THE MAGNETIC FIELD BETWEEN A DOUBLE COIL

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One method of obtaining a uniform magnetic field is by means of a double coil. This consists of two coaxial coils[1], such as those used in the Helmholtz galvanometer.

The magnetic field between the coils may be found by assuming a power series to represent the potential of points off the axis and then applying Laplace's equation[2].

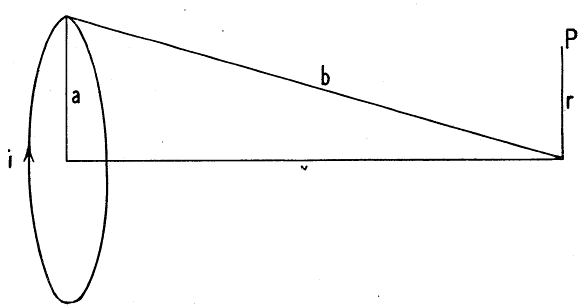

FIG. 1

Fig. 1 shows a single coil carrying a current $= i$ electromagnetic units. Let the radius of the coil be represented by a and the axis be drawn in the x-direction with the origin at the center of the coil. The coil may be replaced by a magnetic shell of strength i. The boundary of the shell coincides with the coil.

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[1]This arrangement of coils may be used in the Kaufman method of finding e/m so as to insure a uniform field over the region traversed by the electron beam. See J. B. Hoag, "Electron Physics", pp. 48-50.

[2]This is essentially the method used in A. Gray, "Absolute Measurements in Electricity and Magnetism", 2nd ed. 1921, p. 211. However, it seems to the writer that the conventions and material used are difficult for the student first approaching the problem, and likely to be confusing unless an undue amount of time is taken. The equations given in this paper follow at once from material conventially given to advanced undergraduate or first-year graduate students.

The magnetic potential at points external to the shell must satisfy Laplace's equation. For points on the axis the potential is[3]

$$V=2\pi i\left(1-\frac{x}{(a^2+x^2)^{\frac{1}{2}}}\right). \tag{1}$$

The potential of a point, such as P, in Fig. 1, a distance r off the axis, must satisfy equation (1) as a boundary condition.

It will now be assumed that the potential at points off the axis may be expressed in a series in r of the form

$$V=2\pi i(A_0+A_1r^2+A_2r^4+\cdots). \tag{2}$$

The condition of symmetry requires that the series be restricted to even powers. Since, when r = 0, V must reduce to equation (1),

$$A_0=\left(1-\frac{x}{(a^2+x^2)^{\frac{1}{2}}}\right). \tag{3}$$

A_1, A_2, . . . are functions of x and may be evaluated with the aid of Laplace's equation.

Laplace's equation in cylindrical coordinates for the conditions represented in Fig. 1 is written in the form[4]

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial r^2}+\frac{1}{r}\frac{\partial V}{\partial r}=0. \tag{4}$$

Substituting in equation (4) the terms obtained by differentiating equation (2), the result is

$$\left(\frac{\partial^2 A_0}{\partial x^2}+\frac{\partial^2 A_1}{\partial x^2}r^2+\cdots\right)+(2A_1+12A_2r^2+\cdots)+(2A_1+4A_2r^2+\cdots)=0$$

By equating the coefficients of the different powers of r separately to zero, it follows that

$$A_1=-\frac{1}{2^2}\frac{\partial^2 A_0}{\partial x^2}, \qquad A_2=\frac{1}{2^2\cdot 4^2}\frac{\partial^4 A_0}{\partial x^4}, \ldots\ldots \tag{5}$$

[3]See Page, L., and Adams, N. I., "Principles of Electricity", p. 264.
*Ibid., p. 86.

Applying equation (3), the coefficients A_1, A_2, . . . may be found in terms of a and x. Equation (2) then becomes

$$V=2\pi i\left\{\left(1-\frac{x}{(a^2+x^2)^{\frac{1}{2}}}\right)-\frac{1}{2^2}\,\frac{3xa^2}{(a^2+x^2)^{\frac{5}{2}}}\,r^2+\frac{3\cdot5}{2^2\cdot4^2}\,\frac{a^2(4x^3-3a^2x)}{(a^2+x^2)^{\frac{9}{2}}}\,r^4-\cdot\cdot\right\}$$

The axial component of the magnetic field is

$$H_x=-\frac{\partial V}{\partial x}.$$

By differentiating the equation for the potential, it is now possible to express the field as follows,

$$H_x=2\pi i\,\frac{a^2}{(a^2+x^2)^{\frac{3}{2}}}\left\{1+\frac{3}{2^2}\,\frac{(a^2-4x^2)}{(a^2+x^2)^2}\,r^2+\frac{3^2\cdot5}{2^2\cdot4^2}\,\frac{(a^4-12a^2x^2+8x^4)}{(a^2+x^2)^4}\,r^4+\cdot\cdot\right\}$$

If now two coils are used, with N/2 turns to each coil,

$$H_x=2\pi N\,\frac{I}{10}\,\frac{a^2}{b^3}\left\{1+\frac{3}{2^2}\,\frac{r^2}{b^4}\,(a^2-4x^2)+\frac{3^2\cdot5}{2^2\cdot4^2}\,\frac{r^4}{b^8}\,(a^4-12a^2x^2+8x^4)+\cdot\cdot\right\}$$

for points on a plane half-way between the coils. The coils are connected in series and the directions of the current in each coil made so that the magnetic fields produced are aiding. For simplicity, b^2 is put in place of a^2+x^2. I represents the current in amperes, $I/10$ replacing i. The magnetic field is in gausses.

The series is convergent for values of r less than b. The final equation for the magnet field may therefore be used for points on a plane half-way between the two coils provided the r coordinates are small compared to b.