1-1-1982

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TOWARD A FORMAL MODEL OF OPTIMAL SELLER BEHAVIOR IN THE REAL ESTATE TRANSACTIONS PROCESS

Working Paper 82-121*

by

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Presented at the ASSA Annual Meeting, New York City, December, 1982. The author wishes to thank the participants in the Finance workshop at the Edwin L. Cox School of Business for their comments on an earlier draft. This research was funded by grants from the Texas Real Estate Research Center and the Edwin L. Cox School of Business Trimester Research Fellowship Program.

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I. Introduction

Most of the existing literature on real estate market behavior ignores the imperfect nature of the transactions process. For example, the bid rent models of the New Urban Economics (Muth [1969], Mills [1972], Alonso [1964]) assume perfect information, costless bidding, instantaneous bidding, universal bidding, and complete rationality on the part of all market participants. Several recent efforts, however, (e.g. Simon [1957], Stull [1978a, 1978b], Courant [1978], Yinger [1978, 1981], Miller and Rice [1978, 1979]) have recognized the fact that real estate market participants (1) are involved in a search without perfect information and with costs, (2) sometimes are constrained by factors which affect their ability to seek an "optimal" transaction, and (3) do not receive or offer all bids at a particular point in time, but rather must undertake this over time and often cannot recall a bid once it has been rejected. These efforts have attempted in various ways to incorporate such characteristics in search models that can examine their effect on market outcomes.

The purpose of this paper is to compare and evaluate the contributions of these existing efforts and to propose an improved model which both better reflects actual market behavior and better provides an analytic framework for normative model development. To this end, Section II will be devoted to an examination of the characteristics of seller behavior as it actually occurs in residential sales. These characteristics are used as the basis for comparison with the existing models in Section III and as the basis for development and evaluation of an improved model in Section IV. Seller behavior and residential market behavior only are treated here, although many of the results would be equally applicable to buyer behavior and non-residential markets.

One salient result from this exercise is that a search model based upon a prototype by Rothschild (1974), in which the prior for the expected bid distribution is updated in a Bayesian fashion as bids are received, is superior with respect to being able to extract the highest expected price from the sale of property, both to models which assume the bid distribution is completely known (e.g. Simon [1957]) and to models which assume the bid distribution is unknown (e.g. Stull [1978a]). This appears to be true especially under conditions of imperfect information in which one is likely to have an incorrect notion of the actual distribution of potential bids. Thus, such a model is suggested as the basis both for predictive models of market behavior and for future efforts at normative model development.

II. A Descriptive Analysis of Seller Behavior in Residential Sales

The purpose of this section is to provide a descriptive analysis of the process of sale to provide a basis for comparison and development of the subsequent normative model. There are basically eight steps through which sellers proceed in the sale of their residence:

1. The decision to sell
2. Seeking a realtor
3. Setting initial asking price and "reservation" price
4. Setting initial acceptable terms of sale
5. Adjustment of asking price and "reservation" price
6. Adjustment of terms of sale

7. Negotiation

8. Final acceptance of offer.

The normative model to be developed later will not consider steps 1 and 2, "decision to sell" and "seeking a realtor." It is assumed the decision to put the home on the market has been already made and either the property is being owner-sold or the realtor has no undue influence on the subsequent transactions process.

Each step will now be discussed in detail.

Step 1: The Decision to Sell

A number of negative ("push") and positive ("pull") factors influence the decision of the household to sell the current residence and move to a new one. These reflect both demand and supply influences and changes in transactions costs.

"Push" factors include neighborhood decline, adverse environmental influences such as noise pollution or congestion, high crime rates, low service provision levels (including schools), adverse financing arrangements (such as an escalating mortgage rate), lowered income (making payment of current mortgage obligations difficult), high insurance and tax costs, a change in job location or a change in age structure of the household such that the unit or neighborhood becomes less suited for the family's needs. Both current conditions and expectations are important here. "Pull" factors include alternative residential sites with reduced numbers of adverse influences as enumerated above. The relative current price per unit of housing services and expectations of future rates of appreciation are also relevant. Job location shifts, of course, are major preemptive factors in many moves, especially inter-city moves.

The existence of transactions costs could also be influential in facilitating or hindering moves, creating "threshold effects" in which the disparity in utility between the old and new residential location may have to be significant before a move is undertaken. These costs include such items as prepayment penalties, loan origination fees, appraisals, inspections, escrow costs, broker commissions, moving costs, and "search costs," which include the psychological costs, time costs, opportunity costs, and out-of-pocket expenses associated with the search.

The above costs and benefits associated with moving are incorporated in the household's preference function. At that point in time in which the net benefits to movement (including transactions costs) are positive, the household decides to place its home on the market and search for a new home. The order in which it does so depends on a number of factors, all of which influence the level of "search costs." In particular it depends upon the household's ability to handle temporarily increased cash flows if the new home is purchased when the old home has not yet sold.

Step 2: Seeking a Realtor

Once the household decides to sell/buy it decides whether to make use of a broker or to handle the sale/purchase itself. This decision has been the subject of a voluminous literature (for example see Jud [1982], Yinger [1981], Courant [1978], Miller [1977]) which we shall not replicate here. Briefly, the decision of whether or not to use a broker is a decision based upon the desire of the household to minimize net search costs. Services provided by a selling broker include improving market exposure, reducing time on market, increasing sales price, providing advice on improving the saleability of the property, handling all tours, acting as an intermediary in negotiations, providing information on and perhaps a source of alternative financing, facilitating the various document signings, and perhaps providing advice on listing price, bid acceptance, and sales term bargaining. Costs include the broker's fee (usually around 6 percent of the ultimate purchase price), and to some extent the loss of personal control over the sales process.[1] The basic decision of the selling household depends upon whether the benefits of a reduced time on market (hence reduced search cost), possibly higher gross sales price, and reduced effort in the sale offset the major cost of a commission and the minor cost of some loss of control. One would expect households experiencing the necessity of quick sale to be more likely to seek out a broker's services. A household with some leisure in the sale process and an alternative avenue toward greater market exposure (via the newspaper, non-MLS listing book, etc.), however, would tend toward owner sales.

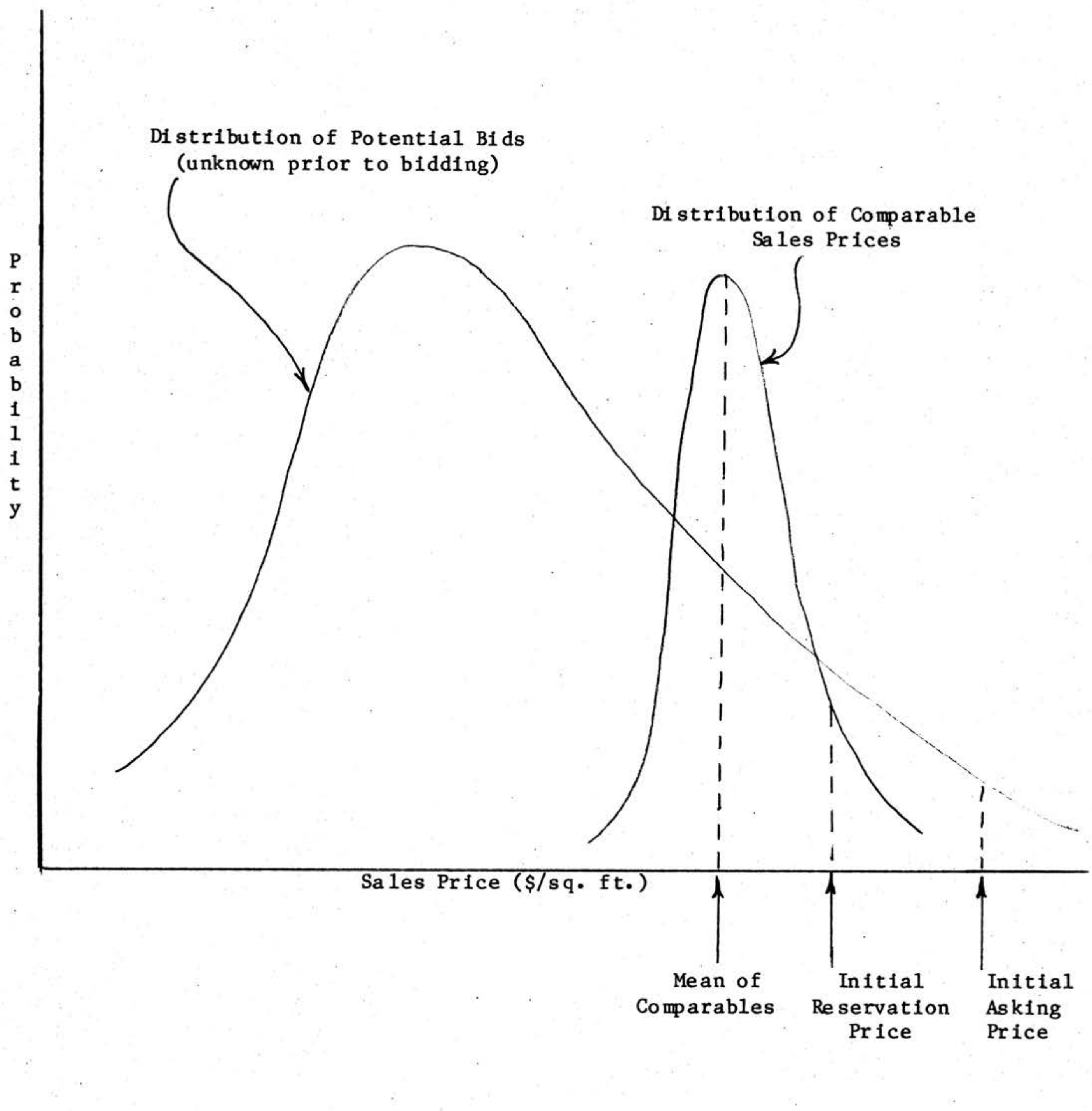
Step 3: Setting Initial Asking Price and "Reservation" Price

There is some probability that households making use of a broker for sale are influenced by the broker's advice on initial asking price and "reservation" price (i.e., price acceptable for sale). This exists because of a general lack of market information and experience by sellers. It introduces the possibility that the goals of the broker, as well as the objectives of the seller, are relevant in analyzing the sale.

Assuming no significant disparity between these sets of goals, however, the process for arriving at an asking price (and "reservation" price) is usually as follows: First a survey is undertaken of recent sales in the neighborhood. Any "unusual" financing or sales arrangements are taken into account. Sales prices are then usually adjusted to a per-square-foot, per-room, or per-bedroom basis to arrive at an expected sales price for the residence. These generally are within a range and create a probability distribution of expected sales price (Figure 1). The initial "reservation" price is most

Figure 1

Setting the Initial Asking and Reservation Prices



frequently selected as somewhat above the average of the adjusted comparable prices, assuming there are not significant outliers.[2] How much above the average is dependent upon the seller's urgency of sale, his confidence that the mean represents the "true market value" of the subject property, and his *ex ante* expectations about the distribution and frequency of potential bids. He is uncertain at first (before bids are received) about what the market will bear for his property, and he hesitates to use the average too quickly for fear he will be undervaluing his home.

The initial asking price is then selected in such a way that it is felt that the price ultimately agreed upon will be at least as high as the previously calculated average. This is where "market feel" and "intuition" play a major role in the positive model. For, most frequently, neither the seller nor his broker go through an explicit analytical exercise to arrive at this figure. The seller and broker, however, recognize certain characteristics of the transaction process.

One of the most important of these is that the frequency of shoppers and serious bidders seems to be dependent on the asking price chosen. If too high a price is selected, few shoppers will appear and few bids will be offered, possibly because shoppers feel the seller is not serious about selling and/or it is unlikely he will lower his bid sufficiently to bring the property within range. On the other hand, if too low a price is selected, the seller may not receive as high a price as he otherwise would.

In order to develop a deliberate, rational strategy for extracting a maximum expected sales price, given the opportunity costs of time on the market, the seller must develop some understanding of both (1) the relationship between asking price and the bid frequency (Figure 2), (2) the relationship between asking price and the bid level (Figure 3), and (3) the existing price

Figure 2

The Relationship Between Asking Price and the Frequency of Bids
(unknown prior to bidding)

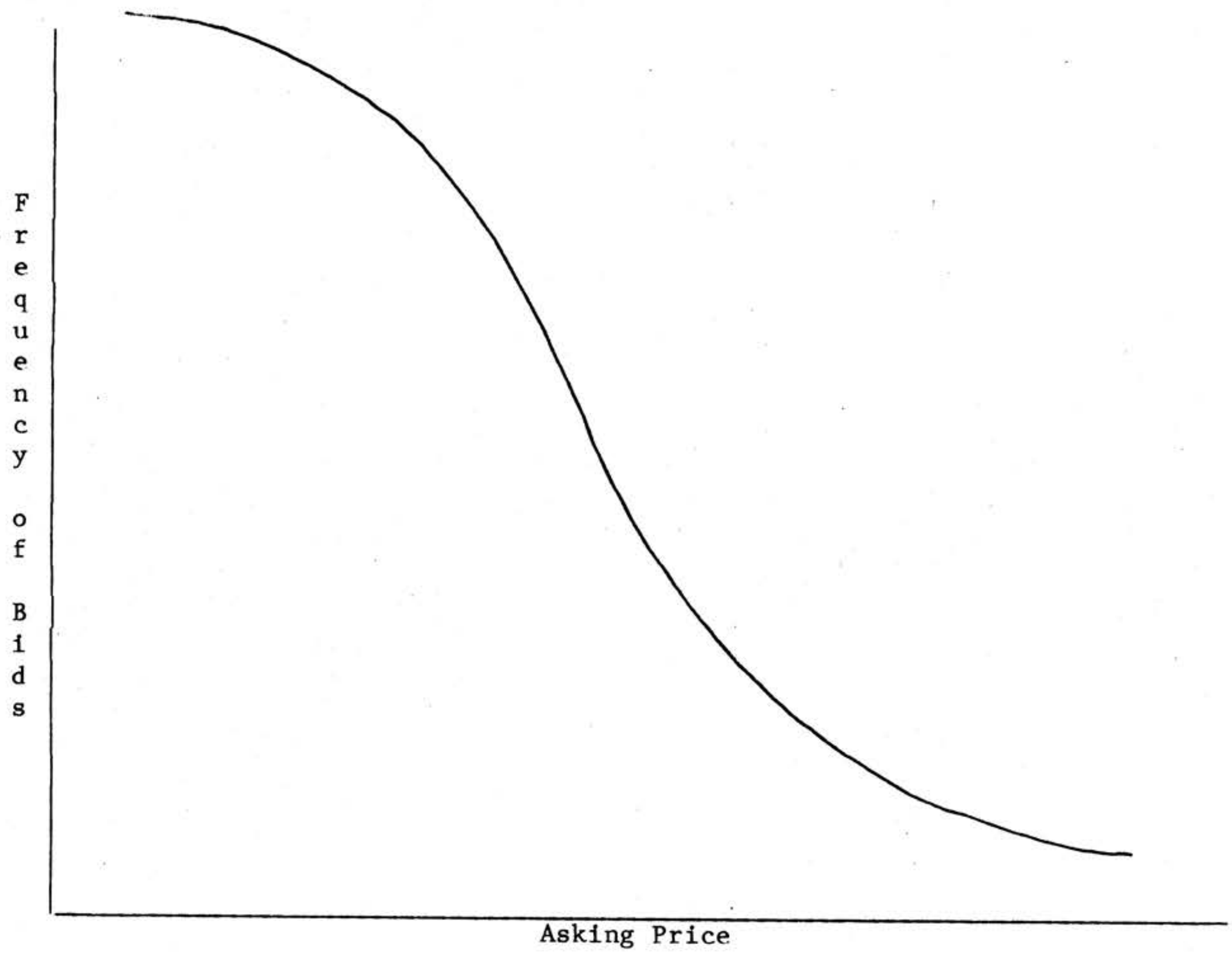
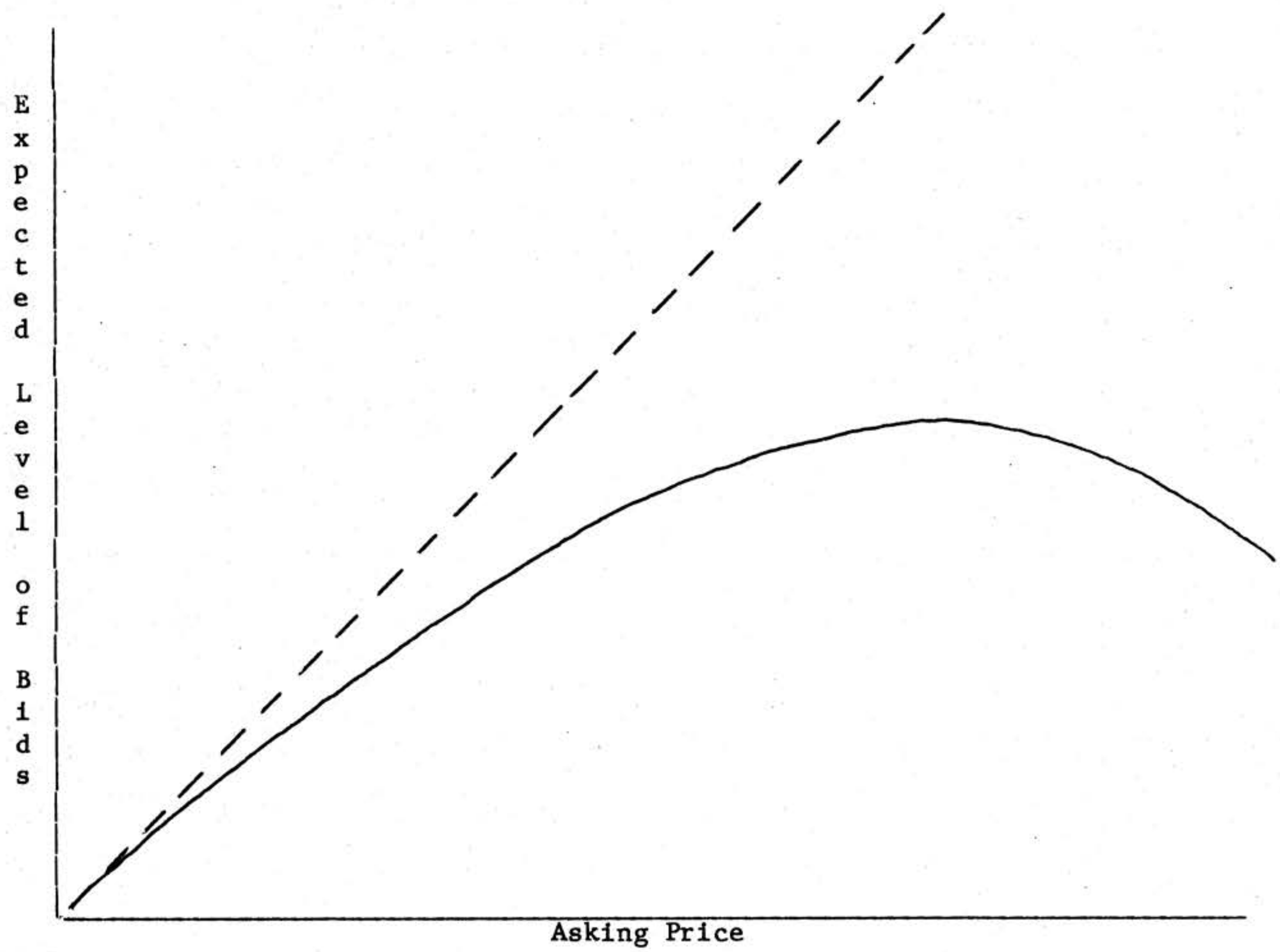


Figure 3

The Relationship Between Asking Price and the Expected Level of Bids
(unknown prior to bidding)



distribution of potential bids (Figure 1). Unfortunately none of these is known initially. Empirical studies have not been carried out to estimate these relationships, and experience with the individual property is insufficient at first to extract them. All the seller and broker have to go on is the broker's "instinct" for market behavior and possibly a developed rough feel for the relationships as bids are received over time.

An informal analysis of MLS sales has determined that sales prices across most price categories tend to be about 90 percent of initial asking price, indicating that sellers average initally asking approximately 10 percent more than they expect to get in the end. If they expect to get the average of the adjusted comparable prices, then they would ask initially 10 percent more than this average. Of course, a number of other variables seem to influence this initial-asking-price premium. A seller in a hurry to sell (with a high opportunity cost to continued time on the market) would post a lower asking price; on the other hand, one with a leisurely attitude about sale would post a higher asking price. The asking price level is also dependent on the bargaining skill of the seller and his available array of non-price terms for negotiation.

Step 4: Setting Initial Acceptable Terms of Sale

The sales price is not the only term of sale in many cases, especially today. Special terms of financing in the form of seller or institutional second mortgages, blended-rate notes, assumptions, or buy-downs often play a major role in determining the ultimate outcome of a sale. Less frequent, but sometimes equally important, are such non-price terms as agreements to provide certain maintenance or improvements prior to sale, lease-purchase arrangements, installment sales, deferred closings, or inclusion of certain personal property in the sale. Each of these, of course, has an imputed value to each potential purchaser which is reflected in the offer price. Some purchasers may be highly sensitive to special financing arrangements (in fact they may be unable to qualify without them); others may be relatively insensitive to them, preferring instead to minimize the offer price. At the same time, some sellers may be unwilling or unable to make "out of the ordinary" concessions (many today are still unwilling to carry the note), while others, usually those with greater urgency of sale or flexibility, would prefer to provide these non-price concessions rather than lowering their price.

Most sellers initially do not offer unusual concessions beyond possibly the "special financing" provided through a blended rate assumption. Yet they may initially have a set of "reservation terms," including willingness to provide financing, depending on their urgency to sell and ability and willingness to handle more flexible arrangements. Most frequently they "cross this bridge when they come to it," deciding upon receipt of a bid whether to include such non-price terms in a negotiation. This is covered later.

Step 5: Adjustment of Asking Price and "Reservation" Price

As shoppers pass through and bids are received, the seller and broker receive information about the relationship between frequency and level of bids and asking price and the distribution of potential bids. They make use of this information, given their costs of continuing to hold the property on the market, to set a new asking price and reservation price. In most circumstances these are lower than previously, since the costs of continued holding often increase over time. However, it is possible that an upward revision in expectations of bid levels or frequency could cause an upward shift in both the asking price and reservation price.

How frequently and how much are the asking and reservation price shifted? Essentially the asking price is shifted relatively infrequently (every few months or so) to provide "market exposure." One characteristic of the market is that, upon initial listing, a whole host of shoppers initially view the property. These are a sample of all the shoppers on the market at that time. After a couple of weeks to a month, however, the volume falls off. Only a sample of the new shoppers coming on the market then views the property. This fact implies that "market exposure" might be a relatively short period of time initially and longer later. However, the cost of continued holding the property is low initially, but increases later. These two factors offset each other, and it is not clear whether the frequency or level of asking price is adjusted more at first or later. Certainly, after the property has been on the market for some time, the holding cost effect dominates, and as time goes on the frequency and level of adjustment in asking price increases.

The above is also true for the reservation price in general, although the seller's reservation price probably adjusts more frequently than the asking price. One would not expect to find too great a disparity between the asking price and reservation price, especially if the asking price influences bid levels and frequency. Raising the asking price but lowering the reservation price might lower the frequency of serious bids and not significantly raise the level of bids, thus lowering the expected sales price. On the other hand, lowering the asking price but raising the reservation price could increase the frequency of bids, but also lower the general level of bids, thus also lowering the expected sales price.

Step 6: Adjustment of Terms of Sale

The non-price asking and reservation terms of sale are also adjusted over time in a manner similar to that above. In the event that the seller sees, through bids received, that one particular non-price dimension, say owner-financing, seems to be particularly important in affecting the frequency of bids and willingness to consider the asking price, he may consider including it in the set of terms offered and certainly in those considered acceptable in the ultimate transaction. This again, of course, depends upon his flexibility, his urgency to sell, and the perceived cost of offering the term adjustment. Again, he would probably adjust the set of asking terms less frequently to allow market exposure, but would be expected to continuously adjust the set of reservation terms.

Step 7: Negotiation

The negotiation phase of the transactions process is extremenly important but is often overlooked. Upon the receipt of each bid, the seller may enter into a bargaining process with the bidder (presumably whether or not the bid is below the reservation price). The elements in bargaining include both the sales price and the non-price terms. The level of a counter-bid is important in that if it is too high, the seller runs the risk of losing the potential seller (which he may be prepared to do, depending on his opportunity costs and his expectations for the future). If it is too low, he consummates the sale, but receives less than he would have otherwise. The seller and buyer both are presumably more willing to "give in" on those dimensions which are relatively less costly to them. They are also willing to be more generous on those dimensions which are further above (below) the reservation price or term level.

A seller might find it advantageous to "hold back" certain dimensions of acceptable sales terms from the initial set of asking terms and even from the set of offered terms early in the bargaining process. He can offer for negotiation first that dimension which has the least marginal value to him and that which he thinks has the greatest marginal value for the bidder, or he may attempt to "mislead" the bidder into thinking that one dimension is of great importance to him when it is not (or vice versa) and then offering it later as a "concession." Presumably, if all dimensions of the bidder's ultimate offer are above the seller's final reservation price and terms levels, the sale will be comsummated. If all are below, the sales will not be consummated. If some are above and some below, the seller must balance the surplus of those above against the deficit of those below to arrive at a decision whether or not to accept the offer.

There is a substantial literature on bargaining theory which is relevant here (for example, see von Neumann and Morgenstern [1953], Luce and Raiffa [1957], Owen [1968], Stahl [1972], Harsanyi [1976], Case [1979]). We will discuss this in more detail in Section III. However, in practice the sophistication applied to negotiating behavior is substantially lower than that espoused as a normative model in the literature.

The broker is often instrumental in the bargaining process and has a stake in consummating the sale. Thus, acting as the intermediary, he continuously counsels the principals toward a middle ground. He is the representative of the seller but does not have a high monetary stake in negotiating toward a high sales price, since his marginal return from doing so is only a small percentage of the increased sales price, while his potential loss is high, in view of competition among brokers for the sale.[3]

Step 8: Acceptance of Offer

Of course, the cost of not accepting an offer is both the time (and associated costs) involved with waiting for additional bids and the possibility that an offer so high will not come again. That has to be weighed against the more certain benefits accrued from the sale.

This has been essentially a descriptive analysis of the way residential properties tend to be sold (although to some extent it describes all real estate markets). The analysis undertaken by the seller and broker to develop an appropriate strategy to maximize return is seen to be relatively crude, in part because the empirical information necessary to make more sophisticated decisions does not exist, in part because neither has the analytic expertise nor the time to make such an analysis, in part because there is so much uncertainty about the future that even information about current conditions could change tomorrow.

It is the contention of this study that this descriptive model can be analytically exposited to allow more precise examination of the influence of the various elements which affect a sale. Furthermore, in certain applications, the descriptive model can be extended to a normative model which can improve sales behavior through the development of explicit optimizing methodologies. Toward this end, in the next section we shall review existing analytic models of the real estate, and related, transaction processes and compare their elements to those of the descriptive model developed here. The final section will develop a model of seller behavior which improves upon existing models and most closely matches (and even improves upon) actual behavior.

III. Existing Models of the Real Estate Transactions Process

There are clearly a number of characteristics of the real estate transactions process as described above which serve as a guide to the proper nature of any formal modeling effort. First, the process is not an auction but represents a sequential bid process without recall. Second, it exists in a world with imperfect (or no) information about the distribution of potential bids or the relationships between asking price and bid frequencies and levels. Third, it consists not only of a "price" transaction, but also a "nonprice" transaction in the form of the non-price terms of the contract. Finally, bargaining becomes a major element of the process once the offer bid and reservation price are within range of each other.

There have been a number of modeling efforts which possess to a greater or lesser degree these characteristics. They are all part of a general class of "optimal stopping" models, so-called because they describe optimal behavior with respect to the acceptance or rejection of bids received over time. In the first part of this section we shall review existing search models in the real estate/urban land economics tradition and compare them to the descriptive model above. Additional contributions from the job search and economics of uncertainty literature will be discussed in the second part of this section. Each model will be discussed in detail and compared along the following dimensions:

1. Whether it models seller, buyer, broker, or market (equilibrium) behavior.
2. Whether the distribution of bids is known, unknown, or learned.
3. The nature of the relationship between level and frequency of bids and asking price.
4. Whether the unit of measurement is money or utility.
5. Whether or not non-price terms are considered.
6. Whether or not bargaining behavior is considered.
7. The manner in which costs of search are included.
8. Whether or not future bids are discounted.
9. Whether the time horizon is finite or infinite.

These results are summarized in Table 1 and compared to the characteristics of the descriptive model of the transactions process developed in Section II.

Table 1

Models of Search Behavior Applied to the Real Estate Market

Characteristic	Descriptive Model of Transactions Process	Previous Models of Real Estate Market Behavior: Simon (1957)	Stull (1978a)	Stull (1978b)	Courant (1978)	Yinger (1978,1981)	Miller and Rice (1978,1979)
Behavior Modeled	Seller	Seller	Seller	Seller (Landlord)	Buyer (w/broker)	Broker	Buyer and Seller/ Equilibrium
Distribution of Bids	Learned	Known	Unknown[a]	Known	Known[b]	Known[c]	Known[d]
Distribution of Bids Dependent on Asking Price?	Learned	No	No	No	No	Yes/Known[c]	No
Frequency of Bids Dependent on Asking Price?	Learned	No	No	Yes/Known	No	Yes/Known[c]	No
Unit of Measurement (Utility or Money?)	Utility	Money	Money	Money	Both (considered identical)	Money	Money
Consideration of Non-Price Terms?	Yes	No	No	No	No	No	No
Bargaining?	Yes	None	None	None	None	None	None
Costs of Search?	Yes	None	None	None	Yes	Yes[e]	Yes[f]
Discounting?	Yes	Required for convergence[g]	No	No[h]	No	No	No
Time Horizon	Infinite	Infinite/Finite	Finite	Finite[i]	Infinite	Infinite	Infinite

[a] Only range known.
[b] Known distribution of housing units as a function of characteristics. Known distribution of utilities derived from housing units.
[c] Rather than bids, considers contacts with buyers/sellers and probabilities of consummating a sale.
[d] Buyers (Sellers) reservation price assumed to increase (decrease) linearly over time.
[e] Plus transaction costs on consummation of a sale.
[f] Increase over time. Limited recall permitted to buyer.
[g] Or a downard shift in the offer distribution. Alternatively a finite time horizon may be set.
[h] However, time t to "fallback rent" determined by discounting expected returns.
[i] As determined by selection of "fallback rent" R.

Models of Real Estate Market/Urban Land Economic Behavior

Herbert Simon, in his seminal text Models of Man (1957), was the first to apply optimal stopping considerations to a model of the real estate transactions process. His intent was

> ... to replace the global rationality of economic man with a kind of rational behavior that is compatible with the access to information and the computational capabilities that are actually possessed by organisms, including man, in the kinds of environments in which such organisms exist."4

He first considers the highly idealized situation of a seller of a home who has complete information about the distribution of bids each time period, is unlimited in search time, and faces no costs of search or information gathering. His objective is to maximize the expected value of the sales price. Simon shows a rational reservation or "acceptance" price, which would optimize seller behavior, would come about only if the probability distribution of offers shifts downward over time "with sufficient rapidity." This condition is made necessary since the model neglects the possibility of a cost element to continued search or a finite time horizon. Simon hypothesizes this downward shift could occur under expectations of falling prices or by discounting future prices to the present (which implies an opportunity cost of capital).

An alternative model is then proposed which limits the search period to one in which the probability of sale is unity after a "sufficiently long" period. The reservation price at this time is set such that sale is insured. Under such conditions, one can work backward using dynamic programming techniques to determine the reservation price trend over time.

Simon acknowledges both of these models to be only rough approximations to reality due to the "perfect knowledge" requirement and contends that in the real world a seller must make approximations to avoid using the information he doesn't have:

> First, he will probably limit the planning horizon by assuming a price at which he can certainly sell and will be willing to sell in the nth time period. Second, he will set his initial acceptance price quite high, watch the distribution of offers he receives, and gradually and approximately adjust his acceptance price downward or upward until he receives an offer he accepts -- without ever making probability calculations."[5]

He does not formally develop such a model.[6]

Stull (1978a) attempts to address the issue of rationality under imperfect information by moving to the other extreme and assuming minimal information on the part of the seller. He argues that it is unrealistic to expect the seller to have even a notion (developed over time) of the distribution of potential bids. Rather he is assumed to know with certainty only that the potential offers will be within a certain range. He then follows a decision strategy with respect to bid acceptance/rejection which maximizes the probability of choosing the highest bidder (rather than maximizing the expected price, as in Simon's model). He is subject to no costs of continued search, but is assumed to have a finite time horizon for the sale, imposed by a limitation on the (known) number of bids. Thus, the solution still has the reservation price property, as in the Simon model. The number of bids to be accepted can be obtained by choosing the largest value of N subject to the condition that the probability of waiting more than t^* periods, q^*, to get N serious offers is appropriately small.

Stull compares his results to a non-optimizing rule-of-thumb strategy of accepting the first serious offer (i.e., within the acceptable range) and to the basic Simon model, an expected-price-maximization strategy with a known uniform distribution of bids within the price range. He finds the first strategy to yield a shorter waiting time to sale but a lower probability of receiving the highest bid except when N is small. Thus only when "time is of the essence" is such a rule-of-thumb appropriate. The Simon model, however, yields (for N>1) a higher expected price and shorter expected waiting time. This is expected, as the seller is using the additional known distribution to improve his performance. Stull does not consider the performance of the Simon model when the distribution of bids is different from that assumed by the seller.

In a second model, Stull (1978b) investigates the asking rent behavior of a landlord who has complete knowledge of the "rental probability" function (the probability a unit will be rented for a given asking rent), which is equivalent to assuming he has complete knowledge of the underlying acceptance rent distribution. He limits his time horizon by exogeneously setting a "fallback rent" R_0 (thought of as the landlord's perception of what the unit is "worth" on the current market) which he asks in all subsequent periods if the unit is not rented by period t. t is considered to be given at first, but is later considered to be endogenously determined by that asking rent strategy which maximizes the landlord's present discounted value of expected returns. Thus it depends upon the landlord's time preference. One major departure of this model from previous models is the assumption that the frequency of bids is dependent on the asking rent, a relationship which is assumed to be completely known.

The landlord's objective is to set his reservation rent (which is considered to be identical to his asking rent) such that he maximizes his expected (undiscounted) return in each period. Again, as in previous models, there are no costs to continued search. The reservation price property is established by virtue of the selection of the fallback rent level and the time period t in which it becomes effective. Stull shows his assumptions result in the standard falling reservation rent trend over time and a higher expected return (but longer expected waiting time) for a higher t (i.e., a longer waiting period to the "fallback rent"). He advocates an empirical test of the assumptions and implications of his model.

Courant (1978) employs a model of buyer search (with or without broker "guidance") in an urban housing market to investigate price dynamics under conditions of racial discrimination. It differs from the above models in that it permits recall, in a sense, since it assumes enough houses are on the market at any given time that one does not have to wait for additional houses to appear to receive an acceptable price offer. Furthermore, if one house is rejected, it can again be "called up" later. It also differs from previous models in that it considers an explicit, constant cost to continued search. Buyers are assumed to be attempting to maximize their expected utilities (rather than sellers maximizing expected monetary profits), although utility and money are assumed to be equivalent (i.e., the marginal utility of money is assumed to be constant). Buyers are also assumed to have perfect knowledge about the distribution of utilities derived from purchasing houses in the market. These distributions, of course, vary from buyer to buyer, depending on their different sets of tastes and preferences.

A buyer is assumed to continue his search among the available alternatives until the expected gain in utility derived from further search will just equal the loss in utility associated with the cost of additional search. There is not an exogenous limit placed on the period of search (or equivalently on the "fallback" utility level), nor is one needed, since the existence of a search cost causes the solution to converge. The solution is a familiar one: the buyer has a "reservation utility" level which falls over time. Furthermore, people stop searching short of their "dream house," even if that dream is defined within a current budget constraint.

Rather than describing buyer or seller behavior, Yinger (1978, 1981) models the behavior of brokers searching for buyers and listings. Brokers are assumed to purchase units of search for buyers and sellers at a cost. The amount of search a broker undertakes affects the probability that any given buyer or seller will come to that broker's office. The relationship between search and this probability is assumed known. It is assumed under competition this relationship is the same for all brokers. Furthermore brokers are assumed to have complete power to set asking prices, which they adjust over time as they receive information about demand in the form of offer prices. Brokers receive payment of a commission for their services, which is the same for all brokers under competition. They also pay transaction costs whenever a sale is consummated. The objective of all brokers is to maximize their expected net income.

At the same time, sellers on the market are sensitive to the rate of commission charged by brokers and reduce their use of broker services as the commission level increases. The relationship between the commission level and the use of a broker is assumed known. Buyers, the third set of actors in the housing market, are responsive to the average market price of housing V, and the likelihood of their buying a house is some (known) decreasing function of V. This function is analogous to the demand curve facing a single seller; the greater the competition in the housing market and the lower buyers' search costs, the more responsive is this function to a change in V. Buyers are assumed to search for housing until they find a house they are willing to buy or until the expected gain from search is less than the cost.

The solution of the model determines (1) the commission level c, (2) the average sale price of housing V (the optimal adjustment of asking price over time is not determined), (3) the optimal number of units of search purchased for buyers S_B, and (4) the optimal number of units of search purchased for listings S_L. Each broker effectively increases search until the expected marginal value product is less than the cost.

Unlike the above models, the Yinger model does not determine a reservation price trend over time. It also assumes all relationships (e.g., between contacts and units of search, between contacts and the commission level, and between the probability of consummating a sale and the average market price of housing) are known. Thus the optimization problem is not dealing with uncertainty or learned relationships and distributions over time.

A final search model in the real estate market literature developed by Miller and Rice (1978,1979) is the only model to consider market equilibrium (i.e., the behavior of both buyers and sellers simultaneously) under conditions of price uncertainty. The objective function of sellers of real estate is to maximize their expected net revenue; that of buyers to minimize their expected net cost. In each time period a buyer (seller) chooses a sample of sellers (buyers) at random (the size of the sample is a given function of the search cost). He then determines which of these has the lowest (highest) reservation price and compares this to his own reservation price. A transaction occurs if the buyer's (seller's) price is not exceeded by (greater than) the lowest seller's (highest buyer's) price in the sample of that period. Otherwise search continues. Each buyer and seller has a cost to continued search which is assumed to increase over time (rather than remain constant as in the previous models). The buyer's and seller's search cost parameters are determined randomly under the assumption they are generated by a uniform distribution. Since costs of continued search are considered, it is not necessary to impose a finite time horizon on buyers or sellers.

The distribution of potential bids is assumed to be known by sellers as is the distribution of available reservation prices by buyers. Indeed, these distributions are assumed to be uniform. Reservation price and bid trends are imposed exogenously and are assumed to be linearly declining or increasing, in contrast to other models in which these are determined endogenously. Simulations using this model estimate transaction prices and time-to-sale under a number of assumptions about market parameters. Results confirm that price dispersion is an expected result of uncertainty in the market and furthermore that price dispersion increases in markets of relatively high transaction costs.

A Critique

The literature discussed above is seen to vary greatly in its adherence to the characteristics of the descriptive search process developed in Section II. No single model represents fully all dimensions considered important as an adequate representation of reality.

Consideration of transactions or search costs is neglected completely in Simon and Stull, while Yinger and Courant assume constant search costs over time and Miller-Rice assume increasing search costs. The lack of consideration of search costs forces Simon and Stull to impose exogenously a finite time horizon on the search period (or alternatively, in Simon's case, require discounting of future receipts or a downward shift in the expected bid distribution over time).

Assumptions about knowledge of the distribution of potential bids also vary greatly. Most of the models, including Simon, Stull (1978b), Courant, Yinger, and Miller-Rice assume it is completely known, which reduces the problem of optimization to a relatively straightforward one. Stull (1978a), on the other hand, takes the other extreme and assumes a completely unknown distribution (with the exception of knowing the range of bids). None of the models incorporate an assumption that a seller (or market participant) can improve upon his knowledge of the bid distribution as bids are received over time, although both Simon and Yinger (1978) acknowledge this "learning" to be an important part of the transactions process. Stull (1978a) rejects the learning hypothesis because he assumes it requires one to have unrealistic a priori knowledge about the functional form of the distribution, with the learning only allowing improved estimates of the parameters.

With the exception of Stull (1978b) and Yinger, none of the existing models acknowledge the possibility that both the level (or distribution) and frequency of bids could be affected by the asking price. Stull (1978b) assumes a relationship between the asking rent level and the likelihood of renting, but this relationship is assumed known a priori. Similarly, Yinger assumes these relationships exist but that all such relationships are fully known at the outset. Finally, none of the real estate search models consider bargaining to be an element in the transactions process. Only Stull (1978a) even mentions such a possibility.

Thus, in order to improve our ability to portray analytically the seller's search process, it will be necessary not only to extract individual elements of the various existing models applied to the real estate market, described above, but also to search more broadly for promising contributions in the existing optimal stopping literature, especially the literature on job search and the economics of uncertainty. The following discussion reports upon several such promising contributions.

Models of Job Search and the Economics of Uncertainty

In the first part of this section we saw that existing search models of the real estate transactions process are inadequate in several dimensions for adequately describing the process. This is particularly true for four areas in which none of the existing models provide any considerations: (1) knowledge of the distribution of potential bids, (2) the nature of the relationship between the level (or distribution) and frequency of bids and the asking price,(3) the existence of both price and non-price terms to the transaction, and (4) the existence of bargaining as an important element in the determination of the ultimate terms of sale.[7] The optimal stopping and bargaining litterature existing outside of the real estate framework, in particular the litterature on job search and the economics of uncertainty, will be surveyed in this part to see what contributions they can provide in these four areas.

a) Knowledge of the Distribution of Potential Bids. There have been several efforts designed to consider the case in which the seller (or job searcher) does not know the distribution of bids (or wage offers) initially but in which each offer provides information which the seller uses to update his prior in a Bayesian fashion. The declining reservation price characteristic has been shown to hold in certain of these cases but not in others. DeGroot (1968), Obregon (1967), McCall (1970), and Lippman and McCall (1976) consider the cases in which the form of the bid distribution is known, but one or more parameters are known only up to a prior probability distribution. DeGroot completely solves the problem when the distribution is known to be normal with a normal variance and unknown precision. Obregon considers the case in which the wage distribution is exponential with unknown parameter, λ, on which a gamma prior distribution is placed. McCall and Lippman and McCall consider the more general case in which the distribution is a known general distribution, the mean and higher moments are updated in Bayesian fashion, and an accept/reject decision is made after a bid is received. In these cases the reservation property is shown to hold. In a similar case considered by

Lippman and McCall in which a decision is made prior to updating, the reservation price property may not hold.

Of course, the problem with these models, as observed by Stull (1978a), is that they require a priori knowledge of the distributional form, which is more knowledge than sellers can usually be expected to possess. Another more promising model of adaptive search, which is less restrictive in this respect, is by Rothschild (1974). Rothschild considers the case in which the prior distribution is a Dirichlet.[8] The general nature of the Dirichlet distribution permits an approximation (in the limit) to any probability distribution of prices. Thus, this case is more general than at first perceived and does not possess the restrictive assumptions of the Degroot and Obregon models. Furthermore, the adaptive expectations process associated with the Rothschild model approximates what actually seems to go on in our description of the transactions process. The seller has a reasonable idea of the range in which he will receive bids, but he is not always totally ignorant of the distribution of the bids as in the Stull (1978a) case. Over time he develops a "feel" for this distribution without any perception of the functional form it is approximating.

Rothschild shows in his solution that the reservation price property holds in the Dirichlet case. Furthermore, when the searcher's prior is Dirichlet, an increase in the dispersion of the expected offer distribution increases the reservation price, as in the case of a known distribution. Thus, he concludes that "optimal search rules from unknown distributions have the same qualitative properties as search rules from known distributions." The Rothschild model holds great promise for application in the real estate case; in Section IV we will explore its potential further.

b) The Relationship Between the Level (or Distribution) and Frequency of Bids and the Asking Price. These relationships, like the bid distribution above, are based upon "learning" and are unknown at the outset. Thus we would expect analytic treatment of this situation to be similar to the "learned" bid distribution case. There are few treatments of this situation in the existing optimal stopping/job search literature. Salop (1973) develops a model in which the searcher has some influence over the distribution of bids through systematic search (using prior information) rather than random search or receipt of bids. This creates a declining reservation price over time as the searcher moves down his queue of bids. However, this case has nothing to do with influence over the distribution of bids caused by asking price behavior.

Lippman and McCall discuss the case in which the number of bids received per period is not one but is rather a random variable (again unrelated, however, to asking price). They consider two cases, that in which the number of offers received is (i) at most one, and (ii) has expected value one. In the first case they found the intuitive result that, as the probability of receiving a job offer in any period q decreases, the reservation price decreases. Hence the less frequently that offers are made the greater, ceteris paribus, the likelihood of their acceptance.

In the second case, they found it was preferable to have exactly one offer per day, rather than a random number with a mean of one per day. Furthermore, the minimal acceptable price is lower when there are a random number of offers. This is because, with the possibility of more than one acceptable offer arriving on the same day, the fact that the expected cost per observation is still c and the fact that the seller can utilize but one acceptable offer mean that the effective cost per utilized acceptable offer has increased.

Note that in both the above situations, the frequency (or level) of bids is still unaffected by the asking price; however they do affect the reservation price. In such a case, asking price behavior is totally independent of the ultimate outcome of the transaction. This situation is unrealistic for our purposes, since it is well known that asking price strategies can affect the ultimate sales price in real estate market transactions. Thus, this is fertile ground for future research efforts.

c) The Existence of Both Price and Non-Price Terms to the Transaction. To the extent that the various non-price terms of a transaction can be translated to monetary (or additive utility) terms, there is little problem with conceptually extending single-term (price) search models to the multi-term case. The objective function then collapses to maximizing an equivalent adjusted expected sales price (or utility).[9] The ultimate set of optimal reservation terms would be at that point where their marginal cost (or utility) is equal, and a seller would constantly be comparing the expected marginal revenue (or utility) from waiting for another bid (derived from the set of all terms, not merely the sales price) with the expected marginal cost. However, practically and empirically, the task of adequately estimating the marginal costs of the various non-price terms would be a formidable one, but one which nonetheless ought to receive attention in future research efforts.

d) Bargaining as a Determinant of the Terms of Sale. Here we rely solely on the bargaining/game theory literature. The situation is one in which a counter-bid has been received by a seller which is below both the asking price and (possibly) the reservation price.[10] Ignoring the existence of non-price dimensions,[11] the objective of the seller is to set his "response price" at such a level that the ultimate transaction price will maximize his return.[12] If the seller sets his response price too low, he could potentially be giving up too much. If he sets it too high, there is a greater likelihood the potential buyer will either lower his bid or drop out of the negotiations and the sale will not be consummated. Thus, the response offers are dependent on the previous sets of offers by the opposing party. The buyer is assumed to have no knowledge of the reservation price of the seller but can eliminate certain ranges as response bids are received. Furthermore, both the seller and buyer are assumed initially to have no understanding of the relationship between their response bids and the subsequent response bids of their opponent. These relationships are learned over time as bids are made. This situation is very analogous to the previous learning situations for bid distributions in which priors are updated in a Bayesian fashion.

This bargaining situation is illustrated in Figures 4 and 5. In Figure 4, as the number of bids received or offered increases, the asking and reservation prices of the seller normally decline and the expected offer and reservation prices of buyers increase. To the left of A, no transaction is possible since the maximum reservation price of the buyer is lower than the minimum reservation price of the seller. Nonetheless the parties do not know this and would "test each other out" through bargaining. The ultimate outcome of such bargaining must be unsuccessful. At point B (and presumably beyond), on the other hand, the transaction will take place without bargaining, since the offer bid of the buyer just equals the asking price of the seller.

Between points A and B, however, bargaining is undertaken and may be successful in effecting a transaction. Between B and D it will always be successful since all price levels between the asking price and the offer bid are feasible to both parties. Nonetheless, each will attempt to extract what he can through prudent bargaining. Between C and D all price levels are feasible to the seller, but there is a possibility at higher bids that the buyer will

Figure 4

Bargaining Domains in the Transactions Process

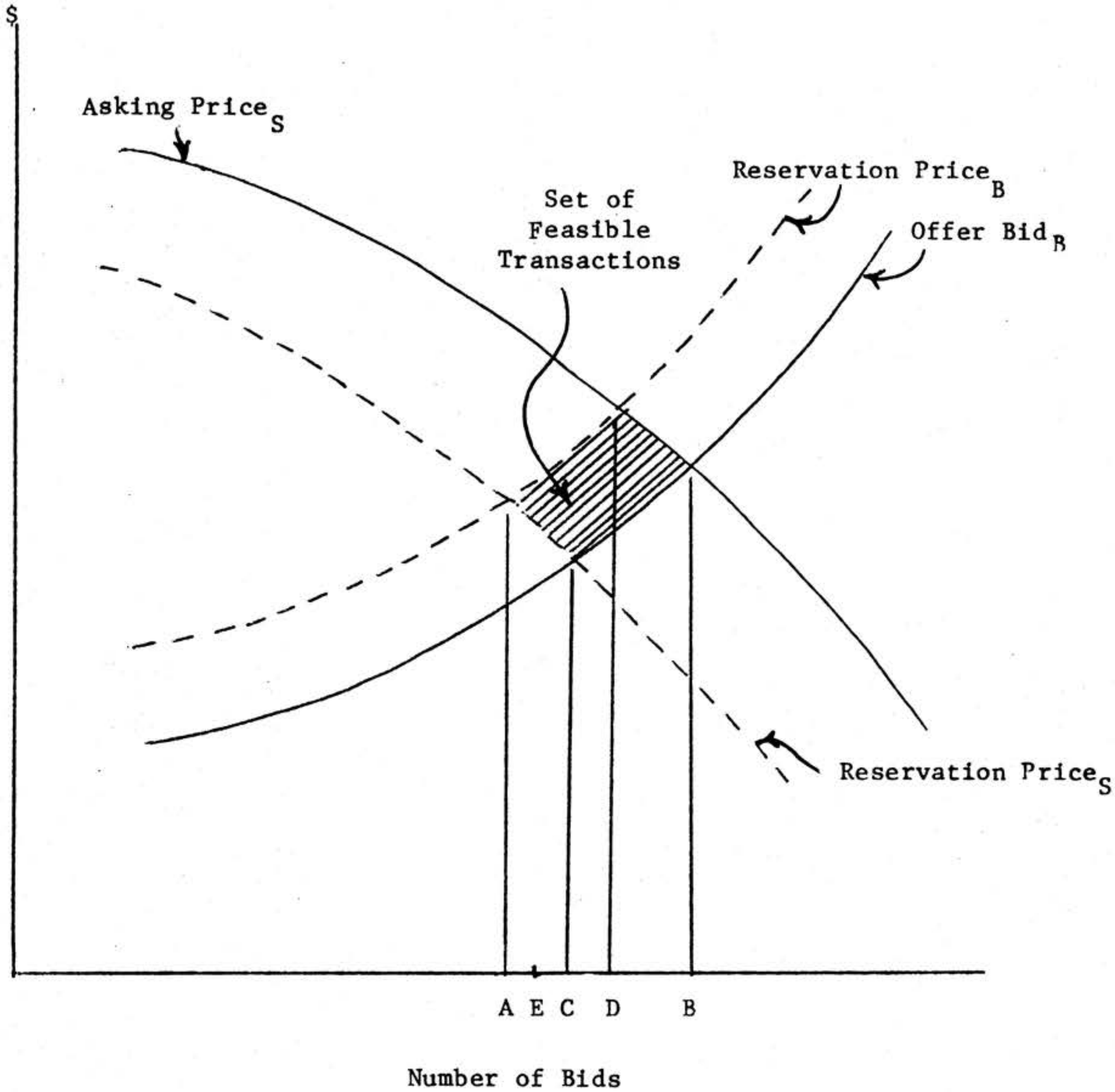
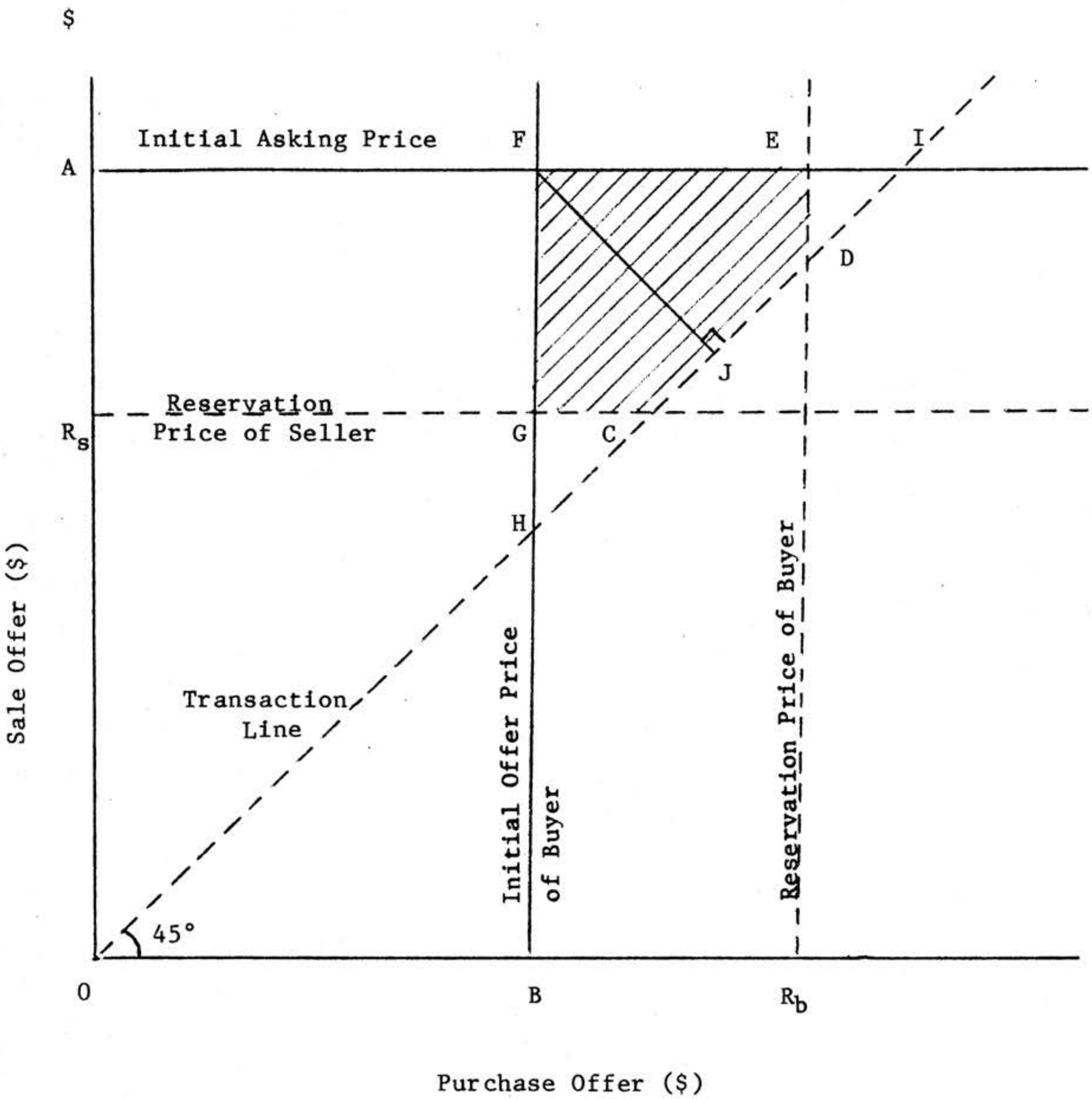


Figure 5

Bargaining Toward a Final Transaction Price



find a response bid by the seller to be infeasible, and if he feels that is his final offer, will exit the negotiations. Between A and C, there exist possibilities that both the buyer and seller will find infeasible price levels.

The bargaining situation at a point such as E is displayed in Figure 5. The asking price of the seller is at A and his reservation price at R_s. The offer price of the buyer is at B and his reservation price at R_b. The initial set of offering prices is at F, the intersection of A and B. The feasible set of subsequent pairs of bids is all bids smaller than A but larger than B bounded by the minimum reservation price of the seller R_s, the maximum reservation price of the buyer R_b, and the transaction line (where the offer price equals the acceptance price) OI. The feasible set of pairs of bids therefore is the shaded area CDEFG and the range of potential transaction prices exists between C and D.

As a rational decision maker, a buyer/seller would not let a potential transaction slip by by refusing in the end to move out of areas CGH or DEI, respectively, although a situation may develop in which a buyer/seller may misestimate his opponent's ultimate reservation price and thus lose the sale. "Bluffing" within CDEFG, on the other hand, is entirely appropriate. The question is, how does the seller optimally proceed from F to CD in such a way that he maximizes his expected profit (i.e., is as close to D as possible)?

Of course, if the marginal utility of money is constant and the same for both the buyer and seller, and is known to be so by both parties, then the logical route to CD would be a symmetric route, intersecting CD perpendicularly at J. In the event that this intersection is between C and H or D and I, both infeasible solutions, the ultimate solution would move to points C or D, respectively. This is the case in which the seller and buyer "split the difference." However, we know that marginal utilities of money are not constant nor are they the same for all individuals. Thus a "splitting the difference" situation would not necessarily be the outcome. Furthermore, neither the seller nor buyer knows what his opponent's marginal utilities are for sure. Each could thus successfully "bluff" his way to a more profitable transaction.[13]

One additional consideration in all this is that both the seller and buyer have knowledge of the "market value" estimate of the property provided through comparables or an appraiser's estimate. This provides an independent guide toward the ultimate transactions price which keeps the bluffing at a restricted level. Case (1979) presents an "export-import" game which is analogous to this situation.

The negotiating game described above is a two person zero-sum or non-zero sum game, depending on whether or not the utility of money is constant and equal for buyer and seller. There are a number of steps (counter-bids) to a final transaction. The game is a very rich and complex one because it involves "learning" behavior by both the buyer and seller, received through counter-bids, and it involves "bluffing" strategies. Additional research efforts should be applied toward better understanding of its nature.

Thus, we find that the broader optimal stopping/job search and bargaining literature has been useful to us in one case, specifically the ability to model a learned distribution of bids. However, in our search for good analogies representing the relationship between the asking price and the level (or distribution) and frequency of bids, the importance of multiple independent terms to the transaction, and the nature of the bargaining process, we are left with relatively little guidance, but several suggestions for future research efforts. In the next section, we incorporate Rothschild's learned distribution of bids in an improved search model of selling behavior and compare its behavior to more traditional models through simulations.

IV. A Proposal For an Improved Model

In this section we shall propose a modest extension to existing models of real estate seller behavior, building upon the contributions noted in the previous section. To test its relative usefulness, comparisons will be made of its performance with the performance of traditional transactions models with respect to expected sales price and time-on-market.

The notable contribution of the proposed model is that it incorporates Rothschild's (1974) suggestion of a "learned" notion of a bid distribution for the relatively general case in which the prior is Dirichlet. Extensions which incorporate consideration of the relationships between the asking price and the frequency and level (or distribution) of bids, multiple terms to the transaction, and negotiation will be left as later exercises.

Model Development

Assume that the seller behaves as if the potential bids for his property are distributed among a finite number of bid levels p_{d1}, p_{d2}, ... p_{dn}, ranging from p_{d1} at the low end to p_{dn} at the high end. The probability distribution of prices is an initially unknown multinominal distribution. This distribution is completely characterized by the vector Π whose ith element Π_i is the actual (unknown) probability that the ith price is chosen. Since Π is a probability distribution,

$$(1) \quad \Pi \; \varepsilon \; \Delta \; \{(x_1, \ldots, x_n) \; \varepsilon \; R^n \mid x_i > 0, \; \Sigma x_i = 1\}.$$

In the limit as n goes to infinity within a given price range, Π can approach any probability distribution over the range.

The seller has a Dirichlet prior distribution F( ) over Δ. He updates this prior recording to Baye's rule each time after he receives more information about the distribution of bids in the form of an offered bid. The total information he has at any point in time is contained in the number of times each bid level has been received N_i, or, equivalently, in the total number of bids received S(N) (or its inverse $\rho = \frac{1}{S(N)}$) and the proportion of time each has been observed $\mu_i = N_i/S(N) = N_i\rho$.

Each time a bid is received, this information set is updated according to the following rule: with the observation of price i, the information set (μ,ρ) becomes

$$(2) \quad h_i(\mu,\rho) = (\frac{\mu_1}{\rho + 1}, \ldots, \frac{\mu_i + \rho}{\rho + 1}, \ldots, \frac{\mu_n}{\rho + 1}, \frac{\rho}{\rho + 1})$$

The objective of the seller is to make optimal use of this information as he accumulates it to maximize his expected net gain from sale. Each bid rejected costs the seller an amount c in the form of time and other opportunity costs. We shall assume for the time being c is constant. Furthermore, we shall assume initially that one bid is received per time period and the level and frequency of bids is independent of the asking price. Thus the asking price becomes an irrelevant consideration; only the optimal reservation price is of concern. Finally, we shall assume a seller cannot recall a bid once it is rejected but replaces a bid once it is received so that it again becomes part of the unknown distribution of bids.[14]

Let $\lambda_i(\mu,\rho)$ be a probability distribution representing the searcher's expected beliefs of the likelihood that the next price observed would be p_{di}. Rothschild shows that $\lim_{\rho \to 0} \lambda_i(\mu,\rho) = \mu_i$ and that, if the prior is Dirichlet, then it is possible to parameterize experience so that $\lambda_i(\mu,\rho) = \mu_i$.

Let $V_0(\mu,\rho) = \Sigma\lambda_i(\mu,\rho)p_{di} = \Sigma\mu_i p_{di}$ be the expected sales price if an offer is accepted, given prior information (μ,ρ). Let $V_T(\mu,\rho)$ be the maximum expected net sales price received by a seller with prior experience (μ,^) who is allowed to accept at most T + 1 bids, but must accept the T + 1^{th} price offer made to him. This would be represented by

$$(3) \quad V_T(\mu,\rho) = \sum_i \lambda_i(\mu,\rho)\max\{p_{di}, V_{T-1}[h_i(\mu,\rho)] - c\},$$

which is interpretable as follows: if a searcher rejects a price offer p_{di}, he receives an updated expected sales price $V_{T-1}[h_i(\mu,\rho)]$ less the cost of rejection c. The maximum possible price for each i is thus $\max\{p_{di}, V_{T-1}[h_i(\mu,\rho)] - c\}$. This quantity is then multiplied by the (prior) probability of observing p_{di}, $\lambda_i(\mu,\rho)$, and added to yield the maximum expected net sales price given information (μ,ρ). Rothschild shows by induction that $V_T(\mu,\rho) > V_{T-1}(\mu,\rho)$ and $V_T(\mu,\rho) < p_{dn}$ for all T, so the $V_T(\mu,\rho)$ converge. Furthermore if $V(\mu,\rho) = \lim_{T\to\infty} V_T(\mu,\rho)$, then $V(\mu,p)$, satisfies

$$(4) \quad V(\mu,\rho) = \sum_i \lambda_i(\mu,\rho)\max\{p_{di}, V[h_i(\mu,\rho)] - c\},$$

which defines the optimal policy for a potentially infinite time period to sale:

If p_{di} is drawn when beliefs are (μ,ρ), accept if

$$(5) \quad p_{di} > V[h_i(\mu,\rho)] - c;$$

otherwise elicit another bid.

The solution is illustrated numerically in the following example. Assume there are two possible prices, $p_{d1} = 1$ and $p_{d2} = 2$, and our prior beliefs, based upon 10 observations, are that these are uniformly distributed (i.e., equal likelihood of occurrence; each appeared 5 times). Thus $V_0(.5, .5, .1) = .5(1) + .5(2) = 1.5$. If we were to sample once again and price p_{d1} appeared, then our updated information set becomes

(6) $h_1(\mu,\rho) = \left(\frac{.5 + .1}{.1 + 1}, \frac{.5}{.1 + 1}, \frac{.1}{.1 + 1}\right) = (.545, .455, .091)$

and the updated expected gross sales price becomes

(7) $V_0[h_1(\mu,\rho)] = .545(1) + .455(2) = 1.455$

If the cost of search = .05, then the updated expected net sales price is .545 (1) + .455 (2) −.05 = 1.405 and $\max\{p_{d1}, V_0[h_1(\mu,\rho)] - c\} = V_0[h_1(\mu,\rho)] - c = 1.405$. Doing the same in the case in which price p_2 appears results in

(8) $h_2(\mu,\rho) = (.455, .545, .091)$

(9) $V_0[h_2(\mu,\rho)] = 1.545$

(10) $V_0[h_2(\mu,\rho)] - c = 1.495$

(11) $\max\{p_{d2}, V_0[h_2,\mu,\rho)] - c\} = p_{d2} = 2$

$V_1(\mu,\rho)$ then becomes

(12) $V_1(\mu,\rho) = \lambda_1(\mu,\rho)\ \{V_0[h_1(\mu,\rho)] - c\} + \lambda_2(\mu,\rho)p_{d2}$

$= .5(1.405) +.5(2)$

$= 1.7025$

Note that $V_1(\mu,\rho) > V_0(\mu,\rho)$ but $V_1(\mu,\rho) < p_{d2}$. This exercise could be undertaken for T = 2, ..., ∞, yielding ultimately the maximum expected net sales price if one were allowed to receive an infinite number of additional bids when his current beliefs are (μ,ρ). This must converge. The seller under these conditions compares this expected net return from rejecting, $V[h_i(\mu,\rho)] - c$, to the bid received, p_{di}, and selects the maximum.

The process begins again upon the receipt of the next bid (if the first one is rejected), whereupon the priors and reservation price are updated.

Model Performance Comparisons

In this section we empirically compare the performance of the "learning" model developed above with two other search models which have been applied to the real estate market -- the Stull (1978a) model based upon an unknown (and unlearned) distribution of bids and the Simon (1957) model, based upon a known (or assumed) distribution of bids. Each is examined from the standpoint of reservation prices over time, expected sales price, and expected time-on-market. The comparisons are carried out under four assumed bid distributions from which bids are randomly selected: a standard uniform distribution, a standard right triangular distribution, a standard left triangular distribution, and a standard symmetric triangular distribution, each with $p_{min} = 0.0$ and $p_{max} = 1.0$. The Simon model assumes the actual bid distribution is always standard uniform. The Stull model, of course, makes no assumptions about the nature of the bid distribution. The prior Dirichlet distribution for the Rothschild model is assumed to approximate that of a standard uniform distribution. Of course, the prior for the Rothschild model adjusts over time as bids are received; that for the Simon model does not.

The three models were rendered as consistent as possible to make comparisons meaningful. The distribution and frequency of bids was assumed independent of asking price, hence rendering asking price an irrelevant consideration. Money was taken as the unit of measurement, and no non-price terms were considered. The possibility of bargaining was not taken into account. Discounting was not considered directly, and the costs of search were assumed to be zero, but the time horizon was assumed to be finite (10 periods).

a) Model I: The Stull Model. The Stull model, a variation on the beauty contest problem, develops a strategy which maximizes the probability of selecting the largest price, rather than maximizing the expected price. The optimal strategy is of the following form: Pass S-1 bids and select the first one thereafter which is greater than those passed. This bid is called a candidate.

The probability of getting the highest price when there are N bids and S-1 are to be passed is

$$(13)\ \eta(S,N) = \sum_{i=S}^{N} \left(\frac{1}{N}\right)\left[\frac{S-1}{i-1}\right]$$

Stull shows η is maximized for N = 10 when S*-1 = 3. Thus, the reservation price for the first three periods is p_{max}, the maximum possible bid from the unknown distribution (which is 1.0 for our simulations). It is during this time the seller samples the market. After period 3 he adjusts his reservation price to the maximum of the three bids observed previously. This, of course, would vary depending on the nature of the bid distribution and the bids actually observed. The expected value of the sample maximum for a sample of size i from a distribution, $E(p_{max_i})$, must satisfy the criterion.

$$(14)\ \text{Prob}\ (p < E[p_{max_i}]) = \frac{i}{1 + i}\ .$$

For a standard uniform distribution this is simply

$$(15)\ E(p_{max_i}) = \frac{i}{1 + i} \text{ and } E(p_{max_3}) = 0.750,$$

for a standard right triangular distribution,

$$(16)\ E(p_{max_i}) = \sqrt{\frac{i}{1 + i}} \text{ and } E(p_{max_3}) = .866,$$

for a standard left triangular distribution,

$$(17)\ E(p_{max_i}) = 1 - \sqrt{\frac{1}{1 + i}} \text{ and } E(p_{max_3}) = 0.500,$$

and finally for a standard symmetric triangular distribution,

$$(18)\ E(p_{max_i}) = 1 - \sqrt{\frac{1}{2(1+i)}} \text{ and } E(p_{max_3}) = 0.646.$$

$E(p_{max_3})$ becomes the expected reservation price for periods 4 through 9. (In period 10, of course, the reservation price must be zero since any observed price must be accepted.) Thus, although the seller has no knowledge, nor does he ever develop any knowledge, of the bid distribution, he nonetheless adjusts his strategy according to information received in the first three bids. This could therefore be considered a crude "learning" strategy. The expected reservation price trend for the Stull model is plotted in Figure 6 and listed in Table 2.

The expected sales price in the Stull model for a general distribution and N periods to required sale is

$$(19)\ E(p_N) = \sum_{i=S*}^{N-1} \frac{S*-1}{i(i+1)} \left[E(p_{max_i}) \middle| Prob(p<E(p_{max_i}) = \frac{i}{i+1}\right] + \frac{S*-1}{(N-1)} \left[E(p_{max_i}) \middle| Prob(p<E(p_{max_i}) = 0.5\right].$$

The first term in the sum is the probability of stopping at (and therefore accepting) the ith serious offer. The second is simply the expected price if accepted, which is the expected value of the sample maximum for a sample of size i from the general distribution. The last term is the same as those previous, except that the relevant expected price is the mean of the bid distribution, since the last offer must be accepted. For N = 10 and a standard uniform distribution, $E(p_{10}) = 0.725$. For the standard right triangular distribution, $E(p_{10}) = 0.846$, and for the standard left triangular distribution, $E(p_{10}) = 0.497$. Finally, for the standard symmetric triangular distribution, $E(p_{10}) = 0.645$. Note that, as expected, the expected sales price varies depending on the bid distribution, since the setting of the reservation price in periods 4-9 depends upon the sampling from the distribution in periods 1-3. Furthermore, as expected, the expected sales price is higher for a rightward-skewed distribution and vice-versa.

Figure 6

Model Comparisons

Reservation Price Trends

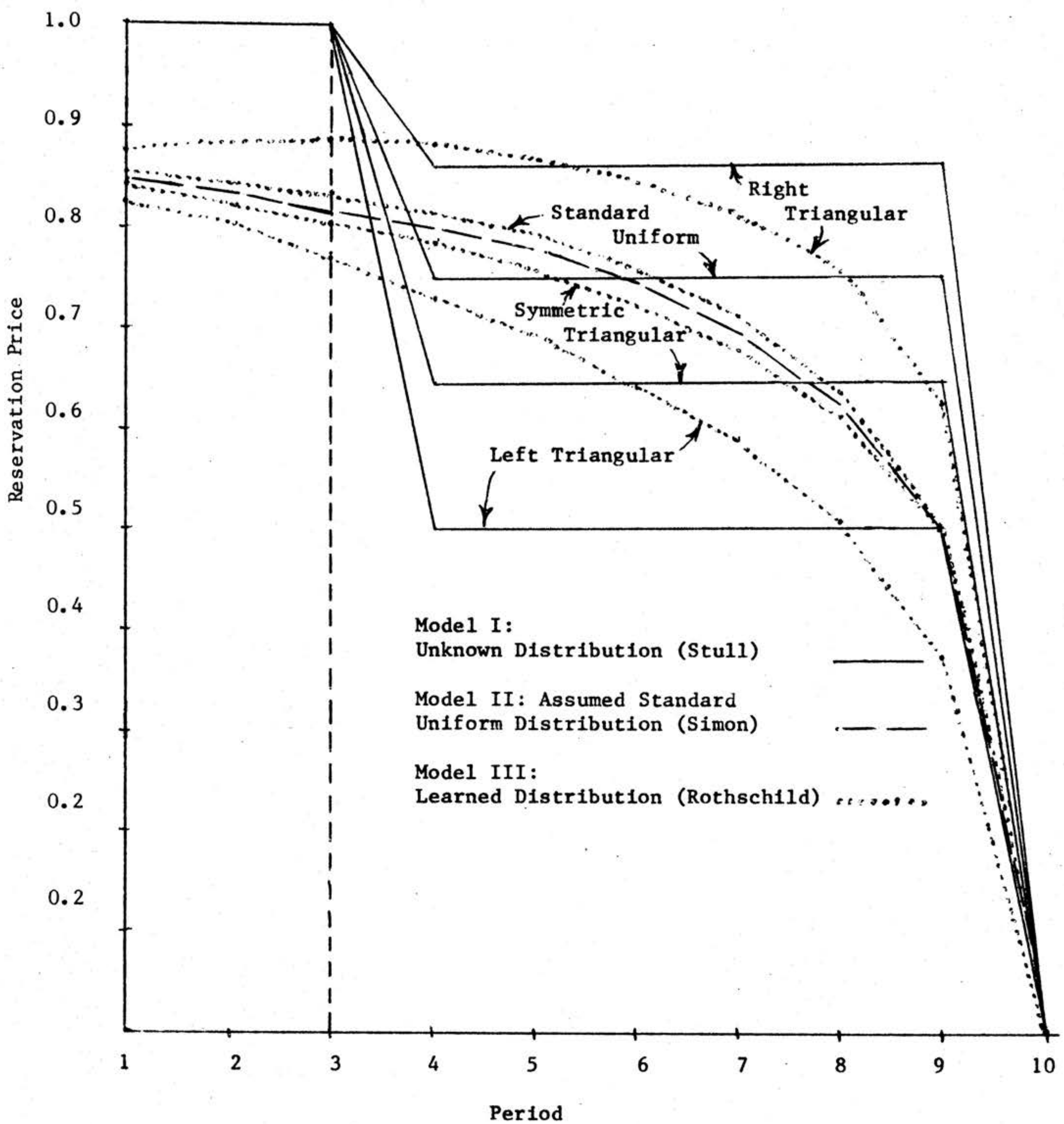


Table 2

Model Comparisons
Reservation Price Trends

	Model I: The Stull Model (Unknown Distribution) — Expected Reservation Price If Actual Bid Distribution Is:				Model II: The Simon Model (Assumed Standard Uniform Distribution)	Model III: The Rothschild Model (Learned Distribution) — Expected Reservation Price If Actual Bid Distribution Is:			
Period	Std. Uniform	Std. Right Triangular	Std. Left Triangular	Std. Symmetric Triangular		Std. Uniform	Std. Right Triangular	Std. Left Triangular	Std. Symmetric Triangular
1	1.000	1.000	1.000	1.000	0.850	0.859	0.881	0.836	0.848
2	1.000	1.000	1.000	1.000	0.836	0.847	0.886	0.803	0.827
3	1.000	1.000	1.000	1.000	0.820	0.834	0.886	0.770	0.806
4	0.750	0.866	0.500	0.646	0.800	0.817	0.880	0.735	0.783
5	0.750	0.866	0.500	0.646	0.775	0.794	0.868	0.696	0.757
6	0.750	0.866	0.500	0.646	0.742	0.761	0.846	0.648	0.724
7	0.750	0.866	0.500	0.646	0.695	0.714	0.811	0.588	0.679
8	0.750	0.866	0.500	0.646	0.625	0.638	0.752	0.506	0.612
9	0.750	0.866	0.500	0.646	0.500	0.500	0.629	0.371	0.500
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Finally, Stull shows the expected time-on-market in his model is

$$(20)\quad E(W_N) = \sum_{i = S^*}^{N-1} i(1/i)\left[\frac{S^*-1}{i-1}\right] + N\left[\frac{S^*-1}{N-1}\right] = N\,(S^*,N)+S^*-1\ .^{15}$$

For N = 10, $E(W_{10}) = 6.990$. Note that this waiting time is independent of the actual distribution of bids since it was derived without any assumption about the bid distribution. The performance of the Stull model with respect to expected sales price and expected time-on-market is summarized in Table 3.

b) Model II: The Simon Model. The Simon model is based upon full knowledge of the bid distribution. It incorporates an expected price maximization strategy, which is more common among optimal stopping models than the Stull strategy of maximizing the probability of selecting the largest price. Since the bid distribution is known (or, more correctly, assumed), the optimal strategy (i.e., reservation price over time) is invariant with the actual distribution encountered. However, model performance, of course, depends upon how well the assumed distribution approximates that actually encountered. The optimal strategy for setting the reservation price in the Simon model is as follows: Accept a bid only if it is greater than the expected return from rejecting it, but always accept the last bid. This implies that there is a direct relationship between the reservation price and the expected sales price.

Let us derive the reservation price and expected sales price for a general distribution using induction. Assume there are N = 2 periods to required sale. The optimal strategy has the following form:

> If the first offer p_a is such that
>
> $p_a > E(p_r)$, accept it. Otherwise accept p_r.

$E(P_r)$, the expected return from rejecting the bid, is simply the expected price of a single sample from the bid distribution, or simply the mean of the

Table 3

Model Comparisons

Expected Sales Price and Expected Time-on-Market
10 Periods to Required Sale

	Actual Bid Distribution			
Model	Standard Uniform	Standard Right Triangular	Standard Left Triangular	Standard Symmetric Triangular
	Expected Sales Price			
I. Unknown Distribution (Stull)	0.725	0.846	0.497	0.645
II. Assumed Standard Uniform Distribution (Simon)	0.861	0.909	0.548	0.694
III. "Learned" Distribution (Rothschild)	0.840	0.913	0.577	0.709
	Expected Time-on-Market			
I. Unknown Distribution (Stull)	6.990	6.990	6.990	6.990
II. Assumed Standard Uniform Distribution (Simon)	4.602	3.038	8.133	6.810
III. "Learned" Distribution (Rothschild)	4.865	3.805	7.917	7.472

bid distribution, $\{E(p_r)|Prob[p<E(p_r)] = 0.5\}$. The expected sales price if the seller follows this strategy is the probability of accepting the first bid times the expected value if it is accepted plus the probability of accepting the second times its expected value, or

$$(21)\ E(p_2) = \frac{1}{2}\{E(p_a)|Prob[p<E(p_a)] = 0.75\} + \frac{1}{2}\{E(p_r)|Prob\ [p<E(p_r)] = 0.5\}$$

In the case of N = 3, the seller again will accept the first offer received only if it is greater than the expected return from rejecting it. This latter expectation is simply $E(p_2)$. Thus, the optimal strategy becomes:

> If the first offer > $E(p_2)$, accept it. Otherwise go on to the second. If the second offer is > $\{E(p_r)|Prob[p<E(p_r)] = 0.5\}$, accept it. Otherwise accept the third.

The reservation price with three periods to required sale then becomes the expected sales price with two periods to required sale. Again, the expected sales price is easily calculated:

$$(22)\ E(p_3) = [1-E(p_2)][E(p_a)|Prob[p<E(p_a)] = \tfrac{1}{2}[E(p_2)+1]] + E(p_2)^2$$

This expression can now be written in general form for any arbitrary N:

$$(23)\ E(p_{N+1}) = [1-E(p_N)][E(p_a)|Prob[p<E(p_a)] = \frac{1}{2}[E(p_N)+1]] + E(p_N)^2.$$

In our simulations, we assume the seller is assuming a standard uniform bid distribution, regardless of experience with bids received. In this case, as shown in Stull,

$$(24)\ \{E(p_a) | \mathrm{Prob}[p<E(p_a)] = \frac{1}{2}\ [E(p_N)+1]\}$$
$$= \frac{1}{2}\ [E(p_N)+1],$$

and expression (23) simplifies to

$$(25)\ E(p_{N+1}) = (\frac{1}{2})[1+E(p_N)^2]$$

The reservation price with N + 1 periods to required sale is simply the expected sales price with N periods to required sale. The reservation price trend for the Simon model is graphed in Figure 6 and charted in Table 2. Note it is invariant with the actual bid distribution encountered. Furthermore, note that it declines at an increasing rate over time and is lower than the Stull model reservation price for the early bids.

The above expected sales price expression is a valid representation of the expected sales price if both the assumed and actual bid distributions are standard uniform. However, if the assumed bid distribution is standard uniform and the actual bid distribution is not, then the expected sales price deviates from expression (25). In such a case, the reservation prices if a certain bid is received remain invariant, but the probability of receiving a certain bid varies depending on the actual distribution encountered. Let us designate $p_{R_{N+1}} = E(p_N)$ to be the reservation price with N + 1 periods to required sale as determined in expression (25) above, assuming a standard uniform distribution. For N = 2,

$$(26)\ E(p_2)' = \mathrm{Prob}(p>p_{R_2})\{E(p_a)|\mathrm{Prob}[p<E(p_a)] = \frac{1}{2}\ [\mathrm{Prob}(p<p_{R_2})+1]\}$$
$$+ \mathrm{Prob}(p<p_{R_2})E(p_1)'$$

where

$$(27)\ E(p_1)'=\{E(p_1)'|\mathrm{Prob}\ (p<E(p_1)') = 0.5\}.$$

For N = 3,

$$(28)\quad E\,(p_3)' = \text{Prob}(p>p_{R_3})\{E(p_a)|\text{Prob}[p<E(p_a)] = \frac{1}{2}\;\;[\text{Prob}\;(p<p_{R_3})+1]\} + \text{Prob}(p<p_{R_3})E(p_2)'.$$

For arbitrary N, this becomes

$$(29)\quad E(p_{N+1})' = \text{Prob}(p>p_{R_{N+1}})\{E(p_a)|\text{Prob}[p<E(p_a)] = \frac{1}{2}[\text{Prob}\;(p<p_{R_{N+1}})+1]\} + \text{Prob}(p<p_{R_{N+1}})E(p_N)'.$$

Now, if in fact the actual distribution encountered is standard uniform, then

$$(30)\quad \text{Prob}(p>p_{R_{N+1}}) = 1 - E(p_N)$$

and

$$(31)\quad \{E(p_a)|\text{Prob}[p<E(p_a)] = \frac{1}{2}\;[\text{Prob}(p<p_{R_{N+1}}) + 1]\} = \frac{1}{2}\;E(p_N) + 1.$$

Thus

$$(32)\quad E(p_N)' = E(p_N),$$

and expression (29) collapses to expression (25).

For N=10 and an assumed standard uniform distribution, if a standard uniform distribution actually occurs, $E(p_{10}) = 0.861$. However, if a standard right triangular distribution is encountered, $E(p_{10}) = 0.909$, and if a standard left triangular distribution is encountered, $E\,(p_{10}) = 0.548$. Finally, if a standard symmetic triangular distribution is encountered, $E(p_{10}) = 0.694$. These results are summarized in Table 3. Note that the Simon model expected-sales-price results dominate those for the Stull model in every case, suggesting that even if a wrong guess is made about the distribution of bids, a seller may be better off assuming a standard uniform distribution and using an expected price maximization strategy than using the beauty contest strategy suggested by Stull.

Finally, let us turn to the expected waiting time, which is simply

(33) $E(W_N) = \text{Prob}(p > p_{R_N})(1) + \text{Prob}(p < p_{R_N})[E(W_{N-1}) + 1]$

For $N = 10$, and if the actual distribution encountered is standard uniform, $E(W_{10}) = 4.602$. If the distribution is standard right triangular, $E(W_{10}) = 3.038$. This is lower, as expected, since the seller would be more likely to receive an acceptable bid earlier under a rightward-skewed distribution. If the distribution is standard left triangular, however, $E(W_{10})$ increases to 8.133. And finally, if the distribution is standard symmetric triangular, $E(W_{10})$ increases to 6.810. This is expected since there are fewer "high end" bids under the standard symmetric triangular distribution than under the standard uniform distribution, rendering a longer search necessary.

Note in Table 3 that, for three of the four distributions, the expected time-on-market is shorter under the Simon Model than under the Stull model, in spite of the fact that the expected sales price is greater. This seems counterintuitive at first, but can be explained by considering the inefficiencies generated in the Stull model by automatically waiting three periods to consider a bid and then only adjusting the reservation price a single time. Only in the standard left triangular distribution case must the seller trade off a higher sales price for a longer waiting time.

c) Model III: The Rothschild Model. The Rothschild model hypothesizes behavior different from that for either the Stull or Simon models. It assumes an initial distribution based upon previous experience or comparison with comparables as does the Simon model, but updates this in a Bayesian fashion over time as bids are received. Thus it reflects learning behavior, as does the Stull model, but with more sophisticated use of the information. The objective is to maximize the expected sales price, like that of the Simon model.

For comparison purposes, we assumed the seller held an initial Dirichlet prior with equal probabilities of occurrence of each of 5 prices spaced uniformly along the interval (0,1). This approximates the standard uniform distribution assumed for the Simon model. It is assumed this prior is derived from either past market experience or observation of bids on comparable properties. One observation of each of the 5 prices p_{di} = (0, 0.25, 0.50, 0.75, 1.00) is assumed to have been made initially. As bids p_t are received from the actual bid distribution with t periods to required sale, they are mapped into the updated Dirichlet prior according to the following mapping:

$$(34)\quad \begin{aligned} 0.0 \leq p_t < 0.2 &\Rightarrow p_t \rightarrow p_{d1} = 0.00 \\ 0.2 \leq p_t < 0.4 &\Rightarrow p_t \rightarrow p_{d2} = 0.25 \\ 0.4 \leq p_t < 0.6 &\Rightarrow p_t \rightarrow p_{d3} = 0.50 \\ 0.6 \leq p_t < 0.8 &\Rightarrow p_t \rightarrow p_{d4} = 0.75 \\ 0.8 \leq p_t \leq 1.0 &\Rightarrow p_t \rightarrow p_{d5} = 1.00.^{16} \end{aligned}$$

The optimal strategy is of the following form in the case in which search costs are zero, there are N periods for search, and the seller must accept the Nth price offer:

If p_N is drawn when beliefs are (μ,ρ), accept if

$$(35)\quad p_N \geq V_{N-1}[h_i(\mu,\rho)]$$

where

$$(36)\quad V_N(\mu,\rho) = \sum_j \mu_j \max\{p_{dj}, V_{N-1}[h_j(\mu,\rho)]\},$$

and $h_i(\mu,\rho)$ is the updated information set, given that price p_N has been observed. Otherwise elicit another bid.

$V_{N-1}(\mu,\rho)$ is simply the maximum expected price obtained by a searcher with prior experience (μ,ρ) who is allowed to search at most N times but must accept the Nth price offer made to him. Working back to N=1,

$$(37)\quad V_o(\mu,\rho) = \sum_j \mu_j p_{dj}$$

is simply the expected value of the sales price upon the receipt of a single bid, given beliefs (μ,ρ).

Expression (35) defines the reservation price with N periods to required sale. After the first bid is received, if he rejects the bid, the seller adjusts his prior and faces a new problem with initial beliefs $h_i(\mu,\rho)$ and N-1 searches permitted. This process continues until the seller must accept the last bid. It should be noted that, as in the Stull model, the reservation price trend is not determined <u>a priori</u>. It varies, depending on actual bid experience, but the expected reservation price trend over time can be calculated, given the set of initial beliefs and the actual bid distribution. The expected reservation price in period N, given beliefs (μ,ρ) prior to bid receipt, $E\left(p_{R_N}/(\mu,\rho)\right)$, is simply

$$(38)\quad E\left(p_{R_N}/(\mu,\rho)\right) = \sum_j \text{Prob}(p_{N-1} \rightarrow p_{d_j})V_{N-1}[h_j(\mu,\rho)],$$

or the average of the possible reservation prices weighted by the probability of receiving each bid. The probability of bid p_{dj} being received is simply $\lambda_j(\mu,\rho) = \mu_j$ in the event that the prior is the actual distribution encountered. For our standard uniform case μ = (0.20, 0.20, 0.20, 0.20, 0.20). However, in the event the actual bid distribution does not conform to the prior, $\text{Prob}(p_{N-1} \rightarrow p_{dj}) \neq \mu_j$. In particular, given our mapping scheme from the actual bid distribution to the Dirichlet, for the standard right triangular distribution,

(39) $\text{Prob}(p_{N-1} \rightarrow p_{dj}) = (0.04, 0.12, 0.20, 0.28, 0.36);$

for the standard left triangular distribution,

(40) $\text{Prob}(p_{N-1} \rightarrow p_{dj}) = (0.36, 0.28, 0.20, 0.12, 0.04);$

and finally, for the standard symmetric triangular distribution,

(41) $\text{Prob}(p_{N-1} \rightarrow p_{dj}) = (0.08, 0.24, 0.36, 0.24, 0.08).$

The expected reservation price trend for each of the four assumed actual distributions is plotted in Figure 6 and listed in Table 2. It is assumed at the beginning of period 1 the seller holds initial beliefs $(\mu,\rho) = (0.20, 0.20, 0.20, 0.20, 0.20, 0.20, 0.20)$. Of course, any set of initial beliefs is possible. The set chosen was felt to approximate adequately the standard uniform distribution assumption of the Simon model. Essentially the calculation of the expected reservation price for each period then involves first the estimation of the set of all possible reservation prices for that period, then weighting those by the probability of occurrence of each.[17]

Note in Figure 6 that if the standard uniform distribution is actually encountered, the expected reservation price trend approximates very closely the reservation price trend in the Simon model. It is slightly higher, on the average, hence one would expect there to be a slightly greater probability of not receiving an acceptable bid in the first nine periods and being forced to accept the tenth bid. Thus the expected sales price would be expected to be somewhat lower than that for the Simon model and the expected time-on-market to be somewhat longer, but because of the close approximation to the Simon reservation price trend, model performance would be expected to be comparable (we shall see later this is, indeed, the case).

In the event the actual bid distribution encountered is not standard uniform, however, the expected reservation price trend for the Rothschild model is seen to shift upward or downward, depending on whether the distribution is skewed upward or downward respectively. In the case of the standard symmetric triangular distribution, the reservation price trend is shifted somewhat lower because fewer "high end" bids are expectd than in the standard uniform case. Because of its sensitivity to differences in sampling experience, we would probably expect the Rothschild model to perform somewhat better than the Simon model in terms of expected sales price whenever the actual bid distribution deviates significantly from the assumed bid distribution. However, the expected time-on-market comparisons would be expected to vary depending on the nature of the actual distribution encountered.[18] Again, we shall see later this is, indeed, the case.

One interesting observation is that, in the case of the standard right triangular distribution, <u>the expected reservation price trend actually increases initially</u>. This intuitively seems appropriate, as one is quickly revising upward his expectations of what he will be offered. Such a phenomenon is observed occasionally in practice. It does not contradict Rothschild's finding that $V_N(\mu,\rho) > V_{N-1}(\mu,\rho)$ since the information set is continuously updated and we are actually comparing $E\left(p_{R_N}/(\mu,\rho)\right)$ to $E\left(p_{R_{N-1}}/h(\mu,\rho)\right)$.

Let us turn now to consideration of expected sales price in the Rothschild model for each of the possible bid distributions. The expected sales price is simply the sum of the probability of accepting a given price times its expected value plus the probability of rejecting a given price times the expected price if the bid is rejected. This occurs for each of the possible outcomes of a bid, so in order to derive the expected sales price, each of these possible outcomes must be weighted by its probability of occurrence each period and summed.

The probability of acceptance of a bid depends upon the nature of our mapping into the Dirichlet distribution and the relationship of the mapped bid to the reservation price p_{Rt}. A price received p_t exists within a certain range of bids in the actual bid distribution (p_{min_i}, p_{max_i}) [see expression (34)], which is mapped in turn into a certain price p_{di} in the Dirichlet. If the reservation price is below this range, then the probability of acceptance Prob(p_t p_{Rt}) is unity. If it is above this range, then the probability of acceptance is zero. If, however, the reservation price is within this range, then Prob(p_t p_{Rt}) is between zero and one. This depends upon the range in question and the nature of the actual bid distribution encountered. These probabilities are tabulated in Table 4. Of course the probability of rejecting a given price is simply one less its probability of acceptance. For the last period (period 10 in our case) the probability of acceptance must be unity.

The expected value of the bid, given that it has been accepted, again depends upon the nature of the mapping into the Dirichlet and the actual bid distribution. If the reservation price p_{Rt} is below the range (p_{min_i}, p_{max_i}), then the expected value is simply the expected price within the range. If p_{Rt} is above the range, then the expected value does not exist, since the probability of acceptance is zero. If, however, p_{Rt} is within the range, then the expected value is simply the expected value within the range (p_{Rt}, p_{max_i}). These relationships are shown in Table 5. For the next-to-the-last period (period 9 in our case), the expected value in the event of bid rejection must be the mean of the actual bid distribution (since the seller must accept the last bid). This is 0.5 for the standard uniform distribution,

Table 4

Probability of Acceptance of a Bid by Reservation Price Range and Nature of Actual Bid Distribution

Rothschild Model

Reservation Price Range	Actual Bid Distribution: Standard Uniform	Standard Right Triangular	Standard Left Triangular	Standard Symmetric Triangular	
$p_R > p_{max}$	1.0	1.0	1.0	1.0	
$p_{min} < p_R \quad p_{max}$	$\frac{p_R - p_{min}}{p_{max} - p_{min}}$	$\frac{p_R^2 - p_{min}^2}{p_{max}^2 - p_{min}^2}$	$\frac{(p_{max}-p_R)(2-p_R-p_{max})}{(p_{max}-p_{min})(2-p_{max}-p_{min})}$	$\frac{(1-p_R)^2 - (1-p_{max})^2}{(1-p_{min})^2 - (1-p_{max})^2}$	for $p_{min} > 0.6$
				$\frac{p_{max}^2 - p_R^2}{p_{max}^2 - p_{min}^2}$	for $p_{max} \quad 0.4$
				$\frac{0.68 - 2p_R^2}{.36}$	for $p_{min} = 0.4$ and $p_R \quad 0.5$
				$\frac{2(1 - p_R)^2 - .32}{.36}$	for $p_{min} = 0.4$ and $p_R > 0.5$
$p_R \quad p_{min}$	0.0	0.0	0.0	0.0	

Note: The subscripts i and t have been dropped for simplicity of exposition.

Table 5

Expected Value of a Bid by Reservation Price Range and Nature of Actual Bid Distribution

Rothschild Model

Reservation Price Range	Actual Bid Distribution: Standard Uniform	Standard Right Triangular	Standard Left Triangular	Standard Symmetric Triangular	
$P_R > P_{max}$	$\frac{P_{max} + P_{min}}{2}$	$\sqrt{\frac{P_{max}^2 + P_{min}^2}{2}}$	$1 - \sqrt{\frac{(1-P_{min})^2 + (1-P_{max})^2}{2}}$	$1 - \sqrt{0.5[1-P_{min})^2 + (1-P_{max})^2}$	for $P_{min} > 0.6$
				$\sqrt{0.5(P_{max}^2 + P_{min}^2)}$	for $P_{max} < 0.4$
				0.5	for $P_{min} = 0.4$
$P_{min} < P_R < P_{max}$	$\frac{P_R + P_{max}}{2}$	$\sqrt{\frac{P_R^2 + P_{max}^2}{2}}$	$1 - \sqrt{\frac{(1-P_R)^2 + (1-P_{max})^2}{2}}$	$1 - \sqrt{0.5[(1-P_R^2) + (1-P_{max}^2)}$	for $P_{min} > 0.6$
				$\sqrt{0.5(P_{max}^2 + P_R^2)}$	for $P_{max} < 0.4$
				$1 - \sqrt{0.5(.34-P_R^2) + (1-P_{max})^2}$	for $P_{min} = 0.4$ and $P_R < 0.5$
				$1 - \sqrt{0.5[(1-P_R)^2 + (1-P_{max})^2]}$	for $P_{min} = 0.4$ and $P_R > 0.5$
$P_R < P_{min}$	–[a]	–[a]	–[a]	–[a]	

Notes: The subscripts i and t have been dropped for simplicity of exposition.
[a]Does not exist since probability of acceptance is zero.

$\sqrt{\frac{1}{2}} = .707$ for the standard right triangular distribution, $1 - \sqrt{\frac{1}{2}} = .293$ for the standard left triangular distribution, and 0.5 for the standard symmetric triangular distribution. For earlier periods, the expected value in the event of bid rejection is simply the average of the possible expected values from the subsequent period weighted by the probability of receipt of each bid.

The results of the expected sales price calculations for the Rothschild model are shown in Table 3. As expected, the Rothschild model performs slightly inferior to the Simon model if the assumed standard uniform bid distribution is the actual distribution (0.840 vs. 0.861). However, when our guess about the actual bid distribution is wrong, in every case the Rothschild model performed superior to the Simon model. This was especially true in the case of the standard left triangular distribution in which the penalty in the Simon model for holding too optimistic expectations about the bid distribution is most severe (i.e., the seller has a much greater probability of being forced to accept the last bid, which is drawn from a downward skewed distribution).[19]

These results imply that the "optimal" strategy to be followed by the seller should depend both on any a priori information about the likely distribution of bids and the seller's degree of risk aversion. In the event that each of the bid distributions has an equal probability of occurrence, a risk neutral seller would prefer the Rothschild strategy (expected sales price = 0.760 vs. 0.753 for the Simon model). Positive risk aversion would also suggest the Rothschild model (so long as the seller does not have substantial confidence that the assumed bid distribution is the actual distribution). Only in the case in which the seller is a risk searcher or for some reason has substantial confidence that the assumed standard uniform distribution is the actual distribution would he select the Simon strategy.[20] The seller would avoid the Simon strategy especially when there is a significant possibility of encountering a downward skewed distribution.

Finally, let us examine the expected time-on-market performance of the Rothschild model. The relationship for expected time-on-market is closely akin to that for expected sales price: It is the sum of the probability of accepting a given price times the period in which it appears plus the probability of rejecting a given price times the expected time-on-market if the bid is rejected. Again, this occurs for each of the possible outcomes of a bid, so in order to derive the expected time-on-market, each of these possible outcomes must be weighted by their probability of occurrence and summed. The probability of acceptance and rejection is calculated according to the procedure described in calculating expected sales price. The expected time-on-market if the bid is rejected is simply the expected time-on-market with t-1 periods to forced sale.

The results of the expected time-on-market calculations for the Rothschild model are shown in Table 3. If the actual bid distribution is standard uniform, the Rothschild strategy is somewhat less efficient than the Simon strategy in that, not only is the expected sales price somewhat lower, the expected time-on-market is somewhat longer (4.865 periods vs. 4.602 in the Simon model). This is expected, since the slightly higher expected reservation price trend in the Rothschild strategy results in a longer expected time period until an acceptable bid is received.

For the other bid distributions, however, the expected time-on-market using the Rothschild strategy is sometimes longer, but sometimes shorter than using the Simon strategy. This does not necessarily relate to the degree of efficiency in search behavior, since the expected sales price is higher under the Rothschild strategy in all cases, but rather has to do with the ability of each strategy to respond to the bid distribution actually encountered. In the case of the standard right triangular distribution, the Rothschild model has a higher expected time-on-market because it quite properly encourages the seller to wait in the expectation of a higher bid in the future. The reverse is true in the case of the standard left triangular distribution. In the case of the standard symmetric triangular distribution, the explanation for the longer expected time-on-market under the Rothschild strategy is somewhat unexpected, since the lowered reservation price trend under this strategy would suggest somewhat earlier acceptance of a price. However, a possible explanation might lie in the fact that a seller under a standard symmetric triangular distribution has an incentive to continued search beyond the expected sales price of 0.694 under the Simon strategy because his probability of obtaining a bid in excess of this amount is greater (at least up to $p_t = 0.750$) than under a standard uniform distribution. This incentive to continued search would be manifest as a longer expected time-on-market.

V. Summary and Conclusions

Our examination of seller behavior in residential sales has revealed that the selling strategy is oftentimes relatively crude, constrained in part by lack of information availability, in part by lack of analytic expertise, in part by the rapid dynamics of the market. However, there clearly exist a number of characteristics of this process which serve both as the basis for an improved descriptive modeling effort and as clues toward the development of a normative model which could potentially improve sales performance. First, the process represents a sequential bid process, normally without recall. Second, it exists in an environment in which there is limited *a priori* information about the distribution of potential bids and the relationships between asking price and bid frequencies and levels. Subsequent information is obtained only through experience as bids are received. Third, the sale consists not only of a "price" but also a set of "non-price" transactions in the form of the non-price terms of the contract. Finally, bargaining and negotiation often play a major role in the determination of the nature of the outcome.

Existing search models, both those narrowly constructed within the real estate/urban land economics tradition and those more broadly constructed in the literature on the economics of job search and uncertainty, provide varying degrees of adherence to the characteristics of the actual real estate transactions process. However, no efforts have explicitly considered a learned relationship between asking price and the frequency and level (or distribution) of bids. This creates the unrealistic situation in which the asking price is irrelevant; only the reservation price is of any concern. Few attempts likewise have been made to estimate empirically the marginal utilities of various non-price terms (such as mortgage terms) in residential transactions. Nor has bargaining behavior in a real estate market context been investigated. These are all fertile grounds for future research efforts.

However, in one area, past research has provided us with a potentially significant improvement in our ability to model the transactions process. Existing real estate search models assume the distribution of potential bids is either fully known or fully unknown, whereas in reality one makes use of previous bids received to improve his reservation price strategy. Such behavior is encompassed in a "learning" model postulated by Rothschild (1974) in which the prior is a Dirichlet distribution which is updated in a Bayesian fashion as bids are received. One of the primary contributions of this paper has been an evaluation of the performance of this model with respect to expected sales price and expected time-on-market and a comparison of its performance with that of two of the more traditional search models of real estate market behavior: a model which assumes a completely known distribution (the Simon [1957] model) and one which assumes a completely unknown distribution (the Stull [1978a] model). The following salient results were obtained from this exercise:

1. The Simon model performs optimally, as expected, when the actual bid distribution is identical to that assumed (in our case a stand uniform distribution).

2. The Rothschild model performance approaches that of the Simon model when the bid distribution is correctly assumed.

3. When the bid distribution is correctly assumed, the Stull model, because of the inefficiencies generated by automatically waiting several periods before considering a bid and then only adjusting the reservation price a single time, performs considerably less well than either of the other models, both with respect to a lower expected sales price and a longer expected time-on-market.

4. When the actual bid distribution is different from that initially assumed (in our case, when it is either right, left, or symmetic triangular rather than standard uniform), the Rothschild model performs the best of the three models with respect to expected sales price. This is because of its efficient use of information to improve reservation price strategy. Performance is especially improved when the actual bid distribution is downward skewed, as is the left triangular distribution.

5. When the actual bid distribution is different from that initially assumed, the Stull model performs substantially less well than the other two models with respect to expected sales price. Thus, the Stull model is dominated whether or not the actual bid distribution is correctly assumed, and its use would not be optimal even under conditions of considerable lack of information about the true distribution of bids.

6. The expected reservation price trend is actually positive for a time in the Rothschild model under conditions in which there is a sufficiently strong upward adjustment in the Dirichlet representation of the bid distribution as bids are received. This is expected and sometimes observed in practice. It does not conflict with negative reservation price trend requirements, which are based upon static assumptions about the bid distribution, because the information set is updated each period. Indeed, observation of a positively sloped reservation price trend in the marketplace would constitute evidence of behavior more consistent with the Rothschild model than with the other two models.

In summary, the results suggest the inferiority of the Stull model and the superiority of the Rothschild model as appropriate models of selling behavior in the real estate market. Stull suggests his model may be more appropriate under conditions of considerable uncertainty in the actual distribution of bids. We have shown that even under such conditions, the Stull model would be expected to perform inferior to both the Simon and Rothschild models. More importantly, the Rothschild model in such cases can provide considerable improvement over the Simon model, which rigidly assumes a single known bid distribution *a priori* (which may, of course, be wrong). Only in the case in which there is considerable confidence that the assumed bid distribution is the actual bid distribution would the Simon model be preferred.

To the extent that individuals engaged in selling behavior in the long run set their reservation price trends in a more or less optimal fashion (regardless of whether or not they actually undertake the calculations developed here), these results suggest they would tend to behave in most circumstances more in accordance with the reservation price strategy predicted by the Rothschild model than either of the other two models. Thus, the exercise here could be viewed as a contribution to our descriptive understanding of seller behavior in the real estate market as well as an improved normative model.

Further research efforts should attempt to empirically test reservation price behavior, in the market (if possible) or in a controlled setting (if necessary), to establish its adherence to one of the three models presented here. Additional efforts should incorporate consideration of the role of both the asking price, non-price terms, and bargaining in determining the ultimate outcome of the transactions process. We will begin to fully understand real estate market behavior only after we improve our understanding of the true nature of the transactions process in the real estate market.

Footnotes

[1]Since we are concentrating on the seller, we do not consider the benefits and costs of brokers used in purchase. The commission is paid indirectly by purchasers (at least in part) through an increased purchase price (see Jud [1982]). Another potential cost to purchasers is loss of control of search through "steering." Benefits include reduced search time and effort, facilitation of document preparation and execution, advice on and possibly sources of financing, facilitation of negotiations, and perhaps providing advice on offering bids and negotiations.

[2]This assumes adjustment of comparable sales prices has been complete. To the extent that quality differences remain, however, there would be an upward or downward adjustment from the mean. There also might be weighted averages used in the event that one comparable is felt to be a much stronger incicator than another.

[3]This is an important point in that the existence of such a disparity of goals between the seller and broker could result in numerous variations in market behavior, depending upon market characteristics. For example, brokers dealing with large volumes of lower-priced properties could develop different strategies of sale than brokers dealing with very few higher-priced properties. Exploration of such possibilities would be interesting, but is beyond the scope of this paper.

[4]Simon [1957], p. 241.

[5]Simon [1957], pp. 259-260.

[6]Simon's model, which assumes perfect knowledge of the bid distribution, has been extensively examined elsewhere in a real estate market context. (See, for example, de Groot [1970], Stull [1978a]). Some of these have been variations on the basic theme. All shall be described in this study as the Simon model.

[7]Of course, a number of additional dimensions to the transactions process could be considered which have not been discussed here. These include such considerations as risk aversion, existence of a dynamic economy, bankruptcy (in which the wealth position of the seller influences his search behavior), optimal search-start strategies, and variable intensity of search. These are considered to be more sophisticated "fine tunings" of the basic model which are beyond the scope of this paper. Inclusion of such considerations would be fruitful, however. The reader is referenced to Lippman and McCall (1976) for a discussion of such extensions.

[8]The Dirichlet or multivariate beta distribution (see de Groot [1970], pp. 49-51) is defined as follows: A random vector $\bar{X} = (X_1,\ldots,X_k)'$ has a Dirichlet distribution with parametric vector $\underline{\alpha} = (\alpha_1,\ldots,\alpha_k)'$ $(\alpha_i>0; i=1,\ldots,k)$ if the probability density function $f(.|\bar{\alpha})$ of $\bar{X}$ satisfies the following properties: Let $\bar{X} = (X_1,\ldots,X_k)'$ be any point in R^k such that $X_i>0$ for $i=1,\ldots,k$ and $\sum_{i=1}^{k} X_i = 1$. Then

$$f(\bar{X} \mid \bar{\alpha}) = \frac{\Gamma(\alpha_1+\ldots+\alpha_k)}{\Gamma(\alpha_1)\ldots\Gamma(\alpha_x)} X_1^{\alpha 1}\ldots X_k^{\alpha k-1}$$

where $\Gamma(\alpha_i) = \int_0^{\alpha} u^{\alpha_i - 1} e^{-u} du \quad \alpha_i>0$

and where it is known $\Gamma(\alpha_i)$ has the following property:

$\Gamma(\alpha_i) = (\alpha_i-1)\Gamma(\alpha_i-1)\ \alpha_i>1$, or for any positive integer n, $\Gamma(n)=(n-1)!$. Also, $f(\bar{X}|\bar{\alpha}) = 0$ at any other point $\bar{X} \varepsilon R^k$.

This distribution can perhaps be illustrated most intuitively using the following example provided by Rothschild (1974): Assume the searcher faces an urn in which there are $S(N) = \Sigma N_i$ pieces of paper; p_{d1} is written on N_1 of these slips, p_{d2} on N_2 of them, etc. We may consider these p_{di} to be price quotations, which, although discrete, could go in the limit to infinity over a range. The searcher does not know the real price distribution Π which generates the price quotations. However, as he samples from the urn (returning the slip to the urn after each sample) he develops a "feel" for the probability of getting price p_{di} from the urn. This is represented by the proportion of times he has observed p_{di} out of the total number of samples taken. If price quotations are really generated by Π, then the strong law of large numbers states that this proportion will, with probability 1, eventually be equal to Π.

[9]See Vandell (1981) for a preliminary effort at developing some notion of the demand and supply influences of the non-rate terms of mortgage credit.

[10]The setting of the initial asking price by the seller is treated in (b) above. Presumably the buyer also behaves according to an optimal utility maximizing calculus in setting his response bid (and of course his reservation price), but from the point of view of the seller this calculus is unknown (at least at first).

[11]Of course, inclusion of the non-price dimensions renders the bargaining stage much more complex.

[12]Note that this is related to the setting of the initial asking price, but is one bidding cycle removed from that situation. The problem of setting the initial asking price is considered in (b) above.

[13]However, in the long run, assuming equal market power and skill in negotiation, one would expect a solution at that point along CD where both the seller and buyer sacrifice equally (in utility terms) at the margin.

[14]This is equivalent to assuming either that there is a large number of potential bidders (the competitive market case) or that a bidder, once rejected, cannot be recalled but could again enter the market with the same bid. An interesting variation of this situation would be the case in which the bid is not replaced and the seller must deal with a dwindling supply of bidders. This would make the rejection of higher bids more costly, thus lowering the reservation price.

[15]This is only when the probability of getting a serious offer each period is unity. Otherwise $E(W) = \frac{1}{q} E(W)$, where q is the probability of getting a serious offer each period.

[16]Of course any mapping is possible. The intent here is to approximate insofar as possible in the Dirichlet distribution the characteristics of the assumed bid distribution of the Simon model. As the number of potential prices in the Dirichlet distribution increases, in the limit it replicates exactly the assumed bid distribution. As the number of potential prices is increased, however, solution of the Rothschild model also becomes a bulkier task. The selection of only five possible prices in the Dirichlet was made necessary by computing time limitations. Simulation of the Rothschild model over 10 periods required 15 minutes computing time.

[17]This would mean for period 1 there would be 5 possible outcomes, hence 5 possible reservation prices; for period 2, 25 possible reservation prices (5 possible outcomes for each of the 5 previous reservation prices); etc.

[18]That is, having a longer time-on-market would not necessarily imply greater inefficiency in search, because the nature of the actual bid distribution may render a longer search time closer to optimality.

[19]In the case of the rightward-skewed distribution, on the other hand, the seller's standards are too pessimistic and he accepts a bid too quickly in the belief he may not see a bid so high again. His penalty is less severe, however, since the bid is drawn from an upward skewed distribution.

[20]The probability of encountering a standard uniform distribution must approach 43 percent if the other three distributions all have an equal probability of occurrence.

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