1-1-1982

Selecting Optimal Portfolios with a Futures Market in a Stock Index

Dwight Grant

Southern Methodist University

Follow this and additional works at: https://scholar.smu.edu/business_workingpapers

Part of the Business Commons

This document is brought to you for free and open access by the Cox School of Business at SMU Scholar. It has been accepted for inclusion in Historical Working Papers by an authorized administrator of SMU Scholar. For more information, please visit http://digitalrepository.smu.edu.
SELECTING OPTIMAL PORTFOLIOS WITH A FUTURES MARKET IN A STOCK INDEX

Working Paper 82-202*

by

Dwight Grant

*This paper represents a draft of work in progress by the author and is being sent to you for information and review. Responsibility for the contents rests solely with the author. This working paper may not be reproduced or distributed without the written consent of the author. Please address all correspondence to Dwight Grant.
Selecting Optimal Portfolios with a Futures Market in a Stock Index

Several futures markets have proposed a futures contract in a stock market index. The Chicago Mercantile Exchange, for instance, has suggested a futures contract in the Standard and Poor's 500 Stock Index [1]. Such a market, if it comes into existence has important implications and substantial potential for people who invest in equities, both individuals and professional portfolio managers. It will very much simplify the process of selecting the optimal portfolio. Furthermore, it will alter the composition and increase the return of the portfolios which they deem best as compared to the preferred portfolio in the absence of a futures contract in the index. To illustrate this effect we will compare portfolio selection with and without such a contract. An article in this journal [2] by Elton, Gruber and Padberg, EGP, provides a convenient data base and benchmark for doing so. In that article they used data for 10 securities to demonstrate a simple procedure for constructing optimal portfolios. We will use the same data to illustrate how a futures contract further simplifies the selection of, and changes the composition of the best portfolio. First, however, we should discuss the important characteristics of a futures market in a stock index.

What is a Futures Market?

When futures contracts in commodities such as wheat are bought (sold), investors agree to accept (make) delivery of a particular type and quantity of wheat at a specific site on a given day. The investors' objective is to alter the way in which changes in the price of wheat affect the values of their portfolios. A futures market in a stock index will be similar to this in some respects but will also differ because of the nature of the index as a commodity. The definition of the index, for example the S&P 500, is analogous to the
"type" of grain. The site of delivery for the index is largely irrelevant because the cost of transporting and storing the index is essentially zero. Indeed, unlike most futures markets, the stock index market is not likely to be what is called a "delivery market." In a delivery market investors can, if they choose, fulfill their contracts by accepting or making delivery of the commodity. A delivery market makes sense when the commodity involved plays an economic role. Wheat, for instance, is grown by farmers and purchased as a raw material by bakers. That is not, in general, the case with stock indices.

Stock indices are weathervanes. Index funds aside, they do not exist and would be expensive to create. The vast majority of investors do not own a specific index, nor do they want to acquire it. Rather, investors recognize that there is a close relationship between changes in the index and changes in the values of their portfolios. This makes a futures contract in the index a useful tool for altering how movements in the market as a whole affect the value of a portfolio, but it does not create any demand for ownership of the index itself. Consequently, any futures market in a stock index is likely to be a cash settlement market. Investors will not deliver or accept the index. Instead all contracts which are outstanding on the "delivery" date will be deemed to be settled by purchase or sale of the index at its value on that day. In all cases, investors will gain or lose the cash difference between the price at which they sell, and the price at which they buy the futures contract. The index itself will not change hands.

To illustrate how the market will likely operate consider this example. Suppose the S&P 500 Index is chosen as the basis for the futures contract. The value of the contract is established by choosing a dollar factor and multiplying the index by that factor. If the index is 100 and the factor is
$500, then the value of one contract in the index is $50,000 \((500)(100)\). A one point change in the index changes the value of the contract by $500. Suppose a portfolio manager wants to reduce the exposure of his portfolio to market risk by a cash amount of $4.0 million. To do so he will promise to deliver -- sell short -- 80 futures contracts \([4.0 \text{ million} = (80)(500)(100)]\). If the value of the index when he closes his position is 95 then the cost of closing will be $3.8 million \([3.8 \text{ million} = (80)(500)(95)]\). The gain on the futures contract would be $200,000 \([200,000 = (80)(500)(5)]\). Assuming that the futures position was taken as a hedge, this gain would be offset, more or less, by a decrease in the value of his portfolio of equities. We have been purposely vague on the question of how an investor should determine what position to take in the futures contract because this is an integral part of the overall portfolio selection procedure which we will now discuss.

Which Securities Should You Buy?

The selection technique which we will describe is appropriate if: 1) the investor is risk averse and is concerned about only the expected value and standard deviation of his portfolio return; 2) there is a risk-free rate of return at which the investor can either borrow or lend; 3) the investor cannot sell short securities; 4) the single index or beta model describes the returns on securities; and 5) there is a futures market in the single index of the beta model.

In order to determine the expected return and risk of potential portfolios the investor must estimate, for each security \(i\), an expected return, \(\bar{R}_i\), a beta, \(\beta_i\), and the security's unsystematic risk, \(\sigma_{i}^2\). In addition, he must determine the risk-free rate of return, \(R_f\), and estimate the variance of return on the index, \(\sigma_m^2\). All of this information is also required by the EGP simple selection model. The only additional information, which is required
when there is a futures market in the index, is an estimate of the expected return on the index, $R_m$. In the EGP method one value for each security determined whether or not that security would be contained in the optimal portfolio. Not only did this simplify the selection procedure, but it also provided a strong intuitive basis for identifying the characteristics of securities which make them desirable investments. In the procedure which we will describe there is an analogous value. It is the risk-adjusted excess rate of return for each security. Its significance was first recognized by Jensen [4] when he measured portfolio performance, and in keeping with convention we will refer to it as alpha, $\alpha_i$. For each security it can be calculated from the estimates already identified:

$$\alpha_i = (\bar{R_i} - R_f) - \beta_i (\bar{R_m} - R_f)$$  \hspace{1cm} (1)

Alpha can be thought of as a bonus return -- it is the rate of return in excess of the risk-free rate of return plus that return which would be appropriate given the security's level of systematic risk, $\beta_i$. The size of alpha determines whether or not a security will be purchased. If it is positive the security will be purchased and if it is negative it will not. This is an intuitively appealing rule which can be extended in a logical way. The proportion of the portfolio which is invested in a security will vary directly with the security's alpha or bonus return.

The Selection Procedure

The simple selection procedure which EGP developed depends upon one of the fundamental ideas in the portfolio selection literature: the separation theorem. The idea is that when there is a risk-free asset the investor can separate his portfolio decision into two stages. First, he can identify a portfolio of risky assets which is best. Best in this case means that when this portfolio is combined with the risk-free asset the combination provides
an expected return which is larger for every level of risk, than that provided by any other portfolio. The second stage of the portfolio selection process involves choosing the preferred combination of the portfolio of risky assets and the risk-free asset -- that is the preferred level of expected return and risk. If the best portfolio of risky assets is too risky then the investor will commit only a portion of his wealth to it and will lend the remainder. If it is not risky enough, he will leverage his investment in the best portfolio of risky assets by borrowing at the risk-free rate of return.

If we examine this process we can identify an important respect in which a futures market in the market index will benefit investors. The first stage in the EGP process is the selection of the best portfolio of risky assets. A security's desirability is determined by its total expected return and total risk. But we know that both return and risk can be divided into market related or systematic components, and firm unique or unsystematic components. In the absence of a futures market in the stock index these components must be considered simultaneously as a package.

This is akin to the situation which would exist if dairies sold only whole unhomogenized milk and would not divide it into cream and skim milk. The consumer would be confronted with the choice of no skim milk and no cream; or skim milk and cream in proportions determined by the cow. It would be impossible for individuals to adjust their purchases to reflect the differences in their tastes for cream and skim milk. In addition, there would be no market in which the relative price of cream and skim milk could change.

The unsystematic and systematic components of return and risk are the cream and skim milk of the example. An investor may prize a security because of his assessment of its unsystematic return and risk characteristics. At the same time he may have a negative view of the expected return on the
market. If the security has a high beta the investor confronts a dilemma. He like to own the security's firm unique return and risk but not its market related return and risk. In the absence of a futures market in the stock index he cannot separate these two components of return -- he would like to purchase cream only, but is forced to buy whole milk or no milk.

A futures market in the stock index will eliminate this problem. Instead of portfolio selection being a two-stage process, it will become a three-stage process. Instead of there being one separation theorem -- that between choosing risky assets and the risk-free asset -- there will be two separation theorems. In the first stage the investor will choose the best portfolio of unsystematic return and risk. Then he will choose the best combination of this portfolio and the index. Finally, he will mix this combination with the risk-free asset. There are two important implications of this additional separation property: 1) Because only the desired characteristics of security returns are added to the portfolio, it will offer a better combination of expected return and risk; and 2) Because we can examine the unsystematic and systematic components of return separately, the selection process is even simpler and more readily understood and interpreted than that presented by EGP. In the next section we will demonstrate how an optimal portfolio should be selected when there is a futures market in the index. To provide a contrast with both the process and the results which follow when a futures contract does not exist, we will employ the same data in our example that EGP used in their example.

Selecting a Portfolio

In Table 1 we have displayed the data which EGP used in their example. As noted above, their model does not require an estimate of $\bar{R}_m$. This is needed when there is a futures market in the index. We assumed that $\bar{R}_m$ was
<table>
<thead>
<tr>
<th>Security No.</th>
<th>Mean Return $E_i$</th>
<th>Bonus Return $a_i$</th>
<th>Beta $\beta_i$</th>
<th>Unsystematic Risk $\sigma^2_{ei}$</th>
<th>$Z_i$</th>
<th>$X_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15</td>
<td>4.87</td>
<td>1.0</td>
<td>50</td>
<td>.0974</td>
<td>.197</td>
</tr>
<tr>
<td>2</td>
<td>17</td>
<td>4.31</td>
<td>1.5</td>
<td>40</td>
<td>.1078</td>
<td>.218</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>1.87</td>
<td>1.0</td>
<td>20</td>
<td>.0935</td>
<td>.189</td>
</tr>
<tr>
<td>4</td>
<td>17</td>
<td>1.74</td>
<td>2.0</td>
<td>10</td>
<td>.1744</td>
<td>.352</td>
</tr>
<tr>
<td>5</td>
<td>11</td>
<td>0.87</td>
<td>1.0</td>
<td>40</td>
<td>.0218</td>
<td>.044</td>
</tr>
<tr>
<td>6</td>
<td>11</td>
<td>-1.70</td>
<td>1.5</td>
<td>30</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>7</td>
<td>11</td>
<td>-4.26</td>
<td>2.0</td>
<td>40</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>-2.10</td>
<td>0.8</td>
<td>16</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>9</td>
<td>7</td>
<td>-3.13</td>
<td>1.0</td>
<td>20</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>10</td>
<td>5.6</td>
<td>-2.48</td>
<td>0.6</td>
<td>6</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>
equal to 10.13%. Using this value we computed a bonus return, \( \alpha_i \), for each security using the formula in equation (1). We indicated earlier that if \( \alpha_i \) is positive, the security enters the portfolio and if it is negative, it does not. Securities 1-5 which have positive \( \alpha_i \)'s, will, therefore, be purchased, while securities 6-10 will not be purchased.

We determine the composition of the portfolio in two steps. First, we are concerned with only the firm unique or unsystematic characteristics of return. The unsystematic expected rate of return of each security is \( \alpha_i \) and the unsystematic variance is \( \varepsilon_i^2 \). The desirability of each security is determined by the ratio of these two values. Let us call the ratio:

\[
Z_i = \frac{\alpha_i}{\varepsilon_i^2}
\]

For example: \( Z_1 = \frac{4.87}{50} = .0974 \)

The \( Z \) values for the 5 securities which will enter the portfolio are reported in Table 1. After making this calculation for each security with a positive bonus return, we sum the \( Z_i \)'s and then divide each \( Z_i \) by that sum. The resulting values, call them the \( X_i \)'s, are the proportions which each security represents in the best portfolio of unsystematic returns. In this example, the sum of the \( Z_i \)'s is .4949. The optimal proportion of this portfolio to invest in security 1 is 19.7\% (\( X_1 = .0974/.4949 = .197 \)). Similarly, for securities 2, 3, 4 and 5, the optimal proportions are 21.8\%, 18.9\%, 35.2\% and 4.4\% respectively. To see how the futures market has affected portfolio selection we can compare these proportions with those which would be optimal if there were not a futures market. EGP found that the values would then be: 23.5\%, 24.6\%, 20.0\%, 28.4\% and 3.5\% respectively. These values are similar, but certainly not identical.
In the second stage of the selection procedure the investor must determine his optimal exposure to systematic return and risk and thus the optimal position to take in the futures market in the index. An investor's preferred exposure to systematic return will depend upon his expectations concerning the index, \( R_m \) and \( \sigma_m^2 \), and upon the unsystematic return and risk characteristics of his portfolio. We will call these unsystematic components \( \alpha_p \) and \( \varepsilon_p^2 \). These values are weighted averages of the unsystematic return and risk characteristics of the individual securities in the portfolio. In this example these values are 2.90 and 5.87.\(^5\) In general, if his view of the market is bullish relative to his views on individual securities, he will prefer substantial market exposure. Conversely, if he feels optimistic about a specific group of stocks but feels bearish about the market, he will choose relatively little market exposure. The optimal commitment to the index \( X_m \), expressed as a fraction of the portfolio's value, is determined by the relative attractiveness, in a return/risk sense, of the market and the portfolio of unsystematic characteristics:

\[
X_m = \frac{[\bar{R}_m - R_f]/\sigma^2}{\alpha_p/\varepsilon_p}
\]

In this example:

\[
X_m = \frac{5.13/10}{2.90/5.87} = 1.0385
\]

This value, \( X_m \), is the preferred index exposure, not the optimal futures market position. One more calculation is required to determine that. Recall that when we bought securities 1-5, we acted as if we were buying only the unsystematic return component of each security. In fact, when you buy a security you buy both its unsystematic and systematic components of return. We could ignore the systematic component because we knew that it could be offset.
by selling short the futures contract in the market index. In essence, if the systematic return which is acquired incidentally when the investor purchases securities for their unsystematic characteristics is too large, it can be offset by selling short the futures contract. If the incidentally acquired systematic return exposure is too small, it can be augmented by purchasing futures market contracts. We know that the preferred market (index) exposure, $X_m$, is 1.038. The market exposure acquired when purchasing the portfolio based on unsystematic returns is a weighted average of the $\beta$'s in the portfolio. In this example that value is 1.461 $[1.461 = .197(1.0) + .218(1.5) + .189(1.0) + .352(2.0) + .044(1.0)]$. Therefore, the optimal futures market position is a short sale of contracts equal in value to 42.3% ($1.038 - 1.461 = - .423$; the minus sign indicates a short sale) of the value of the portfolio.

To recapitulate: the selection of an optimal portfolio of risky assets when there is a futures contract in the market index is a two-stage process. First, the investor selects an optimal portfolio of securities based solely on their unsystematic return and risk characteristics. Then, he determines a preferred market exposure and takes a futures market position which reflects that preference and the market exposure which was acquired when he purchased securities based on their unsystematic return and risk characteristics.

The first two stages of the process identify the optimal portfolio of risky assets, including a futures market position in the index. When this portfolio is combined with the risk-free asset it creates a set of return and risk opportunities for the investor which are best. The last step in the portfolio selection process is to choose that combination which is preferred given the individual investor's attitude toward return and risk.
The Significance of a Futures Market

The example we have developed demonstrates that a futures contract in the market index will simplify the portfolio selection process and alter the composition of the preferred portfolio. The extent to which the portfolio is altered depends upon the investor's expectations. In some cases, it could be very little while in other circumstances the differences could be substantial. It is also true that the optimal portfolio with a futures market will always offer a return/risk combination which is at least as good as, and usually will be better than, that which would be available if the futures market did not exist. Again, the difference can be large or small. In the example we examined it is quite small. The best way to gauge this is with the Sharpe measure of efficiency: \( \frac{(R_p-R_f)}{\sigma} \). Based on the results reported by ECP, when the futures market does not exist this value is 2.011. When it does exist this value is 2.016. This larger value means that for every level of risk the portfolio which contains a futures position offers a level of expected return which is slightly higher than the optimal portfolio which does not contain a futures contract.

Before closing this essay there are several points which warrant mention. Any futures contract in the market index will be of fixed size and sold only in units. Therefore investors, especially smaller investors, may not be able to closely match their preferred futures positions. For example, if the contract size is $50,000 and the investor wants to sell short $75,000 worth of the index, he confronts a dilemma. He can sell short one contract or two, $50,000 worth or $100,000 worth, but not $75,000. Also, we have not discussed the possible implications of performance deposits. These may or may not be serious, depending upon the rules imposed by the exchanges and brokers. Finally, the price of the futures contract depends upon the price of the index.
and the risk-free rate of return. To the extent that the latter varies it will also complicate portfolio choices.

Footnotes

1. This assumption is relaxed and the problem solved without it in [3].

2. If short selling of securities were permitted, the investor would sell short any security with a negative alpha. This too is an intuitively appealing result.

3. This important development is attributed to Tobin. See [7].

4. This value is consistent with the values in the paper by EGP.

5. These are calculated as follows:

\[ \alpha_p = X_1\alpha_1 + X_2\alpha_2 + X_3\alpha_3 + X_4\alpha_4 + X_5\alpha_5 \]

\[ = (.197)4.87 + (.218)4.31 + (.189)1.87 + (.352)1.74 \]

\[ + (.044)0.87 = 2.90 \]

\[ \sigma^2 = X_1\sigma_1^2 + X_2\sigma_2^2 + X_3\sigma_3^2 + X_4\sigma_4^2 + X_5\sigma_5^2 \]

\[ = (.197)^22.50 + (.218)^24.00 + (.189)^22.20 + (.352)^22.10 \]

\[ + (.044)^22.40 = 5.874 \]


The following papers are currently available in the Edwin L. Cox School of Business Working Paper Series.

79-100 "Microdata File Merging Through Large-Scale Network Technology," by Richard S. Barr and J. Scott Turner

79-101 "Perceived Environmental Uncertainty: An Individual or Environmental Attribute," by Peter Lorenzi, Henry P. Sims, Jr., and John W. Slocum, Jr.


80-100 "Implementing the Portfolio (SBU) Concept," by Richard A. Bettis and William K. Hall

80-101 "Assessing Organizational Change Approaches: Towards a Comparative Typology," by Don Hellriegel and John W. Slocum, Jr.

80-102 "Constructing a Theory of Accounting--An Axiomatic Approach," by Marvin L. Carlson and James W. Lamb

80-103 "Mentors & Managers," by Michael E. McGill

80-104 "Budgeting Capital for R&D: An Application of Option Pricing," by John W. Kensinger

80-200 "Financial Terms of Sale and Control of Marketing Channel Conflict," by Michael Levy and Dwight Grant


80-301 "Controlling the Performance of People in Organizations," by Steven Kerr and John W. Slocum, Jr.

80-400 "The Effects of Racial Composition on Neighborhood Succession," by Kerry D. Vandell


80-801 "Comparison of the EEOC Four-Fifths Rule and A One, Two or Three σ Binomial Criterion," by Marion Gross Sobol and Paul Ellard

80-900 "Bank Portfolio Management: The Role of Financial Futures," by Dwight M. Grant and George Hempel

80-902 "Hedging Uncertain Foreign Exchange Positions," by Mark R. Eaker and Dwight M. Grant
| 80-111 | "Sources of Performance Differences in Related and Unrelated Diversified Firms," by Richard A. Bettis |
| 80-112 | "The Information Needs of Business With Special Application to Managerial Decision Making," by Paul Gray |
| 80-113 | "Diversification Strategy, Accounting Determined Risk, and Accounting Determined Return," by Richard A. Bettis and William K. Hall |
| 80-114 | "Toward Analytically Precise Definitions of Market Value and Highest and Best Use," by Kerry D. Vandell |
| 80-115 | "Person-Situation Interaction: An Exploration of Competing Models of Fit," by William F. Joyce, John W. Slocum, Jr., and Mary Ann Von Glinow |
| 80-116 | "Correlates of Climate Discrepancy," by William F. Joyce and John Slocum |
| 80-121 | "Project Abandonment as a Put Option: Dealing with the Capital Investment Decision and Operating Risk Using Option Pricing Theory," by John W. Kensinger |
| 80-122 | "The Interrelationships Between Banking Returns and Risks," by George H. Hempel |
| 80-123 | "The Environment For Funds Management Decisions In Coming Years," by George H. Hempel |
| 81-100 | "A Test of Gouldner's Norm of Reciprocity In A Commercial Marketing Research Setting," by Roger Kerin, Thomas Barry, and Alan Dubinsky |
| 81-200 | "Solution Strategies and Algorithm Behavior in Large-Scale Network Codes," by Richard S. Barr |
| 81-201 | "The SMU Decision Room Project," by Paul Gray, Julius Aronofsky, Nancy W. Berry, Olaf Helmer, Gerald R. Kane, and Thomas E. Perkins |
| 81-300 | "Cash Discounts To Retail Customers: An Alternative To Credit Card Performance," by Michael Levy and Charles Ingene |
| 81-400 | "Merchandising Decisions: A New View of Planning and Measuring Performance," by Michael Levy and Charles A. Ingene |

81-600 "Managerial Uncertainty and Performance," by H. Kirk Downey and John W. Slocum, Jr.

81-601 "Compensating Balance, Rationality, and Optimality," by Chun H. Lam and Kenneth J. Boudreaux


81-800 "The Chinese-U.S. Symposium On Systems Analysis," by Paul Gray and Burton V. Dean


81-900 "Forecasting Industrial Bond Rating Changes: A Multivariate Model," by John W. Peavy, III

81-110 "Improving Gap Management As A Technique For Reducing Interest Rate Risk," by Donald G. Simonson and George H. Hempel


81-112 "The Significance of Price-Earnings Ratios on Portfolio Returns," by John W. Peavy, III and David A. Goodman

81-113 "Further Evaluation of Financing Costs for Multinational Subsidiaries," by Catherine J. Bruno and Mark R. Eaker

81-114 "Seven Key Rules For Successful Stock Market Speculation," by David Goodman

81-115 "The Price-Earnings Relative As An Indicator of Investment Returns," by David Goodman


81-117 "Sequential Information Dissemination and Relative Market Efficiency," by Christopher B. Barry and Robert H. Jennings

81-118 "Modeling Earnings Behavior," by Michael F. van Breda


81-120 "The Price-Earnings Relatives - A New Twist To The Low-Multiple Strategy," by David A. Goodman and John W. Peavy, III.
82-100 "Risk Considerations in Modeling Corporate Strategy," by Richard A. Bettis


82-103 "A Typology of Small Businesses: Hypothesis and Preliminary Study," by Neil C. Churchill and Virginia L. Lewis

82-104 "Imperfect Information, Uncertainty, and Credit Rationing: A Comment and Extension," by Kerry D. Vandell

82-200 "Equilibrium in a Futures Market," by Jerome Baesel and Dwight Grant

82-201 "A Market Index Futures Contract and Portfolio Selection," by Dwight Grant

82-202 "Selecting Optimal Portfolios with a Futures Market in a Stock Index," by Dwight Grant