INDIRECT IMAGING USING COMPUTATIONAL IMAGING TECHNIQUES

Aparna Viswanath
aviswanath@smu.edu

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INDIRECT IMAGING USING
COMPUTATIONAL IMAGING TECHNIQUES

Approved by:

Dr. Marc P. Christensen
Professor

Dr. Prasanna Rangrajan
Assistant Professor

Dr. Duncan MacFarlane
Professor
INDIRECT IMAGING USING
COMPUTATIONAL IMAGING TECHNIQUES

A Thesis Presented to the Graduate Faculty of the
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in
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for the degree of
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in
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by
Aparna Viswanath

B.Tech, , Calicut University, Calicut, India (2013)
M.Tech, , Indian Institute of Space Science and Technology, Trivandrum, India (2016)

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I would like to thank God Almighty for giving me the opportunity to undertake this research and to complete it successfully. I owe my gratitude to my thesis supervisor Dr. Marc Christensen for his words of encouragement and thoughtful guidance. I am grateful to my advisor Dr. Prasanna Rangarajan for constructive criticism provided during the course of my research that helped shape the project to its current state. I would like to thank Dr. Duncan MacFarlane for the continuous support, his constant motivation and insightful comments. I thank Dr. Indranil Sinharoy for being a very supportive mentor and friend and guiding me through my research. I thank my colleague and friend Muralidhar Balaji for his help with some of the experiments, for the sleepless nights we were working together before deadlines and for giving me moral support during my research. I take this opportunity to thank all the professors and students working in the DARPA-REVEAL project in the SMU team for their constructive comments which incented me to widen my research from various perspectives. I thank my friends Ashwini, Vibha, Anna, Brooke and Nathan who helped and encouraged me during my stay at SMU. Last but not the least I thank my parents, sister and friends who supported me through all my endeavors.
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M.Tech, , Indian Institute of Space Science and Technology, Trivandrum, India (2016)

Indirect Imaging Using
Computational Imaging Techniques

Advisor: Dr. Marc P. Christensen
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The work describes various methods employed towards solving the problem of indirect imaging. Computational techniques are employed to indirectly decipher information about an object hidden from view of a camera. Notion of virtualizing the source of illumination and detectors on real world rough surfaces was exploited to construct a non line of sight computational imager. Diversity was explored from the stand point of both illumination of the object and imaging of light reflected off the object.

To understand the impact of scattering by real world rough surfaces, an instrument was developed that allows characterization of isoplanatic angle for different surface types. Various aspects (impact of absorption, multiple scatter, roughness scales, etc) of scattering from a surface was explored and identified. A computational scheme was identified to isolate the contribution of singly scattered light from multiply scattered light by employing techniques borrowed from linear algebra.

An experimental testbed that uses continuous wave(CW) sources to create spatially resolved intensity images of objects completely hidden from line of sight was developed. Ideas were borrowed from imaging correlography, a line of sight imaging technique widely used in satellite imaging. Experiments were carried out to explore limits of this non-line-of-sight computational imager.

A testbed was constructed that could convert real world rough surfaces into virtualized
pattern projectors in order to illuminate the hidden object with known light patterns. A mathematical model was partially developed to understand the capability of identifying depth and 3D information of the hidden object using the virtualized pattern projector. The idea of moving the center of perspective of the imaging system to the plane of the wall was proposed by moving the entrance pupil of the imager to the plane of the wall. This could potentially endow the system with stereo imaging capability. A Zemax simulation was carried out to identify the feasibility of this idea.
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Dedicated to my parents, sister and my husband
Chapter 1

Introduction

Imaging objects that are not in the direct view of the detector is an area that has received a lot of attention in the recent years. Recovery of the hidden object relies on the information encoded in the light scattered off of it.

1.1. Problem formulation

An object in a corridor is obscured from view of the light source and the sensor(Figure1.1). A computational imager is to be built exploiting wave nature of light and the intrinsic property of real-world objects to scatter light at optical scales. A light source to illuminate the object and a detector to capture the light reflected off of the object are two essential components of the imager. However in the indirect imaging scenario, both the light source and the sensor are hidden from the object. The intrinsic ability of rough surfaces to scatter incident coherent light is exploited in visualizing sources and detectors at rough surfaces in the scene, for instance walls, that are accessible both from the object side and from the viewer side. This approach has been exploited in [1] where holographic imaging techniques are used to image the hidden object. A limitation of the holographic approach is the limited geometry within which the approach is feasible. This calls for understanding the limits of the holographic technique. This is explained in more detail in the second chapter.

By exploiting the wave nature of coherent light it is possible to conceive a system that allows to glean further information about the hidden object for instance its range and contour. It could also be possible to improve the resolution and field of view of the imager using concepts from image processing and computer vision. Towards this, experiments were carried that could enable structuring illumination in the hidden volume by borrowing ideas and inspiration from existing techniques used in line of sight imaging.
1.1.1. Literature Survey

Several approaches have been proposed each of which makes use of some aspect of light to decode the hidden information. They can be broadly classified into those that rely on the finite speed of light and on the wave nature of light. The ones relying on the wave nature of light could in turn be classified into schemes that rely on a bare sensor for non line of sight imaging and ones that use a camera for non line of sight imaging. Several groups are involved in using time of flight of light to recover shape of the object hidden around a corner ([2]). The time light takes to reach the object and bounce back is used as a measure to understand the topography of the object. The success of this approach relies on the capability of the sensor to be able to sense very small depth details. For example, one would require a sensor to be able to have an exposure time of 1 picosecond to distinguish between light pulse that came from two points on the object that are separated in depth by about 0.3 mm. Hence the major shortcoming of this approach would be that it cannot resolve spatial details like texture on the object.

The wave nature of coherent light to interfere and produce specific spatial patterns was
explored by several research groups. A very interesting approach to image hidden objects around a corner was proposed by \[3\] \[4\], where the object was illuminated with incoherent light source. The light scattered by the object and reflected off a rough surface like a wall makes its way to a bare detector. In this work, the wall is assumed to behave like an aberrated virtual lens that diverges the light incident on it. Although the approach provides immense insight into the problem, it would require the detectors to be very large in extent or very close to the wall to be able to collect much of the scattered light.

Gigain et. al. \[5\] attacks the problem in a more computational manner by trying to understand the point spread function of the wall to gain insight about the object. The detector in this configuration is shifted axially to get multiple images where the diversity is imparted as phase due to translation to the collected light field. The computational burden that the approach would mean in order to understand the point spread function of the wall would be cumbersome and the resolution with which it can be measured would be limited by the size of a pixel at the location of the wall.

Holographic approaches that involve directing a reference laser beam at the wall, W, in order to create a hologram at the wall to retrieve field information of the object was suggested by \[1\]. Although this work is able to successfully recover the amplitude and phase of the object field, it implicitly assumes that the phase fluctuations imparted by the wall on the field scattered off the object and the reference beam are largely similar. This means that the complex valued reflectivity of the wall as experienced by the object field and that imparted to the reference beam are largely similar. This poses a hard constraint on the architecture that the scene needs to satisfy for the object information to be recovered. Eitan et. al. \[6\] was successfully able to retrieve the hidden object details in a transmissive geometry without using a reference beam using imaging correlography technique and the Fourier domain shower curtain effect.
1.2. Organization of chapters

Imaging of objects that are not in the line of sight of the detector can be termed as indirect imaging. In the following chapters, a series of tasks are describes that seek to better understand the limitations of indirect imaging. The second chapter deals with the description of an experimental test bed designed to compute the isoplanatic angle of a rough surface. From this analysis one can determine materials and geometry of the experiment for indirect imaging. The third chapter tries to introduce the concept of using imaging correlography to aid in indirect imaging of a hidden object in a corridor. The chapter also analyses the performance and limitations of the indirect imager. The fourth chapter deals with a technique devised to structure the illumination around a corner on to the hidden volume that could potentially uncover other object properties like contour, range, etc and superresolving the image obtained using the correlography approach. The last chapter gives a zemax model that can move the entrance pupil of an imaging system to any plane in front of it. Moving the entrance pupil plane displaces the center of perspective of the imaging system which can potentially help the indirect imager to do stereo imaging.
2.1. Introduction

2.1.1. Optical memory effect and isoplanaticity of a surface

Optical memory effect [7] is a concept used to explain the predictable behavior of a light wave incident on a scattering medium after interacting with the medium. This is exhibited as a correlation between the scattered waves upon perturbation of the incident wave. A parameter that describes the angular range over which phase gradients are conserved upon scattering from a rough surface is described as its isoplanatic range. Isoplanaticity arises due to near identical optical path length differences experienced by light beams illuminating the rough sample with an angular separation. For instance, a highly polished surface like
Figure 2.2. Optical path length difference as a function of height of surface from mean height, cosine of angle of incidence and cosine of viewing angle

whereas in the case of a rough mirror, the angular range over which isoplanaticity is exhibited by the surface would be limited.

To understand the isoplanatic angle of a surface one may look at the geometry in Figure 2.2 where a ray of light illuminates a rough surface at an angle of incidence $\beta_i$ and the reflected light is viewed at an angle $\beta_o$ with respect to the plane normal at the point of incidence. The optical path length difference (OPD) experienced by the light ray about a mean surface height is given by the formula.

$$OPD = \Delta h(x, y)\cos(\beta_i) + \Delta h(x, y)\cos(\beta_o)$$ (2.1)

The optical path length difference of a surface is a function of the roughness profile of the surface about a mean surface, cosine of the angle of incidence and cosine of the viewing
angle. For a fixed angle of illumination and fixed viewing direction, the optical path length difference is only a function of the height fluctuation. Let the viewing angle $\beta_o = 0$ so that the viewing direction is aligned with the normal to rough surface.

$$\text{OPD} = \Delta h(x, y)\cos(\beta_i) + \Delta h(x, y) = \Delta h(x, y) [\cos(\beta_i) + 1] \quad (2.2)$$

The phase shift due to the OPD is given by,

$$\text{Phaseshift} = \frac{2\pi}{\lambda} \Delta h(x, y) [\cos(\beta_i) + 1] \quad (2.3)$$

Where, $\lambda$ is the wavelength of light and $\beta_{\text{reference}}$ is a reference angle against which correlation measurement is made. Correlation between reflected light fields captured at the angle of incidences $\beta_i$ and $\beta_{\text{reference}}$ is high when $\beta_i - \beta_{\text{reference}}$ is very small and drops or vanish when the difference increases.

Normalized cross correlation between the two fields, $\mu_A$ is a function of first order characteristic function of the surface height fluctuations $M_h$ and normalized Fourier transform of the intensity distribution across the spot, $\varphi$. $M_h$ is the ratio of rms surface height fluctuations to wavelength.

$$|\mu_A|^2 = |M_h|^2 |\varphi|^2 \quad (2.4)$$

First order characteristic function of the surface height fluctuations $M_h$ is given by [8],

$$M_h = \langle e^{\frac{2\pi i}{\lambda} \sigma_h[(1+\cos(\beta_{\text{reference}}))-(1+\cos(\beta_i))]} \rangle \quad (2.5)$$

where, $\sigma_h$ standard deviation of surface height fluctuations. The aim is to find the largest angular range after which the correlation between the two scattered light intensities drops to $\frac{1}{e^2}$ which will give a sense for the memory effect angular range for the surface with respect
to the reference angle of incidence. Hence,

\[
|\mu_A|^2 = e^{-\left(\frac{2\pi}{\lambda}\right)^2 \sigma_h^2 \left[\cos(\beta_{\text{reference}}) - \cos(\beta_i)\right]^2} |\varphi|^2
\]  

(2.6)

Assuming \( \beta_{\text{reference}} = 0 \), the largest \( \beta_i \) for which the correlation between scattered light intensities drops to \( \frac{1}{e^2} \) can be obtained as,

\[
\langle e^{i \frac{2\pi}{\lambda} \sigma_h [1 - \cos(\beta_i)]} \rangle = \frac{1}{e}
\]  

(2.7)

\[
\beta_i = \cos^{-1}\left(1 - \frac{\lambda}{2\pi\sigma_h}\right)
\]  

(2.8)

Assuming small \( \beta_{\text{reference}} > 0 \),

\[
\langle e^{i \frac{2\pi}{\lambda} \sigma_h [\cos(\beta_{\text{reference}}) - \cos(\beta_i)]} \rangle = \frac{1}{e}
\]  

(2.9)

\[
\cos(\beta_{\text{reference}}) - \cos(\beta_i) = \frac{\lambda}{2\pi\sigma_h}
\]  

(2.10)

The above equation suggests that the isoplanaticity exists in direction cosine space and not in angular space as it is the difference in cosine of the angles of incidences that remains constant.

Let \( \beta_{\text{reference}} - \beta_i \) be very small, as would be the case for a very rough surface. Under this small angle approximation the above equation further simplifies to,

\[
-2 \sin\left(\frac{\beta_{\text{reference}} + \beta_i}{2}\right) = \frac{\lambda}{2\pi\sigma_h}
\]  

(2.11)

\[
\beta_i = \beta_{\text{reference}} - \frac{\lambda}{2\pi\sigma_h \sin(\beta_{\text{reference}})}
\]  

(2.12)
2.1.2. Impact on indirect imaging

By observing the correlation in the intensity of the light scattered off the sample across varying incident angles one can estimate the angular range over which a strong correlation exists.

In the problem geometry in Figure 2.3, there is a laser beam that illuminates the wall. The wall scatters this light in all directions thereby illuminating the hidden object. The hidden object reflects some of this light back, part of which is intercepted by the wall and is imaged by the camera. This setup allows us to only get to the autocorrelation of the object brightness. In order to get the brightness distribution of the object directly one would have to create a virtual hologram at the plane of the wall by illuminating the virtual detector with a reference beam [1](Figure 2.4). Using phase shifting interferometry one may get to the object field information.

However, this geometry is constrained by the large angular separation ($\theta$) between the object beam and the reference beam. The reflectivity of the wall for the two beams coming in at large angular separations would be very different due to the roughness of the wall. The angular extent within which this proposed experiment would work depends on the isoplanatic angle of the wall, which is the angular extent within which the walls reflectivity is largely similar. The complex valued reflectivity experienced at the virtual detector by the latent object field and the reference beam can be assumed to be largely similar only within the isoplanatic angle of the virtual detector surface.

Knowledge of isoplanatic angle of surfaces helps better understand the constraints on the scene geometries for the the above approach. Experimental evidence using the testbed suggests that an angular separation of less than a degree between the latent field and the reference beam is crucial to recovering a reliable holographic representation of the hidden object, hence making the approach less practical for objects that are further inside the hallway.
Figure 2.3. Problem geometry

Figure 2.4. Geometry that allows to create a virtual hologram at the plane of the wall
2.2. Approach

To understand the impact of parameters like roughness scale, absorption and subsurface scatter on the isoplanatic angle, a pairwise comparison of materials is carried out. The comparison is carried out using an experimental testbed borrowing ideas from instrumentation for measuring BRDF [9], and approaches to Fourier Ptychography [10].

The light scattered from the sample may be collected using two different geometries (Figure 2.5).

- Imaging geometry: In this geometry the rough surface is imaged using a lens and a bare detector. The limited angular resolution of collection optics is one drawback of this approach. Also, field information of the captured light is required to characterize memory effect angular range. This is because correlation manifests as a phase ramp in the subjective speckle field.

- Free space geometry: Memory effect angular is range characterized from intensity images captured using a bare detector alone. The correlation manifests as a linear shift in the objective speckle field which can be easily measured. This is the geometry that was used in the experimental test bed.

2.2.1. Experimental test bed

The test bed comprises of laser source (linearly polarized, 532nm, 0.3mm diameter) and a motorized rotating platform upon which the sample and a bare detector are mounted. The laser beam passes through a beam delivery unit that comprises of an optical isolator, beam attenuator, spatial filter and beam alignment unit. The isolator prevents stray laser light from returning to the laser cavity. The beam power control is achieved by using a half waveplate and a non polarizing beam splitter. A balanced spatial filter is designed using two identical plano convex lenses and a pinhole. A beam alignment unit comprising of two mirrors are used to fold the light path so that it is lined up with the holes on the optical table.

The rotating platform is calibrated to rotate the sample along an axis that lies in the plane
Figure 2.5. Approaches to measure angular correlation

Figure 2.6. Experimental testbed to quantify isoplanaticity of materials
Figure 2.7. Envisioned architecture to track change in incidence angle

of the sample and passing through the area of laser illumination on the sample to ensure that the illumination of the same sample area during rotation (Figure 2.6). Calibration of the rotating platform ensures that the same portion of the sample is illuminated when it is subject to rotation. A bare detector, also mounted atop the platform, collects the light scattered in a direction normal to the plane of the sample. This is made possible by placing a folding beam splitter at 45 degrees with respect to the plane of the sample. For smaller angles of incidence of the beam on the sample the input beam would pass through the beam splitter before reaching the sample and therefore gets laterally displaced and illuminates a different area of the sample.

To avoid this from happening one may adopt one of the two solutions. The first would be to use a Pellicle beam splitter, a very thin beamsplitter which displace the beam by very small amounts that may be ignored. But the main disadvantage of using Pellicle beam splitters is their tendency to vibrate which would mean that the system would have to be enclosed in enclosures that would shield the system from temperature fluctuations and provide vibration isolation. The second solution, which was what was adopted, was to use a compensation
glass slab before the beamsplitter, identical in thickness, that would rotate in the opposite
direction as the sample platform rotates to compensate for the lateral displacement at the
beamsplitter. There would still be angles of incidence for which the scattered beam cannot
be measured when the beam skims off the edge of the beamsplitter.
A disadvantage of the above mentioned approach is that only intensity of the scattered field
can be analysed using the current capability of the system. To realize field measurements a
reference beam may be launched to the sample plane of the detector. When a bare detector
collects the intensity measurements of the scattered light one would observe that the scattered
speckle pattern appears to translate across the detector and slowly begin to change in form.
The angular range after which the correlation between the speckle patterns at the detector
drops to $\frac{1}{e^2}$ from a reference speckle pattern as the sample is rotated would give an idea of
the isoplanatic angle of the sample. The main limitation of this system is the finite physical
extent of the sensor area which affects the correlation value computed.
In another configuration, by collecting scattered light at the specular angle corresponding
to each angle of incidence, instead of normal to the sample plane, the same set of angular
frequencies may be collected, thereby overcoming limitations posed by the finite extent of
the detector. This geometry would require the detector to undergo an additional rotation to
reorient itself in the specular angle, corresponding to the angle of illumination. A goniometric
setup (Figure 2.7) is envisioned where the detector rotates about the axis of rotation of the
sample as seen through the folding beamsplitter to mirror the change in incidence angle.

2.3. Experimental findings

There are two approaches to measure angular correlation - one is to image the scattering
surface in which case we get subjective speckle pattern and the second approach is to collect
the objective speckle pattern after scattering from the surface. For the former approach one
would require to access to scattered field information but for the latter the angular correla-
tion can be measured from intensity as the speckle pattern would appear to shift as the angle
of incidence is varied. This can be observed as a translation of the objective speckle pattern
Figure 2.8. Materials used for pairwise comparison of scatter properties

across the detector sensor area. Two main experiments were carried out using the testbed in Figure 2.6. One was to estimate the isoplanatic angle of different materials and to study the impact of different properties like roughness, color and subsurface scatter. The second experiment was focused on how to isolate the singly scattered component of the scattered light from the multiply scattered.

A series of experiments were conducted to understand change in isoplanatic angle as a function of roughness, absorption and subsurface scatter by making a pairwise comparison between different samples (Figure 2.8).

The surface may exhibit roughness of different scales which would have a profound impact on the phase perturbations imparted to the light scattered off it. One would expect a rougher surface to have a smaller isoplanatic angle when compared to the same surface with a more slowly varying height profile. Two reflective ground glass diffusers of the same material but with different grit sizes are compared to draw a parallel between their correlation lengths and isoplanatic angle.

To answer the question as to what color of wall would be more advantageous for the imager, the isoplanatic angles of two sandpapers of the same grit size and material but of different
colors are compared. Black color absorbs more light compared to other colors causing the light that penetrates beneath the surface of the black sample to get absorbed. While in the case of brown sandpaper the light that undergoes subsurface penetration scatters back reducing the isoplanatic angle of the material. From a light throughput standpoint, although a black wall would be less preferred, the superior isoplanatic angular range of black walls when compared to others make them a better choice.

When light penetrates through the surface layer of a sample and undergoes multiple scattering, the total scattered wavefront from the sample gets decorrelated faster with change in incidence angle. To validate this hypothesis, a white matte paper with significant subsurface scattering owing to its fibrous texture is compared with a ground glass diffuser that largely exhibits single surface scattering. A dramatic drop in the isoplanatic angle range can be observed in materials with significant multiple scatter which is evidenced by the boiling of the scattered speckle intensity as the angle of incidence of light to the sample is varied. The isoplanaticity exists not in angular space but in direction cosine space. This can be portrayed by plotting the curves obtained as a function of difference in cosines of the two angles (Figure 2.11).

2.3.1. Estimation of isoplanatic angle for different surface types

Intensity of scattered objective speckle patterns is collected using a bare detector (Figure 2.6). To computationally overcome the limitation posed by the finite extent of the detector,
Figure 2.10. Estimation of isoplanatic angle and pairwise comparison to estimate impact of roughness, color and subsurface scatter.
the correlation between speckle images are computed by subdividing the speckle images into smaller images to mimic a larger detector. This is depicted in Figure 2.9. The metric used for comparison between different samples is the normalized cross correlation between the speckle images.

2.3.1.1. Impact of Roughness scales

To study the impact of roughness scales of different materials, two rough mirrors with different grit values were compared - silver coated N-BK7 ground glass diffuser (1500 grit) and silver coated N-BK7 ground glass diffuser (120 grit). The former sample is smoother due to the higher grit size. It can be seen that speckle pattern appears to translate across the detector as the angle of incidence of the laser is changed and soon begins to boil and deform in structure depending on the roughness of the sample. As the 120 grit diffuser is rougher, the speckle begins to boil at a smaller angular separation. The correlation between the speckle images also indicate a similar trend (Figure 2.10). The 1500 grit diffuser has a larger isoplanatic angle than that of the 120 grit diffuser. As can be noted from the plots, for a value of say -0.5 on the Y axis the angular separation obtained on the X axis for 1500
grit diffuser is close to 2.65 degrees while for 120 grit diffuser it is only 0.45. This shows that the rougher the surface the smaller the isoplanatic angular range it would have.

2.3.1.2. Impact of absorption

Color of a rough surface decides how much of the incident light is absorbed within the material and how much is reflected. A black surface absorbs a lot of the light incident on it, while a different color would result in lesser amount of absorption. The part of light that is reflected from the black sample would hence be mostly singly scattered light. To test this hypothesis, two different colors of sandpapers were chosen of the same grit size, one black in color and the other brown (Figure 2.8). It was found that the isoplanatic angle of the black sandpaper exceeds that of the brown sandpaper as the light that is reflected back from the former is mostly singly scattered due to absorption loss within the material. From Figure 2.10), it can be observed that a value of say -2 on the Y axis is obtained only at the 73rd frame number for black sandpaper while in case of brown sandpaper this value is crossed at the 6th frame itself.

2.3.1.3. Impact of subsurface scatter

To understand the impact of subsurface scatter a mostly singly scattering surface like a black sandpaper and a white matte paper were chosen as object samples (Figure 2.8). The isoplanatic angle for matte paper is much less than that of the sandpaper. The fibrous texture of the paper allows light to penetrate through and undergo subsurface scattering which thereby limits the angular range within which memory effect is observed. From Figure 2.10), using the same argument as in the previous case, a value of say -2 on the Y axis is obtained only at the 73rd frame number for black sandpaper while in case of matte paper this value is crossed at the 6th frame itself.
2.3.1.4. *Isoplanaticity in direction cosine space*

Comparing the plots generated by comparing the speckle images against three different reference incidence angles it can be seen that the angular ranges are different. This is because as the angle of incidence increases the speckle pattern starts to decorrelate much faster. For grazing angles the speckle pattern would be completely decorrelated. The isoplanaticity is observed in direction cosine space and can be observed if one plots the curve against the difference in cosine of the the two angles (Figure 2.11). While in the first plot the X axis is the difference between the two angles, in the second plot the X axis is the cosine of the difference between the two angles to illustrate the difference.

2.3.2. *Isolation of singly scattered light from multiply scattered light*

The scattered light mainly comprises of surface scattered or singly scattered light and light that penetrates into the material of the sample and undergoes multiple scattering events before exiting the surface of the sample (Figure 2.12). The multiple scattering events experienced by sub-surface scattered light results in the isoplanatic angle range for such surfaces to be much more limited. Hence we can see that there are different scales of decorrelation. The singly scattered light remains correlated over a larger angle compared to multiply scattered components.

![Figure 2.12. Single and sub-surface scattering](image-url)
Figure 2.13. Flowchart for isolation of singly scattered light from multiply scattered light using Principal component analysis.
To separate the singly scattered light from multiply scattered light, the angular diversity of incidence beam is exploited, which is realized by rotating the sample about an axis passing through the point of illumination. The sequence of images is cut into a thinner slice and a window is cut from the slice in each image. This is done to mimic the architecture in Figure 2.7 which tries to always track the incidence angle. Using correlation as a metric a brute force registration of the windowed sub-images is done. To do a sub-pixel level registration, the sub-images are registered using an image registration algorithm developed by Keren et.al. The singly scattered component can be accentuated by averaging registered speckle patterns. The mean image is then subtracted from each of the registered sub-images. An Eigen value decomposition (Figure 2.13) is performed on the mean subtracted registered images, yielding Eigen images. The singly scattered component in the registered speckle sub-images remains largely unchanged and can be said to reside in a one dimensional subspace, while the multiply scattered component decorrelates within the measurement window and hence resides in a higher dimensional subspace. To isolate the singly scattered component from a
sub-image, it is expressed as a weighted superposition of these Eigen images. The component of the scattered light associated with the First Eigen image, after adding the mean image, corresponds to the singly scattered light while the orthogonal component corresponds to the multiply scattered light contribution. The analysis was done on 120 grit rough mirror and black sandpaper (Figure 2.14) and it was observed that when the multiply scattered component was removed computationally the isoplanatic angle for the surface also appeared to have increased.

2.4. Conclusion

After pairwise comparison of different materials, the following conclusions may be drawn. Isoplanatic angle increases as the surface of the material exhibits slower height fluctuations. Absorbing walls can help minimize the multiply scattered component in the reflected light and thereby improve the isoplanaticity of the surface. The current capability of the system allows measurement of intensity of the scattered speckle field.
Chapter 3

Indirect imaging based on imaging correlography techniques

3.1. Introduction

3.1.1. Motivation

Traditional imaging techniques allow imaging of objects that are in the line of sight of the camera. The objective of this work was to build a computational imager that makes use of coherent light sources to image objects that are completely hidden from the camera by employing the scattering nature of real world objects, like walls, in the scene. The scenario is depicted in Figure 3.1. To construct an image of an object, a light source is required that would illuminate the object and a detector that captures the light reflected off of the object. However in the indirect imaging scenario, both the light source and the sensor are hidden from the object. The nature of rough surfaces to scatter coherent light incident on it is used to create virtualized sources and detectors at the surface of rough surfaces in the scene, for instance walls, that are accessible both from the object side and from the viewer side. When a laser beam strikes a rough surface it scatters light part of which illuminates the hidden object. Hence it acts as a secondary source of illumination for the object. The object is hence illuminated by a speckle pattern and not necessarily uniformly illuminated (Figure 3.2). After reflection from the object surface a part of the light that makes its way back to a different part of the wall. This light distribution on the wall is imaged by a camera, which is equivalent to having a bare sensor at the plane of the wall, thereby virtualizing a detector at the wall (Figure 3.3).

Hence, the computational camera builds up an image of the hidden object by monitoring the subtle variations in intensity arising from the light bouncing off the hidden object and
Figure 3.1. Problem Statement. Uncovering intensity image of the object B hidden from the view in a corridor

redirected towards the wall. Borrowing insight from a technique that is widely used in satellite imaging, laser correlography [11] [12], the intensity image of the hidden object is retrieved. The novelty in the approach is that an object that is completely obscured from view could be recovered by monitoring the intensity fluctuations in the scattered object light intercepted by the wall and observed by the camera.

3.2. Theoretical background

3.2.1. Interference of two coherent point sources - Young’s Experiment

Light field at any point in space and time can be thought of as a vector with the magnitude of the vector containing information about the intensity or brightness of the light at that point and the phase of the vector conveying information about the direction of propagation of the light vector at the point. In case of incoherent light sources like fluorescent tubes, where the magnitude and phase of the vectors constituting the light field is random across space
Coherent light sources like lasers are highly directional beams which exhibit a high degree of correlation between two points in space and time. This means that knowledge of the phase and amplitude at a point in space or time can give information about the amplitude and phase information in a neighboring point. Two light sources which are coherent with respect to each other when combined gives rise to a light field with bright and dark regions. These regions arise due to the complex wave vector of the light field adding constructively and destructively at different locations in space and time. This property of light called interference was demonstrated by Young in the famous double slit experiment.
The experiment involved illuminating two small pinholes ($P_1$ and $P_2$) (Figure 3.4), that were very separated by a very small distance ($d$), with a light source that was equidistant from them. By the Huygens Fresnel principle, each of the pinholes would act like a point source of light which are coherent with respect to one another due to their close proximity. At a screen kept away from the pinholes dark and bright regions was observed due to interference of light. This interference fringe encodes the information about the separation between these two points sources of light. The distance from the pinholes to a point $P$ can be expressed as,
Figure 3.5. Fringe periodicity as a function of spatial separation between point like sources

\[ d_1 = \sqrt{(x - 0)^2 + \left(y - \frac{D}{2}\right)^2 + (a + b - a)^2} \]

\[ d_1^2 = x^2 + y^2 - yD + \frac{D^2}{2} + b^2 \]

\[ d_2^2 = x^2 + y^2 + yD + \frac{D^2}{2} + b^2 \]

\[ d_1^2 - d_2^2 = 2yD \quad (3.1) \]

Computing the difference between the geometric path traveled by two light rays from the two point sources to a point \( P \), assuming that the separation between the pinholes is very
small compared to distance between the plane of the pinholes and the plane of observation,

\[ d_1 - d_2 = \frac{2yD}{d_1 + d_2} = \frac{yD}{b} \]  \hspace{1cm} (3.2)

If the refractive index of the medium is \( n \), then the optical path length difference (opd) is given as a function of geometric path length difference (gpd),

\[ \text{opd} = n \times \text{gpd} \]  \hspace{1cm} (3.3)

The light field vectors from a point source may be expressed as,

\[ E(x, y, t) = \Re\left[ A(x, y)e^{j(\omega t + \phi(x,y))}\right] \]  \hspace{1cm} (3.4)

Where, \( A \) is the amplitude and \( \phi \) is the phase of the vector. The camera in the observation plane would only be able to sense the intensity information which is modulus squared of the light field. So when the light field arising from the two point sources combine at the detector, intensity as observed at the detector plane would be,

\[ I = |E_{P1} + E_{P2}|^2 = |E_{P1}|^2 + |E_{P2}|^2 + |E_{P1}||E_{P2}||\cos(\phi_1 - \phi_2) \]  \hspace{1cm} (3.5)

The phase difference \( \phi_1 - \phi_2 \) is related to the opd as,

\[ \phi_1 - \phi_2 = \frac{2\pi}{\lambda} \times \text{opd} = \frac{2\pi}{\lambda} \frac{nyD}{b} \]  \hspace{1cm} (3.6)

For values of \( y \) that causes Eqn.3.6 to be equal to 0, 2\( \pi \), 4\( \pi \), etc the camera intensity is given by,

\[ I = |E_{P1}|^2 + |E_{P2}|^2 + |E_{P1}||E_{P2}| \]  \hspace{1cm} (3.7)
For values of $y$ that causes Eqn. 3.6 to be equal to $\pi$, $3\pi$, $5\pi$, etc the camera intensity is given by,

\[ I = |E_{P1}|^2 + |E_{P2}|^2 - |E_{P1}| |E_{P2}| \tag{3.8} \]

Equations 3.7 and 3.8 explains how the intensity at the plane of observation is a pattern of bright and dark regions. The separation between two bright fringes is given by,

\[ \Delta y = \frac{b\lambda}{nD} \tag{3.9} \]

From Eqn. 3.9 it can be inferred that the information about the separation of the two point sources ($D$) that gave rise to the fringe at a distance $b$ from them can be obtained from the distance of separation of the fringe pattern on the detector. It should be noted at this point that the fringe pattern only encodes the distance $D$ between the pinholes and does not give the $(x, y)$ coordinates of the pinholes. The Young’s double slit experiment was replicated in two configurations using a camera with imaging optics imaging a rough surface like a wall where the Youngs fringes are formed (Figure 3.5). The separation between the pinholes could be ascertained by analyzing the fringe patterns on the sensor.

### 3.2.2. Imaging Correlography

When coherent light illuminates a rough surface, each pair of points on the surface would create a fringe pattern which contains the information about the separation between the two points. The scattered light spot is no longer uniform in intensity and phase but has a random granular structure associated with it. This is termed as ”laser speckle” and is caused due to the random height fluctuations in the surface which causes it to be rough in the optical scale. Speckle arises as a result of random optical path length variations introduced by the rough surface as the light rays strikes it.

By the same analogy if one were to think of an object being illuminated by a laser source, every point on the object would act as a point scatterer. The intensity pattern as seen by a detector placed at the observation plane would therefore be a superposition of pair
wise interference of many such point sources that comprises the entire object (Figure 3.5). This is the underlying principle used in imaging correlography [12], a technique used very widely in astronomical imaging to capture the image of satellites. According to [11], a laser correlogram is obtained from the power spectrum of the irradiance pattern scattered from the object when illuminated with sufficiently coherent radiation. The technique relies on a lensless imaging technique where the object is directly illuminated by coherent radiation and the reflected light is captured by bare detectors (Fig.3.6). The idea behind this approach is that the light field at the detector ($E_{detector}$) is related to the field at the object plane ($E_{object}$) through a transform relationship [11] [12] [13].

$$E_{detector} = \mathcal{F}\{E_{object}\}$$ (3.10)
Detectors are only capable of sensing the intensity of the light field incident on them unless holographic techniques are employed. The intensity of light field at the plane of the detector is given by,

\[
I_{\text{detector}} = |E_{\text{detector}}|^2 \\
= | \mathcal{F}\{E_{\text{object}}\} |^2 \\
= \mathcal{F}\{E_{\text{object}}\} \mathcal{F}^*\{E_{\text{object}}\} 
\]  

(3.11)

Inverse Fourier transform of above equation yields,

\[
\mathcal{F}^{-1}\{I_{\text{detector}}\} = \mathcal{F}^{-1}\{\mathcal{F}\{E_{\text{object}}\} \mathcal{F}^*\{E_{\text{object}}\}\} \\
= \text{ACF}(E_{\text{object}}) 
\]  

(3.12)

ACF($E_{\text{object}}$) is the autocorrelation of the object. Hence from just the intensity of the light field incident on the detector one is able to retrieve the autocorrelation of the object.

If the two point sources in the Young’s experiment can be thought of as two delta functions in space $\delta(0, y - \frac{D}{2})$ and $\delta(0, y + \frac{D}{2})$, then the intensity of the fringe pattern produced at observation plane is given by,

\[
I(x, y, a + b) = \mathcal{F}\left\{\delta(0, y - \frac{D}{2})\right\} \mathcal{F}\left\{\delta(0, y + \frac{D}{2})\right\} \\
= 2 + 2 \cos(2\pi D f_y) 
\]  

(3.13)

Where $f_y$ is the frequency of the fringe pattern.

The intensity or brightness of the object can be recovered from its autocorrelation only if
the phase of the electric field at the detector plane is known, i.e., the phase of the Fourier
transform of the object needs to be estimated. As was concluded from the previous section
the intensity of the fringe patterns only gives information about the separation of the point
sources that gave rise to the fringe at that distance and not the exact transverse location of
the point sources. This information is known only if the phase of the complex fringe pattern
is known. In conclusion to recover the object brightness a phase retrieval algorithm [14]
needs to be employed that recovers the phase of the Fourier transform of the object light
field. The autocorrelation of the object brightness gives information about the extent of the
object which is used as a support constraint to extract phase information using the phase
retrieval algorithm.

The intensity pattern at the detector is speckled as the object is rough at optical scales.
Hence the autocorrelation would appear speckled which makes phase retrieval very difficult.
This is mitigated in [12] and [11] by making use of the object translation or rotation.
When the object moves substantially the speckle intensity on the detector also changes. The
ensemble average over time of the modulus squared of the object autocorrelation over these
multiple speckle realizations will cause the resultant to approach the autocorrelation of the
object brightness.

3.3. Approach

The approach adopted to recover object albedo when it is hidden from the line of sight
of the detector borrows ideas from the techniques used in imaging correlography described
in the previous section. The object is located around a corner away from the detector (As
in figure 3.7). Laser light is used to coherently illuminate the hidden object indirectly. This
is accomplished by illuminating a rough surface like a wall in the scene that would scatter
light, part of which makes its way to the object. Every two points on the object would give
rise to a fringe pattern on a part of the wall which is then imaged by the camera focused at
it. So in this configuration, although the camera is no longer directly imaging the hidden
object, the fringe pattern it images contains information about the object which can be used
to retrieve the object brightness. Multiple speckle realizations required to get the incoherent autocorrelation of the object can be achieved by steering the laser spot on the wall. For experimental simplicity this was achieved by rotating the sample that was used as the section of the wall acting as virtual source of illumination of the object. Several calibration techniques are devised along the way to ensure that the speckle statistics at the object plane is such that ensemble averaging the multiple speckle realizations does not violate the mathematical assumptions made in object brightness recovery.

3.4. Experiment

A schematic of the experimental setup with the calibration units is shown in Figure 3.8. The coherent source used is a 532nm continuous wave laser (50mW). The laser source is fed through a beam delivery unit. The beam from the laser enters an isolator to prevent laser light which may be reflected back from any optical component in system from reentering
Figure 3.8. Schematic of experimental setup highlighting calibration steps

Beam attenuation is done using a variable beam attenuator designed using a polarized beam splitter and half wave plate combination to attenuate the laser beam if necessary. A periscope assembly launches the laser light at a desired height above the optical table. A spatial filter assembly comprising of an afocal system with an aperture in the focal plane removes higher order modes in the beam to retrieve a pure Gaussian beam. The beam is steered using two piezo actuated mirrors to control the alignment of the beam. The beam is then fed to a 10X beam expander to increase its size. Increasing the size of the beam allows one to focus the beam using a longer focal length optic and obtain smaller focus spots at standoff. The last section comprises of a steering mirror and lens assembly that is used to create focused spots or large expanded spot on the virtual source as required.

After the beam delivery unit the laser beam strikes a part of the wall (virtual source) and scatters part of the light towards the hidden object. The imager comprises of an IDS UI-328 CMOS sensor with a telecentric lens (TECM55) mounted on it. It images a section of the wall (virtual detector) where the light from the object creates intensity fluctuations. The experiment was conducted with ambient lights turned off so as to improve the contrast of
these intensity fluctuations that are imaged. Also the optical table was masked with black velvet and the test bed was enclosed using black posterboard.

3.4.1. System calibration

There are three additional cameras that were used initially within the experiment purely for calibration purposes.

- Calibration camera I: This imaging camera is calibrated such that it looks head on at the plane of the virtual source and images the laser spot illuminating it. The laser spot is elliptical due to the inclination of the wall. The main purpose of this camera is to assess the size of the laser spot illuminating the virtual source.

- Calibration camera II: It is bare detector placed right below the object in the same plane as the object. The purpose of this camera is to estimate the size of the speckles that illuminate the object plane.

- Calibration camera III: This imaging camera is positioned very close to the virtual detector. It captures the incoherent image of the object which is illuminated using a goose neck lamp with a 532nm color filter. This image would serve as a benchmark for comparison with the final output of the system.

3.4.2. Coherence area of speckle on the object plane

When the hidden object is illuminated using the light scattered off the wall it is no longer a uniform coherent beam illuminating it; it is a partially coherent speckle field that now illuminates the object. For the Fourier transform relationship between the object power spectrum and the intensity at the detector plane to hold true, the size of the speckle illuminating the object should be smaller than a wavelength. Said another way the coherence area of the speckle field illuminating the object should be very small. Coherence area is the area within which field value at two points lying within the area is correlated. A method to estimate the coherence area of speckle relies on specific statistics that needs to be satisfied by the speckle pattern.
The speckle pattern produced by a surface with roughness of the order of several wavelengths is called a "fully developed speckle". The intensity distribution of a fully developed speckle pattern is exponential with phase uniformly distributed between 0 and $2\pi$. According to [8], when the speckle field is fully developed, it allows us to relate the intensity autocorrelation of the light illuminating the object plane with the field correlation at that plane. This allows us to derive a simple expression for the speckle size at the object plane from which the coherence area is computed.

The coherence area of speckle illuminating the object is found to depend on the area of the laser spot illuminating the virtual source, the wavelength of light and on the distance between the virtual source plane and the object. If the virtual source is illuminated with a large spot it results in a smaller speckle size but at the cost of distributed light energy over a larger area which is disadvantageous from a radiometric standpoint. The coherence area at the object plane at a distance of $z$ from the virtual source is given by,

$$A_c = \pi r_s^2 = \frac{\lambda^2 z^2}{\pi \omega^2} \quad (3.14)$$

where, $w$ is the radius of the laser illuminating the virtual source, $r_s$ is the speckle radius and $\lambda$ is the wavelength of light. The derivation of this expression is provided with the supplementary material. The coherence area is proportional to the square of the speckle size. The coherence area may be computed either by estimating the size of laser illuminating the virtual source (using calibration camera I) or by calculating the autocorrelation of the intensity of speckle at the object plane (using calibration camera II).

When the speckle size illuminating object is very small its intensity autocorrelation may be approximated to a delta function, then equation 3.12 still holds true.

3.4.3. Distance from the object to virtual detector

The field at the object is related to the field at the virtual detector by a transform relationship. For the relationship to be Fourier transform, the distance between the object
and the virtual detector should be equal to the far field distance which is given by,

$$Z_{\text{far field}} = \frac{2D^2_{\text{obj}}}{\lambda} \quad (3.15)$$

Where $D_{\text{obj}}$ is the size of the object. Since the object in our experiment is not illuminated by a laser beam, the far field distance is no longer given by the above expression. The far field distance is given by the geometric mean of the coherence area of the speckle field at the object plane and area of the object $A_{\text{obj}}$ which is a much smaller distance.

$$Z_{\text{far field}} = \frac{2\sqrt{A_c A_{\text{obj}}}}{\lambda} \quad (3.16)$$

3.4.4. Multiple speckle realizations

From a single image captured by the detector, the autocorrelation obtained would not resemble the incoherent autocorrelation of the object due to scattering of the laser light. Many speckle realizations is required to get an optimized autocorrelation of the object from the Fourier transform of the intensity imaged by the camera (equation 3.12). The object needs to be illuminated using speckle fields which changes with time such that every point on the object is illuminated with a light field which is not correlated in time. Hence the autocorrelation of the object is obtained by temporal ensemble average of modulus square of the Fourier transform of the intensity recorded by the imager focused at the virtual detector,

$$\frac{1}{N} \sum_{n=1}^{N} |\mathcal{F}^{-1}\{I_{\text{detector}}\}|^2 \quad (3.17)$$

The above expression would have a very large dc peak [12] which can be eliminated by subtracting the mean image from the above expression giving,

$$\frac{1}{N} \sum_{n=1}^{N} |\mathcal{F}^{-1}\{I_{\text{detector}}\}|^2 - \frac{1}{N} \sum_{n=1}^{N} |\mathcal{F}^{-1}\{I_{\text{mean}}\}|^2 \quad (3.18)$$
where, \( I_{\text{mean}} \) is the mean of all the intensity images captured by the detector. The autocorrelation of the speckle pattern detected by calibration camera III, which is a bare detector placed at the virtual detector, gives an estimate of the expected object autocorrelation. Since we take the squared modulus of the Fourier transform of the intensity recorded, the phase terms in the object field are no longer accounted for. This means that even if we are not in the far field, as long as there is a transform relationship between the object and virtual detector plane, be it Fourier or Fresnel, the object information can be recovered from the autocorrelation.

3.4.5. Phase retrieval

Once the optimized autocorrelation of the object is obtained, a phase retrieval algorithm [14] is employed to extract the phase of the Fourier transform of the field at the detector plane. The intensity measured by the detector gives the amplitude of the Fourier transform of the object field. The average energy spectrum of the observed speckle image is estimated by averaging together the squared moduli of many independent speckled autocorrelations. The DC term is subtracted from this to obtain the diffraction-limited autocorrelation of the incoherent object. The square root of the Fourier transform of this incoherent object autocorrelation gives the modulus of the Fourier transform of the object intensity. Image of the hidden object is reconstructed from the Fourier modulus estimates by using the iterative Fourier transform algorithm. Sophisticated phase retrieval techniques may be employed to further improve the results obtained.

3.5. Experimental results

3.5.1. Direct illumination of object with laser beam - Inverse gold standard

One of the preliminary tests to be carried out was to make sure that the speckle image captured would in fact preserve the object information. The object was directly illuminated with a laser beam and the light reflected from it and incident on the virtual detector was
then imaged by the camera. A drywall panel painted with white paint with eggshell enamel sheen was used as the virtual detector. The reconstruction procedure adopted in acquiring the preliminary experimental results is given in Figure 3.11. The estimated autocorrelation of the object is displayed for three different cases (Figure 3.9). In the first case, no object was placed. The laser beam illuminated a white painted wall. Since the beam illuminating the wall was a circular disk shape, the autocorrelation also assumes a circular shape. However there is an elongation of the circular shape in the autocorrelation along the vertical direction. This is because of the relative pose of the white wall and the plane of the virtual detector. This experiment gives an intuition about the impact of object pose on the reconstruction of the image. Also, the camera image acquired does not seem to exhibit any definite pattern -
it has got a mottled speckle like appearance.

In the next two experiments an object with the shape of the letter X and the letter N are used it can be observed that the camera images shows structure. When using the object N it can be seen that the autocorrelation of the letter resembles that of a laterally flipped N. This observation can be attributed to the fact that there is a bounce or reflection of light at the virtual detector which causes lateral inversion of the image.

3.5.2. Bare detector in position of virtual detector Gold standard

In the experiment, the virtual source is a silver foil coated posterboard to throw enough light at the object which is a cutout of the letter N made from the same material. A bare detector is placed in the position of the virtual detector. The light illuminating the object is speckled and non uniform. The bare detector hence captures light reflected from the object in the direction of the virtual detector while the virtual source is rotated in-plane on a rotation stage in discrete steps so that temporally incoherent speckle fields illuminate the hidden object. The autocovariance of the speckle images is computed and run through the phase retrieval algorithm to recover the object brightness distribution.

3.5.3. Indirect imaging using imaging correlography

For the experiment, the virtual source surface was chosen to be a silver foil sample pasted on black posterboard (Figure 3.10) to allow more light throughput to the hidden volume. The virtual source was illuminated with area illumination spanning 5.5 inches. The hidden object used was a cutout of the letter "N" on a highly reflecting surface. A drywall panel painted with white paint with eggshell enamel sheen was used as the virtual detector.

As the estimate of the autocorrelation of the object is corrupted by speckle, averaging over multiple speckle realizations is essential. This can be achieved by illuminating the object with different speckle patterns by steering the virtual source location on the wall or by imaging a slightly different patch of the wall. Subdividing a single image captured by the detector into smaller tiles to fake many smaller detectors would be another workaround the problem
but at the expense of spatial resolution. 50 images of the virtual detector were captured by a CMOS CCD camera for different speckle realizations illuminating the object. Each image in the stack were further subdivided into tiles of 501x501 pixels. Computing a modulus of the Fourier transform of each sub-image results in an estimate of the autocorrelation of the hidden object (Figure 3.11). Averaging over all sub-images removes noise due to speckle and smooths out the autocorrelation estimate. The value of autocorrelation in DC does not provide any information and hence is subtracted. Phase retrieval algorithm is used to assemble the intensity image of the hidden object. The reconstructed image of the letter appears flipped laterally due to reflection off the virtual detector.

It should be noted that black velvet and black posterboards were used as surrounding walls and light blocks to avoid any light from the surroundings other than the object reflection to reach the virtual detector.
3.6. Understanding performance limits

3.6.1. Radiometric analysis

A radiometric analysis was carried out to estimate the light throughput in the system limited to 3-bounce light paths originating from the physical source bounced off the virtual source, the hidden object and the virtual detector finally terminating at the physical detector (Figure 3.13). The hidden object is illuminated by narrowband light scattered off the virtual source which is the only source of light. The following assumptions are made.

- Propagation medium is free space and homogenous
- Propagation distances exceed the spatial extent of the scene components
- Object is planar
- Scene components have a Lambertian BRDF
• Influence of speckle is mitigated by ensemble averaging

The incident irradiance on the virtual source can be expressed as,

\[ E_{VS}^{inc} = \frac{P_{laser}}{A_{beam}} \text{Wm}^{-2} \]  \hspace{1cm} (3.19)

Where \( P_{laser} \) is the incident laser power and \( A_{beam} \) is the area of the laser spot on the virtual source. The radiance from the virtual source in the object direction can be computed as,

\[ L_{VS}^{out} = E_{inc}^{VS} \frac{\rho_{VS}}{\pi} \left| \overrightarrow{n_{VS}} \overrightarrow{PQ} \right| \text{Wm}^{-2} \text{sr}^{-1} \] \hspace{1cm} (3.20)

\( \rho_{VS} \) is the object albedo and \( \frac{1}{\pi} \) is the Lambertian BRDF. \( n \) refers to the normal to the surface described by the subscript. The solid angle subtended by the object of area \( A_{obj} \) at
the virtual source is computed as,

$$\Omega_{obj} = \frac{A_{obj}}{||PQ||^2} \text{sr} \tag{3.21}$$

From the above equations the incident irradiance on the object can be computed as,

$$E_{inc}^{obj} = L_{out}^{V^S} \Omega_{obj} \left| \mathbf{n}_{obj}^T PQ \right| Wm^{-2} \tag{3.22}$$

Similarly computing the incident irradiance on the virtual detector we obtain,

$$E_{inc}^{VD} = L_{out}^{obj} \Omega_{VD} \left| \mathbf{n}_{VD}^T QR \right| Wm^{-2} \tag{3.23}$$

Where,
\[ L_{out}^{obj} = E_{inc}^{obj} \frac{\rho_{obj}}{\pi} \left| \mathbf{n}_{obj} \mathbf{\hat{T}} Q \mathbf{\hat{R}} \right| W m^{-2} \text{sr}^{-1} \] (3.24)

\[ \Omega_{VD} = \frac{A_{VD}}{\left| Q \mathbf{\hat{R}} \right|^2 \text{sr}} \] (3.25)

refers to the radiance from the object along the direction of the virtual detector and the solid angle subtended by the virtual detector area at the object plane respectively. The radiance of the virtual detector in the direction of the camera is,

\[ L_{out}^{VD} = E_{inc}^{VD} \frac{\rho_{VD}}{\pi} W m^{-2} \text{sr}^{-1} \] (3.26)

The flux in each pixel is given by,

\[ \Phi_{pix} = L_{out}^{VD} \times \left( \frac{\pi D_{cam}^2}{4 Z^2} \times m_T^{-2} A_{pix} \right) W \] (3.27)

Where, \( A_{pix} \) is the area of a detector pixel, \( D_{cam} \) is the diameter of the collection aperture, \( m_T \) is the imager magnification and \( Z \) is the distance between the entrance pupil of the camera and the virtual detector. \( m_T^{-2} A_{pix} \) is the area of a detector pixel as seen on the object side and \( \frac{\pi D_{cam}^2}{4 Z^2} \) is the solid angle subtended by the collection aperture at the virtual detector. The radiometric throughput is defined as the ratio of flux per pixel(\( \Phi_{pix} \)) to the laser power \( P_{laser} \),

\[ \text{Throughput} = \left\{ \frac{\rho_{VS} \rho_{obj} \rho_{VD}}{\pi^3} \right\} \times \left\{ \left| P \mathbf{\hat{Q}} \right|^{-2} \left| Q \mathbf{\hat{R}} \right|^{-2} Z^{-2} \right\} \times \left\{ \left| \mathbf{n}_{VS}^{T} P \mathbf{\hat{Q}} \right| \left| \mathbf{n}_{obj}^{T} P \mathbf{\hat{Q}} \right|^2 \left| \mathbf{n}_{VD}^{T} Q \mathbf{\hat{R}} \right| \right\} \times \left\{ \frac{A_{obj}}{A_{beam}} \frac{\pi}{4} D_{cam}^2 N_{pix} \left( m_T^{-2} \right)^2 A_{pix} \right\} \]
The first group of elements in the above equation represents the albedo and Lambertian BRDF loss, the second group represents the propagation loss and the third group represents foreshortening noise. From the above expression it can be inferred that the loss of light due to radiometry is more as the scale of the experiment becomes larger due to propagation loss.

3.6.2. Impact of size of virtual source

When the wall is illuminated by a laser spot, the size of the speckle grain that illuminates the hidden object is given by

$$\text{specklediameter} = \frac{\lambda Z_{\text{ill}}}{\pi D} \quad (3.28)$$

Where, $D$ is the size of the laser spot illuminating the wall, $Z_{\text{ill}}$ is the distance between the virtual source and the object and $\lambda$ is the wavelength of the laser source. The speckle size increases as the size of the virtual source decreases.

There are three regimes of operation when the size of the virtual source is varied (Figure 3.14). For a tightly focused spot on the wall, the speckle size illuminating the object maybe so large that the object may be almost homogenously illuminated by a single speckle grain. This would be almost equivalent to direct illumination of the object. So one would get the diffraction pattern of the illuminated object at the detector. As the size of the virtual source is increased gradually, one can see that there are multiple speckle grains that illuminate the hidden object and so the autocorrelation estimate does not look pristine. It has a mottled speckle like appearance. As the size of the spot is increased still further, such that the object is illuminated by a very large number of speckle grains the edges of the autocorrelation becomes more and more sharper. What we observe at the detector would be speckled speckle due to the object.
3.6.3. Impact of object reflectivity and pose

The object reflectivity and pose are critical. The hidden object should have enough contrast from the surrounding scene that the edges of autocorrelation is sharp and one maybe able to recover the object. Also the object pose is important as it needs to be such that it throws enough light back at the virtual detector plane.

3.6.4. Presence of multiple objects

Presence of multiple objects in the scene would result in light from all of them reaching the virtual detector. Hence the autocorrelation estimate becomes complicated that isolated recovery of the intensity image of each object becomes a challenge. The letters S and N created from silver foil were used as objects and positioned side by side in the same plane. Figure 3.15 shows the geometry of the experiment and the autocorrelation estimate obtained. When the two objects were moved away from one another the impact of aliasing was observed.
as the second order terms begin to wrap around once the object separation exceeds the pixel pitch of the virtual detector.

3.6.5. Impact of size of virtual detector

The resolution of the image obtained depends on the size or extent of the virtual detector. The image captured by the detector is subdivided into smaller subimages to to fake smaller detectors and the autocorrelation estimate was computed for each. One can see that the resolution of the image becomes poorer as the size of the subimage is lowered. The last image in Figure 3.16 can be seen to be heavily pixelated.

3.6.6. Pixel pitch of virtual detector

The field of view of the imager is a function of the size of the pixels on the virtual detector. To verify this theory, the image acquired by the detector was binned (Figure 3.17).
As the bin size increases the extent of the hidden volume accessible to the imager decreases. The autocorrelation estimate obtained hence increases in extent. Therefore to image larger objects one would need to have finer pixels at the virtual detector. Getting fine pixels at the virtual detector means that the magnification of the lens used to image the virtual detector should be close to 1. A telephoto macro lens with a 1:1 magnification at a large working distance close to 0.5m was used (Nikon AF FX Micro-NIKKOR 200mm f/4D IF-ED).

3.6.7. Impact of clutter

To learn the impact of the environment, black enclosures were replaced with white painted walls in the experiment. The light reaching the virtual detector is hence a superposition of light from the object and light from the surroundings. It would become very difficult to discern the light from the object from the light scattered from clutter in the scene. In the experiment conducted the speckle image (Figure 3.18) observed showed very little intensity.
fluctuations and autocorrelation estimate obtained from such an image was heavily unreliable.

In such a scenario the information about the object may be recognized if the object moves by very small amounts while the background remains unchanged. The object in the experiment was stepped through small incremental motion and the speckle images acquired by the detector were subtracted and upon Fourier transforming, fringe patterns could be observed whose periodicity gives us a scaled approximation to the motion of the object. This is again under the assumption that the only moving element in the scene is the object and it moves by very small increments so that the speckle patterns at the virtual detector does not fully decorrelate.

3.6.8. Background subtraction

There is a large part of light in the scene that bounces off surfaces multiple times and adds to an almost incoherent glow at the virtual detector. This may be eliminated by subtracting
Figure 3.18. Impact of clutter

an incoherent estimate of the glow from the speckle images by rapidly scanning the laser beam on the wall while capturing the images. This would wash out the speckle due to the object and the only term that would remain is the estimate of the incoherent glow which can be subtracted. Incoherent glow subtraction is seen to significantly improve the contrast and quality of the autocorrelation estimates.

3.7. Limitations

To lessen the impact of radiometric loss when working with scales of operation beyond 10m, it requires the use of high power lasers, larger collection apertures and detector sizes exceeding medium format. Other approaches that may be possible solutions to the problem could be the use of modulated light sources and lock in detectors. Another limitation of the experiment would be the impact of light scattered off other objects in the scene(clutter) which affects the system SNR. The issue was addressed in the experiment by ensuring the
optical table was coated with non reflective black material and the experimental test bed was enclosed within black poster boards.

### 3.8. Future work

Clutter in the scene poses challenges in terms of recovering object information as the pattern due to scattering from the object at the virtual detector gets swamped by light from other reflecting objects in the scene. Aliasing also poses difficulties for the phase retrieval algorithm. Hence methods to confine illumination on the object needs to be devised. Modulated sources of light or using coherence gating techniques may help to isolate the light scattered off from the object from the background.
4.1. Motivation

Indirect imaging using correlography techniques is fundamentally limited by the extent and resolution of pixels on the virtual detector. The extent of the virtual detector determines the resolution of the imager while the pixel size at the virtual detector determines its field of view. The imaging capabilities of this system could be expanded by developing a technique that would allow us to enhance the size and resolution of the virtual detector. We know from Chapter 2 that within the isoplanatic angle of the hidden object surface the scattered light would be highly correlated. This could be exploited by dithering the point like virtual source across the rough surface of the wall which is almost equivalent to illuminating the hidden objects with slightly different angles of incidence. The focused spot on the wall is small enough that the hidden object is illuminated by a plane wave. Structuring illumination around the corner allows access to spatial frequencies in the object which were previously inaccessible. To exploit this capability the notion of virtualizing an illumination source on the wall was extended to convert it instead into a virtualized pattern projector that illuminates the hidden object with light patterns. By focusing two point like sources on the plane of the virtual source, the scenario becomes comparable to Young’s double pinhole experiment (Figure 4.1). The two light spots behave like two apertures in a Youngs double pinhole experiment with different optical path length differences introduced by the roughness of the wall. Effectively after scattering from the wall the two scattered wavefronts have a large degree of correlation which results in the generation of a linear fringe pattern. Structured illumination of the hidden object could potentially also assist in uncovering other object information like object contour. Hence a rough surface, like a
Figure 4.1. Concept diagram

The experimental test bed designed to structure illumination in the hidden volume is shown in Figure 4.2. The experiment is designed borrowing ideas from a Mach Zehnder interferometer architecture. A Mach Zehnder interferometric architecture is repurposed to create two beams that emerge from the recombination beam splitter $BS_2$ with an angle between them that can be controlled by rotation of the beam splitter.

A laser beam (532nm, 50mW, 0.3mm diameter) is allowed to pass through a beam delivery system comprising of an isolator, beam attenuation unit, spatial filter assembly and a two mirror beam alignment unit. The isolator prevents any back reflected light from entering into the laser cavity. The beam attenuation unit comprises of a half waveplate and a non polarizing beam splitter used to control the laser power reaching the virtual source. It is important to make sure that the power does not burn the wall. A balanced spatial filter is designed using two identical plano convex lenses and a pinhole. The beam alignment unit folds the light path so that it is lined up with the holes on the optical table. The beam from the beam delivery unit is expanded using a 10X beam expander. The size of the beam...
entering the beam expander is about 1.2mm in diameter expanded due to propagation. The output beam size is about 12mm. This beam enters the interferometric arrangement where it is equally split into two identical beams using a 50:50 plate beam splitter and recombined at the exit port of the interferometer. The architecture of the interferometer is not balanced. This allows for the control of the angle between the two exiting beams. The beam splitter $BS_2$ is mounted atop a motorized rotation stage which allows for the separation of the beams in a plane parallel to the plane of the optical table. The beamsplitter is also equipped with other degrees of freedom that allows for precise alignment of the beams. The beam splitter coated surface of the beam splitter is positioned such that it faces away from the interferometer and lies approximately in the plane of the entrance pupil plane of an $f-\theta$ scan lens placed immediately after the interferometer. The 50mm focal length scan lens was used for initial experiments for its $f-\theta$ property. However in subsequent experiments it was found that the approach works with any well corrected optic as it was exceedingly difficult to find scan lenses with long focal lengths.
The scan lens focuses the two beams which come in at two slightly different angles at the plane of the virtual source to create two tightly focused spots on it. The virtual source now behaves as a virtual pattern projector projecting linear fringe patterns onto the hidden volume. The frequency of the fringes can be changed by rotating the beam splitter BS₂ which separates the two beams entering the scan lens in angle thereby adjusting the separation between the two focused spots on the wall.

4.3. Structuring illumination on a hidden object around a corner

The linearity and visibility of the fringe pattern is a function of the spot size at the wall, surface height fluctuations (roughness) within each focused spot, and the coherence time of the laser source.

4.3.1. Estimation of size of focused spot on the wall

A bare sensor is positioned with the image sensor parallel to the plane of the virtual source and slid into the location of the virtual source. The illumination optic is translated along the optical axis to realize different amounts of defocus on the spot illuminating the sensor. An ellipse is fit to each image in the stack of images the sensor captures for different amounts of defocus of the illumination optic. Ellipticity of the focused spot is a function of the relative pose of the wall with respect to the illumination. The size of the focused spot that illuminates the virtual source can then be estimated by fitting a curve to the major and minor axis values obtained. The method adopted closely resembles the procedure used in estimation of $M^2$ parameter of Gaussian beams [15].

4.3.2. Evaluation of structure illuminating hidden volume

To understand the structure illuminating the hidden volume, a bare sensor was positioned within the hidden volume and linear fringe patterns were observed which change in frequency as the distance between the focused spots changes.
4.3.2.1. Moire fringes

A Ronchi resolution target with 1lp/mm features is used as the hidden object. Upon illumination of the target with scattered light from the virtualized pattern projector low frequency Moire fringes appear on the target as a result of multiplication of the spatial frequency on the target with the frequency of the pattern illuminating it. The target was rotated to demonstrate the change in frequency of the Moire fringes (Figure 4.3).

4.3.2.2. Hidden object contouring around a corner

Placing an object with 3D structure like a curved ball in the path of the scattered light from the virtual source results in the curving of the fringe pattern as it gets distorted by the topography of the object (Figure 4.3). The behavior hints at the possibility of recognizing topography around a corner using a fully continuous wave approach.
4.3.3. Impact of spot size

The size of the speckle grains that illuminate the hidden volume is a function of the size of the virtual source amongst others. The ability of the virtualized projector to give rise to pristine looking patterns is limited by the size of the speckle grains. In other words as the number of scattering events increases with increase in size of spot on the wall, the scattered wavefronts due to the two focused spot become less and less correlated. This is evidenced in the distortion and subsequent disappearance of the fringe pattern in the light scattered off the wall. This makes it crucial to be able to realize such tightly focused light spots at standoff.

4.4. Computationally increasing the spatial extent of the Virtual Detector

A bare detector was mounted in place of the virtual detector that records the intensity at the virtual detector plane. It can be observed in the detector image that there are two nearly identical copies of speckle pattern. Illumination of the virtual source plane of the wall with a focused spot that is translated over time results in the speckle pattern observed on the Gold standard detector to shift correspondingly. The patterns observed on this Gold standard detector can be seen as shifted copies of one another. This was confirmed by checking the correlation between the different detector images. As the focused spot on the wall is translated the object is illuminated by a field that varies in angular space. Hence the field scattered from the object also rotates by the same angle if the angular changes are within the isoplanatic angle of the object. Hence the pattern on the detector appears to translate by an amount proportional to the translation of the virtual source spot. In conclusion, by scanning the spot on the virtual source, one can computationally expand the extent of the detector which in turn improves the resolution of the image. The patterns captured by the detector are mosaicked to mimic a virtual detector with a larger collection aperture.

Translation of focused spot on the virtual source plane by very small amounts could result in shifting the speckle imagery at the virtual detector plane to shift by sub-pixel amounts.
This is the key idea of digital superresolution where in sub-pixel shifts between the images captured results in a set of observations that differ in phase. The previously aliased high-frequency content in the scene can then be retrieved using this phase information to reconstruct a higher resolution image.

4.5. Discussion

Illuminating the hidden object with a linear fringe pattern riding atop a speckle pattern results in the appearance of fringe pattern on the autocorrelation estimate also. To verify the hypothesis, the two focused spots on the virtual source were turned one after the other and images were captured by a bare detector positioned in place of a virtual detector (Figure 4.6). The autocorrelation estimates acquired while illuminating the virtual source with one focused spot at a time were compared with that obtained when both focused spots were present simultaneously. As the separation between the two focused spots increases the fre-
frequency of the fringes also increases until a point where the detector resolution is insufficient to record these fluctuations in intensity in the scattered light. It can also be observed that although both the focused spots are of equal power the autocorrelation estimate obtained using the focused spot (point source-2) is not pristine. This is because when the spot is translated across the virtual source there would situations when the object is fully illuminated by a bright speckle grain or partially illuminated or worst case the object lies in the dark areas of speckle illumination.

As a preliminary experiment the virtual source was replaced with a 1” mirror so that the object is illuminated by a pristine fringe pattern. The experiment results are given in Figure 4.7. The inset image shows a magnified view of the autocorrelation estimate after post processing and the fringe patterns can be clearly observed. Post processing involves raising the image intensity values to the power of a natural number. This would help bring out the object information from the background fuss for display purposes. Image correlrography using virtualized pattern projection was carried out in the full geometry. The 1” mirror in the previous experiment was replaced by a white photopaper (Figure 4.8).
4.6. **Future work**

The ability to structure illumination around a corner opens up new capabilities for the computational imager - indirect estimation of object contour, superresolution of imagery obtained, to name a few. Extending the idea a little further, one may also conceive of a scenario wherein the virtual source is illuminated by a collection of spots phased appropriately to shape and steer illumination around a corner which would immensely improve the SNR of the imaging system.
Figure 4.7. Inverse gold standard pattern projector imaging
Figure 4.8. Indirect imaging using virtualized pattern projection
5.1. Motivation

The entrance pupil (EnP) of an imaging system is the image of the limiting aperture of the system as imaged through all the lens elements in front of the aperture. The computational imager designed involves virtualizing a detector plane at a rough surface to look around a corner. In this scenario, the entrance pupil of the system lies somewhere within the barrel of the lens used for imaging, away from the plane of the wall. Light scattered from the plane of the virtual detector reaches the entrance pupil plane and the lens performs a Fourier transform operation, i.e., the field at the detector plane would be the Fourier transform of the field at the entrance pupil (Figure 5.1). In this configuration, due to scattering at the virtual detector, one would not be able to perform stereo imaging around a corner. Stereo imaging involves the construction of a 3D image of a scene from different 2D images captured at different vantage points.

The geometric location of the entrance pupil is the center of perspective of the camera. The camera when rotated around it would be able to capture a larger extent of the angular spectrum. Moving the entrance pupil of the imaging system to the plane of the rough surface, the center of perspective of the imaging system is moved to plane of the rough surface. The detector captures the Fourier transform of the field at the plane of the surface. Hence by moving the imager in a plane parallel to the plane of the rough surface, slides the entrance pupil of the system along the plane of the rough surface (Figure 5.2). This way, one may be able to capture different images of the scene from different vantage points, thereby construct a 3D image of the scene around a corner. The camera in this case no longer images the wall,
instead images objects around the corner. Roughness of wall manifest as aberrations in the entrance pupil of the imaging system.

5.2. Approach

5.2.1. Geometry

One straightforward solution to moving the entrance pupil of the imaging system to a different plane in front of the lens barrel would be to place a physical aperture stop behind the lens system such that the image of this stop as seen through the lens system would be formed at the desired plane(Figure 5.2). In the figure 5.2, $u$ and $v$ represent the object and image distances for the imaging system.
The physical stop placed after the lens would behave as a field stop for the lens system. The system was modeled using Zemax and verified using standard lens equations.

The geometry of the problem was illustrated in Figure 5.3 is used to examine the feasibility of this approach. The object is directly illuminated with a light source. The object is assumed to be 1.5m behind the rough surface. An on axis system was used to demonstrate the concept. So the plane of the virtual detector (rough surface) is treated like a thin phase mask that scrambles the phase of the light incident on it, which is similar to the behaviour of a rough reflector. For the analysis done, the virtual detector plane is considered as a plane.
with zero phase distortions for easiness in modelling the system. The physical aperture is 0.5m behind the proposed location of the entrance pupil.

For the purpose of our study, an off the shelf Tessar lens (Figure 5.4) was used. The object plane was chosen as the origin and all distances in the table (Figure 5.4) were measured from the origin. Throughout the analysis, the numbers output from Zemax was verified using standard imaging relations.

5.2.2. Limiting aperture of an imaging system

As discussed above the entrance pupil of an imaging system is the image of the limiting aperture as seen through the lens elements in front of it. The image of every surface in a lens system as seen through surfaces preceding it subtends an angle at the object point. The image subtending the smallest angle at the object is the entrance pupil of the system and the surface that created this image is the limiting aperture of the system [16].

In Figure 5.5, three different configurations are presented to understand the limiting aperture of an imaging system. In the first case the image of the aperture within the Tessar lens as seen through lens elements prior to it subtends a smaller angle than subtended by the image of external physical aperture. This means that the limiting aperture of the system lies...
somewhere within the lens barrel and is not the external aperture. In the second case the two angles coincide. In this case also the limiting aperture is within the barrel. As it is the first plane that clips or limits the light rays through the system. The light rays pass exactly through the external aperture. For the external aperture to act as the limiting aperture, the angle subtended by its image should be the smallest.

- Location of physical aperture should be such that image of the physical aperture is formed at the plane of the wall

- Size of physical aperture should be such that image of physical aperture subtends an angle at the object less than the angle subtended by entrance pupil of the off the shelf lens system at the object

Figure 5.6 gives the angle subtended by the entrance pupil of an off the shelf Tessar lens (0.407 degrees). The external aperture should be chosen such that its image would subtend an angle that is smaller than this value.
Figure 5.5. Three configurations used to understand limiting aperture of an imaging system.

a) Angle subtended by image of physical aperture is bigger than angle subtended by entrance pupil of the Tessar lens

Physical aperture is not the limiting aperture of the system

b) Angle subtended by image of physical aperture is equal to the angle subtended by entrance pupil of the Tessar lens

c) Angle subtended by image of physical aperture is greater than the angle subtended by entrance pupil of the Tessar lens

Physical aperture is the limiting aperture of the system
5.3. Experiments

<table>
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<th>Distance (in mm)</th>
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</tr>
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</tr>
<tr>
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<td>2072.57</td>
</tr>
<tr>
<td>$Z_{A3}$</td>
<td>2072.57</td>
</tr>
</tbody>
</table>

Pupil magnification, $m_p = \frac{(z_{max} - z_{EnP})}{(z_{max} - z_{obj})} = -0.108$

Imaging relation, $m_p = \frac{1}{m_p} = \frac{1}{(z_{imag} - z_{EnP})} + \frac{1}{(z_{imag} - z_{obj})} = \frac{1}{f}$

$Z_{imag} = 2068.45\text{mm}$

Difference between $Z_{imag}$ from the imaging relations and using $z_{max} = 0.05\text{ mm}$

Figure 5.6. Size of the entrance pupil in an off the shelf Tessar lens

Figure 5.7. Imaging lens and aperture system
An external aperture is positioned behind the off the shelf Tessar lens to virtually move the entrance pupil of the imaging system to the proposed plane of the rough surface (Figure 5.7). From the figure it can be noted that the image plane is before the aperture stop. Placing a physical detector at this location would not be feasible for this system. So a relay lens is required to relay the image onto a detector plane after the external aperture. When choosing the relay lens caution must be exercised to ensure that the aperture stop within the relay lens system does not behave as the limiting aperture for the entire system (Figure 5.8). This means that the angle subtended at the object plane by the image of the aperture within the relay lens as seen through the group of lenses preceding it should be greater than that subtended by the newly positioned entrance pupil of the system. The complete system with the imaging lens, the external aperture and the relay lens is illustrated in Figure 5.9.
5.4. Discussion

From the Zemax simulations carried out it can be inferred that one may be able to reposition the center of perspective of an imaging system in a plane positioned in front of the system. By scanning this imager in a plane parallel to the plane of the entrance pupil it would be possible to get images of the object from different view points hence enabling stereo imaging.

There are additional details that need to be noted when the model is implemented in a real system. The object in the model is a fixed object whose distance behind the rough surface is known. The precision with which this value is known would limit the precision with which one may move the entrance pupil to the plane of the rough surface. An on axis geometry for the model for the sake of simplicity. However, in the indirect imaging model this approach would have to be generalized. In the hallway scene considered the angle subtended by the object at the plane of the virtual detector is very large which would make the system heavily

\[
Pupill\ magnification, \quad m_p = \frac{Z_{\text{img}} - Z_{\text{obj}}}{Z_{\text{obj}} - Z_{\text{pp1}}} = 0.18
\]

Imaging relation, \( m_p = \frac{-1}{m_p (Z_{\text{obj}} - Z_{\text{pp1}})} + \frac{m_p}{(Z_{\text{img}} - Z_{\text{pp1}})} = \frac{1}{f} \)

\[
Z_{\text{img}} = 2115.88 \text{mm}
\]

Difference between \( Z_{\text{img}} \) from the imaging relations and using zemax = 0.19 mm

<table>
<thead>
<tr>
<th>Surface/plane</th>
<th>Distance (in mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Z_{\text{obj}} )</td>
<td>Object plane</td>
</tr>
<tr>
<td>( Z_{\text{pp1}} )</td>
<td>Entrance pupil plane</td>
</tr>
<tr>
<td>( Z_{\text{pp1}} )</td>
<td>First principal plane</td>
</tr>
<tr>
<td>( Z_{\text{img,intermediate}} )</td>
<td>Intermediate image plane</td>
</tr>
<tr>
<td>( Z_{\text{A5}} )</td>
<td>Physical aperture</td>
</tr>
<tr>
<td>( Z_{\text{img}} )</td>
<td>Image plane</td>
</tr>
<tr>
<td>( Z_{\text{pp1}} )</td>
<td>Exit pupil plane</td>
</tr>
<tr>
<td>( Z_{\text{pp1}} )</td>
<td>Second principal plane</td>
</tr>
</tbody>
</table>
off axis. For every object position on axis the position of the external aperture would have to changed very precisely and at the same time it needs to be ensured that it remains to be the limiting aperture of the system. Due to the presence of a physical aperture after the lens the amount of light collected would be limited by its extent. The impact on image reconstruction remains to be ascertained.
Bibliography


