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Junyang Mi

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COORDINATED OPEARTION OF NATURAL GAS AND ELECTRICITY NETWORKS WITH LINE-  
PACK

Approved by:

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Dr. Mohammad Khodayar

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Dr. Jianhui Wang

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Dr. Carlos Davila

COORDINATED OPEARTION OF NATURAL GAS AND ELECTRICITY NETWORKS WITH LINE-  
PACK

A Thesis Presented to the Graduate Faculty of the

Bobby B. Lyle School of Engineering

Southern Methodist University

in

Partial Fulfillment of the Requirements

for the degree of

Master of Science in Electrical Engineering

by

Junyang Mi

(B.S.E.E, Beijing University of Posts and Telecommunications, 2016)

Dec 15, 2018

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Junyang Mi

2018

TO MY DEAREST PARENTS FOR THEIR LIFELONG LOVE AND ENCOURAGEMENT

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Coordinated Operation of Natural Gas and Electricity Networks

Advisor: Carlos Davila

Thesis advisor: Mohammad Khodayar

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This dissertation addresses the coordinated operation of electricity and natural gas networks considering the line-pack flexibility in the natural gas pipelines. The problem is formulated as a mixed integer linear programming problem. The objective is to minimize the operation cost of the electricity and natural gas networks considering the price of the natural gas supply. Benders decomposition is used to solve the formulated problem. The master problem minimizes the startup and shutdown costs as well as the operation cost of the thermal units other than gas-fired generation units in the electricity network. The first subproblem validates the feasibility of the decisions made in the master problem in the electricity network and if there is any violation, feasibility Benders' cut is generated and added to the master problem. The second subproblem ensures the feasibility of the decisions of the master problem in the natural gas transportation network considering the line-pack constraints. The last subproblem ensures the optimality of the natural gas network operation problem considering the demand of the gas-fired generation units and line-pack. The nonlinear line-pack and flow constraints in the natural gas transportation network feasibility and optimality subproblems are linearized using Newton-Raphson technique. The presented case study shows the effectiveness of the proposed approach. It is shown that leveraging the stored gas in the natural gas pipelines would further reduce the total operation cost.

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## LIST OF SYMBOLS

*Indices:*

$b$	Index of line in the electricity network
$c$	Index of coal-fired generation unit
$i$	Index of gas-fired generation unit
$j$	Index of gas load
$k$	Index of iteration
$l$	Index of electricity demand
$m, n$	Indices of nodes in the natural gas transportation network
$p, q$	Indices of bus in the electricity network
$r$	Index of gas compressor
$s$	Index of gas supplier
$t$	Index of time

*Variables:*

$f_{m,n,t}$	Natural gas flow between nodes $m$ and $n$ at time $t$
$f'_{m,n,t}$	Natural gas inflow from node $m$ to $n$ at time $t$
$f''_{m,n,t}$	Natural gas outflow from node $m$ to $n$ at time $t$
$pf_{b,t}$	Power flow in line $b$ at time $t$
$sd_{(\cdot)}$	Shutdown time of generation unit
$su_{(\cdot)}$	Startup time of generation unit
$u_{(\cdot)}$	Commitment of the generation unit

$v_{s,t}^{(\cdot)}$	Gas supply volume from supplier $s$ at time $t$
$y_{(\cdot)}$	Startup indicator of a generation unit
$z_{(\cdot)}$	Shutdown indicator of a generation unit
$\theta_{p,t}$	Voltage angle at bus $p$ at time $t$
$\mu_{(\cdot)}$	Lagrangian multipliers
$\pi_{(\cdot)}$	Nodal natural gas pressure
$F_{(\cdot)}$	Required thermal energy for a generation unit (MBTU)
$H_{r,t}$	Power output of the compressor $r$ at time $t$
$L_{j,t}^{(\cdot)}$	Gas volume of load $j$ at time $t$
$P_{(\cdot)}$	Generation dispatch of the generation unit
$P_{l,(\cdot)}$	Electricity demand
$S_{1,p,t}, S_{2,p,t}$	
$, S_{j,t}$	Positive slack variables
$SD_{(\cdot)}$	Shutdown cost of the generation unit
$SU_{(\cdot)}$	Startup cost of the generation unit
$V_{lp_{m,n,t}}$	The mass of gas in pipeline between nodes $m$ and $n$ at time $t$
$V_{stock_{m,n,t}}$	Change in mass of gas in the pipeline between nodes $m$ and $n$ at time $t$
$Z$	Value of the objective function
<i>Constants:</i>	
$a_i, b_i, c_i$	Empirical parameters of gas-fired generation unit $i$

$a_c, b_c, c_c$	Empirical parameters of coal unit $c$
$a_r, b_r, c_r$	Parameters for gas consumption function of compressor $r$
$cp_c$	Fuel price for coal-fired unit $c$
$gp_{s,t}$	Gas price of supplier $s$ at time $t$
$k_{1,r}, k_{2,r}$	Empirical parameters of compressor $r$
$\hat{L}_{j,t}$	Volume of gas load $j$ at time $t$
$\hat{\mathbf{P}}$	Vector of generation dispatch of all units
$pf_{b,max}$	Maximum power transmitted through line $b$
$csu_{i,t}$	Startup cost of generation unit
$csd_{i,t}$	Shutdown cost of generation unit
$v_{min,s}$	Minimum output volume of supplier $s$
$v_{max,s}$	Maximum output volume of supplier $s$
$x_{p,q}^b$	Reactance of the line $b$ between nodes $p$ and $q$
$\pi_{min,m}$	Minimum pressure at node $m$
$\pi_{max,m}$	Maximum pressure at node $m$
$\pi_{st}$	Pressure under the standard condition
$\varepsilon$	Threshold value
$\alpha_r$	Empirical parameter of compressor $r$
$A_{m,s}$	Elements of node-gas supplier incidence matrix
$B_{m,j}$	Elements of node-gas load incidence matrix
$C_{b,i}$	Elements of bus-gas generator incidence matrix

$C_{b,c}$	Elements of bus-coal generator incidence matrix
$C'_{m,n}$	Pipeline constant for gas flow from nodes $m$ to $n$
$C''_{m,n}$	Pipeline constant for line-pack from nodes $m$ to $n$
$D_{p,l}$	Elements of the bus-load incidence matrix
$De_t$	Total electricity demand of the system at time $t$
$DR_{(.)}$	Ramping down rate limit of the generation unit
$E_{p,b}$	Elements of the bus-line incidence matrix
$F_{m,r}$	Elements of the node-gas compressor incidence matrix
$G_{j,i}$	Elements of gas load-GFG unit incidence matrix
$H_{max,r}$	Maximum power output of compressor $r$
$H_{min,r}$	Minimum power output of compressor $r$
$L_{j,t}$	Gas-fired generation load in the natural gas network
$MF_{(.)}$	Maximum hours that a generation unit is off
$MN_{(.)}$	Minimum hours that a generation unit is on
$NGL$	Total number of gas loads
$NGS$	Total number of gas suppliers
$P_{min,(.)}$	Minimum capacity of generation unit
$P_{max,(.)}$	Maximum capacity of generation unit
$R_{max,r}$	Maximum pressure ratio of compressor $r$
$R_{min,r}$	Minimum pressure ratio of compressor $r$
$T_{off,(.)}$	Minimum off time of the generation unit

$T_{on,(i)}$	Minimum on time of the generation unit
$UR_{(i)}$	Ramping up rate limit of the generation unit



## Chapter 1

### INTRODUCTION

The environmental considerations to reduce the greenhouse gas generation, the reduction in building time and investment cost of combined-cycle generation units, the increase in the installed capacity of renewable generation, and the emergence of shale gas promote gas-fired generation (GFG) technology in the bulk power networks [1], [2]. Natural gas remains as the primary source for electricity generation as the GFG is expected to provide 33% and 34% of the total energy demand in the U.S. in 2018 and 2019 respectively [3]. The increase in the installed capacity of this technology highlights the essence of capturing the interdependence among electricity and natural gas networks.

Considering the interconnection among electricity and natural gas networks, deficiency in natural gas supply could impose risks to the electricity supply adequacy in the bulk power networks. The outages in natural gas pipelines and severe weather conditions could impede the normal operation of the natural gas network and further jeopardize the security and reliability of the electricity networks by mitigating the available GFG capacity. Effective coordination among the electricity and natural gas networks, diversifying the fuel resources, and incorporating effective load shedding strategies could improve the reliability and security of the electricity network that is exposed to such contingencies.

Several research works were focused on the coordinated operation of electricity and natural gas networks. The proposed short-term operation framework in [4], addresses the interdependence among electricity and natural gas networks by incorporating the natural gas network constraints into the security-constrained unit commitment problem. A two-stage nonlinear optimization model for the integrated operation of natural gas and electricity networks is proposed in [5]. At the first stage, a Mixed Inter Linear Programming (MILP) problem is formulated to determine the direction of natural gas flow and in the second stage, the maximum electrical power generated by GFG units in the bulk power network is determined by formulating a nonlinear programming problem knowing the direction of the natural gas flow from the first stage. The intra-day interaction among the electricity and natural gas networks is addressed in [6] by developing a continuous time optimal power flow formulation that incorporates the dynamic optimal gas flow as well as the real-time

GFG consumption in the natural gas transportation network. The electricity and natural gas flows are optimized considering the time-variable gas demand. While such research efforts focused on the steady-state and dynamic operations of the natural gas network, limited attention was dedicated to the line-pack flexibility in the natural gas pipelines. Line-pack allows for temporarily storing natural gas in the pipeline by regulating the pressure at the input and output of the pipeline. It is discussed in [7] that the line-pack contributes to the reduction of the system operation cost as natural gas could be stored at periods with lower price and utilized once the price is increased. Furthermore, line-pack provides significant value to GFGs by leveraging the difference between the spatial and temporal prices of natural gas supply and provide flexibility for the natural gas transportation network [8]. Such flexibility is crucial for the electricity generation with highly volatile renewable generation resources [9].

Line-pack in pipelines could be used as a tool by the gas transmission system operator (GTSO) to improve the reliability and security of the natural gas transportation network as the stored gas in pipelines could compensate for the disturbance in the natural gas injection and withdrawal [10]. In [11] security constrained optimal power and gas flow is formulated as a mixed integer linear programming problem to determine the stabilized operation of the interconnected network exposed to contingencies. While the storage is considered as an asset to the natural gas network, the line-pack is ignored in the presented formulation. Benders decomposition and linearization techniques were used in [12] to solve the security constrained unit commitment considering the dynamic natural gas constraints. However, the economic benefits of line-pack flexibility in the natural gas network are ignored and the approximations were used to calculate the stored mass of natural gas in the pipeline. While the compressibility and gas travel velocity is considered in [13], linearization techniques were used to solve the integrated operation of electricity and natural gas networks while ignoring the impact of intake natural gas compression of GFG units on their generated output power and the operation cost of the electricity network. The contributions of this research are as follows:

- The coordination among natural gas and electricity is presented in the day-ahead operation in which Benders decomposition is used to decompose the problem into feasibility and optimality subproblems for electricity and natural gas networks.
- The impact of line-pack in the natural gas pipelines on the operation cost of the natural gas and electricity network is addressed.

- Newton-Raphson technique is used to handle the nonlinear line-pack and natural gas flow constraints in the natural gas feasibility and optimality subproblems.

The dissertation is organized as follows: Chapter 2 presents the integrated electricity and natural gas operation problem, that includes the unit commitment (UC), economic dispatch (ED) and optimal flow in the natural gas network. Chapter 3 provides a solution algorithm that captures the autonomous operation of electricity and natural gas networks. Benders decomposition and Newton-Raphson linearization techniques are used to solve for feasibility and optimality of the electricity and natural gas network operation. Chapter 4 presents the case study to validate the proposed solution methodology and further to evaluate the impact of line-pack flexibility on the electricity and natural gas network operation. Chapter 5 summarizes the outcomes and presents the conclusion.

## Chapter 2

### PROBLEM FORMULATION

The problem formulation is shown in (1)-(49). The short-term operation problem in the bulk power network is presented as UC and ED problem [14]. As the electricity network is coupled with the natural gas network, the proposed problem should further capture the natural gas transmission network constraints. The objective function (1) minimizes the operation cost of the electricity and natural gas transportation networks. The constraints include the generation unit constraints, the electricity network constraints and the natural gas network constraints [14]. The demand and supply balance in the electricity network is enforced by (2). The GFG units' constraints are shown in (3)-(15). The thermal energy required to produce electricity by a GFG unit is determined using (3). The power dispatch limits for the a GFG unit is imposed by (4). The relationship between startup/shutdown indicators and commitment states are shown in (5). The startup and shutdown costs for a GFG unit are formulated as (6) and (7). The number of hours that a GFG unit is on or off is determined using (8)-(13). The minimum up time and down time are enforced by (10) and (13) respectively. The limits for ramping of a GFG unit are enforced by (14) and (15). Similar constraints are considered for the coal generation units as shown in (16)-(28). The nodal power balance is enforced by (29). The power flow in each line is limited by (30). The power flow in a line is determined by the difference between the voltage angles at the connected buses at two sides of the line and the impedance of the line as shown in (31). The voltage angle for the reference bus is zero as enforced by (32).

$$\min_{P,u,\pi,v,L,H} Z = \sum_i \sum_t [SU_{i,t} + SD_{i,t}] + \sum_c \sum_t [SU_{c,t} + SD_{c,t} + F_{c,t}(P_{c,t}) \cdot cp_{c,t}] + \sum_s \sum_t v_{s,t} \cdot gp_{s,t} \quad (1)$$

$$\sum_i P_{i,t} + \sum_c P_{c,t} = De_t \quad (2)$$

$$F_{i,t}(P_{i,t}) = a_i \cdot P_{i,t}^2 + b_i \cdot P_{i,t} + c_i \quad (3)$$

$$P_{min,i} \cdot u_{i,t} \leq P_{i,t} \leq P_{max,i} \cdot u_{i,t} \quad (4)$$

$$y_{i,t} - z_{i,t} = u_{i,t} - u_{i,(t-1)} \quad (5)$$

$$SU_{i,t} = su_i \cdot y_{i,t} \quad (6)$$

$$SD_{i,t} = sd_i \cdot z_{i,t} \quad (7)$$

$$0 \leq su_{i,t} \leq MN_i u_{i,t}, \quad (8)$$

$$(MN_i + 1)u_{i,t} - MN_i \leq su_{i,t} - su_{i,(t-1)} \leq 1 \quad (9)$$

$$su_{i,t} \geq T_{on,i} \cdot z_{i,t} \quad (10)$$

$$0 \leq sd_{i,t} \leq MF_i(1 - u_{i,t}) \quad (11)$$

$$1 - (MF_i + 1)u_{i,t} \leq sd_{i,t} - sd_{i,(t-1)} \leq 1 \quad (12)$$

$$sd_{i,t} \geq T_{off,i} \cdot y_{i,(t+1)} \quad (13)$$

$$P_{i,t} - P_{i,(t-1)} \leq UR_i(1 - y_{i,t}) + P_{min,i} \cdot y_{i,t} \quad (14)$$

$$P_{i,(t-1)} - P_{i,t} \leq DR_i(1 - z_{i,t}) + P_{min,i} \cdot z_{i,t} \quad (15)$$

$$F_{c,t}(P_{c,t}) = a_c \cdot P_{c,t}^2 + b_c \cdot P_{c,t} + c_c \quad (16)$$

$$-P_{max,c} u_{c,t} \leq P_{c,t} \leq P_{max,c} u_{c,t} \quad (17)$$

$$y_{c,t} - z_{c,t} = u_{c,t} - u_{c,(t-1)} \quad (18)$$

$$SU_{c,t} = csu_c \cdot y_{c,t} \quad (19)$$

$$SD_{c,t} = csd_c \cdot z_{c,t} \quad (20)$$

$$0 \leq su_{c,t} \leq MN_c u_{c,t}, \quad (21)$$

$$(MN_c + 1)u_{c,t} - MN_c \leq su_{c,t} - su_{c,(t-1)} \leq 1 \quad (22)$$

$$su_{c,t} \geq T_{on,c} \cdot z_{c,t} \quad (23)$$

$$0 \leq sd_{c,t} \leq MF_c(1 - u_{c,t}) \quad (24)$$

$$1 - (MF_c + 1)u_{c,t} \leq sd_{c,t} - sd_{c,(t-1)} \leq 1 \quad (25)$$

$$sd_{c,t} \geq T_{off,c} \cdot y_{c,(t+1)} \quad (26)$$

$$P_{c,t} - P_{c,(t-1)} \leq UR_c(1 - y_{c,t}) + P_{min,c} \cdot y_{c,t} \quad (27)$$

$$P_{c,(t-1)} - P_{c,t} \leq DR_c(1 - z_{c,t}) + P_{min,c} \cdot z_{c,t} \quad (28)$$

$$E_{p,b} \cdot pf_{b,t} = C_{p,i} \cdot P_{i,t} + C_{p,c} \cdot P_{c,t} - D_{p,l} \cdot P_{l,t} \quad (29)$$

$$pf_{b,min} \leq pf_{b,t} \leq pf_{b,max} \quad (30)$$

$$pf_{b,t} = \frac{\theta_{p,t} - \theta_{q,t}}{x_{p,q}^b} \quad (31)$$

$$\theta_{slack} = 0 \quad (32)$$

$$h_m(\boldsymbol{\pi}, \mathbf{v}, \mathbf{L}, \mathbf{H}) = \sum_{s=1}^{NGS} A_{m,s} \cdot v_{s,t} - \sum_{j=1}^{NGL} B_{m,j} \cdot L_{j,t} - \sum_{n \in GC(m)} (f'_{m,n,t} - f''_{m,n,t}) - \sum_r^{NC} F_{m,r} \cdot G_r(H_{r,t}) = 0 \quad (33)$$

$$V_{lp_{m,n,t}} = V_{lp_{m,n,t-1}} + f'_{m,n,t} - f''_{n,m,t} \quad (34)$$

$$f_{m,n,t} = \frac{f'_{m,n,t} + f''_{n,m,t}}{2} \quad (35)$$

$$V_{lp_{m,n,t}} = C_{m,n}'' \cdot \frac{2}{3} \left( \frac{\pi_{m,t}^3 - \pi_{n,t}^3}{\pi_{m,t}^2 - \pi_{n,t}^2} \right) \quad (36)$$

$$V_{stock_{m,n,t}} = V_{lp_{m,n,t}} - V_{lp_{m,n,t-1}} \quad (37)$$

$$f_{m,n,t} = \text{sgn}(\pi_{m,t}, \pi_{n,t}) \cdot C_{m,n}' \sqrt{|\pi_{m,t}^2 - \pi_{n,t}^2|} \quad (38)$$

$$\text{sgn}(\pi_{m,t}, \pi_{n,t}) = \begin{cases} 1 & \pi_{m,t} \geq \pi_{n,t} \\ -1 & \pi_{m,t} < \pi_{n,t} \end{cases} \quad (39)$$

$$f_{m,n,t} = \text{sgn}(\pi_{m,t}, \pi_{n,t}) \frac{H_{r,t}}{k_{2,r} - k_{1,r} \left[ \frac{\max(\pi_{m,t}, \pi_{n,t})}{\min(\pi_{m,t}, \pi_{n,t})} \right]^{a_r}} \quad (40)$$

$$v_{min,s} \leq v_{s,t} \leq v_{max,s} \quad (41)$$

$$\pi_{min,m} \leq \pi_{m,t} \leq \pi_{max,m} \quad (42)$$

$$H_{min,r} \leq H_{r,t} \leq H_{max,r} \quad (43)$$

$$R_{min,r} \leq \frac{\max(\pi_{m,t}, \pi_{n,t})}{\min(\pi_{m,t}, \pi_{n,t})} \leq R_{max,r} \quad (44)$$

$$G_r(H_r) = a_r^2 \cdot H_{r,t} + b_r \cdot H_{r,t} + c_r \quad (45)$$

$$L_{j,t} = \rho \cdot G_{j,i} \cdot F_{i,t}(P_{i,t}) \quad (46)$$

$$f_{m,n,t} = -f_{n,m,t} \quad (47)$$

$$f'_{n,m,t} = -f''_{n,m,t} \quad (48)$$

$$f'_{m,n,t} = -f''_{m,n,t} \quad (49)$$

The natural gas network constraints are presented in (33)-(49). The natural gas flow in the pipelines is dependent on the length of pipeline, diameter of pipeline, the temperatures of system, the pressures of the nodes, type of natural gas, altitude change and the friction of pipelines [13]. However, most of above factors are considered as fixed parameters in the daily operation of natural gas transportation network. The nodal supply and demand balance in natural gas network is enforced by (33). Here,  $\boldsymbol{\pi}$ ,  $\boldsymbol{v}$ ,  $\boldsymbol{L}$ ,  $\boldsymbol{H}$ , are the vector of nodal gas pressure, gas supply volume, gas load, and compressor power consumption respectively. The line-pack is the amount of natural gas in a pipeline. The natural gas pipelines can store natural gas and the difference between the mass of natural gas in two consecutive periods is considered as the stored gas. The relationship between in-flow and out-flow at each node of a pipeline and the line-pack is shown in (34). The flow of natural gas in the pipeline is calculated using (35). The line-pack in a pipeline is determined using (36), while the volume of stored natural gas in the pipeline is calculated using (37). Here,  $C''_{m,n}$  is dependent on the geometrical volume and the temperature of the pipeline [10].

The gas flow in the pipelines without compressor is determined using (38) and (39), where  $C'_{m,n}$  is dependent on the operating temperature, length, diameter, roughness of the pipeline as well as the gas type. It is worth noting that the limitation on the nodal pressure would limit the natural gas flow in the pipelines. In the natural gas transportation network, the pressure will drop due to the pipeline resistance, and in order to compensate for the pressure loss, the compressors are used. The gas flow through the compressor is given by (40). The volume of the supplied natural gas is limited by (41), the nodal pressure in the natural gas network is limited

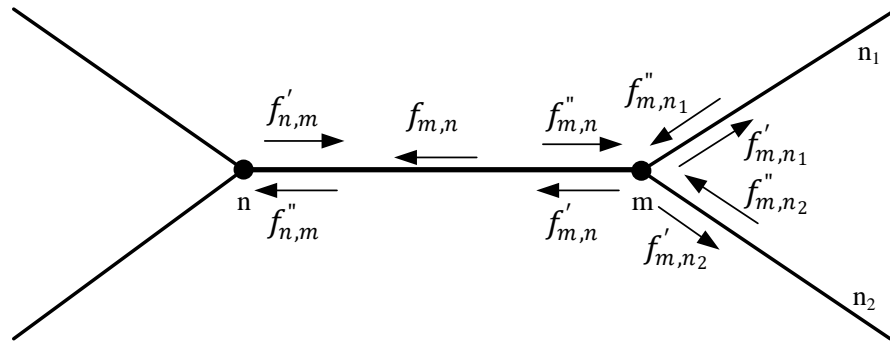


Fig. 2.1. The in-flow, out-flow and gas flow in a natural gas pipeline

by (42), and the power output of the compressor is limited by (43). Furthermore, the pressure ratio is limited by (44). The volume of consumed natural gas to provide the required pressure by the compressor is determined by (45). The volume of natural gas consumed by the GFG unit is determined by (46). The in-flow and out-flow of natural gas in a pipeline satisfy (47)-(49). The in-flow, out-flow, and the flow of natural gas in the pipeline is shown in Fig. 2.1.



## SOLUTION FRAMEWORK

While the proposed mathematical problem in previous section, captures the electricity and natural gas constraints, the information from each infrastructure system may not be readily available to the other system's operator. For example, the electricity network operator may not have access to the natural gas network data to consider the natural gas network constraints. Therefore, the proposed mathematical programming problem is decomposed using Benders decomposition technique to capture the interaction among the electricity and natural gas system operators. Fig. 3.1 shows the flowchart of the presented optimization framework for the short-term operation of electricity and natural gas networks. The steps taken are as follows:

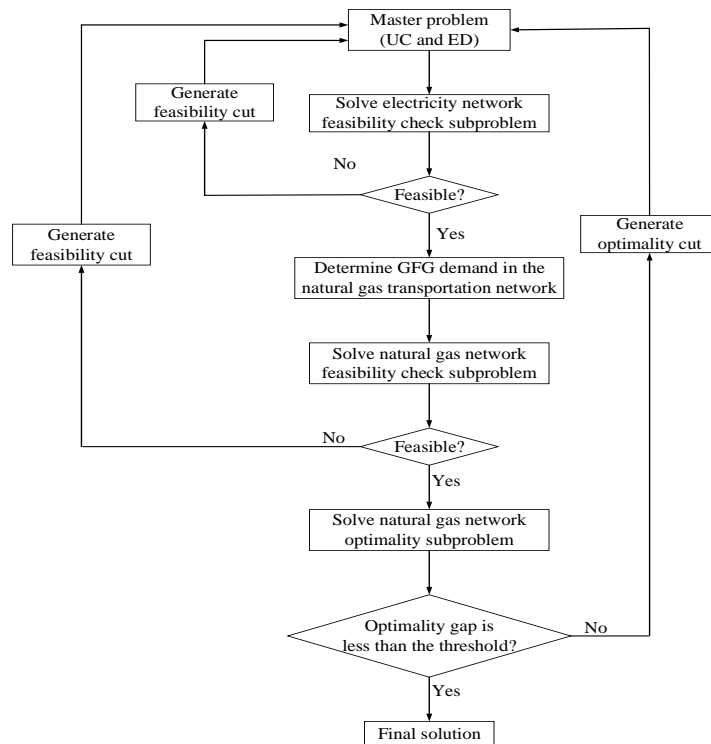


Fig. 3.1. The proposed solution methodology

### 3.1 Master Problem (UC and ED):

Master problem is formulated by the electricity network operator. Here the objective is to minimize the operation cost of the coal generation units as well as the startup and shut down costs of coal and GFG units. The objective function is shown in (50) which is bounded by (51).

$$\min Z \tag{50}$$

$$Z \geq \sum_i \sum_t [SU_{i,t} + SD_{i,t}] + \sum_c \sum_t [SU_{c,t} + SD_{c,t} + F_{c,t}(P_{c,t}) \cdot cp_{c,t}] \tag{51}$$

The constraints in this problem include (51) and (2)-(28). More details on these constraints are presented in [14]. The solution to this problem yields the commitment and dispatch of the coal-fired and GFG units. The solution is passed to electricity network feasibility check subproblem in the next step.

### 3.2 Electricity network feasibility check subproblem:

After solving master problem (UC and ED), the electricity network feasibility check subproblem is formulated to check for any network constraints' violation caused by the solution of the master problem (UC and ED). The feasibility check subproblem is formulated as (52)-(54) and (30)-(32), where the objective is to minimize the mismatch in the nodal supply and demand subjected to DC power flow constraints (30)-(32), and the nodal electricity demand and supply balance (53) and (54). If the value of the objective function is zero, the solution of the master problem satisfies the electricity network constraints. Otherwise, Benders cut (55) is generated and sent to the master problem. Here,  $\hat{\mathbf{P}}$  represents the generation dispatch of all generation units determined by solving the master problem (UC and ED) and  $\omega(\hat{\mathbf{P}})$  is the value of the objective function (52). Similarly, the solution of the master problem will be checked again using the electricity network feasibility check subproblem until the values of all slack variables are zero and the provided solution results in no network violation. At this step, the solution of the master problem is passed to the natural gas network feasibility check subproblem.

$$\min \omega(\hat{\mathbf{P}}) = \sum_t \sum_p (S_{1,p,t} + S_{2,p,t}) \tag{52}$$

*s.t.*

$$E_{p,b} \cdot pf_{b,t} = C_{p,i} \cdot P_{i,t} - D_{p,t} \cdot P_{l,t} + S_{1,p,t} - S_{2,p,t} \quad (53)$$

$$P_{i,t} = \hat{P}_{i,t} \quad : \mu_{1,i,t} \quad (54)$$

$$\omega(\hat{\mathbf{P}}) + \sum_i \sum_t \mu_{1,i,t} (P_{i,t} - \hat{P}_{i,t}) \leq 0 \quad (55)$$

### 3.3 Natural gas network feasibility-check subproblem:

In this step, the feasibility of the natural gas network constraints is checked with the solution passed from the master problem. The feasibility check subproblem is formulated as (56), (34)-(49) and (57). The objective function is to minimize the nodal demand and supply mismatch in the natural gas network as shown in (56) and (57), subjected to the natural gas network constraints (34)-(49). Since the constraints (36), (38), (40), (45) and (57) include nonlinear terms, successive linearization techniques using Newton Raphson method is used to solve the feasibility check subproblem iteratively.

$$\min \omega(\hat{\mathbf{L}}) = \sum_t \sum_j S_{j,t} \quad (56)$$

$$\sum_{s=1}^{NGS} A_{m,s} \cdot v_{s,t} - \sum_{j=1}^{NGL} B_{m,j} \cdot L_{j,t} - \sum_{j=1}^{NGL} B_{m,j} \cdot S_{j,t} - \sum_{n \in GC(m)} (f'_{m,n,t} - f''_{m,n,t}) - \sum_r^{NC} F_{m,r} \cdot G_r(H_{r,t}) = 0 \quad (57)$$

The developed Newton Raphson algorithm is as follows:

- 1) Initiate the nodal pressure  $\pi_{m,t}^0$ , gas supplier volume  $v_{s,t}^0$  and power output of the compressor  $H_{r,t}^0$  and set iteration index  $k = 0$ . Go to step 2.
- 2) Calculate the elements of the Jacobian matrix  $[J_{\pi,t} \ J_{v,t} \ J_{L,t} \ J_{H,t}]$ , using (58)-(62) where the partial derivatives of the required elements are calculated using (63)-(69) and  $\varepsilon, M$  are a small and relatively large numbers respectively to avoid numerical instability. Here the diagonal and off-diagonal elements of  $J_{\pi,t}$  are calculated using (60) and (61) respectively. The derivatives of the natural gas flow with respect to the nodal pressure in the pipelines without compressor are formulated in (63) and (64). Similarly, the derivative of the line-pack with respect to the nodal pressure is formulated in (67) and (68). For the lines with compressor, the partial derivative of the natural gas flow with respect to the nodal pressure is calculated using (65) and

(66). It is worth noting that we ignored line-pack for the pipelines with compressor. The elements of  $J_{v,t}$ ,  $J_{L,t}$  and  $J_{H,t}$  are calculated by (58), (59) and (62) respectively. Go to step 3.

3) Solve (70)-(76) to determine  $\Delta\boldsymbol{\pi}_t^k, \Delta\boldsymbol{v}_t^k, \Delta\boldsymbol{L}_t^k, \Delta\boldsymbol{H}_t^k$ . If the elements of vectors  $\Delta\pi_{m,t}^k \in \Delta\boldsymbol{\pi}_t^k, \Delta v_{s,t}^k \in \Delta\boldsymbol{v}_t^k, \Delta L_{j,t}^k \in \Delta\boldsymbol{L}_t^k, \Delta H_{r,t}^k \in \Delta\boldsymbol{H}_t^k$  are less than threshold  $\epsilon_1$  then go to step 4 otherwise go to step 5. It is worth noting that  $\Delta L_{j,t}^k$  represents the changes in natural gas volume of load  $j$  from iteration  $(k - 1)$  to  $k$  at time  $t$ ;  $\Delta v_{s,t}^k$  represents the changes in natural gas volume of supplier  $s$  from iteration  $(k - 1)$  to  $k$  at time  $t$ ; and  $\Delta\pi_{m,t}^k$  represents the changes in nodal pressure from iteration  $(k - 1)$  to  $k$  at node  $m$  at time  $t$ .

4) The value of objective function (70) is checked. If the value of the objective function (70) i.e.  $\omega^k$  is positive then feasibility Benders cuts (77) are generated. Here,  $\hat{L}_{j,t}$  is the value of the gas load of GFG units in the current iteration of the Newton Raphson method. The Benders cut (70) is transformed to  $\widehat{\Delta P}_{i,t}^k + \sum_m \mu_{2,m,t} \cdot B_{m,j} \cdot G_{j,i} \cdot (P_{i,t} - \hat{P}_{i,t}) \leq 0$  using (3) and (46). The Natural gas network feasibility cut is sent to the master problem. If the value of the objective function is zero, the solution of the master problem is feasible for the natural gas network and the process ends. At this stage, the solution of the master problem is passed to the natural gas transmission optimality subproblem.

5) The vector of variables  $[\boldsymbol{\pi}_t^k \ \boldsymbol{v}_t^k \ \boldsymbol{L}_t^k \ \boldsymbol{H}_t^k]^T$  is updated using (79), increase the iteration index by one, ( $k = k + 1$ ); and go to step 2 to calculate the Jacobian matrix.

$$\frac{\partial h_m}{\partial v_{s,t}} = A_{m,s} \quad (58)$$

$$\frac{\partial h_m}{\partial L_{j,t}} = -B_{m,j} \quad (59)$$

$$\frac{\partial h_m}{\partial \pi_{m,t}} = -\sum_n \frac{\partial (f'_{m,n,t} - f''_{m,n,t})}{\partial \pi_{m,t}} = -\sum_n \frac{\partial \left( f_{m,n} + \frac{V_{stock_{m,n,t}}}{2} \right)}{\partial \pi_{m,t}} \quad (60)$$

$$\frac{\partial h_m}{\partial \pi_{n,t}} = -\frac{\partial \left( f_{m,n} + \frac{V_{stock_{m,n,t}}}{2} \right)}{\partial \pi_{n,t}} \quad (61)$$

$$\frac{\partial h_m}{\partial H_{r,t}} = -\sum_n \frac{\partial f_{m,n,t}}{\partial H_{r,t}} - F_{m,r} \cdot (2a_r + b_r) \quad (62)$$

$$\frac{\partial f_{m,n,t}}{\partial \pi_{m,t}} = \begin{cases} -M, & |\pi_{m,t}^2 - \pi_{n,t}^2| \leq \varepsilon \\ -\frac{C_{m,n}\pi_{m,t}}{\sqrt{|\pi_{m,t}^2 - \pi_{n,t}^2|}}, & |\pi_{m,t}^2 - \pi_{n,t}^2| > \varepsilon \end{cases} \quad (63)$$

$$\frac{\partial f_{m,n,t}}{\partial \pi_{n,t}} = \begin{cases} M, & |\pi_{m,t}^2 - \pi_{n,t}^2| \leq \varepsilon \\ \frac{C_{m,n}\pi_{n,t}}{\sqrt{|\pi_{m,t}^2 - \pi_{n,t}^2|}}, & |\pi_{m,t}^2 - \pi_{n,t}^2| > \varepsilon \end{cases} \quad (64)$$

$$\frac{\partial f_{m,n,t}}{\partial \pi_{m,t}} = \begin{cases} \frac{\alpha_r H_{r,t} k_{1,r} \frac{\pi_{n,t}^{\alpha_r}}{\pi_{m,t}^{\alpha_r+1}}}{\left[ k_{1,r} \left( \frac{\pi_{n,t}}{\pi_{m,t}} \right)^{\alpha_r} - k_{2,r} \right]^2}, & \pi_{n,t} \geq \pi_{m,t} \\ \frac{\alpha_r H_{r,t} k_{1,r} \frac{\pi_{m,t}^{\alpha_r-1}}{\pi_{n,t}^{\alpha_r}}}{\left[ k_{1,r} \left( \frac{\pi_{m,t}}{\pi_{n,t}} \right)^{\alpha_r} - 2, r \right]^2}, & \pi_{n,t} < \pi_{m,t} \end{cases} \quad (65)$$

$$\frac{\partial f_{m,n,t}}{\partial \pi_{n,t}} = \begin{cases} -\frac{\alpha_r H_{r,t} k_{1,r} \frac{\pi_{n,t}^{\alpha_r-1}}{\pi_{m,t}^{\alpha_r}}}{\left[ k_{1,r} \left( \frac{\pi_{n,t}}{\pi_{m,t}} \right)^{\alpha_r} - k_{2,r} \right]^2}, & \pi_{n,t} \geq \pi_{m,t} \\ \frac{\alpha_r H_{r,t} k_{1,r} \frac{\pi_{m,t}^{\alpha_r}}{\pi_{n,t}^{\alpha_r+1}}}{\left[ k_{1,r} \left( \frac{\pi_{m,t}}{\pi_{n,t}} \right)^{\alpha_r} - k_{2,r} \right]^2}, & \pi_{n,t} < \pi_{m,t} \end{cases} \quad (66)$$

$$\frac{\partial V_{stock_{m,n,t}}}{\partial \pi_{m,t}} = \frac{2}{3} \cdot C_{m,n} \left( 1 - \frac{\pi_{n,t}^2}{(\pi_{m,t} + \pi_{n,t})^2} \right) \quad (67)$$

$$\frac{\partial V_{stock_{m,n,t}}}{\partial \pi_{n,t}} = \frac{2}{3} \cdot C_{m,n} \left( 1 - \frac{\pi_{m,t}^2}{(\pi_{m,t} + \pi_{n,t})^2} \right) \quad (68)$$

$$\frac{\partial f_{m,n,t}}{\partial H_{r,t}} = \begin{cases} \frac{1}{k_{1,r} \left( \frac{\pi_{n,t}}{\pi_{m,t}} \right)^{\alpha_r} - k_{2,r}}, & \pi_{n,t} \geq \pi_{m,t} \\ \frac{-1}{k_{1,r} \left( \frac{\pi_{m,t}}{\pi_{n,t}} \right)^{\alpha_r} - k_{2,r}}, & \pi_{n,t} < \pi_{m,t} \end{cases} \quad (69)$$

$$\min \omega^k = \sum_j \sum_t (-\Delta L_{j,t}^k) \quad (70)$$

$$\begin{bmatrix} J_{\pi,t} & J_{v,t} & J_{L,t} & J_{H,t} \end{bmatrix} \begin{bmatrix} \Delta \pi_t^k \\ \Delta v_t^k \\ \Delta L_t^k \\ \Delta H_t^k \end{bmatrix} = -h(\pi_{m,t}^k, v_{s,t}^k, L_{j,t}^k, H_{r,t}^k) \quad \mu_{2,m,t} \quad (71)$$

$$\pi_{min} \leq \Delta\pi_{m,t}^k + \pi_{m,t}^k \leq \pi_{max} \quad (72)$$

$$v_{min} \leq \Delta v_{s,t}^k + v_{s,t}^k \leq v_{max} \quad (73)$$

$$-L_{j,t} \leq \Delta L_{j,t}^k \leq 0 \quad (74)$$

$$H_{min,r} \leq H_{r,t} + \Delta H_{r,t}^k \leq H_{max,r} \quad (75)$$

$$R_{min,r} \leq \frac{\Delta\pi_{m,t}^k + \pi_{m,t}^k}{\Delta\pi_{n,t}^k + \pi_{n,t}^k} \leq R_{max,r} \quad (76)$$

$$\widehat{\Delta L}_{j,t}^k + \sum_m \mu_{2,m,t} \cdot B_{m,j} \cdot (L_{j,t} - \hat{L}_{j,t}) \leq 0 \quad (77)$$

$$\begin{bmatrix} \pi_{m,t}^{k+1} \\ v_{s,t}^{k+1} \\ L_{j,t}^{k+1} \\ H_{r,t}^{k+1} \end{bmatrix} = \begin{bmatrix} \Delta\pi_{m,t}^k \\ \Delta v_{s,t}^k \\ \Delta L_{j,t}^k \\ \Delta H_{r,t}^k \end{bmatrix} + \begin{bmatrix} \pi_{m,t}^k \\ v_{s,t}^k \\ L_{j,t}^k \\ H_{r,t}^k \end{bmatrix} \quad (78)$$

Here  $\widehat{\Delta L}_{j,t}^k$  represents the difference between gas load that could be served by the natural gas transportation network at node  $j$  and the natural gas demand provided by the master problem. If  $\widehat{\Delta L}_{j,t}^k$  is negative this means that part of the natural gas demand imposed by the GFG units could not be served.

### 3.4 Natural gas network optimality subproblem:

In this problem, the operation cost of the natural gas network is minimized. Similar to the feasibility check subproblem, this problem is solved by the natural gas network operator. The problem is formulated as (79), (33)-(49). The objective function is to minimize the total cost of acquiring the natural gas from the suppliers in the natural gas network.

$$\min \sum_s \sum_t gp_{s,t} \cdot v_{lp_{s,t}} \quad (79)$$

Similar to the natural gas feasibility check subproblem, as the constraints (33), (36), (38), (40), and (45) include nonlinear terms, successive linearization using Newton Raphson method is used to solve this problem. The solution algorithm for solving this problem is as follows:

- 1) Initiate the nodal pressure  $\pi_{m,t}^0$ , gas supplier volume  $v_{s,t}^0$  and power output of the compressor  $H_{r,t}^0$  and set iteration index  $k = 0$ . Go to step 2.

2) Calculate the elements of the Jacobian matrix  $[J_{\pi,t} J_{v,t} J_{L,t} J_{H,t}]$ , using (58)-(62) where the partial derivatives of the required elements are calculated using (63)-(69). Go to step 3.

3) Solve (79), (71)-(73), (80), (75), (76) to determine  $\Delta\boldsymbol{\pi}_t^k, \Delta\boldsymbol{v}_t^k, \Delta\boldsymbol{L}_t^k, \Delta\boldsymbol{H}_t^k$ . If the elements of vectors  $\Delta\boldsymbol{\pi}_{m,t}^k \in \Delta\boldsymbol{\pi}_t^k, \Delta v_{s,t}^k \in \Delta\boldsymbol{v}_t^k, \Delta L_{j,t}^k \in \Delta\boldsymbol{L}_t^k, \Delta H_{r,t}^k \in \Delta\boldsymbol{H}_t^k$  are less than threshold  $\epsilon_1$  then go to step 4 otherwise go to step 5. Note that as the feasibility of the natural gas network is guaranteed by enforcing feasibility cuts, the natural gas demand for GFG units at each iteration is enforced by (80).

4) Calculate  $\hat{z}_1$  using (81) and  $\hat{z}_2 = \hat{z}_1 + \boldsymbol{g}p_{s,t} \cdot \hat{v}_{lp_{s,t}}$  where  $\hat{v}_{lp_{s,t}}$  is the solution to the natural gas network optimality subproblem. If (82) is satisfied, then the process ends. Otherwise, generated optimality cut formulated as (83) and send it to the master problem. Note that the optimality Benders cut is reformulated as  $Z \geq \hat{z}_2 + \sum_t \sum_i \mu_{3,j,t} \cdot G_{j,i} \cdot (P_{i,t} - \hat{P}_{i,t})$  considering (3) and (46).

5) The vector of variables  $[\boldsymbol{\pi}_t^k \quad \boldsymbol{v}_t^k \quad \boldsymbol{L}_t^k \quad \boldsymbol{H}_t^k]^T$  is updated using (80), increase the iteration index by one, ( $k = k + 1$ ); and go to step 2 to calculate the Jacobian matrix.

$$L_{j,t}^k + \Delta L_{j,t}^k = \hat{L}_{j,t} \cdot \mu_{3,j,t} \quad (80)$$

$$\hat{z}_1 = \sum_i \sum_t [\widehat{S}U_{i,t} + \widehat{S}D_{i,t}] + \sum_c \sum_t [\widehat{S}U_{c,t} + \widehat{S}D_{c,t} + F_{c,t}(\hat{P}_{c,t}) \cdot cp_c] \quad (81)$$

$$2 * |\hat{z}_2 - \hat{z}_1| / (\hat{z}_1 + \hat{z}_2) \leq \epsilon \quad (82)$$

$$Z \geq \hat{z}_2 + \sum_t \sum_j \mu_{3,j,t} (L_{j,t} - \hat{L}_{j,t}) \quad (83)$$

CASE STUDY

**4.1 Six-bus electricity network with seven-node natural gas network**

The electricity network and natural gas network topologies are shown in Fig. 4.1 and Fig. 4.2 respectively.

The electricity network consists of three generation units and seven transmission lines. Three electricity demands are connected to buses 3, 4 and 5. Two generation units G1 and G2 are GFG units and G3 is coal-fired generation unit. The characteristics of the units and the transmission lines are shown in Table 4.1 and

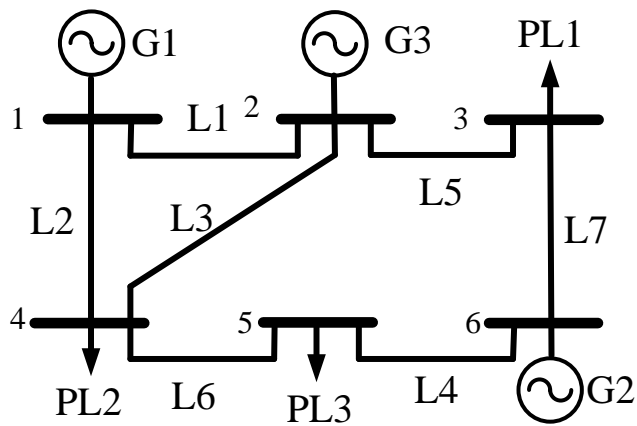


Fig. 4.1. 6-bus electricity network

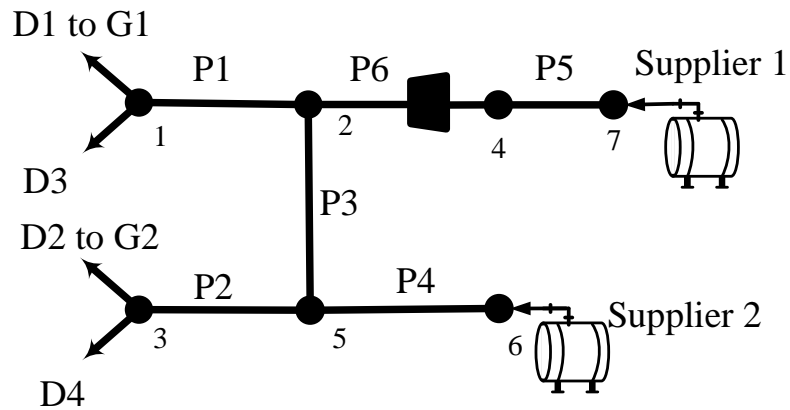


Fig. 4.2. 7-node natural gas network



Table 4.1 Generation unit characteristics for 6-bus network

unit	a(MBtu /MWh <sup>2</sup> )	b(MBtu /MWh)	c(MBtu /h)	Pmin (MW)	Pmax (MW)	Min on(h)	Min off(h)
1	0.0004	13.5	177	100	220	4	4
2	0.005	17.7	137	10	20	2	3
3	0.001	32.6	130	10	100	1	1

Table 4.2 Transmission line characteristics for 6-bus network

Branch	From	To	X(p.u.)	Flow Limit
1	1	2	0.17	200
2	1	4	0.258	100
3	2	4	0.197	100
4	5	6	0.14	100
5	2	3	0.037	100
6	4	5	0.037	100
7	3	6	0.018	100

Table 4.3 Characteristics of the natural gas pipelines in 7-node network

Pipeline	From node	To node	Length(m)	Pipeline constant
1	1	2	120000	2.277
2	3	5	90000	1.708
3	2	5	120000	2.277
4	5	6	95000	1.802
5	4	7	85000	1.613

Table 4.2. Here, 1 kcf of natural gas provides 1.037 MBTU of energy in units G1 and G2. The total peak demand of the system is 256 MW at hour 17 and the hourly total demand profile is shown in Fig. 4.3. The natural gas network has seven nodes, six pipelines, one compressor and two natural gas suppliers, as shown in Fig. 4.2. The characteristics of the natural gas pipelines are shown in Table 4.3. The gas load consists of

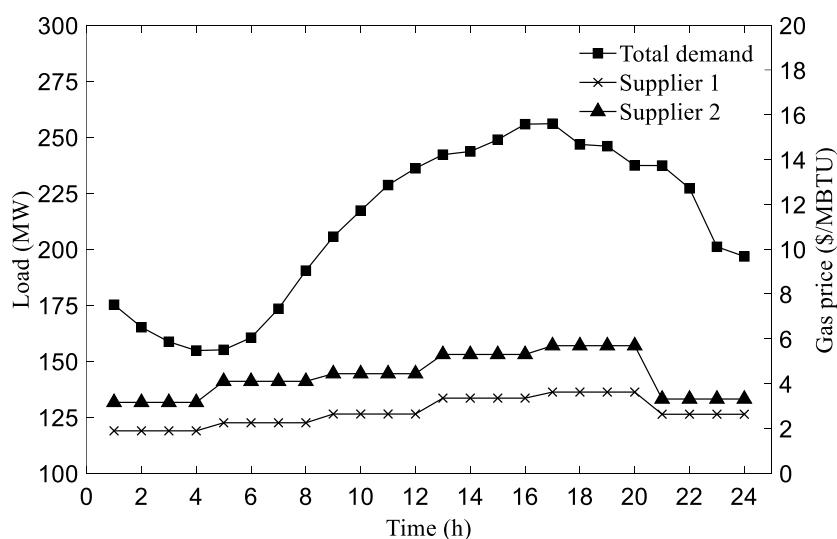


Fig. 4.3. Hourly total electricity demand and the price of natural gas

two residential gas loads D3 and D4; and the demand for generation units G1 and G2. To highlight the impact of line-pack and stored natural gas in pipelines, the price of natural gas is changed every four hours shown in Fig. 4.3. The following cases are considered:

Case 1 – UC and ED without line-pack.

Case 2 – UC and ED with line-pack in the natural gas network.

Case 3 – UC and ED with line-pack and congestion in the natural gas network.

#### 4.1.1 Case 1: UC and ED without line-pack

The hourly commitment of the generation units ignoring the electricity network constraints, is shown in Table 4.4. Once the electricity network feasibility check is considered, the commitments of the generation units are shown in Table 4.5. The impact of considering the natural gas network constraints on the solution of the master problem is shown in Table 4.6.

Fig. 4.4(a) shows the dispatch of the generation units G1-G3 in the operation horizon without considering the electricity and natural gas network constraints. Fig. 4.4(b) shows the dispatch of the generation units G1-G3 considering the electricity network constraints and Fig. 4.4(c) shows the dispatch of the generation units G1-G3 considering the electricity and natural gas network constraints. Comparing Table 4.4 with Table 4.5, the commitment of G3 is changed from hours 13-18 to 11-22 because of the congestion in the electricity

Table 4.4 Hourly unit commitment without electricity network feasibility check

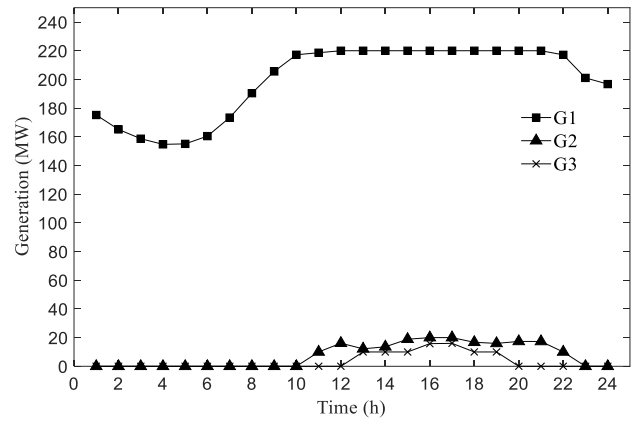
Unit	Hours [h] (1-24)
1	1 1
2	0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 0 0
3	0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 0 0 0 0 0

Table 4.5 Hourly unit commitment with electricity network feasibility check

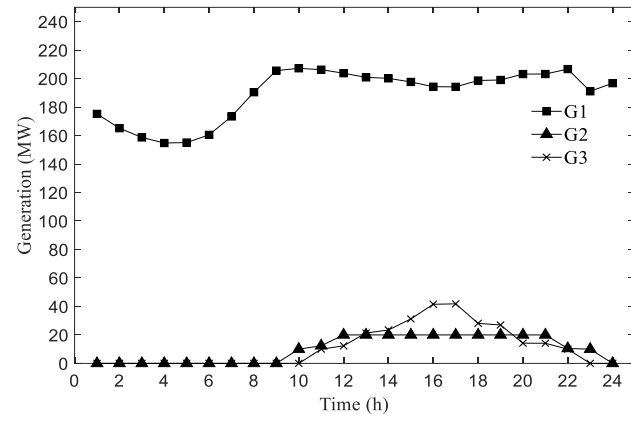
Unit	Hours [h] (1-24)
1	1 1
2	0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0
3	0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0

Table 4.6 Hourly unit commitment with natural gas feasibility check

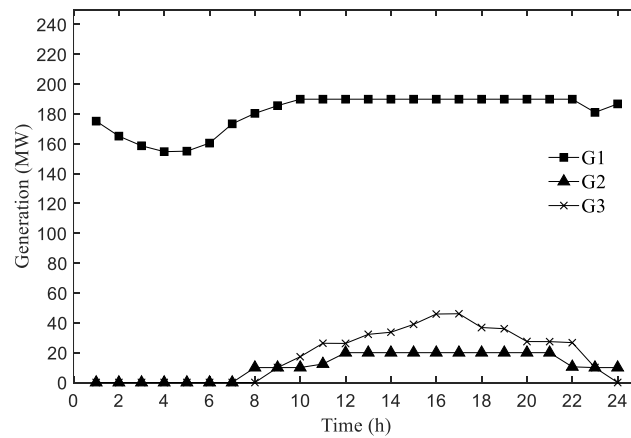
Unit	Hours [h] (1-24)
1	1 1
2	0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
3	0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0



(a)



(b)



(c)

Fig. 4.4. Generation dispatch (a) without electricity network feasibility check, (b) with electricity network feasibility check, (c) with natural feasibility check

network. Here, without considering the electricity network constraints transmission line L2 carries at least

102.3 MW at hours 10-22 which exceeds its maximum limit 100

MW. Therefore, the dispatch of G1 is reduced at hours 10-22 as shown in Fig. 4.4(b). Table 4.6. shows that considering the natural gas network constraints impacts the commitment of the generation units. Ignoring the natural gas network constraints will result in violation of the natural gas flow in pipeline P1. Here, the natural gas flow in pipeline P1 exceeds the limits at hours 8-24. For instance, by ignoring the network constraints, the supplied gas by pipeline P1 to node 1 is 7001 kcf at hour 10; however, considering the pressure limits at the end nodes, the limit for gas flow in pipeline P1 is 6765 kcf which is less than the demand at node 1. In order to eliminate this violation, the natural gas demand of unit G1 is reduced and the dispatch of G3 is increased at hours 8-24 to compensate for the shortage in generation as shown in Fig. 4.4(c). As shown in Fig. 4.4(c), G1, which is the least expensive unit in the electricity network, provides its maximum generation capacity at hours 8-24. The rest of the demand is served by dispatching G2 and G3 as more expensive units. In order to serve the load at hours 12-21, the dispatch of G2 will reach its maximum and G3 further compensates for the unserved demand. Dispatching G1 and G2 will further impact the demand in the natural gas network. Here, the generation dispatch at hour 12 is 189.9 MW, 20 MW and 26.3 MW for G1, G2 and G3 respectively. Therefore, the total natural gas demand at natural gas demand nodes are 1 and 3 are 6737.0 kcf and 2494.0 kcf gas respectively. Considering the natural gas optimality subproblem, the total production costs for G1, G2 and G3 in the operation horizon are \$241,075, \$43,389 and \$113,803 respectively. The total operation cost in this case is \$855,613. The total operation cost of the natural gas network in this case is \$741,810.

#### **4.1.2 Case 2: UC and ED with line-pack in the natural gas network:**

In this case, except the pipelines with compressors, the line-pack for all pipelines is considered. Therefore, for these pipelines, the inflow of the natural gas pipeline is not equal to its outflow. Fig. 4.5 shows the line-pack for pipelines P1 and P3. The line-pack is dependent on the average nodal pressure at two sides of the pipeline. The line-pack of P3 is larger than P1 as the average pressure of nodes 5 and 2 is larger than the average pressure of nodes 2 and 1. It is worth noting that the direction of flow is from the nodes with higher

Table 4.7 Inflow, outflow, line-pack and stored NG in pipeline P1

Time	1	2	3	4	5	6	7	8	9	10	11	12
Inflow	6570.1	6433.0	6339.4	6288.4	6288.7	6364.4	6538.8	6635.8	6729.8	6726.0	6728.5	6736.8
Outflow	6566.6	6430.0	6337.4	6287.4	6288.7	6366.0	6542.0	6637.3	6732.0	6726.0	6728.5	6737.0
Line-pack	323.4	326.4	328.4	329.4	329.4	327.8	324.0	321.9	319.7	319.8	319.7	319.5
Stored NG	3.470	3.007	1.983	1.057	0	-1.606	-3.795	-2.162	-2.185	0.114	-0.059	-0.197
Time	13	14	15	16	17	18	19	20	21	22	23	24
Inflow	6743.1	6745.0	6751.8	6760.1	6763.1	6749.5	6749.7	6738.8	6738.5	6729.4	6645.3	6719.4
Outflow	6743.2	6745.0	6751.9	6760.3	6763.2	6749.2	6749.7	6738.5	6738.5	6729.2	6643.3	6721.1
Line-pack	319.4	319.3	319.1	319.0	318.9	319.2	319.2	319.5	319.5	319.7	321.7	319.9
Stored NG	-0.145	-0.045	-0.160	-0.195	0	0.325	0	0.260	0	0.213	1.965	-1.753

pressure to the nodes with the lower pressure and the direction of the natural gas flow is from node 5 to node 2 and from node 2 to node 1. Table 4.7 presents the in-flow, out-flow, line-pack and the volume of stored natural gas in pipeline P1. The volume of stored natural gas in P1 is calculated by the difference among the line-pack in consecutive periods. Therefore, the flexibility of pipelines to serve the natural gas load is determined by the volume of stored natural gas in the pipeline. As show in Table 4.7, at hours 1-4, pipeline P1 stores natural gas because the price of natural gas at hours 1-4 are \$1.89/MBTU and \$3.16/MBTU for suppliers 1 and 2 respectively. These prices are the lowest in the operation period and therefore, the stored level of natural gas in the system reaches its maximum. At hours 6-9 when the price of natural gas increases, the stored gas in P1 is consumed as shown in Table 4.7. The total volume of stored gas in P1 at hours 10-20 is zero. The stored gas at hours 1-4 and 23-24 is used at hours 6-9 and the gas demand at node 1 increases. At hours 10-20 pipeline P1 does not have enough capacity to store natural gas the in-flow and out-flow limits are reached. In this case, the natural gas operator can store 9.514 kcf of gas in pipeline P1 at hours 1-4 and use it at hours 6-9. Using such storage capacity will reduce the operation cost to \$855,606 from \$855,613 in Case 1. Although the savings is \$7 which is small compared to the total operation cost, it will increase as the limitation on the nodal pressure is further relaxed. For instance, if the maximum nodal pressure increases by 2 times and the minimum nodal pressure is reduced by half, the savings will increase to \$54. It is worth noting that the pipelines do not always store gas as they may reach their maximum flow limits in peak periods. The differences between the natural gas price in two successive periods contributes to the savings in the operation horizon. The savings from line-pack is not significant in this case. However, with the increase in the size of



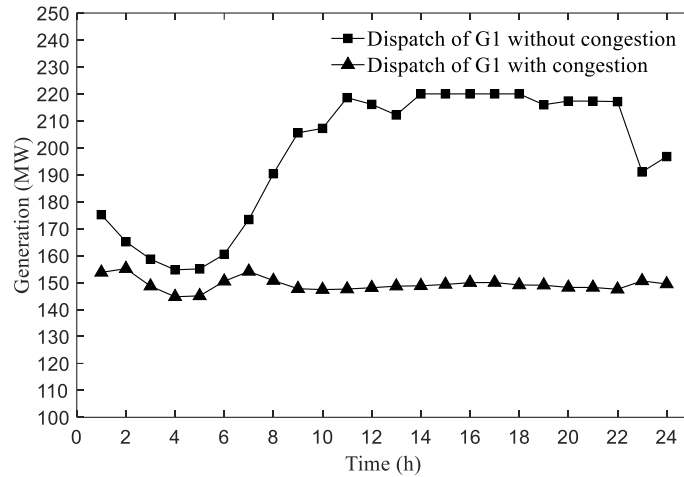


Fig. 4.6. Dispatch of G1 with/without network congestion

generation G3 is much higher than the operation cost of G1 and G2 and therefore, the total operation cost of the electricity and natural gas networks increase. It is worth noting that the outage in P3 between nodes 2 and 5 will result in deficiency in the natural gas supply for G1 and consequently the infeasibility of the electricity network problem as no load shedding is allowed in this case. Fig. 4.5 shows the line-pack of P3 with congestion. As shown in this figure, the line-pack is lower than previous case as the line-pack constant is decreased by 57%. As a result, the natural gas flow in P3 will reach its maximum limit of 1,339 kcf/h which is lower than that in Case 2 (i.e. 5356 kcf/h). Therefore, the congestion in the natural gas pipeline will reduce the natural gas supply for the GFG units and could increase the operation cost of the system and jeopardize the sufficiency of electricity supply.

#### 4.2 30-bus electricity network with 12-node natural gas network

The 30-bus electricity network and 12-node natural gas network are shown in Fig. 4.7 and Fig. 4.8 respectively. The electricity network has 30 buses, 41 lines, 6 generation units and 21 demands. Four generation units G1, G2, G3 and G4 are GFG units that are connected to buses 1, 2, 5 and 8 respectively. Two coal-fired generation units G5 and G6 are connected to buses 11 and 12 respectively. The characteristics of the generation units are shown in Table 4.9. The peak load is 414 MW that occurs at hour 17. The natural gas network has 12 nodes, 10 pipelines, 2 compressors and 3 suppliers. The characteristics of the natural gas pipelines are shown in Table 4.10. The gas load is composed of four residential loads D5-D8 and four GFG

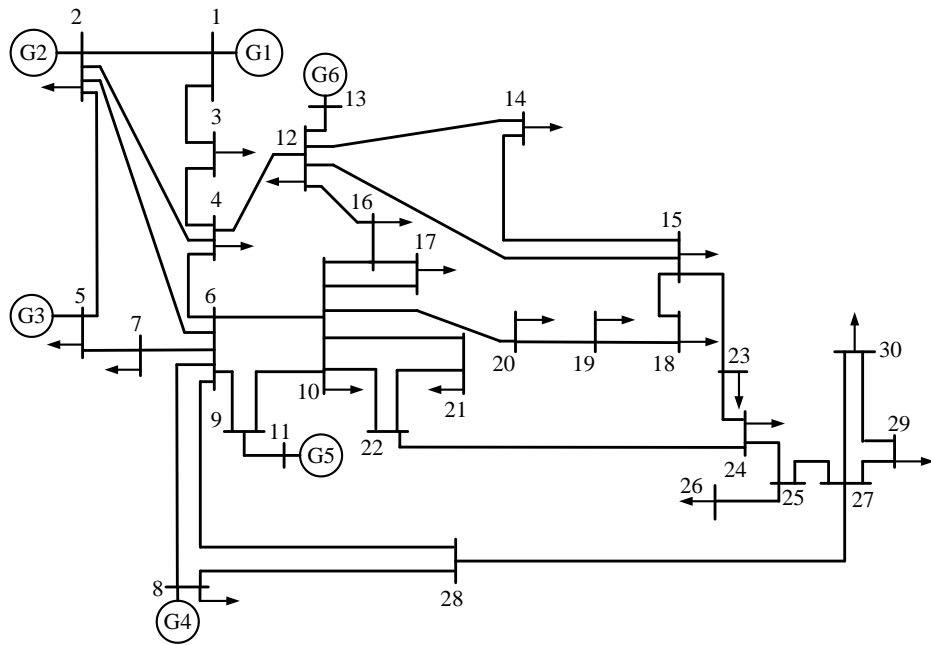


Fig. 4.7. 30-bus electricity network

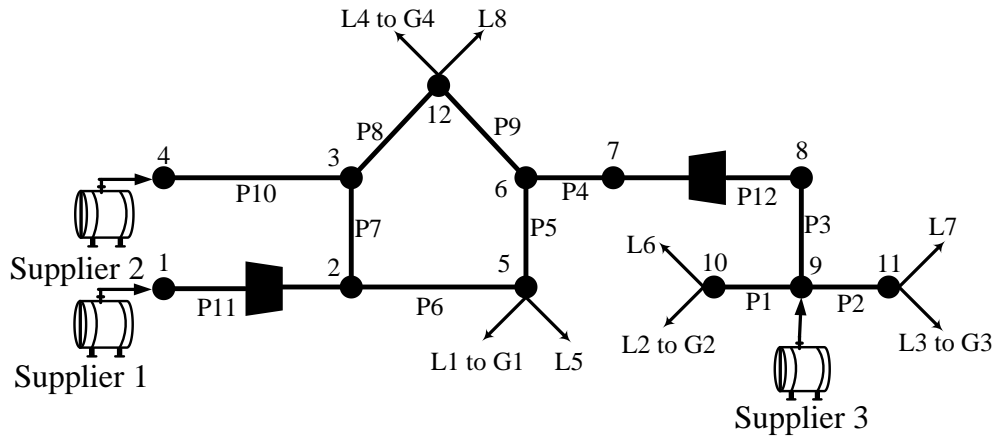


Fig. 4.8. 12-node natural gas network

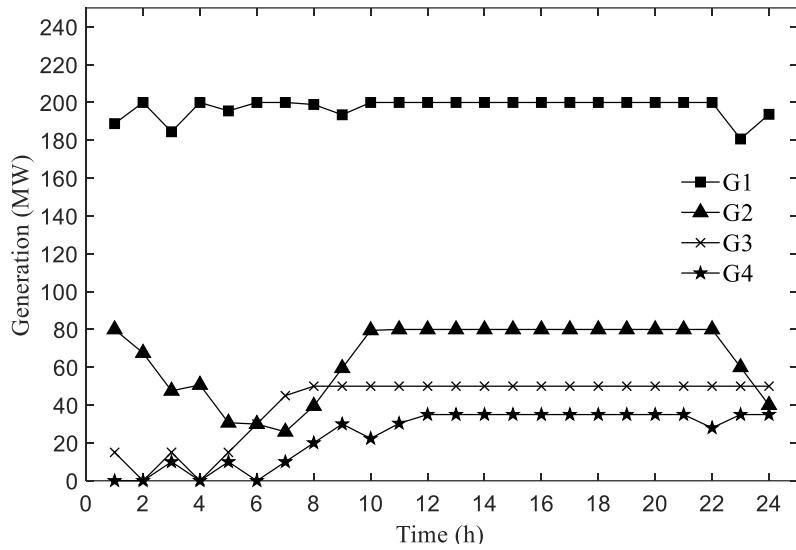
Table 4.9 Characteristics of the generation units for 30-bus network

unit	a(MBtu /MW2h)	b(MBtu /MWh)	c(MBtu /h)	Pmin (MW)	Pmax (MW)	UR (MW)	DR (MW)
1	0.00375	2	0	50	200	65	85
2	0.01750	1.75	0	20	80	12	22
3	0.06250	1	0	15	50	12	12
4	0.00834	3.25	0	10	35	08	16
5	0.025	3	0	10	30	06	09
6	0.025	3	0	12	40	08	16

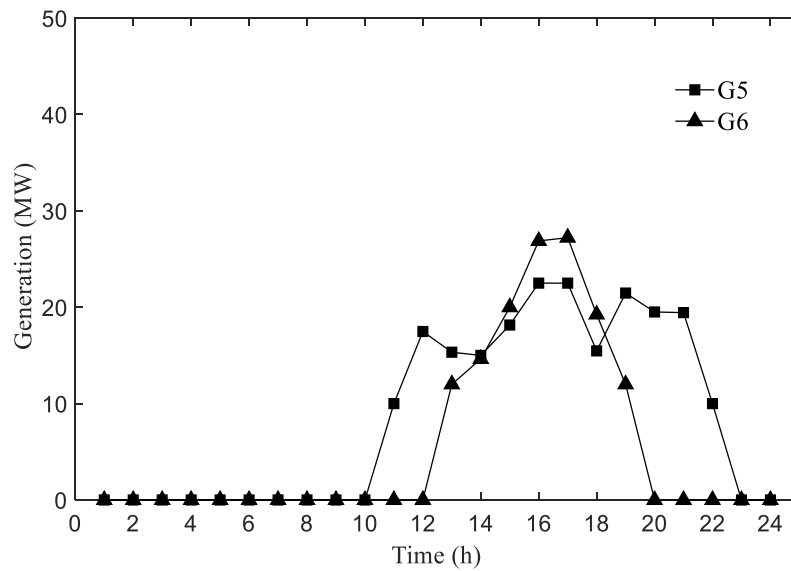
loads D1-D4 for G1-G4 respectively. The price of natural gas for the sources connected to nodes 1 and 4 are the same as that of source 2 in the previous case study. The price of natural gas supply at node 9 is the same







(a)



(b)

Fig. 4.9 Generation dispatch of units in Case 1 (a) GFG units, (b) coal units

reach their maximum capacity for storing natural gas. At hours 6-13 the stored gas is used as the price of natural gas increases. Using such storage capacity will reduce the operation cost to \$989,865 from \$988,174, where storing natural gas in the pipelines can save \$1691.

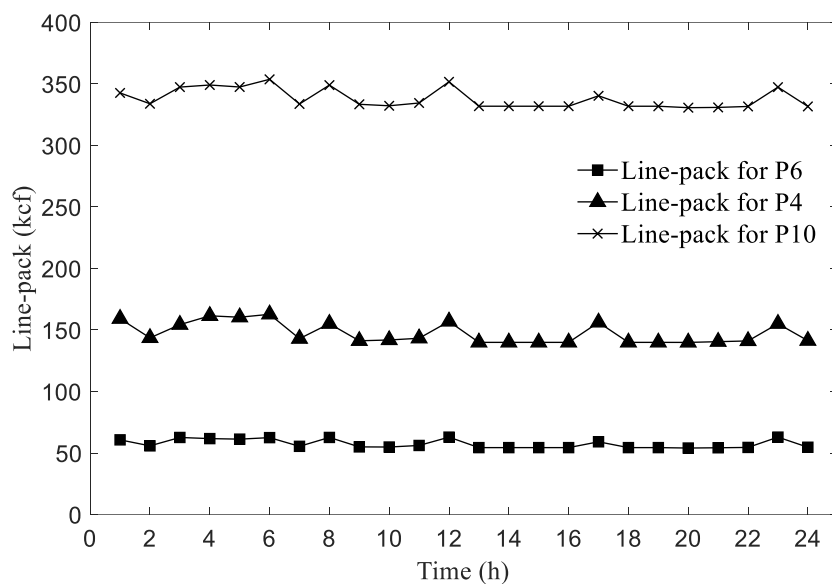


Fig.4.10. Line-pack of pipelines in Case 2

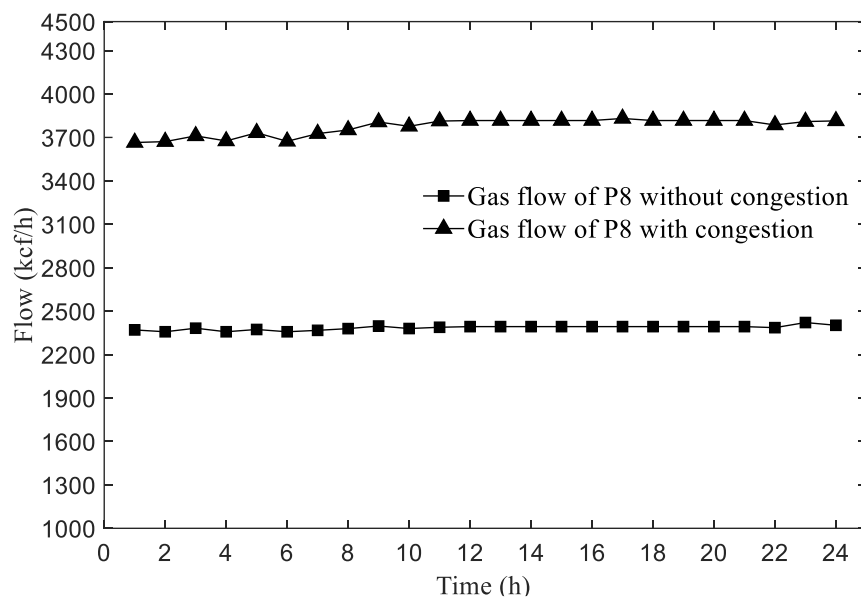


Fig.4.11. Gas flow in pipeline P8 with and without congestion in natural gas network

#### 4.2.3 UC and ED with line-pack and congestion in natural gas network

In this case, a congestion is considered in pipeline P9. Therefore, the pipeline constant and the line-pack constant decreases into 20% and 52.5% of those in Case 2 respectively. In this case, the natural gas supplied

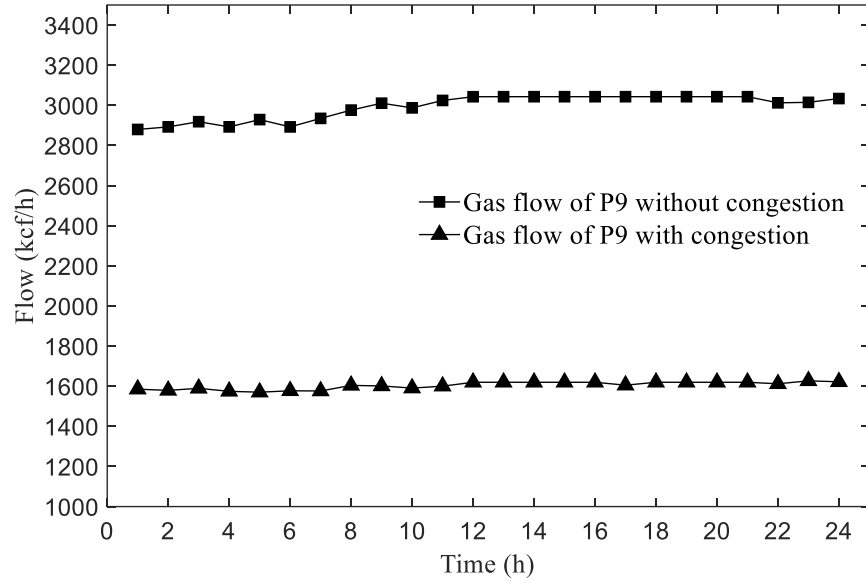


Fig.4.12. Gas flow in pipeline P9 with and without congestion in natural gas network

from supplier 3 through P9 decreases and suppliers 1 and 2 will serve L4 and L8 by increasing the flow in P8 as shown in Fig.4.11. At peak hour 17, the natural gas flow in P8 increases to 3031 kcf/h from 2 kcf/h in Case 2. In contrary, the natural gas flow in some pipelines (e.g. pipeline P9) is decreased as shown in Fig. 4.12. At hour 17, the natural gas flow in pipeline P9 is 1605 kcf/h which is lower than that in Case 2 (i.e. 3042kcf/h). In this case, supplier 3, which is cheapest supplier, serves less hourly load. For example, in this case, supplier 3 serves 9636 kcf at peak hour 17 which is less than that in Case 2 (i.e. 10640 kcf). The total operation cost is \$1,134,176 which is more expansive than that in Case 2 (i.e. \$988,174).

## Chapter 5

### CONCLUSION

This research is focused on the coordinated operation of the electricity and natural gas network considering the line-pack in the natural gas transportation network. Benders decomposition is used to decompose the problem into a master problem solved by the electricity network operator and several subproblems solved by the electricity and natural gas network operators. Here, the master problem addresses the commitment of the generation units while the first subproblem handled by the electricity network operator ensures the feasibility of the provided solution for the electricity network. Once the solution is feasible for the electricity network, the natural gas network operator will ensure the feasibility of the solution for the natural gas network by solving the feasibility subproblem for the natural gas network. Feasibility Benders cuts are generated and added to the master problem in case of any violation exists. Once the solution is feasible for the natural gas network, optimality subproblem is solved and optimality Benders cuts are generated and added to the master problem. The solution process stops once there is no improvement to the lower bound of the objective function. The merit of the proposed method is that the natural gas network data is not shared with the electricity network operator and the Benders cut includes the required information passed from the natural gas transportation network to the electricity network operator. Furthermore, nonlinear natural gas transportation network constraints were captured in the natural gas feasibility and optimality subproblems. Newton-Raphson technique as a successive linearization method is used to solve the feasibility and optimality subproblems iteratively. The value of line-pack is shown by the represented case studies. It is shown that line-pack could contribute to the stored volume of natural gas in the pipelines and further reduces the operation cost of the natural gas network. Furthermore, the congestion in natural gas transportation network would increase the total operation cost as it limits the supply of cheaper GFG units. Congestion in natural gas transportation network could further jeopardize the security of the electricity network.

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