Design and Nonlinear Control of a Haptic Glove for Virtual Palpation

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DESIGN AND NONLINEAR CONTROL OF
A HAPTIC GLOVE FOR VIRTUAL PALPATION

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DESIGN AND NONLINEAR CONTROL OF
A HAPTIC GLOVE FOR VIRTUAL PALPATION

A Dissertation Presented to the Graduate Faculty of the

Bobby B. Lyle School of Engineering

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with a

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by

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This dissertation presents the design, kinematic analysis, and nonlinear control of a Haptic Glove for medical elastographic imaging virtual palpation. Of the 13 degrees of freedom present in the index finger, middle finger, and thumb of the hand, the design fixes 4, constrains 2 and controls 6 with pneumatic air cylinder actuators, allowing uncontrolled, but measured motion in the remaining 1 degree of freedom. Nearly linear bijective transfer functions between the actuator positions and joint angles are found in closed form for all 6 actuated joints. A nonlinear, sliding-mode controller that allows each actuator to be controlled by a single 5/3 proportional valve is designed and implemented. Test results for typical palpation motions are presented and discussed.
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I dedicate this dissertation to my parents.
Robots have been envisioned as electromechanical devices that may replace human beings in performing various specialized tasks. The main obstacle that has been impeding the wider usage of robotics is the difficulty involved in building a machine that has the cognitive skills of a human being. This leads to the emergence of an alternative trend in robotics based on merging human beings and robots to produce a human machine interface that combines the mental skills of the human with the superior physical abilities of the machine.

Haptic interfaces are systems that fit into the category of this class of devices. Such systems include a human-machine interface (the master) that is used to link the operator with a remote man made mechanism (the slave). The interface enables the operator not only to manipulate the slave, but also to feel the reaction forces that the slave experiences with its environment. A variety of systems with different scales, ranging from the human power amplifier technology [17] to micro-surgical systems [26], are currently in the testing and implementation stage. Highly sensitive dexterous devices are being developed, such as the PHANToM at MIT [19].

A novel direction in the area of telemanipulation is based on virtual-reality or virtual-environment systems. In these types of systems, the computer generates the slave mechanism and the remote environment [8, 12, 4, 15, 18, 19]. For example, the Pneumatically-driven Haptic Interface (PHI) developed at Southern Methodist University [8, 15, 27, 28], enables the operator to interact with computer-generated slaves and environments, providing both visual and realistic force feedbacks. Haptic
interfaces can be used in exploration of patient data, assisting and participating in remote medical procedures and surgery, and other exciting possibilities that have been outside the realm of conventional robotics.

Recent developments in ultrasound imaging techniques can produce elastometric imaging scans that include local elasticity properties of tissues. Ultrasound Critical-angle Reflectometry (UCR) was developed specifically to obtain in vivo or in vitro elastometric scans of hard tissues such as bone and cartilage [2, 20]. In addition, recently Acoustic Diffraction Tomography (ADT) was developed to obtain for the first time elastometric scans of soft tissue such as breast and liver tumors [21, 3, 22, 25, 33, 34, 37].

Today, ultrasound plays a key role in the diagnostic of cancerous tumors, providing high sensitivity and specificity [36]. Recent research revealed that mechanical properties of abnormal growths can be an important factor in identifying cancerous tumors [35]. This may play a significant role in early detection of cancer. Nevertheless, a significant number of misdiagnoses still occur [14]. A false-positive result in diagnostic imaging leads to an unnecessary biopsy and patient stress. A false-negative has serious implication as the diagnosis of cancer is delayed, leading to potentially worse clinical outcome for the patient.

Doctors have used their touching (palpating) ability to examine patients since the inception of the medical profession. Yet, this has been limited to direct external examination of a patient. The combination of speed-of-sound (SOS) images (bulk modulus) with elastography (shear modulus) and a haptic interface that embeds the human hand into virtual reality offers a unique technological opportunity in innovative medical imaging and diagnostics. The advent of the developments in computer and ultrasound techniques and nonlinear control methods has made the idea of touchable medical scans a reality. The proposed system will open up the entire human body
to complete physical examination. An example data flow diagram of the proposed virtual tumor palpation system is shown in Fig. 1.1.

The Haptic Glove presented in this research is the prototype haptic device for medical elastographic imaging virtual palpation. Other haptic devices for the human hand have been developed, such as the Exo-Hand by FESTO [6], the Cyber Grasp by Cyber Glove Systems [11], and an exoskeleton proposed by Jo and Bae [16]. However, these designs fail to control all degrees of freedom in the fingers, instead acting upon
the collective movement of the finger joints. This simplified finger-bending model is sufficient to approximate grasping of a virtual object and to provide a realistic force feedback to the operator. Nevertheless, multiple fingertip locations may correspond to the same position of the haptic actuators. The intended virtual palpation usage of the Haptic Glove requires a more accurate model to precisely compute the location of the fingertip and to respond to small movements within the joints. The Haptic Glove design aims to maximize control over not just the overall bend of the finger, but to uniquely determine the precise location of the fingertip using minimal number of sensors and actuators without sacrificing user comfort or freedom of movement.
2.1. Bones and Joints

To aid in the discussion of the Haptic Glove, we will use the established medical nomenclature to refer to each segment of the fingers and thumb within the hand.

A *phalanx* (plural: *phalanges*) is a single segment of the finger between two joints. Each finger is made up of three phalanges while the thumb consists of only two phalanges. Phalanges within a given finger are distinguished by their distance from the hand. The furthest phalanx containing the fingertip and fingernail is called the *distal phalanx*. The next phalanx is called the *intermediate phalanx*, and the phalanx immediately adjacent to the hand is called the *proximal phalanx*. In the case of the thumb, the intermediate name is dropped, and the two thumb phalanges are simply the proximal and distal phalanges. Each finger and thumb has an additional bone in its structure that is contained within the palm of the hand called the *metacarpal*. The finger metacarpals reside within the palm, do not contribute to motion of the hand, and thus can be largely ignored within the context of this research. However, thumb metacarpals have a significant range of movement and will require discussion. One final term that will aid in the naming of joints later is the *carpus*, which refers to the collection of eight bones that make up the human wrist and join the thumb metacarpal and the four finger metacarpals to the arm [1].

Each adjacent pair of bones in the fingers and thumb are connected by a joint and this joint is named according to the names of the two bones it joins. Thus,
a joint connecting two phalanges is called an interphalangeal joint. As before, the two interphalangeal joints in each finger are distinguished as the proximal interphalangeal joint and distal interphalangeal joint. The joints connecting the proximal phalanx of each finger to its respective metacarpal (often referred to as knuckles) are called metacarpophalangeal joints. So, each finger is made of four segments and three joints: the distal phalanx, the intermediate phalanx, the proximal phalanx, the metacarpal, the distal interphalangeal joint, the proximal interphalangeal joint, and the metacarpophalangeal joint. There is a final joint that can be attributed to each
finger, the *carpometacarpal joint* which joins the metacarpals to the carpus inside the wrist. Only the thumb carpometacarpal is capable of discernible movement so it is the only carpometacarpal joint that will be discussed in this research. The thumb carpometacarpal joint is responsible for giving primates the opposable thumbs required to grasp things effectively. All of the phalanges and joints are labeled in Fig. 2.1 [1].

### 2.2. Motion in the Hand

The human hand is capable of many types of motion, though this research is only concerned with motion of the fingers and thumb. Motion occurs at the joints of each finger and each joint is capable of specific types of motion.

![Figure 2.2: Motions of the Fingers (a) Flexion and Extension Motion (b) Abduction and Adduction Motion](image)

The interphalangeal joints can only be bent in one direction, moving the phalanges of a given finger through a plane of possible fingertip positions. The motion that causes an interphalangeal joint to bend, decreasing the angle between adjacent phalanges and curling the finger or thumb into a fist, is called *flexion*. The reverse
of this motion that increases the angle between adjacent phalanges and causes the fingers to straighten is called extension. Flexion and extension are the only motions the two interphalangeal joints on each finger are capable of performing. The metacarpophalangeal joint on each finger is also capable of flexion and extension motion, but can additionally move side to side, and this motion is referred to as abduction and adduction. Abduction and adduction movements are differentiated by whether the body’s structure is spreading out or closing together. In the case of the hand, motion is named relative to middle finger. Spreading fingers away from the middle finger is called abduction. The opposing motion that brings the fingers back towards the middle finger is called adduction [1]. Figure 2.2 shows the different motions of the fingers [7].

![Figure 2.2: Motions of the Fingers](image)

(a) Flexion and Extension Motion (b) Abduction and Adduction Motion

Motions of the thumb carry the same names, but instead of a second interphalangeal joint, the thumb has a carpometacarpal joint that adds a new degree of
freedom. The interphalangeal joint of the thumb is capable of only flexion and extension movement. However, both the metacarpophalangeal and carpometacarpal joints of the thumb are capable of both flexion/extension and abduction/adduction. The abduction/adduction range is very minor for the metacarpophalangeal joint, but significant for the carpometacarpal joint, which has the largest abduction/adduction range of any joint in the hand. The plane created through flexion and extension of the thumb is also rotated away from rest of the fingers by about 90°, and this angle can become bigger or smaller through the abduction and adduction of the thumb carpometacarpal joint [1]. Figure 2.3 shows the different motions of the thumb [7].
The human hand has 27 degrees of freedom [1], but not all are required to simulate virtual palpation. For simplicity, the Haptic Glove only interfaces with the distal phalanges of the index finger, middle finger, and thumb of the user, which are the three primary fingers utilized during palpation. Additionally, certain degrees of freedom are coupled to others to reduce the number of actuators, while other degrees of freedom are either constrained to a fixed position, or left unactuated. The final design, shown in Fig. 3.10, allows for 7 independent degrees of freedom. All 7 degrees are measured by linear potentiometers, but only 6 of the 7 are actuated by pneumatic cylinders [10].

The large variance in human hands makes it challenging to create a single Haptic Glove design that would fit all users. This problem was addressed by incorporating several adjustable regions of the glove design where certain measurements could be lengthened or shortened to best fit the current user’s hand geometry. Statistical measurements of the human hand were acquired from the publicly released 1991 Hand Anthropometry of U.S Army Personnel. It was deemed infeasible to design enough adjustable features with a large enough range to cover the entire population represented by this study. Therefore, a reduced range covering the upper 75% of the female population and the lower 75% of the male population was accommodated, where possible [13]. In theory, concepts from the presented Haptic Glove design could be replicated on a smaller and/or larger Haptic Glove to cover the remaining extremes of the population, but such gloves were not pursued by this research. The final design
has 12 adjustable connections that cover the most important points across the hand.

3.1. Index Finger Substructure

Figure 3.1: Index Finger Substructure - Mechanical

The Haptic Glove design includes 4 main substructures: the index finger, the middle finger, the thumb, and the palm. The index finger substructure, shown in Fig. 3.1 consists of multiple curved plates that partially wrap around each phalanx of the user’s index finger along with a system of linkages above the phalanges that direct the force from two pneumatic actuators into the finger. An initial pivot plate lies above the index finger metacarpophalangeal joint and connects the index finger substructure to the rest of the glove. The proximal phalanx plate is attached to this pivot plate and consists of two parts that are fixed by a bolt in a track. By tightening the bolt at different points along the track, the proximal phalanx plate can accommodate any proximal phalanx length between 36.5 mm and 49 mm. This is the first example of several adjustable features on the Haptic Glove design. The proximal phalanx plate is connected to the pivot plate by a shoulder screw through a
ball-bearing. This connection forms an axle that allows the proximal phalanx plate to bend, tracking the user’s movement as they flex the metacarpophalangeal joint of their index finger. This motion can be actuated by the proximal index finger pneumatic cylinder connected to the pivot plate. The rod of the pneumatic cylinder attaches above the proximal phalanx plate, inducing a moment about the metacarpophalangeal joint. A linear potentiometer is mounted alongside the pneumatic actuator to track the current displacement of the actuator. By measuring the displacement of each actuator with potentiometers, it is possible to calculate the exact bend angle of each joint in the finger, and by extension the location of the fingertip relative to the base of each finger substructure. The details of this computation will be examined in Section 4.1.

Figure 3.2: Gearing-Linkage System Detail

Continuing down the index finger substructure, there is an intermediate phalanx plate with a fixed length of 25 mm followed by a distal phalanx mounting arm. A
finger cup is attached to the distal phalanx mounting arm via another adjustable track that supports distal phalanx lengths between 16 mm and 21 mm. Another series of linkages attach above the intermediate and distal phalanx plates, but unlike the metacarpophalangeal joint, the two interphalangeal joints covered by these plates are not free to move independently. Most humans find it difficult to flex and extend their proximal and distal interphalangeal joints independently, and the design of the Haptic Glove takes advantage of this fact to simplify the index and middle finger substructures. Instead of allowing independent motion in the two interphalangeal joints, the index and middle finger substructures couple the bend angles of the two interphalangeal joints through a series of gears. A simple pair of 16-tooth nylon gears in a 1:1 ratio together with a symmetric linkage structure connecting the proximal phalanx plate and the distal phalanx arm to the intermediate phalanx plate constrains the bend angle of the proximal interphalangeal joint to always match the bend angle of the distal interphalangeal joint. Figure 3.2 shows a more detailed view of this gearing mechanism on the Index Finger substructure. A few parts have been made transparent to allow a better view of the gears within the intermediate phalanx plate.

Empirical data of the proximal and distal interphalangeal joint angles of a volunteer asked to perform a natural flexion of both interphalangeal joints of their index finger is shown in Fig. 3.3. As can be seen from the graph, this movement does not exactly align with the dashed line, which shows the 1:1 interphalangeal joint angle ratio enforced by the Haptic Glove. However, the intermediate phalanx plate is only loosely held against the user’s intermediate phalanx and this same volunteer reported no discomfort in using the Haptic Glove. This suggests that the enforced 1:1 interphalangeal joint angle ratio is a sufficient approximation for the natural simultaneous flexion movement of the interphalangeal joints. The coupled flexion and extension of the interphalangeal joints can be actuated by the distal index finger pneumatic
Figure 3.3: Interphalangeal joint angle dependency in the motion of a volunteer cylinder, which is anchored on the proximal phalanx plate and connected to the interphalangeal linkage system. Like the proximal actuator, the distal actuator also has a linear potentiometer alongside it to measure the displacement of the actuator.

Each actuator requires two pneumatic tubes to the two chambers of the cylinder and each potentiometer requires two voltage wires and a signal wire. It is also necessary to be able to sense the current pressure in each cylinder chamber in order to build a closed-loop controller for the system. To this end, two pressure sensors are introduced in the pneumatic network via two 3-way connectors. It is important to
install the pressure sensors as close to the pneumatic cylinders as possible to reduce the delay in sensing the chamber pressures. A small PCB accompanies each actuator-potentiometer pair to provide power to the potentiometer as well as to each pressure sensor. This PCB also consolidates all signal and power lines to a single plug for each actuator, consisting of power, ground, the cylinder rod displacement signal from the potentiometer, and two pressure signals. The index finger substructure with all of these pneumatic and electrical components is shown in Fig. 3.4

3.2. Middle Finger Substructure

The middle finger substructure, shown in Fig. 3.5 is similar to the index finger substructure, except the overall design is mirrored and the lengths of the phalanges are slightly increased to account for the longer middle finger. The parts connecting the middle finger substructure to the rest of the glove are also considerably different, since they must connect to the far end of the glove, opposite the thumb, and therefore must bridge over the remaining two fingers. Where the index finger has a pivot plate securing the index finger substructure to the rest of the glove, the middle finger has a non-pivoting slider bar that connects the middle finger substructure to the
rest of the glove. Once again, the proximal phalanx is split into pieces that are joined by an adjustable track. This allows the glove to accommodate middle finger proximal phalanges between 38.5 mm and 50.5 mm. The proximal pneumatic cylinder is again mounted before the metacarpophalangeal joint, in this case on the slider bar. Actuation is achieved through the rod of the pneumatic cylinder connected to the proximal phalanx. A gearing linkage nearly identical to that present on the index finger substructure houses the intermediate and distal phalanges. The only difference is the length of the intermediate phalanx plate, which is extended to 31 mm for the middle finer substructure. These fixed-length intermediate phalanges are necessary to allow for a simple gearing linkage and are intended to be oversized for most users. Since the only part that directly wraps around the finger is the finger cup, it isn’t crucial that the user’s phalanx fits exactly in the mechanism. Forces are only applied at the finger tip so haptic sensations are still perceived correctly.
The adjustment on the finger cap of the middle finger substructure accommodates distal phalanges between 17 mm and 23 mm. The distal actuator connects above the proximal phalanx plate and actuates the gear-linkage system just like in the index finger substructure. Once again, potentiometers aligned with both actuators sense the current displacement of the two cylinder rods. The same pneumatic topology and accompanying PCB are also included for each actuator in the middle finger substructure as shown in Fig. 3.6.

3.3. Thumb Substructure

The thumb substructure, shown in Fig. 3.7 is considerably different than the finger substructures. To simplify the design of the Haptic Glove, many degrees of freedom of the thumb are constrained. Whereas the human thumb is typically considered to have 5 degrees of freedom, the thumb substructure allows for only 2 degrees of freedom,
fixing the other 3 degrees of freedom in a position that allows for typical palpation of a virtual object. Only the flexion and extension motions of the interphalangeal joint and metacarpophalangeal joints are left available for the user. The abduction and adduction of the metacarpophalangeal joint, as well as both flexion/extension and the abduction/adduction of the metacarpophalangeal joint are all constrained to a fixed position by the thumb substructure. A series of plates bent at fixed angles connect the thumb substructure to the rest of the glove. These plates wrap around the entire carpometacarpal joint of the thumb. Importantly, one connection between these series of plates is secured by a thumb screw. This permits the user quick removal of the majority of the thumb substructure to allow the entire Haptic Glove to
be put on and taken off. Without this release mechanism, the glove would be impossible to wear and remove. Similar to the finger proximal phalanx plates, the thumb proximal phalanx plate is split into two pieces and the connection between these two pieces forms an adjustable feature that supports thumb proximal phalanx lengths between 28 mm and 34 mm long. The thumb substructure also has a distal phalanx arm similar to the finger substructures that likewise supports a thumb cup. The adjustable attachment point accommodates distal thumb phalanges between 17 mm and 34 mm long. Unlike the finger substructures, there is no gearing linkage system in the thumb substructure. This is because the motion dependency between the two interphalangeal joints on each finger is not present between the interphalangeal and metacarpophalangeal joints of the thumb. Most humans are capable of moving each thumb joint independently to some extent and in fact, tend to favor bending
either the interphalangeal joint or metacarpophalangeal joint and leaving the other joint straight when it comes to overall flexion of the thumb. Therefore, the Haptic Glove design must be able to support either flexion movement independently to accommodate the user’s preference. Each joint therefore has its own actuator and accompanying potentiometer bridging over the two implemented flexion joints. Figure 3.8 shows the complete thumb substructure with the electronics and pneumatic tubes attached.

3.4. Palm Substructure

![Palm Substructure](image)

Figure 3.9: Palm Substructure, viewed from the fingertips.

All 3 finger substructures are anchored to the central palm substructure, shown in Fig. 3.9. To improve clarity, one part from each finger substructure is repeated in the figure. The central plate of the palm substructure is secured over the user’s hand by 3 braces that wrap around to the underside of the palm. One brace tucks between the
user’s index finger and thumb, while the other two braces are on a separate curved plate on the other side of the hand, opposite the thumb. The attachment between this curved plate and the main plate is adjustable to allow for any palm width between 85 mm and 99 mm. All 3 braces that wrap to the underside of the palm can be secured independently at different heights, with ranges that vary depending on the brace. This allows the Haptic Glove to fit palms between 25 mm and 31.5 mm thick.

Each substructure’s connection to the palm substructure is unique. The pivot joint of the index finger substructure allows the entire substructure to rotate. This permits the user to abduct the index finger up to $20^\circ$ away from the middle finger. This abduction/adduction of the index finger is the one degree of freedom not actuated in the Haptic Glove design. However, there is still a linear potentiometer across the top surface of the palm plate that measures the current angle of the index finger abduction. This measurement allows for the computation of the index fingertip position during use. The middle finger substructure attaches to the palm plate via two tracks that allow adjustment left and right by 14 mm, altering the separation between the index and middle finger metacarpophalangeal joints. However, the middle finger substructure does not allow any rotation, so abduction/adduction of the user’s middle finger is constrained. The thumb substructure is similarly connected to the palm plate by two tracks that can be adjusted up and down the length of the palm by 13.5 mm to accommodate users with longer or shorter hands by altering the distance between the index finger metacarpophalangeal joint and the thumb carpometacarpal joint. Combining all substructures together forms the complete Haptic Glove design, shown in Fig. 3.10. In total, the Haptic Glove contains 6 actuators, 7 potentiometers, and 12 adjustable features, and permits the user 7 degrees of freedom across the index finger, middle finger, and thumb. More details about the geometry of individual parts can be found in Appendix A.
Figure 3.10: The complete Haptic Glove design
Chapter 4

EQUATIONS OF MOTION

4.1. Kinematics

An analytical geometric approach was used to find the transformation functions from the actuator positions to the angles in each joint. These transformation functions are required as part of the virtual work derivation to calculate the required force in each cylinder for a desired fingertip force.

4.1.1. Index Finger Kinematics

Figure 4.1 shows the geometry of the index finger substructure of the Haptic Glove. The measurements of relevant fixed dimensions of the structure are shown in millimeters. The index finger substructure also includes two adjustable lengths and two pneumatic actuator cylinders of variable length. Lengths $d_{11}$ and $d_{12}$ account for the adjustable segments, restricted to $36.5 \text{ mm} \leq d_{11} \leq 49 \text{ mm}$ and $20 \text{ mm} \leq d_{12} \leq 25 \text{ mm}$. Each actuator is incorporated into the rest of the Haptic glove design through additional pieces, making calculations based on the exact tip of the actuator rod rather cumbersome. Therefore, geometrical calculations are performed using the parameters $a_{11}$ and $a_{12}$ instead. These variables measure the current actuator displacements plus the relevant structure around the actuators, resulting in more convenient calculations. $a_{11}$ measures the distance between points B and C, which includes the current displacement of the proximal actuator on the index finger. Similarly, $a_{12}$ measures the current displacement along the distal actuator on the index
finger from point F to a small coupler causing a 7 mm offset from point G.

We begin by using the law of cosines to find $\angle BAC$.

$$\angle BAC = \cos^{-1} \left( \frac{(23^2 + 46^2) + (6.25^2 + 33^2) - a_{11}^2}{2\sqrt{23^2 + 46^2}\sqrt{6.25^2 + 33^2}} \right)$$

(4.1)

Defining $\theta_{11}$ as the flexion of the metacarpophalangeal joint from an open palm, we have

$$\theta_{11} = \tan^{-1} \left( \frac{46}{23} \right) + \angle BAC + \tan^{-1} \left( \frac{33}{6.25} \right) - 180^\circ$$

(4.2)

Plugging in and simplifying,

$$\theta_{11} = \cos^{-1} \left( 1.092 - 2.895 \times 10^{-4} a_{11}^2 \right) - 37.29^\circ$$

(4.3)
This equation is graphed in Fig. 4.2. This transformation function is very close to linear and so could be approximated by the linear equation

$$\theta_{11} \approx 1.85a_{11} - 60.6^\circ$$  \hspace{1cm} (4.4)

where $\theta_{11}$ is given in degrees. There is no component interference during the extension of the proximal actuator on the index finger, so the entire 1.5-inch range of motion is available for use. This translates to $32 \text{ mm} \leq a_{11} \leq 70.1 \text{ mm}$, which results in a total bend range of $0^\circ \leq \theta_{11} \leq 72^\circ$ for the metacarpophalangeal joint on the index finger.

![Figure 4.2: Index Finger Metacarpophalangeal Joint $\theta_{11}$ vs. Proximal Actuator Displacement $a_{11}$](image)

Continuing for the interphalangeal joints, we can find $\angle FEG$ using a very similar approach as before, albeit with a more complicated triangle.

$$\angle FEG_1 = \cos^{-1} \left( \frac{((38 - 8.25)^2 + (58 - 24)^2) + (6^2 + 16^2) - (a_{12}^2 + 7^2)}{2 \sqrt{(38 - 8.25)^2 + (58 - 24)^2} \sqrt{6^2 + 16^2}} \right)$$  \hspace{1cm} (4.5)
We also need to find $\angle DEF$, which is defined by fixed pieces and is thus a constant.

$$\angle DEF_1 = \cos^{-1}\left(\frac{(8.25^2 + 24^2) + ((38 - 8.25)^2 + (58 - 24)^2) - (38^2 - 58^2)}{2\sqrt{8.25^2 + 24^2}(38 - 8.25)^2 + (58 - 24)^2}\right) \quad (4.6)$$

Using the above, we can solve for $\angle DEH$. This angle is integral to the gearing mechanism and drives the geometry of the remaining structure of the index finger. The phalanx shells and the arms supporting them were specifically designed to be symmetric, meaning at all times $\angle DEH = \angle DIH$. This equality additionally extends to the other two shell/arm connections around the distal interphalangeal joint. As this angle is so important to the geometry calculations and is located at the gearing interface, it is labeled $\theta_{1g}$.

$$\theta_{1g} = \angle DEH_1 = 360^\circ - \angle DEF_1 - \angle FEG_1 - \tan^{-1}\left(\frac{6}{16}\right) \quad (4.7)$$

Plugging everything in and simplifying,

$$\theta_{1g} = \sin^{-1}\left(1.479 - 6.477 \times 10^{-4} a_{12}^2\right) + 91.66^\circ \quad (4.8)$$

From here, we can compute the distance $\overline{DH}$ using another iteration of the law of cosines.

$$\overline{DH}_1 = \sqrt{33^2 + (8.25^2 + 24^2) - 2(33)\sqrt{8.25^2 + 24^2}\cos \theta_{1g}} \quad (4.9)$$

By symmetry, $\angle EDI = 2\angle EDH$, so we can compute $\angle EDI$ as

$$\angle EDI_1 = 2\cos^{-1}\left(\frac{\overline{DH}_1^2 + (8.25^2 + 24^2) - 33^2}{2\overline{DH}_1\sqrt{8.25^2 + 24^2}}\right) \quad (4.10)$$

Finally, defining $\theta_{12}$ as the flexion of one of the interphalangeal joints (which are forced to be equal by the gearing mechanism) from an open palm, we have

$$\theta_{12} = 2\tan^{-1}\left(\frac{24}{8.25}\right) + \angle EDI_1 - 180^\circ \quad (4.11)$$
After plugging in, this expression is best simplified in terms of $\theta_{1g}$ as

$$\theta_{12} = 2 \cos^{-1}\left(\frac{33.84 - \cos \theta_{1g}}{\sqrt{3081 - 2978 \cos \theta_{1g}}}\right) - 37.94^\circ$$  \hspace{1cm} (4.12)

By combining Eqs. 4.8 and 4.12, we get an expression for $\theta_{12}$ in terms of $a_{12}$. The combined transformation function is graphed in Fig. 4.3. Once again, the transformation is very close to linear, so we could approximate $\theta_{12}$ as

$$\theta_{12} \approx 4.15a_{12} - 132.7^\circ$$  \hspace{1cm} (4.13)

where $\theta_{12}$ is given in degrees. In the case of the distal actuator on the index finger, the gear arms collide with the phalanx shells on both sides of the gearing mechanism at $90^\circ$. This interference has been intentionally inserted into the design as a safety measure ensuring that the Haptic Glove will never exert force beyond a $90^\circ$ bend in the interphalangeal joints even in the case of controller error. As a result, the entire

![Figure 4.3: Index Finger Interphalangeal Joint $\theta_{12}$ vs. Distal Actuator Displacement $a_{12}$](image)

$y = 4.1509x - 132.7$
1-inch range of motion for the distal actuator on the index finger is not available. The bend angle is restricted to $0^\circ \leq \theta_{12} \leq 90^\circ$, corresponding to an actuator range of $32.0 \text{ mm} \leq a_{12} \leq 52.9 \text{ mm}$, utilizing 20.9 mm, or 82% of the 1-inch actuator stroke. This also limits the gear angle as $72.3^\circ \leq \theta_{1g} \leq 146.6^\circ$, although the direction of movement in $\theta_{1g}$ is opposite in sign of $\theta_{12}$, so that $\theta_{1g}$ attains its minimum value as $\theta_{12}$ attains its maximum value, and vice-versa.

The Haptic Glove applies forces at the fingertip. Thus, it is required to know the index fingertip location vector, $\vec{r}_1$ which can be defined using $\theta_{11}$, $\theta_{12}$, $d_{11}$, and $d_{12}$.

$$\vec{r}_1 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = d_{11} \begin{bmatrix} \cos \theta_{11} \\ \sin \theta_{11} \end{bmatrix} + 25 \begin{bmatrix} \cos(\theta_{11} + \theta_{12}) \\ \sin(\theta_{11} + \theta_{12}) \end{bmatrix} + d_{12} \begin{bmatrix} \cos(\theta_{11} + 2\theta_{12}) \\ \sin(\theta_{11} + 2\theta_{12}) \end{bmatrix}$$

(4.14)

Figure 4.4: 2-Degrees of Freedom Planar Workspace of the Index Finger Substructure
Figure 4.4 shows the workspace of the index fingertip. This workspace assumes the origin is centered at the metacarpophalangeal joint with $x$ positive towards the fingertip of the outstretched hand and $y$ positive in the direction of the finger flexion. By measuring actuator lengths $a_{11}$ and $a_{12}$, it is possible to compute $\theta_{11}$ and $\theta_{12}$ and, by extension, find the user’s index fingertip location, $\bar{r}_1$, within this workspace [10].

4.1.2. Middle Finger Kinematics

An equivalent procedure can be applied to the middle finger. All leading indices of the terms introduced in Subsection 4.1.1 are incremented in the following subsections to indicate which finger they correspond to. A leading index of 1 corresponds to the index finger, 2 to the middle finger, and 3 to the thumb.
Figure 4.5 shows the geometry of the middle finger substructure of the Haptic Glove. Once again, the measurements of relevant fixed dimensions of the structure are shown in millimeters. Like the index finger, the middle finger substructure also includes two adjustable lengths and two pneumatic actuator cylinders of variable length. Lengths $d_{21}$ and $d_{22}$ account for the adjustable segments, restricted to $28.5 \text{ mm} \leq d_{21} \leq 50.5 \text{ mm}$ and $22 \text{ mm} \leq d_{22} \leq 28 \text{ mm}$. Similar to the index finger, $a_{21}$ and $a_{22}$ measure the current actuator displacements plus the relevant structure around each actuator. $a_{11}$ measures the distance between point B and a coupler offset from point C by 7 mm and accounts for the displacement of the proximal actuator on the middle finger. $a_{12}$ measures the current displacement along the distal actuator on the middle finger from point F to a similar coupler offset 7 mm from point G.

Following the same procedure as before to find $\angle BAC$. 

$$
\angle BAC_2 = \arccos\left(\frac{(23^2 + 38^2) + (6.25^2 + 33^2) - (a_{21}^2 + 7^2)}{2\sqrt{23^2 + 38^2}\sqrt{6.25^2 + 33^2}}\right)
$$

Defining $\theta_{21}$ like before as the flexion of the metacarpophalangeal joint from an open palm, we have

$$
\theta_{21} = \arctan\left(\frac{38}{23}\right) + \angle BAC_2 + \arctan\left(\frac{33}{6.25}\right) - 180^\circ
$$

Plugging in and simplifying,

$$
\theta_{21} = \arccos\left(1.023 - 3.352 \times 10^{-4}a^2_{11}\right) - 41.91^\circ
$$

This equation is graphed in Fig. 4.6. Like the index finger, this transformation function is also very close to linear and so could be approximated by the linear equation

$$
\theta_{21} \approx 1.96a_{21} - 59.2^\circ
$$

where $\theta_{21}$ is given in degrees. There is no component interference during the extension of the proximal actuator on the middle finger, so the entire 1.5-inch range of motion
is available for use. This translates to \(28.8 \text{ mm} \leq a_{21} \leq 66.9 \text{ mm}\), which results in a total bend range of \(0^\circ \leq \theta_{21} \leq 76.7^\circ\) for the metacarpophalangeal joint on the middle finger.

Figure 4.6: Middle Finger Metacarpophalangeal Joint \(\theta_{21}\) vs. Proximal Actuator Displacement \(a_{21}\)

We next find \(\angle F E G\) using the same method as before.

\[
\angle F E G_2 = \cos^{-1} \left( \frac{(46 - 11.25)^2 + (60 - 25)^2 + (6^2 + 16^2) - (a_{22}^2 + 7^2)}{2 \sqrt{(46 - 11.25)^2 + (60 - 25)^2} \sqrt{6^2 + 16^2}} \right) \quad (4.19)
\]

Once again, \(\angle D E F\) is a constant and can be found by

\[
\angle D E F_2 = \cos^{-1} \left( \frac{(11.25^2 + 25^2) + ((46 - 11.25)^2 + (60 - 25)^2) - (46^2 - 60^2)}{2 \sqrt{11.25^2 + 25^2} \sqrt{(46 - 11.25)^2 + (60 - 25)^2}} \right) \quad (4.20)
\]

Once again, the symmetric design of the middle finger substructure ensures that the gearing angle is matched across the support arms so that \(\angle D E H = \angle D I H\). We
additionally label this gear angle \( \theta_{2g} \), just as before.

\[
\theta_{2g} = \angle DEH_2 = 360^\circ - \angle DEF_2 - \angle FEG_2 - \tan^{-1}\left(\frac{6}{16}\right) \tag{4.21}
\]

Plugging everything in and simplifying,

\[
\theta_{2g} = \sin^{-1}(1.587 - 5.933 \times 10^{-4} a_{12}^2) + 90.01^\circ \tag{4.22}
\]

From here, we compute the distance \( DH \).

\[
\overline{DH}_2 = \sqrt{33^2 + (11.25^2 + 25^2) - 2(33)\sqrt{11.25^2 + 25^2} \cos \theta_{2g}} \tag{4.23}
\]

By symmetry, \( \angle EDI = 2 \angle EDH \), so we can compute \( \angle EDI \) as

\[
\angle EDI_2 = 2 \cos^{-1}\left(\frac{\overline{DH}_2^2 + (11.25^2 + 25^2) - 33^2}{2\overline{DH}\sqrt{11.25^2 + 25^2}}\right) \tag{4.24}
\]

Finally, defining \( \theta_{22} \) as the flexion of one of the interphalangeal joints (which are forced to be equal by the gearing mechanism) from an open palm, we have

\[
\theta_{22} = 2 \tan^{-1}\left(\frac{25}{11.25}\right) + \angle EDI_2 - 180^\circ \tag{4.25}
\]

After plugging in, this expression is best simplified in terms of \( \theta_{2g} \) as

\[
\theta_{22} = 2 \cos^{-1}\left(\frac{15.67 - 18.86 \cos \theta_{2g}}{\sqrt{601 - 590.8 \cos \theta_{2g}}}\right) - 48.46^\circ \tag{4.26}
\]

By combining Eqs. 4.22 and 4.26, we get an expression for \( \theta_{22} \) in terms of \( a_{22} \). The combined transformation function is graphed in Fig. 4.7. Once again, the transformation is very close to linear, so we could approximate \( \theta_{22} \) as

\[
\theta_{22} \approx 4.11 a_{22} - 158.9 \tag{4.27}
\]

where \( \theta_{22} \) is given in degrees. Just like the index finger, the gear arms intentionally collide with the phalanx shells on both sides of the gearing mechanism at 90\(^\circ\) as a safety measure. As a result, the entire 1-inch range of motion for the distal actuator on
the middle finger is also not available. The bend angle is restricted to $0^\circ \leq \theta_{22} \leq 90^\circ$, corresponding to an actuator range of $38.3 \text{ mm} \leq a_{22} \leq 59.4 \text{ mm}$, utilizing $21.1 \text{ mm}$, or 83% of the 1-inch actuator stroke. This also limits the gear angle as $59.8^\circ \leq \theta_{2g} \leq 135.8^\circ$, although the direction of movement in $\theta_{2g}$ is once again opposite in sign of $\theta_{22}$, so that $\theta_{1g}$ attains its minimum value as $\theta_{22}$ attains its maximum value, and vice-versa.

The middle fingertip location vector can be defined using $\theta_{21}$, $\theta_{22}$, $d_{21}$, and $d_{22}$.

$$\vec{r}_2 = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = d_{21} \begin{bmatrix} \cos \theta_{21} \\ \sin \theta_{21} \end{bmatrix} + 31 \begin{bmatrix} \cos(\theta_{21} + \theta_{22}) \\ \sin(\theta_{21} + \theta_{22}) \end{bmatrix} + d_{22} \begin{bmatrix} \cos(\theta_{21} + 2\theta_{22}) \\ \sin(\theta_{21} + 2\theta_{22}) \end{bmatrix}$$

(4.28)

Just like with the index finger, this workspace assumes the origin is centered at the metacarpophalangeal joint with $x$ positive towards the fingertip of the outstretched hand and $y$ positive in the direction of the finger flexion. Figure 4.8 shows the workspace of the middle fingertip [10].
4.1.3. Thumb Kinematics

Since the thumb substructure contains no gearing mechanism, the thumb kinematics are simpler than the fingers. Figure 4.9 shows the geometry of the thumb substructure of the Haptic Glove. As before, measurements are shown in millimeters. Although there is one less moving joint in the thumb substructure, there are still two pneumatic actuators of variable length. Length $d_{31}$ and $d_{32}$ account for the adjustable segments, restricted to $28 \text{ mm} \leq d_{31} \leq 34 \text{ mm}$ and $23 \text{ mm} \leq d_{32} \leq 30 \text{ mm}$. Once again, variables $a_{31}$ and $a_{32}$ measure the current actuator displacements plus the relevant structure around each actuator. Like the previous fingers, $a_{31}$ measures the
Figure 4.9: Haptic Glove Thumb Geometry Detail. Fixed lengths are shown in millimeters. Actuator lengths are labeled $a_{31}$ and $a_{32}$. Adjustable lengths are labeled as $d_{31}$ and $d_{32}$.

distance between point B and a coupler offset from point C by 7 mm and accounts for the displacement of the proximal actuator on the thumb. Likewise, $a_{32}$ measures the current displacement along the distal actuator on the thumb from point E to a similar coupler offset 7 mm from point F.

The same procedure as before can be repeated to find $\angle BAC$.

$$\angle BAC_3 = \cos^{-1} \left( \frac{(28^2 + 44^2) + 32^2 - (a_{31}^2 + 7^2)}{2 \sqrt{28^2 + 44^2} \cdot 32} \right)$$  (4.29)
Defining $\theta_{31}$ like before as the flexion of the metacarpophalangeal joint of the thumb, we have

$$\theta_{31} = \tan^{-1} \left( \frac{44}{28} \right) + \angle BAC - 90^\circ \quad (4.30)$$

Plugging in and simplifying,

$$\theta_{31} = \cos^{-1} \left( 1.107 - 2.996 \times 10^{-4} a_{31}^2 \right) - 32.47^\circ \quad (4.31)$$

Figure 4.10: Thumb Metacarpophalangeal Joint $\theta_{31}$ vs. Proximal Actuator Displacement $a_{31}$

This equation is graphed in Fig. 4.10. As with the previous fingers, this transformation function is also very close to linear and so could be approximated by the linear equation

$$\theta_{31} \approx 1.88a_{31} - 56.4 \quad (4.32)$$

where $\theta_{31}$ is given in degrees. There is no component interference during the extension of the proximal actuator on the thumb, so the entire 1.5-inch range of motion is
available for use. This translates to $29.6 \text{ mm} \leq a_{31} \leq 67.7 \text{ mm}$, which results in a total bend range of $0^\circ \leq \theta_{31} \leq 73.1^\circ$ for the metacarpophalangeal joint on the thumb.

The structure connecting the distal actuator to the interphalangeal joint of the thumb is nearly identical to the structure connecting the proximal actuator to the metacarpophalangeal joint, so we use the same procedure one last time to find $\angle EDF$.

$$\angle EDF_3 = \cos^{-1}\left(\frac{(17^2 + 46^2) + (6.25^2 + 33^2) - (a_{32}^2 + 7^2)}{2\sqrt{17^2 + 46^2} \sqrt{6.25^2 + 33^2}}\right)$$  \hspace{1cm} (4.33)

Defining $\theta_{32}$ flexion of the interphalangeal joint of the thumb, we have

$$\theta_{32} = \tan^{-1}\left(\frac{46}{17}\right) + \angle BAC_3 + \tan^{-1}\left(\frac{33}{6.25}\right) - 180^\circ$$  \hspace{1cm} (4.34)

Plugging in and simplifying,

$$\theta_{32} = \cos^{-1}\left(1.058 - 3.036 \times 10^{-4}a_{32}^2\right) - 31.01^\circ$$ \hspace{1cm} (4.35)

This equation is graphed in Fig. 4.11. This final transformation function is also

![Figure 4.11: Thumb Interphalangeal Joint $\theta_{32}$ vs. Distal Actuator Displacement $a_{32}$](image)
very close to linear just like the other transformation functions and so could also be approximated by the linear equation

\[
\theta_{32} \approx 1.79a_{32} - 46.9 \tag{4.36}
\]

where \(\theta_{32}\) is given in degrees. There is no component interference during the extension of the distal actuator on the thumb either, so the entire 1.5-inch range of motion is available for use. This translates to \(25.7 \text{ mm} \leq a_{32} \leq 63.8 \text{ mm}\), which results in a total bend range of \(0^\circ \leq \theta_{32} \leq 69.2^\circ\) for the interphalangeal joint on the thumb.

The thumb fingertip location vector can be defined using \(\theta_{31}, \theta_{32}, d_{31},\) and \(d_{32}\).

\[
\vec{r}_3 = \begin{bmatrix} x_3 \\ y_3 \end{bmatrix} = d_{31} \begin{bmatrix} \cos \theta_{31} \\ \sin \theta_{31} \end{bmatrix} + d_{32} \begin{bmatrix} \cos(\theta_{31} + \theta_{32}) \\ \sin(\theta_{31} + \theta_{32}) \end{bmatrix} \tag{4.37}
\]
Just like with the index and middle fingers, this workspace assumes the origin is centered at the metacarpophalangeal joint with $x$ positive towards the fingertip of the outstretched thumb and $y$ positive in the direction of the thumb flexion. Figure 4.12 shows the workspace of the thumb fingertip [10].

4.2. Generalized Coordinates and Generalized Forces

The equations of motion for each finger in the glove can be derived from Lagrange's formulation.

$$\mathcal{L} = T - V$$  \hspace{1cm} (4.38)

$$\frac{d}{dt} \left( \frac{\partial \mathcal{L}}{\partial \dot{q}_j} \right) - \frac{\partial \mathcal{L}}{\partial q_j} = Q_j$$  \hspace{1cm} (4.39)

where $\mathcal{L}$ is the Lagrangian function, $T$ is kinetic energy, $V$ is the potential energy, $q_j$ are the generalized coordinates, $\dot{q}_j$ are the generalized velocities, and $Q_j$ are the generalized forces.

The Haptic Glove simulates a force at each fingertip by applying forces to the linkage structure via the two corresponding air cylinder actuators. It is necessary to know the forces required at the actuators to simulate the desired forces felt at each fingertip. Therefore, we select as generalized coordinates the displacement of the cylinders, $q_{j1} = a_{j1}, q_{j2} = a_{j2}$. In order to calculate the generalized forces, we examine the virtual work done by the forces at the fingertips and actuators’ forces. In vector form, the virtual work for each finger is computed as

$$\delta W_j = \sum_n \bar{F}_{jn} \cdot \delta \bar{r}_{jn} = \bar{F}_j \cdot \delta \bar{r}_j - F_{j1} \delta a_{j1} - F_{j2} \delta a_{j2}$$  \hspace{1cm} (4.40)

where $\bar{r}_j$ is the vector $(x_j, y_j)$ computed above. The relationship between the virtual work and the generalized forces $Q_1, Q_2$:

$$\delta W_j = \sum_n Q_{jn} \delta q_{jn} = Q_{j1} \delta q_{j1} + Q_{j2} \delta q_{j2}$$  \hspace{1cm} (4.41)
Combining with the above, the generalized forces become

\[ Q_{j1} = \bar{F}_j \cdot \frac{\partial \bar{r}_j}{\partial a_{j1}} - F_{j1} \quad Q_{j2} = \bar{F}_j \cdot \frac{\partial \bar{r}_j}{\partial a_{j2}} - F_{j2} \] (4.42)

For typical palpation procedures, the motion of the fingertips is minimal. Thus, we can consider the system to be quasi-static. Therefore, the generalized forces for each finger, \( Q_{j1} \) and \( Q_{j2} \), are equal to zero. Equation 4.42 can be used to find the actuator forces, \( F_{j1} \) and \( F_{j2} \) given a desired virtual force on each of the user’s fingertip within the Haptic Glove. The resulting equation for the proximal actuator force for each finger, \( F_{j1} \), is

\[ F_{j1} = \bar{F}_j \cdot \frac{\partial \bar{r}_j}{\partial a_{j1}} \] (4.43)

This can be rewritten as

\[ F_{j1} = \frac{\partial \theta_{j1}}{\partial a_{j1}} \left( F_{jx} \cdot \frac{\partial x_j}{\partial \theta_{j1}} + F_{jy} \cdot \frac{\partial y_j}{\partial \theta_{j1}} \right) \] (4.44)

The equivalent equation for the distal actuator force for each finger, \( F_{j2} \), is

\[ F_{j2} = \bar{F}_j \cdot \frac{\partial \bar{r}_j}{\partial a_{j2}} \] (4.45)

For the index and middle fingers, this can be rewritten as

\[ F_{j2} = \frac{\partial \theta_{j2}}{\partial \theta_{jg}} \frac{\partial \theta_{jg}}{\partial a_{j2}} \left( F_{jx} \cdot \frac{\partial x_j}{\partial \theta_{j2}} + F_{jy} \cdot \frac{\partial y_j}{\partial \theta_{j2}} \right) \] (4.46)

Since the thumb is missing the gear mechanism, the equation for the thumb instead reduces to

\[ F_{32} = \frac{\partial \theta_{32}}{\partial a_{32}} \left( F_{3x} \cdot \frac{\partial x_3}{\partial \theta_{32}} + F_{3y} \cdot \frac{\partial y_3}{\partial \theta_{32}} \right) \] (4.47)

These equations require the partial derivatives of some of the equations derived in section 4.1 for each finger.

4.2.1. Index Finger Forces
Figure 4.13: Forces on the index finger substructure of the Haptic Glove.

Figure 4.13 shows the fingertip and actuator forces of the index finger. The partial derivatives of the fingertip location vector defined by Eq. 4.14 in each direction with respect to each actuator are as follows.

\[
\frac{\partial x_1}{\partial \theta_{11}} = -d_{11} \sin \theta_{11} - 25 \sin(\theta_{11} + \theta_{12}) - d_{12} \sin(\theta_{11} + 2\theta_{12}) \quad (4.48)
\]

\[
\frac{\partial y_1}{\partial \theta_{11}} = d_{11} \cos \theta_{11} + 25 \cos(\theta_{11} + \theta_{12}) + d_{12} \cos(\theta_{11} + 2\theta_{12}) \quad (4.49)
\]

\[
\frac{\partial x_1}{\partial \theta_{12}} = -25 \sin(\theta_{11} + \theta_{12}) - 2d_{12} \sin(\theta_{11} + 2\theta_{12}) \quad (4.50)
\]

\[
\frac{\partial y_1}{\partial \theta_{12}} = 25 \cos(\theta_{11} + \theta_{12}) + 2d_{12} \cos(\theta_{11} + 2\theta_{12}) \quad (4.51)
\]
Each joint angle is dependent on only a single actuator, so we only need the partial derivative of each bend angle with respect to its respective actuator. The partial derivative of the metacarpophalangeal bend angle defined by Eq. 4.3 with respect to the proximal actuator displacement can be simplified to

$$\frac{\partial \theta_{11}}{\partial a_{11}} = \frac{5.789 \times 10^{-4} a_{11}}{\sqrt{1 - (1.092 - 2.895 \times 10^{-4} a_{11})^2}}$$

(4.52)

The interphalangeal bend angle defined by Eq. 4.12 is written in terms of $\theta_{1g}$ to keep the expression tractable. A similar approach can be used to prevent the partial derivative expression from becoming too unwieldy by finding the derivative with respect to the gear angle $\theta_{1g}$ instead.

$$\frac{\partial \theta_{12}}{\partial \theta_{1g}} = \frac{1}{3.765 \cos \theta_{1g} - 3.895} - 1$$

(4.53)

$$\frac{\partial \theta_{1g}}{\partial a_{12}} = \frac{-1.295 \times 10^{-3} a_{12}}{\sqrt{1 - (1.479 - 6.477 \times 10^{-4} a_{12}^2)^2}}$$

(4.54)

By combining Eqs. 4.3, 4.8, 4.12, 4.14, 4.44, 4.46, 4.48, 4.49, 4.50, 4.51, 4.52, 4.53, and 4.54, it is possible to derive expressions for actuator forces $F_{11}$ and $F_{12}$ in terms of adjustable lengths, $d_{11}$ and $d_{12}$, actuator displacements, $a_{11}$ and $a_{12}$, and a desired index fingertip force vector, $\vec{F}_1 = \langle F_{1x}, F_{1y} \rangle$. 

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\[ F_{11} = \frac{0.0005789a_{11}}{\sqrt{1 - (1.092 - 0.0002895a_{11}^2)^2}} \]

\[-F_x \left( d_{11} \sin \left( \cos^{-1} (1.092 - 0.0002895a_{11}^2) - 37.29^\circ \right) \right) \]

\[ + 25 \sin \left( \cos^{-1} (1.092 - 0.0002895a_{11}^2) - 75.23^\circ \right) \]

\[ + 2 \cos^{-1} \left( \frac{33.84 - 44 \cos \left( \sin^{-1} (1.479 - 0.0006477a_{12}^2) + 91.67^\circ \right) \right) \]

\[ \left( \frac{33.84 - 44 \cos \left( \sin^{-1} (1.479 - 0.0006477a_{12}^2) + 91.67^\circ \right) \right) \]

\[ + d_{12} \sin \left( \cos^{-1} (1.092 - 0.0002895a_{11}^2) - 113.16^\circ \right) \]

\[ + 4 \cos^{-1} \left( \frac{33.84 - 44 \cos \left( \sin^{-1} (1.479 - 0.0006477a_{12}^2) + 91.67^\circ \right) \right) \]

\[ \left( \frac{33.84 - 44 \cos \left( \sin^{-1} (1.479 - 0.0006477a_{12}^2) + 91.67^\circ \right) \right) \]

\[ + F_y \left( d_{11} \cos \left( \cos^{-1} (1.092 - 0.0002895a_{11}^2) - 37.29^\circ \right) \right) \]

\[ + 25 \cos \left( \cos^{-1} (1.092 - 0.0002895a_{11}^2) - 75.23^\circ \right) \]

\[ + 2 \cos^{-1} \left( \frac{33.84 - 44 \cos \left( \sin^{-1} (1.479 - 0.0006477a_{12}^2) + 91.67^\circ \right) \right) \]

\[ \left( \frac{33.84 - 44 \cos \left( \sin^{-1} (1.479 - 0.0006477a_{12}^2) + 91.67^\circ \right) \right) \]

\[ + d_{12} \cos \left( \cos^{-1} (1.092 - 0.0002895a_{11}^2) - 113.16^\circ \right) \]

\[ + 4 \cos^{-1} \left( \frac{33.84 - 44 \cos \left( \sin^{-1} (1.479 - 0.0006477a_{12}^2) + 91.67^\circ \right) \right) \]

\[ \left( \frac{33.84 - 44 \cos \left( \sin^{-1} (1.479 - 0.0006477a_{12}^2) + 91.67^\circ \right) \right) \]

(4.55)
\[
F_{12} = \frac{0.001295a_{12}}{\sqrt{1 - (1.479 - 0.0006477a_{12}^2)^2}} - \frac{0.001295a_{12}}{3.705 \cos \left( \sin^{-1} \left( 1.479 - 0.0006477a_{12}^2 + 91.67^\circ \right) \right) - 3.895} - F_{1x} \left( 25 \sin \left( \cos^{-1} \left( 1.092 - 0.0002895a_{11}^2 \right) \right) - 75.23^\circ \right)
\]
\[
+ 2 \cos^{-1} \left( \frac{33.84 - 44 \cos \left( \sin^{-1} \left( 1.479 - 0.0006477a_{12}^2 + 91.67^\circ \right) \right) \right) \right) \right) \right) - 113.16^\circ
\]
\[
+ 4 \cos^{-1} \left( \frac{33.84 - 44 \cos \left( \sin^{-1} \left( 1.479 - 0.0006477a_{12}^2 + 91.67^\circ \right) \right) \right) \right) \right) \right) - 113.16^\circ
\]
\[
+ F_{1y} \left( 25 \cos \left( \cos^{-1} \left( 1.092 - 0.0002895a_{11}^2 \right) \right) - 75.23^\circ \right)
\]
\[
+ 2 \cos^{-1} \left( \frac{33.84 - 44 \cos \left( \sin^{-1} \left( 1.479 - 0.0006477a_{12}^2 + 91.67^\circ \right) \right) \right) \right) \right) \right) - 113.16^\circ
\]
\[
+ 4 \cos^{-1} \left( \frac{33.84 - 44 \cos \left( \sin^{-1} \left( 1.479 - 0.0006477a_{12}^2 + 91.67^\circ \right) \right) \right) \right) \right) \right) - 113.16^\circ
\]
\]
\[(4.56)\]
If desired, it is also possible to substitute the linear approximations for \( \theta_{11} \) and \( \theta_{12} \), given by Eqs. 4.4 and 4.13, in place of Eqs. 4.3 and 4.12. This results in the following simpler approximations for \( F_{11} \) and \( F_{12} \).

\[
F_{11} = F_{1x}(0.032d_{11}\sin(60.6^\circ - 1.85a_{11}) + 0.81\sin(193.3^\circ - 1.85a_{11} - 4.15a_{12}) \\
+ 0.032d_{12}\sin(326.0^\circ - 1.85a_{11} - 8.30a_{12})) \\
+ F_{1y}(0.032d_{11}\cos(60.6^\circ - 1.85a_{11}) + 0.81\cos(193.3^\circ - 1.85a_{11} - 4.15a_{12}) \\
+ 0.032d_{12}\cos(326.0^\circ - 1.85a_{11} - 8.30a_{12}))
\]

(4.57)

\[
F_{12} = F_{1x}(1.81\sin(193.3^\circ - 1.85a_{11} - 4.15a_{12}) \\
+ 0.14d_{12}\sin(326.0^\circ - 1.85a_{11} - 8.30a_{12})) \\
+ F_{1y}(1.81\cos(193.3^\circ - 1.85a_{11} - 4.15a_{12}) \\
+ 0.14d_{12}\cos(326.0^\circ - 1.85a_{11} - 8.30a_{12}))
\]

(4.58)

4.2.2. Middle Finger Forces

For the middle finger, for which the fingertip and actuator forces are shown in Fig. 4.14, the partial derivatives of the fingertip location vector defined by Eq. 4.28 in each direction with respect to each actuator are as follows.

\[
\frac{\partial x_2}{\partial \theta_{21}} = -d_{21}\sin \theta_{21} - 31\sin(\theta_{21} + \theta_{22}) - d_{22}\sin(\theta_{21} + 2\theta_{22}) \\
\frac{\partial y_2}{\partial \theta_{21}} = d_{21}\cos \theta_{21} + 31\cos(\theta_{21} + \theta_{22}) + d_{22}\cos(\theta_{21} + 2\theta_{22})
\]

(4.59)

\[
\frac{\partial x_2}{\partial \theta_{22}} = -31\sin(\theta_{21} + \theta_{22}) - 2d_{22}\sin(\theta_{21} + 2\theta_{22}) \\
\frac{\partial y_2}{\partial \theta_{22}} = 31\cos(\theta_{21} + \theta_{22}) + 2d_{22}\cos(\theta_{21} + 2\theta_{22})
\]

(4.60)

Just like the index finger, each joint angle in the middle is dependent on only a single actuator, so we only need the partial derivative of each bend angle with respect
Figure 4.14: Forces on the index finger substructure of the Haptic Glove.

to its respective actuator. The partial derivative of the metacarpophalangeal bend angle defined by Eq. 4.17 with respect to the proximal actuator displacement can be simplified to

$$\frac{\partial \theta_{21}}{\partial a_{21}} = \frac{6.703 \times 10^{-4} a_{21}}{\sqrt{1 - (1.023 - 3.352 \times 10^{-4} a_{21})^2}}$$  \hspace{1cm} (4.63)

The partial derivative of Eq. 4.26 with respect to the gear angle, $\theta_{2g}$ can be simplified to

$$\frac{\partial \theta_{22}}{\partial \theta_{2g}} = \frac{1}{5.362 \cos \theta_{2g} - 5.455} - 1$$  \hspace{1cm} (4.64)
The partial derivative of Eq. 4.22 with respect to the distal actuator displacement, \( a_{22} \) can be simplified to

\[
\frac{\partial \theta_{22}}{\partial a_{22}} = \frac{-1.187 \times 10^{-3} a_{22}}{\sqrt{1 - (1.587 - 5.933 \times 10^{-4} a_{22}^2)^2}}
\]  

(4.65)

By combining Eqs. 4.17, 4.22, 4.26, 4.28, 4.44, 4.46, 4.59, 4.60, 4.61, 4.62, 4.63, 4.64, and 4.65, it is possible to derive expressions for actuator forces \( F_{21} \) and \( F_{22} \) in terms of adjustable lengths, \( d_{21} \) and \( d_{22} \), actuator displacements, \( a_{21} \) and \( a_{22} \), and a desired middle fingertip force vector, \( \bar{F}_2 = \langle F_{2x}, F_{2y} \rangle \).
\[ F_{21} = \frac{0.0006703 a_{21}}{\sqrt{1 - (1.023 - 0.0003352 a_{21}^2)^2}} \]

\[-F_{2x} \left( d_{21} \sin \left( \cos^{-1} (1.023 - 0.0003352 a_{21}^2) - 41.91^\circ \right) + 31 \sin \left( \cos^{-1} (1.023 - 0.0003352 a_{21}^2) - 90.36^\circ \right) + 2 \cos^{-1} \left( \frac{15.67 - 18.86 \cos \left( \sin^{-1} (1.587 - 0.0005933 a_{22}^2) + 90.01^\circ \right) \right) \right) \]

\[ + d_{22} \sin \left( \cos^{-1} (1.023 - 0.0003352 a_{21}^2) - 138.83^\circ \right) + 4 \cos^{-1} \left( \frac{15.67 - 44 \cos \left( \sin^{-1} (1.587 - 0.0005933 a_{22}^2) + 90.01^\circ \right) \right) \]

\[ + F_{2y} \left( d_{21} \cos \left( \cos^{-1} (1.023 - 0.0003352 a_{21}^2) - 41.91^\circ \right) + 31 \cos \left( \cos^{-1} (1.023 - 0.0003352 a_{21}^2) - 90.36^\circ \right) + 2 \cos^{-1} \left( \frac{15.67 - 18.86 \cos \left( \sin^{-1} (1.587 - 0.0005933 a_{22}^2) + 90.01^\circ \right) \right) \right) \]

\[ + d_{22} \cos \left( \cos^{-1} (1.023 - 0.0003352 a_{21}^2) - 138.83^\circ \right) + 4 \cos^{-1} \left( \frac{15.67 - 18.86 \cos \left( \sin^{-1} (1.587 - 0.0005933 a_{22}^2) + 90.01^\circ \right) \right) \]

\[ (4.66) \]
\[ F_{22} = \frac{0.001187a_{22}}{5.362 \cos \left( \sin^{-1} (1.587 - 0.0005933a_{22}^2) + 90.01^\circ \right) - 5.455} \sqrt{1 - (1.587 - 0.0005933a_{22}^2)^2} \]

\[ -F_{2x} \left[ 31 \sin \left( \cos^{-1} (1.023 - 0.0003352a_{21}^2) \right) - 90.36^\circ \right. \]

\[ + 2 \cos^{-1} \left( \frac{15.67 - 18.86 \cos \left( \sin^{-1} (1.587 - 0.0005933a_{22}^2) + 90.01^\circ \right) \right) \]

\[ \left. + 2d_{22} \sin \left( \cos^{-1} (1.023 - 0.0003352a_{21}^2) \right) - 138.83^\circ \right) \]

\[ + 4 \cos^{-1} \left( \frac{15.67 - 18.86 \cos \left( \sin^{-1} (1.587 - 0.0005933a_{22}^2) + 90.01^\circ \right) \right) \]

\[ + F_{2y} \left[ 31 \cos \left( \cos^{-1} (1.023 - 0.0003352a_{21}^2) \right) - 90.36^\circ \right. \]

\[ + 2 \cos^{-1} \left( \frac{15.67 - 18.86 \cos \left( \sin^{-1} (1.587 - 0.0005933a_{22}^2) + 90.01^\circ \right) \right) \]

\[ \left. + 2d_{22} \cos \left( \cos^{-1} (1.023 - 0.0003352a_{21}^2) \right) - 138.83^\circ \right) \]

\[ + 4 \cos^{-1} \left( \frac{15.67 - 18.86 \cos \left( \sin^{-1} (1.587 - 0.0005933a_{22}^2) + 90.01^\circ \right) \right) \]

\[ \left(4.67\right) \]
Once again, it is possible to substitute the linear approximations for $\theta_{21}$ and $\theta_{22}$, given by Eqs. 4.18 and 4.27, in place of Eqs. 4.17 and 4.26. This results in the following simpler approximations for $F_{21}$ and $F_{22}$.

\[
F_{21} = F_{2x}(0.034d_{21} \sin (59.2^\circ - 1.96a_{21}) + 1.06 \sin (218.1^\circ - 1.96a_{21} - 4.11a_{22})
+ 0.034d_{22} \sin (377.0^\circ - 1.96a_{21} - 8.22a_{22}))
+ F_{2y}(0.034d_{21} \cos (59.2^\circ - 1.96a_{21}) + 1.06 \cos (218.1^\circ - 1.96a_{21} - 4.11a_{22})
+ 0.034d_{22} \cos (377.0^\circ - 1.96a_{21} - 8.22a_{22}))
\]

\[\text{(4.68)}\]

\[
F_{22} = F_{2x}(2.22 \sin (218.1^\circ - 1.96a_{21} - 4.11a_{22})
+ 0.14d_{22} \sin (377.0^\circ - 1.96a_{21} - 8.22a_{22}))
+ F_{2y}(2.22 \cos (218.1^\circ - 1.96a_{21} - 4.11a_{22})
+ 0.14d_{22} \cos (377.0^\circ - 1.96a_{21} - 8.22a_{22}))
\]

\[\text{(4.69)}\]

### 4.2.3. Thumb Forces

The fingertip and actuator forces for the thumb are shown in Fig. 4.15. The partial derivatives of the fingertip location vector defined by Eq. 4.37 in each direction with respect to each actuator are as follows.

\[
\frac{\partial x_3}{\partial \theta_{31}} = -d_{31} \sin \theta_{31} - d_{32} \sin(\theta_{31} + \theta_{32})
\]

\[\text{(4.70)}\]

\[
\frac{\partial y_3}{\partial \theta_{31}} = d_{31} \cos \theta_{31} + d_{32} \cos(\theta_{31} + \theta_{32})
\]

\[\text{(4.71)}\]

\[
\frac{\partial x_3}{\partial \theta_{32}} = -d_{32} \sin(\theta_{31} + \theta_{32})
\]

\[\text{(4.72)}\]

\[
\frac{\partial y_3}{\partial \theta_{32}} = d_{32} \cos(\theta_{21} + \theta_{22})
\]

\[\text{(4.73)}\]

Once again, each joint angle in the middle is dependent on only a single actuator, so we only need the partial derivative of each bend angle with respect to its respective actuator. The partial derivative of the thumb metacarpophalangeal bend angle defined
by Eq. 4.31 with respect to the proximal actuator displacement can be simplified to

\[
\frac{\partial \theta_{31}}{\partial a_{31}} = \frac{5.992 \times 10^{-4} a_{31}}{\sqrt{1 - (1.107 - 2.996 \times 10^{-4} a_{31}^2)^2}}
\] (4.74)

Unlike the finger substructures, there is no gearing mechanism so we only need one more derivative. The partial derivative of the thumb interphalangeal bend angle defined by Eq. 4.35 with respect to the distal actuator displacement can be simplified to

\[
\frac{\partial \theta_{32}}{\partial a_{32}} = \frac{6.071 \times 10^{-4} a_{32}}{\sqrt{1 - (1.058 - 3.036 \times 10^{-4} a_{32}^2)^2}}
\] (4.75)
By combining Eqs. 4.31, 4.35, 4.37, 4.44, 4.47, 4.70, 4.71, 4.72, 4.73, 4.74, and 4.75, it is possible to derive expressions for actuator forces $F_{31}$ and $F_{32}$ in terms of adjustable lengths, $d_{31}$ and $d_{32}$, actuator displacements, $a_{31}$ and $a_{32}$, and a desired thumb fingertip force vector, $F_3 = \langle F_{3x}, F_{3y} \rangle$.

\[
\begin{align*}
F_{31} &= \frac{0.0005992a_{31}}{\sqrt{1 - (1.107 - 0.0002996a_{31}^2)^2}} \left( -F_{3x}d_{31} \sin\left( \cos^{-1}(1.107 - 0.0002996a_{31}^2) - 32.47^\circ \right) \\
&\quad + d_{32}\sin\left( \cos^{-1}(1.107 - 0.0002996a_{31}^2) \\
&\quad + \sin^{-1}(1.058 - 0.0003036a_{32}^2) - 63.48^\circ \right) \\
&\quad + F_{3y}\left( d_{31}\cos\left( \cos^{-1}(1.107 - 0.0002996a_{31}^2) - 32.47^\circ \right) \\
&\quad + d_{32}\sin\left( \cos^{-1}(1.107 - 0.0002996a_{31}^2) \\
&\quad + \sin^{-1}(1.058 - 0.0003036a_{32}^2) - 63.48^\circ \right) \right) \\
\end{align*}
\]

(4.76)

\[
\begin{align*}
F_{32} &= \frac{0.0006701a_{32}}{\sqrt{1 - (1.058 - 0.0003036a_{32}^2)^2}} \left( -F_{3x}d_{32}\sin\left( \cos^{-1}(1.107 - 0.0002996a_{31}^2) \\
&\quad + \sin^{-1}(1.058 - 0.0003036a_{32}^2) - 63.48^\circ \right) \\
&\quad + F_{3y}d_{32}\cos\left( \cos^{-1}(1.107 - 0.0002996a_{31}^2) \\
&\quad + \sin^{-1}(1.058 - 0.0003036a_{32}^2) - 63.48^\circ \right) \right) \\
\end{align*}
\]

(4.77)
Though these expressions aren’t nearly as complicated as those for the index and middle fingers, it is still possible to substitute the linear approximations for $\theta_31$ and $\theta_32$, given by Eqs. 4.32 and 4.36, in place of Eqs. 4.31 and 4.35. This results in the following simpler approximations for $F_31$ and $F_32$.

\[
F_{31} = F_{3x} (0.033d_{31} \sin (56.4^\circ - 1.88a_{31}) \\
+ 0.033d_{32} \sin (103.3^\circ - 1.88a_{31} - 1.79a_{32})) \\
+ F_{3y} (0.033d_{31} \cos (56.4^\circ - 1.88a_{31}) \\
+ 0.033d_{32} \cos (103.3^\circ - 1.88a_{31} - 1.79a_{32})) \tag{4.78}
\]

\[
F_{32} = 0.031F_{3x} d_{32} \sin (103.3^\circ - 1.88a_{31} - 1.79a_{32}) \\
+ 0.031F_{3y} 1.81 \cos (103.3^\circ - 1.88a_{31} - 1.79a_{32}) \tag{4.79}
\]
5.1. Cylinder Chamber Model

A schematic of the pneumatic circuit is shown in Fig. 5.1. The normally closed Enfield 5/3 Proportional Valve has 5 ports and 3 states. When excited by a positive voltage, the valve connects chamber $A$ of the pneumatic cylinder to air supply $P_s$ and
chamber \( B \) to atmospheric pressure \( P_{atm} \). This causes pressure \( P_a \) to rise towards \( P_s \) and \( P_b \) to fall towards \( P_{atm} \), eventually resulting in a positive actuator force \( F \).

When excited by a negative voltage, the valve connects chamber \( B \) to air supply \( P_s \) instead, while chamber \( A \) is connected to atmospheric pressure \( P_{atm} \) through a second vent port. This causes the pressure in the chambers to move in the opposite direction, eventually causing a negative actuator force. The actuator position is tracked by variable \( x \), with \( x = 0 \) representing the center of the actuator range of motion. In addition to changing the geometry of the Haptic Glove structure, the actuator position also changes the volume of each chamber in the cylinder, affecting the chamber pressures and force output of the actuator. In order to develop the controller for the pneumatic valves, a more exact model of the pressure dynamics and flow rates through the valves must be developed.

Each chamber can be modeled as a control volume with the following assumptions:

1. The air is an ideal gas.
2. Pressure and temperature within each chamber are uniformly distributed.
3. The kinetic and potential energy of the air is negligible.
4. The charging and discharging processes are isothermal.

Considering these assumptions and applying the conservation of mass, the ideal gas model, and the conservation of energy to each chamber we can derive expressions for the change of pressure in each chamber [23].

\[
\begin{align*}
\dot{P}_a &= \frac{RT}{V_a} \dot{m}_a - \frac{P_a}{L_a + x} \dot{x} \\
\dot{P}_b &= \frac{RT}{V_b} \dot{m}_b + \frac{P_b}{L_b - x} \dot{x}
\end{align*}
\]  

(5.1)

where \( R \) is the ideal gas constant, \( T \) is the absolute temperature, and \( \dot{m}_a \) and \( \dot{m}_b \) are the mass flows into chambers \( A \) and \( B \), respectively. The volumes of chambers \( A \) and
B are represented by \( V_a \) and \( V_b \), respectively, and can be more explicitly defined as

\[
\begin{align*}
V_a &= (L_a + x)A_a \\
V_b &= (L_b - x)A_b
\end{align*}
\tag{5.2}
\]

where \( A_a \) and \( A_b \) are the cross-sectional areas of the cylinder chambers \( A \) and \( B \) along the tube length, respectively. The pneumatic cylinders used in the Haptic Glove design have constant diameter, however there is an important detail that causes the cross-sectional areas to differ. Chamber \( A \) contains only air, while chamber \( B \) also houses the rod that applies the pneumatic cylinder force to the rest of the Haptic Glove mechanism. This rod displaces the air in chamber \( B \), causing the cross-sectional area of chamber \( B \), \( A_b \), to be a bit smaller than that of chamber \( A \).

\[
\begin{align*}
A_a &= \frac{\pi}{4}D_p^2 \\
A_b &= \frac{\pi}{4}D_p^2 - A_r \\
A_r &= \frac{\pi}{4}D_r^2
\end{align*}
\tag{5.3}
\]

where \( D_p \) is the diameter of the interior wall of the cylinder and \( D_r \) is the diameter of the piston rod. Equations 5.1 and 5.2 both utilize \( L_a \) and \( L_b \), which represent the lengths of chambers \( A \) and \( B \), respectively, when \( x = 0 \). However, the contributions of the tubes and the dead space within the ends of the cylinder must be accounted for. Therefore the lengths are defined as

\[
\begin{align*}
L_a &= L_{0a} + \frac{L}{2} \\
L_b &= L_{0b} + \frac{L}{2}
\end{align*}
\tag{5.4}
\]

where \( L \) is the total length of the pneumatic cylinder and the terms \( L_{0a} \) and \( L_{0b} \) account for the dead volumes on either side of the cylinder and the tubes connecting them to the pneumatic valves. As the cross-sectional area of the tube doesn’t apply to the rest of the dead volume, these lengths are normalized by the cross-sectional
areas of each chamber.

\[
\begin{align*}
L_{0a} &= \frac{V_{0a}}{A_a} \\
L_{0b} &= \frac{V_{0b}}{A_b}
\end{align*}
\] (5.5)

where \(V_{0a}\) and \(V_{0b}\) are the dead volumes in the ends of the cylinder and the tubes that are present in the system regardless of the \(x\) position of the cylinder. The dead volumes can further be defined as

\[
\begin{align*}
V_{0a} &= \frac{\pi}{4} D_t^2 L_{ta} \\
V_{0b} &= \frac{\pi}{4} D_t^2 L_{tb}
\end{align*}
\] (5.6)

where \(D_t\) is the inner diameter of the pneumatic tubing, and \(L_{ta}\) and \(L_{tb}\) are the lengths of the pneumatic tubing connecting the valves to chambers \(A\) and \(B\) of the cylinder, respectively [9].

The force generated by the actuator is caused by the difference in pressure between the two chambers of the cylinder as well as the atmospheric pressure. Additionally, there is friction present between the pneumatic piston and the cylinder wall that this research will not neglect. The force generated due to the pressure differential in the cylinder chambers ignoring friction is denoted by \(F_p\) and the frictional force is denoted by \(F_f\). The pressure force is defined to be positive for an outward pushing force, and the frictional force is oriented opposite to the pressure force so that the total force \(F\), can be written as

\[F = F_p - F_f\] (5.7)

The force component caused by the pressure differential can be computed as

\[F_p = P_a A_a - P_b A_b - P_{atm} A_r\] (5.8)

The force component caused by the frictional force was intended to account for both static and kinetic friction. Ideally, the frictional force and its derivative should be
computed as
\[
F_f = \begin{cases} 
\mu(P_m, F_p, F_u) & |x'| < \delta_v \\
\nu(P_m, x') & |x'| > \delta_v
\end{cases}
\]  
(5.9)

\[
F'_f = \begin{cases} 
\mu'(P_m, F_p, F_u) & |x'| < \delta_v \\
\nu'(P_m, x', x'') & |x'| > \delta_v
\end{cases}
\]  
(5.10)

where \( P_m = \max(P_a, P_b) \), the maximum pressure between chambers \( A \) and \( B \), \( F_u \) is the force from the user pushing inward on the actuator cylinder, and \( \delta_v \) is a sufficiently small velocity denoting the threshold between static and kinetic friction, and functions \( \mu \) and \( \nu \) will be determined experimentally. However, the system is not equipped with adequate sensors to determine the direction of the static friction as the force from the user, \( F_u \), cannot be directly measured in a static state. Therefore, static friction was ignored in the controller design and \( F_f \) only accounted for kinetic friction. Nevertheless, models for \( \mu \) and \( \nu \) will be developed later in Section 7.2.

5.2. Valve Flow

The mass flow through the pneumatic valve, \( \dot{m}_v \), can be modeled as compressible flow through an orifice [23, 24].

\[
\dot{m}_v(P_u, P_d, z) = A_v \psi(P_u, P_d, C_f)
\]  
(5.11)

where \( A_v \) is the aperture area of the valve. The mass flow per unit area, \( \psi(P_u, P_d) \) is a function of the upstream and downstream pressures, \( P_u \) and \( P_d \).

\[
\psi(P_u, P_d, C_f) = \begin{cases} 
C_1C_f \frac{P_u}{\sqrt{T}} & \frac{P_d}{P_u} \leq P_{cr} \\
C_2C_f \frac{P_u}{\sqrt{T}} \left( \frac{P_d}{P_u} \right)^{1/\gamma} \sqrt{1 - \left( \frac{P_d}{P_u} \right)^{(\gamma - 1)/\gamma}} & \frac{P_d}{P_u} > P_{cr}
\end{cases}
\]  
(5.12)

The upstream and downstream pressures are always defined such that \( P_d \leq P_u \). When defined this way, the mass flow always goes from the upstream pressure towards the
downstream pressure according to Eq. 5.12. When the ratio of the downstream pressure to upstream pressure is less than $P_{cr}$, known as the critical pressure ratio and dependent on the fluid under the pressure differential, the mass flow is limited by the geometry of the orifice and maxes out a constant flow rate that is dependent on the upstream pressure, but not the downstream pressure. This state is known as choked flow. When the downstream to upstream pressure ratio rises above the critical pressure ratio, the pressure differential has grown small enough that the orifice is no longer the limiting factor in determining the mass flow. This state is known as subsonic flow, and the mass flow is determined by a more complex expression dependent on downstream to upstream pressure ratio, as well as several other parameters. $C_f$ is a non-dimensional, flow coefficient, that accounts for the exact geometry of the orifice being modeled and must be determined experimentally. $\gamma$ is the ratio of the specific heat of the fluid at constant pressure to the specific heat at constant volume. In the case of air, $\gamma \approx 1.4$. The remaining two parameters, $C_1$ and $C_2$, are constants dependent on both $\gamma$ and the ideal gas constant, $R$.

\[
C_1 = \sqrt{\frac{\gamma}{R} \left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma + 1}{\gamma - 1}}} \tag{5.13}
\]

\[
C_2 = \sqrt{\frac{2\gamma}{R(\gamma - 1)}} \tag{5.14}
\]

The critical pressure ratio can also be expressed in terms of $\gamma$.

\[
P_{cr} = \left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma + 1}} \tag{5.15}
\]

Equation 5.12 governs two flow paths in each pneumatic circuit on the Haptic Glove, namely the flow from the air supply into one chamber of the cylinder, and the flow from the other chamber of the cylinder out to atmosphere, with both flows traveling through the pneumatic valve. Assuming constant supply and atmospheric pressures, an interesting side effect of Eq. 5.12 is that finite vessels experience different mass flow rate curves for inward and outward flowing air. This discrepancy
between the two flow directions is shown in Fig. 5.2. During pressurizing, the upstream pressure is the supply pressure and is assumed to stay constant. Therefore, below the critical pressure, the flow into the vessel stays constant. This inward flow causes the chamber pressure, which is serving as the downstream pressure, to climb, raising the downstream to upstream ratio. Once the ratio crosses the critical pressure ratio threshold, the mass flow into the chamber will slow, following the subsonic flow curve. By contrast, in a depressurizing situation, the chamber pressure serves as the
upstream pressure. Assuming the pressure ratio between the chamber and the atmosphere is enough to cause choked flow, the mass flow out of the chamber will steadily drop proportional to the chamber pressure until subsonic flow is reached at the critical pressure ratio. If the depressurizing flow is allowed to continue, the flow rate will continue dropping at an accelerating rate, following the subsonic flow curve until the chamber pressure finally matches the atmospheric pressure. This distinction in the pressurizing and depressurizing behaviors has an important impact on the pneumatic elements of the Haptic Glove. Since both chambers of each cylinder are controlled by a single valve, any time spent depressurizing one chamber will simultaneously be pressurizing the other chamber. As greater mass flow occurs during pressurizing than depressurizing for vessels of median pressure, the chamber pressures in all pneumatic cylinders on the Haptic Glove have a tendency to climb to higher pressures over time. This also means it is impossible to reach a state where both chambers of a single pneumatic cylinder are near atmospheric pressure once the supply pressure has been connected to the system. One chamber or the other must be under pressure in each pneumatic cylinder at all times [9].

The other parameter that influences Eq. 5.11 is the aperture area of the valve. Taheri et al. used the same Enfield 5/3 Proportional Valves and established model equation to relate $A_v$, the normalized aperture area of the valve, to the spool position $z$ [31].

$$A_v = (a + bz + cz^2 + dz^3 + fz^4) (1 - \tanh(gz + h))$$  (5.16)

The coefficients Taheri found are summarized in Table 5.1. Taheri chose to extend Eq. 5.16 into the negative $z$ range by making it an odd function, substituting the absolute value of $z$ for the input and multiplying the output by the sign of $z$. However, this produces a negative result for all negative spool positions, resulting in the somewhat nonsensical interpretation of a negative aperture area. As it works well with the
controller developed in Chapter 6, this research instead chooses to define Eq. 5.16 only for positive inputs, resulting in a positive aperture area in both flow directions. A controller derivation requires the inverse of Eq. 5.16, but a direct inverse computation is difficult. Instead, an inverse model was created using coefficients found using the least-squares method. The form chosen for this inverse model is given below.

\[ z = \frac{a + bA_v + cA_v^2}{d + fA_v + gA_v^2 + hA_v^3} + k \]  

(5.17)

where \( k = -\frac{a}{d} \) to ensure that zero aperture area corresponds to a spool position of zero. The coefficients used in this model are summarized in Table 5.2 and the comparison with the numeric inverse of Taheri’s model is shown in Fig. 5.3. This model will be used in the controller design in Chapter 6.

Table 5.1: Coefficients for Taheri model given by Eq. 5.16.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.07626</td>
</tr>
<tr>
<td>b</td>
<td>-1.16871</td>
</tr>
<tr>
<td>c</td>
<td>6.63638</td>
</tr>
<tr>
<td>d</td>
<td>-8.15992</td>
</tr>
<tr>
<td>f</td>
<td>3.11639</td>
</tr>
<tr>
<td>g</td>
<td>2.91100</td>
</tr>
<tr>
<td>h</td>
<td>-19.94045</td>
</tr>
</tbody>
</table>

controller developed in Chapter 6, this research instead chooses to define Eq. 5.16 only for positive inputs, resulting in a positive aperture area in both flow directions. A controller derivation requires the inverse of Eq. 5.16, but a direct inverse computation is difficult. Instead, an inverse model was created using coefficients found using the least-squares method. The form chosen for this inverse model is given below.

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(5.17)

where \( k = -\frac{a}{d} \) to ensure that zero aperture area corresponds to a spool position of zero. The coefficients used in this model are summarized in Table 5.2 and the comparison with the numeric inverse of Taheri’s model is shown in Fig. 5.3. This model will be used in the controller design in Chapter 6.
Table 5.2: Coefficients for the inverse model developed by this research and given by Eq. 5.17.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>-0.12785</td>
</tr>
<tr>
<td>b</td>
<td>-37.0931</td>
</tr>
<tr>
<td>c</td>
<td>36.4598</td>
</tr>
<tr>
<td>d</td>
<td>0.0239176</td>
</tr>
<tr>
<td>f</td>
<td>7.15272</td>
</tr>
<tr>
<td>g</td>
<td>-6.25325</td>
</tr>
<tr>
<td>h</td>
<td>-0.748084</td>
</tr>
</tbody>
</table>

Figure 5.3: Comparison of the direct inverse of Taheri’s aperture model and the developed inverse model
Chapter 6

CONTROLLER DESIGN

In order to generate a desired force at the user’s fingertip, a controller must be developed to regulate the force produced by each pneumatic cylinder on the glove. The input to this controller is the desired actuator force and the output is the voltage signal to be sent to the pneumatic valves. This chapter focuses on the development of a sliding-mode controller with an integral term to control the force output of each pneumatic actuator.

Equation 5.1 defines a first-order control system with two variables, $P_a$ and $P_b$. The output of this system, $F$, is defined by Eq. 5.7. We define the system error as

$$e(t) = F(t) - F_d(t)$$

(6.1)

where $F_d$ is the desired force output of the pneumatic actuator. We define a sliding surface, $s$, by

$$s(t) = e(t) + \lambda \int e(t) dt$$

(6.2)

where $\lambda$ is a positive constant designating the weight of the integral error. The mass flow of air into each chamber of the pneumatic cylinder is given by Eq. 5.11. However, each chamber can be in two different dynamic configurations, depending on the current position of the controller. The upstream and downstream pressure of each configuration changes the inputs to Eq. 5.12, so it is important to identify exactly what pressure sources each chamber is connected to. When the spool position of the valve is positive, chamber $A$ of the cylinder is connected to the air supply while chamber $B$ is connected to exhaust. This causes air flow into $A$ and out of $B$. When
the spool position of the valve is negative, chamber $A$ of the cylinder is connected to exhaust while chamber $B$ is connected to the air supply. This causes air flow into chamber $A$ and out of chamber $B$. These different flow configurations are summarized in the next two equations.

$$
\dot{m}_a = \begin{cases} 
A_m A_v \psi(P_s, P_a, C_{f_{in}}) + \dot{m}_{aL} & z > 0 \\
-A_m A_v \psi(P_a, P_{atm}, C_{f_{out}}) + \dot{m}_{aL} & z < 0 
\end{cases} \quad (6.3)
$$

$$
\dot{m}_b = \begin{cases} 
-A_m A_v \psi(P_b, P_{atm}, C_{f_{out}}) + \dot{m}_{bL} & z > 0 \\
A_m A_v \psi(P_s, P_b, C_{f_{in}}) + \dot{m}_{bL} & z < 0 
\end{cases} \quad (6.4)
$$

These equations introduce several other variables that require explanation. Aperture area $A_v$ defined by Taheri’s model in Eq. 5.16 is normalized so we must multiply by the maximum aperture area, $A_m$, to compute the true area. The flow constant introduced in Eq. 5.12 is dependent on the geometry the air flows through. Since the path the air takes to enter each chamber from the pressurized supply is slightly different than the path to vent the chamber, we introduce two flow coefficients, $C_{f_{in}}$ and $C_{f_{out}}$, to differentiate the two flow geometries. $C_{f_{in}}$ describes the flow coefficient of the flow from the air supply into a given cylinder chamber while $C_{f_{out}}$ describes the flow coefficient of the flow from a given cylinder chamber out to the atmosphere [32].

The final variables introduced by Eqs. 6.3 and 6.4 are $\dot{m}_{aL}$ and $\dot{m}_{bL}$, which account for the continuous leakages present in the valve that were discovered during experimentation. It was assumed that these leaks can be accurately modeled as compressible flows through an orifice, just like the intended flows through the valve. This means Eq. 5.11 can also be used to define the leakage flow both in and out of each chamber in the pneumatic cylinder. Two distinct leaks are accounted for in this research. The first is a leakage flow from the air supply into the chamber, and the second is a leakage flow from the chamber out to atmosphere. Any leakage flow
between the two chambers is negligible within the cylinder and was ignored within
the valve. There may also be a leak present from the air supply straight to the
atmosphere, but such a leak has no impact on the Haptic Glove and so was ignored.
Both assumed leakage flows for each single chamber are accounted for together in Eq.
6.5.

\[
\begin{align*}
\dot{m}_{aL} &= A_L \left( \psi(P_s, P_a, C_{fin}) - \psi(P_a, P_{atm}, C_{fout}) \right) \\
\dot{m}_{bL} &= A_L \left( \psi(P_s, P_b, C_{fin}) - \psi(P_b, P_{atm}, C_{fout}) \right) 
\end{align*}
\]  

(6.5)

where \(A_L\) is the cross-sectional leakage area and must be determined experimentally.

A sliding-mode controller for the valve aperture are is created by setting the
derivative of Eq. 6.2 to zero and solving for \(A_v\). Since the pneumatic system can
operate in two configurations depending on the valve spool position, it is easiest to
compute two different controllers, one for each configuration, and actively switch
between them as dictated by the system configuration. When the spool position is
positive \((z > 0)\) air is flowing into chamber \(A\) and out of chamber \(B\) and the resulting
control law becomes

\[
\begin{align*}
u_{eqa} &= -\dot{F}_d + \dot{F}_f + \lambda(F_d - F) + \left( \frac{P_a A_a}{L_{ea}} + \frac{P_b A_b}{L_{ea}} \right) \dot{x} - A_L RT \left( \frac{\psi_a}{L_{ea}} - \frac{\psi_b}{L_{eb}} \right) \\
&= \frac{\dot{F}_d + \dot{F}_f + \lambda(F_d - F) + \left( \frac{P_a A_a}{L_{ea}} + \frac{P_b A_b}{L_{ea}} \right) \dot{x} - A_L RT \left( \frac{\psi_a}{L_{ea}} - \frac{\psi_b}{L_{eb}} \right)}{A_m RT \left( \frac{\psi(P_s, P_a, C_{fin})}{L_{ea}} + \frac{\psi(P_b, P_{atm}, C_{fout})}{L_{eb}} \right)} 
\end{align*}
\]  

(6.6)

When the spool position is negative \((z < 0)\) air is flowing into chamber \(B\) and out
of chamber \(A\) and the resulting control law becomes

\[
\begin{align*}
u_{eqb} &= -\dot{F}_d + \dot{F}_f + \lambda(F_d - F) + \left( \frac{P_a A_a}{L_{ea}} + \frac{P_b A_b}{L_{ea}} \right) \dot{x} - A_L RT \left( \frac{\psi_a}{L_{ea}} - \frac{\psi_b}{L_{eb}} \right) \\
&= \frac{\dot{F}_d + \dot{F}_f + \lambda(F_d - F) + \left( \frac{P_a A_a}{L_{ea}} + \frac{P_b A_b}{L_{ea}} \right) \dot{x} - A_L RT \left( \frac{\psi_a}{L_{ea}} - \frac{\psi_b}{L_{eb}} \right)}{A_m RT \left( \frac{\psi(P_s, P_{atm}, C_{fout})}{L_{ea}} + \frac{\psi(P_s, P_b, C_{fin})}{L_{eb}} \right)} 
\end{align*}
\]  

(6.7)

The numerators of the two control laws, Eqs. 6.6 and 6.7, are identical except for
the leading negative sign in Eq. 6.7. Furthermore, each individual factor the product
in the denominator of both equations is positive. Therefore, the overall sign of both 
equations is dependent on the shared numerator. When this numerator is positive, 
only Eq. 6.6 results in a positive control law, indicating that a positive valve aperture 
area in the positive spool configuration would result in an effective controller. When 
the numerator is negative, only Eq. 6.7 results in a positive control law, indicating 
that a positive valve aperture area in the negative spool configuration would result 
in an effective controller. Since it is impossible for both control laws to be positive 
at the same time, which control law to implement is always unambiguous.

Equations 6.6 and 6.7 introduce several new variables for ease of readability. Variables \( L_{ea} \) and \( L_{eb} \) simplify the control law expressions by replacing recurring terms 
and can be thought of us as the normalized lengths of chambers A and B, respectively, 
including the dead volumes of each chamber.

\[
L_{ea} = \frac{V_0 a}{A_a} + \frac{L}{2} + x \quad \text{(6.8)}
\]
\[
L_{eb} = \frac{V_0 b}{A_b} + \frac{L}{2} - x \quad \text{(6.9)}
\]

The other two new variables, \( \psi_a \) and \( \psi_b \), are the net normalized mass flow into chambers A and B, respectively, through both configurations simultaneously and account 
for the leakages through the valve.

\[
\psi_a = \psi \left( P_s, P_a, C_{fin} \right) - \psi \left( P_a, P_{atm}, C_{fout} \right) \quad \text{(6.10)}
\]
\[
\psi_b = \psi \left( P_s, P_b, C_{fin} \right) - \psi \left( P_b, P_{atm}, C_{fout} \right) \quad \text{(6.11)}
\]

Despite their complexity, the purpose of each term in Eqs. 6.6 and 6.7 is readily ap-
parent and the control laws can be understood conceptually. In each control law, the 
umerator consists of 5 addends. The first addend accounts for the derivative of the 
desired force. The second addend accounts for the derivative of the frictional force in 
the cylinder. The third addend is the integral term utilizing the integral coefficient
\( \lambda \) and accounts for the steady state error. The fourth addend is the only term including the derivative of the cylinder extension and accounts for the cylinder velocity. The fifth addend is the only term including the cross-sectional area of the valve leak and accounts for the expected leakage within the valve. The denominators of each control law specify the means of affecting the system state for each configuration. Specifically, the denominator of Eq. 6.6 accounts for the combined flows of mass into chamber \( A \) and out of chamber \( B \) which are the two flows directly influenced by the valve aperture area when the spool position is positive. Likewise, the denominator of Eq. 6.7 accounts for the two flows influenced by the valve aperture area for negative spool positions.

To account for uncertainties in the system, sliding mode controllers introduce a robustness term. To prevent excess chattering, we also add a boundary layer to the sliding surface, which smooths out the discontinuity of a sign-dependent \( \kappa \) term. Once again, depending on the current system configuration, there are two possible results.

\[
\begin{align*}
    u_a &= u_{eqa} - \kappa \text{sat} \left( \frac{e(t)}{\phi} \right) \\
    u_b &= u_{eqb} + \kappa \text{sat} \left( \frac{e(t)}{\phi} \right)
\end{align*}
\]

(6.12) (6.13)

where \( \text{sat}(x) \) is the saturation function defined by

\[
\text{sat}(x) = \begin{cases} 
-1 & x < -1 \\
    x & -1 \leq x < 1 \\
    1 & x \geq 1
\end{cases}
\]

(6.14)

The robustness term \( \kappa \) defines the weight of the uncertainty term relative to the rest of the controller, and \( \phi \) defines the thickness of the boundary layer. Finally, we can plug the active control law, \( u_a \) or \( u_b \) into the inverse model defined by Eq. 5.17 to get the corresponding normalized voltage to send to the pneumatic valves to realize the controller [5, 29, 30, 38]. Note that although the control signal and corresponding
voltage will always be positive due to our construction of the two control laws for the two system configurations, in order to achieve a negative spool position, the valve must be sent a negative voltage. Therefore, whenever the active controller is defined by Eq. 6.7, the final voltage computed by Eq. 5.17 must be negated to utilize the negative spool configuration of the system.
Chapter 7
EXPERIMENTAL VALIDATION

7.1. Fabrication and Experimental Setup

Of the 40 parts that make up the Haptic Glove, 34 were machined out of 6061 Aluminum on a 3-axi milling machine. The other 6 parts were fabricated in ABS plastic on a Stratsys uPrint SE 3D Printer. A base plate to hold the pneumatic manifold, valves, and valve drivers was also machined out of 6061 Aluminum, and a test stand for the Haptic Finger tests, discussed in Section 7.5, was machined out of Delrin acetal homopolymer. Engineering drawings of all machined parts are included in Appendix A. Hardware used in the assembly of the glove was purchased from McMaster-Carr (200 Aurora Industrial Pkwy. Aurora, OH 44202), and pneumatic connectors were purchased from Grainger (8321 John W. Carpenter Fwy, Dallas, TX 75247). An SMC Pneumatics 1/4” Inlet, 5/32” Outlet Manifold was used to split the pressurized air supply to 6 Enfield Technologies LS-V05s Proportional Pneumatic Control Valves. Each valve was controlled by an Enfield Technologies LS-C21 Analog Valve Driver. The valves direct air into 6 Bimba Round Body 5/16” Bore Double-Acting Air Cylinders. The two cylinders actuating the gearing mechanism acting on the interphalangeal joints of the index and middle fingers have a 1”-stroke. The other four cylinders have a 1 1/2”-stroke. Each actuator is accompanied by two Honeywell 100PAAB5 TruStability® Board Mount Pressure Sensors that measure the pressure in each chamber of the actuator cylinder. Also alongside each actuator is a P3 America Series LMC13 Linear Motion Conductive Plastic Potentiometer to measure
the current displacement of the actuator. The 1”-stoke actuators are measured by LMC13-25 potentiometers while the 1 1/2”-stroke actuators are measured by LMC13-50 potentiometers. An additional P3 America LMC8-11 Linear Motion Conductive Plastic Potentiometer attached to the palm substructure of the Haptic Glove measures the abduction/adduction motion of the index finger. Figure 7.1 shows the physically realized Haptic Glove design on a user’s hand.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$</td>
<td>25.4 mm, 38.1 mm</td>
<td>stroke-length of cylinder</td>
</tr>
<tr>
<td>$D_p$</td>
<td>7.94 mm</td>
<td>inner diameter of piston</td>
</tr>
<tr>
<td>$D_r$</td>
<td>3.18 mm</td>
<td>diameter of cylinder rod</td>
</tr>
<tr>
<td>$D_t$</td>
<td>2.38 mm</td>
<td>inner diameter of pneumatic tubes</td>
</tr>
<tr>
<td>$L_{ta}$</td>
<td>840 mm</td>
<td>length of tube to chamber $A$</td>
</tr>
<tr>
<td>$L_{tb}$</td>
<td>840 mm</td>
<td>length of tube to chamber $B$</td>
</tr>
<tr>
<td>$A_m$</td>
<td>5 mm$^2$</td>
<td>valve cross-sectional area</td>
</tr>
<tr>
<td>$A_L$</td>
<td>0.05 mm$^2$</td>
<td>leak cross-sectional area</td>
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<tr>
<td>$R$</td>
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<td>ideal gas constant</td>
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<tr>
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<td>atmospheric pressure</td>
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<tr>
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<td>supply pressure</td>
</tr>
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</tr>
<tr>
<td>$P_{cr}$</td>
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<td>critical pressure ratio</td>
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<td>sub-sonic flow coefficient</td>
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<tr>
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<td>flow coefficient out of chamber</td>
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</tr>
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<td>$\kappa$</td>
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</tr>
<tr>
<td>$\phi$</td>
<td>2 N</td>
<td>boundary layer thickness</td>
</tr>
</tbody>
</table>

Table 7.1: Haptic Glove experimental parameters
Figure 7.1: The fabricated Haptic Glove
The controllers developed in Chapter 6 and defined by Eqs. 6.6 and 6.7 were coded in LabVIEW 2013 and implemented in an NI CompactRIO FPGA Board. Details about the LabVIEW implementation of the controller can be found in Appendix B. Input signals from the pressure sensors and linear potentiometers were delivered to the FPGA System via an NI-9205 C Series Voltage Input Module. The output signals that control the valve drivers were transmitted via a pair of NI-9263 C Series Voltage Output Modules. Custom printed circuit boards were designed and ordered to power and organize the outputs from the pressure sensors and linear potentiometers associated with each actuator. An additional PCB was designed and ordered to provide power to the valve drivers and the various electronic sensors. Details about the Haptic Glove wiring can be found in Appendix C.

Table 7.1 summarizes the parameters used in the relevant Haptic Glove and Haptic Finger experiments covered in the next sections. Some of these parameters are defined by the Haptic Glove design or the ambient environment and others had to be determined experimentally.

7.2. Friction Identification

In order to implement Eqs. 5.9 and 5.10, we need to develop a model for the friction present in the actuator cylinders. To accomplish this, a force sensor was attached to the end effector of a fixed cylinder in an extended position. The actuator was then retracted by manually applying a force to the other side of the force sensor. Figure 7.2 shows the experimental setup for the friction tests. This test was repeated several times at 3 different chamber pressures.

Figure 7.3 shows the results of a typical friction test. For this test, both chambers of the cylinder were set to 0.342 MPa, which is about halfway between the atmospheric pressure and the supply pressure. The force applied to the force sensor
increases without causing movement in the actuator position until about 0.8s into the experiment. At this point static friction is overcome and the cylinder begins to retract as can be seen in Fig. 7.3a. The actuator becomes fully retracted at about 2.2s, at which point the force applied to the force sensor becomes irrelevant until the end of the test. The minor increase in velocity during actuator motion is accompanied by steadily increasing force. Figure 7.3b shows the viscous friction coefficient, $\nu_v$, computed as the force on the sensor divided by the velocity of the actuator rod. This calculation becomes unstable in regions of the test where the actuator was not moving, so the only region of interest lies between about 0.8s to about 2.2s. This shows a fairly consistent viscous friction coefficient of about 0.17 N·s/mm at 0.342 MPa.

Figure 7.4 shows the results of 15 similar friction tests repeated at varying pressures. Five tests, shown in shades of blue, were conducted with both chambers set to atmospheric pressure of 0.101 MPa. Five other tests, shown in shades of red, were conducted with both chambers set to 0.583 MPa, which is near the maximum pressure. Finally, the remaining five tests, shown in shades of green were conducted with
both chambers set to 0.342 MPa, which is near the middle of the possible pressure ranges. The best fit line to each cluster of data is also shown in the figure. The slopes of these best fit lines give the viscous friction coefficients of the frictional force present in the cylinders, while the y-intercepts give the kinetic friction coefficients, which are not dependent on velocity [40, 39].

Figure 7.5 shows the models developed from the friction tests for the range of pressures used in this research. The best fit line through the average kinetic friction
at each tested pressure is shown in Fig. 7.5a. This friction term is constant across all velocities, provided that there is sufficient velocity to be considered a kinetic state. The best fit line through the average viscous friction at each tested pressure is shown in Fig. 7.5b. This friction term varies with the velocity. Combining the best fit lines from these first two graphs and assuming that the friction behaves according to the larger of the two chamber pressures, we finally have a model for $\nu$ first defined by Eq. 5.9.

$$\nu = 0.0404P_m\dot{x} + 0.0006\dot{x} + (9.4748 P_m + 0.3374) \text{sgn}(\dot{x}) \quad (7.1)$$

where $P_m$ is the maximum pressure between chambers A and B and $x'$ is the velocity of the actuator’s displacement. The derivative of 7.1 is also required by the controller. Since the velocity of the actuator displacement cannot change signs without first coming to a stop and thereby entering a static friction state, it can be assumed that the term $\text{sgn}(x')$ is constant during states where kinetic friction must be considered.
Figure 7.5: Friction Models vs. Pressure (a) Kinetic Friction (b) Viscous Friction (c) Static Friction with Error Bars
We therefore have

\[ \dot{\nu} = 0.0404(\dot{P}_m \dot{x} + P_m \ddot{x}) + 0.0006 \ddot{x} + 9.4748 \dot{P}_m \text{sgn}(\dot{x}) \]  

(7.2)

During each friction test, the initial application of force rises without causing motion in the actuator rod. However, at some point, the static friction is overcome and there is a sudden drop in the force as the actuator rod begins to retract. The maximum force value reached before the onset of motion was recorded as the static friction present at each pressure. For the test shown in Fig. 7.3a, the static friction was found to be about 5.83 N. Figure 7.5c shows the summary of the static friction for all 15 conducted friction tests, as well as a logarithmic model for the static friction. From this data, we create the model for \( \mu \), first defined by Eq. 5.9.

\[ \mu = (3.4691 \ln(P_m) + 9.6707) \text{sgn}(F_p - F_u) \]  

(7.3)

where \( P_m \) is once again the maximum pressure between chambers A and B and \( F_u \) is the force of the user’s fingertip on the actuator. Assuming the signum term does not vary with time, the derivative of Eq. 7.3 is

\[ \dot{\mu} = \frac{3.4691}{P_m} \text{sgn}(F_p - F_u) \]  

(7.4)

However, as stated before in 5.1, the Haptic Glove does not have adequate sensors to determine \( F_u \). Because of this, it is impossible to determine the sign of the static friction and therefore Eqs. 7.3 and 7.4 cannot be properly implemented in the controller. Therefore, static friction was ignored in the controller and \( \mu \) was set to 0 for all experiments. Kinetic friction, however, was accounted for by the implemented controller. It was observed that noise within the potentiometers would occasionally cause velocities as high as 1.2 mm/s to be detected by the controller. Therefore, \( \delta_v \) was set to 2 mm/s for the purposes of detecting motion within the actuator and activating the kinetic friction model within the controller.

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7.3. Open Loop Step Response

Figure 7.6 shows the open loop step response of a single pneumatic cylinder-valve system to a 2.5 V 1 Hz square wave input. The two outputs shown are the absolute pressures of the two cylinder chambers as recorded by the Honeywell pressure sensors embedded in the pneumatic circuit. As can be seen from the graph, there is a brief delay in the system that was measured to be about 8 ms. Additionally, there is an unusual rapid pressure oscillation that occurs in both chambers immediately after the transition onset. This oscillation is stronger in the chamber that is transitioning down from a high pressure to a low pressure, and the oscillation rapidly decays over a period of about 50 ms. This oscillation is superimposed onto a more typical exponential decay curve converging to the expected steady-state pressures. The minimum steady state value for both chambers is the atmospheric pressure of 0.101 MPa and the maximum
steady state value for both chambers is the supply pressure of about 0.591 MPa.

Figure 7.7: Single cylinder step response to a 1 Hz square wave input of varying voltage with a 838 mm tube (a). Detail view of falling edge region(b).

To better understand the origin of this excitation in the system, additional open loop step responses with varying parameters were recorded. Figure 7.7 shows the step responses to inputs of varying voltages, maintaining a constant 1 Hz square wave for each test. Because the rise/fall time of each response is affected by the magnitude of the input, the system did not reach steady-state for the 1.5 V and 2 V tests within a single time period. Despite this, the previously identified oscillation is present across
all voltage levels and furthermore, the period of oscillation is constant at about 8 ms, regardless of voltage level. As expected, voltage also had no impact on the 8 ms response delay.

Figure 7.8: (a) Single cylinder step response to a 2.5 V 1 Hz square wave input with varying tube lengths. (b) Detail view of falling edge region.

Figure 7.8 shows the effect of changing the length of the pneumatic tubes connecting the valves to the actuating cylinders. Once again, the oscillation is present in all five experiments, but this time the oscillation frequency changed depending on the tube length. The longer the tube length, the greater the period of each oscillation.
The oscillation peaks recede too quickly in the 178 mm tubes to discern an oscillation frequency, but there are enough distinct peaks in the other tests to establish oscillation periods. The 368 mm tubes produce oscillations of period 5 ms. The 838 mm tubes, which are used in the final Haptic Glove design, produce oscillations of period 8 ms. The 1268 mm tubes produce oscillations of period 10 ms and the 1778 mm tube had a period of about 13 ms. Additionally, the oscillations diminished quicker in the shorter tube lengths and lingered in the response for longer tube lengths. The response delay was also affected by the varying tube lengths, with shorter tubes having shorter time delays.

We can conclude from these step response tests that the unusual oscillations present in every step response are related to pressure waves within the tubes themselves. This makes the system difficult to control, but the following section proves that the controller developed in Chapter 6 has acceptable performance for typical palpation frequencies.

7.4. Single Cylinder Controller Performance

The effectiveness of the controller was determined through a series of experiments on a simplified test setup consisting of only a single 1.5 inch (38 mm)-stroke cylinder. The cylinder rod was fixed in place at 19 mm, 0 mm, and -19 mm. In each x position, the controller was applied to the pneumatic system with the desired force set to a 10 N sinusoid at varying frequencies between 0.1 Hz and 50 Hz. The results are summarized by the Bode plot in Fig. 7.9

For a majority of the range of frequencies tested, the controller performs very similarly in all three x positions. For frequencies less than about 5 Hz, the controller performs very well in both magnitude and phase, reaching its maximum magnitude error of -1 dB, or 89% of the desired magnitude, at 2.5 Hz for x = 19 mm and its
maximum phase lag of 11.52° at 1.5 Hz for $x = -19$ mm. As the desired frequency extends above 5 Hz, the magnitude of the system response begins to climb considerably upwards, finally reaching a maximum magnitude error of 5.2 dB, or about 182% of the desired magnitude, at 35 Hz for $x = 19mm$. By changing the parameters of the leakage area, $A_{vL}$, and flow coefficients, $C_{f_{in}}$ and $C_{f_{out}}$, it is possible to lower this maximum magnitude error and improve performance in the 5 Hz to 50 Hz range. However, this comes at the cost of a worse performance for frequencies less than 5 Hz, and typical palpation motion will rarely require responses above 5 Hz. This also suggests that the parameters of the leakage area, $A_{vL}$, and flow coefficients, $C_{f_{in}}$ and
are not static, but depend on the frequency components of the desired valve performance. Beyond 50 Hz, both the magnitude and phase plots begin dropping rapidly as expected in typical control systems. At around 60 Hz, the output becomes unstable and it is no longer feasible to identify sinusoidal characteristics in the pressure sensors, but this is well beyond the palpation frequency of humans. One final point of interest is the unique phase curve of the system at $x = -19$ mm. Between 10 Hz and 35 Hz, the response of the system at this position diverges from the other position responses and acquires a phase lead as high as 14.4°. Though this is not a large deviation in phase, it is notable that the system in the other two tested positions remained much truer in phase to the desired signal. The cause of this deviation for $x = -19$ mm is unknown, but it is hypothesized to be a resonance effect between the chamber volumes and the pneumatic tubes connecting the cylinder to the valve. Outside of this frequency window, all three positions behave as expected. A phase lag of less than 12 degrees is maintained until about 25 Hz, after which a sharp increase in phase lag is observed as the signal period gets closer to the delay time. Overall, the system performs quite well at frequencies under 5 Hz, and the shortcomings observed around 35 Hz can be explained by the unusual step response of the system discussed in 7.3.

7.5. Haptic Finger Controller Performance

The final piece of the project requiring verification is the control of an entire finger of the glove using the generalized forces derived in Section 4.2. This is accomplished on a simplified testbed called the Haptic Finger, shown in Fig. 7.10. The Haptic Finger testbed consists of the index finger structure removed from the entire Haptic Glove, and the two pneumatic cylinders and corresponding valves used to control the forces within the index finger. The testbed has a sensor that measures the force of
Figure 7.10: Haptic Finger testbed consisting of two cylinder subsystems controlling the index finger structure.

the fingertip and has the ability to adjust both the mounting angle and the joint angles. The Haptic Finger system is controlled by a modified LabView program (see Appendix B) that contains two copies of the single cylinder controller developed in Chapter 6.

Figure 7.11 shows a typical performance of the Haptic Finger system. The fingertip of the system was held fixed against a horizontal surface. As negative vertical fingertip forces would cause the fingertip to lift off the surface, a haversine was chosen as the desired waveform. For this particular test, the Haptic Finger system was mounted at an upward angle of 50°, and the metacarpophalangeal angle, $\theta_{11}$, was set to 15°. The desired force function was a haversine wave with an amplitude of
Figure 7.11: Haptic Finger typical performance. $F_d = 9$ N, 1 Hz haversine wave. Mount angle = $50^\circ$, $\theta_1 = 15^\circ$. (a) Forces. (b) Chamber pressures. (c) Valve signals.
9 N and a frequency of 1 Hz. Equations 4.55 and 4.56 were utilized to derive the corresponding desired cylinder forces $F_{11}$ and $F_{12}$. Each of these desired forces were used as inputs to the dual cylinder controllers. Pressure sensors in the two cylinders were used to compute the expected force output from each cylinder, and these forces were put through the Lagrange Equation to determine the resultant vertical force in the fingertip. Both the desired and measured forces in the fingertip as well as the individual cylinders are shown in Fig. 7.11a. The pressures present in each chamber of the two cylinders during the test are plotted in Fig. 7.11b. Both plots indicate that in this configuration, the required distal actuator force, $F_2$, is higher than the proximal actuator force, $F_1$, and therefore a larger pressure differential is created in cylinder 2 to create the desired force at the fingertip. Finally, the voltage signals sent to each pneumatic valve during this performance are shown in Fig. 7.11c. These signals include a 0.1 V, 50 Hz square dither to help combat valve stiction.

There is a limit to how much force can be generated from the fixed pressure supply used in the Haptic Finger System. Figure 7.12 shows the performance of the Haptic Finger when the desired force at the fingertip is greater than what is achievable given the supply pressure and geometric parameters. Other than the amplitude of the desired force, which was increased to 18 N, all other parameters were kept the same as the typical test. As can be seen in Fig. 7.12a, both individual cylinder forces get clipped at the same value of about 25.7 N. This is the force that results when chamber $A$ of the cylinder is set to the maximum supply pressure of about 0.62 MPa and chamber $B$ of the cylinder is set to atmospheric pressure, which in the testing lab came to about 0.101 MPa. The system is not capable of producing any more force in either cylinder, so the desired cylinder forces are not met. As a result, the fingertip force gets clipped at about 13.7 N. These pressure saturation effects are more apparent in Fig. 7.12b, which shows the two chamber pressures of
Figure 7.12: Haptic Finger oversaturated performance. $F_d = 18$ N, 1 Hz haversine wave. Mount angle = $50^\circ$, $\theta_1 = 15^\circ$. (a) Forces. (b) Chamber pressures. (c) Valve signals.
both cylinders during this test. The greatest positive cylinder force is generated when chamber \( A \) is at the maximum pressure and chamber \( B \) is at the minimum pressure, and this saturation indeed happens several times throughout the test. Any time the pressures reach their respective limiting values, the corresponding force reaches its maximum point and the output is clipped. It can be seen in Fig. 7.12c that as the pressure saturation occurs, there is an increase in the control effort as the controller tries to compensate for the sudden increase in error from the desired behavior.

The Haptic Index Finger was also tested at higher frequencies. Figure 7.13 shows the results of the same typical test repeated, except with a 3 Hz desired force wave. It can be seen that the controller begins to struggle to reach the maximum and minimum desired force of the wave, but an overall sinusoidal shape is maintained. Minor chattering is occasionally present on some periods of the actuator force wave, but the chattering is greatly reduced on the fingertip force.

Figure 7.14 shows the same test repeated at 5 Hz and the trends of the 3 Hz test remain present. The downward slopes of the actuator forces seem to always have a smaller slope in absolute value than the desired downward slope. In a careful analysis, it was discovered the primary reason the corresponding signal for these instances was not more negative was the leakage term of the controller. Although the leakage area term, \( A_L \), can be increased to improve performance on this 3 Hz response, it worsens performance at lower frequencies. It can be concluded that the leakage rate in the valve is dependent on the rate at which pressure within the tubes is changing. However, this research assumed a model where leakage was independent of the pressure derivative, and used static parameters that gave the best overall performance. Additional factors not being accounted for by this research, such as the friction present in the Haptic Finger joints, or the non-uniformity of the pressure within the cylinder chambers and tubes, are likely impacting this performance as well.
Figure 7.13: Haptic Finger 3 Hz performance. $F_d = 9$ N, 3 Hz haversine wave. Mount angle = $50^\circ$, $\theta_1 = 15^\circ$. (a) Forces. (b) Chamber pressures. (c) Valve signals.
Figure 7.14: Haptic Finger 5 Hz performance. $F_d = 9$ N, 5 Hz haversine wave. Mount angle $= 50^\circ$, $\theta_1 = 15^\circ$. (a) Forces. (b) Chamber pressures. (c) Valve signals.
Figure 7.15: Haptic Finger 10 Hz performance. $F_d = 9$ N, 10 Hz haversine wave. Mount angle = 50°, $\theta_1 = 15°$. (a) Forces. (b) Chamber pressures. (c) Valve signals.
Figure 7.15 shows the performance of the Haptic Finger given a desired 10 Hz haversine wave. The time scale has been reduced to prevent clutter, so that only a half second of the response is shown. At this scale, tiny fluctuations in the forces and pressures are more noticeable. Despite the challenges discussed in the previous tests, the resulting forces on both the actuators and the fingertip have a clear, repeating sinusoid shape.

The Haptic Finger was also tested with different shaped desired force waves. The performance on a 1 Hz triangle wave is shown in Fig. 7.16. A non-negative desired force was required to keep the Haptic Finger flat on the testbed. Thus, the triangle equivalent to the haversine function was used, with linear segments alternating between 0 N and 9 N. While there is some expecting rounding of the corners of the wave, the fingertip force follows the desired force quite well for triangle waves.

Figure 7.17 shows the Haptic Finger performance on a square wave input. Once again, a square wave alternating between 0 N and 9 N was used to avoid non-negative fingertip forces. Due to the approximation of the force derivative being used in the controller, a slight overshoot is present on the rising edges of the desired actuator forces. Once again, the system is a little more sluggish on the falling edges of the response, caused by the same overestimation of the leakage during more rapid depressurization of the cylinder chambers and other simplifying assumptions discussed above.

All of the Haptic Finger results discussed so far were performed with the same geometric parameters, with the system mounted at 50° and the metacarpophalangeal angle, \( \theta_{11} \), set to 15°. However, other geometric parameters were tested. Figure 7.18 shows the performance of the Haptic Finger when the metacarpophalangeal angle, \( \theta_{11} \), set to 40° instead. This also resulted in a decreased interphalangeal angle, \( \theta_{12} \) to keep the fingertip resting on the horizontal surface. In this configuration, the force
Figure 7.16: Haptic Finger triangle wave performance. $F_d = 9$ N, 1 Hz positive triangle wave. Mount angle = 50°, $\theta_1 = 15^\circ$. (a) Forces. (b) Chamber pressures. (c) Valve signals.
Figure 7.17: Haptic Finger square wave performance. $F_d = 9$ N, 1 Hz positive square wave. Mount angle = 50°, $\theta_1 = 15°$. (a) Forces. (b) Chamber pressures. (c) Valve signals.
Figure 7.18: Haptic Finger alternate $\theta_1$ performance. $F_d = 6$ N, 1 Hz haversine wave. Mount angle = 50°, $\theta_1 = 40°$. (a) Forces. (b) Chamber pressures. (c) Valve signals.
of the distal actuator, $F_2$ is required to be even greater in proportion to $F_1$, the force of the proximal actuator, as shown in Fig. 7.18a. As can be seen in Fig. 7.18b, this results in the pressure of chamber $A$ of the distal actuator, $P_{2a}$ being periodically clipped by the available supply pressure. In these instances, an increase in the distal signal, $u_2$ can be seen in Fig. 7.18c to compensate the lack of pressure change in one chamber. Because the pressure in chamber $B$ has not yet reached atmospheric pressure, it is able to drop much lower than the pressures required in the proximal actuator chambers, and the overall force created by the distal actuator continues to rise, albeit a bit slower than before.

Figure 7.19 shows the performance of the Haptic Finger mounted at 30° instead of 50° as it was for all previous tests. Similar to the previous result, this configuration requires a large enough distal actuator force that the pressure in chamber $A$ of the distal actuator is periodically clipped by the available supply pressure. Once again, the distal signal $u_2$ can be seen compensating for this and dropping the pressure in chamber $B$ enough to maintain an increase in distal actuator force $F_2$.

Finally, Fig. 7.20 shows the Haptic Finger Performance mounted at a 70° upward angle. At this extreme angle, the configuration requires a greater force out of the proximal actuator than the distal actuator, unlike any other test case presented. The result is a role-reversal of many of the trends discussed so far. This time the proximal actuator experiences the clipping of its pressure in chamber $A$ as it is limited by the available supply pressure. Just like with the distal actuator, the distal signal increases in time with this saturation point to compensate. Chamber $B$ of the proximal actuator continues to drop even as chamber $A$ reaches its maximum value, and as a result an increase of force in the proximal actuator is maintained.
Figure 7.19: Haptic Finger smaller mount angle performance. $F_d = 5$ N, 1 Hz haversine wave. Mount angle = $30^\circ$, $\theta_1 = 20^\circ$. (a) Forces. (b) Chamber pressures. (c) Valve signals.
Figure 7.20: Haptic Finger bigger mount angle performance. $F_d = 16$ N, 1 Hz haversine wave. Mount angle = $70^\circ$, $\theta_1 = 5^\circ$. (a) Forces. (b) Chamber pressures. (c) Valve signals.
Chapter 8

CONCLUSION

This dissertation presents the original design of a pneumatically-actuated Haptic Glove composed of 4 substructure assemblies. Of the 13 degrees of freedom present in the index finger, middle finger, and thumb of the hand, the Haptic Glove fixes 4 degrees of freedom, constrains 2 degrees of freedom using gearing linkage systems, and actuates 6 degrees of freedom with pneumatic cylinders, allowing uncontrolled but measured motion in the abduction/adduction of the index finger, the remaining degree of freedom. Nearly linear bijective transfer functions between the actuator displacements and the joint angles were derived in closed forms for all 6 actuated joints. By considering the Haptic Glove as a quasistatic system, Lagrange’s Equations were used to convert these transfer functions into actuator force equations for each finger of the Haptic Glove that take fingertip force vectors as input. Models were developed for the chamber pressures and cylinder forces, as well as the choked airflow through the pneumatic valves. A nonlinear, sliding-mode controller that allows for each actuator to be controller by a single 5/3 proportional valve and accounts for friction within the cylinder, changing chamber volumes due to movement in the actuator rods, and leakage within the valve between the cylinder chambers and both the supply pressure and atmospheric pressure was developed and implemented in LabVIEW 2013. The Haptic Glove design was fabricated in machined aluminum and 3D-printed ABS, and outfitted with pneumatic actuators and electronic pressure and position sensors. Tests were performed to identify the friction within the pneumatic cylinders as well as the open-loop step response of the chamber pressures to a step
voltage input into the electronic valves. An unusual open loop step response was observed and determined to be related to pressure waves within the tubes themselves. Controller validation tests were conducted and presented on both a single pneumatic cylinder and an entire finger structure. The single-cylinder controller performs well on sinusoidal desired forces less than 5 Hz, with the maximum error of -1 dB occurring at about 2.5 Hz. The experiments on the performance of the index finger substructure showed that the force on the fingertip is able to adequately track sine waves, triangle waves, and square waves at frequencies ranging from 1 Hz to 10 Hz in a variety of mount and joint angle configurations by utilizing the proximal and distal actuators simultaneously. In situations where one chamber of an actuator reaches the maximum pressure available due to the air supply limitation, an increase in control effort to compensate was observed as expected. The ability of the controller to track different desired waveforms in different angle configurations demonstrates that the full Haptic Glove could produce haptic feedback for 3 fingers as required during typical palpation movements.
Appendix A

CAD DRAWINGS

The Haptic Glove design includes 42 part designs. The Base Plate holds the valves, valve controllers, and pneumatic manifold. The Single Finger Test Stand serves as a mount for the Index Finger Substructure of the Haptic Glove, allowing for the Haptic Finger tests covered in Chapter 7. Three sizes of finger cups were designed as part of this research. All three finger cups are interchangeable between the index finger, middle finger, and thumb fingertips, allowing the user to attach the most comfortable selection for his or her fingertip sizes. The other 37 parts form the rest of the Haptic Glove. In addition to the three finger cups, three other parts were printed in ABS plastic using additive manufacturing technology - the Outer Palm Clamp, the Palm Gripper-Flat, and two copies of the Palm Gripper-Curved. As a result, no engineering drawings were made for these parts. Engineering drawings for the rest of the parts are included in this appendix. The original drawings have been resized to fit these pages, so scale information may no longer be accurate.
The information contained in this drawing is the sole property of SMU. Any reproduction in part or as a whole without the written permission of SMU is prohibited.

Title: Index Finger Proximal Phalanx 1A

Material: 6061 Aluminum

Finish unless specified:
All surfaces to be stock or machined finish. Break all sharp edges and chamfer holes to .030" max.

Comments:
Screw heads need to be completely counter-sunk.

Dimensions are in inches.
Metric sizes in [brackets].

Tolerances:
Angular: ±1 DEG
X ± .050
XX ± .020
XXX ± .010
XXXX ± .005

Unless otherwise specified:

Dimensions are in inches. Metric sizes in [brackets].

Tolerances:
Angular: ±1 DEG
X ± .050
XX ± .020
XXX ± .010
XXXX ± .005

Material:
6061 Aluminum

Finish unless specified:
All surfaces to be stock or machined finish. Break all sharp edges and chamfer holes to .030" max.

Comments:
Screw heads need to be completely counter-sunk.

Dimensions are in inches. Metric sizes in [brackets].

Tolerances:
Angular: ±1 DEG
X ± .050
XX ± .020
XXX ± .010
XXXX ± .005

Material:
6061 Aluminum

Finish unless specified:
All surfaces to be stock or machined finish. Break all sharp edges and chamfer holes to .030" max.

Comments:
Screw heads need to be completely counter-sunk.

Dimensions are in inches. Metric sizes in [brackets].

Tolerances:
Angular: ±1 DEG
X ± .050
XX ± .020
XXX ± .010
XXXX ± .005

Material:
6061 Aluminum

Finish unless specified:
All surfaces to be stock or machined finish. Break all sharp edges and chamfer holes to .030" max.

Comments:
Screw heads need to be completely counter-sunk.

Dimensions are in inches. Metric sizes in [brackets].

Tolerances:
Angular: ±1 DEG
X ± .050
XX ± .020
XXX ± .010
XXXX ± .005

Material:
6061 Aluminum

Finish unless specified:
All surfaces to be stock or machined finish. Break all sharp edges and chamfer holes to .030" max.

Comments:
Screw heads need to be completely counter-sunk.
All surfaces to be stock or machined finish. Break all sharp edges and chamfer holes to .030" max.

2-56 threaded holes must accept at least 0.165" of screw

MATERIAL
6061 Aluminum

UNLESS OTHERWISE SPECIFIED:

DIMENSIONS ARE IN INCHES
METRIC SIZES IN [BRACKETS]

TOLERANCES:

ANGULAR: ±0.050
X: ±0.020
XX: ±0.010
XXX: ±0.005

SMU LYLE
SCHOOL OF ENGINEERING

REV
1

SHEET 1 OF 1
2/13/19

UNLESS OTHERWISE SPECIFIED:
SCALE: 2:1
Third Angle Projection
All surfaces to be stock or machined finish. Break all sharp edges and chamfer holes to .030" max.
3 x Ø 0.089 THRU ALL
4-40 UNC THRU ALL

Ream Ø 0.1563

R0.246 (flexible)

R0.125

6061 Aluminum

UNLESS OTHERWISE SPECIFIED:

All surfaces to be stock or machined finish. Break all sharp edges and chamfer holes to .030" max.
The 1/2" radius can be replaced by 1/4" if required.
All surfaces to be stock or machined 1/2. Break all sharp edges and chamfer holes to 0.030" max.
All surfaces to be stock or machined. Break all sharp edges and chamfer holes to .030" max.
All surfaces to be stock or machined finish. Break all sharp edges and chamfer holes to .030" max.

MATERIAL: 6061 Aluminum

SCALE: 2:1
THIRD ANGLE PROJECTION

UNLESS OTHERWISE SPECIFIED:
FINISH UNLESS SPECIFIED:

5 DEG

SMU LYLE
SCHOOL OF ENGINEERING

DRAWN
 Reviewed By

REV

9/4/18

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THIRD ANGLE PROJECTION

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THIRD ANGLE PROJECTION

UNLESS OTHERWISE SPECIFIED:
SCALE: 2:1
THIRD ANGLE PROJECTION

UNLESS OTHER
3 x ø 0.089 THRU ALL
4-40 UNC THRU ALL

75° Bend Down
2.072

ø0.1885 ±0.0000
ø 0.257

0.374
0.374

2.529

0.536

75° Bend Down
2.072

0.374

1.374

2.529

2.844

X4 R0.217 0.217

flexible

A

REV

SCALE: 2:1
Third Angle Projection
SH EET 1 OF 1

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MATERIAL
Aluminum 6061

FINISH UNLESS SPECIFIED
All surfaces to be stock or machined finish. Break all sharp edges and chamfer holes to .030" max.

TOLERANCES:
ANGULAR: ±1DEG
X: ± .050
XX: ± .020
XXX: ± .010
XXXX: ± .005

UNLESS OTHERWISE SPECIFIED:

DRAWN Matt Galla 1/30/19

DIMENSIONS ARE IN INCHES
METRIC SIZES IN [BRACKETS]

Comments:
1/8" thick.
Bend away from viewer

SMU LYLE
SCHOOL OF ENGINEERING

TITLE:
Thumb Carpometacarpal

SIZE Part Number
A 1301

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THE INFORMATION CONTAINED IN THIS DRAWING IS THE SOLE PROPERTY OF SMU. ANY REPRODUCTION IN PART OR AS A WHOLE WITHOUT THE WRITTEN PERMISSION OF SMU IS PROHIBITED.
7 x \( \phi 0.129 \) THRU ALL
\[ \phi 0.225 \times 100^\circ \]

**Bent out of page (towards viewer) 30° around a 0.584" radius centered 0.989" from top**

**TITLE:** Thumb Metacarpal 1

**SIZE** | **Part Number** | **REV**
--- | --- | ---
A | 1302 | 1

**DIMENSIONS ARE IN INCHES**

**METRIC SIZES IN [BRACKETS]**

**TOLERANCES:**
- \( \pm 0.050 \)
- \( \pm 0.020 \)
- \( \pm 0.010 \)
- \( \pm 0.005 \)
- \( \pm 0.0005 \)

**MATERIAL:** 6061 Aluminum

**FINISH UNLESS SPECIFIED:**
- All surfaces to be stock or machined finish.
- Break all sharp edges and chamfer holes to 0.030" max.

**COMMENTS:**
- 1/8" Thick.
- All holes are countersunk 100 degree-angle 4-40
**Title:** Thumb Metacarpal 2

**Material:** 6061 Aluminum

**Finish Unless Specified:**
- All surfaces to be stock or machined finish.
- Break all sharp edges and chamfer holes to .030" max.

**Dimensions:**
- 2.795
- 0.276
- 0.276
- 0.551
- 0.236
- 0.591
- 1.043
- 0.827
- 0.250
- 0.313
- 4 x R0.236 flexible
- 0.787
- 0.531
- 1.752
- 0.256
- 0.256
- 0.313

**Tolerances:**
- Angular: ±1 DEG
- X: ± .050
- X: ± 0.020
- XX: ± .010
- XXX: ± .005
- XXXX: ± .0005

**Notes:**
- 4 x Ø 0.089 THRU ALL
- 4-40 UNC THRU ALL
- 1/8" Thick

**Scale:** 2:1

**Drawing by:** Matt Galla

**SMU LYLE SCHOOL OF ENGINEERING**

**Sheet 1 of 1**

**Title Block Information:**
- **Title:** Thumb Metacarpal 2
- **Material:** 6061 Aluminum
- **Finish Unless Specified:**
  - All surfaces to be stock or machined finish.
  - Break all sharp edges and chamfer holes to .030" max.
- **Tolerances:**
  - Angular: ±1 DEG
  - X: ± .050
  - X: ± 0.020
  - XX: ± .010
  - XXX: ± .005
  - XXXX: ± .0005
- **Notes:**
  - 4 x Ø 0.089 THRU ALL
  - 4-40 UNC THRU ALL
  - 1/8" Thick
- **Scale:** 2:1
- **Third Angle Projection**
DIMENSIONS ARE IN INCHES
METRIC SIZES IN [BRACKETS]
TOLERANCES:
ANGULAR: ±1DEG
X ± .050
XX ± .020
XXX ± .010
XXXX ± .005
XXXXX ± .0005
MATERIAL
6061 Aluminum
FINISH UNLESS SPECIFIED
All surfaces to be stock or machined finish.
Break all sharp edges and chamfer holes to .030” max.

THUMB PROXIMAL PHALANX 1B

SMU LYLE
SCHOOL OF ENGINEERING

TITLE:
Thumb Proximal Phalanx 1B

SIZE | Part Number
A | 1305

DRAWN | Matt Galla
REV | 1

SCALE: 2:1
Third Angle Projection

UNLESS OTHERWISE SPECIFIED:

2 x 0.070  0.354
2-56 UNC  0.301

0.236
0.177
0.669
0.472
0.118
0.709
0.413
0.630

2 x R0.125 flexible

0.177 slot center

0.098
0.709
0.669
1.142

R0.063 flexible

R0.472

0.118
0.472
UNLESS OTHERWISE SPECIFIED:
FINISH UNLESS SPECIFIED:

All surfaces to be stock or machined finish. Break all sharp edges and chamfer holes to .030" max.

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SCHOOL OF ENGINEERING

MATERIAL
6061 Aluminum

SCALE: 1.5:1
Third Angle Projection

Ronald D. Smith
Adjunct Professor

Michael W. Gunter
Adjunct Professor

SHEET 1 OF 1
3/24/20

UNLESS OTHERWISE SPECIFIED:
SCALE: 1.5:1
THUMB DISTAL PHALANX

Matt Galla

1307
SMU LYLE
SCHOOL OF ENGINEERING
DIMENSIONS ARE IN INCHES
METRIC SIZES IN [BRACKETS]

MATERIAL

6061 Aluminum

FINISH UNLESS SPECIFIED:
All surfaces to be stock or machined finish.
Break all sharp edges and chamfer holes to .030" max.

3 x Ø 0.096 THRU ALL
√ Ø 0.172 X 82°
20 degree angle location is critical

All surfaces to be stock or machined finish. Break all sharp edges and chamfer holes to .030" max.

MATERIAL
6061 Aluminum

UNLESS OTHERWISE SPECIFIED:
FINISH UNLESS SPECIFIED:

SCALE: 1:1
Third Angle Projection
All surfaces to be stock or machined finish. Break all sharp edges and chamfer holes to .030" max.

UNLESS OTHERWISE SPECIFIED:

SCALE: 1:1
REV

Part Number 1405

REV 1

Third Angle Projection

MATERIAL

6061 Aluminum

FINISH UNLESS SPECIFIED

Part Number

All dimensions are in inches. Metric sizes in [brackets].

TOLERANCES:

ALL SURFACE

MACHINE CLASS IN [B] CHARACTERS

ANGULAR: ±1 DEG

X: ±.001

Y: ±.005

Z: ±.015

±.020

±.010

±.005

±.0005

SMU LYLE

SCHOOL OF ENGINEERING

TITLE: Middle Finger Slider A

SIZE

B

COMMENTS

DRAWN

MATT GALLA

1405

SMU LYLE

SCHOOL OF ENGINEERING

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Middle Finger Slider B

Part Number: 6061 Aluminum

Material: 6061 Aluminum

Dimensions: All surfaces to be stock or machined finish. Break all sharp edges and chamfer holes to .030" max.

Finish: UNLESS SPECIFIED

Tolerances:
- Angular: ±1° .005
- Linear: ±.001

Drawn by: Matt Galla

Scale: 2:1

Third Angle Projection

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**Title:** Thumb Slider

**Material:** 6061 Aluminum

**Finish Unless Specified:**
All surfaces to be stock or machined finish. Break all sharp edges and chamfer holes to 0.030" max.

**Dimensions:**
- Bend Down 1.228 in.

**Notes:**
- 1/8" Sheet Metal.
- Stud Press-Fit into 0.250 diameter hole

**Tolerances:**
- Angular: ±1°
- X: ±0.05
- X: ±0.02
- XX: ±0.01
- XXX: ±0.005
- XXXX: ±0.0005

**Title Block:**
- Title: Thumb Slider
- Size: A
- Part Number: 1407
- Scale: 1:1
- Third Angle Projection

**Drawn by:** Matt Gallia

**Scale:** 1:1

**Sheet:** 1 of 1

**Notes:**
- Dimensions are in inches. Metric sizes in brackets.
- Tolerances: Angular ±1°.
- 1/8" Sheet Metal stud Press-Fit into 0.250 diameter hole.

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DIMENSIONS ARE IN INCHES
METRIC SIZES IN [BRACKETS]
TOLERANCES:
ANGULAR: ±1DEG
X: ± .050
XX ± .020
XXX ± .010
XXXX ± .005

MATERIAL
6061 Aluminum

FINISH UNLESS SPECIFIED
All surfaces to be stock or machined finish. Break all sharp edges and chamfer holes to .030" max.

UNLESS OTHERWISE SPECIFIED:

DRAWN Matt Galla 12/3/18
COMMENTS:
0.125" Thick.
Need 2 Copies.

TITLE: Link - Actuator
SIZE Part Number 1501
SCALE: 2:1 Third Angle Projection
REV 1
2

A

DIMENSIONS ARE IN INCHES
METRIC SIZES IN [BRACKETS]

TOLERANCES:
ANGULAR: ±1DEG
X ± .050
XX ± .020
XXX ± .010
XXXX ± .005

MATERIAL
6061 Aluminum

FINISH UNLESS SPECIFIED
All surfaces to be stock or machined finish.
Break all sharp edges and chamfer holes to .030" max.

0.197" Thick.
Need 4 Copies

DRAWN
Matt Galla
12/3/18

COMMENTS:

TITLE:
Link - Gear

SIZE
Part Number
A
1502

SCALE: 2:1 Third Angle Projection

REV
1

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0.125" Thick.
Need 2 Copies
Matt Galla
6061 Aluminum

UNLESS OTHERWISE SPECIFIED:
DRAWN
DIMENSIONS ARE IN INCHES
MATERIAL
FINISH UNLESS SPECIFIED

METRIC SIZES IN [BRACKETS]

TOLERANCES:
ANGULAR: ±1DEG
X. ± 0.050
XX ± 0.020
XXX ± 0.010
XXXX ± 0.005

MATERIAL
6061 Aluminum

FINISH UNLESS SPECIFIED
All surfaces to be stock or machined finish. Break all sharp edges and chamfer holes to 0.030" max.
**TITLE:** Joint R

**SIZE** | **Part Number** | **REV**
---|---|---
A | 1504 | 1

**DIMENSIONS ARE IN INCHES**

**METRIC SIZES IN [BRACKETS]**

**TOLERANCES:**
- ANGULAR: ±1DEG
- X: ±0.005
- XX: ±0.005
- XXX: ±0.005
- XXXX: ±0.005

**MATERIAL**
6061 Aluminum

**FINISH UNLESS SPECIFIED**
All surfaces to be stock or machined finish. Break all sharp edges and chamfer holes to .030" max.

**COMMENTS:**
Need 2 copies

**DRAWN**
Matt Galla

**SCALE:** 2:1

**THIRD ANGLE PROJECTION**

**SHEET 1 OF 1**

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**UNLESS OTHERWISE SPECIFIED:**

**DIMENSIONS ARE IN INCHES**
**METRIC SIZES IN [BRACKETS]**

**TOLERANCES:**
- **ANGULAR:** ±1DEG
  - X: ±0.05
  - Y: ±0.02
  - Z: ±0.01
  - XXX: ±0.005
  - XXXX: ±0.0005

**MATERIAL**
- 6061 Aluminum

**FINISH UNLESS SPECIFIED**
- All surfaces to be stock or machined finish.
- Break all sharp edges and chamfer holes to .030" max.

**COMMENTS:**
- Need 3 Copies

**DRAWN**
- Matt Galla

**DATE**
- 4/23/19

**TITLE:**
- Joint L

**SIZE**
- A

**PART NUMBER**
- 1505

**REV**
- 1

**SCALE:** 2:1
**THIRD ANGLE PROJECTION**

**SCHOOL OF ENGINEERING**

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**PROPRIETARY AND CONFIDENTIAL**
DIMENSIONS ARE IN INCHES
METRIC SIZES IN [BRACKETS]

TOLERANCES:
ANGULAR: ±1DEG
X: ± .050
XX: ± .000
XXX: ± .0005
XXX: ± .00005

MATERIAL
6061 Aluminum

UNLESS OTHERWISE SPECIFIED:
FINISH UNLESS SPECIFIED
All surfaces to be stock or machined finish.
Break all sharp edges and chamfer holes to .030" max.

Note all 3 screw holes are different sizes

DRAWN: Matt Gallia  9/15/20

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PROPRIETARY AND CONFIDENTIAL
SMU LYLE
SCHOOL OF ENGINEERING

TITLE: Joint Special
SIZE: A  Part Number: 1506
REV: 1  SCALE: 2:1 Third Angle Projection SHEET 1 OF 1
SMU LYLE
SCHOOL OF ENGINEERING

UNLESS OTHERWISE SPECIFIED:
DRAWN

DIMENSIONS ARE IN INCHES
METRIC SIZES IN [BRACKETS]
TOLERANCES:
ANGULAR: ±10°
X: ±0.050
XX: ±0.020
XXX: ±0.010
XXXX: ±0.005

MATERIAL
6061 Aluminum

FINISH UNLESS SPECIFIED
All surfaces to be stock or machined finish. Break all sharp edges and chamfer holes to .030" max.

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SMU LYLE
SCHOOL OF ENGINEERING

TITLE: MM Sensor Joint
SIZE A
Part Number 1507
REV 1

SCALE: 5:1  Third Angle Projection  SHEET 1 OF 1
DIMENSIONS ARE IN INCHES
METRIC SIZES IN [BRACKETS]

TOLERANCES:
- ANGULAR: ±1DEG
- X: ±.050
- XX: ±.020
- XXX: ±.010
- XXXX: ±.005

MATERIAL
6061 Aluminum

FINISH UNLESS SPECIFIED
All surfaces to be stock or machined finish.
Break all sharp edges and chamfer holes to .030" max.

UNLESS OTHERWISE SPECIFIED:

DRAWN: Matt Galla 3/06/20

COMMENTS:
Need 4 Copies

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PROPRIETARY AND CONFIDENTIAL

SMU LYLE
SCHOOL OF ENGINEERING

TITLE: LMC Sensor Support Short

SIZE Part Number 1508

SCALE: 3:1 Third Angle Projection SHEET 1 OF 1

REV 1
DIMENSIONS ARE IN INCHES
METRIC SIZES IN [BRACKETS]
TOLERANCES:
ANGULAR: ±1°
X: ±.050
XX: ±.020
XXX: ±.010
XXXX: ±.005
MATERIAL
6061 Aluminum
FINISH UNLESS SPECIFIED
All surfaces to be stock or machined finish. Break all sharp edges and chamfer holes to .030" max.

SMU LYLE
SCHOOL OF ENGINEERING

TITLE: Cylinder Collar
SIZE | Part Number | REV
A | 1510 | 1
SCALE: 2:1 Third Angle Projection SHEET 1 OF 1

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2 x \( \phi 0.136 \) THRU ALL 8-32 UNC THRU ALL

4 x \( \phi 0.107 \) THRU ALL 6-32 UNC THRU ALL

4 x \( \phi 0.063 \) THRU ALL M2x0.4 - 6H THRU ALL

24 x \( \phi 0.107 \) THRU ALL 6-32 UNC THRU ALL

2 x \( \phi 0.136 \) THRU ALL 8-32 UNC THRU ALL

DIMENSIONS ARE IN INCHES
METRIC SIZES IN [BRACKETS]

ANGLES:
- \( \pm 1.0^\circ \)
- \( \pm 0.05^\circ \)
- \( \pm 0.02^\circ \)
- \( \pm 0.01^\circ \)
- \( \pm 0.005^\circ \)
- \( \pm 0.0005^\circ \)

MATERIAL
- Aluminum

FINISH UNLESS SPECIFIED
- All surfaces to be stock or machined finish.
- Break all sharp edges and chamfer holes to 0.030" max.

COMMENTS:
- 12.5" x 12.5" plate
- 1/4" Thick

UNLESS OTHERWISE SPECIFIED:

DRAWN Matt Galla

REV 1

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SIZE Part Number
A 1701
Appendix B

LABVIEW BLOCK DIAGRAMS

The controller derived in Chapter 6 was coded in LabVIEW 2013. The completed controller was implemented on an F.P.G.A. Board. A total of 7 custom VI’s were designed, and the details of each of these custom VI’s can be found in this appendix.
Appendix C
HAPTIC GLOVE WIRING DIAGRAM

The full Haptic Glove has many different input and signal wires, as well as multiple power and ground wires to power all of the electronic components. The FPGA board has 19 analog inputs through an NI-9205 C Series Voltage Input Module. These inputs include the 6 pairs of pressure sensors for each chamber of the 6 pneumatic cylinders and the 7 potentiometer voltages that indicate the current position of the 6 actuators and the 1 unactuated degree of freedom in the abduction/adduction of the index finger. The FPGA board also has 6 analog outputs that each have a unique ground wire, totaling 12 output wires connected through a pair of NI-9263 C Series Voltage Output Modules. These 6 signals control the 6 Enfield LS-C21 Analog Valve Drivers. The custom Power PCB developed for this research requires a +12 V, -12 V, and a ground line as inputs and provides 6 +12 V terminals, 6 -12 V terminals, 7 5 V terminals, and a total of 14 ground terminals. Each valve driver requires a +12 V, GND, and -12 V input, and receives a pair of wires carrying the command signal from the FPGA board. Six identical custom Glove PCBs are utilized to distribute power and organize signals to each pneumatic actuator and linear potentiometer pair. Each Glove PCB is powered by a +5 V line and a ground line and sends 3 signals to the FPGA board: the 2 voltages from the Honeywell 100PAAB5 TruStability Board Mount Pressure Sensors, which measure the pressure in the two chambers of each pneumatic cylinder, and the voltage from the P3 America LMC13-25 or LMC13-50 Linear Motion Conductive Plastic Potentiometers, which measures the current displacement of each actuator. A full wiring diagram is included on the next page.
REFERENCES


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[34] Vargheese, T. Quasi-static ultrasound elastography. Ultrasound clinics 4, 3 (2009), 323.


