1-1-1985

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Working Paper 85-904*

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This research has been supported in part by NSF Grant #8007103A1

Abstract

Although the application of decision analysis in practice has become increasingly popular, a major limitation restricting its use is the difficulty in measuring a decision maker's (DM) single or multi-attribute utility (MAU) function. The assessment process can be complex, tedious, and, in the multi-attribute case, generally involves: (1) identifying relevant independence assumptions, (2) assessing conditional utility functions, (3) assessing scaling constants, and (4) checking for consistency. Some of the complexities encountered include the DM's inability to quantitatively respond in a meaningful and consistent manner to hypothetical gambles, and the analyst's difficulty in selecting an appropriate functional form best describing the assessed conditional utility functions. A simplified procedure that mitigates these difficulties by obtaining conditional utility functions and scaling constants via methematical programming models is proposed. Using a general function for the conditional utility functions, qualitative and quantitative responses to hypothetical gambles, and a nonlinear programming formulation, parameters of the function are determined which best fit and describe a DM's expressed risk attitudes and preferences for a given attribute. Scaling constants are calculated via linear programming by minimizing inconsistencies in expressed preferences to pairwise consequence vectors, assuming a general multilinear multi-attribute utility functional form. The procedure circumvents performing certain independence tests, simplifies the query process, and eliminates the problem of inconsistent responses by accepting them as input into the model.

INTRODUCTION

The measurement of single and multi-attribute utility functions in practice has been a challenge for the decision analyst. The actual measures are subject to modeling errors in functional form, parameter estimation errors, and also to measurement errors due to faulty communication between the analyst and the decision maker (DM). The process of measurement per se, can be tedious and time consuming [9]. Recently considerable attention has been focused on simplifying the measurement of both single and multi-attribute utility (MAU) and value (MAV) functions [4], [8], [9], [10]. Keeney & Raiffa (ch. 4, [9]), for example, discuss the respective importance of determining certain qualitative and quantitative risk characteristics and restrictions of the DM's utility function prior to selecting a specific functional form. Qualitative characteristics include monotonicity, boundedness, continuity, and risk properties such as risk aversion. Quantitative restrictions are determined by comparing responses to various gambles over the attribute in question. Based upon such responses from the DM, a specific utility function is chosen by the analyst from a collection of functions having such characteristics that represent the DM's expressed risk attitudes. When several functions characterize the DM's expressed risk attitudes, the choice of a functional form is then often determined by the function having the best fit or the one which is most mathematically expedient.

A number of researchers have focused on first measuring a value (ordinal utility) function in multi-attribute problems for subsequent conversion into a cardinal utility function, to simplify the assessment process by minimizing the number of responses to hypothetical gambles

[8], [9], [10], [12]. Kirkwood and Sarin [10], for example, introduce a methodology that yields a precise functional form for the measured value function and minimizes the required interaction between the DM and the analyst when certain perference properties are exhibited by the DM. Keelin [8] develops a general process of value function measurement with implications for utility function properties which are analogous to the risk characteristics and restrictions developed for utility functions.

Combining conditional utility functions on individual attributes into a real-valued multi-attribute utility function can also lead to measurement difficulties. Various approaches exist (e.g., [9], [14], and [16] for the determination of appropriate scaling constants to achieve this aggregation. An experimental examination of the more commonly used methods for scaling additive utility functions showed that the methodology used in the measurement process affects the vlaues of the scaling constants [15].

Irrespective of the methodology selected in assessing a multi-attributed utility function, there exists the problem of satisfying certain conditions specific to a given MAU function. The querying procedures that reveal the DM's preference or risk properties have been well documented (e.g., [6]) but a general process for determining a specific functional form for the conditional utility functions for each attribute have not been devised. Furthermore, a simpler, more meaningful procedure to obtain the scaling constants may be useful. The work described in this paper is directed toward these goals by substituting some linear and non-linear programs for several of the current procedural steps in assessing utility functions. The proposed methodology is aimed at reducing the time and effort required by the analyst and DM, while providing a more common framework for utility construction. Using a general summed exponential functional form, or any form preferred by the analyst, the construction of single attribute utility functions is performed with the aid of a nonlinear programming (NLP) formulation. The NLP estimates the parameters of the general utility form by fitting observed response values to gambles subject to the DM's expressed qualitative and quantitative risk characteristics. Once all single attribute utility functions have been determined, a linear program is used to compute the scaling constants for a general multilinear utility function based on expressed preferences of pairwise consequence vectors.

DETERMINING SINGLE-ATTRIBUTE UTILITY FUNCTIONS

2.1 The Summed Exponential Utility Function

When a decision analyst is attempting to map a single dimensional utility function of a DM, he should have a set of readily available acceptable forms for the utility function. In general, a collection of various functions exist that adequately represent all reasonable qualitative restrictions on the risk characteristics of the utility function. For example, the logarithmic $\ln(x^b)$, $b>0$, and other functions, such as $(x+b)^{-c}$ with $c>0$, $-e^{-ax}+bx$ with $a, b>0$ are some common decreasing risk-averse utility functions on the random variable (attribute) x, for the restrictions on the parameters specified [6, p.173]. Loosely speaking, risk is a measure of the DM's reaction to uncertainty. Risk measures, as risk premium, are used to express a decision maker's risk characteristics. For example, a risk prone DM prefers choosing a gamble rather than settling for a guaranteed outcome equal to the expected value of the gamble in question. A risk averse DM prefers taking the sure outcome of a gamble's value over the gamble.

If a common set of plausible shapes representing most DM's utility functions may be ascertained or assumed prior to the actual measurement of the DM's utility function, it may be possible to define a single general functional form for the single attribute utility function whose parameters can be adjusted to fit the particular qualitative and quantitative risk characteristics of the DM. This suggested form is a requirement for the use of the NLP defined subsequently, and will also be an aid in testing the consistency of the DM's responses.

For convenience, our subsequent discussion will be arbitrarily limited to monotonically decreasing utility functions, although little change in the methods discussed would be required to accomodate increasing utility functions. The most commonly observed classes of single attribute utility functions are:

1) Fully increasing risk averse.
2) Fully decreasing risk averse.
3) Constantly risk averse.
4) Fully increasing risk prone.
5) Fully decreasing risk prone.
6) Constantly risk prone.
7) Risk prone for small attribute values and risk averse for large attribute values.
8) Risk averse for small attribute values and risk prone for large attribute values.

Through appropriate parameterization, a summed exponential function can represent all of the above characteristic types of utility functions. It also allows easy mathematical manipulation relative to other functional forms in terms of model development and optimization.

The summed exponential utility function for an attribute i is represented as

$$u_i(x) = a_i - b_i \exp(c_i x) - d_i \exp(e_i x).$$

The summed exponential satisfies the above eight utility classes for various values of the parameters. Using the risk aversion function $r(x)$,

$$r(x) = \frac{-u''(x)}{u'(x)} \qquad (2)$$

It can be verified that for a = 5, b = 1, c = 1.8, d = 9.7, and e = 4.3, the risk function is negative at x = 0.01 and positive at x = .99. Hence, these parameters yield the 7th utility class listed earlier. Parameters are obtainable for the other seven classes.

Several recent studies have investigated various functional representations of single and mult-attribute utility functions. Moskowitz, Ravindran, Klein, and Eswaren [13] in a quality control environment addressed the problem of using differing single- and bi-attribute functions. They conclude that functional form had little impact on the final quality acceptance plan selected, and less impact than does the selection of scaling constants. A more general study conducted by Keefer and Pollock [7], outlines a procedure to aid in the construction of a MAU function. Included are the testing of preferential, utility, and probabilistic independence, parameter estimation for a common one dimensional utility function, selection of the MAU form, and determination of the scaling constants. Sensitivity analysis is performed on a bicriterion model. Optimal solutions were found to be sensitive to scaling constant selection, parameter estimation of the single attribute utility functions, and the form selected for the multiple attribute utility function. Demonstrations of solution differences through extreme point changes are provided, with the conclusion being, in contrast to [13], that selecting the proper form of utility function is as important as obtaining accurate probability and utility data. Therefore a general functional form that satisfies the proper qualitative characteristics of the DM's risk attitudes is as important as the fitting of the observed data to the selected function. The ability of the summed exponential to represent a wide spectrum of risk attitudes thus makes it a desirable function for mapping utility functions.

2.2 Construction of the Single Attribute Utility Functions

2.2.1 Risk Considerations. The functional form of a one dimensional utility function should be a construct consistent with the risk characteristics expressed and/or exhibited by the DM. Risk aversion $r(x)$ and the rate of change of risk aversion are often the measures used as the characteristic constraints when selecting the utility form that will be fit to the observed certainty equivalents to gambles expressed by the DM. Von Neumann and Morganstern [17] provide the basis for the questions to ask, and Keeney and Raiffa [9] discuss the appropriate parametric families of utility functions derived from the constraints on the risk characteristics. The measures used to determine the risk properties are the risk premiums at various levels of a given attribute. The risk premium is the difference between the certainty equivalent of a given lottery and the expected (actuarial) value of the same lottery.

In describing our methodology, the lotteries we shall consider will be two outcome gambles, with a 50% chance of each occurring. A lottery will be denoted $A_i < x, y >$, meaning that attribute i has a 50% chance of realizing a value of x, and a 50% chance of realizing a value of y. The certainty equivalent of a lottery is a sure amount of an attribute such that the DM is indifferent between choosing the sure amount and the lottery. The risk premium, which is the certainty equivalent less the expected value of the gamble is denoted as P(x,h) for the lottery A_i < x - h, x + h >. Two results in [9] are as follows:

Theorem 1. For decreasing utility functions, the following are equivalent.

A) A DM is risk averse (prone, neutral).

B) The risk premium for all nondegenerate lotteries is positive (negative, zero).

C) The risk aversion function r(x) is positive (negative, zero).

D) The utility function U(x) is concave (convex, linear).

Theorem 2. The risk aversion function r(x) for a utility function U(x) is increasing (constant, decreasing) IFF the risk premium P(x,h) is an increasing (constant, decreasing) function of x for all h.

Given these two results, quantitative restrictions on the DM's utility function are determined by obtaining his/her certainty equivalent for a series of lotteries < x - h, x + h > while varying x (the level of the attribute) and maintaining a constant h. If the risk premiums at the various levels are observed to increase, then the observations can be extrapolated to include all possible < x - h, x + h > lotteries. With the extrapolation, Theorem 2 may be used to decide on which one of the eight types of utility functions is appropriate.

The specification and curve fitting procedure involves three basic steps: Step 1) Elicit qualitative information on risk properties, including such characteristics as monotonicity, boundedness, and continuity.

Step 2) Select an admissible utility function satisfying these properties.

Step 3) Use the quantitative (certainty equivalent) responses to gambles as observations in fitting the selected utility function.

Step 1) involves considerable questioning of the DM in order to assure proper application of Theorem 2. Step 2 requires the availability of appropriate utility functional forms that are consistent with the known or determinable risk properties of the DM. Step 3 requires solving a system of linear or nonlinear equations to obtain the parameters of the utility function that fit the observations.

2.2.2 A Nonlinear Programming Model. We propose to reduce the process of determining a proper utility function to two steps, by combining function selection (Step 2) with the curve fitting process. Moreover, our procedure essentially eliminates the analyst's task of selecting an admissible and/or 'best' admissible functional form by using a general utility function having a rich variety of risk properties. This may be accomplished with the use of a general summed exponential utility function and a nonlinear programming model formulation. The process does not reduce the curve fitting effort, but aids in reducing the number of responses required of the DM. Consider the following NLP to be solved for each attribute:

MINIMIZE:

$$\sum_{j=1}^{r} [U(x_j) - Q(x_j)]^2 \quad (3.A)$$

SUBJECT TO:

$$r(x_m) \leq \text{ or } \geq r(x_{m+1}) \quad \text{for } m = 1, \ldots, q-1 \quad (3.B)$$

$$u''(x_m) < \text{ or } > \text{ZERO for } m = 1, \ldots, q \quad (3.C)$$

$$\text{MAX } u'(x) \leq \text{ZERO}, \quad (3.D)$$

WHERE:

x_j = The observations (certainty equivalents),

n = The number of observations (x_j) determined by the von Neumann-Morgenster^N Method,

x_m = The certainty equivalent responses used to determine risk properties,

q = The number of certainty equivalent responses used to determine the qualitative characteristics of the utility function,

$r(x_m)$ = The risk function in (2) evaluated at the mth value of the attribute (i) under consideration,

$u''(x_m)$ = The second derivative of the utility u evaluated at x_m, and

$Q(x_j)$ = The observed utility value at x_j.

It should be noted that the parameters of the utility form selected are the decision variables for this NLP. For the summed exponential, a, b, c, d, and e would be the decision variables. Also, the objective function minimizes the sum of the squared differences between the predicted and observed ability forms. Hence, only the 'n' certainty equivalent (CE) responses for which the utility of the attribute at the CE value is known, is used in the objective. The 'q' CE responses are for small lotteries at various attribute levels and the utilities at these levels are not determined.

Equation 3A represents a least squares curve fitting criterion. This may be changed as desired. Equation 3B represents the decreasing (or increasing) nature of the risk function. This set of equations is generated by the type of questions required in Step 1 of the standard approach. Fewer responses may be required, however, becuase any number of responses may be incorporated into the constraint set. Inconsistent responses to risk premium questions will indicate a form other than the classes provided by the functional form selected. Thus, inconsistent responses will not yield a feasible region for fitting responses and provide automatic consistency checks. However, irregularities or disturbances in the risk function are permitted during the curve fitting step in the attribute values between responses. This feature may or may not be desirable depending upon the certainty of the analyst about the prior estimates of possible utility forms of the DM. Technically, only three responses are neseccary to define any of the eight utility function classifications that were listed earlier. If the analyst wishes to validate his prior assumptions regarding the utility function or test the consistency of the DM, more responses are required. As long as a feasible region can be found, the analyst's assumptions are valid and the DM is consistent, but only within the responses provided.

Equation 3C defines whether the DM is risk prone or risk averse at certain levels of the attribute. 3D enforces the monotonically decreasing nature of the utility function. The constraint forms an optimization problem itself, and as such requires special attention. Bracken [1] had discussed the nature of model formulations having optimization problems embedded in the constraint set. When using the summed exponential utility function, the following shortuct procedure can be taken. The maximum of the function in 3C must occur at either an extreme point or an interior point where the second derivative is equal to zero. Since we are dealing only with a single-attribute utility function, one can enumerate all possible maxima, and include a constraint for each. With the summed exponential, this amounts to u (x) = 0 for x at the lowest attribute value, x at the highest attribute value, and for two interior points where

$$u''(x) = b\,c^2 \exp(c\,x) + d\,e^2 \exp(e\,x) = 0. \quad (4)$$

The solutions for (4) are

$$x = \frac{\ln(-bc^2/de^2)}{(e-c)} \quad \text{and } x = \frac{\ln(-de^2/bc^2)}{(c-e)}.$$

Further constraints in the model may be added to represent upper and lower bounds on the utility, (e.g., $u_i(x_{i\ max}) = 1$). The proposed method and formulation is simple from the viewpoint of the analyst and the DM, given satisfactory computing support. Even though the formulation is not complex, the NLP may be difficult to solve for a global optimum. Care must therefore be given to the optimization algorithm selected. Our testing was performed using the COMPLEX search method of Box [2], because of its ability to overcome some local minimum difficulties. This search procedure does not guarantee a global optimum, but a near optimal. A probable optimal fit may be achieved by varying the starting points and comparing the final solutions obtained. Other difficulties involve the starting feasible region. A phase one (preliminary) approach was used to find a starting feasible point, but this was not guaranteed. Another difficulty is the possible nonconvexity of the feasible region. This difficulty was mitigated by using different starting points for each optimization problem. This allows an algorithm to search small segments of the entire feasible region, increasing the chance of finding global optimality, rather than local optimality.

2.2.3 Illustration of the NLP Approach. In order to demonstrate the differences between existing approaches and our propsed nonlinear programming methodology for determining a DM's utility function, let us consider the multi-attribute problem of selecting a new car. For simplicity, we shall limit the attributes of price, operating costs, size, and styling. For price, let us derive the single attribute utility function by the NLP approach.

The first step will require that several questions regarding the DM's qualitative risk characteristics be asked. A minimum of three is sufficient to employ the procedure. Keeney and Raiffa [9] recommend a minimum of ten lottery responses for the standard procedure. We will assume the TABLE 1 responses for our example. Our lotteries use hypothetical values for the attributes. When reassuring a specific utility function, choices of actual and hypothetical values should all be on the same curve that approximates the actual utility. In some applications when actual lotteries are used to adjust for "wealth effects," this may be easy when the attribute is money income, but it may be difficult for other applications. This is a problem with some other methods of fitting multi-attribute utility functions.

Insert Table 1 about here

Lotteries 1, 2, and 3 are directed at finding the risk properties, and lotteries 4, 5, and 6 are used to generate three utility value observations using the vonNeumann-Morgenstern method. Also included are the extreme values of the attributes, which are assigned utilities of 0 and 1 as utility observations. These observed utility values give us the following least squares objective function, using price in thousands of dollars:

$$\text{MIN } \{u\ (4) - 1\}^2 + u\{(7.8) - .75\}^2 + u\{(10) - .5\}^2$$
$$+ \{u\ (12.8) - .25\}^2 + u\{(14) - 0\}^2$$

The selected certainty equivalent responses for lotteries 1, 2, and 3 in TABLE 1 yield the following constraints by using expected values from the lotteries (since risk premium is measured as a deviation from expected value).

TABLE 1

CERTAINTY EQUIVALENT RESPONSES TO OBTAIN
QUALITATIVE RISK CHARACTERISTICS
AND UTILITY OBSERVATIONS

Lottery #	Lottery	Certain Equivalent	Risk Premium
1	5000, 6000	5675	175
2	8000, 9000	8600	100
3	11,000, 12,000	11,530	30
4	4000, 14,000	10,000	-
5	4000, 10,000	7,800	-
6	10,000, 14,000	12,800	-

Set 3A represents decreasing risk aversion; thus $r(5.5) < r(8.5)$ and $r(8.5) < r(11.5)$. For the summed exponential, the first constraint in set 3A, for example would thus be:

$$\frac{-b\,c^2 \exp(5.5c) - d\,e^2 \exp(5.5e)}{-b\,c \exp(5.5c) - d\,e \exp(5.5e)} < \frac{-b\,c^2 \exp(8.5c) - d\,e^2 \exp(8.5e)}{-b\,c \exp(8.5c) - d\,e \exp(8.5e)}$$

Set 3B, which is used to represent risk aversion, the constraints are:

$u''(5.5) \leq 0$,

$u''(8.5) \leq 0$,

$u''(11.5) \leq 0$.

For the summed exponential, the first constraint in set 3B would be:

$$-b\,c^2 \exp(5.5c) - d\,e^2 \exp(5.5e) < 0.$$

Set 3C would be identical regardless of the DM's responses, since it enforces the decreasing nature of the utility function. Using the summed exponential function, the first constraint of set 3C (see also eqn. 4) would be:

$$-b\,c \exp(4\,c) - d\,e \exp(4e) \leq 0.$$

The locally optimal solution to this NLP is

$$u(x) = 2.51 - 1.002 \exp(.0089x) - .328 \exp(.105x).$$

2.2.4 Applications. The methodology outlined above employing a general summed exponential utility function and NLP formulation was applied in both a laboratory and field setting. The experimental setting involved a bicriterion quality control model developed in [11], where the utility functions of 73 subjects were measured on each of the two attributes of the model, yielding 146 summed exponential utility functions. With the attributes scaled between zero and one, and fitting five observations in a least squares framework, the average least squares error was .0075, with a range from .0000 to .19735. In this case consistency checks were performed prior to the final curve fitting, and the subjects were requested to revise their responses accordingly.

The second application was in a field study [3] of the utility functions (on profit) of fourteen auditors in several "big 8" accounting firms. Again, the proposed method using the summed exponential yielded excellent fits consistent with the risk characteristics of the auditors. Several auditors displayed Friedman-Savage (F-S) type utility functions [5]. In these cases, although the fits were good using a single set of parameters, a two piece approximation using different parameters for the convex and concave regions yielded significantly better results, and is therefore recommended for individuals exhibiting F-S type utility functions. This two piece approximation can still use the NLP presented by approximating the attribute level where the utility shifts risk properties by using several lotteries, then fitting the proper shape on the attribute from its minimum level to shift level, then also from its shift level to maximum level.

A simulation study was also performed to compare the sum of squares fit of the summed exponential solutions to solutions obtained with some other functional forms (i.e., cubic and quadratic) over a variety of conditions. No major differences in fit were noted, although the cubic exhibited a slightly better fit, on the average, under the conditions tested.

OBTAINING THE SCALING CONSTANTS OF MULTI-ATTRIBUTE UTILITY FUNCTIONS

Constructing a multi-attribute utility function from a set of single-attribute conditional utility functions involves choosing a proper multi-attribute functional form which should be based on certain independence conditions. Often a common heuristic employed is to determine attribute weights that are then combined additively with the single-attribute functions into a multi-attribut utility function.

There are a variety of methods used to obtain the scaling constants of MAU function. Many of these are described and compared in [15] for additive MAU functions. For example, Keeney and Raiffa [9] propose obtaining a set of k independent equations with k unknown scaling constants, which are generated from responses to tradeoffs (certainty considerations) or gambles (uncertainty considerations). This procedure has several potential drawbacks. First, the DM's responses are to tradeoffs or gambles at their best or worst levels. Responding to such extreme conditions is cognitively complex, and ignores any information provided by the DM's previously measured single-attribute utility functions. Our procedure, utilizing and extending the LINMAP concepts of Srinivasan and Shocker [16], simplifies the process of obtaining the scaling constants by requiring only pairwise preferences to consequence vectors in the relevant ranges of interest, and reducing the need for independence testing to determine an appropriate MAU function.

3.1 Forms of the Multiple Attribute Utility Function

A reasonable general multi-attribute utility function is the multilinear form, i.e.,

$$U(X) = \sum_{i=1}^{m} k_i u_i(x_i) + \sum_{i=1}^{m} \sum_{j\ i}^{m} k_{ij} u_i(x_i) u_j(x_j)$$

$$+ \sum_{i=1}^{m} \sum_{j>i}^{m} \sum_{\ell>j}^{m} k_{ij}\ u_i(x_i) u_j(x_j) u_\ell(x_\ell)$$

$$+ \ \ldots + k_{1,2,3,\ldots m}\ u_1(x_1) u_2(x_2) \ldots u_m(x_m)$$

where m is the number of attributes,

X is the attribute vector, and

x_i is the value of the ith attribute.

given the set of attributes $X \equiv \{x_1,\ldots x_m\}$ with $m \geq 2$, the independence conditions for the multilinear utility function are that each attribute x_j be utility independent of its complement $\bar{x}_j$.

That is, $\bar{x}_j$ = X with x_j EXCLUDED).

Utility independence implies that the values of all remaining attributes at any given value is independent of the level of the values of all remaining attributes. Operationally, utility independence (UI) of an attribute from all others indicates that the DM can define a utility function on a single attribute independent (without any knowledge) of the values of the remaining attributes.

The multilinear utility function is a generalization of both the multiplicative and additive utility functions. The independence conditions for these special cases are more stringent than for the more general multilinear form. The multiplicative form is similar to the multilinear form, but the conditions that each attribute $x_1,\ldots,x_m$ must be mutually utility independent (MUI) implies the form:

$$U(X) = \sum_{i=1}^{m} k_i u_i(x_i) + k \sum_{i=1}^{m} \sum_{j\ i} k_i k_j u_i(x_i) u_j(x_j)$$

$$+ k^2 \sum_{i=1}^{m} \sum_{j>i} \sum_{\ell>j} k_i k_j k_\ell u_i(x_i) u_j(x_j) u_\ell(x_\ell)$$

$$+ \ldots. + k^{n-1} k_1 k_2 \ldots k_n u_1(x_1) u_2(x_2) \ldots u_m(x_m).$$

If the multiplicative form is appropriate, fewer scaling constants are required. The constraints insuring consistency in the scaling constants, however, are no longer linear. Testing for the utility independence conditions for the multiplicative function involves more questions than those required by the multilinear form.

A further simplification of the multilinear form is the additive form, i.e.,

$$U(X) = \sum_{i=1}^{m} k_i u_i(x_i),$$

with all $0 \leq k_i \leq 1$.

The additive form has only one constraint on the scaling constants. The sum of the scaling constants is set equal to one so that U(X) is between zero and one, as is each $u_i(x_i)$. The additive utility function requires additive independence (AI) among all attributes for UI to hold. Additive independence holds if preferences over lotteries on attributes $x_1,...,x_m$ depend only on their marginal probability distributions and not on their joint probability distribution. This property permits the elimination of the interaction terms needed in the other utility forms to accurately represent the utility when taking expectations of the utility function. Scaling constants for an additive utility function are simpler to determine, and also provide much latitude in consistency testing. Unfortunately, additive independence involves more testing than utility independence.

3.2 Proposed Linear Programming Formulation. The proposed method begins with the use of the general multilinear form and stated pairwise preferences by the DM similar to those proposed in [16]. From this, scaling constant determination can be formulated into a simple linear program. The framwwork is to present the DM with a series of pairwise alternate decision vectors and obtain preference responses. Each preference indicated by the DM provides a strong statement about the U(X). That is, $U(X_1)$ is greater than $U(X_2)$ if the consequence vector X_1 is preferred to consequence vector X_2. A series of these statements provides a set of linear inequalities. Let each inequality be expressed as $U(X_1) - U(X_2) + E^- - E^+ = 0$. The E^+ and E^- terms represent differences between the utility of the two consequences. When X_1 is preferred to X_2, the variable E^+ should be ≥ 0, but when the stated preference is violated by the structure of the utility function, E^- will become 0. Hence, to obtain the most consistent scaling constants for the DM, the sum of the E^- terms for all preference statements is to be minimized.

Construct a formal LP by letting X_{1j} denote the preferred consequence of the pair (X_{1j}, X_{2j}). Thus:

$$\text{MINIMIZE } Z = \sum_{i=1}^{p} E_i^- \qquad \text{(5A)}$$

Subject to:

$$U(X_{1j}) - U(X_{2j}) + E_j^- \geq 0; \quad j = 1....p \qquad \text{(5B)}$$

$$k_i \leq 1 \text{ for } i = 1,..., m$$

$$k_{ij} = d_{ij} - k_i \text{ for } i = 1,...,m; \quad j = i + 1,...,m \qquad \text{(5C)}$$

$$k_{ij\ell} = d_{ij\ell} - d_{ij} - d_{i\ell} - k_{i\ell} - k_j - k_\ell; \quad \text{for } i = 1,...,m; \; j = i+1,...m, \; \ell = j+1,...m$$

.
.
.

$k_{1234...m} = 1 - \Sigma$ of all other k variables,

All d_i, d_{ij}, d_{ij} are between 0 and 1,

$d_{ij} \geq k_i$,

$d_{ij} \geq k_j$,

.
.

$d_{1234...m} \geq$ all k variables,

where the k variables are the scaling constants for the multilinear MAU function and the d variables represent the utility of all included attributes at their maximum values, and all omitted attributes at their minimum values [6], and the E^- variables take the form of slack variables.

Constrants set 5B is used to establish the direction of the preference reponses as stated by the DM. Set 5C constrains the scaling of the MAU function between 0 and 1.

3.3 Example

Let us now return to the new car selection example. Using the new general summed exponential and the earlier proposed procedure, assume that all the single-attribute utility functions have been determined and are as follows:

u_1 (Price = P) = 2.51 - 1.002 EXP (o.0089P) - p.328EXP(0.150P)

u_2 (Operating Cost = C) = 2.06 - EXP(-.045C) - 0.087EXP(.275C)

u_3 (Room = R) = 2.16 - EXP(-0.049R) - 0.18EXP(0.33R)

u_4 (Style = S) = 3.35 - EXP(0.034S) + 1.464EXP(0.3S)

Where P is price in thousands of dollars, C is operating costs in \$ per mile, R is roominess (expressed as 6 minus the number of passengers), and S is a subjective rating in the range 1 to 10. The utility (U(X)) would be represented in multilinear form as:

$U(\text{Price, Operating Cost, Room, Style}) = U(P,C,R,S) = k_1u_1(P) + k_3u_3(R) + k_4u_4(S) + k_{12}u_1(P)u_2(C) + k_{13}u_1(P)u_3(R) + k_{14}u_1(P)u4(S) + k_{23}u_2(C)u_3(R) + k_{24}u_2(C)u_4(S) + k_{34}u_3(R)u_4(S) + k_{123}u_1(P)u_2(C)u_3(R) + k_{124}u_1(P)u_2(C)u_4(S) + k_{134}u_1(P)u_3(R)u_4(S) + k_{234}u_2(C)u_3(R)u_4(S) + k_{1234}u_1(P)u_2(C)u_3(R)u_4(S)$.

Next the DM is asked to compare the pairs of alternatives and state his preferences as shown in Table 2. It probably makes sense to generate the alternatives randomly from among undominate solutions, and insuring a wide spectrum of consequence pairs. The consequences are then used to determine the single attribute utility values leaving only the scaling constants (k values) and error term (E^- values) as decision variables in the linear program. For the example the LP is:

$$\text{MIN} = \sum_{j=1}^{8} E_j^-$$

Subject to: $U(7,8,6.5,2.5,3.25) - U(10,2.5,2.5,3.25) + E_1^- - E_1^- \geq 0$

$\cdot$
$\cdot$
$\cdot$

$U(7.8,5,2.5,6) - U(8,5,2.5,1) + E_8^- - E_8^- \geq 0$

and the constraints of set 5.0 for the multilinear form.

The first constraint, for example, would appear as:

$$.265k_1 - .2340k_2 + .0817k_{12} + .1624k_{13} + .1609k_{14} - .1410k_{23}$$

$$- .1397k_{24} + .05k_{123} + .0496k_{124} + .098k_{134} - .855k_{234} + .030k_{1234}$$

$$+ E_1^- \geq 0$$

Since k variables range between -1 and +1, variable substitution is made to allow negative values in the LP and then the problem is solved. The result yields $k_1 = .365$, $k_2 = .294$, $k_3 = .047$, $k_4 = .052$, $k_{13} = .022$, $k_{24} = .219$ and all other scaling constants equal to zero.

Insert Table 2 about here

TABLE 2

PAIRWISE PREFERENCE COMPARISONS

Alternative 1	Alternative 2	Preference Response
(7.8, 6.5, 2.5, 3.25)	(10, 2.5, 2.5, 3.25)	1
(10, 2, 3.5, 10)	(10, 2, 4.0, 3.5)	1
(10, 7.5, 4.5, 2)	(12.3, 7.5, 2.5, 2.0)	2
(10, 3.5, 2.5, 6)	(10, 5, 2.5, 3.25)	2
(14, 5, 1.0, 2)	(10, 5, 4.5, 2)	1
(4, 5, 3.5, 6)	(6.5, 5, 3.5, 1)	2
(7.8, 2, 3.5, 10)	(7.8, 6.0, 3.5, 1.5)	2
(7.8, 5, 2.5, 6)	(8, 5, 2.5, 1.0)	2

Modifications to the procedure presented may include the incorporation of weights into the objective function based upon strength of preferences between alternatives. The strength of preference weights would place large costs on certain preferences and small costs on vague preferences. This would ential obtaining strength-of-preference information from the DM, which would probably reduce the number of pairwise comparisons to be made to achieve a given level of accuracy. For example, if the DM rates his preferences on a scale of 1 to 10 and gives a rating of 10 to preference #1 (Table 2) and a rating of 1 to all other preferences, then the objective function is

$$\text{MIN } Z = 10E_1^- + E_2^- + E_3^- \ldots + E_8^-.$$

The model can be changed to include terms that encourage, but do not enforce, the additive form of the utility function. This can be accomplished by bringing the scaling constants associated with the interactive terms into the objective and multiplying by a penalty cost. The objective is then $\text{MIN } Z = \sum_{i=1}^{8} E_i^- + P(k_{12} + k_{13} + k_{14} + \ldots k_{1234})$ where P is a positive constant.

There exist several advantages to the linear programming method just described. Response to extreme attribute values are avoided. The information obtained in measuring the single-attribute utility functions is incorporated directly into the construction of the MAU function. The cognitive burden on the DM is reduced to pairwise preferences and elimination of the need for testing the DM for AI or MUI. Another advantage of the model formulation is that inconsistencies are accepted with the value of the objective function (Z) providing a measure of inconsistency; i.e., $c = (Z/1+Z)$ [16].

SUMMARY

Two mathematical programs to aid in the measurement of single and multi-attributed utility functions have been presented. Each formulation may be used independently or conjointly to help reduce the cognitive burden and interaction between the analyst and the DM. The NLP formulation in conjunction with a general summed exponential function is used to describe and fit a DM's single attribute utility function. The richness of the risk properties of the summed exponential alleviates the analyst's burden of choosing an admissible utility functional form, while providing easy mathematical manipulation in formal decision models. The LP formulation, based on [16], and assuming (but not necessarily restricted to) a multilinear MAU form, permits easy cognitive and mathematical determination of the scaling constants of the MAU function as well as special cases of the multilinear form without stringent testing of the associated independence conditions.

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