Network Generating Models for Equipment Replacement

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NETWORK GENERATING MODELS FOR EQUIPMENT REPLACEMENT

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*This paper represents a draft of work in progress by the authors and is being sent to you for information and review. Responsibility for the contents rests solely with the authors. This working paper may not be reproduced or distributed without the written consent of the authors. Please address all correspondence to Julius S. Aronofsky.
We discuss the generation and solution of network equipment replacement models from a kernel (or base case) network. Using the IFPS (Interactive Financial Planning System) modeling language, we implemented an efficient network generator. We discuss a special version of the Dijkstra shortest path labeling algorithm to solve these problems. A specific example is given to illustrate the method.

1. INTRODUCTION

Over the past 35 years steady progress has been made in developing equipment replacement methods. As pointed out by Dean [18], equipment is normally replaced for two reasons. First, degradation or deterioration occurs or obsolescence takes place and the equipment is subject to replacement because newer equipment offers improved, faster, or cheaper return on planned investment. Second, complete or partial failure may occur in the original unit or units which in turn forces the decision of immediate replacement or repair of single or group units.

In this paper we are concerned with the first type of planned replacement. The problem involves 1) determining the optimum point in time or cumulative usage to replace all or part of the existing units and 2) choosing the best of the available equipment to be purchased for replacement or to cope
with a forecasted future demand. The various approaches to this type of problem are reviewed briefly in the following sub-section.

1.1 Approaches to Equipment Replacement Problems

Equipment replacement approaches for the first and second types mentioned above are reviewed briefly. For a comprehensive review of the literature see the survey article by Pierskalla and Voelker [37] and Rapp [39].

Consider the equipment replacement problem faced by a production shop which must keep a machine or facility operating over a finite time horizon. As the machine ages, its costs of operation and maintenance increase, while its salvage value decreases. At some point in time, it may be replaced by a new machine. The process repeats for this new machine. As a result, a chain of replacement decisions are made [37], [39], [42], [43], [46].

The problem then is to determine the optimal time to replace the machines in sequence. The replacement decisions are based on machine prices, maintenance and operation costs, and salvage functions. It is perhaps useful to classify the methods into the categories of Capital Budgeting/Cash Flow, Dynamic Programming and Markovian Processes, and Shortest Path Methods.

1. Capital Budgeting/Cash Flow

It was recognized some time ago that equipment replacement could be viewed as another capital investment opportunity; that is, a cost incurred in the expectation of future benefits, usually monetary benefits. Usually the model is organized in the form of a discounted cash flow where the time for equipment retirement or replacement is essentially the only decision variable [14], [18]. The problem of optimal retirement was first investigated by Hotelling [26] with significant extensions by Terborgh [47]. See [18] for
review material. The following formulation will be useful later on in this paper.

Define:

\[ T \] = The time of the replacement.

\[ V(T) \] = Net present value piece of equipment retained in service from time 0 to T,

\[ C(0) \] = Initial investment at time 0,

\[ S(T) \] = Salvage value of equipment at time T,

\[ Q(t)dt \] = Return earned by the equipment in the time interval \((t, t+dt)\),

\[ i \] = The discount rate.

Then

\[ V(T) = \int_0^T e^{-it}Q(t)dt + S(T)e^{-iT} - C(0) \] (1)

An advantage attributed to this approach is that the model and assumptions, as indicated in Equation (1), are well known to practitioners involved in capital budgeting. Perhaps a disadvantage is that the method is not a true optimizer and frequently a large number of cases have to be enumerated and compared separately. In order to ease the computational burden, Aronson, Aronofsky and Gray [6] developed interactive software that permits analysis of proposed replacement schedules with different factors for inflation, escalation, and technological improvement.

2. Dynamic Programming and Markovian Processes

The sequential nature of the equipment replacement process lends itself to the application of dynamic programming. Such approaches are due to Bellman [11, 12], Bellman and Dreyfus [13] and Howard [27] and are also discussed by Hillier and Lieberman [25] and Wagner [49]. These formulations were the first
to apply an optimization technique to this decision making problem. A chain of sequential decisions can be made over the planning period. Dynamic programming liberates the restrictions of the cash flow models and can eliminate some of the enumerations in the optimization of a problem. However, dynamic programming methods are computationally inefficient. For the probabilistic failure case over an infinite horizon, the problem can be modeled as a Markovian decision process [37] and solved by the value determination/policy improvement algorithm [27]. An important feature of this formulation is that the decision is dependent upon the "state" of the equipment.

Forward algorithms are a new approach for solving sequential decision making problems. Instead of a backward recursion relationship, a forward recursion formulation of the problem is established. In addition, decision (previously called planning) horizon procedures indicate when the addition of the next period data in the forward recursion will have no effect on early period decisions. Thus computation can stop.

Forward algorithms and decision horizon procedures based on the results of the Wagner and Whitin [50] algorithm for the dynamic lot size model and those of Lundin and Morton [32] are due to Sethi and Chand [16, 42] and Bean, Lohmann and Smith [10].

3. Control Theory

Early optimal control theory [44] approaches to the determination of the optimal maintenance schedule and the sale date of a machine for continuous models are due to Masse [33] and Naslund [34]. The necessary conditions for an optimal control policy are called the maximum principle [38], [44]. Thompson [48] discusses an application of the maximum principle to a simple maintenance model resulting in a bang-bang policy, that is, a policy where the maintenance is applied at its maximum level for a period of time, after which
zero maintenance is used. Extensions of Thompson's model are due to Arre and Lele [8] (technological progress), Kamien and Schwartz [29] (modification of the probability of failure), Sethi and Morton [43] and Sethi [41] (chain of replacement decisions), and Tapiero [46] (technical obsolescence). See also [1], [2], [3] and [40].

4. Shortest (Longest) Path Networks

It is not difficult to cast the capital budgeting formulation of the equipment replacement problem into the structure of a network [36], [49]. For instance consider the example shown in Figure 1 for a company planning its equipment replacement over a 5 year planning horizon. Referring back to Equation (1), let

\[ V_{ij} = \text{Net present value of the benefit of a machine purchased} \]
\[ \text{at the start of period } i \text{ and sold at the beginning} \]
\[ \text{of period } j. \]

In order to transform this problem into a shortest route network, let

\[ d_{ij} = -V_{ij} \]

and refer to the directed network of Figure 1. Nodes 1 and 6 represent the start and end of the planning period and each intermediate node \( j = 2, 3, 4, 5 \) represents the beginning of year \( j \) or the end of year \( j-1 \). From inspection of Figure 1, from every node \( i \) there is a directed arc to all nodes \( j \) only if \( j > i \).

It is not our intent to discuss network solutions in this Section but it is apparent that the determination of the shortest path from node 1 to 6 automatically determines the period or periods when equipment replacement takes place.

To date, the shortest path network has received only limited attention. It is mentioned briefly in Hillier and Lieberman [25] as a vehicle for
explaining a dynamic programming algorithm. Bean, Lohmann and Smith [10] also show a network, but do not use the shortest path for formulation or solution. Also, some of the current Operations Research textbooks, such as [36] and [49] illustrate the network model for replacement but only to describe the shortest path problem.

1.2 Organization of this Paper

The purpose of this paper is to examine carefully the network representation of the shortest path type and expand its utility to more realistic problems. Three areas are identified:

1. Establish a nomenclature and mathematical structure that will permit representation of more advanced replacement problems into a directed network format.

2. Develop means for generating nodes, arcs, and arc data values for replacement problems that require networks of substantial size.

3. Improve upon existing algorithms to solve shortest (and longest) path directed networks that lead to efficient solutions to advanced replacement problems.

All three items are discussed in this paper. However, emphasis is given to item 2 on network generation. A specific interactive financial planning language, called IFPS, was of considerable help in the network generation. Since the financial planning language is designed to manipulate tables or matrices, it is worthwhile to establish a method for transforming the network nodes, arc, and data into a suitable matrix form for matrix generation in IFPS and then back into the network form for efficient algorithmic solution.

Finally, all items are illustrated with a numerical solution.
2. NETWORK FORMULATION

Our model, similar to that in [42], is a network based discretization and generalization of a model originally due to Bowman and Fetter [14]. We assume that machine deterioration and technological obsolescence are deterministic. We further make the simplifying assumption that there is only one challenger available to replace the defender at any point in time. Technological advances are reflected by the fact that the challenging machine has a different purchase price, revenues and costs. The selection of alternative challengers based on different technologies has been addressed in [16] and is the subject of a companion paper [5]. In this model we do not consider the difficult issue of capacity expansion in conjunction with the equipment replacement decision. Although not required by the network model, we assume an exponential decay function in the salvage value of the equipment over time. A minor modification to the IFPS model is required to incorporate a linear decay function. For now, we assume that there is no limitation on the working life of a machine. We relax this assumption later. Normally, equipment replacement decisions are made annually, so periods are years. Except for the purchase, we assume cash flows to occur at the end of the period.

2.1 Notation

\[ P_i = \text{the price of a new machine at the start of period } i. \]
\[ R_{ij} = \text{the revenue for period } j, \text{ of a machine purchased at the start of period } i. \]
\[ C_{ij} = \text{the maintenance and operating cost for period } j, \text{ of a machine purchased at the start of period } i. \]
\[ CF_{ij} = R_{ij} - C_{ij} = \text{cash flow for period } j, \text{ of a machine purchased at the start of period } i. \]
\( S_{ij} \) = the salvage value of a machine purchased at the start of period \( i \) and sold at the end of period \( j \). By definition,
\[
S_{i,i-1} = P_i.
\]
\( \alpha \) = exponential salvage drop factor, \( 0 < \alpha < 1 \), \( S_{ij} = \alpha S_{i,j-1} \).
\( d \) = discount rate. Alternatively, we could use \( d_i \) for period \( i \).
\( r \) = escalation rate for the cost, revenue and cash flow streams. Alternatively we could use \( r_i \) for period \( i \).
\( T \) = fixed time horizon = problem length.
\( V_{ij} \) = The net present value of the benefit of a machine purchased at the beginning of period \( i \) and selling it at the beginning of period \( j \).

We compute \( V_{ij} \) as
\[
V_{ij} = \sum_{k=i}^{j-1} \frac{CF_{ik}}{(1+d)^k} + \frac{S_{i,j-1}}{(1+d)^{j-1}} - \frac{P_i}{(1+d)^{i-1}}
\]

2.2 The Kernel

We now define the kernel problem and network. Using the price of the machine at the start of period 1, \( P_1 \), and the cash flows of a machine purchased in period 1, \( CF_{1j}, j = 1, \ldots, T \), we can calculate \( V_{1j}, j=2, \ldots, T+1 \).

A network representation of the kernel problem is shown in Figure 2. The arcs from node 1 to node \( j \) represent the purchase of a new machine at the start of period 1, operating it through the beginning of period \( j \) when it is sold for its salvage value.

We can use the kernel for network generation in the following way. Given a forecast of prices for new equipment \( P_t, t = 2 \ldots T \), and assuming that the individual cash flows of revenue and cost, or the total cash flow increase, with escalation over the base case of a period 1 purchase, we have
\[ C_{ij} = (1 + r)^{i-1} C_1(j-i+1) \]

\[ R_{ij} = (1 + r)^{i-1} R_1(j-i+1) \]

\[ CF_{ij} = R_{ij} - C_{ij} = (1 + r)^{i-1} [R_1(j-i+1) - C_1(j-i+1)] \]

\[ = (1 + r)^{i-1} CF_1(j-i+1) \]

If dictated by the model, separate escalation factors can be used for the cost and revenue cash flows. So, the \( V_{ij} \), for \( i=2, \ldots, T \), \( j=i+1, \ldots, T+1 \), can be generated from the \( P_i \), the \( CF_{ij} \) and the escalation factor \( r \). The complete network representation is shown in Figure 3. Researchers [10], [25], [36], [49] have also discussed this model, but have not utilized its potential. A network representation of a \( T \) period problem has \( T+1 \) nodes and \( T(T+1)/2 \) arcs. A 20 period problem has 21 nodes, 210 arcs; for 25 periods there are 26 nodes and 325 arcs. If the maximum life of a machine is \( m \) periods, \( m < T \), then a \( T \) period problem has \( T+1 \) nodes and \( m(T+1) - m(m+1)/2 \) arcs.

The arc values, \( V_{ij} \), of the network can be generated from the kernel network as required, rather than inputted and stored explicitly. If the price of the machine in period \( t > 1 \) is defined in terms of \( P_1 \), then an open-ended network can be generated in the sense that a \( T+1, T+2, \ldots \) period network can be generated by functions identical to those used in generating a \( T \) period problem. Ideally, one would like to know when to stop generating data and use a finite horizon model. Intuitively, the end effects are minimal if the longest possible finite horizon model is used, but this is not always the case. A forward algorithm and decision horizon theorems are presented in [42] to determine how many periods of data are necessary. The time of the last replacement decision is monotonically increasing in the problem length. The early replacement decisions are fixed for any longer finite horizon problem.
2.3 Matrix Representation: An Example

Consider the example data shown in Table 1 which assigns specific values for the symbols given above. These are the kernel data which we will use for network generation in Section 5.

3. NETWORK GENERATION

Problems of real interest will require an acyclic network of substantial size. This means there must be a software support system that starts with the kernel and generates all of the data needed. There already exists a tradition of using software support systems for generating networks (NETGEN [31], NETGEN II [21]) and matrices ([7], [22]).

Over the past decade, software systems have emerged under the general name of financial planning systems [15], [23], [30], [35]. For example, see [9], [28] and [45]. The features of most interest to us for the current work are

1. Matrix manipulation/Electronic Tablet.
2. Command Driven.

IFPS was chosen because it not only meets all of the above criteria but also has a GENERATE command, useful for model generation. In terms of implementation, IFPS is efficient and capable of not only generating the network using escalation factors from cash flows, but also capable of using the special labeling algorithm described in the next section to solve the problem. The IFPS optimization model is discussed in a companion paper [4].
4. ALGORITHMS FOR SOLUTION

Because this problem is modeled as a special structure network, efficient solution procedures can be developed. Algorithms for finding the shortest or longest path in an acyclic graph are applicable. Because the arc data are given as benefits, a longest path formulation is used. These include the Dijkstra [20] algorithm and critical path approaches [24]. However, these methods require a complete statement of the network initially. Because of the special structure of the network, we can use the following labeling algorithm which eliminates the expensive scanning operations and network data structures required by the other methods:

Special Structure Longest Path Algorithm:

1. Initialization
   
   Set $\text{Label}(1) = 0$
   $\text{Pred}(1) = 0$
   
   Compute $\text{CF}_{1j}$ for $j=2,\ldots,T$
   $\text{V}_{1j}$ for $j=2,\ldots,T+1$
   
   Set $\text{Label}(j) = \text{V}_{1j}$ for $2,\ldots,T+1$
   $\text{Pred}(j) = 1$ for $j = 2,\ldots,T+1$

2. For $i = 2$ to $T$
   
   For $j = i+1$ to $T+1$
   
   Find $\text{CF}_{ij}$ by (3) in terms of $\text{CF}_{i}(j-i+1)$.
   Find $\text{V}_{ij}$ by (2).
   
   Set $\text{Pred}(j) = i$, if $\text{Label}(i) + \text{V}_{ij} > \text{Label}(j)$
   
   Set $\text{Label}(j) = \text{Maximum} \{\text{Label}(j), \text{Label}(i) + \text{V}_{ij}\}$

The algorithm permanently labels the nodes starting from node 1 through $T+1$ in numerical sequence. Because the permanently labeled set is predictably
augmented, only the label and predecessor node functions need be stored. This leads to the following Theorem:

**Theorem:**

The labeling algorithm above produces an optimal solution to the shortest path equipment replacement problem in \( T \) iterations. The solution time is on the order of \( T^2 \).

**Proof:**

The fact that the labeling algorithm finds an optimal solution is based on the original Dijkstra algorithm [20]. See also [17] and [19]. Since there are \( T \) nodes to be labeled following the initialization, and the nodes are labeled in numerical sequence, it takes \( T \) iterations to execute the algorithm. The four assignments in Step 2 are performed exactly \( T(T-1)/2 \) times. Thus, the algorithm is on the order of \( T^2 \).

This algorithm requires only \( 5T+2 \) memory storage locations to store the cash flow and solution data for the \( T(T+1)/2 \) arcs. \( T \) memory locations are required each for the price, cash flow and arc values \( V_{ij} \). \((T+1)\) locations are required for each of the label and predecessor functions.

Not only is the algorithm efficient, but instead of specifying and storing the entire network, the arc values are generated on the fly from the kernel and discarded following their use. Then, we can solve the network problem either with a special structure FORTRAN code, modify an existing critical path method code to accept negative arc values, or use an IFPS model [4].

In IFPS, we use the GENERATE statement to generate the actual statements which define the model from the kernel.

In addition to generating problems with the IFPS model, we used Harris and Maggard's [24] critical path method code CRIT. We generated the arc values from our IFPS model and with a utility program (MAKCPM), converted the
data for CRIT. Our equipment replacement model can be readily solved by standard critical path software if the $V_{ij}$ are positive. Otherwise, a minor modification to the code is required.

5. EXAMPLE

Consider the problem data shown in Table 1. In Figure 4, we show the IFPS kernel model. See the Appendix for a line-by-line description of the model and the special functions used. In Figure 6, we show the IFPS network generation statements. The GENERATE statement in line 270 generates the network in matrix form. Lines 280-360 are replicated starting at line 400, using the # as an index which is replaced with 2 - 10. In Table 2, we show the arc values (PRESENT VALUE 1 - PRESENT VALUE 10) generated by the model. Using the STORENT command, we set up a data file containing the arc values, and converted it to a form that CRIT would accept as input. The critical path from node 1 to 6 to 11 indicates that the machine should be replaced at the start of year 6 and retained until the end of the tenth year.

6. SUMMARY AND CONCLUSIONS

We have presented a network generating model for solving finite horizon equipment replacement problems. The effectiveness of the model is enhanced by the fact that modeling language software and critical path computer codes are available to the management science community.

In early tests on an IFPS satisficing equipment replacement model [6], the participants stated that although the model was a worthwhile tool for decision support, an optimizing model would be a major improvement. Our network and IFPS generating model meets this need. It optimizes and is easy to explain. A companion paper [5] will focus on the changing technology
environment, where the decision maker must choose among alternatives as well as the time to replace existing equipment.

Financial planning systems manipulate matrices in a user friendly form. They have certain features for ease of use which we have found effective for network generation and solution.

The generation of a complete network model from the kernel is the key to a simple formulation tool. We can define and solve large open-ended equipment replacement problems through the combination of the features of network optimization and modeling languages. The longest path format of the problem also enables us to use existing shortest path or critical path method codes to solve such problems.
APPENDIX: Description of the IFPS Model

<table>
<thead>
<tr>
<th>Line</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>COLUMNS Statement - the columns are to be numbered consecutively from 1 - 11.</td>
</tr>
<tr>
<td>20-100</td>
<td>Comments start with an asterisk (*).</td>
</tr>
<tr>
<td>110 - 170</td>
<td>Global Economic Data.</td>
</tr>
<tr>
<td>110</td>
<td>TIME - Column Counter.</td>
</tr>
<tr>
<td>120</td>
<td>TIME 1 - Indirect Address Variable.</td>
</tr>
<tr>
<td>130</td>
<td>INFLATION RATE - The cash flow escalation factor.</td>
</tr>
<tr>
<td>140</td>
<td>INFLATION - 1+INFLATION RATE.</td>
</tr>
<tr>
<td>150</td>
<td>DISCOUNT - The discount rate / cost of capital.</td>
</tr>
<tr>
<td>160</td>
<td>SALVAGE DROP FACTOR - Used to decrease the value of the asset by 15% per period.</td>
</tr>
<tr>
<td>170</td>
<td>PRICE schedule.</td>
</tr>
<tr>
<td>180 - 250</td>
<td>Kernel Network Problem to define arcs 1-2, 1-3, ... 1-11, in columns 1, 2, ..., 10.</td>
</tr>
<tr>
<td>190</td>
<td>INVESTMENT 1 - The investment made when purchasing a machine at the start of period 1.</td>
</tr>
<tr>
<td>200 - 210</td>
<td>CASH FLOW 1 - Cash flow schedule ( = CF_{1j} ).</td>
</tr>
<tr>
<td>220 - 230</td>
<td>SALVAGE SCHEDULE 1 - ( S_{1j} ).</td>
</tr>
<tr>
<td>240 - 250</td>
<td>SALVAGE SCHEDULE 1 - ( V_{1j} ) - takes the net present value of the cash flow minus the net present value of the investment plus the net present value of the salvage for a machine purchased at the start of period 1 and sold at the end of periods 1, 2, ..., 10.</td>
</tr>
<tr>
<td>260 - 360</td>
<td>Generates the ( V_{1j} ) for Years 2 through 10.</td>
</tr>
<tr>
<td>270</td>
<td>GENERATE - for each of lines 280 - 360, starting at line 400, create 9 new lines by replacing the # with the numbers 2, 3, ..., 10.</td>
</tr>
<tr>
<td>280</td>
<td>INDEX # - used to offset the columns and perform indirect addressing with the VMATRIX function.</td>
</tr>
<tr>
<td>290</td>
<td>TIME # - used to perform indirect addressing with the VMATRIX function.</td>
</tr>
</tbody>
</table>
INVESTMENT # - The investment made when purchasing a machine at the start of period # for # = 2 through 10.

CASH FLOW # - Cash flow schedule = CF#j for # = 2 through 10.

SALVAGE SCHEDULE # - S#j for # = 2 through 10.

PRESENT VALUE # - V#j - takes the net present value of the cash flow minus the net present value of the investment plus the net present value of the salvage for a machine purchased at the start of period # and sold at the end of periods #, #+1, ..., 10, for # = 2 through 10.

**IFPS FUNCTIONS and KEY WORDS Used**

<table>
<thead>
<tr>
<th>FUNCTION</th>
<th>LINE</th>
<th>FIRST USED</th>
<th>MEANING</th>
</tr>
</thead>
<tbody>
<tr>
<td>PREVIOUS</td>
<td>110</td>
<td></td>
<td>Take the value from the previous column.</td>
</tr>
<tr>
<td>FOR n</td>
<td>200-210</td>
<td></td>
<td>Repeat this expression for n columns.</td>
</tr>
<tr>
<td>NPVC</td>
<td>240</td>
<td></td>
<td>Net Present Value at the specified discount rate.</td>
</tr>
<tr>
<td>XPOWERY(a,b)</td>
<td>250</td>
<td></td>
<td>$a^b$, a raised to the power of b.</td>
</tr>
<tr>
<td>V MATRIX(a,b)</td>
<td>310</td>
<td></td>
<td>Use the value of variable a in the column addressed by the value of the variable b in this column.</td>
</tr>
</tbody>
</table>
References


[22] Fourer, R., "Modeling Languages vs. Matrix Generators for Linear Programs" Department of Industrial Engineering and Management Sciences, Northwest University, Evanston, IL (April 1981).


Time Horizon, \( T = 10 \)
Escalation Rate, \( r = .066 \)
Discount Rate, \( d = .12 \)
Salvage Drop Factor, \( \alpha = .85 \)

<table>
<thead>
<tr>
<th>Period</th>
<th>Variable</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Price, ( p_t )</td>
<td>$1000</td>
<td>1050</td>
<td>1103</td>
<td>1158</td>
<td>1216</td>
<td>1276</td>
<td>1340</td>
<td>1407</td>
<td>1477</td>
<td>1551</td>
</tr>
<tr>
<td></td>
<td>Cash Flow, ( CF_{1j} )</td>
<td>300.0</td>
<td>345.0</td>
<td>396.8</td>
<td>456.3</td>
<td>479.1</td>
<td>431.2</td>
<td>388.1</td>
<td>232.8</td>
<td>139.7</td>
<td>83.82</td>
</tr>
<tr>
<td></td>
<td>Net Present Value of Benefit, ( V_{1j} )</td>
<td>26.79</td>
<td>118.9</td>
<td>262.4</td>
<td>447.0</td>
<td>638.9</td>
<td>796.6</td>
<td>926.1</td>
<td>985.2</td>
<td>1009</td>
<td>1016</td>
</tr>
</tbody>
</table>

Table 1: Matrix Representation of the Kernel Network for the Example Equipment Replacement Problem. The notation corresponds to that of Section 2.
<table>
<thead>
<tr>
<th>From</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>26.79</td>
<td>118.9</td>
<td>262.4</td>
<td>447.0</td>
<td>638.9</td>
<td>796.6</td>
<td>926.1</td>
<td>985.2</td>
<td>1009</td>
<td>1016</td>
</tr>
<tr>
<td>2</td>
<td>28.94</td>
<td>119.2</td>
<td>257.8</td>
<td>435.0</td>
<td>618.7</td>
<td>769.8</td>
<td>839.6</td>
<td>950.4</td>
<td>973.4</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>30.77</td>
<td>119.1</td>
<td>252.9</td>
<td>432.0</td>
<td>598.9</td>
<td>743.5</td>
<td>862.0</td>
<td>916.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>32.31</td>
<td>118.7</td>
<td>247.8</td>
<td>411.0</td>
<td>579.5</td>
<td>717.8</td>
<td>831.2</td>
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Table 2: The Arc Values, $V_{ij}$, for the Generated Network Equipment Replacement Example Problem. These values correspond to variables PRESENT VALUE 1 Through PRESENT VALUE 10 of the IFPS Model. Columns 2-11 above correspond to columns 1-10 of the IFPS model shown in Figures 4 and 5.
DESCRIPTION OF THE FIGURES

Figure 1: Shortest Path Equipment Replacement Model over a five year planning horizon.

Figure 2: Kernel Network Representation of an Equipment Replacement Problem. The value of arc \((1,j)\) is \(V_{1j}\).

Figure 3: The Complete Network Model of an Equipment Replacement Problem. A longest path from node 1 to node \(T + 1\) defines an optimal equipment replacement policy. The value of arc \((i,j)\) is \(V_{ij}\).

Figure 4: The IFPS Kernel Model.

Figure 5: The IFPS Model Lines for Generating the Network.
Figure 1: Shortest Path Equipment Replacement Model over a Five Year Planning Horizon.
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Figure 3: The Complete Network Model of an Equipment Replacement Problem. A longest path from node 1 to node T+1 defines an optimal equipment replacement policy. The value of arc (i,j) is $V_{ij}$. 

The Complete Network Model of an Equipment Replacement Problem. A longest path from node 1 to node T+1 defines an optimal equipment replacement policy. The value of arc (i,j) is $V_{ij}$. 

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**Figure 3:** The Complete Network Model of an Equipment Replacement Problem. A longest path from node 1 to node T+1 defines an optimal equipment replacement policy. The value of arc (i,j) is $V_{ij}$. 

---
10 COLUMNS 1-11
20 * MODEL ARCEX - NETWORK GENERATING EQUIPMENT REPLACEMENT MODEL.
30 * THE NUMBER OF COLUMNS IS SET TO 11 TO ALLOW FOR VARIABLE REFERENCES
40 * FOR THE GENERATED ARC VALUES.
50 * THIS IS A 10 YEAR PROBLEM.
60 * COLUMN 1 REPRESENTS THE BENEFIT FOR THE ARC FROM NODE 1 TO 2,
70 * PRESENT VALUE 1 = V(1,2) IN COLUMN 1, V(1,3) IN COLUMN 2, ETC.
80 *
90 * -------------- GLOBAL ECONOMIC DATA ------------------------------
100 *
110 TIME = PREVIOUS + 1
120 TIME 1 = TIME + 1
130 INFLATION RATE = .066
140 INFLATION = 1 + INFLATION RATE
150 DISCOUNT = .12
160 SALVAGE DROP FACTOR = .85
170 PRICE = 1000, PREVIOUS * 1.05
180 * ------------ KERNEL NETWORK PROBLEM -----------------------------
190 INVESTMENT 1 = PRICE, 0
200 CASH FLOW 1 = 300, PREVIOUS * 1.15 FOR 3, PREVIOUS * 1.05,
210 PREVIOUS * .9 FOR 2, PREVIOUS * .6
220 SALVAGE SCHEDULE 1 = INVESTMENT 1 * SALVAGE DROP FACTOR,
230 * PREVIOUS * SALVAGE DROP FACTOR
240 PRESENT VALUE 1 = NPVC(CASH FLOW 1, DISCOUNT, INVESTMENT 1) +
250 SALVAGE SCHEDULE 1 / XPOWERY(1+DISCOUNT, TIME)

Figure 4: The IFPS Kernel Model.
260 * ------------ GENERATE ARC VALUES FROM NODES 2-10 ---------------
270 GENERATE L280 THRU L360 AT L400 BY 10 FOR 2 THRU 10
280 INDEX # = # - 1
290 TIME # = 0 FOR INDEX #, PREVIOUS + 1
300 INVESTMENT # = 0 FOR INDEX #, PRICE, 0
310 CASH FLOW # = 0 FOR INDEX #, VMATRIX(CASH FLOW 1, TIME #) * ~
320 XPOWERY(INFLATION, INDEX #)
330 SALVAGE SCHEDULE # = 0 FOR INDEX #, INVESTMENT # * SALVAGE DROP FACTOR, ~
340 PREVIOUS * SALVAGE DROP FACTOR
350 PRESENT VALUE # = NPVC(CASH FLOW #, DISCOUNT, INVESTMENT #) + ~
360 SALVAGE SCHEDULE # / XPOWERY(1 + DISCOUNT, TIME)

Figure 5: The IFPS Model Lines for Generating the Network.
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