Summer 2021

Response Modification and Seismic Protection of Yielding Structures Equipped with Inerters and Hysteretic Dampers

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RESPONSE MODIFICATION AND SEISMIC PROTECTION OF YIELDING STRUCTURES
EQUIPPED WITH INERTERS AND HYSTERETIC DAMPERS

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EQUIPPED WITH INERTERS AND HYSTERETIC DAMPERS

A Dissertation Presented to the Graduate Faculty of
Lyle School of Engineering
Southern Methodist University
in
Partial Fulfillment of the Requirements
for the degree of
Doctor of Philosophy
with a
Major in Civil Engineering
by
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Aug 04, 2021
ACKNOWLEDGMENTS

This work could not have been accomplished without the wisdom of my wonderful advisor, Professor Makris, who opened the door of real research to me when I started my Ph.D. study under his supervision. His support, patience, wisdom, and friendship made the challenging journey of graduate studies a very special and enjoyable one for me. I will always be thankful for his trust in me to explore various opportunities, and for all I learned from him about my professional career and life. I would like to thank my other committee members, Dr. El Shamy, Dr. Hurmuzlu, Dr. Story, and Dr. Whittaker, for their time and interest in this study. I am very thankful for the sincere friendship and help of many of my friends and colleagues at the University of Central Florida and Southern Methodist University: Specifically Dr. Mehrdad Aghagholizadeh, Dr. Eleftheria Efthymiou, Konstantinos Kalfas, Dr. Soroush Mokhtari, and Dr. Mehdi Alirezaei. I am also forever grateful to my family for their unconditional love and support. Thanks for being my source of motivation and encouragement.
This study investigates the seismic response of structures with sustainable, long-stroke response modification devices. The main thrust of the dissertation is the investigation of the seismic response of yielding structures equipped with supplemental rotational inertia, or inerter-device. The last chapter of this dissertation investigates the seismic response of multistory yielding steel structures equipped with pressurized sand dampers.

Inerters are mechanical devices with resisting force proportional to the relative acceleration of their end nodes. This class of response modification devices complements the traditional fluid viscous damping devices with resisting force proportional to the relative velocity at their end-nodes. Mass-amplification is the main benefit of inerter-based devices, which provide a high level of vibration control with small amount of actual mass.

This study first reviews the seismic response of elastic structures equipped with supplemental rotational inertia. The generalized equations of motion of structures equipped with inerter-devices are installed in the bottom story. A time-domain formulation for the response analysis of a single-degee-of-freedom structure and a two-degree-of-freedom 2DOF structure equipped with inerter are developed. The seismic performance of supplemental rotational inertia system
compared to traditional energy dissipation mechanism. Both a single inerter and a pair of clutching inerters that can only resist the motion of the structure are examined.

The nonlinear behavior of structures equipped with supplemental rotational inertia is investigated by using the Bouc-Wen hysteretic model. The effect of rigid and compliant inerters supports are examined. The post-yielding behavior of the system is investigated, and the advantages and challenges associated with using supplemental rotational inertia are discussed.

The seismic performance of high-rise yielding structures equipped with the novel response-modification strategy, the outrigger-inerter system, is studied. The proposed seismic control mechanism uses inerters vertically within a conventional core-to-external column outrigger system. To study the seismic behavior of the outrigger-inerter system, a new material developed in C++. This new material is used to represent the behavior of inerters in the OpenSees platform. Both single inerter and a pair of clutching inerters are examined.

This research concludes that supplemental rotational inertia effectively controls the seismic response of structures and could emerges as an attractive response modification strategy with potential to replace the traditional energy dissipation systems in building structures. The last chapter of this dissertation studies the seismic response analysis of the 9-story SAC building equipped with pressurized sand dampers. Sand dampers are low-cost energy dissipation devices wherein the material enclosed within the damper housing is pressurized sand. The strength of the pressurized sand damper is proportional to the externally exerted pressure on the sand via prestressed steel rods. The strong pinching behavior of the pressurized sand dampers is characterized with a previously developed 3-parameter Bouc-Wen hysteretic model. The model was implemented in the open source code OpenSees with a C++ algorithm and used to analyze the seismic response of a 9-story SAC steel building subjected to several strong ground motions.
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This work is dedicated to my lovely parents Ahmadali and Mahnaz.
Chapter 1
Introduction

1.1 Overview

Response modification of structures is by now a widely accepted alternative strategy for resisting earthquake shaking. In conventional seismic design, the acceptable performance of a structure during earthquake shaking is based on the lateral force-resisting system being able to absorb and dissipate energy in a stable manner for a large number of cycles. Alternative design procedures have been developed which incorporate earthquake protection systems in the structure. These systems may take the form of seismic isolation systems or supplemental energy dissipation devices (Soong and Dargush 1997; Constantinou et al. 1998). Structural control systems can be employed to enhance the response of structures to seismic loads.

Traditional response modification devices such as base isolation system (BIS), viscous damper (VD), and tuned mass damper (TMD), modify stiffness, damping, and mass and provide passive counter forces (Kelly et al. 1972; Skinner et al. 1974; Clough and Penzien 1975; Robinson and Greenbank 1976; Whittaker et al. 1991; Aiken et al. 1993; Skinner et al. 1993; Kelly 1997; Soong and Dargush 1997; Constantinou et al. 1998; Makris and Chang 2000a, b; Black et al. 2002, 2003, 2004; Symans et al. 2008; Kelly and Konstantinidis 2011; among others). Viscous dampers offer large values of supplemental damping to structures; however, they may suffer from the
viscous heating issue (Makris 1998; Makris et al. 1998; Black and Makris 2007) and potential leaking during prolonged cyclic loading. A linear TMD consists of a secondary mass with spring and damping elements that are tuned to the dominant natural frequency of the primary structure. Tuning a TMD to the fundamental frequency of the structure assures the transfer of considerable amount of kinetic energy from the primary structure to the secondary mass, which is eventually dissipated through a damping element. The efficiency of a TMD is limited by the secondary mass attached at the top of the primary structure. Buckling-restrained braces, BRBs, which are yielding braces that increase the strength and stiffness of a structure and offer stable energy dissipation (Watanabe et al. 1988; Wada et al. 1989; Black et al. 2002, 2003, 2004; FEMA 547 2006) are widely used but their displacement capacity is limited to the inelastic elongation of the steel inner core. Moreover, their pre-yielding elasticity stiffens the structure, and this may attract additional forces prior to yielding (Makris et al. 2021). In this dissertation, two alternative energy dissipation devices are studied: 1) inerter; and 2) pressurized sand dampers.

In early 1980s mechanical snubbers used on safety-related piping and components of nuclear power plants which are similar to inerter. The “inert” was theoretically introduced by Smith (2002), who coined this term for any mechanical arrangement in which the output force is proportional only to the relative acceleration between its end-nodes. The constant of proportionality of the inerter is coined the "inertance" = \(M_R\) (Smith 2002) and has units of mass [\(M\)]. Several kinds of inerter were patented, and a growing number of publications have proposed the use of rotational inertia dampers for the wind and seismic protection of civil structures. Arakaki et al. (1999a, b) proposed a ball–screw assembly to modify the seismic response of structures. Following this work and systematic theoretical and experimental studies in vehicle mechanics and
dynamics (Papageorgiou and Smith 2005; Papageorgiou et al. 2009; Chen et al. 2009; Kuznetsov et al. 2011), Hwang et al. (2007) proposed a rotational inertia damper (RID) in association with a toggle bracing for vibration control of building structures. By the ball–screw mechanism, the story drift is converted into rotational motion in the damper, and kinetic energy is generated by a rotating mass in the damper. Input energy can be dissipated by a viscous fluid damper (rotational inertia–viscous damper, RIVD). The damper's performance depended heavily on the length of the ball–screw lead, the effective mass, effective damping, and consequently, the damper's efficiency significantly increased as the lead decreased. Wang et al. (2011) considered four kinds of basic suspension layouts with their corresponding transfer functions: 1) a traditional suspension with a spring and a damper, 2) a basic parallel inerter arrangement, 3) a basic serial inerter arrangement, and 4) a serial inerter arrangement with centering springs. From the simulation results, inerters were deemed effective in suppressing vibrations from both traffic and earthquakes. Saitoh et al. (2012) studied the performance of a so-called ‘gyro-mass’ provided for mitigating displacements of base isolation systems. Saitoh et al. (2012) focused on three types of base isolation systems incorporating gyro-masses, and a model proposed in this study, called ‘Model II’ exhibits a significant decrease in the relative displacement of the object with respect to the base at low frequencies as well as almost the same decrease in the response acceleration at high frequencies as a conventional base isolation system. Ikago et al. (2012) studied the dynamic response of a single-degree-of-freedom (SDOF) structure equipped with a tuned viscous mass damper (TVMD)—that is, the viscous mass damper (VMD) or rotational inertia–viscous damper (RIVD) in series with an additional spring element. The effectiveness of TVMD is shown by examining the amplitudes of the primary system subjected to harmonic excitation through shake table tests.
by using a small-scale specimen. A seismic control system incorporated with TVMDs is not proportionally damped, and it needs a complex-valued eigenvalue analysis, which is not common among structural designers. Ikago et al. (2012) proposed a seismic response estimation method based on the square root of the sum of the square (SRSS) of the maximum modal responses derived from the undamped real eigenvalue analysis, which gives a good approximation in practical term compared to exact complex-valued eigenvalue analysis. At about the same time, Takewaki et al. (2012) examined the response of SDOF and multi-degree-of-freedom (MDOF) structures equipped with supplemental rotational inertia offered from a ball-screw-type device that sets in motion a rotating flywheel. He showed the influence of inertial dampers on the ground-motion by the influence coefficient vector, \( \{ \eta \} \). It is demonstrated that if an inertial damper is taken out from one story, the inertial dampers above that story do not influence the input acceleration above that story. Garrido et al. (2013) proposed a rotational inertia double-tuned mass damper (RIDTMD) that consists of a tuned mass damper (TMD) in which the typical viscous damper is replaced with a tuned viscous mass damper. Marian and Giaralis (2014) proposed the tuned mass-damper-inerter (TMDI), which constitutes a generalization of the linear TMD. Attention is focused on providing analytical and numerical evidence to demonstrate its enhanced performance compared to the TMD. Lazar et al. (2014) introduced tuned inerter damper (TID) with a similar configuration to that of TMD, which can be considered as a special case of TMDI with zero attached mass. A generalized framework for computation of the response of TID controlled structures has been developed for \( n \)-DOF systems, and the best structural response was obtained with the inerter installed at the bottom story level, connected to the ground. Ishii et al. (2014) stated that oil dampers require very stiff supporting members in order to dissipate energy. Ishii et al. (2014) proposed a method for
externally installing TVMD to high-rise-buildings and pointed out that the installation of TVMD needs to meet two requirements: 1) The supporting members of the TVMD need to be flexible and 2) The TVMD can dissipate more energy when the relative displacement between the TVMD and the support members is large. Tuned viscose mass damper systems have been developed and installed in a few structures (Sugimura et al. 2012; Ogino and Sumiyama 2014).

More recently, Makris and Kampas (2016) introduced the implementation of two parallel-rotational-inertia systems together with the use of a clutch (pair of clutching inerters) so that the rotating flywheels only resist the motion of the structure without inducing any deformations. The benefits of using a pair of counter-rotating inerters were subsequently examined on a 2DOF elastic structure (Makris and Moghimi 2019). De Domenico and Ricciardi (2018) addressed a vibration control system combining the conventional base-isolation system (BIS) with an inerter-based device that so-called ‘enhanced BIS.’ By attaching an inerter-based device, a TMDI, or a TID, to the isolation floor, it is demonstrated that the displacement demand of base-isolated structures can be significantly reduced. Taflanidis et al. (2019) examined the multi-objective design of inerter-based vibration absorbers (IVAs), focusing on the three most widely considered IVAs in the literature—that is the TVMD, the TMDI, and the TID, for seismic risk mitigation of building structures aiming to quantify in a practical context the compromise between the competing objectives of improving seismic performance and avoiding large IVA control forces. Javidialesaadi and Wierschem (2018) proposed a one-directional rotational inertia viscous damper (ODRIVD), which is similar to a pair of clutching inerters (Makris and Kampas 2016; Makris and Moghimi 2019). The ODRIVD allows for energy to be passively transferred from a primary structure to a rotational flywheel in a one-directional fashion.
There is a growing number of publications have proposed the use of inerter-based systems for the wind and seismic protection of civil structures (Hoang and Warnitchai 2005; Chen et al. 2014; Giaralis and Taflanidis 2018; Pietrosanti et al. 2017; Wen et al. 2017; Makris 2017, 2018; De Domenico and Ricciardi 2018; among others). Fig. 1.1 shows the most widely considered two-terminal-inerter-based systems in the literature.

**Fig. 1.1. Inerter-based device configurations**
1.2 Objectives and Scope

The goal of this doctoral research is to investigate the seismic response of structures with new response modification devices. This study compare the seismic response of structures equipped with these new response modification devices with seismic response of the same structures when they are equipped with traditional response modification systems.

The objectives of this dissertation are:

• A comprehensive study to identify the advantages and limitations of the use of inerters on the seismic response of multi-degree-of-freedom structures for the both cases of single inerter and a pair of clutching inerters.

• Study the seismic performance of high-rise yielding structures equipped with the novel response-modification strategy, the outrigger-inerter system in which the inerters are installed vertically within a conventional core-to-external column outrigger system.

• Introduce a preliminary study of the seismic response of the 9-story SAC building equipped with low-cost pressurized sand dampers—a new type of low-cost energy dissipation devices where the material enclosed within the damper housing is pressurized sand.
Chapter 2

Review of Seismic Response of SDOF Elastic Structures with Inerters

This chapter reviews the seismic response of a single-degree-of-freedom (SDOF) elastic structure with supplemental rotational inertia. Makris and Kampas (2016) introduced the implementation of two parallel-rotational-inertia systems together with the use of a clutch (pair of clutching inerters). Supplemental rotational inertia suppresses displacements effectively at the expense of transferring appreciable forces at the support of the inerter. Both single inerter and a pair of clutching inerters are examined.

As briefly mentioned in chapter 1, the "inerter" was theoretically introduced by Smith (2002), who coined this term for any mechanical arrangement in which the output force is proportional only to the relative acceleration between its end-nodes. For instance, the driving spinning top in Fig. 2.1 is a physical realization of the inerter because the driving force is only proportional to the relative acceleration between nodes 1 and 2. The constant of proportionality of the inerter is coined the "inertance" $= M_R$ (Smith 2002) and has units of mass $[M]$. The unique characteristic of the inerter is that it has an appreciable inertial mass as opposed to a marginal gravitational mass. Accordingly, if $F_1, u_1$ and $F_2, u_2$ are the forces and displacements at the end-nodes of the inerter with inertance, $M_R$, its constitutive relation is (Saitoh 2012; Makris 2018; Makris and Moghimi 2019):
\[
\begin{bmatrix}
F_1(t) \\
F_2(t)
\end{bmatrix} =
\begin{bmatrix}
M_R & -M_R \\
-M_R & M_R
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_1(t) \\
\ddot{u}_2(t)
\end{bmatrix}
\]

\[
(2.1)
\]

Smith and coworkers developed and tested both a rack-and-pinion inerter and a ball-screw inerter (Papageorgiou and Smith 2005; Papageorgiou et al. 2009). Upon its conceptual development and experimental validation, the inerter was implemented to control the suspension vibrations of racing cars under the name J-damper (Chen et al. 2009; Kuznetsov et al. 2011).

![Physical realization of inerter](image)

**Fig. 2.1.** Physical realization of inerter in which the force output is proportional only to the relative acceleration of Nodes 1 and 2 and is the mechanical analog of the capacitor in a force current/velocity-voltage analogy. (Image by Nicos Makris.)
2.1 Dynamics of Supplemental Rotational Inertia

Figure 2.2(a) depicts a SDOF structure in which the mass, \( m_1 \), is engaged with a flywheel with radius \( R_1 \) and mass \( m_{w1} \) that can rotate about an axis O. Consider the case of a stiff chevron frame as a support of the inerter, the deformation of which is negligible to the translational displacements, \( u_1(t) \), of the SDOF structure. Concentric to the flywheel, there is an attached pinion with radius \( \rho_1 \) engaged with a linear rack connected to the bottom of the mass \( m_1 \) of the SDOF structure. With this arrangement when the mass \( m_1 \) undergoes a positive displacement, \( u_1(t) \), the flywheel is subjected to a clockwise rotation, \( \theta_1(t) \). Because there is no slipping between the rack and the pinion

\[
\theta_1(t) = \frac{u_1(t)}{\rho_1}
\]  

(2.2)

For a positive displacement, \( u_1(t) \), to the right, the internal force, \( F_I(t) \) at the rack-pinion interface opposes the motion (to the left) [Fig 2.1(a)]. Moment equilibrium of the flywheel about point O is

\[
I_{W1}\ddot{\theta}_1(t) = F_I(t)\rho_1
\]  

(2.3)

where \( I_{W1} = (1/2)m_{w1}R_1^2 \) = moment of inertia of the flywheel about point O.

![Fig. 2.2.](image)

(a) An elastic SDOF structure with supplemental rotational inertia; (b) An elastic SDOF structure with supplemental damping \( c_1 = c_c + c_d \).
Substituting Eq. (2.2) into Eq. (2.3) gives:

\[
F_I(t) = \frac{1}{2} m_{W1} \frac{R_1^2}{\rho_1^2} \ddot{u}_1(t) = M_R \ddot{u}_1(t)
\]  

(2.4)

Equation (2.4) gives the inertial force, \(F_I(t)\), at the rack-pinion interface—that is, the force transferred to the stiff chevron frame. The constant of proportionality, \(M_R = (1/2) m_{W1}(R^2/\rho_1^2)\) is the inertance of the supplemental rotational inertia system and has units of mass [M]. The inertance, \(M_R\), can be amplified by adding two (or more) flywheels in series, in which the first flywheel is a gearwheel (Smith 2002; Makris and Kampas 2016) as shown in Fig. 2.3. The suppression coefficient assumes the form (Makris and Kampas 2016)

\[
\sigma = \frac{1}{2} \frac{m_{W1}}{m} \frac{R_1^2}{\rho_1^2} + \frac{1}{2} \frac{m_{W2}}{m} \frac{R_2^2}{\rho_2^2} + \cdots + \frac{1}{2} \frac{m_{Wn}}{m} \frac{R_n^2}{\rho_n^2}
\]  

(2.5)

For a ratio \(R_j/\rho_j \approx 10\), each term in Eq. (2.5) is two orders of magnitude larger than the previous term; therefore, for any number \(n\) of flywheels selected, the suppression coefficient is merely governed by the last term of Eq. (2.6).

![Diagram of rack and flywheels](image)

**Fig. 2.3.** More than one flywheel in series that amplify the effect of supplemental rotational inertia (Makris and Kampas 2016)
\[ \sigma = \frac{1}{2} \frac{m_{wn}}{m} \frac{R_1^2 R_2^2 \ldots R_n^2}{\rho_1^2 \rho_2^2 \ldots \rho_n^2} \]  \hspace{1cm} (2.6)

Accordingly, regardless of how small the ratio \( m_{wn}/m \) is, the suppression coefficient \( \sigma \) can assume any desired value with the sufficient size and number of flywheels.

2.2 Equation of Motion of an Elastic SDOF Structure with Inerters Supported on a Stiff Chevron Frame

With reference to Fig. 2.2(a), this section reviews the dynamic response of an elastic SDOF structure with vibrating mass, \( m_1 \), that is engaged with an inerter with inertance, \( M_R \), supported on a stiff chevron frame. Dynamic equilibrium of the vibrating mass when subjected to a ground excitation, \( \ddot{u}_g(t) \), gives:

\[ m_1 \left[ \ddot{u}_1(t) + \ddot{u}_g(t) \right] = -F_i(t) - c_1 \dot{u}_1(t) - k_1 u_1(t) \]  \hspace{1cm} (2.7)

where \( F_i(t) \) = internal force from the flywheel given by Eq. (2.4). By introducing the nominal frequency, \( \omega_1 \), viscose damping ratio, \( \xi_1 \) and inertance ratio, \( \sigma \)

\[ \omega_1^2 = \frac{k_1}{m_1}, \hspace{1cm} 2\xi_1 \omega_1 = \frac{c_1}{m_1}, \hspace{1cm} \sigma = \frac{M_R}{m_1} \]  \hspace{1cm} (2.8)

Equation (2.7) assumes the form

\[ \ddot{u}_1(t) + \frac{2\xi_1 \omega_1 \dot{u}_1(t)}{(1+\sigma)} + \frac{\omega_1^2 u_1(t)}{(1+\sigma)} = -\frac{\ddot{u}_g(t)}{(1+\sigma)} \]  \hspace{1cm} (2.9)

The engagement of the flywheel in a rotational motion not only lengthens the vibration period of the structure (Chen et al. 2014) but also suppresses the level of input ground motion (Makris and Kampas 2016). The solution of the Eq. (2.9) is computed numerically via a state-space formulation...
(Konstantinidis and Makris 2005; Pitilakis and Makris 2010; Vassiliou and Makris 2012; Aghagholizadeh and Makris 2018; Makris and Moghimi 2019). The state-vector of the system is

\[ \{y(t)\} = (y_1(t), y_2(t))^T = (u_1(t), \dot{u}_1(t))^T \]  

\[ (2.10) \]

where the superscript, $T$, stands for the transpose of the line vector, $<$ >, and the time-derivative state-vector, $\{\dot{y}(t)\}$, is expressed solely in terms of the state-variables appearing in the state-vector given by Eq. (2.10)

\[ \{\dot{y}(t)\} = \begin{cases} \ddot{u}_1(t) \\ \ddot{u}_1(t) \end{cases} = \begin{cases} y_2(t) \\ \frac{1}{1+\sigma} \left[ -\ddot{u}_g(t) - 2\xi_1\omega_1y_2(t) - \omega_1^2y_1(t) \right] \end{cases} \]  

\[ (2.11) \]

Figure 2.4(a) plots the relative displacement, $u_1(t)$, velocity, $\dot{u}_1(t)$, force transferred to the first floor by the pinion [opposite of force transferred to the chevron frame given by Eq. (2.4)], and absolute acceleration of the first story show in Fig. 2.2(a) with $T_0 = 1.0$ s, when subjected to a one-sine acceleration pulse with acceleration amplitude $a_p = 0.5g$ and pulse duration $T_p = 0.5$ s. In the interest of simplicity in this analysis, zero damping is assumed ($\xi_1 = 0$). The shaded stripes in Fig. 2.4 correspond to the time-segments where the magnitude of the relative velocity of the first story, $\dot{u}_1(t)$, reduces on its way to reach a peak displacement. During this interval, the flywheels have built angular momentum and now, as the translating mass tends to move slower, the flywheels may drive the mass, therefore inducing deformations—a situation that is not desirable.

One challenge with the proposed concept is that the rotating flywheels should only resist the motion of the structure without inducing any deformations. This is feasible with the use of a clutch so that the pinion of the first gearwheel that is engaged to the rack is unable to drive the
Fig. 2.4. Response of an elastic SDOF structure with stiff chevron frame: (a) single inerter, which may induce deformations; (b) pair of clutching inerters that can resist only the motion.
rack and only the motion of the translating rack can drive the pinion-gearwheel (Makris and Kampas 2016). This is similar to the motion of a bicycle, where the cyclist can drive the wheel through the pedals; yet, when the bicycle is rolling, the pedals may remain idle. Without loss of generality, assume that upon initiation of motion, the structure moves to the left; therefore, the front gearwheel rotates counterclockwise and the force on $m_1$ from the gearwheel is to the right (positive). As the mass, $m_1$, keeps moving to the left, it will slow down and at the instant where the gearwheel will tend to drive $m_1$ due to its angular momentum, the force transmission needs to become idle. With the proposed arrangement, upon $m_1$ reaching its first maximum displacement, to the left $\dot{u}_1(t) < 0$; the front gearwheel keeps rotating freely counterclockwise without inducing any force to the structure. When the motion reverses and the structure starts moving to the right $\dot{u}_1(t) > 0$, a second, parallel rotational inertia system (back flywheels) is needed to oppose the motion, and during the course of this motion, the first gearwheel of the back system engaged to the rack and rotates clockwise.

Clearly, with the two parallel front and back rotational inertia systems, the flywheels only resist the motion of the structure and do not give back any energy to the structure. During the time-period when one of the flywheel systems is rotating idle, its rotation needs to decelerate appreciably so that when it is again engaged into motion, it will be capable of resisting the motion through its rotational inertia. This can be achieved by appending an induction generator to the axis of the flywheel. With this arrangement, part of the earthquake-induced energy is converted into electricity. Two parallel-rotational-inertia systems and the use of clutching ineters have been extensively investigated by Makris and Kampas (2016). Fig. 2.4(b) plots the same response
quantities as presented in Fig. 2.4(a); however now, the rotating flywheels only resist the motion of the structure (when the flywheels rotate idle, the transmitting force is zero and in this way throughout the response history, the force from the flywheels and velocity always have opposite signs).

2.3 Response Spectra of a SDOF Structure with a Stiff Chevron Frame

Figure 2.5 shows the recorded acceleration time histories used for the response spectrum analysis presented in this study: (a) Gilroy Array 6/230 ground motion recorded during the 1979 Coyote Lake, USA earthquake, (b) Cholame Number 2/360 ground motion recorded during the 2004 Parkfield California earthquake, (c) Newhall/360 ground motion recorded during 1994 Northridge, California earthquake, (d) Takarazuka/000 ground motion recorded during the 1995 Kobe, Japan earthquake, and (e) Pacoima Dam/164 ground motion recorded during the 1971 San Fernando, California earthquake.

The seismic response of an elastic SDOF structure equipped with an inerter is compared with the response of the same elastic SDOF structure where the inerter is replaced by a supplemental viscous damper [Fig. 2.2(b)]. In this case, the value of the damping coefficient, $c_1 = c_c + c_d$, where $c_c$ is the damping originating from the SDOF structure and $c_d$ is the damping associated with supplemental viscous damper. Together with the drift response, $u_1$ (relative displacement), of interest are the total acceleration of the structure, $\ddot{u}_1 + \ddot{u}_g$ which is the normalized base shear of the structure, and the normalized force transferred to the mounting of the flywheel, $F_1/m_1g$ or to the mounting of the supplemental damper $c_d\dot{u}_1/m_1g = 2\xi_d\omega_1\dot{u}_1/g$. 

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Fig. 2.5. Acceleration time histories recorded during: (a) 1979 Coyote Lake, California earthquake; (b) 2004 Parkfield, California earthquake; (c) 1994 Northridge, California earthquake; (d) 1995 Kobe, Japan earthquake; and (e) 1971 San Fernando, California earthquake.
Fig. 2.6. Response spectra of an elastic SDOF structure equipped with inters (heavy solid lines) or supplemental viscous damping (dashed lines) supported on a stiff frame when excited by the Gilroy Array 6/230 ground motion recorded during the 1979 Coyote Lake earthquake: (a) Single inerter; (b) Pair of clutching initters.
Fig. 2.7. Response spectra of an elastic SDOF structure equipped with ineters (heavy solid lines) or supplemental viscous damping (dashed lines) supported on a stiff frame when excited by the Cholame Number Array 2/360 ground motion recorded during the 2004 Parkfield earthquake: (a) Single inerter; (b) Pair of clutching ineters.
Fig. 2.8. Response spectra of an elastic SDOF structure equipped with inerters (heavy solid lines) or supplemental viscous damping (dashed lines) supported on a stiff frame when excited by the Newhall /360 ground motion recorded during the 1994 Northridge earthquake: (a) Single inerter; (b) Pair of clutching inerters.
Fig. 2.9. Response spectra of an elastic SDOF structure equipped with inerters (heavy solid lines) or supplemental viscous damping (dashed lines) supported on a stiff frame when excited by the Takarazuka/000 ground motion recorded during the 1995 Kobe: (a) Single inerter; (b) Pair of clutching inerters.
Fig. 2.10. Response spectra of an elastic SDOF structure equipped with inerters (heavy solid lines) or supplemental viscous damping (dashed lines) supported on a stiff frame when excited by the Pacoima Dam/164 ground motion recorded during the 1971 San Fernando: (a) Single inerter; (b) Pair of clutching inerters.
The response spectra shown in Figs. 2.6–2.10 are the results of the solution of Eq. (2.9) for single inerter (left plots) and a pair of clutching inerters (right plots) [see Makris and Kampas 2016 for details]. If $\sigma = 0$ (thin line), the solution offers the response of the structural systems without any control devices. For the structural system shown in Fig. 2.2(a), values of the normalized inertance $\sigma = 0.5$ and $\sigma = 1.0$ are used. For the structural system shown in Fig. 2.2(b) values of $\xi_c = 2\%$ and $\xi_d = 23\%$ are used so that $\xi_1 = \xi_c + \xi_d = 0.02 + 0.23 = 0.25$. The parameter $F_{MD}(t)$ is the resisting force from the response modification device (either a fluid damper with damping constant, $c_d$, or an inerter with inertance constant, $M_R$).

Figure 2.6 presents response spectra for the three aforementioned configurations of the SDOF structure subjected to the GilroyArray 6/230 ground motion recorded during the 1979 Coyote Lake, USA earthquake. Across the spectra, two shaded strips are indicated. The shaded strip for $0.5M \leq T_1 \leq 1.0M$ represents the period range for low-rise structures and the shaded strip for $T_1 \geq 2.0M$, corresponds to seismic isolated structures or tall buildings.

The first observation in Fig. 2.6 is that supplemental rotational inertia is most effective in suppressing the displacement of the structure, $u_1$, in particular for long period structures. If two parallel rotational inertia systems (pair of clutching inerters, right plots) are used, the effectiveness of supplemental rotational inertia [Fig. 2.2(a)] in suppressing $u_1$ outperforms the supplemental damping for which $\xi_1 = 25\%$ in the entire period range. In the period range $0.5s \leq T_1 \leq 1.0s$ the base shear of the structure, $V_1$, is lower when supplemental rotational inertia is used. This situation reverses in the neighborhood of $T_1 = 2.0s$. The forces transferred at the support of the flywheels (chevron frame) are appreciable; however, when a pair of inerters is used, these forces are reduced appreciably (see bottom plots of Fig. 2.6).
Figs. 2.7−2.10 present response spectra for the three aforementioned configurations of the SDOF structure when subjected to the Cholame Number 2/360 ground motion recorded during the 2004 Parkfield California earthquake; The Newhall/360 ground motion recorded during 1994 Northridge, California earthquake; The Takarazuka/000 ground motion recorded during the 1995 Kobe, Japan earthquake; The Pacoima Dam/164 ground motion recorded during the 1971 San Fernando, California earthquake; respectively, and reveals similar trends as those observed from the response spectra shown in Fig. 2.5.

2.4 Conclusions

In this chapter, the potential advantages and also limitations of using supplemental rotational inertia for the seismic protection of SDOF elastic structures are reviewed. The proposed concept employs a rack pinion-flywheel system whose resisting force is proportional to the relative acceleration between the vibrating mass and support of the flywheels. The cases of single inerter and a pair of clutching inerters that can only resist the motion of the structure without inducing any deformation (Makris and Kampas 2016) supported on a stiff chevron frame were investigated and the corresponding equations of motion were reviewed. This chapter shows that the supplemental rotational inertia controls the displacements of SDOF structures along with a wide range of the response spectrum. When the chevron frame that supports the rotational inertia system is stiff, the use of two parallel-rotational-inertia systems offers improved results for the response of the SDOF structure. The proposed seismic protection strategy can accommodate large relative displacements without suffering from the issue of viscous heating (Makris 1998; Makris et al. 1998; Black and Makris 2007) and potential leaking that challenges the implementation of fluid dampers under prolonged cyclic loading.
Chapter 3
Seismic Response of MDOF Elastic Structures with Inerters and their Optimal Placement

Given the effectiveness of supplemental rotational inertia to suppress the seismic displacements of SDOF systems (Makris and Kampas 2016), the seismic response of the multi-degree-of-freedom (MDOF) structure shown in Fig. 3.1 is investigated. It is important to find the best location along the height of the structure to install inerter-devices in order to get the best seismic performance. To this aim, it is shown that by modifying the stiffness and damping matrices of the system in Fig. 3.1(b), the exact same structural response for a structure equipped with inerter-devices in any arbitrary story other than first floor could be obtained. A two-degree-of-freedom, 2DOF, structure is used as the demonstration of this interesting behavior. It is also shown that by locating supplemental rotational inertia at the first story, the level of ground shaking at the base level is suppressed given that the input ground acceleration is multiplied with a quantity that is always lower than unity (Makris and Moghimi 2019). Lazar et al. (2014) also showed that the best structural response is obtained when inerters are installed at the bottom story level. Finally the advantages and limitations of using supplemental rotational inertia for the seismic protection of elastic 2DOF structures is investigated. A 2DOF structure can be viewed as the idealization of a structure supported on solitary columns, known in modern architecture as a structure on *pilotis*. Both cases of a single inerter and a pair of clutching inerters that can only resist the motion of the
structure are examined. The effect of the compliance of the chevron frame as the support of inerter-devices on the seismic performance of structures is also investigated.

3.1 Equations of Motion of an Elastic MDOF Structure with Inerters Supported on a Stiff Chevron Frame and an Elastic MDOF Structure with Additional Mass at the Top Story

With reference to Fig. 3.1(a), if supplemental rotational inertia is mounted at the $j^\text{th}$ story, the equations of motion of a structure equipped with inerters supported on a stiff chevron frame can be expressed in matrix form as

$$\begin{bmatrix} M_a \end{bmatrix}\{\ddot{u}\} + \begin{bmatrix} C_a \end{bmatrix}\{\dot{u}\} + \begin{bmatrix} K_a \end{bmatrix}\{u\} = -\{m_a\} \ddot{u}_g$$ (3.1)

in which the subscript $a$ indicates the structure shown in Fig. 3.1(a). By multiplying Eq. (3.1) from the left by the inverse of the mass matrix, $\begin{bmatrix} M_a \end{bmatrix}^{-1}$, the acceleration vector, $\{\ddot{u}\}$, is expressed as

$$\{\ddot{u}\} = -\left(\begin{bmatrix} M_a \end{bmatrix}^{-1}\begin{bmatrix} C_a \end{bmatrix}\right)\{\dot{u}\} - \left(\begin{bmatrix} M_a \end{bmatrix}^{-1}\begin{bmatrix} K_a \end{bmatrix}\right)\{u\} - \begin{bmatrix} M_a \end{bmatrix}^{-1}\{m_a\} \ddot{u}_g$$ (3.2)

Equation (3.2) can be expressed as

$$\{\ddot{u}\} = -\left[\tilde{C}\right]\{\dot{u}\} - \left[\tilde{K}\right]\{u\} - \{1\} \ddot{u}_g$$ (3.3)

where damping matrix $\left[\tilde{C}\right] = \begin{bmatrix} M_a \end{bmatrix}^{-1}\begin{bmatrix} C_a \end{bmatrix}$, and stiffness matrix $\left[\tilde{K}\right] = \begin{bmatrix} M_a \end{bmatrix}^{-1}\begin{bmatrix} K_a \end{bmatrix}$.

Now with reference to Fig. 3.1(b), the equations of motion of a structure with additional mass at the top story can be expressed in matrix form as

$$\begin{bmatrix} M_b \end{bmatrix}\{\ddot{u}\} + \begin{bmatrix} C_b \end{bmatrix}\{\dot{u}\} + \begin{bmatrix} K_b \end{bmatrix}\{u\} = -\{m_b\} \ddot{u}_g$$ (3.4)
Fig. 3.1. (a) schematic multi degree of freedom (MDOF) structure equipped with inerter supported on a stiff chevron frame in an arbitrary story other than first story; (b) schematic multi degree of freedom (MDOF) structure with an additional mass at the top story.

where the subscript $b$ indicates the structure shown in Fig. 3.1(b). It is found that by modifying the stiffness and damping matrices of the system shown in Fig. 3.1(b), the exact same structural response as system shown in Fig. 3.1(a) could be obtained. In this regard, the modified stiffness and damping matrices are:

$$[K_{eq}] = [M_b][K_a]^{-1}[K_a] = [M_b][\bar{R}]$$  \hspace{1cm} (3.5)
and

\[
[C_{eq}] = [M_a]^{-1}[C_a] = [M_b][\hat{C}]
\] (3.6)

Therefore, Eq. (3.4) can assumes the form

\[
[M_b]\{\ddot{u}\} + [C_{eq}]{\dot{u}} + [K_{eq}]{u} = -[m_b]{\ddot{u}}_g
\] (3.7)

By multiplying Eq. (3.7) from the left by the inverse of the system shown in Fig 3.1(b), \([M_b]^{-1}\), the acceleration of each story become explicit expressions of the displacements, and velocities of the stories

\[
\{\ddot{u}\} = -[M_b]^{-1}[C_{eq}]{\dot{u}} - [M_b]^{-1}[K_{eq}]{u} - [M_b]^{-1}[m_b]{\ddot{u}}_g
\] (3.8)

Equation (3.8) can be expressed in terms of the equivalent stiffness matrix and equivalent damping matrix as defined in Eqs. (3.5) and (3.6)

\[
\{\ddot{u}\} = -[M_b]^{-1}[M_b][\hat{C}]{\dot{u}} - [M_b]^{-1}[M_b][\hat{K}]{u} - \{1\}{\ddot{u}}_g
\] (3.9)

Therefore, Eq. (3.9) assumes the form

\[
\{\ddot{u}\} = -[\hat{C}]{\dot{u}} - [\hat{K}]{u} - \{1\}{\ddot{u}}_g
\] (3.10)

It is clear that Eq. (3.10) for the structure shown in Fig. 3.1(b) is the same as Eq. (3.3) which is obtained from the structure in Fig. 3.1(a).

The above formulation is validated with the response analysis of a two-degree-of-freedom structure is used as demonstration [Fig. 3.2]. The equations of the motion for the 2DOF structure shown in Fig. 3.2(a) can be expressed in matrix form as
\[
\begin{bmatrix}
  m_1 + M_R & -M_R \\
  -M_R & m_2 + M_R 
\end{bmatrix}
\begin{bmatrix}
  \ddot{u}_1(t) \\
  \ddot{u}_2(t) 
\end{bmatrix}
+ \begin{bmatrix}
  c_1 + c_2 & -c_2 \\
  -c_2 & c_2 
\end{bmatrix}
\begin{bmatrix}
  \dot{u}_1(t) \\
  \dot{u}_2(t) 
\end{bmatrix}
+ \begin{bmatrix}
  k_1 + k_2 & -k_2 \\
  -k_2 & k_2 
\end{bmatrix}
\begin{bmatrix}
  u_1(t) \\
  u_2(t) 
\end{bmatrix}
= \begin{bmatrix}
  m_1 \\
  m_2 
\end{bmatrix}
\ddot{u}_g(t) \tag{3.11}
\]

The inverse of the mass matrix assumes the form

\[
\begin{bmatrix}
  m_1 + M_R & -M_R \\
  -M_R & m_2 + M_R 
\end{bmatrix}^{-1}
= \begin{bmatrix}
  \frac{m_2 + M_R}{\psi} & \frac{M_R}{\psi} \\
  \frac{M_R}{\psi} & \frac{m_1 + M_R}{\psi} 
\end{bmatrix}
\tag{3.12}
\]

in which \( \psi = m_2M_R + m_1(m_2 + M_R) \). By multiplying Equation (3.11) from the left with the inverse of the mass matrix

\[
\begin{bmatrix}
  \ddot{u}_1(t) \\
  \ddot{u}_2(t) 
\end{bmatrix}
+ \begin{bmatrix}
  \frac{c_1(m_2+M_R)+c_2m_2}{\psi} & -\frac{c_2m_2}{\psi} \\
  -\frac{c_2m_1+c_1M_R}{\psi} & \frac{c_2m_1}{\psi} 
\end{bmatrix}
\begin{bmatrix}
  \dot{u}_1(t) \\
  \dot{u}_2(t) 
\end{bmatrix}
+ \begin{bmatrix}
  \frac{k_1(m_2+M_R)+k_2m_2}{\psi} & -\frac{k_2m_2}{\psi} \\
  -\frac{k_2m_1+k_1M_R}{\psi} & \frac{k_2m_1}{\psi} 
\end{bmatrix}
\begin{bmatrix}
  u_1(t) \\
  u_2(t) 
\end{bmatrix}
= \begin{bmatrix}
  -1 \\
  1 
\end{bmatrix}
\ddot{u}_g(t) \tag{3.13}
\]

**Fig. 3.2.** (a) A two-degree-of-freedom (2DOF) structure equipped with ineters supported on a stiff chevron frame in the second story; (b) A two-degree-of-freedom (2DOF) structure with an additional mass at top story.
Equation (3.13) can be expressed as

\[
\begin{bmatrix}
\dot{u}_1 \\
\dot{u}_2
\end{bmatrix} = -\begin{bmatrix}
\hat{c} \\
\hat{K}
\end{bmatrix} \begin{bmatrix}
\dot{u}_1 \\
\dot{u}_2 \\
1
\end{bmatrix} - \begin{bmatrix}
1
\end{bmatrix} \ddot{g}(t) 
\] (3.14)

Where damping matrix, \([\hat{c}]\), and stiffness matrix, \([\hat{K}]\), are

\[
\hat{c} = \begin{bmatrix}
\frac{c_1(M_2+M_R)+c_2m_2}{\psi} & -\frac{c_2m_2}{\psi} \\
-\frac{c_2m_1+c_1M_R}{\psi} & \frac{c_2m_1}{\psi}
\end{bmatrix} \quad \hat{K} = \begin{bmatrix}
\frac{k_1(M_2+M_R)+k_2m_2}{\psi} & -\frac{k_2m_2}{\psi} \\
-k_2m_1+k_1M_R & \frac{k_2m_1}{\psi}
\end{bmatrix} 
\] (3.15)

Now, with reference to Fig. 3.2(b), the equations of the motion of the 2DOF structure with added mass at the second story can be written in matrix form as

\[
\begin{bmatrix}
m_1 & 0 \\
0 & m_2+M
\end{bmatrix} \begin{bmatrix}
\ddot{u}_1(t) \\
\ddot{u}_2(t)
\end{bmatrix} + \begin{bmatrix}
C_{eq} \\
K_{eq}
\end{bmatrix} \begin{bmatrix}
\dot{u}_1(t) \\
\dot{u}_2(t)
\end{bmatrix} + \begin{bmatrix}
K_{eq} \\
K_{eq}
\end{bmatrix} \begin{bmatrix}
u_1(t) \\
u_2(t)
\end{bmatrix} = -\begin{bmatrix}
m_1 \\
(\dot{M} + M)
\end{bmatrix} \ddot{g}(t) 
\] (3.16)

where the equivalent damping matrix \([C_{eq}]\), and the equivalent stiffness matrix \([K_{eq}]\), defined as

\[
[C_{eq}] = \begin{bmatrix}
m_1 & 0 \\
0 & m_2+M
\end{bmatrix} \begin{bmatrix}
\frac{m_1(c_2m_2+c_1(M_2+M_R))}{\psi} & -\frac{c_2m_1m_2}{\psi} \\
\frac{(c_1M_R-c_2m_1)(m_2+M)}{\psi} & \frac{c_2m_1(m_2+M)}{\psi}
\end{bmatrix} 
\] (3.17)

\[
[K_{eq}] = \begin{bmatrix}
m_1 & 0 \\
0 & m_2+M
\end{bmatrix} \begin{bmatrix}
\frac{m_1(k_2m_2+k_1(M_2+M_R))}{\psi} & -\frac{k_2m_1m_2}{\psi} \\
\frac{(k_1M_R-c_2m_1)(m_2+M)}{\psi} & \frac{k_2m_1(m_2+M)}{\psi}
\end{bmatrix} 
\] (3.18)
Where again $\psi = m_2 M_R + m_1 (m_2 + M_R)$. The inverse of the mass matrix in Equation (3.16) is

$$
\begin{bmatrix}
m_1 & 0 \\
0 & m_2 + M
\end{bmatrix}^{-1} = \begin{bmatrix}
\frac{1}{m_1} & 0 \\
0 & \frac{1}{m_2 + M}
\end{bmatrix}
$$

(3.19)

By multiplying Eq. (3.16) from the left with the inverse of the mass matrix, Eq. (3.19)

$$
\begin{bmatrix}
\ddot{u}_1(t) \\
\ddot{u}_2(t)
\end{bmatrix} + \begin{bmatrix}
\frac{c_1 (m_2 + M_R) + c_2 m_2}{\psi} & -\frac{c_2 m_2}{\psi} \\
-\frac{c_2 m_1 + c_1 M_R}{\psi} & \frac{c_2 m_1}{\psi}
\end{bmatrix} \begin{bmatrix}
\ddot{u}_1(t) \\
\ddot{u}_2(t)
\end{bmatrix} + \begin{bmatrix}
\frac{k_1 (m_2 + M_R) + k_2 m_2}{\psi} & -\frac{k_2 m_2}{\psi} \\
-\frac{k_2 m_1 + k_1 M_R}{\psi} & \frac{k_2 m_1}{\psi}
\end{bmatrix} \begin{bmatrix}
u_1(t) \\
u_2(t)
\end{bmatrix} = -\begin{bmatrix}
1 \\
1
\end{bmatrix} \ddot{u}_g(t) 
$$

(3.20)

Equation (3.20) can be expressed as

$$
\begin{bmatrix}
\ddot{u}_1 \\
\ddot{u}_2
\end{bmatrix} = -[\hat{C}] \begin{bmatrix}
\ddot{u}_1 \\
\ddot{u}_2
\end{bmatrix} - [R] \begin{bmatrix}
u_1 \\
u_2
\end{bmatrix} - \begin{bmatrix}
1 \\
1
\end{bmatrix} \ddot{u}_g(t) 
$$

(3.21)

It is shown that Eq. (3.21) for the 2DOF structure shown in Fig. 3.1(b) is the same as Eq. (3.14) which is obtained from the 2DOF structure in Fig. 3.1(a).

### 3.2 The Open "Soft" First Story: From Aesthetics, Functionality and Seismic Isolation to a Lateral Failure Mechanism

As it is mentioned earlier, a 2DOF system can be viewed as the idealization of a structure supported on solitary columns, known in modern architecture as a structure on *pilotis*. In this configuration, only the first story (*pilotis*) is engaged to a rotational flywheel system in an effort to investigate to what extent the use of supplemental rotational inertia (use of inerters as a retrofit strategy) can limit large displacements versus the use of large values of supplemental damping.
The next section compares the computed response quantities of the 2DOF system shown in Fig. 3.3(a) with those when the *pilotis* is retrofitted with large values of supplemental damping [Fig. 3.3(b)] and with those from the “classical” two-degree-of-freedom system shown in Fig. 3.3(c) that has been used to introduce the linear theory of seismic isolation, (Kelly 1997; Kelly and Konstantinidis 2011).

Fig. 3.3. (a) 2DOF structure engaged with a rotational flywheel system;(b) 2DOF structure retrofitted with supplemental damper at first soft-story; and (c) classical two-degree-of-freedom seismic isolation system.
Prior to world war II, structural engineers at the California Institute of Technology (Martel 1929) and at Stanford University (Jacobsen 1938) viewed the flexible first story—that is LeCorbusier’s *pilotis* (LeCorbusier 1925; 1986-translated in English by F. Etchells 1931)—as a way to lengthen the period of the structure and reduce the shear forces at the base of the structure. Because of the low lateral stiffness of the solitary columns of the "soft" first story, the deformation demands would be concentrated in these first-story columns which essentially isolate the superstructure from the ground shaking. This early concept of seismic isolation (Martel 1929; Green 1935; Jacobsen 1938) assumed that the solitary columns would remain elastic during the earthquake shaking. In the late 1960s, Fintel and Khan (1969) after examining first-story failures from several earthquakes, further advanced the concept of the soft first-story seismic isolation by indicating that the solitary columns will deform inelastically; therefore, offering some energy dissipation. In their remarkable paper Fintel and Khan (1969) recognized that the yielding columns of the soft, first-story will convert the first-story into a mechanism; therefore, they suggested the construction of core stabilizing walls that will support the weight of the structure via elastomeric (polychloroprene= neoprene) bearings with low lateral stiffness. Accordingly, the Fintel and Khan (1969) paper suggested an early concept of seismic isolation with bilinear behavior where recentering is offered by the neoprene bearings and dissipation is offered by the yielding first-story columns. Soon after the Fintel and Khan paper and immediately after the devastating 1971 San Fernando California earthquake, comprehensive numerical studies at the University of California, Berkley (Chopra et al. 1972) showed that the displacement demands due to a "soft" first story may be exceedingly large to the extent that the effect of the gravity loads from the upper levels during this sideways deformation lead to collapse of the entire structure. This lateral failure mechanism
was responsible for the spectacular failure of the Olive View Hospital during the 1971 San Fernando, California earthquake shown in Fig. 3.4 (Bertero et al. 1978); and it became clear to the civil engineering profession that the *pilotis*, while an architectural concept with several aesthetic and functionality advantages, it has poor seismic performance (Bertero et al. 1978; Arnold 1984; Bertero et al. 1991; Repapis et al. 2006; Antonopoulos and Anagnostopoulos 2017, among others). Despite its poor seismic performance, the *pilotis* is widely used by architects around the world.

**Fig. 3.4.** The iconic soft-story failure of the Olive View Hospital during the 1971 San Fernando, California earthquake. Photograph is available to the public by the USGS.
(Arnold 1984); therefore, the chapter examines to what extent supplemental rotational inertia can control the displacements of the two-story structure shown in Fig. 3.3(a).

### 3.3 Response Spectra of the 2DOF Structure with a Stiff Chevron Frame

With reference to Fig. 3.3(a), dynamic equilibrium of the entire structure above the chevron frame gives

\[
m_2[\ddot{u}_1(t) + \ddot{u}_2(t) + \ddot{u}_g(t)] + m_1[\ddot{u}_1(t) + \ddot{u}_g(t)] = -k_1u_1(t) - c_1\dot{u}_1(t) - F_I(t)
\]  

(3.22)

where \(F_I(t)\) is the internal force from the flywheel given by Eq. (2.4). Dynamic equilibrium of the second story gives

\[
m_2[\ddot{u}_1(t) + \ddot{u}_2(t) + \ddot{u}_g(t)] = -k_2u_2(t) - c_2\dot{u}_2(t)
\]  

(3.23)

The solution of the system of equations given by Eqs. (3.22) and (3.23) is computed numerically via a state-space formulation [see Makris and Moghimi 2019 for details].

The seismic response of the 2DOF structure equipped with an inerter at the first story is compared with the seismic response of the same 2DOF structure where the inerter is replaced with a supplemental viscous damper. In this case the value of the damping coefficient \(C_1 = C_c + C_d\), where \(C_c\) is the damping originating from the first-story columns and \(C_d\) is the damping originating from the supplemental viscous damper. Together with the drift responses \(u_1\) and \(u_2\) (relative displacements), of interest are the total acceleration of the first story, \(\ddot{u}_1 + \ddot{u}_g\), the total acceleration of second story, \(\ddot{u}_1 + \ddot{u}_2 + \ddot{u}_g = V_2/m_2\), which is the normalized shear force just above the first story, the total base shear of the structure given by

\[
m_2[\ddot{u}_1(t) + \ddot{u}_2(t) + \ddot{u}_g(t)] = -k_2u_2(t) - c_2\dot{u}_2(t)
\]  

(3.24)
and the normalized force transferred to the mounting of the flywheel, \( F_1(t) / (m_1 + m_2)g \), or to the mounting of the supplemental damper, \( c_d\dot{u}_1 / (m_1 + m_2)g = 2\xi_d\omega_1\dot{u}_1 / g \).

Figures 3.5 shows the response spectra for the 2DOF structure in Fig. 3.3(a) equipped with a single inerter (left plots) and a pair of clutching inerters (right plots). When \( \sigma = 0 \) (thin line), the solution offers the response of the structural systems shown in Figs. 3.3(b) and 3.3(c). For the structural system shown in Fig. 3.3(a) values of the normalized inertance \( \sigma = 0.5 \) and \( \sigma = 1.0 \) are used. For the structural system shown in Fig. 3.3(b) values of \( \xi_c = 5\% \) and \( \xi_d = 20\% \) are used so that \( \xi_1 = \xi_c + \xi_d = 0.05 + 0.20 = 0.25 \). In all spectra, the period of the superstructure is \( T_2 = 0.2 \text{ sec} \), with viscous damping ratio \( \xi_2 = 0.02 \), and mass ratio, \( \mu = 0.5 \). Fig. 3.5 presents response spectra for the three configurations of the 2DOF structure shown in Fig. 3.3(a), (b) and (c) when subjected to the Gilroy Array 6/230 ground motion recorded during the 1979 Coyote Lake, USA earthquake. Across the spectra two shaded strips are indicated. The first strip is for \( 0.5 \leq T_1 \leq 1.0 \) and it represents the period range of \( T_1 \) for a 2DOF structure with the first story being a pilotis. The second shaded strip in the long period range, \( T_1 \geq 2.0 \), corresponds to seismic isolated structures.

The first observation in Fig. 3.5 is that supplemental rotational inertia is most effective in suppressing the displacement of the first story, \( u_1 \), in particular for long period structures. When two parallel rotational inertia systems (pair of inerters, right plots) are used, the effectiveness of supplemental rotational inertia [Fig. 3.3(a)] in suppressing \( u_1 \) outperforms the effectiveness of large values of supplemental damping (\( \xi_1 = 25\% \)) along the entire frequency range. At the same time, in the period range \( 0.5 \leq T_1 \leq 1.0 \) the base–shear of the entire structure, \( V_1 \), is lower when supplemental rotational inertia is used. This situation reverses in the neighborhood of \( T_1 = 1.5 \).
upon which supplemental damping results in lower base shears. At the same time the forces transferred at the support of the flywheels (chevron frame) are appreciable; however, when a pair of inerters is used these forces are comparable to the case where large values of supplemental damping is used (see bottom plots of Fig. 3.5).

In view of the results presented in Fig. 3.5, supplemental rotational inertia \(0.5 \leq \sigma \leq 1.0\), emerges as an attractive alternative to suppress both displacements and base shears of structures supported on *pilotis*—that is \(0.5s \leq T_1 \leq 1.0s\). Among the three configurations examined, seismic isolation \(T_1 \geq 2.0s\) is most effective in reducing floors accelerations and base shears at the expense of producing the largest displacements, \(u_1\). However, isolation systems are designed to accommodate these high displacements above isolators.

Figures 3.6 and 3.7 present response spectra for the three configurations of the 2DOF structure shown in Fig. 3.3(a), (b) and (c) when subjected to the Takarazuka/000 ground motion recorded during the 1995 Kobe Japan earthquake and the Cholame Number Array 2/360 ground motion recorded during the 2004 Parkfield, respectively. Figs. 3.6 and 3.7 reveal similar trends as those observed from the response spectra shown in Fig 3.5.

3.4 Response Spectra of the 2DOF structure with a Compliant Chevron Frame with Finite Stiffness and Damping

Now the case where the chevron frame that supports the rotational inertia system shown in Fig. 3.3(a) has a finite stiffness, \(k_f\), and damping constant, \(c_f\), is considered. Because of its compliance,
Fig. 3.5. Response spectra of two-degree-of-freedom structure equipped with supplemental rotational inertia (heavy solid lines) or supplemental damping (dashed lines) supported on a stiff frame when excited by Gilroy Array 6/230 ground motion recorded during 1979 Coyote Lake earthquake.
Fig. 3.6. Response spectra of two-degree-of-freedom structure equipped with supplemental rotational inertia (heavy solid lines) or supplemental damping (dashed lines) supported on a stiff frame when excited by Takarazuka/000 ground motion recorded during 1995 Kobe, Japan earthquake.
Fig. 3.7. Response spectra of two-degree-of-freedom structure equipped with supplemental rotational inertia (heavy solid lines) or supplemental damping (dashed lines) supported on a stiff frame when excited by the Cholame Number Array 2/360 ground motion recorded during 2004 Parkfield earthquake.
under the force transferred by the mounting of the flywheel, the chevron frame deforms and this
deformation, influences the resisting force, $F_I(t)$, from the flywheel. Accordingly, $F_I(t)$, is no
longer expressed with Eq. (2.4)—that is for a rigid chevron frame [see Makris and Moghimi 2019
for details].

Figure 3.8 shows the response spectra of the 2DOF structure in Fig. 3.3(a) equipped with
a single inerter (left plots) and a pair of clutching inerers (right plots) supported on a compliant
chevron frame. Again, when $\sigma = 0$ (thin line), the solution offers the response of the structural
systems shown in Figs. 3.3(b) and 3.3(c). For the structural system shown in Fig. 3.3(a), values of
the normalized inertance, $\sigma = 0.5$ and $\sigma = 1.0$ are used. The compliance of the chevron frame is
expressed with the relaxation time, $\lambda = c_f/k_f = 0.05$, while the stiffness of the chevron frame
compared to the supplemental inertance $M_R$, is expressed with the dimensionless product $\lambda \omega_R =
0.5$ [see Makris and Moghimi 2019 for details]. For the structural system shown in Fig. 3.3(b),
values of $\xi_c = 0.05$ and $\xi_d = 0.20$ are used so that $\xi_1 = \xi_c + \xi_d = 0.05 + 0.20 = 0.25$. When
supplemental damping $c_d$, is used, the compliance of the chevron frame is $\lambda_D = (c_d + c_f)/k_f =
0.5$. In all spectra, the period of the superstructure is $T_2 = 0.2s$ with viscous damping ratio $\xi_2 =$
0.02 and mass ratio $\mu = 0.5$.

Figure 3.8 present response spectra for the three configuration of the 2DOF structure shown
in Fig. 3.3 when the chevron frame has finite stiffness $k_f$ and damping $c_f$ and is subjected to the
Gilroy Array 6/230 ground motion recorded during the 1979 Coyote Lake, USA earthquake.
Across the spectra the same two shaded strips are indicated as explained when discussing the
spectra shown in Figs. 3.5–3.7.
Fig. 3.8. Response spectra of two-degree-of-freedom structure equipped with supplemental rotational inertia (heavy solid lines) or supplemental damping (dashed lines) supported on a compliant frame when excited by Gilroy Array 6/230 ground motion recorded during 1979 Coyote Lake earthquake.
Fig. 3.9. Response spectra of two-degree-of-freedom structure equipped with supplemental rotational inertia (heavy solid lines) or supplemental damping (dashed lines) supported on a compliant frame when excited by Takarazuka/000 ground motion recorded during 1995 Kobe earthquake.
Fig. 3.10. Response spectra of two-degree-of-freedom structure equipped with supplemental rotational inertia (heavy solid lines) or supplemental damping (dashed lines) supported on a stiff frame when excited by the Cholame Number Array 2/360 ground motion recorded during 2004 Parkfield earthquake.
The first observation in Fig. 3.8 is that even when a compliant chevron frame is used, supplemental rotational inertia remains an effective strategy in suppressing displacements of the first story, \( u_1(t) \), along the entire frequency range. Interestingly, Fig. 3.8 reveals, that the compliance of the chevron frame reduces the effectiveness of the pair of inerterers (right plots) when compared to the case of a single inerter (left plots). Figs. 3.9 and 3.10 show the response spectra for the three configuration of the 2DOF structure shown in Fig. 3.3 when subjected to the Takarazuka/000 ground motion recorded during the 1995 Kobe Japan earthquake, and the Cholame Number Array 2/360 ground motion recorded during the 2004 Parkfield, respectively. They reveal similar trends as those observed from the time history analysis in Fig. 3.8.

In summary, the results presented in Figs. 3.8–3.10 in association with the results presented in Figs. 3.5–3.7 (for a stiff chevron frame) reveal that supplemental rotational inertia is an effective response modification strategy for controlling the response of a structure with a soft first-story at the expense of transferring appreciable forces at the support of the inerter. Accordingly, assuming that the chevron frame is properly designed, supplemental rotational inertia is a competitive alternative to the use of supplemental damping, in particular for cases with large relative displacement demands.

3.5 Conclusion

In this chapter, the potential advantages and also limitations of using supplemental rotational inertia for the seismic protection of elastic moment-resisting frames are reviewed. It is found that by modifying the stiffness and damping matrices of the system shown in Fig 3.1(b), the exact same structural response could be obtained as system shown in Fig 3.1(a). The response of
elastic 2DOF (Makris and Moghimi 2019) structures in which the first story employs a rack–pinion–flywheel system, are examined. The cases of single inerter and a pair of clutching inerters that can only resist the motion of the structure without inducing any deformation (Makris and Kampas 2016) supported on a stiff and a compliant chevron frame were investigated and the corresponding equations of motion were derived [see Makris and Moghimi 2019 for details]. This chapter shows that the supplemental rotational inertia controls the displacements of the first story along with a wide range of the response spectrum. When the chevron frame that supports the rotational inertia system is stiff, the use of two parallel-rotational-inertia systems offers improved results for the response the 2DOF structure. However, as the compliance of the chevron frame that supports the inerters increases, the use of a single rotational inertia system offers a more favorable response. The proposed seismic protection strategy can accommodate large relative displacements without suffering from the issue of viscous heating (Makris 1998; Makris et al. 1998; Black and Makris 2007) and potential leaking that challenges the implementation of fluid dampers under prolonged cyclic loading.
Chapter 4

Seismic Response of Yielding Structures with Inerters

This chapter investigates the seismic response of one- and two-degree-of-freedom yielding structures with inerters installed in the first story. A stable nonlinear response analysis methodology is implemented by using state-space formulation. Given that the engagement with an inerter lengthens the apparent pre-yielding period of the inelastic structure, this chapter shows that when a yielding structure is equipped with supplemental rotational inertia, the equal-displacement rule is valid starting from lower values of the pre-yielding period. The effectiveness of single inerter and a pair of clutching inerters is examined, and it is concluded that a single inerter suppresses the displacement response of inelastic structures effectively by outperforming the response modification with supplemental damping in particular when the supporting frame of the response modification devices is compliant.

A growing number of publications have examined the response of elastic structures invariably. In particular, the recent study of Makris and Moghimi (2019) concluded that while the use of inerters may reduce the first-story displacements of a 2DOF elastic structure effectively, without introducing excessive base shears; under certain strong ground motions, the first-story displacements are large enough suggesting that an inelastic model for the structure is more appropriate. In view of this finding and given the increasing number of strong acceleration records
in urban areas, this chapter examines the inelastic response of a single degree of freedom (SDOF) and a two degree of freedom (2DOF) yielding structure equipped with inerters. Our interest in the inelastic response of a 2DOF yielding structure is primarily motivated by the need to understand to what extent the engagement of an inerter at the first story aggravates the inelastic deformation of the superstructure (Moghimi and Makris 2021).

4.1 Equation of Motion of a Yielding SDOF Structure with Inerters Supported on a Stiff Chevron Frame

With reference to Figs. 4.1(a) and (b), this chapter first examines the dynamic response of a yielding SDOF structure with mass, \( m_1 \), preyielding stiffness, \( k_1 \), postyielding stiffness, \( \alpha k_1 \), and yielding displacement, \( u_{y_1} \), that is engaged with an inerter with inertance, \( M_R \), supported on a stiff chevron frame. Dynamic equilibrium of the vibrating mass when subjected to a ground excitation, \( \ddot{u}_g(t) \), gives:

\[
m_1 \left[ \ddot{u}_1(t) + \ddot{u}_g(t) \right] = -F_{s_1}(t) - c_1 \dot{u}_1(t) - F_I(t)
\]

where \( F_I(t) \) = internal force from the flywheel given by Eq. (2.4) and \( F_{s_1}(t) \) = inelastic restoring force of the structure and is described by the Bouc-Wen model (Wen 1976; Baber and Wen 1981).

\[
F_{s_1} = \alpha k_1 u_1(t) + (1 - \alpha) k_1 u_{y_1} z_1(t)
\]

in which \( \alpha = \) postyielding-to-preyielding stiffness ratio; and \(-1 \leq z_1(t) \leq 1 = \) dimensionless internal variable described by

\[
\dot{z}_1(t) = \frac{1}{u_{y_1}} [\ddot{u}_1(t) - \beta \dot{u}_1(t) |z_1(t)|^n - \gamma |\dot{u}_1(t)| z_1(t) |z_1(t)|^{n-1}]
\]

In Eq. (4.3), constants \( \beta, \gamma \) and \( n = \) model parameters. In this study \( \beta = \gamma = 0.5 \) and \( n = 10 \).
Substitution of Eqs. (2.4) and (4.2) into Eq. (4.1) gives:

\[ m_1 \ddot{u}_1(t) + M_R \dot{u}_1(t) + c_1 \dot{u}_1(t) + \alpha k_1 u_1(t) + (1 - \alpha) k_1 y_1 z_1(t) = -m_1 \ddot{g}(t) \quad (4.4) \]

By using the nominal frequency, \( \omega_1 \), viscose damping ratio, \( \xi_1 \) and inertance ratio, \( \sigma \), defined in Eq. (2.8), Eq. (4.4) assumes the form

\[ (1 + \sigma) \ddot{u}_1(t) + 2\xi_1 \omega_1 \dot{u}_1(t) + \alpha \omega_1^2 u_1(t) + (1 - \alpha) \omega_1^2 y_1 z_1(t) = -\ddot{g}(t) \quad (4.5) \]
The solution of the system of equations given by Eqs. (4.5) and (4.3) is computed numerically via a state-space formulation (Makris and Kampas 2016; Makris and Moghimi 2019; Moghimi and Makris 2021). The state-vector of the system is

\[
\{y(t)\} = \langle y_1(t), y_2(t), y_3(t) \rangle^T = \langle u_1(t), \dot{u}_1(t), \ddot{z}_1(t) \rangle^T
\]  

(4.6)

and the time-derivative state-vector, \(\dot{y}(t)\), is expressed solely in terms of the state-variables appearing in the state-vector given by Eq. (4.6)

\[
\{\dot{y}(t)\} = \begin{cases} 
\dot{u}_1(t) \\
\ddot{z}_1(t)
\end{cases} = \begin{bmatrix} 
\frac{1}{1+\sigma}[-\ddot{y}_g(t) - 2\xi_1 \omega_1 y_2(t) - \alpha \omega_1^2 y_1(t) - (1 - \alpha) \omega_1^2 u_{y_1} y_3(t)] \\
\frac{1}{u_{y_1}}[y_2(t) - \beta y_2(t) |y_3(t)|^n - \gamma |y_2(t)||y_3(t)||y_3(t)|^{n-1}]
\end{bmatrix}
\]

(4.7)

The numerical integration of the time-derivative of the state-vector, \(\dot{y}(t)\), given by Eq. (4.7) is performed with standard ordinary differential Equation (ODE) solvers available in MATLAB.

From the five parameters that appear in the bilinear idealization shown in Fig. 4.1(b) \((k_1 = pre\text{-yielding stiffness, } \alpha k_1 = post\text{-yielding stiffness, } u_{y_1} = yield\text{ displacement, } Q_1 = strength\text{ and } F_{y_1} = yield\text{ force})\), only three parameters are needed to fully describe the bilinear behavior (see, for instance, Makris and Kampas 2013). In this study, the pre-yielding stiffness, \(k_1 = m_1 \omega_1^2 = m_1 4\pi^2/T_1^2\), the post-yielding stiffness, \(k_2 = \alpha k_1\), and the strength of the structure \(Q_1\) are selected.

With reference to Fig. 4.1(b), \(F_{y_1} = k_1 u_{y_1} = Q_1 + \alpha k_1 u_{y_1}\). Accordingly,

\[
u_{y_1} = \frac{Q_1}{k_1 - \alpha k_1} = \frac{Q_1}{m_1} \frac{T_1^2}{4\pi^2(1-\alpha)}
\]

(4.8)
Makris and Kampas (2016) extensively explains the advantages of using a two-parallel-rotational system. The sequential engagement of the two parallel rotational inertial systems that can only resist the motion is expressed mathematically as

\[ \frac{F_I(t)}{m_1} = \sigma \ddot{u}_1(t) \quad \text{when } \text{sgn} \left[ \frac{\ddot{u}_1(t)}{\dot{u}_1(t)} \right] > 0 \quad (4.9a) \]

and

\[ \frac{F_I(t)}{m_1} = 0 \quad \text{when } \text{sgn} \left[ \frac{\ddot{u}_1(t)}{\dot{u}_1(t)} \right] \leq 0 \quad (4.9b) \]

Accordingly, the equation of motion given in Eq. (4.5) is modified to

\[ (1 + \delta \sigma)\dddot{u}_1(t) + 2\xi_1 \omega_1 \ddot{u}_1(t) + \alpha \omega_1^2 u_1(t) + (1 - \alpha) \omega_1^2 u_y1 z_1(t) = -\ddot{u}_g(t) \quad (4.10) \]

in which

\[ \delta = \begin{cases} 1, & \text{when } \text{sgn} \left[ \frac{\ddot{u}_1(t)}{\dot{u}_1(t)} \right] > 0 \\ 0, & \text{when } \text{sgn} \left[ \frac{\ddot{u}_1(t)}{\dot{u}_1(t)} \right] \leq 0 \end{cases} \quad (4.11) \]

Figure 4.2(a) plots the relative displacement, \( u_1(t) \), velocity, \( \dot{u}_1(t) \), force transferred to the stiff chevron frame given by Eq. (2.4), and absolute acceleration of the vibrating mass shown in Fig. 4.1(a) with \( T_1 = 1.0s, Q_1/m_1 = 0.1g \), when subjected to a one-sine acceleration pulse with acceleration amplitude \( a_p = 0.5g \) and pulse duration \( T_p = 0.5s \). The heavy dark line is when an inerter with inerance ratio \( \sigma = 0.5 \) is engaged; whereas, the thin line is when there is no inerter. In the interest of simplicity in this analysis, zero damping \( \zeta_1 = 0 \) is assumed. The shaded stripes in Fig. 4.2 correspond to the time-segments in which the magnitude of the relative velocity of the vibrating mass, \( \dot{u}_1(t) \), decreases on its way to reach a peak displacement. During the previous (accelerating) time-interval, the flywheels have built angular momentum, and as the translating mass tends to move slower, the flywheels may drive the mass, therefore inducing deformations—a
Fig. 4.2. Response of a yielding SDOF structure equipped with inerters that are supported by a stiff frame: (a) Single inerter which may induce deformations; (b) pair of inerters which can only resist the motion of the structure.
situation that is not desirable. The first observation from Fig. 4.2 is that when the inerter is engaged, both relative displacements and velocities are suppressed. Fig. 4.2(b) shows that when a pair of clutching inerters are used (right plots) the peak displacement is slightly less than when a single inerter is used (left plots).

Clearly, with the two parallel rotational inertia systems, the flywheels only resist the motion of the structure and do not give back any energy to the structure. During the period when one of the flywheel systems is rotating idly, its rotation needs to decelerate appreciably so that when it is again engaged into motion, it will be capable of resisting the motion through its rotational inertia. This can be achieved by appending an induction generator to the axis of the flywheel, therefore providing an opportunity for energy harvesting. With this arrangement, part of the earthquake-induced energy is converted into electricity.

4.2 Response Spectra of a Yielding SDOF Structure with Inerters Supported on a Stiff Chevron Frame

The seismic response of an yielding SDOF structure equipped with an inerter as described by Eq. (4.5) or Eqs. (4.10) and (4.11) is compared with the response of the same SDOF yielding structure where the inerter is replaced with a supplemental viscous damper with a damping constant $c_d$ [Fig. 4.1(c)]. In this case, the value of the damping coefficient, $c_1 = c_c + c_d$, where $c_c$ is the damping originating from the columns of the SDOF structure and $c_d$ is the damping originating from the supplemental viscous damper. Together with the drift response, $u_1$ (relative displacement), of interest are the normalized base shear at the columns of the inelastic frame, $V_{columns}/(m_1g)$, the normalized base shear at the foundation of the inelastic structure,
\(V_b/(m_1 g) = (\ddot{u}_1 + \ddot{u}_g)/g\), which is the total acceleration of the first story normalized by \(g\); and the normalized force transferred to the mounting of the flywheel, \(F_t/m_1 g\) or to the mounting of the supplemental damper \(c_d \dot{u}_1/m_1 g = 2\xi_d \omega_1 \dot{u}_1/g\). From Eq. (4.5) or Eqs. (4.10) and (4.11), the normalized base shear at the columns is \([2\xi_c \omega_1 \dot{u}_1(t) + \alpha \omega_1^2 u_1(t) + (1 - \alpha) \omega_1^2 u_{y1} H_1(t)]/g\).

The response spectra shown in Fig. 4.3 are the results of the solution of Eq. (4.5) for a single inerter (left plots) or Eqs. (4.10) and (4.11) when a pair of clutching inerters is used (right plots) to modify the response of an elastoplastic (\(\alpha = 0\)) SDOF structure with normalized strength \(Q_1/m_1 = 0.1 g\) and \(\xi_c = 0\) when subjected to the Gilroy Array 6/230 ground motion recorded during the 1979 Coyote Lake earthquake. When \(\sigma = 0\) (thin black line), the solution offers the response of the elastoplastic SDOF structure without any response modification device. The two heavier solid black lines are for values of normalized inertance, \(\sigma = 0.5\) and \(\sigma = 1.0\). The dashed lines are when the yielding SDOF structure is equipped with the supplemental damping \(\xi_d = \xi_1 = 0.25\) given that the viscous damping of the yielding SDOF is taken to be equal to zero (\(\xi_c = 0\)). Together with the response of the elastoplastic oscillator (black lines), the spectra shown in Fig. 4.3 plot for comparison of the response of an elastic oscillator (red lines). When \(\sigma = 0\), the thin red line meets the thin black line (no inerter, \(\sigma = 0\)) at a value of the pre-yielding period \(T_1 \approx 1.4 sec\), confirming the equal-displacement rule (Veletsos et al. 1965; Moghimi and Makris 2021).

The equal-displacement rule also holds when the inelastic and the corresponding elastic structure are engaged with an inerter (heavier black and red lines). According to Eq. (4.5), when the yielding structure engages with an inerter, the apparent pre-yielding period of the structure lengthens, and in this case, the equal-displacement rule is valid starting from lower pre-yielding...
periods ($T_1 \approx 1.15 \text{ sec}$ for $\sigma = 0.5$ and $T_1 \approx 0.95 \text{ sec}$ for $\sigma = 1.0$ for the case of a single inerter). Fig. 4.3 reveals that the use of inerter supported on a stiff frame suppress effectively the displacements of SDOF yielding structures, while the resulting base shears are systematically lower than when large values of supplemental damping ($\xi_a = 0.25$) are used. Furthermore, the forces transferred to the mounting of the inerter are appreciably lower than the corresponding forces originating from an elastic structure.

All these observations indicate that supplemental rotational inertia (use of inerter) emerges as an attractive response modification strategy for elastoplastic structures with larger pre-yielding periods. In the case of an elastoplastic structure, the use of a pair of clutching inerter (right plots) does not offer any additional benefits when compared to the case where a single inerter is used. A pair of clutching inerter have advantages when suppressing the response of elastic structures.

Figure 4.4 plots the same response quantities as Fig. 4.3 for a bilinear structure with $\alpha = 0.05$ and $\xi_c = 0$. Fig. 4.4 reveals trends similar to those observed in Fig. 4.3, supporting the finding that the use of inerter emerges as the most attractive response modification strategy for yielding structures with larger pre-yielding periods, while a pair of inerter does not offer any additional benefits than when a single inerter is employed. The right plots in Fig. 4.4 indicate that a pair of inerter is attractive to reduce the columns shears and base shears of an elastic frame.

The response spectra shown in Fig. 4.5 are for a single inerter (left plots) or when a pair of clutching inerter (right plots) is used to modify the response of an elastoplastic ($\alpha = 0$) SDOF structure with normalized strength $Q_1/m_1 = 0.1g$ and $\xi_c = 0$ when subjected to the Cholame Number Array 2/360 ground motion recorded during the 2004 Parkfield California earthquake.
Fig. 4.5 reveals trends similar to those observed in Fig. 4.4, supporting the finding that the use of inerters emerges as the most attractive response modification strategy for yielding structure with larger pre-yielding periods; while, a pair of clutching inerters (right plots) does not offer any additional benefits than when a single inerter is employed.

Figure 4.6 plots the same response quantities as Fig. 4.5 for a bilinear structure with $\alpha = 0.05$ and $\xi_c = 0$, and reveals the same trends as those discussed earlier.

4.3 Equations of Motion of a Yielding 2DOF Structure with Inerters Supported on a Stiff Chevron Frame

With reference to Fig. 4.1(d), this section examines the dynamic response of a yielding 2DOF structure with floor masses $m_1$ and $m_2$, pre-yielding and post-yielding stiffness at the first story $k_1$ and $\alpha k_1$ respectively with a yield strength, $Q_1$, and a yield displacement, $u_{y1}$. The pre-yielding and post-yielding stiffness at the second story are $k_2$ and $\alpha k_2$ respectively with a yield strength, $Q_2$, and a yield displacement $u_{y2}$. Only the first floor is engaged to an inerter with inertance, $M_R$ supported on a stiff chevron frame. Dynamic equilibrium of the entire structure above the chevron frame gives

$$m_2[\ddot{u}_1(t) + \ddot{u}_2(t) + \ddot{g}(t)] + m_1[\ddot{u}_1(t) + \ddot{g}(t)] = -F_{s1}(t) - c_1\dot{u}_1(t) - F_I(t) \quad (4.12)$$

where again $F_I(t) = \text{internal force from the flywheel (inerter) given by Eq. (2.4)}$ and $F_{s1}(t) = \text{inelastic restoring force of the structure at the first story described by the Bouc-Wen model (Wen 1976; Baber and Wen 1981)}$ expressed by Eqs. (4.2) and (4.3). Dynamic equilibrium of the second story gives
Fig. 4.3. Response spectra of an elastoplastic ($\alpha = 0$) SDOF structure equipped with inerters (heavy solid black lines) or supplemental viscous damping (dashed lines) supported on a stiff frame when excited by the Gilroy Array 6/230 ground motion recorded during the 1979 Coyote Lake earthquake: (a) Single inerter; (b) Pair of clutching inerters. Red lines are for the corresponding elastic structures without (thin red lines) or with (heavier red lines) inerters.
Fig. 4.4. Response spectra of a bilinear ($\alpha = 0.05$) SDOF structure equipped with inerters (heavy solid black lines) or supplemental viscous damping (dashed lines) supported on a stiff frame when excited by the Gilroy Array 6/230 ground motion recorded during the 1979 Coyote Lake earthquake: (a) Single inerter; (b) Pair of clutching inerters. Red lines are for the corresponding elastic structures without (thin red lines) or with (heavier red lines) inerters.
Fig. 4.5. Response spectra of an elastoplastic ($\alpha = 0$) SDOF structure equipped with inerters (heavy solid black lines) or supplemental viscous damping (dashed lines) supported on a stiff frame when excited by the Cholame Number 2/360 ground motion recorded during the 2004 Parkfield California earthquake: (a) Single inerter; (b) Pair of clutching inerters. Red lines are for the corresponding elastic structures without (thin red lines) or with (heavier red lines) inerters.
Fig. 4.6. Response spectra of a bilinear ($\alpha = 0.05$) SDOF structure equipped with inerters (heavy solid black lines) or supplemental viscous damping (dashed lines) supported on a stiff frame when excited by the Cholame Number 2/360 ground motion recorded during the 2004 Parkfield California earthquake: (a) Single inerter; (b) Pair of clutching inerters. Red lines are for the corresponding elastic structures without (thin red lines) or with (heavier red lines) inerters.
\[ m_2 [\ddot{u}_1 (t) + \ddot{u}_2 (t) + \ddot{g}(t)] = -F_{s2}(t) - c_2 \ddot{u}_2 (t) \]  \tag{4.13}

where \( F_{s2}(t) \) = inelastic restoring force of the structure at the second story described by the Bouc-Wen model.

\[ F_{s2} = \alpha k_2 u_2 (t) + (1 - \alpha) k_2 u_y z_2 (t) \]  \tag{4.14}

where \( \alpha = \) postyielding-to-preyielding stiffness ratio; and \(-1 \leq z_2 (t) \leq 1 = \) dimensionless internal variable described by

\[ \dot{z}_2 (t) = \frac{1}{u_y} [\dddot{u}_2 (t) - \beta \dot{u}_2 (t) |z_2 (t)|^n - \gamma |\dddot{u}_2 (t)| z_2 (t) |z_2 (t)|^{n-1}] \]  \tag{4.15}

Following the notation introduced by Kelly (1997), the nominal frequencies and nominal damping ratios are

\[ \omega_1^2 = \frac{k_1}{m_1 + m_2}, \quad \omega_2^2 = \frac{k_2}{m_2} \]  \tag{4.16}

\[ 2 \xi_1 \omega_1 = \frac{c_1}{m_1 + m_2}, \quad 2 \xi_2 \omega_2 = \frac{c_2}{m_2} \]  \tag{4.17}

Furthermore, the mass ratio, \( \mu \), and the inertance ratio, \( \sigma \), are defined as

\[ \mu = \frac{m_2}{m_1 + m_2}, \quad \sigma = \frac{M_R}{m_1 + m_2} \]  \tag{4.18}

Eqs. (4.12) and (4.13) can be expressed in matrix form in terms of the parameters defined in Eqs. (4.16)-(4.18).

\[
\begin{bmatrix}
1 + \sigma & \mu \\
1 & 0
\end{bmatrix}
\begin{bmatrix}
\dddot{u}_1 (t) \\
\dddot{u}_2 (t)
\end{bmatrix}
+ \begin{bmatrix}
2 \xi_1 \omega_1 & 0 \\
0 & 2 \xi_2 \omega_2
\end{bmatrix}
\begin{bmatrix}
\dot{u}_1 (t) \\
\dot{u}_2 (t)
\end{bmatrix}
+ \begin{bmatrix}
\alpha \omega_1^2 & 0 \\
0 & \alpha \omega_2^2
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_1 (t) \\
\ddot{u}_2 (t)
\end{bmatrix}
+ \begin{bmatrix}
(1 - \alpha) \omega_1^2 u_y \\
0
\end{bmatrix}
\begin{bmatrix}
\ddot{z}_1 (t) \\
\ddot{z}_2 (t)
\end{bmatrix}
= -\begin{bmatrix}
1 \\
1
\end{bmatrix}
\dddot{u}_g (t) \]  \tag{4.19}
By multiplying Eq. (4.19) from the left with the inverse of the normalized mass matrix

\[
\begin{bmatrix}
1 + \sigma & \mu \\
1 & 1
\end{bmatrix}^{-1} = \begin{bmatrix}
\frac{1}{1+\sigma-\mu} & -\mu \\
\frac{1}{1+\sigma-\mu} & \frac{1}{1+\sigma}
\end{bmatrix}
\] (4.20)

the relative accelerations \( \ddot{u}_1(t) \) and \( \ddot{u}_2(t) \) of each story become explicit expressions of the relative displacements and velocities of the stories.

\[
\ddot{u}_1(t) = -\frac{1-\mu}{\psi} \ddot{u}_g(t) - \frac{2\xi_1\omega_1}{\psi} \ddot{u}_1(t) + \frac{2\mu\xi_2\omega_2}{\psi} \ddot{u}_2(t) - \frac{\alpha_1\omega_2^2}{\psi} u_1(t) +
\]

\[
\frac{\alpha \mu \omega_2^2}{\psi} u_2(t) - \frac{(1-\alpha)\omega_2^2 u_1}{\psi} z_1(t) + \frac{(1-\alpha)\mu \omega_2^2 u_2}{\psi} z_2(t)
\] (4.21)

and

\[
\ddot{u}_2(t) = -\frac{\sigma}{\psi} \ddot{u}_g(t) + \frac{2\xi_1\omega_1}{\psi} \ddot{u}_1(t) - \frac{2(1+\sigma)\xi_2\omega_2}{\psi} \ddot{u}_2(t) + \frac{\alpha \omega_2^2}{\psi} u_1(t) -
\]

\[
\frac{\alpha (1+\sigma) \omega_2^2}{\psi} u_2(t) + \frac{(1-\alpha)\omega_2^2 u_1}{\psi} z_1(t) - \frac{(1-\alpha)(1+\sigma) \omega_2^2 u_2}{\psi} z_2(t)
\] (4.22)

where \( \psi = 1 + \sigma - \mu = 1 + (M_R - m_2)/(m_1 + m_2) > 0 \).

The solution of the system of equations given by Eqs. (4.21) and (4.22) is computed numerically via a state-space formulation (Konstantinidis and Makris 2005; Pitilakis and Makris 2010; Vassiliou and Makris 2012; Moghimi and Makris 2021; among the others). The state-vector of the system is

\[
\{y(t)\} = (y_1(t), y_2(t), y_3(t), y_4(t), y_5(t), y_6(t))^T = (u_1(t), \ddot{u}_1(t), u_2(t), \ddot{u}_2(t), z_1(t), z_2(t))^T
\] (4.23)

and the time-derivative state-vector, \( \dot{y}(t) \) is expressed solely in terms of the state-variables appearing in the state-vector given by Eq. (4.23).
\[ \dot{y}(t) = \begin{cases} 
\ddot{u}_1(t) \\
\ddot{u}_1(t) \\
\ddot{u}_2(t) \\
\ddot{z}_1(t) \\
\ddot{z}_2(t) 
\end{cases} \begin{bmatrix} y_2(t) \\
y_2(t) + \frac{1}{\psi} \ddot{u}_g(t) - \frac{2\xi_1\omega_1}{\psi} y_2(t) + \frac{2\mu\xi_2\omega_2}{\psi} y_4(t) - \frac{a\omega_1^2}{\psi} y_1(t) \\
y_4(t) - \frac{1-\mu}{\psi} \ddot{u}_g(t) - \frac{2\xi_1\omega_1}{\psi} y_2(t) + \frac{2(1+\alpha)\xi_2\omega_2}{\psi} y_4(t) + \frac{a\omega_1^2}{\psi} y_1(t) \\
y_3(t) - \frac{a\omega_2^2}{\psi} y_3(t) + \frac{(1-\alpha)\omega_2^2y_1}{\psi} y_5(t) - \frac{(1-\alpha)(1+\sigma)\omega_2^2y_2}{\psi} y_6(t) \\
y_5(t) - \beta (y_2(t)|y_5(t)|^n - y y_2(t)|y_5(t)|y_5(t)|^{n-1}) \\
y_6(t) - \beta (y_4(t)|y_6(t)|^n - y y_4(t)|y_6(t)|y_6(t)|^{n-1}) 
\end{bmatrix} \] (4.24)

With reference to Fig. 4.1(d), the entire base shear of the structure is

\[ V_1(t) = -F_{s1}(t) - c_1 \dot{u}_1(t) - M_R \ddot{u}_1(t) = m_2 \ddot{u}_2(t) + (m_1 + m_2) \left( \ddot{u}_1(t) + \ddot{u}_g(t) \right) \] (4.25)

The sequential engagement of the two parallel-rotational-inertial system with clutching inerters that can only resist the motion is expressed mathematically as

\[ \frac{F_I(t)}{m_1+m_2} = \sigma \ddot{u}_1(t) \quad \text{when} \quad \text{sgn} \left[ \frac{\ddot{u}_1(t)}{\dot{u}_1(t)} \right] > 0 \] (4.26a)

and

\[ \frac{F_I(t)}{m_1+m_2} = 0 \quad \text{when} \quad \text{sgn} \left[ \frac{\ddot{u}_1(t)}{\dot{u}_1(t)} \right] < 0 \] (4.26b)

Accordingly, for the two-parallel-rotational-inertia system with clutching inerters that only resist the motion of the structure, the equation of motion given in Eq. (4.19) is modified to

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\[\begin{bmatrix} 1 + \delta \sigma & \mu \{ \ddot{u}_1(t) \} \\ \mu & 1 \{ \ddot{u}_2(t) \} \end{bmatrix} + \begin{bmatrix} 2\xi_1 \omega_1 & 0 \\ 0 & 2\xi_2 \omega_2 \end{bmatrix} \{ \ddot{u}_1(t) \} + \begin{bmatrix} \alpha \omega_1^2 & 0 \\ 0 & \alpha \omega_2^2 \end{bmatrix} \{ u_1(t) \} + \begin{bmatrix} (1 - \alpha) \omega_1^2 u_{y_1} \\ 0 \end{bmatrix} \{ z_1(t) \} = -\begin{bmatrix} 1 \\ 1 \end{bmatrix} \ddot{u}_g(t) \] (4.27)

where \( \delta \) can be computed by Eq. (4.11).

### 4.4 Response Spectra of a Yielding 2DOF Structure with Inerters Supported on a Stiff Chevron Frame

The seismic response of the yielding 2DOF structure is equipped with an inerter at the first story [Fig. 4.1(d)] as described by Eq. (4.19) or Eqs. (4.27) and (4.11) is compared with the seismic response of the same 2DOF structure where the inerter is replaced with a supplemental viscous damper with a damping constant \( c_d \). Together with the drift response \( u_1 \) and \( u_2 \) (relative displacements), of interest are the normalized shear above the first story, \( V_2/mg \), the total normalized base shear at the ground level, \( V_1/(m_1 + m_2)g \) which is essentially the left-hand side or the right-hand side of Eq. (4.12) and the normalized inerter force, \( F_i/(m_1 + m_2)g \) transferred to the support of inerter.

Figs. 4.7 and 4.8 present response spectra for the 2DOF structures in Fig. 4.1(d) when subjected to the GilroyArray 6/230 ground motion recorded during the 1979 Coyote Lake, USA earthquake; and the Cholame Number 2/360 ground motion recorded during the 2004 Parkfield California earthquake; respectively.

The response spectra in Figs. 4.7 and 4.8 are the results of the solution of Eq. (4.19) for a single inerter or the solution of Eqs. (4.27) and (4.11) when a pair of clutching inerters is used.
The thin solid lines in Fig. 4.7 are for the yielding 2DOF structure without any response modification device, whereas the heavier solid lines are for the case where inerters with $\sigma = 0.5$ and $\sigma = 1.0$ are used. The dashed lines are for when supplemental damping, $\xi_d$, is used so that $\xi_1 = \xi_c + \xi_d = 0.02 + 0.23 = 0.25$. $F_{MD}(t)$ is the resisting force from the response modification device (either fluid damper with damping constant, $c_d$, or inerter with inertance constant, $M_R$).

The first observation is that supplemental rotational inertia supported on a stiff frame is most efficient in reducing displacements for structures with larger pre-yielding periods, $T_1$; nevertheless, the displacement reduction achieved is comparable to the reduction achieved with large values of supplemental damping. The use of a pair of clutching inerterst, where only the structure can drive the inerterst, has a marginal effect in further suppressing displacements and forces transferred at the support of the inerterst.

4.5 Equations of Motion of a Yielding 2DOF Structure with Inerterst Supported on a Compliant Chevron Frame

This section examines the dynamic response of the 2DOF structure shown in Fig. 4.1(d), yet now the chevron frame that supports the inerter has finite stiffness, $k_f$, and damping constant, $c_f$. Because of its compliance, under the force transferred by the mounting of the inerter, the chevron frame deforms; therefore, the force from the inerter is no longer expressed with Eq. (2.4), which is for a rigid frame, but by (Makris and Kampas 2016; Makris 2018)

$$F_i(t) + \frac{c_f}{k_f} \frac{dF_i(t)}{dt} + \frac{M_R}{k_f} \frac{d^2F_i(t)}{dt^2} = M_R \left( \frac{d^2u_1(t)}{dt^2} + \frac{c_f}{k_f} \frac{d^3u_1(t)}{dt^3} \right)$$  \hspace{1cm} (4.28)

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Fig. 4.7. Response spectra of a yielding 2DOF structure equipped with inerter (heavy solid lines) or supplemental viscous damping (dashed lines) supported on a stiff frame when excited by the Gilroy Array 6/230 ground motion recorded during the 1979 Coyote Lake earthquake: (a) Single inerter; (b) Pair of clenching inerters.
Fig. 4.8. Response spectra of a yielding 2DOF structure equipped with inerters (heavy solid lines) or supplemental viscous damping (dashed lines) supported on a stiff frame when excited by the Cholame Number 2/360 ground motion recorded during the 2004 Parkfield California earthquake: (a) Single inerter; (b) Pair of clutching inerters.
Equation (4.28) is the constitutive law of a spring–dashpot parallel connection \((k_f, c_f)\) that is connected in series with an inerter \((M_R)\). This mechanical network is also known as the tuned inerter damper (TID, Lazar et al. 2014), and was coined recently as the inertoviscoelastic fluid A (Makris 2018). The term "fluid" expresses that the mechanical network undergoes infinite displacement under static loading. By defining the relaxation time, \(\lambda = c_f/k_f\) and the rotational frequency \(\omega_R = \sqrt{k_f/M_R}\) (Makris 2017, 2018), Eq. (4.28) assumes the form

\[
F_I(t) + \lambda \frac{dF_I(t)}{dt} + \frac{1}{\omega_R^2} \frac{d^2F_I(t)}{dt^2} = M_R \left( \frac{d^2u_1(t)}{dt^2} + \lambda \frac{d^2u_1(t)}{dt^3} \right)
\] (4.29)

where, the product \(\lambda \omega_R\) is dimensionless. The mechanical system described by Eq. (4.29) becomes critically damped when \(\lambda \omega_R = 2\) (Makris 2018). In the special case in which the damping within the chevron frame is neglected, \(c_f = 0\), Eq. (4.29) reduces to

\[
F_I(t) + \frac{1}{\omega_R^2} \frac{d^2F_I(t)}{dt^2} = M_R \frac{d^2u_1(t)}{dt^2}
\] (4.30)

which is the constitutive equation of the inertoelastic fluid (Makris 2017).

The equations of motion of a 2DOF yielding structure in which its inerters are supported on a compliant chevron frame with finite stiffness, \(k_f\), and damping, \(c_f\), are also given again by Eqs. (4.12) and (4.13); however, for this case, the force from the flywheel, \(F_I(t)\), in Eq. (4.12) is described by Eq. (4.29) rather than by Eq. (2.4). By using the nominal frequencies, nominal damping, mass, and inertance ratios defined by Eqs. (4.16)–(4.18), the equations of motion of a yielding 2DOF structure with a compliant chevron frame is expressed in a matrix form.
\[
\begin{bmatrix}
1 & \mu
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_1(t) \\
\ddot{u}_2(t)
\end{bmatrix}
+ \begin{bmatrix}
2\xi_1\omega_1 & 0 \\
0 & 2\xi_2\omega_2
\end{bmatrix}
\begin{bmatrix}
\dot{u}_1(t) \\
\dot{u}_2(t)
\end{bmatrix}
+ \begin{bmatrix}
\alpha\omega_1^2 & 0 \\
0 & \alpha\omega_2^2
\end{bmatrix}
\begin{bmatrix}
u_1(t) \n
\end{bmatrix}
+ \begin{bmatrix}(1-\alpha)\omega_1^2u_{y1} \\
0
\end{bmatrix}
\begin{bmatrix}
z_1(t) \\
z_2(t)
\end{bmatrix}
= \begin{bmatrix}
\frac{f_i(t)}{m_1+m_2} \\
0
\end{bmatrix}
\] (4.31)

in which \( F_i(t) \) is the solution of Eq. (4.29).

By multiplying Eq. (4.31) from the left with the inverse of the normalized mass matrix
\[
\begin{bmatrix}
1 & \mu \\
1 & 1
\end{bmatrix}^{-1} = \begin{bmatrix}
\frac{1}{1-\mu} & -\frac{\gamma}{1-\mu} \\
-\frac{1}{1-\mu} & \frac{1}{1-\mu}
\end{bmatrix}
\] (4.32)

the relative accelerations, \( \ddot{u}_1(t) \) and \( \ddot{u}_2(t) \) are expressed as

\[
\ddot{u}_1(t) = -\ddot{u}_g(t) - \frac{1}{1-\mu} f_i(t) + \frac{2\xi_1\omega_1}{1-\mu} \dot{u}_1(t) + \frac{2\mu\xi_2\omega_2}{1-\mu} \dot{u}_2(t) - \frac{\alpha\omega_2^2}{1-\mu} u_1(t) + \frac{\alpha\mu\omega_2^2}{1-\mu} u_2(t) - \frac{(1-\alpha)\omega_1^2u_{y1}}{1-\mu} z_1(t) + \frac{(1-\alpha)\omega_2^2u_{y2}}{1-\mu} z_2(t)
\] (4.33)

and

\[
\ddot{u}_2(t) = \frac{1}{1-\mu} f_i(t) + \frac{2\xi_1\omega_1}{1-\mu} \dot{u}_1(t) - \frac{2\xi_2\omega_2}{1-\mu} \dot{u}_2(t) + \frac{\alpha\omega_1^2}{1-\mu} u_1(t) - \frac{\alpha\omega_2^2}{1-\mu} u_2(t) + \frac{(1-\alpha)\omega_1^2u_{y1}}{1-\mu} z_1(t) - \frac{(1-\alpha)\omega_2^2u_{y2}}{1-\mu} z_2(t)
\] (4.34)

where \( f_i(t) = F_i(t) / (m_1 + m_2) \) has units of acceleration.

In this case, the state-vector of the system is
\[
\{y(t)\} = \{ y_1(t), y_2(t), y_3(t), y_4(t), y_5(t), y_6(t), y_7(t), y_8(t) \}^T = \{ u_1(t), \dot{u}_1(t), u_2(t), \dot{u}_2(t), f_i(t), \ddot{f}_i(t), z_1(t), z_2(t) \}^T
\] (4.35)
From Eq. (4.29) it is evident that the time-derivative of \( y_6(t) \), that is \( \dot{y}_6(t) = \ddot{f}_1(t) \), involves the third derivative of \( u_1(t) \) which is given by

\[
\dddot{u}_1(t) = -\ddot{u}_g(t) - \frac{1}{1-\mu} \dot{f}_1(t) - \frac{2\xi_1\omega_1}{1-\mu} \dddot{u}_1(t) + \frac{2\mu\xi_2\omega_2}{1-\mu} \dddot{u}_2(t) - \frac{\alpha\omega_1^2}{1-\mu} \dot{u}_1(t) + \frac{\alpha\mu\omega_2^2}{1-\mu} \dddot{u}_2(t) - \frac{(1-\alpha)\omega_1^2 u_{y1}}{1-\mu} \dot{z}_1(t) + \frac{(1-\alpha)\mu\omega_2^2 u_{y2}}{1-\mu} \dot{z}_2(t) \quad (4.36)
\]

In terms of the state variables given by Eq. (4.35), Eq. (4.36) assumes the form

\[
\dddot{u}_1(t) = -\left( \frac{1}{1-\mu} y_6(t) + \dddot{u}_g(t) \right)
\]

\[
- \frac{2\xi_1\omega_1}{1-\mu} \left( \frac{1}{1-\mu} y_5(t) - \dddot{u}_g(t) - \frac{2\xi_1\omega_1}{1-\mu} y_2(t) + \frac{2\mu\xi_2\omega_2}{1-\mu} y_4(t) - \frac{\alpha\omega_1^2}{1-\mu} y_1(t) \right)
\]

\[
+ \frac{2\mu\xi_2\omega_2}{1-\mu} \left( \frac{1}{1-\mu} y_5(t) + \frac{2\xi_1\omega_1}{1-\mu} y_2(t) - \frac{2\xi_2\omega_2}{1-\mu} y_4(t) + \frac{\alpha\omega_1^2}{1-\mu} y_1(t) \right)
\]

\[
- \frac{\alpha\omega_2^2}{1-\mu} y_2(t) + \frac{\alpha\mu\omega_2^2}{1-\mu} y_4(t) - \frac{(1-\alpha)\omega_1^2}{1-\mu} (y_2(t) - \beta y_2(t) |y_7(t)|^n - \gamma |y_2(t)| y_7(t) |y_7(t)|^{n-1})
\]

\[
+ \frac{(1-\alpha)\mu\omega_2^2}{1-\mu} (y_4(t) - \beta y_4(t) |y_8(t)|^n - \gamma |y_4(t)| y_8(t) |y_8(t)|^{n-1})
\]

The solution of the system of differential equations given by Eqs (4.33), (4.34), and (4.29) is computed by integrating the time-derivative of the state-vector given by Eq. (4.35).
The time-derivative of the state-vector of the system is given by Eq. (4.38) when

\[
\{\dot{y}(t)\} = \begin{bmatrix}
\dot{u}_1(t) \\
\ddot{u}_1(t) \\
\ddot{u}_2(t) \\
\ddot{u}_2(t) \\
\dddot{f}_1(t) \\
\dddot{f}_1(t) \\
\dddot{z}_1(t) \\
\dddot{z}_2(t)
\end{bmatrix} = \begin{bmatrix}
\dot{y}_2(t) \\
y_3(t) - \frac{1}{1-\mu} y_5(t) - \frac{2\xi_1\omega_1}{1-\mu} y_2(t) + \frac{2\mu\xi_2\omega_2}{1-\mu} y_4(t) - \frac{\alpha\omega_2^2}{1-\mu} y_4(t) - \frac{\alpha\omega_2^2}{1-\mu} y_4(t) + \frac{1}{1-\mu} y_5(t) + \frac{2\xi_1\omega_1}{1-\mu} y_2(t) - \frac{2\xi_2\omega_2}{1-\mu} y_4(t) + \frac{\alpha\omega_2^2}{1-\mu} y_4(t) - \frac{\alpha\omega_2^2}{1-\mu} y_4(t) + \frac{1}{1-\mu} y_5(t) + \frac{2\xi_1\omega_1}{1-\mu} y_2(t) - \frac{2\xi_2\omega_2}{1-\mu} y_4(t) - \frac{\alpha\omega_2^2}{1-\mu} y_4(t) + \frac{1}{1-\mu} y_5(t) + \frac{2\xi_1\omega_1}{1-\mu} y_2(t) - \frac{2\xi_2\omega_2}{1-\mu} y_4(t) - \frac{\alpha\omega_2^2}{1-\mu} y_4(t) + \frac{1}{1-\mu} y_5(t) + \frac{2\xi_1\omega_1}{1-\mu} y_2(t) - \frac{2\xi_2\omega_2}{1-\mu} y_4(t) - \frac{\alpha\omega_2^2}{1-\mu} y_4(t) + \frac{1}{1-\mu} y_5(t) + \frac{2\xi_1\omega_1}{1-\mu} y_2(t) - \frac{2\xi_2\omega_2}{1-\mu} y_4(t) - \frac{\alpha\omega_2^2}{1-\mu} y_4(t) + \frac{1}{1-\mu} y_5(t) + \frac{2\xi_1\omega_1}{1-\mu} y_2(t) - \frac{2\xi_2\omega_2}{1-\mu} y_4(t) - \frac{\alpha\omega_2^2}{1-\mu} y_4(t) + \frac{1}{1-\mu} y_5(t) + \frac{2\xi_1\omega_1}{1-\mu} y_2(t) - \frac{2\xi_2\omega_2}{1-\mu} y_4(t) - \frac{\alpha\omega_2^2}{1-\mu} y_4(t) + \frac{1}{1-\mu} y_5(t) + \frac{2\xi_1\omega_1}{1-\mu} y_2(t) - \frac{2\xi_2\omega_2}{1-\mu} y_4(t) - \frac{\alpha\omega_2^2}{1-\mu} y_4(t) + \frac{1}{1-\mu} y_5(t) + \frac{2\xi_1\omega_1}{1-\mu} y_2(t) - \frac{2\xi_2\omega_2}{1-\mu} y_4(t) - \frac{\alpha\omega_2^2}{1-\mu} y_4(t) + \frac{1}{1-\mu} y_5(t) + \frac{2\xi_1\omega_1}{1-\mu} y_2(t) - \frac{2\xi_2\omega_2}{1-\mu} y_4(t) - \frac{\alpha\omega_2^2}{1-\mu} y_4(t) + \frac{1}{1-\mu} y_5(t) + \frac{2\xi_1\omega_1}{1-\mu} y_2(t) - \frac{2\xi_2\omega_2}{1-\mu} y_4(t) - \frac{\alpha\omega_2^2}{1-\mu} y_4(t) + \frac{1}{1-\mu} y_5(t) + \frac{2\xi_1\omega_1}{1-\mu} y_2(t) - \frac{2\xi_2\omega_2}{1-\mu} y_4(t) - \frac{\alpha\omega_2^2}{1-\mu} y_4(t) + \frac{1}{1-\mu} y_5(t) + \frac{2\xi_1\omega_1}{1-\mu} y_2(t) - \frac{2\xi_2\omega_2}{1-\mu} y_4(t) - \frac{\alpha\omega_2^2}{1-\mu} y_4(t) + \frac{1}{1-\mu} y_5(t) + \frac{2\xi_1\omega_1}{1-\mu} y_2(t) - \frac{2\xi_2\omega_2}{1-\mu} y_4(t) - \frac{\alpha\omega_2^2}{1-\mu} y_4(t) + \frac{1}{1-\mu} y_5(t) + \frac{2\xi_1\omega_1}{1-\mu} y_2(t) - \frac{2\xi_2\omega_2}{1-\mu} y_4(t) - \frac{\alpha\omega_2^2}{1-\mu} y_4(t) + \frac{1}{1-\mu} y_5(t) + \frac{2\xi_1\omega_1}{1-\mu} y_2(t) - \frac{2\xi_2\omega_2}{1-\mu} y_4(t) - \frac{\alpha\omega_2^2}{1-\mu} y_4(t) + \frac{1}{1-\mu} y_5(t) + \frac{2\xi_1\omega_1}{1-\mu} y_2(t) - \frac{2\xi_2\omega_2}{1-\mu} y_4(t) - \frac{\alpha\omega_2^2}{1-\mu} y_4(t)
\end{bmatrix}
\]

When the two parallel-rotational-inertia systems (pair of clutching ineters) is employed that can only resist the motion of the structure without inducing any deformation (the pinion of the gearwheel that is engaged in the rack of the first story is unable to drive the rack, and only the motion of the translating rack can drive the pinion), the normalized force, \( f_i(t) = F_i(t)/(m_1 + m_2) \) appearing in Eqs. (4.33) and (4.34) is given by Eq. (4.29) when \( \text{sgn} \left[ F_i(t)/\dot{u}_1(t) \right] \geq 0 \) and by

\[
f_i(t) = \frac{F_i(t)}{m_1 + m_2} = 0 \quad \text{when} \quad \text{sgn} \left[ \frac{F_i(t)}{\dot{u}_1(t)} \right] < 0 \quad (4.39)
\]

The time-derivative of the state-vector of the system is given by Eq. (4.38) when \( \text{sgn} \left[ F_i(t)/\dot{u}_1(t) \right] > 0 \) and by
\[
\{ \dot{y}(t) \} = \begin{cases} 
\ddot{u}_1(t) \\
\ddot{u}_2(t) \\
\dot{z}_1(t) \\
\dot{z}_2(t) 
\end{cases} = \begin{cases} 
y_2(t) \\
y_4(t) \\
y_5(t) \\
y_6(t) 
\end{cases} 
\]

\[
\begin{array}{c}
- \ddot{y}_g(t) - \frac{2\xi_1\omega_1}{1-\mu} y_2(t) + \frac{2\mu\xi_2\omega_2}{1-\mu} y_4(t) - \frac{\alpha\omega_1^2}{1-\mu} y_1(t) + \frac{\alpha\mu\omega_2^2}{1-\mu} y_3(t) \\
\frac{(1-\alpha)\omega_1^2 u_1}{1-\mu} y_5(t) + \frac{(1-\alpha)\mu\omega_2^2 u_2}{1-\mu} y_6(t) \\
\end{array} 
\]

\[
\begin{array}{c}
\frac{2\xi_1\omega_1}{1-\mu} y_2(t) - \frac{2\xi_2\omega_2}{1-\mu} y_4(t) + \frac{\alpha\omega_1^2}{1-\mu} y_1(t) - \frac{\alpha\omega_2^2}{1-\mu} y_3(t) \\
+ \frac{(1-\alpha)\omega_1^2 u_1}{1-\mu} y_5(t) - \frac{(1-\alpha)\mu\omega_2^2 u_2}{1-\mu} y_6(t) \\
\end{array} 
\]

\[
\begin{array}{c}
\frac{1}{u_1} [y_2(t) - \beta y_2(t) |y_5(t)|^n - \gamma |y_2(t)| y_5(t) |y_5(t)|^{n-1}] \\
\frac{1}{u_2} [y_4(t) - \beta y_4(t) |y_6(t)|^n - \gamma |y_4(t)| y_6(t) |y_6(t)|^{n-1}] \\
\end{array} 
\]

(4.40)

when \( \text{sgn} \left[ F_f(t)/\dot{u}_1(t) \right] < 0 \).

The response of the 2DOF structure with supplemental rotational inertia in Fig. 4.1(d) with a compliant chevron frame with finite stiffness, \( k_f \) and damping \( c_f \) is compared with the response of a heavily damped 2DOF structure with supplemental viscous damping at the first story, as shown in Fig. 4.1(f). When supplemental viscous damping, \( c_d = 2\xi_d m_1 \omega_1 \), is used to modify the response of the yielding structure, it results in a mechanical network of a spring-dashpot parallel connection \((k_f, c_f)\) that is connected in series with a dashpot \((c_d)\). This mechanical network is known in the literature as the Jeffreys fluid (Jeffreys 1929; Bird et al. 1987; Makris and Kampas 2009) and its constitutive law is:

\[
F_d(t) + \frac{c_d + c_f}{k_f} \frac{dF_d(t)}{dt} = c_d \left( \frac{du_1(t)}{dt} + \frac{c_f}{k_f} \frac{d^2u_1(t)}{dt^2} \right) 
\]

(4.41)

By defining the relaxation time \( \lambda_d = (c_d + c_f)/k_f \) and recognizing that \( c_f/k_f = \lambda \), Eq. (4.41) assumes the form

\[
f_d(t) + \lambda_d \frac{df_d(t)}{dt} = 2\xi_d \omega_1 \left( \frac{du_1(t)}{dt} + \lambda \frac{d^2u_1(t)}{dt^2} \right) 
\]

(4.42)

where \( f_d(t) = F_d(t)/(m_1 + m_2) \).

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For the case of a 2DOF structure with supplemental damping supported by a compliant chevron frame with finite stiffness shown in Fig. 4.1(f), the state-vector of the system is

\[
\{y(t)\} = \begin{bmatrix} y_1(t), y_2(t), y_3(t), y_4(t), y_5(t), y_6(t), y_7(t) \end{bmatrix}^T = \begin{bmatrix} u_1(t), \dot{u}_1(t), u_2(t), \dot{u}_2(t), f_d(t), z_1(t), z_2(t) \end{bmatrix}^T
\]

(4.43)

The solution of the system of differential equations given by Eqs. (4.33), (4.34) in which \(F_l(t)\) is replaced by \(F_d(t)\) and Eq. (4.42) is computed by integrating the time-derivative of the state-vector given by Eq. (4.43)

\[
\begin{bmatrix} \dot{y}_1(t) \\ \dot{y}_2(t) \\ \dot{y}_3(t) \\ \dot{y}_4(t) \\ \dot{y}_5(t) \\ \dot{y}_6(t) \\ \dot{y}_7(t) \end{bmatrix} = \begin{bmatrix} \frac{1}{1-\mu} y_5(t) - \dot{u}_d(t) - \frac{2\xi_1 \omega_1}{1-\mu} y_2(t) + \frac{2 \mu \xi_2 \omega_2}{1-\mu} y_4(t) - \frac{\alpha \omega_2^2}{1-\mu} y_1(t) \\ \frac{1}{1-\mu} y_5(t) + \frac{2\xi_1 \omega_1}{1-\mu} y_2(t) - \frac{2 \xi_2 \omega_2}{1-\mu} y_4(t) + \frac{\alpha \omega_2^2}{1-\mu} y_1(t) - \frac{\alpha \omega_2^2}{1-\mu} y_3(t) \\ \frac{1}{1-\mu} y_5(t) + \frac{(1-\alpha) \omega_1^2 u_y}{1-\mu} y_4(t) - \frac{(1-\alpha) \omega_2^2 u_y}{1-\mu} y_7(t) \\ \frac{1}{1-\mu} \right \}
\]

(4.44)
4.6 Response Spectra of a Yielding 2DOF Structure with Inerters Supported on a Compliant Chevron Frame

The response spectra in Figs. 4.9 and 4.10 are the results of the solution of the Eq. (4.37) (single inerter supported on a compliant frame) and of Eqs. (4.38) and (4.40) when a pair of clutching inerters is used. Again, when $\sigma = 0$ (thin line), the solution offers the response of the 2DOF yielding structure without any seismic protection devices. The heavier solid lines are for the case where inerters with $\sigma = 0.5$ and $\sigma = 1.0$ are used. The compliance of the chevron frame is expressed with the relaxation time, $\lambda = c_f/k_f = 0.05$; whereas the stiffness of the chevron frame compared with the supplemental inertance, $M_R$, is expressed with the dimensionless product $\lambda \omega_R = 0.5$. For the structural system in Fig. 4.1(f), values of $\xi_c = 2\%$ and $\xi_d = 23\%$ are used so that $\xi_1 = \xi_c + \xi_d = 0.02 + 0.23 = 0.25$. When supplemental damping, $c_d$ is used, the compliance of the chevron frame is $\lambda_d = (c_d + c_f)/k_f = 0.5$. In all spectra $Q_1/(m_1 + m_2) = 0.1g$, $Q_2/m_2 = 0.2g$, $u_{y2} = u_{y1}/2$ and $m_1 = m_2$ so that $\mu = 1/2$.

Figure 4.7 present the response spectra for the two 2DOF yielding structural configurations shown in Fig. 4.1 when subjected to the Gilroy Array 6/230 ground motion recorded during the 1979 Coyote Lake, California earthquake; while, Fig. 4.9 presents the response spectra of the same 2DOF yielding structural configurations when subjected to the Cholame Number Array 2/360 ground motion recorded during the 2004 Parkfield, California earthquake. The main observation is that when the inerters are supported on a compliant frame, they are much more efficient in suppressing inelastic displacement response.
Fig. 4.9. Response spectra of a yielding 2DOF structure equipped with inerters (heavy solid lines) or supplemental viscous damping (dashed lines) supported on a compliant chevron frame when excited by the Gilroy Array 6/230 ground motion recorded during the 1979 Coyote Lake earthquake: (a) Single inerter; (b) Pair of inerters.
Fig. 4.10. Response spectra of a yielding 2DOF structure equipped with inerters (heavy solid lines) or supplemental viscous damping (dashed lines) supported on a compliant chevron frame when excited by the Cholame Number 2/360 ground motion recorded during the 2004 Parkfield California earthquake: (a) Single inerter; (b) Pair of Inerters.
4.7 Conclusion

This chapter investigates the advantages and challenges associated with using supplemental rotational inertia for the seismic protection of yielding SDOF and 2DOF structures. The response analysis of a SDOF elastoplastic and bilinear structure reveals that when the yielding structure is equipped with supplemental rotational inertia (inerters), the equal-displacement rule is valid starting from lower values of the pre-yielding period, given that the presence of inerters lengthens the apparent pre-yielding period. Furthermore, inerters suppress effectively the inelastic displacements of SDOF yielding structures, while the resulting base shears are systematically lower than when large values of supplemental damping ($\xi_d = 0.25$) are used. The forces transferred at the mounting of the inerters are appreciably lower than the corresponding forces originating from an elastic structure. Consequently, the implementation of inerters emerges as an attractive response modification strategy for elastoplastic and bilinear SDOF structures with larger pre-yielding periods. The use of a pair of clutching inerters does not offer any additional benefits compared to the case where a single inerter is used. The pair of clutching inerters are attractive when suppressing the response of elastic structures. Inerters are also most effective in suppressing the inelastic response of 2DOF yielding structures without aggravating the inelastic response of the superstructure. The effectiveness of inerters of suppressing the inelastic response of the 2DOF yielding structure outperforms the effectiveness of large values of supplemental damping ($\xi_d = 25\%$) appreciably when the support frame of the response modification device is compliant. The proposed seismic protection strategy can accommodate large relative displacements without suffering from the issue of viscous heating and potential leaking that challenges the implementation of fluid dampers subjected to prolonged cyclic loading.
Chapter 5

Response Modification of Tall Buildings with Outrigger Frames and Inerters

This chapter investigates the seismic performance of a high-rise yielding structure equipped with an outrigger-inerter system. The proposed seismic control mechanism uses inerters installed along the vertical direction within a conventional core-to-external column outrigger system. Both a single inerter and a pair of clutching inerters are examined. A new material is developed to implement the outrigger-inerter system in OpenSees (McKenna et al. 2000). The twenty-story benchmark SAC building (Gupta and Krawinkler 1999; Chopra and Goel 2002), is equipped with the outrigger-inerter and the seismic response of the structure is compared to the seismic response of the same structure when it is equipped with traditional central core systems such as steel braced frame or concrete shear wall. The outrigger-inerter system with a pair of inerters (PI) outperforms the effectiveness of the outrigger-inerter with single inerter (SI) in suppressing the inter-story drift ratio, base shears and base moments. The proposed response-modification strategy is attractive to improve the seismic response of high-rise structures.

5.1 Introduction

Response modification is the main part to be addressed in the seismic design of building structures. To protect high-rise structures from severe ground motions, researchers have investigated various response control mechanisms such as viscous dampers, viscoelastic dampers,
and tuned mass dampers (Kareem et al. 1999; Spencer and Nagarajaiah 2003; Smith and Willford 2007).

The efficiency of inerter-based systems in low-to-medium rise structures has been discussed in previous chapters and also verified by several researchers (Ikago et al. 2012; Takewaki et al. 2012; Lazar et al. 2014; Marian and Giaralis 2014; Makris and Kampas 2016; De Domenico and Ricciardi 2018; Makris and Moghimi 2019, and references reported therein) and at present, it has enjoyed a handful of full-scale implementations (Sugimura et al. 2012; Ogino et al. 2014). However, since inerters are installed inter-story, such high performance cannot be expected from inerters in tall buildings because, generally, there are not sufficient inter-story drifts to dissipate a large amount of input energy in high-rise buildings (Ishii et al. 2014; Asai et al. 2015). To solve this problem, this chapter proposes a novel response modification strategy, the outrigger-inerter system, in which inerters are installed vertically within a conventional core-to-external column outrigger system, and the advantages and limitations of the use of inerters in association with outriggers in tall buildings is investigated.

The outrigger-braced system has been widely utilized in tall buildings. It consists of a stiff central core such as a steel braced frame or a concrete shear wall and outriggers such as deep girders or trusses connecting the central core to the perimeter columns at one or more levels. When lateral loads are applied, the outriggers and perimeter columns resist the rotation at the core and reduce drifts and base moment. The magnitude of reduction in drifts and base moment depends on the flexural rigidity of the core, outriggers, and columns. It also depends on the location of outriggers within the height of the structure (Smith and Salim 1981). Considering uniform lateral loading, the optimum location for single and multiple outriggers has been investigated, and a
simple formulation to calculate the optimum locations of outriggers proposed (Taranath 1975; McNabb et al. 1975; Smith and Salim 1981; Smith and Coull 1991; Hoenderkamp and Snijder 2000, among others). Figs. 5.1(a)–(c) show the schematic configuration of the outrigger system; the schematic seismic response of the system; and the bending moment diagrams of the central core with and without outriggers when subjected to uniform lateral loading. Fig. 5.1(c) illustrates that the core moment decreases at every level that the structure equipped with outriggers. Wu et al. (2003) examined the optimal design of high-rise structures equipped with outriggers when subjected to triangular lateral load distributions.

![Fig. 5.1](image-url)

**Fig. 5.1.** (a) The schematic configuration of an outrigger system; (b) the schematic response to the uniform lateral loading; and (c) the schematic bending moment diagrams of the structure with and without two outriggers in different levels when subjected to uniform lateral loading (Smith and Salim 1981).
In recent years, damped outrigger systems have been proposed for tall buildings. In this system, viscous dampers are installed between outrigger walls and perimeter columns in a frame-core-tube structure to enhance structural dynamic performance (O’Neill 2006; Smith and Willford 2007; Zhou et al. 2016, among others). Asai et al. (2015) examined the outrigger system employing tuned viscose mass damper (TVMD) for high-rise buildings and concluded that the outrigger-TVMD system works well to reduce the structural responses better than the classical tuned mass damper system.

A novel response modification strategy, the outrigger-inerter system, is proposed in this chapter. The proposed seismic control mechanism uses inerters vertically within a conventional core-to-external column outrigger system. To study the seismic behavior of the outrigger-inerter system, a new material developed in C++ such that the restoring force, \( F_I(t) \), is proportional to the relative acceleration of its two terminals [Eq. (2.1)]. This new material is used to represent the behavior of inerters in the OpenSees platform.

OpenSees (the Open System for Earthquake Engineering Simulation) is a proprietary object-oriented framework primarily written in C++, and it was developed by McKenna et al. (2000) at the National Science Foundation-sponsored Pacific Earthquake Engineering (PEER) Center. In the next section, using the C++ language, a new “uniaxial material” is developed to allow users to implement inerters together with other elements and materials such as yielding material to model building structures equipped with inerters for both cases of single inerter and pair of clutching inerters that can only resist the motion of the structure. The new material helps to perform time history analysis for the seismic performance evaluation of high-rise buildings equipped with the proposed outrigger-inerter system.
5.2 Model New Uniaxial Material, Inerter, in C++ Programming Language for OpenSees

Equation (2.4) shows how the restoring force of inerter, $F_I(t)$, is proportional to the relative acceleration of its end-nodes. Unlike displacement and velocities, there is no built-in function in OpenSees to obtain the accelerations at the end of elements, therefore, based on the inerter’s properties expressed in Eqs. (2.4) and (4.9) a new material is developed in C++ for numerical modeling of structures equipped with supplemental rotational inertia in open-source OpenSees. To this aim, the approximation from the Newmark-Beta was used to compute these accelerations at each time step following equations (Chopra 2017):

\[
\dot{u}(t_{i+1}) = \dot{u}(t_i) + \Delta t (1 - \gamma) \ddot{u}(t_i) + \Delta t \gamma \ddot{u}(t_{i+1}) \tag{5.1}
\]

and

\[
u(t_{i+1}) = u(t_i) + \Delta t \dot{u}(t_i) + \Delta t^2 (0.5 - \beta) \ddot{u}(t_i) + \Delta t^2 \beta \dddot{u}(t_{i+1}) \tag{5.2}
\]

where $\Delta t$ is the time step; the subscript, $i$, stands for the current step, and subscript, $i + 1$, is for the next step. The parameters $\beta$ and $\gamma$ define the variation of acceleration over a time step and determine the stability and accuracy characteristics of the method. By substituting displacement step, $\Delta u(t_i) = u(t_{i+1}) - u(t_i)$, velocity step, $\Delta \dot{u}(t_i) = \dot{u}(t_{i+1}) - \dot{u}(t_i)$, and acceleration step, $\Delta \ddot{u}(t_i) = \ddot{u}(t_{i+1}) - \ddot{u}(t_i)$, in Eqs. (5.1) and (5.2), incremental velocity, and acceleration assumes the form (Hessabi 2017):

\[
\Delta \dot{u}(t_i) = \frac{\gamma}{\beta \Delta t} \Delta u(t_i) - \frac{\gamma}{\beta} \dot{u}(t_i) + \Delta t \left(1 - \frac{\gamma}{2 \beta}\right) \ddot{u}(t_i) \tag{5.3}
\]

and

\[
\Delta \ddot{u}(t_i) = \frac{1}{\beta \Delta t^2} \Delta u(t_i) - \frac{1}{\beta \Delta t} \dot{u}(t_i) - \frac{1}{2 \beta} \ddot{u}(t_i) \tag{5.4}
\]
Equations (5.1)–(5.4) are used to develop the C++ code to calculate acceleration at each time step, and Eqs. (2.4) and (4.9) are used to compute the resisting force, \( F_I \), for given inertance, \( M_R \), in Tcl file. The new “Uniaxial Material” is called “Inerter Material”. To add a new material, it is needed to provide a new subclass of the “Uniaxial Material” class together with an interface function to parse the input and create the new material (https://opensees.berkeley.edu/).

5.3 Validation of the Developed New Material for OpenSees

To evaluate the accuracy of the new C++ developed material, “Inerter Material”, time history analysis has been performed for the elastic SDOF oscillator shown in Fig. 2.2(a) equipped with (a) a single inerter and (b) a pair of clutching inerter. The results obtained from OpenSees compared with the response of the same elastic SDOF structure achieved from MATLAB for both cases of a single inerter and a pair of clutching inerter that can only resist the motion of the structure. Fig. 5.2 plots the relative displacement, \( u(t) \), velocity, \( \dot{u}(t) \), acceleration, \( \ddot{u}(t) \), the force transferred to the chevron frame, \( F_I(t) \), given by Eqs. (2.4) and (4.9), and base shear of the SDOF structure shown in Fig. 5.2 equipped with supplemental rotational inertia with inertance ratio \( \sigma = 0.5 \) and fundamental period \( T = 1 \text{ sec} \) when subjected to the Gilroy Array 6/230 ground motion recorded during the 1979 Coyote Lake, California earthquake. In the interest of simplicity in this analysis, zero damping is assumed (\( \xi = 0.0 \)). The solid black lines are the solution obtained from the numerical integration performed with standard ordinary differential equation (ODE) solvers available in MATLAB, and the red dashed lines are for the structural response computed in OpenSees based on the developed new material, “Inerter Material”, described in the last section. Fig. 5.2 verifies that the structural response of numerical simulation in OpenSees is in good agreement with the solution of mathematical equations obtained from MATLAB in the case of
single inerter [Fig. 5.2(left plots)] and a pair of clutching inerters that can only resist the motion of the structure [Fig. 5.2(right plots)].

Fig. 5.2. Comparison of the seismic response of a SDOF structure equipped with inerters in MATLAB (black solid lines) and OpenSees (red dashed lines): (a) Single inerter, which may induce deformations; (b) pair of clutching inerters which can only resist the motion of the structure.
5.4 Model Description of the Structure Equipped with the Outrigger-Inerter in OpenSees

To investigate the seismic performance of the outrigger-inerter, the twenty-story moment-resisting steel frame (MRF) designed for the SAC Phase II Project is selected as a representative of high-rise structure to be equipped with an outrigger-inerter system. This structure that is well-known to the literature (Gupta and Krawinkler 1999; Chopra and Goel 2002) was designed to meet the seismic code (pre-Northridge Earthquake) for the greater area of Los Angeles, California. This section describes details of the system modeled in OpenSees for seismic analysis.

The exterior frame in the north-south (N−S) direction is selected for the aim of this study. With twenty stories above the ground level and two basements, this benchmark structure is 88.03 m tall. Typical floor-to-floor height, center-of-beam to center-of-beam, is 3.96 m. The height of the basement levels is 3.65 m, and the first floor is 5.49 m. Three bays are considered for the structure, the middle bay is 6.10 m wide, and the perimeter bays are 9.10 m. The beams and columns are wide-flange steel sections, and all beam-column connections are fully restrained except for the basement level, which is pinned. The column bases are modeled as pinned and secured to the ground. Columns splice are at levels 1, 4, 7, 10, 13, 16, and 18 in all spans at 1.83 m above the beam-column joint. The seismic mass of the ground level is $2.66 \times 10^5$ kg, for the first level, is $2.82 \times 10^5$ kg, for the second through the nineteenth level, is $2.76 \times 10^5$ kg, and for the twentieth level is $2.92 \times 10^5$ kg. The seismic mass of the entire structure above the ground, $M_s$, is $5.55 \times 10^6$ kg. Table 5.1 shows the geometric and physical characteristics pertinent to the twenty-story benchmark SAC building. The built-in material “Steel01” in OpenSees is used to construct a uniaxial bilinear steel material object with strain hardening, as shown in Fig. 5.3.
Table 5.1 Geometric and physical characteristics pertinent to the SAC building.

<table>
<thead>
<tr>
<th>Columns (345 MPa)</th>
<th>Beams (248 MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>• B2–4th level</td>
<td>W27×368</td>
</tr>
<tr>
<td>• 5th–10th level</td>
<td>W27×281</td>
</tr>
<tr>
<td>• 11th–13th level</td>
<td>W27×217</td>
</tr>
<tr>
<td>• 14th–16th level</td>
<td>W27×178</td>
</tr>
<tr>
<td>• 17th–18th level</td>
<td>W27×146</td>
</tr>
<tr>
<td>• 19th–20th level</td>
<td>W27×114</td>
</tr>
<tr>
<td>• B2–4th level</td>
<td>W30×99</td>
</tr>
<tr>
<td>• 5th–10th level</td>
<td>W30×108</td>
</tr>
<tr>
<td>• 11th–13th level</td>
<td>W30×99</td>
</tr>
<tr>
<td>• 14th–16th level</td>
<td>W24×131</td>
</tr>
<tr>
<td>• 17th–18th level</td>
<td>W27×84</td>
</tr>
<tr>
<td>• 19th level</td>
<td>W24×62</td>
</tr>
<tr>
<td>• 20th level</td>
<td>W21×50</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>Seismic Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Basements level height</td>
<td>3.65 m</td>
</tr>
<tr>
<td>• Ground level height</td>
<td>2.66 × 10⁵ kg</td>
</tr>
<tr>
<td>• 1st–19th level height</td>
<td>5.49 m</td>
</tr>
<tr>
<td>• 2nd–19th level height</td>
<td>2.82 × 10⁵ kg</td>
</tr>
<tr>
<td>• 20th level</td>
<td>2.92 × 10⁵ kg</td>
</tr>
</tbody>
</table>

Restrains:
- Columns are pinned at the base level.
- The structure is laterally restrained at the ground level.
- Column splices are at 1.83 m with respect to beam-to-column joints.

Properties of steel are defined with the elastic modulus of $E_0 = 210 \text{ GPa}$, a strain hardening ratio, post-yield to pre-yield modulus ratio, $\alpha = 0.05$, a yield strength, $\sigma_y = 248 \text{ MPa}$ for beams and

![Material Behavior](image)

**Fig. 5.3.** Built-in “Steel 01” material behavior in OpenSees.
\[ \sigma_y = 345 \text{ MPa} \] for columns. All beams and columns are modeled with the *nonlinear beam-column element*, which is the force-based beam-column element (FBE) in the OpenSees framework. Five integration points along the member length are considered, and each section at the beam-column ends is discretized to 100 fibers.

To investigate the advantages and limitations of the outrigger-inerter, the twenty-story building is equipped with the proposed system. Truss element with the cross-section area 0.03 \( m^2 \), is used to model braces in the central core of the structure, steel braced frame, and two inerters are installed vertically at the two ends of the outriggers and external columns. The seismic efficacy of the outrigger-inerter system depends on the number, location, and stiffness of outriggers, shear stiffness of the central core, the axial stiffness of external columns and their distance from the core, and the inertance of inerters. Smith and Coull (1991) proposed that the optimum locations of outriggers in a \( n \)-outrigger structure are at \( n/(n + 1) \) height of the structure to minimize the top story deflection. However, the development of their analyses is based on some assumptions, including that the structure behaves linearly, columns only bear axial forces, the sectional properties of columns, outriggers, braces do not change in the height of the structure, and the structure is subjected to a uniform loading.

Figure 5.4 shows the twenty-story building structure equipped with the outrigger-inerter system. In this study, first the case when the structure is equipped with one outrigger-inerter system is investigated in which the outrigger is located at the top floor where the maximum deformation occurs when the structure is subjected to lateral loads. The case of multi-level outrigger-inerter systems when two outrigger-inerters are installed at different levels of the structure is also
explored. In all analyses, both cases of single inerter and pair of clutching inerters that can only resist the motion of the structure are examined.

Fig. 5.4. The twenty-story building equipped with the multi-level outrigger-inerter system
5.5 Seismic Response of the Structure Equipped with the Outrigger-Inerters

With reference to Fig. 5.4, this section first examines the dynamic response of the twenty-story structure equipped with one outrigger-inerter system located at the top story as described in the previous section. The number and locations of the outrigger-inerter systems should be adjusted. To the best of the author’s knowledge, there are only a few studies have been done on the dynamic response of the outriggers to find the optimum number and locations of this system in the height of the structure. However, in practice, outriggers are usually located in mechanical floors instead of their optimum locations. In the case of one outrigger-inerter, this seismic control system is located at the top floor in order to obtain the maximum reduction in inter-story drifts. For the structural system shown in Fig. 5.4 the normalized inertance ratio $\sigma = M_R/M_s = 0.1$ is used. The inertance, $M_R$, can be amplified by adding two (or more) flywheels in series, in which the first flywheel is a gearwheel (Smith 2002; Makris and Kampas 2016). In all analyses, a small amount of damping is also added to the structure ($\xi = 0.02$). Rayleigh Damping Command in OpenSees is used to apply 2% damping to the structure.

Figures 5.5–5.7 present the peak inter-story drifts, base shears, $V_b$, and base Moments, $M_b$, of the twenty-story structure equipped with one outrigger-inerter system for a single inerter (left plots) and a pair of clutching inerters (right plots) when subjected to the Array 2/360 ground motion recorded during the 2004 Parkfield California earthquake; the Nishi-Akashi/000 ground motion recorded during the 1995 Kobe Japan earthquake; and the Beverly Hills-Mulhol/009 ground motion recorded during the 1994 Northridge California earthquake, respectively. All ground motion records are collected from the Pacific Earthquake Engineering Research center.
Fig. 5.5. Time-history analysis of the twenty-story building equipped with an outrigger-inerter system located at the top story, with damping ratio, $\xi = 2\%$, and mass ratio, $\sigma = M_R / M_s = 10\%$, when subjected to the Array 2/360 ground motion recorded during the 2004 Parkfield earthquake: (a) Single inerter which may induce deformation; and (b) pair of inerter that can only resist the motion.
Fig. 5.6. Time-history analysis of the twenty-story building equipped with an outrigger-inerter system located at the top story, with damping ratio, $\xi = 2\%$, and mass ratio, $\sigma = 10\%$, when subjected to the Nishi-Akashi/000 ground motion recorded during the 1995 Kobe earthquake: (a) Single inerter which may induce deformation; and (b) pair of inerters that can only resist the motion.
Fig. 5.7. Time-history analysis of the twenty-story building equipped with an outrigger-inerter system located at the top story, with damping ratio, $\xi = 2\%$, and mass ratio, $\sigma = 10\%$, when subjected to the Beverly Hills-Mulhol/009 ground motion recorded during the 1994 Northridge earthquake: (a) Single inerter which may induce deformation; and (b) pair of inerters that can only resist the motion.
(PEER) ground motion database (https://ngawest2.berkeley.edu). The black lines are for the traditional central core structure, steel braced frame; the green lines are when an outrigger-inerter system with a single Inerter is used, and the red lines are for the case when the structure is equipped with an outrigger-inerter system with a pair of clutching inerters that can only resist the motion of the structure.

The first observation in Fig. 5.5 is that the outrigger-inerter system is effective in suppressing the inter-story drifts, in particular for the top floors. When two parallel rotational inertial systems, pair of clutching inerters, are used, the effectiveness of the proposed system in suppressing the inter-story drifts outperforms the effectiveness of the system with a single inerter. At the same time, base shear, $V_b$, and base moment, $M_b$, of the structure is lower when the outrigger-inerter system is employed. Again, when the structure is equipped with the proposed system with a pair of clutching inerters, the seismic performance of the structure in reducing base shear, and base moment, forces improved compared to the case of the outrigger-inerter system with a single inerter. Figs. 5.6 and 5.7 reveal similar trends as those observed from the time history analysis in Fig. 5.5.

To investigate the seismic performance of structures with multi-level outrigger-inerter systems, now consider the case when the twenty-story structure is equipped with two outrigger-inerters in the different heights of the building. Given that the structure with one outrigger-inerter system at the top story suppresses the inter-story drifts effectively [Figs. 5.5–5.7], one outrigger-inerter is still kept at the top floor. The second outrigger-inerter is located in the lower half of the height of the structure as it is shown in Fig. 5.2 in order to further reduce base shear, $V_b$, and base moment, $M_b$. Figs. 5.8–5.10 plot the peak inter-story drifts, base shears $V_b$, and base moments
Fig. 5.8. Time-history analysis of the twenty-story benchmark building equipped with multi-level outrigger-inerter systems, with damping ratio, $\xi = 2\%$, and mass ratio, $\sigma = 10\%$, when subjected to the Array 2/360 ground motion recorded during the 2004 Parkfield earthquake: (a) Single inerter which may induce deformation; and (b) pair of inerter that can only resist the motion.
Fig. 5.9. Time-history analysis of the twenty-story benchmark building equipped with multi-level outrigger-inerter systems, with damping ratio, $\xi = 2\%$, and mass ratio, $\sigma = 10\%$, when subjected to the Nishi-Akashi/000 ground motion recorded during the 1995 Kobe earthquake: (a) Single inerter which may induce deformation; and (b) pair of inerterst that can only resist the motion.
Fig. 5.10. Time-history analysis of the twenty-story benchmark building equipped with multi-level outrigger-inerter systems, with damping ratio, $\xi = 2\%$, and mass ratio, $\sigma = 10\%$, when subjected to the Beverly Hills-Mulhol/009 ground motion recorded during the 1994 Northridge earthquake: (a) Single inerter which may induce deformation; and (b) pair of inerters that can only resist the motion.
$M_b$, of the twenty-story structure equipped with multi-level outrigger-inerter systems for a single inerter (left plots) and a pair of clutching inerter systems (right plots) when subjected to the Array 2/360 ground motion recorded during the 2004 Parkfield California earthquake; the Nishi-Akashi/000 ground motion recorded during the 1995 Kobe Japan earthquake; and the Beverly Hills-Mulhol/009 ground motion recorded during the 1994 Northridge California earthquake, respectively.

The time history response shown in Fig. 5.8 reveals that the structure equipped with two outrigger-inerter systems effectively decrees the inter-story drifts, base shear, $V_b$, and base moment, $M_b$, forces. Again, when a pair of clutching inerter systems that can only resist the motion are employed, the seismic performance of the structure outperforms the seismic efficiency of the structure equipped with the outrigger-inerter systems with a single inerter system. The seismic response of the structure equipped with two outrigger-inerter systems slightly improved compared to the case when the structure equipped with one outrigger-inerter system at the top story. Therefore, it can be concluded that for the twenty-story frame, using two outrigger-inerter systems may not provide a drastic improvement in the seismic performance of the structure compared to the case of one outrigger-inerter system. Figs. 5.9 and 5.10 reveal similar trends as those observed from the time history analysis in Fig. 5.8.

5.6 Conclusions

This chapter investigates the potential advantages of using a novel response-modification strategy, the outrigger-inerter system, for the seismic protection of tall building structures. The proposed seismic control mechanism employs inerter systems vertically within a conventional core-to-external
column outrigger system. The chapter examines the response of both cases of a single inerter and a pair of clutching inerters that can only resist the motion of the structure. A new material based on the concept of the inerter is developed to implement the outrigger-inerter system in OpenSees. The time history analyses show the proposed system suppresses inter-story drifts effectively. Furthermore, the seismic control strategy decreases base shear and base moment forces in the structure. When two parallel inerters are used [a pair of clutching inerters], the effectiveness of the proposed system outperforms the efficacy of the outrigger-inerter with a single inerter. In the case that the structure is equipped with two outrigger-inerter systems, the seismic response of the structure slightly improved.

In view of these findings, the use of the proposed outrigger-inerter system emerges as an attractive response modification strategy for high-rise building structures.
Chapter 6

Seismic Response of the 9-story SAC Buildings Equipped with Pressurized Sand Dampers

This chapter investigates the seismic response analysis of the 9-story SAC building equipped with pressurized sand dampers—a new type of low-cost energy dissipation devices where the material enclosed within the damper housing is pressurized sand. The strength of the pressurized sand damper is proportional to the externally exerted pressure on the sand via prestressed steel rods, therefore, the energy dissipation characteristics of a given pressurized sand dampers can be adjusted according to a specific application. The strong pinching behavior of the pressurized sand dampers is characterized with a previously developed 3-parameter Bouc-Wen hysteretic model (Makris et al. 2021) which for this study is implemented in the open source code OpenSees with a C++ algorithm and it is used to analyze the seismic response of the 9-story SAC building subjected to several strong ground motions that exceed the design response spectrum for all soil categories. The paper shows that for the family of strong ground motions used in this study, pressurized sand dampers with strength of the order of 5% to 10% of the weights of their corresponding floors are capable to keep the interstory drifts of the 9-story SAC building at or below 1%.

6.1 Introduction

In the early 1970s a new concept for seismic protection, by modifying the earthquake response of structures with specially designed supplemental energy-dissipation devices was
brought forward in the seminal papers by Kelly et al. (1972) and Skinner et al. (1974) and was implemented in important structures that were under design at that time such as the South Rangitikei Rail Bridge (Beck and Skinner 1973; Skinner et al. 1974; Kelly 1997), the Union House Building in Auckland (Boardman et al. 1983) and the Wellington Central Police Station in Wellington (Charleson et al. 1987), New Zealand. The 1972 paper by Kelly et al. marks the beginning of the use of passive energy dissipation (response modification) devices for the seismic protection of structures which today find world-wide applications. Supplemental passive energy dissipation devices enhance the ability of a framing structure to dissipate the earthquake induced kinetic energy; therefore, limiting inelastic structural deformations and damage (Constantinou and Symans 1993; Whittaker et al. 1993). Devices most commonly used for the response modification of structures include viscous fluid dampers, viscoelastic fluid and viscoelastic solid dampers, friction dampers, metallic yielding dampers together with buckling-restrained braces (Soong and Dargush 1997; Constantinou et al. 1998; Hanson and Soong 2001; Black et al. 2002, 2004).

A half century after the first application of supplemental energy dissipation devices (torsionally yielding steel beam dampers) at the stepping piers of the South Rangitikei Rail Bridge (Kelly et al. 1972; Skinner et al. 1980), viscous fluid dampers and buckling-restrained braces have emerged as the two types of passive energy dissipation devices that today enjoy the widest implementations. Viscous fluid dampers that generate fluid flow through orifices or values were originally developed for shock isolation in military applications and their technology was gradually transferred to civil applications in the 1980s (Constantinou et al. 1998; Symans et al. 2008). A potential challenge with fluid dampers is whether they can maintain their long-term integrity when placed in civil structures which are subjected to a verity of dynamic displacements.
ranging from impulsive shocks to prolonged fluctuating displacement histories (Matier and Ross 2013). Early theoretical studies on the problem of viscous heating of fluid dampers have been presented by Makris (1998) and Makris et al. (1998), which have been confirmed experimentally by Black and Makris (2006, 2007) and have uncovered the potential failure of fluid dampers due to viscous heating. Buckling-restrained braces (BRBs) are yielding braces that offer supplemental hysteretic energy dissipation while increasing the strength of the structure (Watanabe et al. 1988; Wada et al. 1989; Black et al. 2002, 2003, 2004; FEMA 547 2006). Because of their distributed yielding that leads to stable hysteretic behavior, buckling-restrained braces enjoy worldwide acceptance and they have been proven to be dependable response modification devices for specific applications where the displacement demands are relatively small (few centimeters) [Sabelli et al. 2003; Fahnestock et al. 2007].

**Fig. 6.1.** Schematic of a pressurized sand-damper in which energy is dissipated from the shearing action of the sand as the sphere mounted on the damper piston is plowing through the pressurized sand. The pressure on the sand is exerted with external post-tensioned steel rods that their tensile force can be easily monitored real-time with strain-gauges.
Recently an innovative low-cost, long-stroke pressurized sand damper was developed and tested by Makris et al. (2021). Given that the material surrounding the moving piston and enclosed within the damper housing is pressurized sand as illustrated in Fig. 6.1, the pressurized sand damper does not suffer from the challenge of viscous heating and failure of its end-seals; therefore, it can be implemented in harsh environments with extreme high or low temperatures. Furthermore, its symmetric force output is velocity independent and it can be continuously monitored with standard inexpensive strain gauges installed along the post-tensioned rods that exert the pressure on the sand as shown in Fig. 6.1.

Fig. 6.2. View of the prototype pressurized sand-damper mounted on the experimental set-up during cyclic testing at the University of Patras, Greece.
A prototype pressurized sand damper was built and tested in the structures Laboratory of the University of Patras, Greece at various exerted pressures, $p$, stroke amplitudes, $u_0$, and frequencies, $f_0$, by employing the experimental setup shown in Fig. 6.2 (Makris et al. 2021). Fig. 6.3 shows selective recorded force-displacement loops from the prototype pressurized sand damper subjected to different pressures and stroke amplitudes. The recorded loops exhibit a repeatable stable behavior with a pronounced pinching that manifests at large strokes. In view of this fail-safe behavior at larger displacement amplitudes in association with the other attractive features of the pressurized sand damper outlined earlier, this paper presents a comprehensive seismic response analysis of the nine-story moment-resisting steel building designed for the SAC Phase II Project (SAC Venture Guidelines 2000). This structure that is well-known to the literature (Gupta and Krawinkler 1998; Chopra and Goel 2002; Aghagholizadeh and Makris 2018) was designed to meet the seismic code (pre-Northridge Earthquake) and represents typical medium-rise buildings designed in the greater area of Los Angeles, California.

### 6.2 Mathematical Model of the Pressurized Sand Damper

Using arguments from dimensional analysis (Langhaar 1951; Barenblatt 1996; Makris and Black 2003a, b) in association with the versatility of the Bouc-Wen model (Wen 1976; Baber and Noori 1985; Constantinou and Adnane 1988; Charalampakis and Koumousis 2008), recently Makris et al. (2021) showed that the strong nonlinear behavior and the pronounced pinching effect at larger strokes of the pressurized sand damper can be satisfactorily approximated with

$$F_d(t) = \Pi_{SD} p R^2 \eta \text{sgn}[\dot{u}(t)] + \zeta z(t)$$

where $\Pi_{SD} p R^2 = Q$ is the strength of the pressurized sand damper, $p$ is the externally exerted pressure on the sand, $R$ is the radius of the moving sphere, $\Pi_{SD}$ is a dimensionless damper constant,
Fig. 6.3. Selected force-displacement loops of the pressurized sand damper shown in Fig. 6.2 recorded at various exerted pressures, $p$, stroke amplitudes, $u_0$, and driving frequencies, $f_0$. 
\( \dot{u}(t) \) is the velocity of the damper piston and \( z(t) \) is a dimensionless internal time-dependent variable of the Bouc-Wen model that is controlled by

\[
\dot{z}(t) = \frac{dz(t)}{dt} = \frac{1}{cR} [\dot{u}(t) - \beta \dot{u}(t)|z(t)|^n - \gamma |\dot{u}(t)| z(t) |z(t)|^{n-1}]
\]  

(6.2)

The exponent, \( n \), appearing in Eq. (6.2) controls the transition from the elastic to the yielding regime and it is set equal to one \( (n = 1) \) given that its effect is immaterial. Parameters \( \beta \) and \( \gamma \) control the shape of the hysteretic loop, whereas parameter \( c \) expresses the ratio of the yield displacement of the damper to the radius of the sphere, \( R \), and it is set equal to \( 1/4 \) \( (c = 0.25) \). Parameters \( \eta \) and \( \zeta \) in Eq. (6.1) together with the parameters \( \beta \) and \( \gamma \) in Eq. (6.2) are essentially the only four parameters of the proposed model that need to be identify with nonlinear regression analysis (Makris et al 2021). The hysteretic damper model described by Eqs. (6.1) and (6.2) is frequency independent given that the friction stresses that develop along the steel-sphere interface are essentially rate-independent.

Fig. 6.4. Measured force from the pressurized sand-damper during cyclic testing as the sphere passes by the displacement origin at pressure levels \( p = 1.0, 2.0, 3.0, 4.0 \) and \( 5.0 \) MPa.
Figure 6.4 plots the measured strength of the damper $Q = \Pi_{SD}pR^2$ that is the output force from the pressurized sand damper during cyclic testing as the sphere mounted on the piston passes by the displacement origin at pressure levels $p = 1.0, 2.0, 3.0, 4.0, \text{ and } 5.0 \text{ MPa}$. The data appearing in Fig. 6.4 include the data initially presented in Makris et al (2021), together with additional experimental data that were obtained during the course of this study. Linear regression analysis of the recorded data yields a value for the dimensionless damper constant $\Pi_{SD} = Q/pR^2 = 5.12$. In view of the linear dependence of the strength, $Q$, to the exerted pressure, $p$ (as is suggested by dimensional analysis), Fig. 6.5 plots all the force-displacement loops recorded during our experimental campaigns for all frequencies and exerted external pressures normalized to the strength of the pressurized sand damper $Q = \Pi_{SD}pR^2 = 5.12 \text{ pR}^2$. Given the normalization; at small displacement amplitudes the normalized damper output force $F_d/Q$ rides essentially along

![Fig. 6.5. Recorded force-displacement loops at various amplitudes, exerted pressures and frequencies normalized to the strength of the pressurized sand damper $Q = 5.12 \text{ pR}^2$](image)
the line ±1, therefore parameter \( \eta \) is set equal to one \( (\eta = 1) \) and the hysteretic model described by Eqs. (6.1) and (6.2) reduces to a 3 parameter model in which only parameters \( \zeta \), \( \beta \) and \( \gamma \) need to be identified from nonlinear regression analysis.

![Normalized force-displacement loops](image)

**Fig. 6.6.** Normalized force-displacement loops to the strength of the pressurized sand damper: \( Q = \Pi_{SP} p R^2 \) recorded at all exerted pressures and all cyclic frequencies for stroke amplitudes: (a) \( \pm 4\text{cm} \); (b) \( \pm 6\text{cm} \); and (c) \( \pm 8\text{cm} \) (solid black lines). Predictions of the 3-parameter \( (\zeta, \beta \text{ and } \gamma) \) hysteretic model described by Eqs. (6.1) and (6.2) with frequencies \( f_0 = 0.10\text{Hz} \) (thin solid red lines) and \( f_0 = 10.0\text{Hz} \) (heavy dashed red lines).
Figure 6.6 plots the performance of the calibrated hysteretic model described by Eqs. (6.1) and (6.2) to capture the overall recorded behavior (at all pressures and all frequencies) as the pressurized sand damper undergoes cyclic motion with displacement amplitudes \( u_0 = 4.0 \text{ cm}, 6.0 \text{ cm} \) and \( 8.0 \text{ cm} \). The optimal values of the parameters \( \zeta, \beta \) and \( \gamma \) of the nonlinear hysteresis model that resulted from nonlinear regression analysis that best fit the entire families of all the recorded force-displacement loops with stroke amplitude \( u_0 = 4.0 \text{ cm}, 6.0 \text{ cm} \) and \( 8.0 \text{ cm} \) are shown in each subplot. When both displacement and velocity histories are symmetric, the hysteretic model described by Eqs. (6.1) and (6.2) is rate-independent.

The reason that the optimal values of parameters, \( \zeta, \beta \) and \( \gamma \) depend on the stroke-amplitude, \( u_0 \), is due to a “first passage effect” that is similar to the scragging effect in elastomeric bearings where larger values of bearing stiffness are observed in the first half-cycle of loading of an untested bearing than in subsequent cycles (Thompson et al. 2000; Morgan et al. 2001). In the pressurized sand damper, as the sphere attached to the damper piston moves to larger strokes, it further compresses the sand towards the stroke-end and in subsequent cycles of the same amplitude, \( u_0 \), the moving sphere encounters less resistance which translates to a milder pinching effect. This first passage effect essentially vanishes after the first \( \frac{3}{4} \) of a cycle as shown in Fig. 6.3.

**6.3 Development and Verification of an OpenSees Routine for Pressurized Sand Dampers**

Given that the aim of the paper is to examine the response of multistory structures equipped with pressurized sand dampers, the first task is the development of a C++ routine that offers the force output from the nonlinear hysteretic model described by Eqs. (6.1) and (6.2), which was implemented in the open source structural analysis software OpenSees (McKenna et al. 2000). The
developed C++ algorithm follows essentially the incremental formulation presented by Haukaas (2003). Accordingly at time-step \( t_{n+1} \), the force output of the damper is

\[
F_d(t_{n+1}) = Q[s\text{gn}(\dot{u}(t_{n+1})) + \zeta z(t_{n+1})]
\]

(6.3)

and the rate equation for \( z(t_{n+1}) \) is discretized by a backward Euler scheme as summarized in Appendix I.

The verification of the C++ algorithm that was implemented in OpenSees is presented herein with the response analysis of an elastic single-degree-of-freedom (SDOF) structure with mass \( m \), stiffness \( k \), and viscous damping \( c \) shown in Fig. 6.7 that is equipped with a pressurized sand damper with strength \( Q \) supported on a non-compliant chevron frame. The SDOF elastic structure has natural frequency \( \omega_0 = 2\pi/T_0 = \sqrt{k/m} \) and viscous damping ratio \( \xi = c/2m\omega_0 \) and is subjected to earthquake induced excitation, \( \ddot{u}_g(t) \). Dynamic equilibrium of the SDOF structure gives

\[
m\ddot{u}(t) + c\dot{u}(t) + k u(t) + F_d(t) = -m\ddot{u}_g(t)
\]

(6.4)

where \( F_d(t) \) is the hysteretic damping force offered by the pressurized sand damper given by Eq. (6.1). Upon dividing with the mass, \( m \), Eq. (6.4) in association with Eq. (6.1) gives

\[
\ddot{u}(t) + 2\xi\omega_0 \dot{u}(t) + \omega_0^2 u(t) + \frac{Q}{m}[s\text{gn}[\dot{u}(t)] + \zeta z(t)] = -\ddot{u}_g(t)
\]

(6.5)

where \( z(t) \) is the dimensionless internal variable offered by Eq. (6.2) and parameter \( n = 1 \). Accordingly, the state-vector of the system \( \{y(t)\} \) is expressed as

\[
y(t) = \langle y_1(t), y_2(t), y_3(t) \rangle^T = \langle u(t), \dot{u}(t), z(t) \rangle
\]

(6.6)
where the superscript, \( T \), stands for the transpose of the line vector, \(< >\). The time-derivative state-vector, \( \{ \dot{y}(t) \} \), is expressed by

\[
\{ \dot{y}(t) \} = \begin{bmatrix}
\dot{u}(t) \\
\dot{u}(t) \\
\dot{z}(t)
\end{bmatrix} = \begin{bmatrix}
y_2(t) \\
-\ddot{u}_g(t) - 2\xi_o \omega_o y_2(t) - \omega_o^2 y_1(t) - \frac{Q}{m} \text{sgn}[y_2(t)] + \zeta y_3(t) \\
\frac{1}{c_R} [y_2(t) - \beta |y_2(t)|^{n-1} - y |y_2(t)| y_3(t) |y_3(t)|^{n-1}]
\end{bmatrix}
\]  

(6.7)

The numerical solution obtained with the C++ algorithm outlined in Appendix I, and implemented in the open source software OpenSees (McKenna et al. 2000), is compared against a numerical solution obtained with MATLAB in which the time derivative of the state-vector \( \{ \dot{y}(t) \} \) offered by Eq. (6.7) is integrated with standard ODE solvers available in MATLAB (2020).
\( T_o = 0.5 \text{sec}, \xi_0 = 0.03, c = 0.25, \zeta = 0.025, \beta = -3.80, \gamma = 3.43, n = 1 \)

**Fig. 6.8.** Displacement time histories [(a) and (c)] of the SDOF structure shown in Fig. 6.7 without damper (black solid lines) and with damper (colored lines) together with the corresponding captured force-displacement loops [(b) and (d)] when subjected to the Cholame 2/360 ground motion recorded during the 2004 Parkfield earthquake (left) and the Nishi/000 ground motion recorded during 1995 Kobe earthquake (right). The numerical solutions obtained with the C++ algorithm implemented in OpenSees and with MATLAB are essentially identical.
Figure 6.8 plots the relative-to-the-ground displacement response of the SDOF structure shown in Fig. 6.7 with $T_0 = 0.5\, sec$, and $\xi = 0.03$ without and with a pressurized sand damper with $Q/mg = 0.05$ [subplots (a) and (b)] and $Q/mg = 0.10$ [subplots (c) and (d)] when subjected to the Cholame 2/360 ground motion recorded during the 2004 Parkfield California earthquake; and the Nishi-Akashi/000 ground motion recorded during the 1995 Kobe, Japan earthquake.

The numerical solution obtained with C++ algorithm implemented in OpenSees and with MATLAB (2020) are essentially identical. Fig. 6.8 indicates that the high damper strength configuration ($Q/mg = 0.1$) results to smaller displacements and also smaller peak forces since the pinching phenomenon is less pronounced at smaller displacements.

Fig. 6.9. Elastic response spectra of the six recorded ground motions used in this study together with the design elastic spectra for soil class D and E (ASCE/SEI 7-13).
The seismic response analysis of the 9-story SAC building that follows in this study uses the six strong ground motions appearing at the bottom of Figs. 6.8, 6.13 and 6.14. The elastic response spectra for viscous damping ratio $\xi = 5\%$ of these 6 historic ground motions exceed by far at the preyielding period of the structure, $T_1 = 2.3$ sec, the design elastic spectra for soil class D and E (ASCE/SEI 7-13).

### 6.4 Seismic Response of the 9-Story SAC Building Equipped with Pressurized Sand Dampers

The 9-story SAC building (Gupta, and Krawinkler 1999; Chopra and Goel 2002) was designed to meet the seismic code (pre-Northridge earthquake) and represents typical medium-rise buildings designed for the greater area of Los Angeles, California.

This moment-resisting, steel building is 40.82 m tall with nine-stories above ground level and a basement as shown in Fig. 6.10. The bays are 9.15 m wide, with five bays in north-south (N-S) and east-west (E-W) directions. Floor-to-floor height of each story is 3.96 m, except for the basement and first floor which are 3.65 and 5.49 m, respectively, as shown in Fig. 6.10. Column splices are on the first, third, fifth, and seventh floors and located 1.83 m above the beam-column joint. The column bases are modeled as pinned connections, and it is assumed that the surrounding soil and concrete foundation walls are restraining the structure in horizontal direction at the ground level. The columns are 345-$MPa$ wide-flange steel sections, and the floor beams are composed of 248 $MPa$ wide-flanges steel sections. All beam column connections of the frames are rigid except for the corner columns which are pinned in order to avoid bi-axial bending of the members. In this study, the exterior frame in N-S direction is chosen for the 2-D validation of our planar analysis. The nonlinear response of the nine-story MDOF structure is computed with the nonlinear built-in model “Steel01” in OpenSees which essentially is a bilinear model, at the stress-strain
Fig. 6.10. Top: Nine-story moment-resisting steel frame designed for the SAC Phase II Project equipped at all levels with pressurized sand dampers supported on a non-compliant chevron frame. Bottom: Geometric and physical characteristics pertinent to the nine-story SAC building. The indicated seismic mass is the entire mass of each floor of the SAC building.
level. Accordingly, an elastic modulus of $E = 210 \text{ GPa}$, a strain hardening ratio (post-yield to elastic, pre-yield modulus ratio), $\alpha = 0.03$, and a yield strength, $\sigma_y = 248 \text{ MPa}$ for beams and $\sigma_y = 345 \text{ MPa}$ for columns have been used.

Figure 6.11 plots the computed push-over curve (base shear vs roof displacement) of the nine-story moment resisting steel building without the hysteretic damper, which is essentially identical with the push-over curve presented in past investigations (Gupta, and Krawinkler 1999; Chopra and Goel 2002). The resulting preyielding period is $T_1 \approx 2.3 \text{ sec}$. The C++ routine summarized in Appendix I that returns the force output of the pressurized sand damper given the
time history of the interstory displacement was implemented in OpenSees for the response analysis of the 9-story moment-resisting SAC building equipped with pressurized sand dampers shown in Fig. 6.10.

Figure 6.6 indicates that depending on the stroke amplitude \((u_0 = 4.0\, cm, 6.0\, cm\) and \(8.0\, cm\)), neighboring, yet different values of the parameters \(\beta, \gamma\) and \(\zeta\) of the Bouc-Wen hysteretic model are needed to best fit the recorded force displacement loops at each given stroke amplitude. When the values of parameters \(\beta, \gamma\) and \(\zeta\) identified for a lower stroke amplitude (say \(u_0 = 4.0\, cm\)) are used to model the damper response at higher amplitudes (say \(u_0 = 6.0\, cm\) or \(u_0 = 8.0\, cm\)), then a more pronounced pinching effect is produced by the hysteretic model. Accordingly, in order to be on the conservative side and avoid the generation of unrealistically large hysteretic forces at every analysis the values of the model parameters, \(\beta, \gamma\) and \(\zeta\) are those associated with displacement amplitudes at or above the interstory displacements of the building when equipped with dampers. As an example, Fig. 6.12(left) shows with heavy dark bars the interstory displacement of the 9-story SAC steel frame without dampers when subjected to the Cholame 2/360 ground motion recorded during the 2004 Parkfield, California earthquake. Given that the interstory displacements at the 8\(^{th}\) and 9\(^{th}\) level marginally exceed \(6.0\, cm\), whereas all the other interstory displacements are below \(6.0\, cm\), the analysis when the 9-story SAC building is equipped with dampers uses the parameters for \(\beta = -3.80\), \(\gamma = 3.43\) and \(\zeta = 0.025\) identified from cyclic testing of the damper with stroke amplitude \(u_0 = 6.0\, cm\) [see Fig. 6.6(b)]. Fig. 6.12(left) shows that interstory displacements of the 9-story SAC building when equipped with pressurized sand dampers (gray bars) are all below \(6.0\, cm\), therefore the choice of the the parameter values \(\beta =

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−3.80, \( \gamma = 3.43 \) and \( \zeta = 0.025 \) is appropriate. The same applies to the response analysis of the 9-story SAC building equipped with pressurized sand dampers shown in Fig. 6.12(right) when

\[
T_1 = 2.27 \text{sec}, \xi = 0.02, c = 0.25, \zeta = 0.025, \beta = -3.80, \gamma = 3.43, n = 1
\]

Fig. 6.12. Peak interstory displacements of the 9-story SAC steel building without (heavy dark bars) and with (gray bars) pressurized sand dampers with strength \( Q = 0.05 \text{mg} \) (a); and \( Q = 0.10 \text{mg} \) (b) when subjected to the Cholame 2/360 ground motion recorded during the 2004 Parkfield earthquake (left) and the Nishi/000 ground motion recorded during 1995 Kobe, Japan earthquake (right) shown at the bottom of Fig. 6.8.
subjected to the Nishi-Akashi/000 ground motion recorded during the 1995 Kobe, Japan earthquake. Fig. 6.12 shows that the pressurized sand dampers are effective in reducing interstory displacements and when their strength, $Q$ is 10% of the weight of their corresponding floors all drifts are below 1% of the story height.

Fig. 6.13(left) shows with heavy dark bars the interstory displacements of the 9-story SAC steel frame without dampers when subjected to the Poe Road/270 ground motion recorded during the 1987 Superstition Hills, California earthquake. All interstory displacements other than the one of the first level are below 6.0cm; therefore, the analysis when the SAC building is equipped with pressurized sand dampers uses the parameters $\beta = -3.80$, $\gamma = 3.43$ and $\zeta = 0.025$ identified from cyclic testing of the damper with stroke amplitude $u_0 = 6.0cm$ [see Fig. 6.6(b)]. Fig. 6.13(left) shows that the interstory displacements of the 9-story SAC building when equipped with pressurized sand dampers (gray bars) are all below 6.0cm, therefore the aforementioned choice of parameters $\beta$, $\gamma$ and $\zeta$ is appropriate. The same applies to the response analysis of the 9-story SAC building with pressurized sand dampers shown in Fig. 6.13(right) when subjected to the Gilroy Array 6/230 ground motion recorded during the 1979 Coyote Lake, California earthquake. Fig. 6.13 shows that the pressurized sand dampers are effective in reducing interstory drifts at or below 1% of the story height.

Figure 6.14 shows the interstory displacements of the 9-story SAC steel frame without and with dampers when subjected to the El Centro Array 5/140 ground motion recorded during the 1979 Imperial Valley earthquake (left) and the Newhall/360 ground motion recorded during 1994 Northridge earthquake (right). Again the pressurized sand dampers with strength $Q_i = 0.05m_ig$ or $Q_i = 0.10m_ig$ are effective in suppressing interstory drifts except at the first level which
Fig. 6.13. Peak interstory displacements of the 9-story SAC steel building without (heavy dark bars) and with (gray bars) pressurized sand dampers with strength $Q = 0.05\,cm$ (a); and $Q = 0.10\,cm$ (b) when subjected to the Poe Road/270 ground motion recorded during 1987 Superstition Hills earthquake (left) and the Gilroy Array 6/230 ground motion recorded during the 1979 Coyote Lake earthquake (right).
Fig. 6.14. Peak interstory displacements of the 9-story SAC steel building without (heavy dark bars) and with (gray bars) pressurized sand dampers with strength $Q = 0.05\,cm$ (a); and $Q = 0.10\,mg$ (b) when subjected to the El Centro Array 5/140 ground motion recorded during the 1979 Imperial Valley earthquake (left) and the Newhall/360 ground motion recorded during 1994 Northridge earthquake (right).
experiences drifts of the order of 1.2% of the floors height level when the damper strength \( Q_1 = 0.10 m_1 g \). In this case at the first floor dampers with strength larger than \( Q_1 = 0.10 m_1 g \) need to be installed to reduce the first story displacement below 1% of the floor height.

### 6.5 Conclusion

The need to limit inelastic deformations and damages during the earthquake shaking of multistory buildings has prompted during the last four decades the use of supplemental energy dissipation devices. At present viscous fluid dampers and buckling-restrained braces have emerged as the two types of passive energy dissipation devices that enjoy the widest implementations.

This paper investigates the seismic response analysis of the 9-story SAC building when equipped with pressurized sand dampers—a new type of low-cost, sustainable energy dissipation devices where the material enclosed within the damper-housing is pressurized sand. The strength of the pressurized sand damper is proportional to the externally exerted pressure on the sand via prestressed steel rods and can be adjusted at will by monitoring the axial strains on the steel rods with standard inexpensive strain gauges. The strong pinching behavior of the pressurized sand damper is characterized with a previously developed 3-parameter Buck-Wen hysteretic model which in this work is implemented in the open source code OpenSees with a C++ algorithm and is used to analyze the seismic response of yielding buildings.

The inelastic response analysis study used six strong recorded ground motions that exceed the design response spectrum for all soil categories at the preyielding period of the 9-story SAC building. The paper concludes that pressurized sand dampers with strength of the order of 5% to 10% of the weights of corresponding floors are capable to keep interstory drifts of the 9-story SAC building at or below 1%.
REFERENCES


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APPENDIX I

Modeling the Pressurized Sand Damper in the Open Source Code OpenSees

The procedure implemented in OpenSees to model the hysteretic damper described by Eqs. (6.1) and (6.2) is summarized in Appendix I.

While \(|z_{n+1}^{\text{old}} - z_{n+1}^{\text{new}}| > tol\)

- Evaluate function \( f(z_{n+1}) \)
  \[
  \psi = \beta + \gamma \text{sgn}[(u_{n+1} - u_n)z_{n+1}]
  \]
  \[
  \phi = 1 - |z_{n+1}|^n \psi
  \]
  \[
  f(z_{n+1}) = z_{n+1} - z_n - \frac{\phi}{u_y}(u_{n+1} - u_n)
  \]

- Evaluate function derivatives (prime denotes derivative with respect to \(z_{n+1}\))
  \[
  \phi' = -n|z_{n+1}|^{n-1} \text{sgn}(z_{n+1})\psi - |z_{n+1}|^n \psi
  \]
  \[
  f'(z_{n+1}) = 1 - \frac{\phi'}{u_y}(u_{n+1} - u_n)
  \]

- Obtain trial value in the Newton scheme:
  \[
  z_{n+1}^{\text{new}} = z_{n+1} - \frac{f(z_{n+1})}{f'(z_{n+1})}
  \]

- Update \(z_{n+1}\) (and store the old value for the convergence check)
  \[
  z_{n+1}^{\text{old}} = z_{n+1} \quad \text{and} \quad z_{n+1} = z_{n+1}^{\text{new}}
  \]

Compute the force described in Eq. (6.1).