \sqrt{s}) \left( \frac{1+v}{2} \right)$$

As $\pi_H(\bar{P}_{a=1}(s)) > \pi_H(P_{a=1}(s))$, all the prices $P \leq \bar{P}_{a=1}(s)$ are ruled out. Intuitive criterion suggests that the high type chooses the least-cost signaling price which is just enough to prevent low type to mimic high type’s prices. This is satisfied by $P^H = \bar{P}_{a=1}(s)$ because $\pi_H(\bar{P}_{a=1}(s))$ is the highest for the price range $P \geq \bar{P}_{a=1}(s)$ and sufficient to separate from the low type.

Therefore, the high type’s best response is to charge $P^H = \max \left\{ \frac{1+v+c_H}{2}, \bar{P}_{a=1}(s) \right\}$. $P^H = \frac{1+v+c_H}{2}$ when:

$$\frac{1+v+c_H}{2} \geq \bar{P}_{a=1}(s) \Rightarrow s \leq \left( \frac{c_H}{1+v} \right)^2$$

Hence, $P^H = \bar{P}_{a=1}(s)$ when $\left( \frac{c_H}{1+v} \right)^2 \leq s \leq \frac{(1-v)^2-4d}{(1+v)^2}$. That is, the high type charges full information price when $s$ is very low and switches to a higher signaling price, $\bar{P}_{a=1}(s)$ for intermediate values of $s$.

Similarly, when $d \geq \overline{d}_1(s)$, the sophisticated consumers are dominant in the market and
the low type charges $v$. The high type signals its quality such that

$$\pi_L(P^L = v, a = 0) \geq P(1 + v - P) - d$$

$$\Rightarrow P \geq \overline{P}(d) = \frac{1 + v}{2} + \frac{\sqrt{(1 - v)^2 - 4d}}{2}$$

or

$$P \leq \underline{P}(d) = \frac{1 + v}{2} - \frac{\sqrt{(1 - v)^2 - 4d}}{2}$$

Again, intuitive criterion suggests that the high type’s best response is:

$$P^H = \max \left\{ \frac{1 + v + c_H}{2}, \overline{P}(d) \right\}$$

This generates the following threshold:

$$\frac{1 + v + c_H}{2} \geq \frac{1 + v}{2} + \frac{\sqrt{(1 - v)^2 - 4d}}{2}$$

$$\Rightarrow d \geq \frac{[(1 - v)^2 - c_H^2]}{4} = \overline{d}_2$$

If $d \geq \overline{d}_2$, the high type charges the full information price because a high penalty along with higher proportion of sophisticated consumers is enough to prevent low type to mimic high type’s price. If $d \leq \overline{d}_2$, high type switches to signaling price, $\overline{P}(d)$.

**Lemma 4.4.** A summary of pricing and advertising strategy of the high type monopolist is as follows:

1. For $d \leq \overline{d}_1(s)$,
   
   (a) $P^H = \frac{1 + v + c_H}{2}, a = 1$ if $0 \leq s \leq \left( \frac{c_H}{1 + v} \right)^2$.
   
   (b) $P^H = \overline{P}_{a = 1}(s), a = 1$ if \( \left( \frac{c_H}{1 + v} \right)^2 \leq s \leq \frac{(1 - v)^2 - 4d}{(1 + v)^2} \).
2. For \( \bar{d}_1(s) \leq d \leq \bar{d}_2 \) and \( \frac{(1-v)^2-4d}{(1+v)^2} \leq s \leq 1 \),

   (a) \( P^H = \bar{P}(d), a = 1 \)

3. For \( \bar{d}_2 \leq \bar{d}_1(s) \leq d \) and \( \frac{(1-v)^2-4d}{(1+v)^2} \leq s \leq 1 \),

   (a) \( P^H = \frac{1+v+c_H}{2}, a = 1 \)

Lemmas 4.3 and 4.4 show the equilibrium for \( r < v \) case, summarized in Proposition C.2 and Figure C.2.

Figure C.2: Separating Perfect Bayesian Equilibrium
\((r < v)\)
Proposition C.2. If \( r < v \), the unique separating equilibrium satisfying the intuitive criterion is defined as:

1. For \( d \leq \bar{d}_1(s) \),
   
   (a) \( P^H = \frac{1+v+cu}{2}, a = 1 \) and \( P^L = \frac{1+v}{2}, a = 1 \) if \( 0 \leq s \leq \left( \frac{cu}{1+v} \right)^2 \).
   (b) \( P^H = \bar{P}_{a=1}(s), a = 1 \) and \( P^L = \frac{1+v}{2}, a = 1 \) if \( \left( \frac{cu}{1+v} \right)^2 \leq s \leq \frac{(1-v)^2-4d}{(1+v)^2} \).

2. For \( \bar{d}_1(s) \leq d \leq \bar{d}_2 \),
   
   (a) \( P^H = \bar{P}(d), a = 1 \) and \( P^L = v, a = 0 \).

3. For \( \bar{d}_2 \leq \bar{d}_1(s) \leq d \),
   
   (a) \( P^H = \frac{1+v+cu}{2}, a = 1 \) and \( P^L = v, a = 0 \).

Proof of Proposition 4.4

The low type’s strategy determines the penalty to avoid deception. The sole basis of the low type’s strategy is to maximize its profit function.

(i) When \( r > v \):

(a) For \( 0 \leq s \leq \left( \frac{r-v}{r+v} \right)^2 \),

\[
\pi_L(P^L, s, d) = \begin{cases} 
(1-s) \left( \frac{1+v}{2} \right)^2 - d & \text{if } d \leq \bar{d}_1(s) \\
\left( \frac{1-s}{r} \right) \left( \frac{r+v}{2} \right)^2 & \text{if } d \geq \bar{d}_1(s)
\end{cases}
\]

At \( d = \bar{d}_1(s) \),

\[
(1-s) \left( \frac{1+v}{2} \right)^2 - d = \left( \frac{1-s}{r} \right) \left( \frac{r+v}{2} \right)^2
\]

that is, the entire benefit from false advertising is extracted and profit under deception is equal to that under no-deception by the low type. Therefore, \( \bar{d}_1(s) \) is the minimum required penalty to avoid deception.

(b) For \( \left( \frac{r-v}{r+v} \right)^2 \leq s \leq \left( \frac{1-v}{1+v} \right)^2 \),

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\[
\pi_L(P_L, s, d) = \begin{cases} 
(1 - s) \left( \frac{1 + v}{2} \right)^2 - d & \text{if } d \leq \frac{(1-v)^2 - (1+v)^2 s}{4} \\
v & \text{if } d \geq \frac{(1-v)^2 - (1+v)^2 s}{4}
\end{cases}
\]

At \( d = \frac{(1-v)^2 - (1+v)^2 s}{4} \),

\[(1 - s) \left( \frac{1 + v}{2} \right)^2 - d = v \]

that is, the entire benefit from false advertising is extracted and profit under deception is equal to that under no-deception by the low type. Therefore, \( d = \frac{(1-v)^2 - (1+v)^2 s}{4} \) is the minimum required penalty to avoid deception.

(c) For \( \left( \frac{1-v}{1+v} \right)^2 \leq s \leq 1 \),

\[\pi_L(P_L, s, d) = v\]

As there is no deception, no penalty is required.

(ii) When \( r < v \):

(a) For \( 0 \leq s \leq \left( \frac{1-v}{1+v} \right)^2 \),

\[
\pi_L(P_L, s, d) = \begin{cases} 
(1 - s) \left( \frac{1 + v}{2} \right)^2 - d & \text{if } d \leq \bar{d}_1(s) \\
v & \text{if } d \geq \bar{d}_1(s)
\end{cases}
\]

At \( d = \bar{d}_1(s) \),

\[(1 - s) \left( \frac{1 + v}{2} \right)^2 - d = v \]

that is, the entire benefit from false advertising is extracted and profit under deception is equal to that under no-deception by the low type. Therefore, \( \bar{d}_1(s) \) is the minimum required penalty to avoid deception.

(b) For \( \left( \frac{1-v}{1+v} \right)^2 \leq s \leq 1 \),
\[ \pi_L(P^L, s, d) = v \]

As there is no deception, no penalty is required.

**Proof of Proposition 4.5**

There are two parts of the proof: (i) when \( r > v \) (ii) when \( r < v \)

(i) The expected social welfare function depends on the proportion of sophisticated consumers:

(a) For \( 0 \leq s \leq \left( \frac{c_H}{1+v} \right)^2 \),

\[
\mathbb{E}(SW(s,d)) = \begin{cases} 
\frac{3}{8}r[(1+v)^2 - c_H^2] + (1-r) \left[ \frac{v(1-s)(1+v)}{2} - d \right] & \text{if } d \leq \tilde{d}_1(s) \\
\frac{3}{8}r[(1+v)^2 - c_H^2] + (1-r) \left[ \frac{v(1-s)(r+v)}{2r} \right] & \text{if } d \geq \tilde{d}_1(s) 
\end{cases}
\]

At \( d = \tilde{d}_1(s) \),

\[
\frac{v(1-s)(1+v)}{2} - d = \frac{v(1-s)(r+v)}{2r}
\]

This implies that \( \mathbb{E}(SW(s,d)) \) is equal under deception as well as no-deception by low type. Therefore, \( \tilde{d}_1(s) \) is the optimal penalty.

(b) For \( \left( \frac{c_H}{1+v} \right)^2 \leq s \leq \frac{r(c_H^2 + 4d_2(s) - (1-r)(r-v^2))}{(r+v)^2} \),

\[
\mathbb{E}(SW(s,d)) = \begin{cases} 
\frac{r[(1-v^2)(1+h)]}{2} \left( \frac{3+v^2}{4} - c_H \right) + (1-r) \left[ \frac{v(1-s)(1+v)}{2} - 2d \right] & \text{if } d \leq \tilde{d}_1(s) \\
rSW_H + (1-r) \left[ \frac{v(1-s)(r+v)}{2r} \right] & \text{if } \tilde{d}_1(s) \leq d \leq \tilde{d}_2(s) \\
\frac{3}{8}r[(1+v)^2 - c_H^2] + (1-r) \left[ \frac{v(1-s)(r+v)}{2r} \right] & \text{if } d \geq \tilde{d}_2(s) 
\end{cases}
\]
where $SW^H_I$ is expressed as:

$$
\left( \frac{1 + v}{2} - \sqrt{\left( \frac{1 + v}{2} \right)^2 - \left( \frac{r + v}{2} \right)^2 \left( 1 - \frac{s}{r} \right) - d} \right) \left( \frac{3(1 + v)}{4} + \sqrt{\left( \frac{1 + v}{2} \right)^2 - \left( \frac{r + v}{2} \right)^2 \left( 1 - \frac{s}{r} \right) - d - c_H} \right)
$$

At $d = \tilde{d}_1(s)$,

$$\frac{v(1 - s)(1 + v)}{2} - d = \frac{v(1 - s)(r + v)}{2r}$$

that is, deceptive advertising is prevented. At $d = \tilde{d}_2(s) = \frac{s(r+v)^2+(1-r)(r-v^2)-r c_H^2}{4r}$,

$$SW^H_I = \frac{3}{8} r [ (1 + v)^2 - c_H^2 ]$$

that is, signaling distortion is removed and full-information outcome is reached under high-quality state. Therefore, $d = \tilde{d}_2(s)$ is the optimal penalty.

(c) For $\frac{v \left( c_H^2 + 4 \tilde{d}_2(s) \right)}{(r+v)^2} \leq s \leq \left( \frac{r-v}{r+v} \right)^2$,

$$
\mathbb{E}(SW(s,d)) = \begin{cases} 
  r \left[ \frac{(1-v^2)(1+v)}{2} \left( \frac{3+v^2}{4} \right) - c_H \right] + (1-r) \left[ \frac{v(1-s)(1+v)}{2} - d \right] & \text{if } d \leq \tilde{d}_1(s) \\
  r SW^H_I + (1-r) \left[ \frac{v(1-s)(r+v)}{2r} \right] & \text{if } \tilde{d}_1(s) \leq d \leq \tilde{d}_2(s) \\
  \frac{3}{8} r [ (1 + v)^2 - c_H^2 ] + (1-r) \left[ \frac{v(1-s)(r+v)}{2r} \right] & \text{if } d \geq \tilde{d}_2(s)
\end{cases}
$$

where $SW^H_I$ is expressed as:

$$
\left( \frac{1 + v}{2} - \sqrt{\left( \frac{1 + v}{2} \right)^2 - \left( \frac{r + v}{2} \right)^2 \left( 1 - \frac{s}{r} \right) - d} \right) \left( \frac{3(1 + v)}{4} + \sqrt{\left( \frac{1 + v}{2} \right)^2 - \left( \frac{r + v}{2} \right)^2 \left( 1 - \frac{s}{r} \right) - d - c_H} \right)
$$

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Similar to previous cases, \( d = \bar{d}_1(s) \) prevents deception. At \( d = \bar{d}_2(s) \),

\[
SW_{II}^H = \frac{3}{8} r[(1 + v)^2 - c_H^2]
\]

that is, signaling distortion is removed. Therefore, \( \bar{d}_2(s) \) is optimal penalty.

(d) For \( \left( \frac{r-v}{r+v} \right)^2 \leq s \leq \left( \frac{1-v}{1+v} \right)^2 \),

\[
E(SW(s, d)) = \begin{cases} 
  r \left[ \frac{\sqrt{(1-v)(1+v)}}{2} \left( \frac{3+\sqrt{5}}{2} - c_H \right) \right] + (1-r) \left[ \frac{\sqrt{(1-s)(1+v)}}{2} - d \right] & \text{if } d \leq \bar{d}_1(s) \\
  r SW_{II}^H + (1-r) \left[ \frac{\sqrt{(1-s)(r+v)}}{2r} \right] & \text{if } \bar{d}_1(s) \leq d \leq \bar{d}_2 \\
  \frac{3}{8} r[(1 + v)^2 - c_H^2] + (1-r)v & \text{if } d \geq \bar{d}_2
\end{cases}
\]

where \( SW_{III}^H \) is expressed as:

\[
\left( \frac{1+v}{2} - \sqrt{\left( \frac{1+v}{2} \right)^2 - d} \right) \left( \frac{3(1+v)}{4} + \sqrt{\left( \frac{1+v}{4} \right)^2 - \frac{d}{4}} - c_H \right)
\]

Again, \( d = \bar{d}_1(s) \) prevents deception. At \( d = \bar{d}_2 \),

\[
SW_{III}^H = \frac{3}{8} r[(1 + v)^2 - c_H^2]
\]

that is, signaling distortion is removed. Therefore, \( \bar{d}_2 \) is the optimal penalty.

(e) For \( \left( \frac{1-v}{1+v} \right)^2 \leq s \leq 1 \),

\[
E(SW(s, d)) = \begin{cases} 
  r \left[ \left( \frac{1+v}{2} - \sqrt{\left( \frac{1+v}{2} \right)^2 - d} \right) \left( \frac{3(1+v)}{4} + \sqrt{\left( \frac{1+v}{4} \right)^2 - \frac{d}{4}} - c_H \right) \right] + (1-r)v & \text{if } d \leq \bar{d}_2 \\
  \frac{3}{8} r[(1 + v)^2 - c_H^2] + (1-r)v & \text{if } d \geq \bar{d}_2
\end{cases}
\]
At $d = \bar{d}_2$, 

$$
\begin{align*}
    r \left\{ \left( \frac{1 + v}{2} \right)^2 - \sqrt{\left( \frac{1 + v}{2} \right)^2 - d} \right\} \left\{ \frac{3(1 + v)}{4} + \sqrt{\left( \frac{1 + v}{4} \right)^2 - \frac{d}{4} - c_H} \right\} = \frac{3}{8} r [(1 + v)^2 - c_H^2]
\end{align*}
$$

This implies that $\mathbb{E}(SW(s, d))$ is equal under signaling case and full-information outcome. Therefore, $\bar{d}_2$ is the optimal penalty.

(ii) For $r < v$, the expected social welfare functions depend on the value of $s$:

(a) For $0 \leq s \leq \left( \frac{c_H}{1 + v} \right)^2$, 

$$
\mathbb{E}(SW(s, d)) = \begin{cases} 
    \frac{3}{8} r [(1 + v)^2 - c_H^2] + (1 - r) \left[ \frac{v(1-s)(1+v)}{2} - d \right] & \text{if } d \leq \bar{d}_1(s) \\
    \frac{3}{8} r [(1 + v)^2 - c_H^2] + (1 - r)v & \text{if } d \geq \bar{d}_1(s)
\end{cases}
$$

At $d = \bar{d}_1(s)$, 

$$
\frac{v(1-s)(1+v)}{2} - d = v
$$

This implies that $\mathbb{E}(SW(s, d))$ is equal under deception as well as no-deception by low type. Therefore, $\bar{d}_1(s)$ is the optimal penalty.

(b) For $\left( \frac{c_H}{1 + v} \right)^2 \leq s \leq \left( \frac{1 - v}{1 + v} \right)^2$, 

$$
\mathbb{E}(SW(s, d)) = \begin{cases} 
    r \left[ \left( \frac{1 - \sqrt{2}}{2} \right)^2 - \sqrt{\left( \frac{1 + v}{2} \right)^2 - d} \right] \left\{ \frac{3(1 + v)}{4} + \sqrt{\left( \frac{1 + v}{4} \right)^2 - \frac{d}{4} - c_H} \right\} + (1 - r) \left[ \frac{v(1-s)(1+v)}{2} - d \right] & \text{if } d \leq \bar{d}_1(s) \\
    r \left[ \left( \frac{1 + v}{2} \right)^2 - \sqrt{\left( \frac{1 + v}{2} \right)^2 - d} \right] \left\{ \frac{3(1 + v)}{4} + \sqrt{\left( \frac{1 + v}{4} \right)^2 - \frac{d}{4} - c_H} \right\} + (1 - r)v & \text{if } \bar{d}_1(s) \leq d \leq \bar{d}_2 \\
    \frac{3}{8} r [(1 + v)^2 - c_H^2] + (1 - r)v & \text{if } d \geq \bar{d}_2
\end{cases}
$$
Again, at \( d = \bar{d}_1(s) \),

\[
\frac{v(1 - s)(1 + v)}{2} - d = v
\]

Therefore, \( \bar{d}_1(s) \) prevents deception. At \( d = \bar{d}_2 \),

\[
\begin{align*}
&\frac{(1 + v)}{2} - \sqrt{\left(\frac{1 + v}{2}\right)^2 - d} \left\{ \frac{3(1 + v)}{4} + \sqrt{\left(\frac{1 + v}{4}\right)^2 - \frac{d}{4} - c_H} \right\} = \frac{3}{8}r[(1 + v)^2 - c_H^2] \\
&\text{that is, signaling distortion is removed and full-information outcome is reached under} \\
&\text{high-quality state. Therefore, } \bar{d}_2 \text{ is the optimal penalty.}
\end{align*}
\]

(c) For \( \left(\frac{1-v}{1+v}\right)^2 \leq s \leq 1 \),

\[
\mathbb{E}(SW(s, d)) = \begin{cases} 
\frac{3}{8}r[(1 + v)^2 - c_H^2] + (1 - r)v & \text{if } d \leq \bar{d}_2 \\
\frac{3}{8}r[(1 + v)^2 - c_H^2] + (1 - r)v & \text{if } d \geq \bar{d}_2 
\end{cases}
\]

Similar to previous case, \( \bar{d}_2 \) is the optimal penalty.

**Proof of Proposition 4.6**

There are two parts of the proof: (i) when \( r > v \) (ii) when \( r < v \).

(i) The expected consumer surplus function depends on the proportion of sophisticated consumers:

(a) For \( 0 \leq s \leq \left(\frac{c_H}{1+v}\right)^2 \),

\[
\mathbb{E}(CS(s, d)) = \begin{cases} 
\frac{(1+v-c_H)^2}{8} + (1 - r) \left[ \frac{v(1 - s)(v - 1)}{2} - d \right] & \text{if } d \leq \bar{d}_1(s) \\
\frac{(1+v-c_H)^2}{8} + (1 - r) \left[ \frac{v(1 - s)(v - r)}{2} \right] & \text{if } d \geq \bar{d}_1(s) 
\end{cases}
\]
At $d = \bar{d}_1(s)$,

$$\frac{v(1-s)(v-1)}{2} - d = \frac{v(1-s)(v-r)}{2}$$

This implies that $\mathbb{E}(CS(s,d))$ is equal under deception as well as no-deception by low type. Therefore, $\bar{d}_1(s)$ is the optimal penalty.

(b) For $(\frac{c_H}{1+v})^2 \leq s \leq \frac{r(c_H^2+4d_2(s))-(1-r)(r-v^2)}{(r+v)^2}$,

$$\mathbb{E}(CS(s,d)) = \begin{cases} 
    r \left[ \frac{(1+v)^2(1-\sqrt{\frac{8}{s}})^2}{8} \right] + (1-r) \left[ \frac{v(1-s)(v-1)}{2} - d \right] & \text{if } d \leq \bar{d}_1(s) \\
    r \left[ \frac{1+v-\sqrt{(1-v)^2-\frac{(1-s)(r+v)^2}{r} - 4d}}{8} \right]^2 + (1-r) \left[ \frac{v(1-s)(v-r)}{2} \right] & \text{if } \bar{d}_1(s) \leq d \leq \bar{d}_2(s) \\
    r \left[ \frac{(1+v-c_H)^2}{8} \right] + (1-r) \left[ \frac{v(1-s)(v-r)}{2} \right] & \text{if } d \geq \bar{d}_2(s) 
\end{cases}$$

At $d = \bar{d}_1(s)$,

$$\frac{v(1-s)(v-1)}{2} - d = \frac{v(1-s)(v-r)}{2}$$

that is, deceptive advertising is prevented. At $d = \frac{s(r+v)^2+(1-r)(r-v^2)-rc_H^2}{4r}$,\[ \left( 1 + v - \sqrt{(1-v)^2 - \frac{(1-s)(r+v)^2}{r} - 4d} \right)^2 = \frac{(1 + v - c_H)^2}{8} \]

that is, signaling distortion is removed and full-information outcome is reached under high-quality state. Therefore, $d = \frac{s(r+v)^2+(1-r)(r-v^2)-rc_H^2}{4r}$ is the optimal penalty.

(c) For $\frac{r(c_H^2+4d_2(s))-(1-r)(r-v^2)}{(r+v)^2} \leq s \leq \left( \frac{r-v}{r+v} \right)^2$,
\[ \mathbb{E}(CS(s, d)) = \begin{cases} 
\frac{r \left[ \frac{(1+v)^2(1-\sqrt{s})^2}{s} \right] + (1-r) \left[ \frac{v(1-s)(v-1)}{2} - d \right]}{8} & \text{if } d \leq \bar{d}_1(s) \\
\frac{r \left[ \frac{(1+v-\sqrt{(1-v)^2-\frac{(1-s)(v+1)^2}{r}})^2}{s} \right] + (1-r) \left[ \frac{v(1-s)(v-r)}{2} \right]}{8} & \text{if } \bar{d}_1(s) \leq d \leq \bar{d}_2(s) \\
\frac{r \left[ \frac{(1+v-c_H)^2}{s} \right]}{8} & \text{if } d \geq \bar{d}_2(s) 
\end{cases} \]

Similar to previous cases, \( d = \bar{d}_1(s) \) prevents deception. At \( d = \bar{d}_2(s) \),

\[
\frac{(1 + v - \sqrt{(1 - v)^2 - \frac{(1-s)(v+1)^2}{r} - 4d})^2}{8} = \frac{(1 + v - c_H)^2}{8}
\]

that is, signaling distortion is removed. Therefore, \( \bar{d}_2(s) \) is optimal penalty.

(d) For \( \left( \frac{r-v}{r+v} \right)^2 \leq s \leq \left( \frac{1-v}{1+v} \right)^2 \),

\[ \mathbb{E}(CS(s, d)) = \begin{cases} 
\frac{r \left[ \frac{(1+v)^2(1-\sqrt{s})^2}{s} \right] + (1-r) \left[ \frac{v(1-s)(v-1)}{2} - d \right]}{8} & \text{if } d \leq \bar{d}_1(s) \\
\frac{r \left[ \frac{(1+v-\sqrt{(1-v)^2-4d})^2}{s} \right]}{8} & \text{if } \bar{d}_1(s) \leq d \leq \bar{d}_2 \\
\frac{r \left[ \frac{(1+v-c_H)^2}{s} \right]}{8} & \text{if } d \geq \bar{d}_2 
\end{cases} \]

Again, \( d = \bar{d}_1(s) \) prevents deception. At \( d = \bar{d}_2 \),

\[
\frac{(1 + v - \sqrt{(1 - v)^2 - 4d})^2}{8} = \frac{(1 + v - c_H)^2}{8}
\]

that is, signaling distortion is removed. Therefore, \( \bar{d}_2 \) is the optimal penalty.

(e) For \( \left( \frac{1-v}{1+v} \right)^2 \leq s \leq 1 \),
\[
\mathbb{E}(\text{CS}(s,d)) = \begin{cases} 
    r \left[ \frac{(1+v-\sqrt{(1-v)^2-4d})^2}{8} \right] & \text{if } d \leq \bar{d}_2 \\
    r \left[ \frac{(1+v-c_H)^2}{8} \right] & \text{if } d \geq \bar{d}_2 
\end{cases}
\]

At \( d = \bar{d}_2 \),

\[
\frac{(1+v-\sqrt{(1-v)^2-4d})^2}{8} = \frac{(1+v-c_H)^2}{8}
\]

This implies that \( \mathbb{E}(\text{CS}(s,d)) \) is equal under signaling case and full-information outcome. Therefore, \( \bar{d}_2 \) is the optimal penalty.

(ii) For \( r < v \), the expected consumer surplus functions depend on the value of \( s \):

(a) For \( 0 \leq s \leq \left( \frac{cu}{1+v} \right)^2 \),

\[
\mathbb{E}(\text{CS}(s,d)) = \begin{cases} 
    r \left[ \frac{(1+v-c_H)^2}{8} \right] + (1-r) \left[ \frac{v(1-s)(v-1)}{2} - d \right] & \text{if } d \leq \bar{d}_1(s) \\
    r \left[ \frac{(1+v-c_H)^2}{8} \right] & \text{if } d \geq \bar{d}_1(s) 
\end{cases}
\]

At \( d = \bar{d}_1(s) \),

\[
\frac{v(1-s)(v-1)}{2} - d = 0
\]

This implies that \( \mathbb{E}(\text{CS}(s,d)) \) is equal under deception as well as no-deception by low type. Therefore, \( \bar{d}_1(s) \) is the optimal penalty.

(b) For \( \left( \frac{cu}{1+v} \right)^2 \leq s \leq \left( \frac{1-u}{1+v} \right)^2 \),
\[ \mathbb{E} (CS(s, d)) = \begin{cases} 
    r \left[ \frac{(1+v)^2(1-\sqrt{s})^2}{8} \right] + (1 - r) \left[ \frac{v(1-s)(v-1)}{2} - d \right] & \text{if } d \leq \bar{d}_1(s) \\
    r \left[ \frac{(1+v-\sqrt{(1-v)^2-4d})^2}{8} \right] & \text{if } \bar{d}_1(s) \leq d \leq \bar{d}_2 \\
    r \left[ \frac{(1+v-c_H)^2}{8} \right] & \text{if } d \geq \bar{d}_2 
\end{cases} \]

Again, at \( d = \bar{d}_1(s) \),

\[
\frac{v(1-s)(v-1)}{2} - d = 0
\]

Therefore, \( \bar{d}_1(s) \) prevents deception. At \( d = \bar{d}_2 \),

\[
\frac{(1 + v - \sqrt{(1-v)^2-4d})^2}{8} = \frac{(1 + v - c_H)^2}{8}
\]

that is, signaling distortion is removed and full-information outcome is reached under high-quality state. Therefore, \( \bar{d}_2 \) is the optimal penalty.

(c) For \( \left( \frac{1-v}{1+v} \right)^2 \leq s \leq 1 \),

\[ \mathbb{E} (CS(s, d)) = \begin{cases} 
    r \left[ \frac{(1+v-\sqrt{(1-v)^2-4d})^2}{8} \right] & \text{if } d \leq \bar{d}_2 \\
    r \left[ \frac{(1+v-c_H)^2}{8} \right] & \text{if } d \geq \bar{d}_2 
\end{cases} \]

Similar to previous case, \( \bar{d}_2 \) is the optimal penalty.

**Proof of Proposition 4.7**

There are two parts of the proof: (i) low type’s profit function and (ii) high type’s profit function

(i) The profit function of low type monopolist is expressed as follows:
\[ \pi_L(P^L, s, d) = \begin{cases} 
 v & \text{if } P^L = v \text{ and } a = 0 \\
 \left( \frac{1-s}{r} \right) \left( \frac{r+v}{2} \right)^2 & \text{if } P^L = \frac{r+v}{2} \text{ and } a = 0 \\
 (1-s) \left( \frac{1+v}{2} \right)^2 - d & \text{if } P^L = \frac{1+v}{2} \text{ and } a = 1 
\end{cases} \]

For low and intermediate values of \( s \), the low type falsely advertises its product and its profit function is \( \pi_L = (1-s) \left( \frac{1+v}{2} \right)^2 - d \). The marginal profitability of regulation is defined as the rate of change in profits due to change in regulation, i.e., optimal penalty. In the case of false advertising, the marginal profitability of regulation for the low type is:

\[ \frac{\partial \pi_L}{\partial d} = -1 < 0 \]

Therefore, as \( d \) increases, the profits of low type will fall and hence, the low type always lobbies against the regulation. Also, the profits fall with \( s \):

\[ \frac{\partial \pi_L}{\partial s} = - \left( \frac{1+v}{2} \right)^2 < 0 \]

This implies that as \( s \) increases, the profits fall and hence, the incentive to lobby against regulation falls. For high values of \( s \), the profit function is \( \pi_L = v \). There is no impact of \( d \) on profits so marginal profitability of regulation is zero.

(ii) The profit function of high type monopolist is expressed as follows:

\[ \pi_H(P^H, s, d) = \begin{cases} 
 \left( \frac{1+v-c_H}{2} \right)^2 & \text{if } P^H = P^F \\
 \left[ \left( \frac{1+v}{2} \right) (1-\sqrt{s}) \right] \left[ \left( \frac{1+v}{2} \right) (1+\sqrt{s}) - c_H \right] & \text{if } P^H = \bar{P}_{a=1}(s) \\
 \left[ \frac{1+v}{2} - \sqrt{\left( \frac{1+v}{2} \right)^2 - \frac{1-s}{4r} (r+v)^2} - d \right] - c_H & \text{if } P^H = \bar{P}_{a=0}(s,d) \\
 \left[ \frac{1+v}{2} - \sqrt{\left( \frac{1+v}{2} \right)^2 - d} \right] \left[ \frac{1+v}{2} + \sqrt{\left( \frac{1+v}{2} \right)^2 - d} - c_H \right] & \text{if } P^H = \overline{P}(s,d) 
\end{cases} \]
For low values of $s$, the high type charges monopoly price: $P^H = \frac{1+v+c_H}{2}$. As its price and profits are not affected by $d$, it is indifferent about regulation. For intermediate and high values of $s$, high type signals quality through high prices, which are dependent on $d$. For intermediate $s$, signaling price is $\bar{P}_{a=0}(s, d) = \frac{1+v}{2} + \sqrt{\frac{(1+v)^2 - (1-s)(r+v)^2}{2} - 4d}$. A change in $d$ has a negative effect on price:

$$\frac{\partial \bar{P}_{a=0}(s, d)}{\partial d} = -\frac{1}{\sqrt{(1+v)^2 - (1-s)(r+v)^2 - 4d}} < 0$$

Therefore, increase in $d$ reduces the signaling price which increases the profit:

$$\frac{\partial \pi_H}{\partial d} = \frac{\sqrt{(1+v)^2 - (1-s)(r+v)^2 - 4d} - c_H}{\sqrt{(1+v)^2 - (1-s)(r+v)^2 - 4d}} > 0$$

Similarly, for high values of $s$, the signaling price is $P(s, d) = \frac{1+v}{2} + \sqrt{\frac{(1-v)^2 - 4d}{2}}$. Again, $d$ has a negative effect on price and positive effect on profit:

$$\frac{\partial P(s, d)}{\partial d} = -\frac{1}{\sqrt{(1+v)^2 - 4d}} < 0$$

$$\frac{\partial \pi_H}{\partial d} = \frac{\sqrt{(1+v)^2 - 4d} - c_H}{\sqrt{(1+v)^2 - 4d}} > 0$$

Hence, marginal profitability of regulation is positive which implies that high type lobbies for regulation. Also, the signaling price is increasing in $s$:

$$\frac{\partial \bar{P}_{a=0}(s, d)}{\partial s} = \frac{(r+v)^2}{4r\sqrt{(1+v)^2 - (1-s)(r+v)^2 - 4d}} > 0$$

$\bar{P}_{a=0}(s, d)$ increases with $s$ and converges to $\bar{P}(s, d)$ which is the maximum signaling price. This implies that as $s$ increases, signaling price rises and profit falls with profit lowest at $\bar{P}(s, d)$. Hence, higher the $s$, stronger the incentive to lobby for regulation.
Proof of Proposition 4.8

The high type monopolist’s ideal price is \( P^H = \frac{1+v+c_H}{2} \), the full information price. This price along with the message \( a = 1 \) is the best response of the high type when all consumers in the market have intermediate level of sophistication because advertising message is enough for such consumers to believe that the product is of high-quality.

For some values of penalty, the low type has an advantage to pool with the high type and falsely advertise its product as high-quality. For that range of penalty, it charges:

\[
\max \pi_L(P^L) = P^L(1 + v - P^L)(1 - s) - d
\]

\[
\Rightarrow P^L = \frac{1+v}{2}
\]

However, when the penalty increases to the point where false advertising becomes expensive, the low type deviates to charge \( v \) because penalty is more than the gain from false advertising. Such a deviation occurs when:

\[
\pi_L(P^L = \frac{1+v}{2} , a = 1) \leq \pi_L(P^L = v , a = 0)
\]

\[
\Rightarrow d \geq \frac{(1-v)^2}{4} = \bar{d}_1(0)
\]

Hence, I have the following lemma:

**Lemma 4.3.** The pricing and advertising strategy of the low type monopolist is summarized as follows:

1. For \( d \leq \bar{d}_1(0) \), \( P^L = \frac{1+v}{2} , a = 1 \)
2. For \( d \geq \bar{d}_1(0) \), \( P^L = v , a = 0 \).
BIBLIOGRAPHY


