A Restrained Beam Sub-model Assembly (RBSA) For Progressive Collapse Analysis

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A RESTRAINED BEAM SUB-MODEL ASSEMBLY (RBSA) FOR PROGRESSIVE COLLAPSE ANALYSIS

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A RESTRAINED BEAM SUB-MODEL ASSEMBLY (RBSA)
FOR PROGRESSIVE COLLAPSE ANALYSIS

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The progressive collapse failure mode occurs in buildings when a load-carrying element is lost due to extreme events such as explosions caused by terrorist attacks or vehicular impacts. Guidelines and many research efforts have been established in order to limit and prevent total collapse of buildings after losing a load-carrying component (e.g., a column). Two famous examples of progressive collapse are the 1968 Ronan Point apartment building kitchen explosion on the 18th floor and the 1995 Alfred P. Murrah Federal Building bombing in Oklahoma City. Both examples resulted in fatalities and injuries. A small number of these injuries and fatalities were due to the actual explosion while a larger number of lives were claimed during the subsequent progressive collapse event.

Progressive collapse guidelines have been developed to enhance the robustness of buildings against progressive collapse. Two of the main progressive collapse guidelines are the General Services Administration, “Progressive Collapse Analysis and Design Guidelines,” (GSA) and the Department of Defense (DoD) Unified Facilities Criteria 4-023-03 “Design of Building to Resist Progressive Collapse” (UFC 4-023-03) guidelines. In addition to these guidelines, many research efforts have been performed to investigate the ability of buildings to resist progressive collapse, such as development of a beam sub-model that has springs to simulate connection’s elements behavior, experimentation to facilitate the load-deflection relationship and connection behavior.

One of the most important findings in progressive collapse research is the effect of the load-deflection relationship at the connection where the column removal scenario occurs. This research focuses on the development of a beam sub-model that has longitudinal and rotational springs to simulate the surrounding
frames and connection behavior. The sub-model equations approximate the vertical displacement of the connection subjected to the column removal and include nonlinear geometry (NLG) effects and some nonlinear material (NLMG) behavior. The derived equation is examined by simulating the behavior of both moment frame and braced frame. Additionally, the derived equations are used to simulate three different beam boundary condition scenarios: simply supported (PFP), Fixed-Fixed-Fixed (FFF), and modified catenary Pin-Pin-Pin (PPP); the results from these simulations are validated against existing experimental results. For the cases considered in this thesis, the RBSA model matches the finite element with in a range of 0% to 10%. Finally, the axial force contribution in the linear region is investigated.
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This is dedicated to my wonderful, patient spouse and my supportive immediate and extended family.
CHAPTER 1
INTRODUCTION

The American Society of Civil Engineers (ASCE) Standard 7 (ASCE 7) commentary identifies progressive collapse as “the spread of an initial local failure from element to element, eventually resulting in the collapse of an entire structure or a disproportionately large part of it” [1]. One of the most famous examples of progressive collapse is the Ronan Point apartment building collapse in 1968 where an explosion occurred on the 18th floor in a corner kitchen leading to a failure in the outer wall panel and the corner bay. The entire structure was affected as the bays above and below the corner bay started to fail gradually [2]. Another famous example of progressive collapse was the Alfred P. Murrah Federal Building bombing in Oklahoma City in 1995. A truck full of explosives detonated and caused catastrophic blast damage to one side of the building, which led to a gradual collapse that killed almost 170 people and injured over 800 people [3]. Since these events, many designs guidelines have been produced and developed to prevent progressive collapse including the General Services Administration (GSA) Progressive Collapse Analysis and Design Guidelines [4] and the Department of Defense (DoD) Unified Facilities Criteria (UFC) 4-023-03 [1]. The main goal of these design guidelines is to enhance the ability of buildings to prevent total collapse after losing one or more load-carrying elements. Along with these guidelines, a number of research efforts focused on progressive collapse have been performed. The UFC guidelines have been improved to provide more economic designs [5]. Sub-models including springs to simulate the behavior of the connections of complicated systems have been investigated to simplify progressive collapse analysis [7-12]. Numerous progressive collapse experiments have been conducted in order to facilitate the understanding of load-deflection relationships, connection behavior and rotation, and development of catenary action of the effected area after the removal of the middle column [13-19]. While many efforts recognize the contribution of axial force in ultimately resisting loads, axial force contribution has often been neglected in the presumed linear region of the load-deflection curve (i.e. until yield) [9, 17, 20]. Axial force can provide
development of catenary action inside beams, which can enhance the load-carrying capacity of the system.

The aim of this research is to develop an intuitive beam sub-model that can approximate the behavior of two beams supported at their ends and subjected to a middle column removal scenario. Specifically, this research seeks to capture key contributions in the static load-deflection curve from surrounding frames, connections, and the beams directly supporting the load. The goal of the sub-model is to facilitate analysis without the use of extremely complicated finite element models; the proposed beam sub-model contains longitudinal and rotational springs to simulate the behavior of connections and produces results that compare favorably to the results from complicated finite element models. The derived load-deflection equation is first examined by simulating the behavior of an idealized simply supported beam (PFP) Fig. (3.3a) and an idealized Fix-Fix-Fix beam (FFF) Fig. (3.3b); simulating the nonlinear geometric behavior of the PFP, FFF, and Pin-Pin-Pin (PPP) beam Fig. (3.3c); and simulating the nonlinear material and geometry of the PFP, FFF and PPP beams. Subsequently, moment frame and braced frame are analyzed to include the effect from surrounding frame elements and connections. The proposed sub-model is validated against existing experimental results that have been collected from frames subjected to a column removal scenario with different types of connections. Finally, the contribution of axial force in the linear region is examined to investigate the validity of axial force negligence.
The objective of the UFC progressive collapse design criteria is to lower the potential of progressive collapse for new and existing DOD facilities [1]. The design procedures of UFC to resist progressive collapse have been divided into three design methods; the tie forces (TF) method, the alternate path (AP) method, and the enhanced local resistance (ELR) method.

In the TF method, the structure is tied together mechanically. Tie forces can be specified by the existing structural components that have been designed using established design techniques to carry the standard loads applied on the structure. Three types of horizontal ties and one type of vertical tie must be provided. The three horizontal ties are longitudinal, transverse, and peripheral ties. Vertical ties are needed in columns and load-bearing walls. In order to use the TF method, the structure must meet minimum criteria governing the number of bays in each direction and the length of load-bearing walls. By following the Load and Resistance Factor Design (LRFD) procedure for tie forces, the design tie strength $\phi R_n$ must be greater than or equal to the required tie strength $R_u$. Uniform, non-uniform, and cladding loads are considered in the TF method. A floor load combination equation, $1.2D + 0.5L$, is used to determine the floor load $w_F$. The tie strength for all longitudinal, transverse, peripheral, and vertical ties are computed and compared with their respective required tie strength to determine if the design is acceptable.

The AP method must be used for the exclusion of specific vertical load-bearing elements to verify that the structure can bridge over removed elements and follows the LRFD procedure where $\phi R_n \geq R_u$. The AP method contains three analysis procedures; linear static (LSP), nonlinear static (NSP) and nonlinear dynamic (NDP). The LSP analysis required the calculation of deformation-controlled actions $Q_{UD}$ and force-controlled actions $Q_{UF}$; these values must be smaller than the expected deformation capacity and strength of the component, respectively. In the NSP and NDP analyses, a finite element model needs to be created in order to
capture the force-displacement behavior and calculate $Q_{UD}$ and $Q_{UF}$, which are once again compared with the expected component deformation capacity and strength, respectively. For the NDP model, unfactored loads are used. The NDP model needs to be initially unloaded. The gravity loads to the entire model are then gradually increased until equilibrium is reached. After the structure reaches equilibrium, the column is removed and the analysis proceeds until the full displacement has been achieved or one vertical motion cycle has occurred at the column section removal location.

In the ELR method, the objective is to ensure that a mechanism of a ductile failure can form when the column or wall is loaded laterally to failure. In order to achieve this objective, the column or wall must not fail in shear until reaching maximum flexure strength. As with the previous methodologies, the LRFD procedure is used. The flexural demand of ELR is the nominal flexural strength $R_n$ of the column or wall under the risk categories (RC), which depends on nature of occupancy. The existing nominal flexure strength is determined using the column design defined after the application of the AP method to the design of the structure. If the baseline nominal flexural controls, then the column needs to be redesigned as required to meet the flexural demand of this controlling condition.

Progressive collapse known as a dynamic and nonlinear event, therefore, the load cases for the linear static procedure (LSP) and nonlinear static procedure (NSP) involve the use of factors to consider the inertial and nonlinear effects. McKay proposes a method in order to show that the increase factors used in the UFC are overly conservative [5]. In the static procedures, McKay identified four major issues:

1) Both LSP and NSP use the same load enhancement factor but LSP uses load increase factor (LIF) in order to account for the dynamic and the nonlinear effects while the NSP uses a dynamic increase factor (DIF) to account for only the dynamic effects. Thus, the LIF should be higher than DIF.

2) Most of the NSP cases are not suitable for an increase load factor of 2.0 except if the structure is designed to remain elastic, which is usually not the case.
3) Even with different performance levels, the load enhancement factors do not differ and thus cannot be described as functions of the wanted building performance level and the building features.

4) Variation of capacity increase factors (m-factor) in LSP cause the design to be overly conservative. Therefore, McKay introduced a method to calculate the LIF and DIF to make the design more economic.

The LSP procedure proposed by McKay is:

1) The most preventive structure element needs to be selected from bays directly around damage location at all floor levels.

2) The m-factor for the element selected on step 1 needs to be obtained using UFC tables.

3) The LIF is calculated using the m-factor from step 2 into the LIF equation found previously.

4) Determine the effective load multiplier by dividing LIF from step 3 by the m-factor from step 2.

5) Complete the linear static analysis by multiplying the effective load multiplier by the progressive collapse load (1.2D + 0.5L).

The NSP procedure proposed by McKay is:

1) The most preventive structure element needs to be selected from bays directly around damage location at all floor levels.

2) The yield rotation of the flexural component or column needs to be computed following the procedure in ASCE 41[6]. Then the ratio of allowable plastic rotation to the yield rotation of the chosen element in step 1 has to be computed.

3) With the ratio substituted into the DIF equation developed previously, the DIF could be calculated to be used in the NSP analysis.

4) The DIF from step 3 need to be applied to the progressive collapse load in order to complete NSP analysis. Both LIF and DIF found in the previse proposed procedures give a solution factor less than the overly conservative factor, 2.0, used in UFC.
Many researchers used a progressive collapse sub-model consist of beams in springs in order to simplify the column removal senior [7-12]. Meng et al. developed a three-bar spring component composed of a series of individual components by dividing and simplifying a top-seat angle with a double web angle (TSDWA) connection according to its geometry [7]. Previous research was used to establish the constitutive relationships of the equivalent springs. The resulting equivalent element model was able to predict the nonlinear mechanical behavior of this type of relation under an interior column failure scenario, including arching and flexural behavior in the initial phase and catenary action in the greater deformation phase, with both good numerical efficiency and good accuracy. Guo et al. have simplified a 4-bay continuous beam supported by five columns with the middle column removed to a 2-bay beam supported by two columns [8]. The 2 bays have been sub-modeled using horizontal restrained springs, whose stiffnesses depend on the bending rigidity of the columns, and rotational springs, whose stiffnesses depend on the bending rigidity of composite joints under hogging moment. Stylianidis and Nethercot have modeled the end and middle joints using compression and tension lateral springs and rotational springs to simulate compression in the end joints and tension in the bottom and top of beam; rotational spring were used to simulate the flexure behavior of the joints [9]. Stoddart et al. used the component method which can predict the moment-rotation behavior of common semi-rigid steel connections by assuming all components act composed as a single rotational spring for a double span situation following the removal of the center support [10]. Additionally, an axial spring was included under the assumption that tension was applied on the connections. As the rotational springs behave independently from the axial springs, an adequate rotation capacity was always available at the connection, which ensures that the axial load is the dominant failure mechanism. Cheng and Wu have used rotational springs at the bottom and at the top of the column as well as at each end connection of the beam to simulate the rotational behavior of the connections in a real frame structure [11]. Instead of analyzing a complicated frame system subjected to a middle column removal action, Stylianidis et al. developed an idealized system considering beam-to-column connections as rotational springs having a rotational stiffness and a linear elastic spring to simulate the axial behavior of the
Many experiments have been performed to investigate the behavior of connections against progressive collapse event [13-19]. Zhong et al. have conducted an experimental study on different connection types including welded unreinforced flange-bolted web (WUF), TSDWA and double web angle (DWA) [13]. The experiments mainly focused on the fracture modes and resistance mechanisms of the three samples subjected to internal column removal. Different failure modes were discovered for the three specimens. The WUF failed with multiple turbulences and discontinuous damage. The TSDWA failed due to a fracture of the tension angles connection. The DWA failed due to a fracture of the double web angles at the bolt hole. Both the WUF and the TSDWA connections developed the same mechanism to prevent collapse.

Wang et al. have investigated the response of steel substructure and connection behaviors under a column loss event [14]. The method proposed in their research represents a more efficient solution than the solid element based simulation strategy. The welded flange plate double web angle connection (WFPDWA) showed greater resistance to progressive collapse than DWA connections. The failure can be shifted from joints to the beam with connections using haunch plates due to the effective firming on the load carrying capacity.

Ghorbanzadeh et al. proposed a new joint configuration for the design or modification of pinned beam-column joints against progressive collapse using a set of plates and stainless steel pins (SSPs) [15]. The study indicated that the greatest displacement of SSP rises significantly for longer SSPs but the ductility is reduced. The case studies revealed that the proposed joint specifications are an important retrofit scheme for improving a steel joint’s tie force resistance and rotation capacity in the event of a column loss.

Daneshvar and Driver have investigated a shear connection performance, WT connection, under a column removal scenario using a numerical study and experimental data to validate the results [16]. After conducting the tests, the governing failure has been found to be either the shear of the bottom bolt or WT bottom hole rupture.

Yang and Tan have stated that failure usually begins from beam-column joints which undergo severe
loads in most progressive collapse events [17]. Their research examines seven different types of connections under middle column-removal scenario including web cleat, top and seat angle, top and seat with web angle (TSWA), fin plate, flush end plate, extended end plate and TSWA. The behavior and failure modes along with the ability to deform in catenary modes of these different connections have been provided. The web cleat connection showed the best behavior in developing a catenary action, while the flush end plate, fin plate, and TSWA connections can develop catenary action before failure. Failure and formation of catenary action are controlled by the tensile capacity of beam-column joints after the occurrence of large rotations.

Yang et al. have conducted four experiments in order to investigate the progressive collapse resistance of composite frames with web cleat and flush end plate connections [18]. The results showed that the beam-column joint failure often controls the internal composite frames' overall resistances. Additional reinforcing bars will aid in the formation of flexural and catenary action according to the observations. Because of the additional bars, the compressive horizontal reactions were minimized which suggests that the reinforcing ratio will have an effect on compressive arch formation. With additional reinforcement bars, the joint rotation angles were not affected.

Sadek et al. have presented an experimental and computational study of two steel beam-column moment connections, welded unreinforced flange-bolted web (WUF-B) and reduced beam section (RBS) connections, under a column removal scenario [19]. Both specimens are subjected to a prescribed vertical displacement applied gradually at the location of the removed center column to examine their behavior with the inclusion of catenary action development in the beam. In the early stage of the response, the beam behavior was controlled by flexure for both WUF-B and RBS connections. After the vertical displacement was increased, the connection experienced yielding and catenary action after the tensile axial force developed. The WUF-B connection failed with three different modes: 1) local buckling of the top flanges of the beams nears the center column, 2) sequential shear fracture of the lowest and middle bolts connecting the web of the beam to the shear tab at the center column, and 3) fracture in the bottom flange near the weld access hole. The RBS connection failed with
two distinctive modes: 1) bottom flange fracture in the middle of the reduced section, and 2) web fracture until the capacity of the column was exceeded. The results from the experiments showed agreement with the computational results. This study shows that using a sub-model consists of beam and springs can be used to model the behavior of both connection types.

Several studies have found that the catenary action increases the load resistances toward the progressive collapse event [13, 16, 18]. Additionally, several studies have found that the maximum rotational capacity of connections is much higher than the recommended rotational capacities provided by design guidelines [14, 17, 18, 19]. Axial force in the linear region up to yield is often ignored due to its small contribution [9, 17, 20].
CHAPTER 3
RESTRAINED BEAM SUB-MODEL ASSEMBLY (RBSA) DEVELOPMENT

Fig. (3.1) Representative moment frame model; (a) 2 bay moment frame; (b) 2 bay moment frame after middle column removal

Fig. (3.2) Representative braced frame model; (a) 2 bay braced frame; (b) 2 bay braced frame after middle column removal

The 2 bay moment frame in Fig. (3.1) and braced frame in Fig. (3.2) represent a typical frame prior to column removal Fig. (3.1a, 3.2a) and subsequent to the progressive collapse event after removing the middle column Fig. (3.1b, 3.2b). Both frames assumed to have beams with the same span $s$, where $2s = L$. In order to
develop a general solution for the frames’ load-deflection curve after removing the middle column, a simplified version of both systems is needed. Assuming the remaining columns provide rigid vertical support, both end joints of the continuous beam have 2 degrees of freedom (DOF). A longitudinal DOF where the end joints move horizontally due to the normal force developed inside the beam (and in the brace element, in the case of the braced frame) and a rotational DOF where the end joints rotate due to the moment caused by the applied load. Discrete springs can represent effects from support conditions and the surrounding frame on the two-span continues beam. The RBSA model of the continuous beam is shown in Fig. (3.4).

![Diagram of RBSA model](image)

**Fig. (3.3)** Famous different types of beams; (a) Pin-Fix-Pin (PFP) beam; (b) Fix-Fix-Fix (FFF) beam; (c) Pin-Pin-Pin (PPP) beam

The mechanics for the RBSA model in Fig. (3.4) are presented in three main sections. The first section will discuss the behavior of the RBSA model considering only geometric nonlinearity. The second section will discuss the behavior of the beam geometric nonlinearity and the connection’s material nonlinearity. The third section will discuss the validation of the equations developed in the first section against three different types of
beams Fig. (3.3) including simply supported (PFP), Fix-Fix-Fix (FFF) and modified catenary (PPP) beams where the letters in the parentheses represent the end points of each span of the beams.

![Beam Diagram](image)

**Fig. (3.4)** RBSA model; (a) 2 spans beam with longitudinal and rotational springs at end joints; (b) Deformed configuration up to the cut

### 3.1 Equivalent Linear Rotational and Longitudinal Moment Frame Stiffnesses for RBSA

Consider the RBSA model in Fig. (3.4) and the simplified moment frame in Fig. (3.5). The left joint in the RBSA model has 2 DOF, the longitudinal and rotational DOFs, which depend on the stiffnesses of the longitudinal spring and the rotational spring, respectively. The upper left joint of the simplified frame, Joint II, has 3 DOF. The only DOF needed from the simplified moment frame is the rotational DOF. From the rotational DOF and with the use of the stiffness matrix method of structural analysis along with Hooke’s law, the rotational stiffness of the moment frame $K_{rmf}$ can be calculated [21]. In the simplified moment frame shown in Fig. (3.5), Joint I the pinned joint, only has a rotational DOF that annotated with the number 1. Joint II has longitudinal, vertical, and rotational DOFs that are annotated with the numbers 2, 3 and 4 respectively. Joint III the vertical roller joint, only has a vertical DOF that is annotated with the number 5. The annotations 6 and 7 are the horizontal and the vertical reactions, respectively, of joint I, and the other annotations 8 and 9 are the
horizontal and the rotational reactions, respectively, of joint III. Each member in the frame has a unique stiffness matrix.

![Fig. (3.5) Half of the simplified moment frame](image)

The global stiffness matrix consists of 6 columns and 6 rows. The first three columns correspond to the horizontal, vertical, and rotational DOFs of the starting joint of the member, respectively. The last three columns correspond to the horizontal, vertical, and rotational DOFs of the ending joint of the member, respectively. Similarly, the first and the last three rows of the global stiffness matrix represent DOF of the starting and the ending joints of the member respectively. Each member has an angle of rotation $\theta$ from the horizontal axis and has either a horizontal span or a vertical height.

Let $\cos \theta = C$, $\sin \theta = S$ and $L = \text{member length}$ which can be either $s$ (for horizontal member) or $h$ (for vertical member). The global stiffness matrix is as follows [21]:
The global stiffness matrix of each member in Eq. (1) contains two important parts: the displacements stiffness matrix \( K_{ff} \) and the reactions stiffness matrix \( K_{fs} \). Thus, \( K_{ff} \) for the whole structure is the sum of \( K_{ff} \) for each member; similarly, \( K_{fs} \) for the whole structure is the sum of \( K_{fs} \) for each member. In order to determine \( K_{ff} \) for the whole structure, the values in the global stiffness matrix that correspond to the displacement DOFs have to be extracted and combined to form \( K_{ff} \). On the other hand, the values those fall into the columns that represent the displacement DOFs and into the rows that represents the reactions DOFs have to be extracted and combined to form \( K_{fs} \).

For column member starting with Joint I and ending with Joint II, which has a height of \( h \), a cross sectional area of \( A_c \) and a second moment of inertia \( I_c \): \( \theta = 90^\circ \rightarrow C = 0 \) and \( S = 1 \), the stiffness matrix in Eq. (1) becomes:

\[
[K_c] = \begin{bmatrix}
\frac{6}{12EIC_c} & 7 & \frac{1}{6EI_c} & 2 & 3 & \frac{4}{6EI_c} \\
\frac{h}{h^3} & 0 & \frac{-6EI_c}{h^2} & -\frac{12EI_c}{h^3} & 0 & \frac{-6EI_c}{h^2} \\
0 & \frac{EA_c}{h} & 0 & 0 & \frac{-EA_c}{h} & 0 \\
-\frac{6EI_c}{h^2} & 0 & \frac{4EI_c}{h} & \frac{6EI_c}{h^2} & 0 & \frac{2EI_c}{h} \\
-\frac{12EI_c}{h^3} & 0 & \frac{6EI_c}{h^2} & \frac{12EI_c}{h^3} & 0 & \frac{6EI_c}{h^2} \\
0 & \frac{-EA_c}{h} & 0 & 0 & \frac{EA_c}{h} & 0 \\
-\frac{6EI_c}{h^2} & 0 & \frac{2EI_c}{h} & \frac{6EI_c}{h^2} & 0 & \frac{4EI_c}{h}
\end{bmatrix}
\]
For the beam member starting with Joint II and ending with Joint III, which has a span of \( s \), a cross sectional area of \( A_b \) and a second moment of inertia \( I_b \): \( \theta = 0^\circ \rightarrow C = 1 \) and \( S = 0 \), the stiffness matrix in Eq. (1) becomes:

\[
[K_b] = \begin{bmatrix}
\frac{2}{E A_b s} & 0 & 0 & -\frac{E A_b}{s} & 0 & 0 \\
0 & \frac{12 E I_b}{s^3} & \frac{6 E I_b}{s^2} & 0 & -\frac{12 E I_b}{s^3} & \frac{6 E I_b}{s^2} \\
0 & \frac{6 E I_b}{s^3} & \frac{4 E I_b}{s^2} & 0 & -\frac{6 E I_b}{s^3} & \frac{2 E I_b}{s} \\
\frac{E A_b}{s} & 0 & 0 & \frac{E A_b}{s} & 0 & 0 \\
0 & -\frac{12 E I_b}{s^3} & -\frac{6 E I_b}{s^2} & 0 & \frac{12 E I_b}{s^3} & -\frac{6 E I_b}{s^2} \\
0 & \frac{6 E I_b}{s^3} & \frac{2 E I_b}{s^2} & 0 & -\frac{6 E I_b}{s^3} & \frac{4 E I_b}{s} \\
\end{bmatrix}
\]

As explained previously, \( K_{ff} \) will contain the values that fall into the columns and rows annotated with 1, 2, 3, 4, and 5, which correspond to the displacement DOFs from Eq. (2, 3). \( K_{fs} \) will contain the values that fall into the columns annotated with 1, 2, 3, 4, and 5, and fall into the rows annotated with 6, 7, 8 and 9, which correspond to the reaction DOFs from Eq. (2, 3).

\[
[K_{ff}] = \begin{bmatrix}
\frac{4 E I_c}{h} & \frac{6 E I_c}{h^2} & 0 & \frac{4 E I_c}{h} & 0 \\
\frac{6 E I_c}{h^2} & \frac{12 E I_c}{h^3} + \frac{E A_b}{s} & 0 & \frac{6 E I_c}{h^2} & 0 \\
0 & 0 & \frac{E A_c}{h} + \frac{12 E I_b}{s^3} & \frac{6 E I_b}{s^2} - \frac{12 E I_b}{s^3} & 0 \\
\frac{2 E I_c}{h} & \frac{6 E I_c}{h^2} & \frac{6 E I_b}{s^2} & \frac{4 E I_c}{h} + \frac{4 E I_b}{s} & -\frac{6 E I_b}{s^2} \\
0 & 0 & -\frac{12 E I_b}{s^3} & -\frac{6 E I_b}{s^2} & \frac{12 E I_b}{s^3} \\
\end{bmatrix}
\]
With the implementation of Hooke’s law \( \{P\} = [K_{ff}]\{\delta\} \) where \( \{P\} \) contains the external forces with or opposite the direction of the displacement DOF \( \{M1, P_x2, P_y3, M4, P_y5\} \) and \( \{\delta\} \) contains the displacement of each of the DOF \( \{\theta_I1, \delta_{IIx}2, \delta_{IIy}3, \theta_{II4}, \delta_{IIly}5\} \). Here, \( M1 \) is the external moment of Joint I; \( P_x2, P_y3, \) and \( M4 \) are the external lateral force, vertical force, and moment, respectively, of Joint II; \( P_y5 \) is the external vertical force of Joint III; \( \theta_I1 \) is the rotation of Joint I; \( \delta_{IIx}2, \delta_{IIy}3 \) and \( \theta_{II4} \) are the longitudinal displacement, the vertical displacement, and the rotation, respectively, of Joint II; and \( \delta_{IIly}5 \) is the vertical displacement of Joint III. All of the external forces and moments are considered as known values, while all of the displacements and rotations are considered as unknown values. \( \{\delta\} = \) The unknown displacements can be found by taking the inverse of \([K_{ff}], Eq. (4)\), and multiplying it by \( \{P\}\):

\[
\begin{bmatrix}
P_1 \\
P_2 \\
P_3 \\
P_4 \\
P_5 = -\frac{P}{2}
\end{bmatrix} = \begin{bmatrix}
\frac{1}{h^2} & \frac{2}{h^3} & 3 & \frac{4}{h^2} & 5 \\
-\frac{6EI_c}{h^2} & -\frac{12EI_c}{h^3} & 0 & -\frac{6EI_c}{h^2} & 0 \\
0 & 0 & -\frac{EA_c}{h} & 0 & 0 \\
0 & -\frac{EA_c}{h} & 0 & 0 & 0 \\
0 & 0 & \frac{6EI_b}{s^2} & \frac{2EI_b}{s} & -\frac{6EI_b}{s^2}
\end{bmatrix}^{-1} \begin{bmatrix}
\theta_I1 \\
\delta_{IIx}2 \\
\delta_{IIy}3 \\
\theta_{II4} \\
\delta_{IIly}5
\end{bmatrix}
\]

Solving Eq. (6) for \( \{\delta\} \), using MATLAB gives:
\[
\{\delta\} = \begin{bmatrix}
\frac{s^2 P(A_b h^3 - 6 I_c s)}{8E(A_b l_b h^2 + 3A_b l_c s h^2 + 3l_b l_c s)} \\
3l_c s^3 Ph \\
-\frac{P h}{2A_c E} \\
-\frac{s^2 P(A_b h^3 - 3 I_c s)}{4E(A_b l_b h^3 + 3A_b l_c s h^2 + 3l_b l_c s)} \\
\frac{P(36 l_c l_b^2 s h + 12 A_b l_b^2 h^4 + 12 A_c l_c l_b s^4 + 4A_b A_c l_b s^3 h^3 + 36 A_b l_c l_b s^3 h^3 + 3A_b A_c l_c s^4 h^2)}{24 A_c E l_b (A_b l_b h^3 + 3A_b l_c s h^2 + 3l_b l_c s)}
\end{bmatrix}
\]

\( (7) \)

Multiplying Eq. (5) by Eq. (7), yields the simplified moment frame reactions:

\[
\{P_{6-9}\} = \begin{bmatrix}
P_x 6 \\
P_y 7 \\
P_x 8 \\
M 9
\end{bmatrix} = \begin{bmatrix}
\frac{3A_b l_c s^2 Ph}{4(A_b l_b h^3 + 3A_b l_c s h^2 + 3l_b l_c s)} \\
\frac{P}{2} \\
\frac{3A_b l_c s^2 Ph}{4(A_b l_b h^3 + 3A_b l_c s h^2 + 3l_b l_c s)} \\
\frac{sP(2A_b l_b h^3 + 3A_b l_c s h^2 + 6l_b l_c s)}{4(A_b l_b h^3 + 3A_b l_c s h^2 + 3l_b l_c s)}
\end{bmatrix}
\]

\( (8) \)

After finding all frame displacements and reactions using Hooke’s law, the necessary moment frame rotational stiffness \( K_{rmf} \) for Joint II can be easily determined. The moment frame rotational stiffness \( K_{rmf} \) of Joint II will be equal to the negative moment on that joint developed by multiplying the horizontal reaction of Joint I by the height of the column \( (P_x 6 \times h) \) divided by the rotation of Joint II \( (\theta_{II}4) \):

\[
K_{rmf} = \frac{3 E I_c}{\frac{h}{1 + \frac{3 I_c s}{A_b h^2}}}
\]

\( (9) \)
The second term inside the parentheses in the denominator is small compared to unity, which will make the value within the parentheses equal to one. Therefore, Eq. (9) can be rewritten as

\[ K_{rmf} = \frac{3 E I_c}{h} \] (10)

In the upper joints in Fig. (3.1), the elements that provide longitudinal frame stiffness are the beam and the column. The beam longitudinal stiffness is considered in the RBSA model by explicitly modeling a beam element, while the longitudinal stiffness of the column is not considered in the RBSA model as no column is provided. Thus, the prescribed moment frame longitudinal stiffness \( K_{smf} \) comes only from the column, with the column acting as a cantilever, which makes the moment frame longitudinal stiffness \( K_{smf} \) approximately equal the longitudinal stiffness of the tip of a cantilever. Applying the matrix method on a cantilever, the moment frame longitudinal stiffness \( K_{smf} \) is:

\[ K_{smf} = \frac{3 E I_c}{h^3} \] (11)

Fig. (3.6) The column-braced system of the braced frame
3.2 Equivalent Linear Rotational and Longitudinal Braced Frame Stiffnesses for RBSA

Consider the RBSA model in Fig. (3.4) and the column-braced system of the braced frame in Fig. (3.6). The joint that connects the beam and the column with the brace element is usually considered as a pinned connection, which simulates a simple connection where the braced frame rotational stiffness $K_{rbf} = 0$.

The braced frame longitudinal stiffness $K_{sbf}$ can be calculated using the method of virtual work. In the frame shown in Fig. (3.6), the column and the brace element act as a truss system. Therefore, the virtual work method will only include the contribution of axial load:

$$
\Delta_s = n_{br} \frac{N_{br} L}{E A_{br}} + n_{co} \frac{N_{co} L}{E A_{co}}
$$

(12)

The geometry of Fig. (3.6) gives:

$$
\cos \theta = \frac{h}{L}
$$

(13)

$$
\sin \theta = \frac{h \tan \theta}{L}
$$

(14)

Solving for the axial loads in both the column and the brace element using the real structure yields:

$$
N_{br} = \frac{P L}{h \tan \theta}
$$

(15)

$$
N_{co} = \frac{P}{\tan \theta}
$$

(16)
Similarly, solving the axial loads in both the column and the brace element using the virtual structure, where \( P = 1 \), yields:

\[
n_{br} = \frac{L}{h \tan \theta} \quad (17)
\]

\[
n_{co} = \frac{1}{\tan \theta} \quad (18)
\]

Substituting Eqs. (15-18) into Eq. (12):

\[
\Delta_s = \frac{P h}{E (\tan \theta)^2} \left( \frac{1}{A_{br} (\cos \theta)^3} + \frac{1}{A_{co}} \right) \quad (19)
\]

Therefore, the braced frame longitudinal stiffness \( K_{sbf} \) can be found by dividing the applied load \( P \) by Eq. (19):

\[
K_{sbf} = \frac{E (\tan \theta)^2}{h \left( \frac{1}{A_{br} (\cos \theta)^3} + \frac{1}{A_{co}} \right)} \quad (20)
\]

From Fig. (3.2a), \( K_s \) is the total longitudinal stiffness for the end joints for either the moment frame or the braced frame, which includes the frame longitudinal stiffness \( K_{sf} \) and the connection longitudinal stiffness \( K_{sc} \), as both act as two springs in series. \( K_r \) is the total rotational stiffness for the end joints for either the moment frame or the braced frame which includes the frame rotational stiffness \( K_{rf} \) and the connection rotational
stiffness $K_{rc}$, as both also act as two springs in series. Thus, the total longitudinal stiffness $K_s$ and the total rotational stiffness $K_r$ are:

$$K_s = \frac{1}{\frac{1}{K_{sf}} + \frac{1}{K_{sc}}}$$  \hspace{1cm} (21)

$$K_r = \frac{1}{\frac{1}{K_{rf}} + \frac{1}{K_{rc}}}$$  \hspace{1cm} (22)

### 3.3 RBSA Axial-Flexure Nonlinearity Geometry

From Fig. (3.4a), the RBSA model will undergo nonlinear geometric effect after the shape of the beam and the stiffness of the joints change due to the displacement occurring following the middle column removal scenario. In order to find an approximate solution for the displacement of the middle joint $\delta(x)$ after removing the column, a cut has been made in Fig. (3.4a) at a distance $x$ from the left joint; the deformed structure up to that cut is shown in Fig. (3.4b).

Using the equilibrium of the forces in the $x$ direction from Fig. (3.4b):

$$\sum F_x = 0 \Rightarrow -A_x + N(x) \cos \theta - V(x) \sin \theta = 0$$  \hspace{1cm} (23)

Assuming that the rotational angle $\theta$ is very small. Eq. (23) becomes:

$$-A_x + N(x) - V(x) \theta = 0$$  \hspace{1cm} (24)
Equilibrium of the forces in the $y$ direction from Fig. (3.4b) gives:

$$+1 \sum F_y = 0 \Rightarrow \frac{P}{2} - N(x) \sin \theta - V(x) \cos \theta = 0 \quad (25)$$

Likewise, by assuming the rotational angle $\theta$ is very small. Eq. (25) becomes:

$$\frac{P}{2} - N(x) \theta - V(x) = 0 \quad (26)$$

From Eq. (24), assume that $\theta$ is small and that $V(x) \theta \ll Ax$ and $V(x) \theta \ll N(x)$, thus the axial force can be taken as a constant value, $A_x = N$

Enforcing equilibrium of the moment about the cut from Fig. (3.4b) gives:

$$+\sum M_{cut} = 0 \Rightarrow M(x) - \frac{P x}{2} + A_x \delta + M_r = 0 \quad (27)$$

From Eq. (27), the moment at the cut $M(x)$ needs to be calculated using the moment-curvature relation [22]:

$$\kappa(x) = \frac{y''(x)}{[1 + (y'(x))^2]^{3/2}} = \frac{M(x)}{EI} \quad (28)$$

Assuming the square of the slope $y'(x)$ in the denominator is small compared to unity, Eq. (28) becomes:
\[ \kappa (x) = y''(x) = \frac{M(x)}{EI} \]  

(29)

And Eq. (27) becomes:

\[ EI \ y''(x) - A_x \ y(x) + M_r = \frac{P \ x}{2} \]  

(30)

Assuming \( A_x = N \) and by dividing by the flexural rigidity \( EI \):

\[ y''(x) - \lambda^2 \ y(x) + \frac{M_r}{EI} = \frac{P x}{2 EI} \]  

(31)

Where

\[ \lambda^2 = \frac{N}{EI} \]  

(32)

The solution for the displacement along the span of the beam will contain homogeneous and particular solutions as follows:

\[ y(x) = y_h(x) + y_p(x) \]  

(33)

The homogeneous and particular solutions of Eq. (33), respectively, are:

\[ y_h(x) = A \cosh \lambda x + B \sinh \lambda x \]  

(34)
\[ y_p(x) = C x + D \quad (35) \]

Hence, Eq. (33) develops into:

\[ y(x) = A \cosh \lambda x + B \sinh \lambda x + C x + D \quad (36) \]

Substituting the particular solution given in Eq. (35) into Eq. (31) leads to:

\[ y_p''(x) - \lambda^2 y_p(x) = \frac{P}{2 EI} \frac{M_r}{EI} \quad (37) \]

\[ - \lambda^2 (C x + D) = \frac{P}{2 EI} \frac{M_r}{EI} \quad (38) \]

By substituting Eq. (32) into Eq. (38), the constants C and D can be solved for:

\[ C = -\frac{P}{2 N} \quad (39) \]
\[ D = \frac{M_r}{N} \quad (40) \]

The displacement as a function of \( x \) in Eq. (36) thus becomes:

\[ y(x) = A \cosh \lambda x + B \sinh \lambda x - \frac{P x}{2 N} + \frac{M_r}{N} \quad (41) \]
Boundary conditions are needed in Eq. (41) in order to solve for the constants A and B and also solve for the unknown rotational moment $M_r$. Due to the pinned condition at the left support ($x = 0$), $y(0) = 0$, which allows the constant A to be solved for:

$$A = -\frac{M_r}{N} \quad (42)$$

At the middle support ($x = s$), the slope $y'(s) = -\frac{M_r}{K_r}$, which allows the constant B to be solved for:

$$B = \frac{M_r}{N} \frac{1}{\lambda \cosh \lambda s} \left( \lambda \sinh \lambda s - \frac{N}{K_r} \right) + \frac{P}{2N} \frac{1}{\lambda \cosh \lambda s} \quad (43)$$

Therefore, Eq. (41) becomes:

$$y(x) = \frac{M_r}{N} \left[ \frac{\sinh \lambda x}{\lambda \cosh \lambda s} \left( \lambda \sinh \lambda s - \frac{N}{K_r} \right) - \cosh \lambda x + 1 \right] + \frac{P}{2N} \frac{\sinh \lambda x}{\lambda \cosh \lambda s} - x \quad (44)$$

Assuming that the slope at both the end and middle joints are equal, $y'(0) = -\frac{M_r}{K_r}$, which allows $M_r$ to be obtained as:

$$M_r = \frac{P K_r (\cosh \lambda s - 1)}{2(K_r \lambda \sinh \lambda s + N \cosh \lambda s - N)} \quad (45)$$

Thus, the solution in Eq. (44) for the displacement prior yield along the span of the beam after substituting in the solution of $M_r$ from Eq. (45) is:
\[ y(x) = \frac{P K_r (\cosh \lambda s - 1)}{2N(K_r \lambda \sinh \lambda s + N \cosh \lambda s - N)} \left[ \frac{\sinh \lambda x}{\lambda \cosh \lambda s} \left( \lambda \sinh \lambda s - \frac{N}{K_r} \right) - \cosh \lambda x + 1 \right] + \frac{P}{2N} \left( \frac{\sinh \lambda x}{\lambda \cosh \lambda s} - x \right) \quad (46) \]

The only unknown left in Eq. (46) is the axial force \( N \). Therefore, using the constitutive relationship, \( \sigma = E \varepsilon \) along with the definition of axial stress \( N = \sigma A \) yields:

\[ \varepsilon = \frac{N}{EA} \quad (47) \]

Assuming a large displacement, the normal strain-displacement relationship is as follows [22]:

\[ \varepsilon = \frac{du}{dx} + \frac{1}{2} (y'(x))^2 \quad (48) \]

Thus, by combining Eqs. (47) and (48)

\[ \frac{N}{EA} = \frac{du}{dx} + \frac{1}{2} (y'(x))^2 \quad (49) \]

Multiplying each side of Eq. (49) by \( dx \) and integrating along the span of the beam from 0 to \( s \) gives:

\[ \int_0^s \frac{N}{EA} \, dx = \int_0^s du + \int_0^s \frac{1}{2} (y'(x))^2 \, dx \quad (50) \]

\[ \frac{Ns}{EA} = u(s) - u(0) + \int_0^s \frac{1}{2} (y'(x))^2 \, dx \quad (51) \]
For the model in Fig. (3.4a), the middle connection has no longitudinal displacement: \( u(s) = 0 \). On the other hand, the left joint is connected to a longitudinal spring that has a stiffness \( K_s \). Therefore, the left connection has a longitudinal displacement of \( u(0) = N/K_s \). With the beam having a stiffness of \( K = EA/s \), Eq. (51) becomes:

\[
\frac{N}{K_{eq}} = \frac{1}{2} \int_0^s (y'(x))^2 \, dx
\]  

(52)

Where \( K_{eq} \) is the equivalent stiffness of the stiffnesses of the spring and the beam since both are considered as two springs in series. Thus, \( \frac{1}{K_{eq}} = \frac{1}{K} + \frac{1}{K_s} \). With the first derivative of Eq. (46) substituted into Eq. (52), Eq. (52) can be solved for \( N \) using any numerical programming platform such as, MATLAB.

The moment frame rotational stiffness \( K_{rmf} \) in Eq. (10) is a linear rotational stiffness. The linear \( K_{rmf} \) will not give an accurate approximation for the moment frame rotational stiffness when considering nonlinear geometry of the RBSA model as axial force develops. Thus, to approximate the effect of the axial force, the matrix method has been used on a cantilever column with a horizontal point load applied on the tip of the column cantilever. The \( K_{rmf} \) of the cantilever has been found to be:

\[
K_{rmf} = \frac{2 E I_c}{h}
\]  

(53)

The \( K_{rmf} \) in Eq. (10) considers only an applied vertical load and does not consider any axial force effect, which makes the stiffness in Eq. (10) higher than the necessary stiffness. Therefore, the coefficient 3 appearing in front of Eq. (10) needs to be reduced to account for axial force. In Eq. (53) the rotational stiffness only considers the horizontal force applied, which simulates the effect of the axial force, but it does not consider
the vertical force applied on the frame in Fig. (3.1). Thus, the coefficient 2 appearing in front of Eq. (53) needs to be increased to account for the vertical force applied on the frame in Fig. (3.1). Accordingly, after adding both stiffnesses and take their average value, the adjusted nonlinear geometry moment frame rotational stiffness $K_{r mf}$ is calculated in Eq. (54):

$$K_{r mf} = \frac{2.5 E I_c}{h}$$

(54)

Solving for $N$ and using the stiffnesses in Eqs. (21-22), Eq. (46) can be used to provide an approximate solution for the load-deflection relationship for different scenarios with different values of longitudinal stiffnesses ($K_{sf}$ and $K_{se}$) and rotational stiffnesses ($K_{rf}$ and $K_{rc}$).

3.4 Nonlinear Geometry of the Beam and Nonlinear Material of the Connections

This section follows the same methodology used in Section 3.3 with the addition of connection material nonlinearity. By taking the material nonlinearity of the connections into consideration, the left and the middle joints in Fig. (3.4a) accommodate the development of plastic hinges. When any joints reach the point of plastic hinge development, the rotation moment $M_r$ is limited and equal to the yield moment $M_{ry}$. Therefore, the previous derivations up to Eq. (42) are the same and $M_r$ is a fixed value equal to the yield moment $M_{ry}$. Hence, the only constant that needs to be solved for is the integration constant B. In order to solve for B after reaching the yield moment, an extra boundary condition is needed. In the configurations considered in this thesis, both joints develop the same yield moment $M_{ry}$, thus, both joints have the same slope. Therefore, at both joints (at $x = 0$ and $x = s$) the slope will be equal:

$$y'(0) = y'(s)$$

(55)
Substitute Eqs. (39) and (42) into Eq. (55):

\[ B \lambda = B \lambda \cosh \lambda s - \frac{M_r}{N} \lambda \sinh \lambda s \] \hspace{1cm} (56)

Solving Eq. (56) for \( B \) gives:

\[ B = \frac{M_r \sinh \lambda s}{N (\cosh \lambda s - 1)} \] \hspace{1cm} (57)

Considering the material nonlinearity of the connections, the equation for displacement beyond yield becomes:

\[ y(x) = \frac{M_{r,y}}{N} \left( 1 - \cosh \lambda x + \frac{\sinh \lambda s}{\cosh \lambda s - 1} \sinh \lambda x \right) - \frac{P x}{2 N} \] \hspace{1cm} (58)

By substituting the first derivative of Eq. (58) into Eq. (52), the axial force beyond the yield of the can be calculated.

### 3.5 RBSA Equations Validation Against Three Different Types of Beams

The RBSA equations can be used to simulate the deflection behavior of three famous types of beams: simply supported (PFP), Fix-Fix-Fix (FFF) and Pin-Pin-Pin (PPP). From the derived RBSA equations, Eq. (44) and (46), the famous linear deflection equations of PFP and FFF beams can be obtained. The deflection of the PFP, FFF and PPP beams considering geometric nonlinearity can be also obtained. Additionally, from Eq. (58),
the deflection beyond yield of the PFP, FFF and PPP beams considering both geometric and material nonlinearities can be also obtained.

3.5.1 RBSA Equations Validation Against Linear Geometry and Material of PFP and FFF Beams

By substituting \( \lambda = \sqrt{N/EI} \) into Eq. (44) and by taking the limit as both \( M_r \) and \( N \) approach zero, the linear deflection of Eq. (44) taken at \( x = s \), where \( s = L/2 \), reduces to the deflection equation for a PFP beam [23]:

\[
y(s) = -\frac{P s^3}{6 E I} = \frac{P L^3}{48 E I}
\]

(59)

By substituting \( \lambda = \sqrt{N/EI} \) into Eq. (46) and by taking the limit as \( K_r \) approach infinity and \( N \) approach zero, the linear deflection of Eq. (46) taken at \( x = s \), where \( s = L/2 \), reduces to the deflection equation for a FFF beam [23]:

\[
y(s) = -\frac{P s^3}{24 E I} = \frac{P L^3}{192 E I}
\]

(60)

3.5.2 RBSA Equations Validation Against Nonlinear Geometry of PFP, FFF and PPP Beams

By taking the limit of Eq. (44) as \( M_r \) approaches zero, the deflection of a PFP beam considering geometric nonlinearity is:

\[
y(x) = \frac{P}{2N} \left( \frac{\sinh \lambda x}{\lambda} \cosh \lambda s - x \right)
\]

(61)

Taking the limit of Eq. (52) as \( K_r \) approaches infinity, solving for \( N \), and substituting the solution for \( N \) into Eq. (61) gives the full behavior of a PFP beam considering geometric nonlinearity.
Taking the limit of Eq. (46) as $K_r$ approaches infinity, deflection of a FFF beam considering geometric nonlinearity is:

$$y(x) = \frac{M_r}{N} \left( \frac{\sinh \lambda s \sinh \lambda x}{\cosh \lambda s} - \cosh \lambda x + 1 \right) + \frac{P}{2N} \left( \frac{\sinh \lambda x}{\lambda \cosh \lambda s} - x \right)$$  \(62\)

Where $M_r = \frac{P}{2 \lambda \sinh \lambda s} (\cosh \lambda s - 1)$.

Taking the limit of Eq. (52) as $K_s$ approaches infinity, solving for $N$, then substituting the solution for $N$ into Eq. (62) gives the full behavior of a FFF beam considering geometric nonlinearity.

Taking the limit of Eq. (46) as $K_r$ approaches zero and $K_s$ approaches infinity and solving for $N$ in Eq. (52), deflection of a PPP beam considering geometric nonlinearity is shown in Eq. (63) which matches the previously published work of Deputy, L. and Story, B. [24] and Deputy et al [25]:

$$y(s) = s^3 \sqrt[3]{\frac{P}{EA}}$$  \(63\)

### 3.5.3 RBSA Equations Validation Against Nonlinear Geometry and Material of PFP, FFF and PPP Beams

Considering both material and geometric nonlinearities for the FFF beam scenario, both connections are assumed to be yield at the same time. Thus, by using Eq. (62) and by setting the rotational moment $M_r$ equal to the yield rotational moment $M_{ry}$, the behavior of the FFF beam beyond yield is obtained. Considering both material and geometric nonlinearities for the PFP beam scenario, only the middle connection will yield. Therefore, using Eq. (58) and using the half of the yield rotational moment $M_{ry}$, the behavior of the PFP beam beyond yield is obtained. Accordingly, the yield rotational moment of the PFP beam $M_{ry-PFP}$ is equal to half of
the yield rotational moment of the FFF beam $M_{ry-FFF} = \frac{M_{ry-PPP}}{2}$. Considering both material and geometric nonlinearities for the PPP beam scenario, the rotational stiffness of both connections is zero, which means that there is also zero moment at the connections. Consequently, the yield will be happening within beam itself through the development of an axial hinge, meaning that the axial force $N$ will reach the yield axial force $N_y$.

Thus, by taking the limit of Eq. (46) as $K_r$ approaches zero and by substituting $N = N_y = AF_y$, the deflection of the PPP beam beyond yield will be equal to Eq. (64) which matches the previously published work of Deputy, L. and Story, B. [24] and Deputy et al [25] beyond yield

$$y(s) = \frac{P s}{2 A F_y}$$ (64)
CHAPTER 4
RESULTS AND DISCUSSION

The models in Chapter 3 provide a method to establish nonlinear load-deflection curves that may be used to facilitate progressive collapse analysis and design. The moment frame model in Fig. (3.1) has been assumed to have semi-rigid connections, while the braced frame model in Fig. (3.2) has been assumed to have simple connections. Both the moment frame and the braced frame have been divided into two cases: an ideal case and a real case. For the moment frame, the ideal case considers the longitudinal and rotational stiffnesses of the connections as infinite. For the braced frame, the ideal case considers the longitudinal stiffness of the connections as infinite and the rotational stiffness of the connections as zero.

Using Eq. (46) prior yield along with the longitudinal and rotational stiffnesses from Eqs. (21) and (22), respectively, for the analyses considering geometric nonlinearity of the moment frame and the braced frame, respectively. Using Eq. (58) beyond yield along with the longitudinal and rotational stiffnesses from Eqs. (21) and (22), respectively, for the analyses considering geometric and material nonlinearity of the braced frame only as the moment frame will be unstable system. Eq. (21) includes the longitudinal stiffnesses of the frame and connection and Eq. (22) includes the rotational stiffnesses of the frame and the connections as well. The deflections of the middle joints of Figs. (3.1b) and (3.2b) can be calculated using the RBSA model rather than an entire frame model. This chapter presents results comparing the methods developed in this thesis to various finite element analyses. Additionally, a comparison between the RPSA model, equation and the PFP, FFF and PPP beams will be provided.

4.1 Define the Model Properties

The section properties for all elements used in both the moment frame and braced frame models as well as the RBSA model are given in Table (1)
Table 1. Sections properties

<table>
<thead>
<tr>
<th>Element Type</th>
<th>Cross-Section</th>
<th>$F_y$ (ksi)</th>
<th>$E$ (ksi)</th>
<th>$A$ ($in^2$)</th>
<th>Length (in)</th>
<th>$I$ ($in^4$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>W16X45</td>
<td>50</td>
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<td>640</td>
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<tr>
<td>Brace</td>
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<td>50</td>
<td>29000</td>
<td>8.84</td>
<td>135</td>
<td>170</td>
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</tbody>
</table>

4.2 Validation of RBSA Model and Equation Against PFP, FFF, and PPP Beams

Consider the three different types of beams in Fig. (3.3), this section will be divided into three cases:

Section 4.2.1 will discuss results considering linear material and geometry for the entire system (LMG), Section 4.2.2 will discuss results considering nonlinear beam geometry and linear connection material (NLG), and Section 4.2.3 will discuss results considering nonlinear beam geometry and nonlinear connection material (NLMG). The LMG case will include only the PFP and FFF beams; the PPP beam has been excluded from the LMG case due to instability.

4.2.1 LMG Validation of RBSA Model and Equation for PFP and FFF beams

For the PFP beam, the results from RBSA equation Eq. (59) given in Section 3.5.1 that derived from Eq. (44) were compared with the results from the SAP2000 analysis for the RBSA and PFP beam models. Fig. (4.1) shows the results for the fully-linear PFP beam. For the FFF beam, the results from RBSA equation Eq. (60) given in Section 3.5.1 were compared with the results from the SAP2000 analysis for the RBSA and FFF beam models. Fig. (4.2) shows the results for the fully-linear FFF beam. Figs. (4.1) and (4.2) show that the results calculated using the RBSA equation closely matches the results obtained from the SAP2000 LMG RBSA model and the SAP2000 LMG PFP and FFF beams.
4.2.2 NLG Validation of the RBSA Model and Equation for PFP, FFF, and PPP Beams

For the PFP beam, the results calculated using the RBSA equation Eq. (61) given in Section 3.5.2 that were compared with results from SAP2000 analysis for the NLG RBSA and NLG PFP beam models. Fig. (4.3)
shows the results for the NLG PFP beam models. For the FFF beam, the results calculated using the RBSA equation Eq. (62) given in Section 3.5.2 were compared with results from SAP2000 analyses for the NLG RBSA and NLG FFF beam models. Fig. (4.4) shows the results for the NLG FFF beam models. For the PPP beam, the results calculated using the RBSA equation Eq. (63) given in Section 3.5.2 were compared with the results from SAP2000 analyses for the NLG RBSA and NLG PPP beam models. Fig. (4.5) shows the results for the NLG PPP beam models. Fig. (4.3) and (4.4) show that the RBSA equation has around 6% error compared with the results obtained from the SAP2000 analyses. This error is due to the assumption used in Eq. (31) that $A_x = N$. 20 inches is a large displacement value, and system will likely develop either a rotational hinge or an axial hinge before reaching the desired displacement. Therefore, the results are an illustrative results assuming linear material behavior to show that the RBSA model give the same results as the models in SAP2000. Fig. (4.5) shows that the RBSA equation closely matches the results obtained from the SAP2000 NLG RBSA and NLG PPP beam analyses.

![Fig. (4.3) NLG load-deflection curve for PFP beam](image)

Fig. (4.3) NLG load-deflection curve for PFP beam
4.2.3 NLMG Validation of the RBSA Model and Equation for PFP, FFF, and PPP Beams

For the PFP beam, the load-deflection curve was created using Eq. (61) for the pre-yield region and Eq. (58) for the post-yield region. The results were compared with results from SAP2000 analyses for the NLMG
RBSA and NLMG PFP beam models. Fig. (4.6) shows results for the NLMG PFP beams. For the FFF beam, the load-deflection curve was created using Eq. (62) for the pre-yield region and Eq. (62) for the post-yield region after substituting the yield rotational moment \( M_{r,y} \) for \( M_r \). The results were compared with the results from SAP2000 analyses from the NLMG RBSA and NLMG FFF beam models. Fig. (4.7) shows the results for the NLMG FFF beams. For the PPP beam, the load-deflection curve was created using Eq. (63) for the pre-yield region and Eq. (64) for the post-yield region. The results were compared with the results from SAP2000 analyses for the NLMG RBSA and NLMG PPP beam models. Fig. (4.8) shows the results for the NLMG PPP beams. Figs. (4.6-4.8) show that the RBSA equation closely matches the results obtained from the SAP2000 analyses.

![Fig. (4.6) NLMG load-deflection curve for PFP beam](image)

Fig. (4.6) NLMG load-deflection curve for PFP beam
Fig. (4.7) NLMG load-deflection curve for FFF beam

Fig. (4.8) NLMG load-deflection curve for PPP beam
4.3 Frame Comparison with RBSA Model and Equation for NLG of the Beam and LMG of the Connections

In this section, the RBSA equation given in Eq. (46) is compared with finite element frame analysis results generated using SAP2000. SAP2000 models were created for the frames shown in Fig. (3.1) and (3.2) as well as the RBSA model shown in Fig. (3.4a). As stated previously, the frames will be divided into two cases; an ideal case and a real case. A table at the end of the each section will be provided to show a comparison of results from Eq. (46) and SAP200 for all models considered.

All of the SAP2000 models consider nonlinear geometry for the members and linear geometry for the connections. The moment frame model has a total of three columns and two beams; the braced frame has a total of three columns and two beams with two brace elements. The middle column in both frames has been removed. The RBSA model has 2 beams. Each beam has longitudinal springs at both ends and rotation springs at both ends as well as the middle of the beam. A displacement-controlled analysis with an increasing prescribed displacement at the location of the removed column was used. By considering geometric nonlinearity (NLG), axial force develops inside the restrained beams.

4.3.1 Idealized Case

The ideal moment frame considers the longitudinal stiffness and the rotational stiffness of the connections as infinite. Thus, the only stiffnesses that are considered in the RBSA model and equation are the longitudinal stiffness of the frame from Eq. (11) and the rotational stiffness of the frame from Eq. (54). Therefore, by using infinite connection stiffness in Eq. (21), the longitudinal stiffness $K_s$ in Eq. (21) of the longitudinal spring in the RBSA model will be equal to Eq. (11): $K_s = K_{smf} = 9.60 \text{ kip/in}$. Similarly, by using infinite rotational stiffness for the connections in Eq. (22), the rotational stiffness $K_r$ of the rotational spring in the RBSA model will be equal to Eq. (54): $K_r = K_{rmf} = 0.26 \times 10^6 \text{ kip/in/\text{rad}}$. Fig. (4.9) illustrates the results of the RBSA equation given in Eq. (46) and SAP2000 analysis of the moment frame and RBSA models.
The results are shown in Fig. (4.9) and indicate that the RBSA equation matches the outcomes of the RBSA model. The moment frame results show agreement with both the RBSA equation and model. The results from the SAP2000 analyses and the RBSA equation at two different displacement values are presented in Table (2). Percentage error between the applied loads from the SAP2000 models and the RBSA equation are given. The error at 10 inches of displacement shown is almost zero, which indicate that the RBSA equation accurately simulates the behavior of the RBSA model and the ideal moment frame up to a displacement of 10 inches. The percentage of error at 20 inches of displacement shown is almost 6%, which indicates that the RBSA equation does not accurately simulate the behavior of the RBSA model and the ideal moment frame at displacement greater than 10 inches. The 6% error is a result of the assumption used in Eq. (31) that $A_x = N$. 20 inches of displacement is a large displacement; the system is likely to develop either a rotational hinge or an axial hinge before reaching the desired displacement value. Therefore, the results are an illustrative results assuming linear material behavior to show that the RBSA model give the same results as the models in SAP2000.

Fig. (4.9) NLG load-deflection curve for ideal moment frame

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The ideal braced frame considers infinite longitudinal stiffness and zero rotational stiffness for the connections. The rotational stiffness of the braced frame is assumed to be zero as simple connections have been used. By substituting infinite longitudinal stiffness for the connections in Eq. (21), the longitudinal stiffness $K_s$ of the longitudinal spring in the RBSA model will be equal to Eq. (20): $K_s = K_{sbf} = 327 \text{ kip/in}$. By taking the limit of the rotational stiffnesses of the connections to approach infinity and the braced frame to approach zero in Eq. (22), the stiffness $K_r$ of the rotational spring in the RBSA model will be zero: $K_r = 0$. Fig. (4.10) shows the results of the RBSA equation given in Eq. (46) and the results of the SAP2000 analyses for the braced frame and RBSA models. The results in Fig. (4.10) indicate that the results from the RBSA equation show agreement with the results from the SAP2000 analyses. The results from the SAP2000 analyses and the RBSA equation at two different displacement values are given in Table (2). Percentage errors between the applied loads for the RBSA equation and the SAP2000 analyses are given. The error at 10 inches of displacement is almost zero, which indicates that the RBSA equation accurately simulates the behavior of the RBSA model and the ideal braced frame up to a displacement of 10 inches. The error at 20 inches of displacement is around 3.5%, which indicate that the RBSA equation is less accurate in simulating the behavior of the RBSA model and the ideal braced frame at displacements greater than 10 inches.
Fig. (4.10) NLG load-deflection curve for ideal braced frame

<table>
<thead>
<tr>
<th>Case</th>
<th>Frame (SAP2000)</th>
<th>RBSA Model</th>
<th>Error %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Load (kips)</td>
<td>Kₛ (kip/in)</td>
<td>Load (kips)</td>
</tr>
<tr>
<td>Moment Frame</td>
<td>111</td>
<td>9.60</td>
<td>111</td>
</tr>
<tr>
<td></td>
<td>212</td>
<td>0.26</td>
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</tr>
<tr>
<td>Braced Frame</td>
<td>4.60</td>
<td>327</td>
<td>37.8</td>
</tr>
<tr>
<td></td>
<td>36.5</td>
<td>0.00</td>
<td>37.8</td>
</tr>
</tbody>
</table>

4.3.2 Realistic Case

The real moment frame used an assumed value for the longitudinal stiffness of the connections of $K_{smc} = 200 \text{ kip/in}$. Three different connection rotational stiffnesses were used, with each rotational stiffness representing a specified percentage of the moment frame rotational stiffness $K_{rmf} = 0.26 (10^6) \text{ kip – in/rad}$:

1) $K_{rmc1} = 0.2 K_{rmf} = 51555.56 \text{ kip – in/rad}$

2) $K_{rmc2} = 0.5 K_{rmf} = 128889 \text{ kip – in/rad}$
3) \( K_{rmc3} = 0.8 \ K_{rmf} = 206222.2 \ \text{kip} - \text{in/rad} \)

By substituting the longitudinal stiffness of the connections \( K_{smc} \) and of the moment frame \( K_{smf} \) into Eq. (21), the stiffness \( K_s \) of the longitudinal spring in the RBSA model will be 9.11 \( kip/in \). Similarly, by substituting each rotational stiffness of the connections \( K_{rmc} \) and of the moment frame \( K_{rmf} \) into Eq. (22), the stiffness \( K_r \) of the three rotational springs in the RBSA model will be 4.3 \( (10^3) \ kip - \text{in/rad} \), 8.6 \( (10^3) \ kip - \text{in/rad} \), and 11.5 \( (10^3) \ kip - \text{in/rad} \), respectively. Figs. (4.11-13) show the results from the RBSA equation and the SAP2000 analyses for the moment frame and RBSA models. These results show that the RBSA equation shows agreement with the results from the SAP2000 moment frame and RBSA models. The results from the SAP2000 models and the RBSA equation at two different displacement values are given in Table (3). The error for a displacement of 10 inches is less than 5\% for all cases, which indicates that the RBSA equation accurately simulates the behavior of the RBSA model and the ideal moment frame up to a displacement of 10 inches. The error at a displacement of 20 inches is between 8\% and 10\% for all cases, which indicates that the RBSA equation does not accurately simulate the behavior of the RBSA model and the real moment frame for displacements greater than 10 inches. The errors are due to the assumption which used in Eq. (31) that \( A_x = N \). 20 inches is a large displacement, and the system will likely develop either a rotational hinge or an axial hinge before reaching the desired displacement. Therefore, the results are an illustrative results assuming linear material behavior to show that the RBSA model give the same results as the frames models in SAP2000.
Fig. (4.11) NLG load-deflection curve for real moment frame (1st case)

Fig. (4.12) NLG load-deflection curve for real moment frame (2nd case)
The real braced frame will use an assumed value of 500 kip/in for the longitudinal stiffnesses of the connections and 100 kip – in/rad for the rotational stiffnesses of the connections. The rotational stiffness of the braced frame is assumed to be zero as the connections are simple connections. By substituting the longitudinal stiffness of the connections $K_{sb\text{c}}$ and of the braced frame $K_{sb\text{f}}$ into Eq. (21), the stiffness $K_s$ of the longitudinal spring in the RBSA model will be $K_s = 198$ kip/in. By setting the rotational stiffness of the braced frame to zero and substituting the rotational stiffness of the connections $K_{r\text{bc}}$ into Eq. (22), the stiffness $K_r$ of the rotational spring in the RBSA model will be $K_r = 0$. Fig. (4.14) shows the results from the RBSA equation and the results from the SAP2000 analyses for the braced frame and RBSA models. The results indicate that the RBSA equation shows agreement with the results of both SAP2000 analyses. The results from the SAP2000 analyses and RBSA equation at two different displacement values are given in Table (3). The percentage errors between the applied loads for the RBSA equation and the SAP2000 analyses are provided. The error at a displacement of 10 inches is almost zero, which indicates that the RBSA equation accurately simulates the behavior of the RBSA model and the real braced frame up to a displacement of 10 inches. The error at a displacement of 20 inches is approximately 2.5%, which indicates that the RBSA equation is less accurate.
accurate in simulating the behavior of the RBSA model and the real braced frame for displacements greater than 10 inches.

Fig. (4.14) NLG load-deflection curve for real braced frame

<table>
<thead>
<tr>
<th>Case</th>
<th>Frame (SAP2000)</th>
<th>RBSA Model</th>
<th>Error %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Load (kips)</td>
<td>Disp. (in)</td>
<td>$K_\delta$ (kip/in)</td>
</tr>
<tr>
<td>Moment (1st)</td>
<td>26.1</td>
<td>10</td>
<td>9.11</td>
</tr>
<tr>
<td></td>
<td>50.0</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>Moment (2nd)</td>
<td>48.0</td>
<td>10</td>
<td>9.11</td>
</tr>
<tr>
<td></td>
<td>92.0</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>Moment (3rd)</td>
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<td>10</td>
<td>9.11</td>
</tr>
<tr>
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<td>117</td>
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<td>10</td>
<td>198</td>
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<tr>
<td></td>
<td>23.9</td>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>
4.4 Frame Comparison with RBSA Model and Equation for NLG of the Beam and NLM of the Connections

By substituting the longitudinal and rotational stiffnesses that were used for the ideal and real braced frames in Sections 4.3.1 and 4.3.2, respectively, into Eq. (46), the total rotational stiffness $K_r$ will always be zero due to treating the connections as simple connections. Thus, the frame rotational stiffness $K_{rbf}$ will also always be zero. The results considering the nonlinear geometry of the beam and the nonlinear material of the connection for both the ideal braced frame and the real braced frame will be identical to the results obtained in Sections 4.3.1 and 4.3.2. Accordingly, refer to Fig. (4.10) for the results from the ideal braced frame and to Fig. (4.14) for the results from the real braced frame. Similarly, refer to Table (2) for the results and the associated errors for the ideal braced frame and to Table (3) for the results and the associated errors for the real braced frame.

4.5 Validation of the RBSA Equation Against Existing Progressive Collapse Experiments

In this section, the RBSA equations, Eqs. (46) and (58), will be compared with existing progressive collapse experimental results. Yang and Tan have studied seven different types of connections under the middle column-removal scenario [17]. Four of the seven connections are simple connections and the remaining three connections are semi-rigid connections. One connection from each category has been chosen to compare with the results obtained from Eqs. (46) and (58). The web cleat connection has been selected from the simple connection category, and the top and seat with web angle (TSWA) with a 12 mm angle has been selected from the semi-rigid connection category. For the web cleat connection, the longitudinal stiffness assumed to fit the obtained results as $K_s = 50 \, kip/in$ and the rotational stiffness is $K_r = 333.4 \, kip-in/rad$; these values were obtained from the recorded moment and rotation at the connection, respectively. The web cleat load-deflection behavior will be calculated using Eq. (46) with these values for $K_s$ and $K_r$. For the TSWA connection, the
longitudinal stiffness assumed to fit the obtained results as $K_s = 140 \text{ kip/in}$ and the rotational stiffness is $K_r = 4772.6 \text{ kip – in/rad}$; these values were obtained from the recorded moment and rotation at the connection.

The TSWA load-deflection behavior will be calculated using Eq. (46) for the pre-yield region and Eq. (58) for the post-yield region using these values for $K_s$ and $K_r$. The yield moment $M_{ry}$ in Eq. (58) is $180 \text{ kip – in}$. Figs. (4.15) and (4.16) show the results for the web cleat connection and the TSWL connection, respectively, compared to the results calculated using the RBSA equations. The results shown in Figs. (4.15) and (4.16) show that the RBSA equations approximately simulate the behavior of both connection types under a column removal scenario if the longitudinal stiffness $K_s$ and the rotational stiffness $K_r$ are known.

![Graph of load vs displacement](image)

**Fig. (4.15)** Web cleat connections result vs. RBSA equation
4.6 Importance of Axial Force in the Elastic Region

The contribution of axial force in the elastic region is often neglected in design and analysis procedures as a linear geometry model is typically implemented in this range [9, 17, 20]. In order to simplify the investigation, this section employs the use of the LMG PFP beam equation, Eq. (59), which neglects the effect of axial force, and the NLG PFP beam equation, Eq. (61), which includes the effect of axial force. Eq. (61) will be divided into two different cases: a case where the connections are considered to be rigid and a case where the connections are considered to be flexible by giving the longitudinal spring a finite value of $K_s = 500 \text{ kip/in}$.

The LMG yield load $P_{yL}$ will be obtained from Eq. (59) for different beam spans in order to compare it with the NLG yield load $P_{yNR}$, which considers the connection rigidity, and with the NLG yield load $P_{yNF}$, which considers the connection flexibility from Eq. (61). The Figs. (4.17-20) show the load-deflection curves for Eqs. (59) and (61) with rigid connections, and Eq. (61) with flexible connections, respectively. The yield loads $P_y$ neglecting and including axial force are shown for all cases. Table (4) tabulates the results from Figs. (4.17-20) with extra beam span cases. The percentage difference between the LMG yield load $P_{yL}$ and NLG yield load
where the connections assumed to be flexible and percentage difference between the LMG yield load \( P_{yL} \) and NLG yield load \( P_{yNR} \) where the connections assumed to be rigid are also given. A W16X45 was used as the beam cross-section.

The results shown in Figs. (4.17-20) and given in Table (4) show significant variation between the LMG yield load \( P_{yL} \) and both NLG yield loads \( P_{yNF} \) and \( P_{yNR} \) for longer beam spans. This indicates that the axial force in the elastic region should be taken into consideration in the analysis and design procedures particularly for beams with long spans.

**Fig. (4.17)** Yield load when axial load is neglected \( P_{yL} \) and included \( P_{yNF} \) and \( P_{yNR} \) for 120” beam span
Fig. (4.18) Yield load when axial load is neglected $P_{yL}$ and included $P_{yNF}$ and $P_{yNR}$ for 240'' beam span

Fig. (4.19) Yield load when axial load is neglected $P_{yL}$ and included $P_{yNF}$ and $P_{yNR}$ for 360'' beam span
Fig. (4.20) Yield load when axial load is neglected $P_{yL}$ and included $P_{yNF}$ and $P_{yNR}$ for 480'' beam span

Table 4. Yield point for vireos beam spans with considering and neglecting axial load

<table>
<thead>
<tr>
<th>Beam span, s (in)</th>
<th>LMG</th>
<th>NLG</th>
<th>% Different</th>
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<td>Axial Load Neglected</td>
<td>Axial Load Included (Flexible Connections)</td>
<td>Axial Load Included (Rigid Connections)</td>
</tr>
<tr>
<td></td>
<td>$P_{yL}$ (kips)</td>
<td>$\delta_{yL}$ (in)</td>
<td>$P_{yNF}$ (kips)</td>
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<tr>
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<td>62</td>
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<td>180</td>
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<td>240</td>
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<td>16.8</td>
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</tbody>
</table>
CHAPTER 5
SUMMARY AND CONCLUSION

This research presents the development of the RBSA model, which is able to approximate the behavior of two beams supported at their ends and subjected to a middle column removal scenario. The RBSA model was developed to simulate the behavior of a moment frame and a braced frame under a middle column removal scenario by modeling the longitudinal and rotational stiffnesses of the frame joints with longitudinal springs at the end joints of the beams and rotational springs at the ends joints and middle joints of the beams. Both frame types were divided into two cases for analysis and validation: an ideal case and a real case. The ideal case of the moment frame assumed that the longitudinal and the rotational stiffnesses of the connections were infinite. The ideal case of the braced frame assumed that the longitudinal stiffness of the connections was infinite and the rotational stiffness of the connections was zero. The real case for both frame types assumed all stiffnesses have finite non-zero values. RBSA equations have been derived using the RBSA model. Eq. (46) simulates the prior-yield behavior and Eq. (58) simulates the post-yield behavior. The longitudinal and the rotational moment frame stiffnesses have been derived in Eqs. (11) and (54), respectively. The longitudinal stiffnesses of the moment frame connections were assumed to be finite non-zero values. The rotational stiffnesses of the moment frame connections were obtained by applying different percentages of Eq. (54). The longitudinal braced frame stiffness was derived and is given in Eq. (20). The rotational braced frame stiffness was considered to be zero as the connections used in the braced frame were simple connections. The longitudinal and rotational stiffnesses of the braced frame connections were assumed to be finite non-zero values. The total longitudinal and rotational stiffnesses, $K_s$ and $K_r$, of the RBSA model is defined in Eqs. (21) and (22), respectively. LMG derivations of the PFP and FFF beams were given in Eqs. (59) and (60), respectively. NLG derivations of the PFP, FFF, and PPP beams equations were given in Eqs. (61), (62) and (63), respectively. NLMG derivation of the PPP beam equation was given in Eq. (64). Therefore, the following conclusions can be drawn:
1. The RBSA model is capable of approximating the LMG of the PFP and FFF beams which demonstrated by a very small percentage margin of error as shown in Figs. (4.1) and (4.2).

2. The RBSA model is capable of approximating the NLG of the PFP, FFF and PPP beams, which demonstrated by around 6% percentage of error for PFP and FFF beams, while the PPP beam show almost no error as shown in Figs. (4.3-5).

3. The RBSA model is capable of approximating the NLMG of the PFP, FFF and PPP beams which validated by almost zero percentage margin of error as shown in Figs. (4.6-8).

4. The RBSA model is capable of approximating the NLG of the ideal moment and braced frames which demonstrated by a small percentage margin of error ranging between 0% and 6% for ideal moment frame, while the error ranging between 2.2% and 3.6% for ideal braced frame as shown in Figs. (4.9) and (4.10) and Table (2).

5. The RBSA model is capable of approximating the NLG of the real moment and braced frames which demonstrated by a percentage margin of error ranging between 3% and 10% for all of the cases of the real moment frame and around 3% for the braced frame as shown in Figs. (4.11-14) and Table (3).

6. The RBSA model is capable of approximating the NLMG of the ideal and real braced frame which demonstrated by a small percentage margin of error lower than 4% as shown in Figs. (4.10) and (4.14) and Table (2) and (3).

7. The RBSA model is capable of accommodating existed progressive collapse experiments after identifying the rotational stiffness $K_r$ from the available information and fitting the longitudinal stiffness $K_s$ as illustrated in Figs. (4.15) and (4.16).

8. The RBSA model is capable of conforming that the neglecting of axial load contribution at the linear geometry range show a high percentage of different between the LMG yield load $P_{yL}$ and both NLG yield loads $P_{yNF}$ and $P_{yNR}$ when the span of the beam increases in the results displayed in Figs. (4.17 –
20) and Table (4). Therefore, axial load in the elastic region should be taken into consideration in the analysis and design especially when the span of the beam rather large.
BIBLIOGRAPHY


