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ASSET RETURNS, VOLATILITY AND THE OUTPUT SIDE

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by

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Abstract

This paper derives relationships linking the nominal rate of interest, the expected return on the stock market and the ex-ante volatility of the stock market return with the marginal return on investment. The data supports the relations linking the nominal interest rate with the real and nominal marginal returns on investment. The data also supports a negative relationship between ex-ante volatility and the real marginal return on investment and a weak positive relationship between expected stock returns and the real marginal return on investment.
A recent trend in research related to financial markets has been to relate the movements in these markets to contemporaneous or adjacent movements in macroeconomic variables. For example, studies have looked at stock returns or bond yields as well as their volatilities to uncover patterns of co-movements between them and macroeconomic variables such as output, industrial production and investment.\textsuperscript{1} Other studies have looked at the relationship between financial variables and the stages of the business cycle.\textsuperscript{2} In general, these studies have found a systematic pattern of joint movements between the above financial variables and macroeconomic variables.

The majority of these studies have been exploratory in nature in the sense that they do not rely on an a priori parameterized model of how financial asset returns should move with macroeconomic variables. Instead, any models used have generally been postulated on an ad hoc basis taking into account previously documented empirical findings. The reason for this is that there are few models currently of how asset returns move with macroeconomic quantities.\textsuperscript{3} It is generally difficult to solve these models to obtain closed form and comparative static solutions except under very specific assumptions.\textsuperscript{4}

In this paper, I derive relationships linking the nominal rate of interest, the ex-ante return on the stock market and the ex-ante volatility of the stock market to the marginal return on investment in the aggregate economy. These relationships are based upon a production-based asset pricing model exposited in Sharathchandra (1989) (hereafter S(1989)). In S(1989), I derive and test a first order condition which relates the marginal return on investment to the return on the market under the assumption of logarithmic utility.

In the current paper, I do not solve the model but instead I examine the implications of the first order condition for the joint movements of the interest rate, the market return and the market volatility with the marginal
return on investment. Under further assumptions, these implications take the form of regression relations which are then tested using the data. This research can therefore be viewed as an attempt to go beyond an investigation of relationships based upon an ad hoc specification of a model but at the same time it is not a test of a fully parameterized model.

This paper documents the following results. The nominal riskfree rate of return is positively related to the nominal marginal return on investment while it is negatively related to the real marginal return on investment. The latter result is similar to that observed for stock returns and appears to be an inflation effect. There is a negative relationship between the real marginal return on investment and the ex-ante variance of the stock market. However, there appears to be, at best, a weak positive relationship between the ex-ante real return on the market and the real marginal return on investment.

This paper is organized as follows. In section I, I briefly outline the model in S(1989) and derive the empirical relations from it. In section II, I test the various relations derived in section I. These include the relations linking the marginal return on investment to the riskfree rate, the expected return on the market and the expected variance of the market. Several other relations are also explored. Section III concludes the paper by summarizing the results and discussing further possible research.

I. Derivation of Relationships

A. Details of the model

The model used in this paper is the same as in Sharathchandra (1989) and I will just present the main details. There is an infinitely-lived representative investor who owns a single firm. The objective of the investor is to maximize his lifetime expected utility of consumption of a single good while
the objective of the firm is to maximize its current market value.

Specifically, the investor's problem is

$$\begin{align*}
\text{Max} & \sum_{t=0}^{\infty} \beta_t^i E_t [U(C_{t+1})] \\
\text{subject to} & C_j + V_j \cdot x_j = (V_j + \Pi_j) \cdot x_{j-1}, j=t, t+1, \ldots\ldots\ldots (1a)
\end{align*}$$

where $\beta$ is the subjective discount rate, $C_t$ is consumption at time $t$, $V_t$ is the value of the asset at time $t$ that the investor is holding, $\Pi_t$ is the dividend paid by the asset at time $t$ while $x_t$ and $x_{t-1}$ are the number of shares of the asset that the investor holds at the end of time periods $t$ and $t-1$ respectively. The numeraire is the consumption good at time $t$.

The firm's objective is

$$\begin{align*}
\text{Max} & \sum_{i=1}^{\infty} \beta_t^i E_t \frac{U'(C_{t+1})}{U'(C_t)} \\
\text{subject to} & \Pi_j = f_j - i_j \\
f_j = \lambda_j k_j \\
k_{j+1} = (1-\delta) k_j + i_j, j = t, t+1
\end{align*}$$

where $\Pi_t$ are the dividends paid out by the firm at time $t$, $f_t$ is the output at time $t$, $i_t$ is the investment at time $t$, $k_t$ is the capital stock at time $t$, $\lambda_t$ is a random variable denoting the state of productivity at time $t$ and $\delta$ the depreciation rate of the capital stock.

The firm's first order condition is

$$\beta_t E_t \left[ \frac{U'(C_{t+1})}{U'(C_t)} \right] \cdot (-k^{t+1} + (1-\delta)) = 1$$

where $f_{t+1}$ is the marginal product of capital at time $t+1$. The entire term...
\( k_{t+1} \) is simply the total return on the marginal unit of investment.

By making use of the fact that \( k_{t+1} = \lambda_{t+1} \cdot \alpha \cdot k_{t+1} \cdot (1-\delta) \) and substituting for \( \lambda_{t+1} \), we can write (3) as

\[
\beta E_t \left[ \frac{U'(C_{t+1})}{U'(C_t)} \cdot (\alpha \cdot \frac{\tilde{f}_{t+1}}{k_{t+1}} + (1-\delta)) \right] = 1
\]

where \( \frac{\tilde{f}_{t+1}}{k_{t+1}} \) is the output to capital ratio at time \( t+1 \). If one further assumes that the representative investor's preferences can be described by logarithmic utility, i.e., \( U(C_t) = \log C_t \), then we can write (4) as

\[
E_t \left[ \frac{1}{1+r_{mt+1}} \cdot (\alpha \cdot \frac{\tilde{f}_{t+1}}{k_{t+1}} + (1-\delta)) \right] = 1
\]

where \( r_{mt+1} \) is the real rate of return on the market from time \( t \) to time \( t+1 \).

B. Interest Rate Relations

In order to derive relations involving the riskfree rates of return, we do not need to assume logarithmic utility and hence we use equation (4) as our starting point. Following Ferson (1989), equation (4) can be rewritten (using the law of iterated expectations) as

\[
E_t \left[ \frac{\bar{m}_{t+1}}{I(t,t+1)} \cdot (1+\bar{f}_{It+1}) \right] = 1
\]

where \( \bar{m}_{t+1} = \beta \cdot \frac{U'(C_{t+1})}{U'(C_t)} \) is the real marginal rate of substitution of consumption and \( \bar{I}(t,t+1) \) is the ratio of price deflators at time \( t+1 \) and time \( t \),

\( 1+\bar{f}_{It+1} = (\alpha \cdot \frac{\tilde{f}_{t+1}}{k_{t+1}} + (1-\delta) \cdot \bar{I}(t,t+1)) \) is the nominal total return on the marginal unit of investment and \( r_{Ft+1} \) is the nominal riskfree rate of return from time \( t \) to time \( t+1 \) which is known at time \( t \).
Expanding the LHS of equation (6) and transposing terms, we get

\[ E[1 + \tilde{r}_{It+1} | r_{Ft+1}] = [1 - \text{cov}(\tilde{m}_{t+1}, \tilde{r}_{It+1} | r_{Ft+1})] \cdot (1+r_{Ft+1}) \]  

(7)

Denoting \( \text{cov}(\tilde{m}_{t+1}, \tilde{r}_{It+1} | r_{Ft+1}) \) as \( C_{mt} \), we can write the regression equation corresponding to (7) as

\[ 1 + \tilde{r}_{It+1} = (1-C_{mt}) + (1-C_{mt}) \cdot r_{Ft+1} + \tilde{\epsilon}_{t+1} \]  

(8)

where \( E[\tilde{\epsilon}_{t+1} | r_{Ft+1}] = 0 \). Equation (8) relates the nominal return on the marginal unit of investment from time to time \( t+1 \) to the nominal riskfree rate of return, \( r_{Ft+1} \), which is known at time \( t \).

A relation similar to equation (8) can be derived for the real riskfree rate of return. Again, following Ferson (1989) and using the law of iterated expectations, we can write equation (4) as

\[ E[\tilde{m}_{t+1} \cdot (1+\tilde{r}_{It+1}) | r_{Ft+1}] = 1 \]  

(9)

where \( 1 + \tilde{r}_{It+1} = \alpha \cdot \frac{\tilde{r}_{t+1}}{k_{t+1}} + (1-\delta) \) is the real total return on the marginal unit of investment. From the stochastic Euler Equation for any financial asset, we can write

\[ E [\tilde{m}_{t+1} \cdot (1+\tilde{r}_{Ft+1}) | r_{Ft+1}] = 1 \]  

(10)

where \( \tilde{r}_{Ft+1} \) is the real riskfree rate of return from time to time \( t+1 \) which is not known at time \( t \). Expanding the LHS of equation (9), we have

\[ E[1 + \tilde{r}_{It+1} | r_{Ft+1}] = [1 - \text{cov}(\tilde{m}_{t+1}, \tilde{r}_{It+1} | r_{Ft+1})] \cdot \frac{1}{E_t[\tilde{m}_{t+1}]} \]  

(11)

A similar expansion of equation (10) gives us

\[ E[1 + \tilde{r}_{Ft+1} | r_{Ft+1}] = [1 - \text{cov}(\tilde{m}_{t+1}, \tilde{r}_{Ft+1} | r_{Ft+1})] \cdot \frac{1}{E_t[\tilde{m}_{t+1}]} \]  

(12)
Combining equations (11) and (12) gives us

\[ E[1 + \tilde{r}_{t+1}^* | r_{Ft+1}] = \left[ \frac{1 - \text{cov}(\tilde{m}_t, \tilde{r}_{It+1}^* | r_{Ft+1})}{1 - \text{cov}(\tilde{m}_t, \tilde{r}_{Ft+1}^* | r_{Ft+1})} \right] \cdot E[1 + \tilde{r}_{Ft+1}^* | r_{Ft+1}] \]  

(13)

If we assume that \( \text{cov}(\tilde{m}_t, \tilde{r}_{Ft+1}^* | r_{Ft+1}) = \lambda \), a constant, then we can write (13) as

\[ 1 + \tilde{r}_{It+1}^* = (1 - C_{mt}) \cdot E[1 + \tilde{r}_{Ft+1}^* | r_{Ft+1}] + \tilde{n}_{t+1} \]  

(14)

where \( C_{mt} = \frac{\text{cov}(\tilde{m}_t, \tilde{r}_{It+1} - \tilde{r}_{Ft+1} | r_{Ft+1})}{1 - \lambda} \) and 

\[ E[\tilde{n}_{t+1} | r_{Ft+1}] = 0 \]

Since both \( C_{mt} \) and \( E[1 + \tilde{r}_{Ft+1}^* | r_{Ft+1}] \) are functions of \( r_{Ft+1} \), equation (14) implies that \( \tilde{r}_{It+1}^* \) can be predicted by (a possibly nonlinear function of) \( r_{Ft+1} \).

C. Expected Return and Volatility Relation

We now make use of the assumption of logarithmic utility which gives us equation (5). If we expand the LHS of equation (5), we have

\[ E_t\left[1 + \tilde{r}_{It}^*\right] E_t\left[\frac{1}{1 + \tilde{r}_{mt+1}^*}\right] = 1 - \text{cov}_t(\tilde{r}_{It+1}^*, \frac{1}{1 + \tilde{r}_{mt+1}^*}) \]  

(15)

Denoting \( \text{cov}_t(\tilde{r}_{It+1}^*, \frac{1}{1 + \tilde{r}_{mt+1}^*}) \) as \( C_{rt}^* \) and making use of the approximate relation\(^5\)

\[ E_t\left[\frac{1}{1 + \tilde{r}_{mt+1}^*}\right] \approx \frac{1 + \text{Var}_t[\tilde{r}_{mt+1}^*]}{1 + E_t[\tilde{r}_{mt+1}^*]} \]  

(15)

we can write (15) as

\[ E_t[1 + \tilde{r}_{It+1}^*] = (1 - C_{rt}^*) \cdot \frac{1 + E_t[\tilde{r}_{mt+1}^*]}{1 + \text{Var}_t[\tilde{r}_{mt+1}^*]} \]  

(17)
Using a first order Taylor expansion for \( \frac{1}{1 + \text{Var}_t[\hat{r}_{mt+1}]} \) and ignoring the resulting higher order product term in the numerator, we can write equation (17) as

\[
E_t[1 + \hat{r}_{It+1}] = (1 - C_{rt}) \cdot (1 + E_t[\hat{r}_{mt+1}] - \text{Var}_t[\hat{r}_{mt+1}])
\]  

(18)

The corresponding regression relation is

\[
1 + \hat{r}_{It+1} = (1 - C_{rt}) + (1 - C_{rt})E_t[\hat{r}_{mt+1}] - (1 - C_{rt}) \text{Var}_t[\hat{r}_{mt+1}] + \xi_{t+1} \]  

(19)

where \( E_t[\xi_{t+1}] = 0 \).

Equation (19) relates the real return on the marginal unit of investment to the expected real return on the market and the expected volatility of the real return on the market. Thus, the variables which, at time \( t \), predict the real return on the market and the volatility of the real return on the market at time \( t+1 \) should also predict the real return on the marginal unit of investment at time \( t+1 \). Since \( C_{rt} \) is generally negative and much smaller than 1 in absolute value, equation (19) implies that \( \hat{r}_{It+1} \) is positively related to the ex-ante real return on the market at time \( t+1 \) and negatively related to the ex-ante volatility of the market at time \( t+1 \).

II. Empirical Results

A. Description of Data

The data used in this study is quarterly and in all of the regressions, they typically run from the fourth quarter of 1949 to the fourth quarter of 1984 (49:4-84:4).

The data on the output to capital ratio series, \( \frac{f_t}{k_t} \), is the same as that used in S(1989) and the reader is referred to that paper for details of construction of the series.
The quarterly rate of return on the market is obtained from the CRSP Index files while the quarterly riskfree rate of return is from the CRSP Bond files. The total real market return and the total real riskfree rate are obtained from the corresponding nominal series by dividing by one plus the change in the seasonally unadjusted consumer price index (PZUNEW) which is obtained from CITIBASE. The implicit GNP deflator (GD), also from CITIBASE, is used to compute the nominal return on the marginal unit of investment from the corresponding real return.

The volatility of the market is computed from daily price data on the S&P 500. The volatility of the market return in quarter t is computed as

\[
\text{Var}(r_{mt}) = \frac{N_t}{2} \sum_{i=1}^{N_t} r_{mi}^2 + 2 \sum_{i=1}^{N_t-1} r_{mi} \cdot r_{mi+1}
\]

where \( r_{mi} \) is the return on the S&P on day i of quarter t and \( N_t \) is the number of trading days in quarter t. Equation (20) computes the variance assuming first order serial correlation in the S&P Index. Though this variance is computed using nominal returns, there is little error in using it as the variance of real returns given the extent to which movements in the market return dwarf movements in inflation.

Among the variables used to form estimates of the ex-ante return on the market are the term premium and the dividend yield. These are used based upon the results of earlier studies which indicate that these variables can predict stock returns. The term premium in this study is defined as the excess of the average yield of all bonds with maturities of at least 10 years (FYGL from CITIBASE) over the 3-month riskfree rate. The dividend yield variable is the dividend yield on the S&P 500 obtained from CITIBASE (FSDXP). The term premium and dividend yield are denoted as UTS and DP, respectively.
Unlike other studies, I was unable to obtain any predictive power in the default premium for forecasting stock returns. I used the difference in yields of BAA corporate bonds and AAA corporate bonds as the default premium (FYBAAC and FYAAAC respectively from CITIBASE). It appears that the predictive ability of this variable depends on the data set used to obtain the yields. I, accordingly, did not use this variable in the estimation.

B. Results from Testing of Interest Rate Relations

The results of testing equation (8) are presented in Table 1. The results are presented for the overall period 49:4-84:4 and three subperiods 54:1-84:4, 54:1-72:4 and 73:1-84:4. The choice of the 54:1-84:4 subperiod is motivated by the fact that it avoids the time period of the Treasury Accord in the early 1950’s when interest rates were pegged at a set level. I further divide the 54:1-84:4 period into two subperiods - 54:1-72:4 and 73:1-84:4. The former subperiod is generally considered to be a period of low inflation and stable real interest rates while the latter subperiod is well known for its high and volatile inflation and interest rates.

Panel A presents the results of regressing $1 + r_{It}$ (henceforth NRATIO$_t$) on $r_{Ft}$ (henceforth $R_{Ft}$). NRATIO$_t$ is calculated assuming a value for $\delta = 0.016389$ which was calculated for the data in S(1989) and a value for $\sigma = 0.30$. Figure 1 contains a time series graph of NRATIO. The regressions in Panel A have only one independent variable, $R_{Ft}$, which is equivalent to an assumption that the covariance term $C_{mt}$ is a constant across time. Due to the presence of autocorrelation in many of these regressions as witnessed by the D-W and Q statistics, the standard errors are corrected for heteroscedasticity and autocorrelation using 12 lags of the residuals.

The regressions in Panel A generally indicate that there is a positive relationship between NRATIO$_t$ and $R_{Ft}$. Recall that the coefficient of $R_{Ft}$ is
(1-C_{mt}) which is typically positive since C_{mt} is much smaller than 1. Under the assumption that C_{mt} = C_m (a constant), it follows that the coefficients of the constant term and of RF_t must be equal and invariant through time. The $\chi^2$ statistic (1) tests this hypothesis and it is rejected in all the four periods. Thus the data does not support a constant C_{mt}.

If C_{mt} = a_0 + a_1 RF_t, then equation (8) can be written as

$$NRATIO_t = a_0 + a_1 RF_t + a_2 RF_t^2 + \tilde{\epsilon}_{t+1}$$

(21)

where $a_0 = a_1 + a_2 = 0$. The $\chi^2$ statistic (3) tests this null hypothesis and it is not rejected in any of the periods. However, there appears to be a great deal of multicollinearity between RF_t and RF_t^2. A test of the null hypothesis $a_2 = 0$, which leads to the $\chi^2$ statistic (2), is unable to reject the null presumably due to multicollinearity between RF_t and RF_t^2. Such multicollinearity could very well be responsible for the $\chi^2$ (3) statistic not being significant. On the whole, the data appear to indicate a time-varying covariance term C_{mt}.

The high autocorrelations in Panel A raise the question of whether one or both of the series in the regressions may be nonstationary. Granger and Newbold (1986) point out that such nonstationarity can give rise to "spurious" regressions with high R^2's and low D-W statistics. Granger and Newbold argue that in such a situation a high R^2 means little except that the model is in some way misspecified.

In order to address this possibility, I use a "whitened" version of NRATIO_t which I call NRRES_t to run the same regressions as in Panel A. I derive the series NRRES as the residual series from fitting an AR (2) process to NRATIO (Panel B). The NRRES series has very little autocorrelation of any order. Keeping in mind that NRATIO_t is the nominal marginal return on investment at time t, the variable NRRES_t can be viewed as a "naive" unanticipated
nominal marginal return on investment at time t. A time series plot of NRRES is displayed in Figure 2.

The regressions in Panel C indicate that, except for the period 73:1-84:4, the variable NRRESₜ has a weak positive relationship with RFₜ. In fact, in the periods 49:4-84:4 and 54:1-84:4, the coefficient of RFₜ is close to being significant at the 10% level. Thus, the regressions in Panel C do not contradict the results of Panel A which indicate a positive relationship between the nominal marginal return on investment and the nominal riskfree rate of return.

Table 2 presents the results of testing equation (14). Recall that

\[ 1 + \tilde{r}_t^{*} = \alpha \cdot \frac{f_t}{k_t} + (1-\delta) \] and since \( \alpha \) and \( \delta \) are constants, we can use \( \frac{f_t}{k_t} \) in the regressions. I denote \( \frac{f_t}{k_t} \) as RATIOₜ and it is graphed in Figure 3. Under the assumption that \( C_{mt} = C^*_{m} \), a constant over time, and that \( E[1 + \tilde{r}_t^{*}|RF_t] = \alpha_0 + \alpha_1 RF_t \), we can write equation (14) as

\[ RATIOₜ = \alpha_0 + \alpha_1 RF_t + \tilde{\eta}_{t+1} \] (22)

The results of equation (22) are given in Panel A. A strong negative relationship between RATIO and RF is indicated in the periods 49:4-84:4 and 54:1-84:4. In the subperiod 54:1-72:4, the coefficient of RF is negative but insignificant while in the subperiod 73:1-84:4, the coefficient is significant. If \( C_{mt} \) is a linear function of RFₜ, then equation (22) will have a \( RF_t^2 \) term in it. I do a \( \chi^2 \) test of the null hypothesis that the coefficient of \( RF_t^2 \) is zero. The results in the last column of Panel A indicate non-rejection of the null which indicates that the \( RF_t^2 \) term does not matter. However, as in Table 1, there is strong multicollinearity between RFₜ and \( RF_t^2 \) and that is certainly a factor in the null hypothesis not being rejected.

The D-W and Q statistics indicate a very high degree of autocorrelation in the residuals and, as before, I use a "whitened" version of RATIO which I call RRES. RRES is the residual series obtained by fitting an AR (2) model to
RATIO (Panel B). This residual series is largely free of autocorrelation as can be seen by looking at Figure 4. Interestingly, the AR (2) model for RATIO has an $R^2$ of 0.96 which indicates the extent of the predictive ability of lagged values of RATIO for the current value. The coefficients of $RATIO_{t-1}$ and $RATIO_{t-2}$ also suggest that the RATIO series has a root close to unity.

Panel C presents the results of the regressions of Panel A with RRES substituted for RATIO. The coefficients of RF are uniformly negative in all the periods and very significantly so. In fact, the coefficients in the two subperiods 54:1-72:4 and 73:1-84:4 are more significant than the corresponding coefficients in Panel A.

The results of Panel A and Panel C taken together indicate a strong negative relationship between the real marginal return on investment and the nominal riskfree rate. Such a pattern has already been documented for stocks by Fama and Schwert (1977) who argue that this is an inflation effect in the sense that stock returns are negatively related to anticipated inflation. It could be interesting to see if such an inflation effect holds for the marginal return on investment as well. This is investigated in Table 3.

Panel A of Table 3 presents a predictive relationship for inflation (denoted CPI). This is a "naive" prediction equation using only the first and third lags of inflation. I use the fitted value from this regression (denoted CPIFIT) as an estimate of anticipated inflation. Both CPI and CPIFIT have been graphed over time in Figure 5. The graph indicates that the predicted value, CPIFIT, has a fairly good ability to track the movements of CPI. This is also indicated by the prediction regression's $R^2$ of 0.55.

Panels B and C explore the relationship between anticipated inflation (CPIFIT) and RATIO and RRES respectively. Since CPIFIT is itself an estimated variable from an earlier regression, one needs to take into account its sample distribution from the earlier regression when computing standard errors for
its coefficients. This is done using the methods of Murphy and Topel (1985) and Pagan (1984).

The results of Panels B and C indicate a fairly consistent negative relationship between anticipated inflation and the real marginal return on investment. Particularly in the longer time periods, 49:4–84:4 and 54:1–84:4, the relationship between RATIO/RRES and CPIFIT mirrors the corresponding relationship with RF. Thus, it appears that the negative relationship documented in Table 2 between nominal interest rates and the real marginal return on investment is at least partly due to an inflation effect. This is an interesting result as it shows certain common patterns of behavior between stock returns and the marginal return on investment.

The next table (Table 4) explores the behavior of the "risk premium" on the marginal return on investment. Starting with equation (14), we can write it as

\[ E[1+\bar{F}_{t+1} | r_{Ft+1}] = E[1+\bar{F}_{t+1} | r_{Ft+1}] - Cmt \cdot E[1+\bar{F}_{t+1} | r_{Ft+1}] \]  

(23)

Equation (23) implies the following relation

\[ E[\bar{F}_{t+1} - \bar{F}_{t+1} | r_{Ft+1}] = -Cmt \cdot E[1+\bar{F}_{t+1} | r_{Ft+1}] \]  

(24)

Denoting \( \bar{F}_{t+1} - \bar{F}_{t+1} \) as \( \bar{R}_{Ft+1} \) and making the assumptions that \( Cmt = Cm \) (a constant) and that \( E[1+\bar{F}_{t+1} | r_{Ft+1}] = \beta_0 + \beta_1 RF_{t+1} \), we can write equation (24) as

\[ \bar{R}_{Ft+1} = \beta_0 + \beta_1 RF_{t+1} + \nu_t \]  

(25)

In an economy with no inflation uncertainty, the real riskfree rate is simply the certainty equivalent of the real marginal return on investment. Hence the variable \( RP \) can be considered as a "risk premium" that the market places on
the marginal return on investment taking into account the uncertainty in the production process. A time series plot of the variable RP is shown in Figure 6.

The results of the regression corresponding to equation (25) are presented in Table 4. The coefficient of RF is negative in all the periods. The coefficient is highly significant only in the last subperiod 73:1-84:4. In the overall period, 49:4-84:4, the coefficient is close to significance at the 5% level. However, if we take out the period of the Treasury Accord, it becomes insignificant (periods 54:1-84:4 and 54:1-72:4). On the whole, there is evidence of a weak negative relationship between the risk premium variable, RP, and the nominal riskfree rate.

If, instead of assuming a constant $C^*_m$, we let $C^*_m = \beta_0 + \beta_1 RF_{t+1}$, a linear function of $RF_{t+1}$, then from equation (24) we have

$$RF_{t+1} = b_0 + b_1 RF_{t+1} + b_2 RF_{t+1} + \nu_t$$

(26)

I tested for the significance of the coefficient $b_2$ using a $\chi^2$ test, the null hypothesis being $b_2 = 0$. The null is rejected in the long period 54:1-84:4 though it is not rejected in any of the subperiods or the overall period 49:4-84:4. Again, multicollinearity between $RF_{t+1}$ and $RF_{t+1}^2$ appears to be a factor. Nevertheless, there is some evidence that the second power of $RF_{t+1}$ is related to $RP_{t+1}$ either because $C^*_m$ is a linear function of $RF_{t+1}$ or perhaps because $E[1+\nu_{t+1} | F_{t+1}]$ is a second order function of $RF_{t+1}$.

All in all, the results of Table 4 suggest that the risk premium on the marginal return on investment varies through time in a way that appears to be negatively related to the nominal rate of interest.
C. Results from Testing of Expected Return and Volatility Relations

We now proceed to the testing of the second set of relations viz. those relating the marginal return on investment to the expected return on the market and the expected variance of the market return. We will start with equation (19). The LHS variable is \( \text{RATIO} \) and, as earlier, the output to capital ratio \( \frac{f_t}{k_t} \) can be used directly as \( \text{RATIO} \). The two RHS variables are \( E_t[F_{mt+1}] \) and \( \text{Var}_t[F_{mt+1}] \) and in order to run the regression based on equation (19), we need estimates of these quantities based on the information available at time \( t \).

The time series properties of the quarterly variance series, denoted \( \text{VAR} \), (autocorrelations and partial autocorrelations which are not shown in the tables) indicate that quarterly variance can be modeled as a stationary AR (1) process. Furthermore, the nominal riskfree rate (known at the beginning of the quarter) has some predictive power for the variance in the same quarter. Accordingly, I use the one-quarter lagged variance and the nominal riskfree rate to form an estimate of the ex-ante variance each quarter.

The first regression in Panel A of Table 5 presents the regression used to forecast the variance each quarter. Both the lagged variance and riskfree rate come in highly significant and the regression has a \( R^2 \) of 0.28. The residuals also appear reasonably free of autocorrelation. The fitted value from this regression is called \( \text{VARFIT} \) and is the measure of ex-ante variance. Figure 7 is a graph of both the variables \( \text{VAR} \) and \( \text{VARFIT} \) over time. The fitted value series, \( \text{VARFIT} \), appears to do a reasonable job of tracking the movements in the original series, \( \text{VAR} \). The series \( \text{VARFIT} \), however, seems to lag the series \( \text{VAR} \) at several points in time and this is clearly due to the fact that we are using the lagged variance as a predictor.

In order to come up with an estimate of the expected return on the market, I use some of the variables that have been documented in the literature as
being able to forecast stock returns. Among these, I use the nominal riskfree rate (RF), the term premium (UTS) and the dividend yield (DP). As mentioned earlier, I was unable to extract any forecasting ability out of the default premium variable and hence I did not include it in the regression. To the above variables, I added the lagged variance of the market which I find to have significant predictive power.

The second regression in Panel A of Table 5 is the forecasting equation for the real return on the market. The forecasting variables are all significant and have signs that are consistent with the findings of earlier studies. The riskfree rate, RF, is significantly negative while the three lagged variables VAR, UTS and DP are all significantly positive.\footnote{The predictive regression has a $R^2$ of 0.20 and the residuals do not appear to display any significant autocorrelation. The fitted value from this regression is called RVWFIT and is an estimate of the expected real return on the market. Both RVW and RVWFIT are graphically displayed in Figure 8.}

Equation (19) can therefore be written as

$$RATIO_{t+1} = a_0 + a_1 \text{VARFIT}_{t+1} + a_2 \text{RVWFIT}_{t+1} + \xi_{t+1}$$  \hspace{1cm} (27)

The results of this regression are presented in Panel B. The results are presented for the overall period 49:4-84:4 and for two roughly equal subperiods, 49:4-67:4 and 68:1-84:4. Since we are dealing with a new set of variables such as the market return and its variance which presumably are not affected to the same extent as interest rates by the Fed's policy or the inflation rate, I decided not to use the earlier division of periods. Over the entire period 49:4-84:4, the variables VARFIT and RVWFIT have the predicted signs as they come in negative and positive respectively. As predicted, the constant term is also positive. The constant and VARFIT have coefficients that are significant while that of RVWFIT is insignificant. The same pattern of
coefficients is repeated in the 49:4-67:4 subperiod with the difference being that the coefficient of VARFIT is no longer significant. Finally, in the 68:1-84:4 subperiod, the coefficient of RVWFIT actually becomes negative and significant at the 10% level which is counter to the predicted sign. The constant is still significantly positive while VARFIT becomes insignificantly negative. It should be noted that the standard errors are corrected for using fitted values from earlier regressions, VARFIT and RVWFIT, as regressors.

As before, due to the high autocorrelation in the residuals, I ran the same regression using RRES in place of RATIO. These results are presented in Panel C. The results of Panel C are, if anything, even more strongly in favor of the predictions in equation (19). The constant and RVWFIT are significantly positive and VARFIT significantly negative in the overall period 49:4-84:4 and the subperiod 68:1-84:4. In the subperiod 49:4-67:4, all the coefficients have the predicted sign but none of them is significant.

One prediction of equation (19) is that the coefficients in equation (27) should be equal in absolute magnitude. In short, we must have $a_0 = -a_1 = a_2$. The $\chi^2$ statistic for the test of this joint restriction is given in the last column of Table 5. The values of the $\chi^2$ statistic indicate rejection of the restriction in all the three periods. One possible reason for this rejection is that the covariance terms $C_{rt}$ in equation (19) may be time-varying which means that the coefficients in equation (27) are also time-varying.

On the whole, the results in Panels B and C indicate a positive relationship between RATIO and the expected real return on the market, RVWFIT, and a negative relationship between RATIO and the ex-ante variance of the market return, VARFIT, with the negative relationship being more pronounced. The negative relationship is consistent with earlier studies such as Schwert (1989) which find that the volatility of the stock market increases during a recession.11
Tables 6 and 7 present regression results of RATIO/RRES against VARFIT and RVWFIT respectively. The purpose of these regressions is simply to investigate if the coefficients in these univariate regressions differ in any appreciable and systematic way from the corresponding coefficients in the bivariate regressions of Table 5. Interactions between VARFIT and RVWFIT could cause the relationships documented in Table 5 to be simply an artifact of the nature of the independent variables. It is therefore useful to see if these relationships persist in direct one on one regressions.

The results in Tables 6 and 7 are qualitatively very similar to those seen in Table 5. They have the same patterns of coefficient values and t-statistics as those in Table 5. Generally speaking, the coefficients of VARFIT in Table 6 are somewhat less negative and the coefficients of RVWFIT in Table 7 somewhat less positive than the corresponding coefficients in Table 5. The t-statistics follow a similar ordering. However, this does not affect the conclusions drawn from Table 5 and those results do not appear to be due to any interactions between VARFIT and RVWFIT.

As mentioned earlier, another implication of equation (19) is that the variables which, at time t, help predict the return on the market and its variance should also predict the marginal return on investment. Hence the variables used to form VARFIT and RVWFIT should themselves jointly be significant predictors of RATIO. This implication is tested in the regressions in Table 8.

First of all, I look only at the predictive ability of the variables that are used to predict VAR. Panel A presents the regressions over the period 49:4-84:4 of RATIO_t and RRES_t on the variables used to predict VAR_t - RF_t and VAR_{t-1}. Both the independent variables have significantly negative coefficients which is consistent with the negative coefficients of VARFIT in the
earlier tables. Thus, the variables used to predict \( \text{VAR}_t \) also predict \( \text{RATIO}_t \) and \( \text{RRES}_t \).

Panel B presents regressions over the period 48:4-84:4 of \( \text{RATIO}_t \) and \( \text{RRES}_t \) on the predictors of \( \text{RVW}_t \). The results here are less clearcut but are nonetheless interesting. Note that two of the predictors of \( \text{RVW}_t \) - \( RF_t \) and \( \text{VAR}_{t-1} \) - are also predictors of \( \text{VAR}_t \). As can be seen in Panel A of Table 5, \( RF_t \) and \( \text{VAR}_{t-1} \) have opposite signs in the prediction equation for \( \text{RVW}_t \) while they have the same signs in the prediction equation for \( \text{VAR}_t \). In Panels A and B of Table 8, \( RF_t \) and \( \text{VAR}_{t-1} \) have the same signs. While this explains the strong negative relationship between \( \text{RATIO}_t \) and \( \text{VARFIT}_t \), it also points to reasons for the weak positive relationship between \( \text{RATIO}_t \) and \( \text{RVWFIT}_t \). In addition \( UT_{St-1} \) and \( DP_{t-2} \) have the same positive sign in the prediction equation for \( \text{RVW}_t \) in Panel A of Table 5 while they have opposite signs (\( UT_{St-1} \) negative and \( DP_{t-2} \) positive) in the regression of \( \text{RATIO}_t \) in Panel B of Table 8. Clearly this is another factor in the weak relationship between \( \text{RATIO}_t \) and \( \text{RVWFIT}_t \). Interestingly in the same Panel B, \( UT_{St-1} \) and \( DP_{t-2} \) have the same signs in the regression of \( \text{RRES}_t \) with \( UT_{St-1} \) significantly positive. This appears to explain the stronger relationship noted between \( \text{RVWFIT}_t \) and \( \text{RRES}_t \) in Table 5.

I repeated the regressions in Panels A and B of Table 8 for the subperiods 49:4-67:4 and 68:1-84:4. The results (not reported) are consistent with the corresponding results in Table 5. Overall, the results in Table 8 indicate that the variables which predict RVW and VAR also predict \( \text{RATIO}/\text{RRES} \).

D. Discussion of Results

The first set of relationships that are obtained and tested use the first order condition in equation (4). Equation (8) presents a positive relationship between \( RF \) and \( \text{NRATIO} \) which is not surprising given that the former is
just the certainty equivalent of the latter. The data appear to support this
relation which is in line with casual observations of lower interest rates
during recessions and higher interest rates during periods of expansion.

The results of testing equation (14) indicate that the relationship be-
tween RATIO and RF is negative, possibly because RF is a proxy for expected
inflation. This puts RATIO in the same category as common stock returns in
terms of its co-movements with expected inflation. Several papers have argued
that a negative relationship between stock returns and expected inflation is
observed only because inflation is proxying for another variable which is neg-
avely related to stock returns.\footnote{12} Given the negative relationship between
RATIO and expected inflation documented in this paper, it would be useful to
construct a model which could explain all of these interactions. It is
possible, given the predictive nature of stock returns, that the relationship
between stock returns and expected inflation is being partly determined by the
corresponding relationship between RATIO and expected inflation.

There appears to be some evidence in the data that the "risk premium" on
the marginal return on investment is negatively related to RF (Equations (25)
and (26) and Table 4). A similar but stronger relationship has been docu-
mented for stocks and a weaker version has been documented for long-term
bonds.\footnote{13}

The second set of relationships uses the first order condition in equation
(5). The data support a significant negative relationship of RATIO with the
VARFIT but only a weak positive relationship of RATIO with RVWFIT. A priori,
there are reasons to expect that stock returns would be positively related to
RATIO since stock returns typically reflect the anticipated fundamentals in
the economy. The negative relationship between VARFIT and RATIO is harder to
see at an intuitive level. It arises from the inverse relationship between
RATIO_{t+1} and the discount factor, \( \frac{1}{1+r_{mt+1}} \) in equation (5). This relationship
is inverse because the conditional expectation of the product of these two quantities is always unity. Now note that the discount factor is a convex function of the discount rate and hence any increase in the variability of the discount rate will serve to increase the average level of the discount factor. In other words, the discount factor is positively related to the variance of the discount rate. This positive relationship combined with the earlier inverse relationship between RATIO and the discount factor gives rise to the negative relationship between RATIO and VARFIT. Empirical findings that stock returns are more volatile during recessionary periods are consistent with this interpretation.

The variables which predict the market return and its variance are jointly able to predict RATIO. The signs of the coefficients of these variables in the prediction regression involving RATIO, while explaining the negative relationship between RATIO and VARFIT, give some clues about why there is only a weak positive relationship between RATIO and RVWFIT. Some of these variables have coefficients with the same sign as and others have coefficients with the opposite sign to what they had in the prediction regression to RVW. Thus, the predicted value RVWFIT has several conflicting effects which tend to dilute its positive relationship to RATIO. It is conceivable that if another set of variables is used to predict RVW, a stronger positive relationship between RATIO and RVWFIT might be observed.

III. Conclusion

The present paper has derived relations linking the marginal return on investment to several financial variables. To the extent that output in the form of GNP is a determinant of the level of the marginal return on investment, the relations can be seen as linking output to various financial
variables. The relations are tested using actual U.S. data and there appears to be considerable evidence for these relations.

The approach taken in this paper is somewhere in between testing an ad hoc specification based on earlier empirical findings and testing a fully parameterized model which has been solved to obtain closed form solutions. I make use of the first order condition of a model of utility maximization and derive implications from this first order condition by making certain auxiliary assumptions.

A richer production function that the one used in this paper could give us implications that are perhaps easier to test and possibly, depending upon the parameters of the production function, better supported by the data. Another possible extension would be to disaggregate the production side into various industrial groupings and to repeat the analysis of the paper for each grouping. It is quite possible that there are different production functions corresponding to the various groupings that fit the data best.
Footnotes


3Some examples of such models are Balvers, Cosimano and McDonald (1990), Cochrane (1991), Rouwenhorst (1989) and Sharathchandra (1989).

4See Abel (1988) and Balvers, Cosimano and McDonald (1990).

5Since we have

\[ E_t[(1+r_{mt+1}^*)] \cdot \frac{1}{(1+r_{mt+1}^*)} = 1 \]

we can write

\[ E_t[1+r_{mt+1}^*] \cdot E_t[\frac{1}{1+r_{mt+1}^*}] = 1 - \text{cov}_t(r_{mt+1}^*, \frac{1}{1+r_{mt+1}^*}) \]

which implies

\[ E_t[\frac{1}{1+r_{mt+1}^*}] \approx \frac{1+\text{Var}_t[r_{mt+1}^*]}{1+r_{mt+1}^*} \]

6I thank Ken French for providing me with the data. Though the returns computed using the price data do not include dividends, French, Schwert and Stambaugh (1987) argue that it makes little difference, if any, to the computation of the volatility of returns.


8I tried a series of values for \( \alpha \) from 0.2 to 0.5. The results changed very little for this range. Estimates of \( \alpha \) is \( S(1989) \) ranged from 0.21 to 0.34. I used \( \alpha = 0.30 \) to be in this ballpark.

9Over the period 49:1-84:4, I find no evidence of nonstationarity in the variance both by looking at the autocorrelations and by performing unit root tests. Poterba and Summers (1986) also find the monthly variance follows a stationary AR(1) process. They consider both the 1950-84 and the 1928-84 periods.

However, French, Schwert and Stambaugh (1987) and Pagan and Schwert (1990) argue that volatility is a nonstationary process. The periods they consider are 1928-84 and 1834-1987 respectively.

10Earlier literature also indicates a negative sign for RF (Fama and Schwert (1977), Ferson (1989)) and positive signs for UTS and DP (Campbell and Shiller (1988), Chen (1991), Chen, Roll and Ross (1986), Fama and French (1988, 1990)).
It has been argued that, during a recession, the value of equity falls relative to the value of debt and this could create additional volatility due to the leverage effect. Schwert (1989), however, argues that his results indicate that leverage is not enough to explain the change in volatility during a recession.


See Ferson (1989).

Actually the set of variables used to predict RVW already includes the variables used to predict VAR.

If the true relationship between RATIO and the expected market return is positive, then as we form better estimates (RVWFIT) we are likely to see a more positive relationship than we have currently. On the other hand, if the true relationship is weak or non-existent, then better estimates of the expected return (RVWFIT) will only show a weaker relationship with RATIO.
References


TABLE 1

NRATIO_t = a_0 + a_1 RF_t (Panel A)

NRRES_t = b_0 + b_1 RF_t (Panel C) where NRRES_t is the residual from

NRATIO_t = c_0 + c_1 NRATIO_{t-1} + c_2 NRATIO_{t-2} (Panel B)

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Dependent Variable</th>
<th>Independent Variables</th>
<th>R^2</th>
<th>Durbin-Watson Statistic</th>
<th>Q-Statistic</th>
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a $\chi^2$ statistic for the null hypothesis that $a_0 = a_1$
b $\chi^2$ statistic for the null hypothesis that $a_2 = 0$ where $a_2$ is the coefficient of $RF_t$ as follows:

NRATIO_t = $a_0 + a_1 RF_t + a_2 RF_t$
c $\chi^2$ statistic for null hypothesis that $a_0 - a_1 + a_2 = 0$ in above equation
d t-statistic for null hypothesis that $a_0 - a_1 + a_2 = 0$ in above equation
e p-values
\[
RATIO_t = a_0 + a_1 RF_t \quad \text{(Panel A)}
\]
\[
RRES_t = b_0 + b_1 RF_t \quad \text{(Panel C)}
\]
where \(NRRES_t\) is the residual from
\[
RATIO_t = c_0 + c_1 RATIO_{t-1} + c_2 RATIO_{t-2} \quad \text{(Panel B)}
\]

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\(\chi^2\) statistic for the null hypothesis that \(a^2 = 0\) where \(a^2\) is the coefficient of \(RF_t^2\) as follows: \(RATIO_t = a_0 + a_1 RF_t + a_2 RF_t^2\)

b t-statistics calculated using heteroscedasticity and autocorrelation consistent standard errors

c p-values
TABLE 3

\[ \text{RATIO}_t = a_0 + a_1 \text{CPIFIT}_t \] (Panel B)

\[ \text{RRES}_t = b_0 + b_1 \text{CPIFIT}_t \] (Panel C) where CPIFIT_t is the fitted value from

\[ \text{CPI}_t = c_0 + c_1 \text{CPI}_{t-1} + c_2 \text{CPI}_{t-2} \] (Panel A)

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Dependent Variable</th>
<th>Independent Variables</th>
<th>(-R^2)</th>
<th>Durbin-Watson Statistic</th>
<th>Q-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A</td>
<td>\text{CPI}_t</td>
<td>Constant</td>
<td>CPI(_t)-1</td>
<td>CPI(_t)-3</td>
<td></td>
</tr>
<tr>
<td>49:4-84:4</td>
<td>0.1762</td>
<td>0.5013</td>
<td>0.3245</td>
<td>0.55</td>
<td>1.99</td>
</tr>
<tr>
<td></td>
<td>(2.79)(^a)</td>
<td>(7.15)(^a)</td>
<td>(4.72)</td>
<td></td>
<td>(0.59)(^c)</td>
</tr>
<tr>
<td>Panel B</td>
<td>\text{RATIO}_t</td>
<td>Constant</td>
<td>CPIFIT(_t)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>49:4-84:4</td>
<td>0.3584(^b)</td>
<td>-0.2689(^b)</td>
<td>0.22</td>
<td>0.16</td>
<td>507.89</td>
</tr>
<tr>
<td></td>
<td>(5.49)(^b)</td>
<td>(-4.16)(^b)</td>
<td></td>
<td></td>
<td>(0.00)</td>
</tr>
<tr>
<td>54:1-84:4</td>
<td>0.3745(^b)</td>
<td>-0.2856(^b)</td>
<td>0.36</td>
<td>0.24</td>
<td>394.44</td>
</tr>
<tr>
<td></td>
<td>(6.59)(^b)</td>
<td>(-5.10)(^b)</td>
<td></td>
<td></td>
<td>(0.00)</td>
</tr>
<tr>
<td>54:1-72:4</td>
<td>0.2660(^b)</td>
<td>-0.1769(^b)</td>
<td>0.08</td>
<td>0.24</td>
<td>483.58</td>
</tr>
<tr>
<td></td>
<td>(2.38)(^b)</td>
<td>(-1.59)(^b)</td>
<td></td>
<td></td>
<td>(0.00)</td>
</tr>
<tr>
<td>73:1-84:4</td>
<td>0.6364\times10^{-1}</td>
<td>0.1848\times10^{-1}</td>
<td>-0.02</td>
<td>0.17</td>
<td>130.92</td>
</tr>
<tr>
<td></td>
<td>(0.98)(^b)</td>
<td>(0.29)(^b)</td>
<td></td>
<td></td>
<td>(0.00)</td>
</tr>
<tr>
<td>Panel C</td>
<td>\text{RRES}_t</td>
<td>Constant</td>
<td>CPIFIT(_t)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>49:2-84:4</td>
<td>0.2343\times10^{-1}</td>
<td>-0.2318\times10^{-1}</td>
<td>0.03</td>
<td>2.20</td>
<td>19.34(^b)</td>
</tr>
<tr>
<td></td>
<td>(3.00)(^b)</td>
<td>(-3.00)(^b)</td>
<td></td>
<td></td>
<td>(0.97)</td>
</tr>
<tr>
<td>54:1-84:4</td>
<td>0.2475\times10^{-1}</td>
<td>-0.2453\times10^{-1}</td>
<td>0.04</td>
<td>2.39</td>
<td>23.94(^b)</td>
</tr>
<tr>
<td></td>
<td>(3.70)(^b)</td>
<td>(-3.71)(^b)</td>
<td></td>
<td></td>
<td>(0.88)</td>
</tr>
<tr>
<td>54:1-72:4</td>
<td>0.1163\times10^{-1}</td>
<td>-0.1154\times10^{-1}</td>
<td>-0.01</td>
<td>2.37</td>
<td>24.90(^b)</td>
</tr>
<tr>
<td></td>
<td>(0.73)(^b)</td>
<td>(-0.73)(^b)</td>
<td></td>
<td></td>
<td>(0.41)</td>
</tr>
<tr>
<td>73:1-84:4</td>
<td>0.4559\times10^{-1}</td>
<td>-0.4496\times10^{-1}</td>
<td>0.10</td>
<td>2.40</td>
<td>21.25(^b)</td>
</tr>
<tr>
<td></td>
<td>(5.96)(^b)</td>
<td>(-5.96)(^b)</td>
<td></td>
<td></td>
<td>(0.27)</td>
</tr>
</tbody>
</table>

\(^a\) t-statistics calculated using heteroscedasticity and autocorrelation consistent standard errors

\(^b\) t-statistics calculated using heteroscedasticity and autocorrelation consistent standard errors which are also corrected for the dependent variable being an estimated value from an earlier regression

\(^c\) p-values
### TABLE 4

\[ R_{Ft} = a_0 + a_1 R_{Ft} \]

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Dependent Variable</th>
<th>Independent Variables</th>
<th>(-R^2)</th>
<th>Durbin-Watson Statistic</th>
<th>Q-Statistic</th>
<th>(\chi^2) Statistic&lt;sup&gt;a&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>49:4-84:4</td>
<td>(R_{Ft})</td>
<td>Constant, (R_{Ft})</td>
<td>0.13</td>
<td>0.87</td>
<td>154.86 (c)</td>
<td>0.90 (c)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1235 \times 10^{-1})</td>
<td>((5.87)^b)</td>
<td>(-0.3785) (b)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1071 \times 10^{-1})</td>
<td>((5.78)^b)</td>
<td>(-0.2987) (b)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>54:1-84:4</td>
<td>(R_{Ft})</td>
<td>Constant, (R_{Ft})</td>
<td>0.10</td>
<td>0.85</td>
<td>205.90 (c)</td>
<td>5.89 (c)</td>
</tr>
<tr>
<td>54:1-72:4</td>
<td>(R_{Ft})</td>
<td>Constant, (R_{Ft})</td>
<td>0.00</td>
<td>2.04</td>
<td>20.98 (c)</td>
<td>0.00 (c)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.7929 \times 10^{-2})</td>
<td>((6.97)^b)</td>
<td>(-0.9889 \times 10^{-1})</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.2184 \times 10^{-1})</td>
<td>((7.22)^b)</td>
<td>(-0.7410) (b)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup> \(\chi^2\) statistic for the null hypothesis that \(a_2 = 0\) where \(a_2\) is the coefficient of \(R_{Ft}^2\) as follows:

\[ R_{Ft} = a_0 + a_1 R_{Ft} + a_2 R_{Ft}^2 \]

<sup>b</sup> t-statistics calculated using heteroscedasticity and autocorrelation consistent standard errors

<sup>c</sup> p-values
TABLE 5

\[
\text{RATIO}_t = a_0 + a_1 \text{VARFIT}_t + a_2 \text{RVWFIT}_t \quad \text{(Panel B)}
\]

\[
\text{RRES}_t = b_0 + b_1 \text{VARFIT}_t + b_2 \text{RVWFIT}_t \quad \text{(Panel C)}
\]

where \( \text{VARFIT}_t \) and \( \text{RVWFIT}_t \) are, respectively, the fitted values from \( \text{VAR}_t = c_0 + c_1 \text{RF}_t + c_2 \text{VAR}_{t-1} \) (Panel A) and \( \text{RVW}_t = d_0 + d_1 \text{RF}_t + d_2 \text{VAR}_{t-1} + d_3 \text{UTST}_{t-1} + d_4 \text{DP}_{t-2} \) (Panel A)

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Dependent Variable</th>
<th>Independent Variables</th>
<th>(- R^2)</th>
<th>Durbin-Watson Statistic</th>
<th>Q-Statistic</th>
<th>( X^2 ) Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>49:4-84:4</td>
<td>( \text{VAR}_t )</td>
<td>Constant RF ( \text{VAR}_{t-1} )</td>
<td>0.28</td>
<td>2.04</td>
<td>47.95</td>
<td>(0.04)</td>
</tr>
<tr>
<td></td>
<td>( \text{RVW}_t )</td>
<td>Constant RF ( \text{VAR}_{t-1} )</td>
<td>0.20</td>
<td>1.79</td>
<td>35.44</td>
<td>(0.35)</td>
</tr>
<tr>
<td>49:4-84:4</td>
<td>( \text{RATIO}_t )</td>
<td>Constant VARFIT RVWFIT</td>
<td>0.38</td>
<td>391.33</td>
<td>24.40</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \text{RRES}_t )</td>
<td>Constant VARFIT RVWFIT</td>
<td>0.10</td>
<td>256.59</td>
<td>18.90</td>
<td></td>
</tr>
</tbody>
</table>

\( a, b, c, d \) denote t-statistics.
TABLE 5 CONTINUED

<table>
<thead>
<tr>
<th>a</th>
<th>$X^2$ statistic for the null hypothesis that $a_0 = -a_1 = a_2$ (Panel B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>t-statistics calculated using heteroscedasticity and autocorrelation consistent standard errors</td>
</tr>
<tr>
<td>c</td>
<td>t-statistics calculated using heteroscedasticity and autocorrelation consistent standard errors which are also corrected for the dependent variable being an estimated value from an earlier regression</td>
</tr>
<tr>
<td>d</td>
<td>p-values</td>
</tr>
</tbody>
</table>
### TABLE 6

\[ \text{RATIO}_t = a_0 + a_1 \text{VARFIT}_t \] (Panel A)

\[ \text{RRES}_t = b_0 + b_1 \text{VARFIT}_t \] (Panel B)

where \( \text{VARFIT}_t \) is from Panel A of Table 5

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Dependent Variable</th>
<th>Independent Variables</th>
<th>( R^2 )</th>
<th>Durbin-Watson Statistic</th>
<th>( Q )-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>49:4-84:4</td>
<td>RATIO(_t)</td>
<td>Constant (-1.0102)</td>
<td>0.30</td>
<td>0.36</td>
<td>524.40 (0.00)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>VARFIT(_t) (0.9169\times10^{-1})</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>49:4-67:4</td>
<td>RATIO(_t)</td>
<td>Constant (-0.1085)</td>
<td>-0.01</td>
<td>0.09</td>
<td>377.81 (0.00)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>VARFIT(_t) (0.8998\times10^{-1})</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>RRES(_t)</td>
<td>Constant (-0.1052)</td>
<td>0.02</td>
<td>1.89</td>
<td>23.24 (0.51)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>VARFIT(_t) (0.5213\times10^{-3})</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>RRES(_t)</td>
<td>Constant (-0.8681\times10^{-1})</td>
<td>0.05</td>
<td>2.52</td>
<td>20.95 (0.64)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>VARFIT(_t) (0.3958\times10^{-3})</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( a \) t-statistics calculated using heteroscedasticity and autocorrelation consistent standard errors which are also corrected for the dependent variable being an estimated value from an earlier regression.

\( b \) p-values


<table>
<thead>
<tr>
<th>Time Period</th>
<th>Dependent Variable</th>
<th>Independent Variables</th>
<th>$R^2$</th>
<th>Durbin-Watson Statistic</th>
<th>Q-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RATIO$\text{t}$</td>
<td>Constant RVWFIT$\text{t}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>49:4–84:4</td>
<td>Constant</td>
<td>0.8664x10$^{-1}$</td>
<td>-0.01</td>
<td>0.05</td>
<td>1054.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(90.50)$^a$</td>
<td></td>
<td></td>
<td>(0.00)$^b$</td>
</tr>
<tr>
<td>49:4–67:4</td>
<td>Constant</td>
<td>0.8876x10$^{-1}$</td>
<td>0.08</td>
<td>0.15</td>
<td>278.28</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(98.69)</td>
<td></td>
<td></td>
<td>(0.00)</td>
</tr>
<tr>
<td>68:1–84:4</td>
<td>Constant</td>
<td>0.8416x10$^{-1}$</td>
<td>0.26</td>
<td>0.32</td>
<td>194.48</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(86.06)</td>
<td></td>
<td></td>
<td>(0.00)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Dependent Variable</th>
<th>Independent Variables</th>
<th>$R^2$</th>
<th>Durbin-Watson Statistic</th>
<th>Q-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RRES$\text{t}$</td>
<td>Constant RVWFIT$\text{t}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>49:4–84:4</td>
<td>Constant</td>
<td>-0.6025x10$^{-4}$</td>
<td>0.01</td>
<td>2.12</td>
<td>18.35</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-0.84)$^a$</td>
<td></td>
<td></td>
<td>(0.98)</td>
</tr>
<tr>
<td>49:4–67:4</td>
<td>Constant</td>
<td>0.5993x10$^{-4}$</td>
<td>-0.01</td>
<td>1.88</td>
<td>22.52</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.64)</td>
<td></td>
<td></td>
<td>(0.55)</td>
</tr>
<tr>
<td>68:1–84:4</td>
<td>Constant</td>
<td>-0.1677x10$^{-3}$</td>
<td>-0.00</td>
<td>2.43</td>
<td>20.56</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-2.49)</td>
<td></td>
<td></td>
<td>(0.66)</td>
</tr>
</tbody>
</table>

$^a$ t-statistics calculated using heteroscedasticity and autocorrelation consistent standard errors which are also corrected for the dependent variable being an estimated value from an earlier regression.

$^b$ p-values
TABLE 8
Regressions of $\text{RATIO}_t$ and $\text{RRES}_t$ on variables used to predict $\text{VAR}_t$ (Panel A) and $\text{RVW}_t$ (Panel B)

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Dependent Variable</th>
<th>Independent Variables</th>
<th>$R^2$</th>
<th>Durbin-Watson Statistic</th>
<th>Q-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>49:4-84:4</td>
<td>$\text{RATIO}_t$</td>
<td>Constant</td>
<td>0.9172x10^{-1}</td>
<td>-0.3144</td>
<td>-0.2055</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\text{RF}_t$</td>
<td>(-4.97)</td>
<td>(-3.23)</td>
<td>0.00</td>
</tr>
<tr>
<td>49:4-84:4</td>
<td>$\text{RRES}_t$</td>
<td>Constant</td>
<td>0.4210x10^{-3}</td>
<td>-0.2243x10^{-1}</td>
<td>-0.2831x10^{-1}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\text{RF}_t$</td>
<td>(-2.99)</td>
<td>(-2.03)</td>
<td>0.94</td>
</tr>
<tr>
<td>Panel B</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>49:4-84:4</td>
<td>$\text{RATIO}_t$</td>
<td>Constant</td>
<td>0.9172x10^{-1}</td>
<td>-0.4458</td>
<td>-0.2223</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\text{RF}_t$</td>
<td>(-6.71)</td>
<td>(-3.09)</td>
<td>0.00</td>
</tr>
<tr>
<td>49:4-84:4</td>
<td>$\text{RRES}_t$</td>
<td>Constant</td>
<td>0.4278x10^{-5}</td>
<td>-0.1317x10^{-1}</td>
<td>-0.3023x10^{-1}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\text{UTS}_t$-1</td>
<td>-0.1829x10^{-2}</td>
<td>0.6841x10^{-3}</td>
<td>1.62</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\text{DP}_t$-2</td>
<td>0.6841x10^{-3}</td>
<td>1.62</td>
<td>0.77</td>
</tr>
</tbody>
</table>

*a t-statistics calculated using heteroscedasticity and autocorrelation consistent standard errors
b p-values
Figure 1

NRATIO

1.045 1.04 1.035 1.03 1.025 1.02 1.015 1.01 1.005 1.0

1949:4 - 1984:4

49 53 57 61 65 69 73 77 81
Figure 5
Actual and Fitted Inflation
1.05
1.04
1.03
1.02
1.01
1.00
0.99
0.98
1949:4 - 1984:4
CPI + CPFIT
49 53 57 61 65 69 73 77 81
Figure 6
Excess Return on Investment

1949:4 – 1984:4
Figure 7
Actual and Fitted Variances

1949:4 – 1984:4

VAR
+
VARFIT
Figure 8

Actual and Fitted Returns

0.3 0.2 0.1 0.0 -0.1 -0.2

RVW RVFIT

1949:4 - 1984:4
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Business Information Center
Edwin L. Cox School of Business
Southern Methodist University
Dallas, Texas 75275
"Organizational Subcultures in a Soft Bureaucracy: Resistance Behind the Myth and Facade of an Official Culture," by John M. Jermier, John W. Slocum, Jr., Louis W. Fry, and Jeannie Gaines

"Global Strategy and Reward Systems: The Key Roles of Management Development and Corporate Culture," by David Lei, John W. Slocum, Jr., and Robert W. Slater

"Multiple Niche Competition - The Strategic Use of CIM Technology," by David Lei and Joel D. Goldhar

"Global Strategic Alliances," by David Lei and John W. Slocum, Jr.

"A Theoretical Model of Household Coupon Usage Behavior And Empirical Test," by Ambuj Jain and Arun K. Jain

"Household's Coupon Usage Behavior: Influence of In-Store Search," by Arun K. Jain and Ambuj Jain

"Organization Designs for Global Strategic Alliances," by John W. Slocum, Jr. and David Lei

"Option-like Properties of Organizational Claims: Tracing the Process of Multinational Exploration," by Dileep Hurry

"A Review of the Use and Effects of Comparative Advertising," by Thomas E. Barry


"Designing Global Strategic Alliances: Integration of Cultural and Economic Factors," by John W. Slocum, Jr. and David Lei

"The Components of the Change in Reserve Value: New Evidence on SFAS No. 69," by Mimi L. Alciatore