1-1-1992

Interest Rate Swaps : A Bargaining Game Solution

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We thank Professor Andrew Chen for his valuable comments and suggestions on an earlier draft of this paper.

INTEREST RATE SWAPS:
A BARGAINING GAME SOLUTION

Working Paper 92-0701*

by

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Abstract

In this paper we provide models for characterizing the equilibrium swap rates for two types of interest rate swaps. The first is a fixed rate for floating rate swap between a risky firm and a riskless institution. The second is a swap between two risky firms with an intermediary guaranteeing the performance. This swap is modeled as a cooperative game between the players in the context of competitive intermediary services. In specific, we determine the payoff space and invoke the Nash Bargaining solution for characterizing the equilibrium swap rates. In addition to being descriptive of the prevailing swap rates, our models can be used by intermediaries and firms to determine equilibrium swap rates.

I. Introduction

For more than a decade swaps have been a popular financial tool available to corporations seeking to alter their interest and/or exchange rate exposure. With the growing popularity of swaps, the swap market has evolved not only in terms of the availability of different types of swap instruments but also in terms of the institutional arrangements for entering into swap contracts. When swaps were initially introduced, swap contracts were directly negotiated by the two counterparties. Subsequently, intermediaries began to play a significant role in swap contracts. In many instances, the intermediary, in addition to bringing the two counterparties together, would also guarantee the performance of the counterparties to each other. Either party to the contract could then behave as if it were dealing with a riskless counterparty. The intermediaries were compensated for their services of providing the information to bring the two counterparties together and the guarantee of performance of the counterparties in terms of the spreads they received. Recently, however, due to increased competition among intermediaries, the spreads have been narrowing.[1]

In this paper we provide models for characterizing equilibrium rates for two types of interest rate swaps. The first is a fixed for floating rate swap between a risky firm and a riskless intermediary. The second is a fixed for floating rate swap between two risky firms through a riskless intermediary. The interest rate swap market has grown significantly since its beginning in the early 80's and the total outstanding volume now exceeds a trillion dollars. The pricing of the interest-rate swap contracts or in other words the setting of the fixed and floating rates that the counterparties pay each other either directly or through an intermediary are influenced by many factors. The factors include the prevailing rates in the financial markets, the creditworthiness of counterparties and the institutional arrangements underlying the contract.

The analysis of the pricing of interest-rate swaps has attracted attention of many financial economists in the past. Bicksler and Chen (1986) developed an approach to price interest rate swaps in a stochastic interest rate environment. They considered a context where two default risk free counterparties contracted with each other directly. In a recent paper, Cooper and Mello (1991) explicitly accounted for the default risk in pricing of swaps. In their scenario, the payor of floating rate is riskless while the payor of fixed rate is risky. Using the option pricing approach, they are able to characterize the solution to the problem of pricing interest rate swaps in the presence of default risk.

In this paper we characterize equilibrium swap rates when both the counterparties are risky. We also explicitly incorporate the intermediary in our analysis. In particular, the intermediary operating in a competitive market for services of intermediation provides a guarantee for the performance of the counterparties to each other. Most swap transactions are carried out on a give-up name basis. The counterparties to a swap can therefore find credit and other information about each other. Also, the process of arriving at the swap rates can involve more than one round of negotiations. The two counterparties may not always directly negotiate with each other but may deal with each other only through an intermediary. In both cases, however, they are dealing with a nearly "full information" situation. In this situation, the final outcome depends on the mutual interaction among the firms interested in a swap and the intermediary institution; each of whom can be assumed to be in pursuit of its own self-interest.

Given this strategic interdependence, we have chosen to model this situation using a game theoretic approach. Our game theoretic solution to the swap problem provides the optimal swap rates that the two counterparties acting in their own self interest will agree upon. It is not our claim that the actual process of pricing swap transactions is played out as a bargaining game. We do, however, believe that the agreed swap rates will be as if they were an outcome of the bargaining game. Given our assumptions, if the actual rates did not conform to the game theoretic solution at least one of the counterparties would not agree to transact. It is in this sense that we believe our model to be descriptive of the observed swap rates.

The rest of the paper is organized as follows. Section II provides the details of the economic setting we consider. Section III details the Nash bargaining game and characterizes its solution. Section IV concludes the paper.

II. The Economic Scenario

In this section we set out in detail the essential features of the economic context we analyze in this paper. We begin by describing the capital market context in which the players in the game operate. We then describe the role of the intermediary in the capital markets. We also provide the details of the possible actions available to each player and outline their payoffs and objective functions.

A. The Capital Market Context

There exist capital markets where all firms in the economy can raise both fixed rate and floating rate funds. In the capital markets considered here all floating rates on funds borrowed will be indexed to the same "base" floating or stochastic rate, $\tilde{R}$, where $\tilde{}$ denotes a random variable. In other words, for all firms, floating rates are quoted as $\tilde{R}$ + spread, where the spread is determined at the time the debt/swap contract is entered into and remains constant through the life of the contract. While this feature of the economic context we consider does limit some flexibility, it allows us to minimize analytical complexity to a great extent.[2] In practical terms this assumption means that if for a particular firm, floating rate is priced off say LIBOR then the floating rates for all other firms will also be priced off LIBOR. In other words, the rates on all floating rate debts will be perfectly correlated.

Within the context of such capital markets we will focus attention on two firms whom we will call firm A and firm B. We will consider a situation where firm A currently has fixed rate debt outstanding and firm B has floating rate debt outstanding. At this time, however, firm A prefers to have a floating rate liability and firm B prefers to have a fixed rate liability. It is not the purpose of our paper to delve into the reasons why firm A might desire floating rate funds or firm B might desire fixed rate funds. We take as given the fact that these firms have their respective preferences. Our goal is to model the rates they would pay each other in case they do decide to enter into a swap contract.

Since firm A desires floating rate funds and firm B desires fixed rate funds, both of them could potentially obtain their desired type of funds in the capital markets and retire their existing liabilities.[3] If they do, there would be no necessity for a swap. Our analysis, however, allows for the possibility that the prevailing structure of rates available to both firms across floating and fixed rate markets may be such that firm A may find it advantageous not to directly issue floating rate debt in the capital market and retire its existing fixed rate liability but keep the existing liability and swap it for floating rate debt. In the scenario we are considering a swap will be feasible between firm A and firm B only if firm B also finds it advantageous not to directly issue fixed rate debt in the capital market and retire its existing liability but keep it and swap it for fixed rate debt.

In relation to the possible structures of prevailing rates in the capital markets that we allow for, two issues need to be noted. First, we do not attempt to analyze or explain why such structures of rates might exist in the capital markets. We also do not enter the debate as to whether such a structure of rates may arise due to existence of arbitrage opportunities as postulated by Bicksler and Chen (1986) or due to market incompleteness as postulated by Smith, Smithson and Wakeman (1988). Whatever may be the reason underlying the existence such structure of rates, the purpose of our paper is to provide insight into the process of rate negotiation in a swap if the structure of rates makes one feasible and attractive.

Second, we assume that firm A has <u>comparative advantage</u> in fixed rate market whereas firm B has comparative advantage in the floating rate market. As an illustration of such comparative advantage, consider the rates shown in Table 1. These rates indicate that in the fixed rate market firm A can raise funds at a rate 120 basis points lower than that available to firm B. On the other hand, firm A can raise funds in the floating rate market at a rate only 50 basis points lower than that available to firm B. In this case, firm A has a comparative advantage in the fixed rate market and firm B in the floating rate market. In the rest of the paper we will focus on situations where there exists such a comparative advantage.

Insert Table 1 here

B. Role of the Third Party

There exist institutions in the capital markets who participate in swap activities. The participation of these institutions in swap activities can be of two types. One, they could use their knowledge of the firms in the market to bring firms A and B together where both firms could contract with each other and the institution (Z) will stand as a guarantor of the performance of both counterparties in the swap transaction. The service provided by Z therefore entails a riskless guarantee to each counterparty to the swap transaction. In this case we will call Z to be functioning as an intermediary. Alternatively, Z could participate in the swap transaction by entering into the swap by itself. In this case Z is in fact a counterparty. Here too we assume that payments from Z to A are riskless. Note that this feature of our analysis is the same as the one considered by Cooper and Mello (1991). In case of both these types of participation by Z in swap transactions, the swap payments received by both firm A and B are riskless.

The intermediary charges a fee proportional to the amount of swap transaction. The market for institutional services is assumed to be competitive. This assumption ensures that all intermediaries would charge the same fee. The intermediary collects his fee from the net cash flow that passes through him between the two counterparties. Since the payments made by the counterparties are risky, the cash flow received by the intermediary in the form of his fee is also risky. In the context of our model, therefore, this assumption implies that in any swap transaction the payoffs to Z must have an exogenously fixed net present value. The other implication of this assumption is that both firm A and firm B have a wide choice of institutions. Thus, if one institution offers to bring A and B together and provide a performance guarantee and another institution offers to enter into the swap on its own account, both firms will choose the better of the two offers.

C. Actions and Contractual Payments

Given the capital market context and the availability of institutional services described in subsections A and B above, we will provide here the details of the actions available to firms A and B and the associated contractual payments. Table 2 provides the actions available to and the consequent contractual payments required of both firms A and B.

Insert Table 2 here

In the Table 2:

F_m^A denotes the payment required to be made by firm A on fixed rate debt raised in the market.

F_Z^A denotes the fixed rate payment required to be made by A when a swap is entered into with Z as a counterparty.

F_S^A denotes the fixed rate payment required to be made by A when a swap is entered into with Z as an intermediary.

$\tilde{Y}_m^A$ denotes the floating rate payment required to be made by A to the investors in the market.

$\tilde{Y}_Z^A$ denotes the floating rate payment required to be made by A when a swap is entered into with Z as a counterparty.

$\tilde{Y}_S^A$ denotes the floating rate payment required to be made by A when a swap is entered with Z as an intermediary.

δ_m^A denotes the spread over $\tilde{R}$ that A agrees to pay when it contracts to pay $\tilde{Y}_m^A$.

Analogous definitions hold for δ_Z^A and δ_S^A. Also, analogous definitions hold for firm B. Without loss of generality, we assume firm A to be more creditworthy than firm B. This implies:

(1)

$$F_j^A < F_j^B \qquad j = m,\ Z \text{ or } S$$

and

$$\tilde{Y}_j^A < \tilde{Y}_j^B \qquad j = m,\ Z \text{ or } S$$

alternatively,

$$\delta_j^A < \delta_j^B \qquad j = m,\ Z \text{ or } S.$$

The maturity of the debt and swap contracts considered here is one period. This implies that F_j^A, F_j^B, $\tilde{Y}_j^A$ and $\tilde{Y}_j^B$ are terminal payouts consisting of both principal and interest.

We assume that both firms A and B will seek to maximize the values of their respective shareholders' wealth. The value of shareholders' equity is, however, equal to the value of a call option on the firm value with the exercise price of the call being the payment to debt holders or the gross payment to swap counterparties. The values of shareholders' wealth as a consequence of each of the six actions listed in Table 2 for both firms A and B are given by:

$$W_K(F_j^K) = C(\tilde{V}_K,\ F_j^K) \qquad j = m,\ Z \text{ or } S \text{ and } K = A \text{ or } B$$

(2) and

$$W_K(\tilde{Y}_j^K) = C(\tilde{V}_K,\ \tilde{Y}_j^K) \qquad j = m,\ Z, \text{ or } S \text{ and } K = A \text{ or } B.$$

where

$W_K(F_j^K)$ denotes the value of the shareholders' wealth for firm K when it contracts to make payment F_j^K.

$W_K(\tilde{Y}_j^K)$ denotes the value of the shareholders' wealth of firm K when it contracts to make payment $\tilde{Y}_j^K$.

$\tilde{V}_K$ denotes the value of the firm K.

$C(\tilde{V}_K, F_j^K)$ denotes the value of a call option on firm value V_K with the exercise price F_j^K.

$C(\tilde{V}_K, \tilde{Y}_j^K)$ denotes the value of a call option on firm value $\tilde{V}_K$ with a stochastic exercise price $\tilde{Y}_j^K$.

The institution Z on its own part simply tries to ensure that the present value of all the cash flows received by itself is equal to the fixed fee it receives as described earlier. This follows directly from the assumption of perfect competition in the market for institutional services.

In the next section we provide the formal model of the swap process.

III. Swap Bargaining Games

As discussed in subsection A of section II above, firm A has fixed rate debt outstanding but desires to have a floating rate liability while firm B has floating rate debt outstanding but desires to have a fixed rate liability. The firms then have a choice of either swapping with Z as counterparty, or swapping with each other with Z as intermediary or not swapping at all. There are three swaps possible in this situation: Firm A could swap with Z, firm B could swap with Z, and both firms could swap with each other through Z. Firm B's swap with Z has been analyzed by Cooper and Mello (1991). We analyze the remaining two. In the rest of this section we provide formal models of both these swaps.

A. Firm A's Swap with Z

In this case, firm A has issued fixed rate debt in the market and has agreed to pay F_m^A. A will then swap with Z. Z pays firm A riskless amount F_m^A. Firm A, in return, will agree to pay Z a floating rate $\tilde{Y}_Z^A = \tilde{R} + \delta_Z^A$. Our problem is to determine a fair δ_Z^A. The payoffs to both parties A and Z are given in Table 3. Following Cooper and Mellow (1991), we assume that the swap contract is a contract for the net cash flows due in the swap, and not for an exchange of gross amounts. Swaps are also assumed to be subordinate to debt in bankruptcy.

Insert Table 3 here

Insert Figure 1 here

B_Z^A are the cash flows to the bondholders of A if it issued fixed rate debt at rate F_m^A and swapped it with Z for floating rate debt at rate $\tilde{Y}_Z^A$. E_Z^A are the cash flows to equity holders of A in the same scenario. Z_Z^A are the cash flows to Z. $CM(V_A, \tilde{Y}_Z^A, F_m^A)$ is the value of the call option on the minimum of V_A and $\tilde{Y}_Z^A$ with exercise price F_m^A. $P(\tilde{Y}_Z^A, F_m^A)$ is the value of the put option on $\tilde{Y}_Z^A$ with exercise price F_m^A. Payoffs to Z are the same as those to a portfolio $CM(V_A, \tilde{Y}_Z^A, F_m^A) - P(\tilde{Y}_Z^A, F_m^A)$, i.e., buy a call on the minimum of V_A and $\tilde{Y}_Z^A$ with exercise price F_m^A and write a put on $\tilde{Y}_Z^A$ with exercise price F_m. Given perfect competition in the market for institutional services Z will set $\tilde{Y}_Z^A = \tilde{R} + \delta_Z^A$, such that[4]

$$\text{(3)} \qquad CM(\tilde{V}_A, \tilde{Y}_Z^A, F_m^A) - P(\tilde{Y}_Z^A, F_m^A) = \beta = \text{Value of } Z_Z^A = \text{Fixed fee.}$$

As discussed before equation (3) just restates the implication of perfect competition in the market for institutional services. Let $\tilde{Y}_Z^A$ (or δ_Z^A) be the solution to (3) and denote by B_F^A the payoff to debt holders of firm A before the swap. The following Lemma then shows that the debt holders of firm A will be better off after the swap.

<u>Lemma 1</u>: Value of B_Z^A > Value of B_F^A

<u>Proof</u>: From the table above notice that the payoffs in B_F^A are identical to those in B_Z^A except in state three where the payoffs are:

$$B_F^A = V_A < V_A + (F_m^A - \tilde{Y}_Z^A) = B_Z^A$$

Hence, Value of B_Z^A > Value of B_F^A.

Since the total value of the firm remains constant, Lemma 1 above must imply that shareholders lose the value that is gained by the debt holders. In other words, there is wealth transfer from shareholders to debt holders. Proposition I below formalizes this intuition.

Proposition I: $W_A(F_m^A) > W_A(\tilde{Y}_Z^A)$

Proof: Follows directly from Lemma 1 and the discussion above.

It is clear that a swap entails a loss in wealth to the shareholders. The incentive of the shareholders to enter into a swap then lies in the fact that they desire floating rate funds. Presumably in some other aspect of the firm's operations there are benefits to be derived from floating rate funds. These are the benefits that give rise to the desire for floating rate funds in the first place. Firm A, therefore, would enter into a swap transaction if the benefits to be derived from having floating rate funds are greater than the loss to shareholder wealth from the swap transaction.

B. Swap Between A and B through Z

We now turn our attention to the case where both firms A and B could engage into an interest rate swap arrangement. In this case the firm A has issued fixed rate debt while it desires floating rate funds. The firm B, on the other hand, has issued floating rate debt while its real desire is to obtain fixed rate funds. The firms A and B, therefore, could arrange an interest rate swap with each other through Z as the intermediary.

Such a swap arrangement will become a reality only if all the parties involved are satisfied with the details of the contractual arrangement. In specific, the following conditions must be true:

(1) the firms A and B are agreeable to the swap rates, i.e., the floating rate that the firm A pays, and the fixed rate that the firm B pays;

(2) the intermediary Z is adequately compensated for the services it provides, in bringing together the right parties and in providing the guarantee for payment of interest from one party to another.

If in case the above conditions are not met, the firms A and B may still be interested in swap arrangements. But then they will have to consider two separate swaps with Z as the counterparty. Thus, firm A may enter into a swap arrangement with Z which is distinct and independent of the swap arrangement that firm B may reach with Z.

Insert Figure 2 here

The above description evidently deals with a situation of strategic interdependence in which the outcome depends on the mutual interaction between rational players; each of whom pursuing its own interests. This is clearly a situation where the game theoretic approach is appropriate and beneficial.[5] Moreover, since the swaps are brokered on a "give-up" name basis the situation clearly allows for communication between parties and involves full information bargaining between parties to reach suitable binding arrangements. Hence, in this subsection, we formulate a cooperative game model to the interest rate swap. In specific, we propose two-person bargaining game between firms A and B, where the role of intermediary Z is captured in terms of the conditions it imposes on the swap rates.

As we discussed in subsection B above, the market for intermediary services is competitive. This competition ensures that the NPV of the cash flows to Z is equal to the fixed fee β. Figure 2 shows the cash flows that Z encounters. Notice that the NPV of cash flows related to A is represented by the L.H.S. of equation (3) above. Similarly, the NPV of cash flows related to B is captured by the L.H.S. of the analogous equation developed by Cooper and

Mello (1991) and restated in equation 3A in footnote 4. Thus, the NPV of Z's cash flows with both A and B is obtained by summing the L.H.S.'s of 3 and 3A. That is:

$$(4) \qquad \bar{F}_S^B - \bar{Y}_S^A - PX\,(\tilde{V}_B, \tilde{Y}_S^A, F_S^B) + CM\,(\tilde{V}_A, \tilde{Y}_S^A, F_S^B) - P(\tilde{Y}_S^A, F_S^B) - \beta = 0$$

where:

$\bar{F}_S^B$ is the value of a default risk free claim on F_S^B, and

$\bar{Y}_S^A$ is the value of a default risk free claim on $\tilde{Y}_S^A$

$PX(\tilde{V}_B, \tilde{Y}_S^A, F_S^B)$ is the value of a put option on the maximum of $\tilde{V}_B$ and $\tilde{Y}_S^A$ with exercise price equal to F_S^B.

β is the NPV of the fee Z charges for the swap.

The above equation has two unknowns, viz. F_S^B and $\tilde{Y}_S^A$, and defines a relation between them. Obviously, if one rate decreases the other must increase to ensure that equation (4) holds, i.e., NPV is equal to β. Equation (4) in fact defines the set of feasible swap rates as depicted by curve SS in Figure 3. In the appendix we provide conditions necessary for curve SS to be convex. It is interesting to note that one specific pair of swap rates on curve SS, denoted by point Q will also simultaneously satisfy the conditions imposed by equation (3) and (3A) in footnote 4. All points on SS other than Q do satisfy equation (4) but not (3) and (3A). In equation (4) the NPV shortfall caused by lowering one rate is compensated by excess due to increasing the other rate. Since Z is indifferent between all points on the curve SS, it defines the feasible set of swap rates for A and B to negotiate upon. The process of negotiation that leads to the specific pair of swap rates acceptable to both A and B is modeled below.

Insert Figure 3 here

Prior to the stage where the bargaining game is played, firms A and B have identified the ideal way of raising funds in the capital market using the process described in subsection A. Thus, the shareholder wealth positions of firms when they begin the game are: $W_A(F_m^A)$ for firm A and $W_B(\tilde{Y}_m^B)$ for firm B.

The initial shareholder wealth levels $W_A(F_m^A)$ and $W_B(\tilde{Y}_m^B)$ are represented by point 0 in Figure 4. Note that point 0 is on the inside of the straight line representing the aggregate firm value (V_A+V_B). The difference between the aggregate firm value and the sum of the shareholder wealth levels $W_A(F_m^A)$ + $W_B(\tilde{Y}_m^B)$ represented at point 0 is the sum wealths of bondholders of firms A and those of firm B. To clarify the relation between curve nn and point 0 let us refer back to curve SS in Figure 3. As we discussed earlier, each point on curve SS represents a pair of feasible swap rates $(\tilde{Y}_S^A, F_S^B)$. Corresponding to each such pair exist shareholder wealth levels $W_A(\tilde{Y}_S^A, F_S^B, F_m^A)$ and $W_B(\tilde{Y}_S^A, F_S^B, \tilde{Y}_m^B)$. Each point on curve nn in Figure 4 therefore represents shareholder wealth levels corresponding to a pair of feasible swap rates. Curve nn is hence labeled the Payoff Space. It is clear that the sum $(W_A + W_B)$, where we have suppressed the arguments of W_A and W_B for expositional convenience, corresponding to any point on curve nn is less than the sum of the shareholder wealths represented by point 0. The aggregate shareholder wealth therefore declines after the swap. This is a consequence of Lemma 1 and the analogous Lemma of Cooper and Mello (1991).

Insert Figure 4 here

The goal of the bargaining process is to choose a suitable point on the curve nn as the final outcome that is acceptable to both the parties. For clarity, we have redrawn the payoff space of Figure 4 as Figure 5. In developing the bargaining game model, we need to describe what happens in case of conflict, i.e., when the firms cannot arrive at an agreement on the final outcome. In conflict situation the firms will act on their own in achieving their individual objectives. This means that firm A will compare two options: (1) arranging independently for a swap with Z as the counterparty to convert its fixed rate obligation into a floating rate obligation, or (2) directly raise floating fund in the marketplace and liquidate the fixed rate debt. The firm A will then naturally choose the better alternative of these two. Thus, the firm A's wealth in case of conflict is given by

$$\begin{aligned} \bar{W}_A &= \text{Conflict Payoff of Firm A} \\ &= \text{Max}(W_A(\tilde{Y}_Z^A),\ W_A(\tilde{Y}_m^A)) \end{aligned}$$

With similar arguments we can determine the wealth of B in case of conflict as

$$\begin{aligned} \bar{W}_B &= \text{Conflict Payoff of Firm B} \\ &= \text{Max}(W_B(F_Z^B),\ W_B(F_m^B)) \end{aligned}$$

Referring to Figure 5, we can see that the conflict payoffs, $\bar{W}_A$ and $\bar{W}_B$, put further constraints on the payoff space. The principle of individual rationality dictates that no firm may agree to a final outcome where the payoff it receives is less than the conflict payoff. Hence, we are left with the segment pq as the undominated set of payoffs. In determining the final payoff, we may only pay attention to this set.

Insert Figure 5 here

The line segment pq may also be seen as the Pareto set since it is not possible to move from one point to another point on the segment while simultaneously improving the wealth of both parties. Von Neumann and Morgenstern (1947) called this the negotiation set and argued that the entire segment pq should be seen as the cooperative solution to the game. From a practical viewpoint, however, one needs to restrict the solution to a single point.

John Nash (1950, 1953) proposed the first, and arguably the most significant, unique solution to a two-person bargaining game. The Nash model is based on four postulates that embody certain notions of fairness and reasonableness. Based on these postulates of joint efficiency, symmetry, linear invariance and independence of irrelevant alternatives, Nash proves the existence and the uniqueness of the solution (see Luce and Raiffa (1957) and Harsanyi (1977)).

In our present situation, this solution, $W^* = (W_A, W_B)$, is obtained by solving the following maximization problem:

$$\text{(P1)} \qquad (W_A^* - \bar{W}_A) \cdot (W_B^* - \bar{W}_B) = \text{Max}[(W_A - \bar{W}_A) \cdot (W_B - \bar{W}_B)]$$

$$\text{S.T.} \qquad W \in \text{Negotiation Set}$$

$$W_A \geq \bar{W}_A \text{ and } W_B \geq \bar{W}_B$$

Another way of defining (P1) follows. Let $H(W_A, W_B) = 0$ be the equation of segment pq representing the negotiation set. Let H_A and H_B be the first partial derivatives of H with respect to W_A and W_B. Using the Lagrangian multipliers we can see that the maximization problem of (P1) is equivalent to

$$\text{(P2)} \qquad \begin{aligned} &H(W_A, W_B) = 0 \\ &H_A(W_A^* - \bar{W}_A) = H_B(W_B^* - \bar{W}_B) \end{aligned}$$

which give a necessary and sufficient condition for the Nash solution (see Harsanyi (1977)).

Having found the solution $W^* = (W_A^*, W_B^*)$ that is acceptable to both A and B, the next step is to solve for underlying swap rates, $\tilde{Y}_S^A$ and F_S^B, such that

$$W_A(\tilde{Y}_S^A, F_S^B, F_m^A) = W_A^* \quad \text{and}$$
$$W_B(\tilde{Y}_S^A, F_S^B, \tilde{Y}_m^B) = W_B^*.$$

Thus, Y_S^A and F_S^B are the equilibrium swap rates that we expect will be agreeable to A, B and Z, when firm A swaps with B through the intermediary Z.

IV. Summary and Conclusion

In this paper we have analyzed two types of interest rate swap transactions. The first, which is an extension of Cooper and Mello (1991) approach, is the swap between a firm and a riskless intermediary where the firm has issued floating rate debt and swaps it for fixed rate debt. We characterize the equilibrium swap rate in this case. The second, which is more complex and therefore more interesting, is the swap between two risky firms arranged through an intermediary that guarantees the performance of both parties to each other. The equilibrium swap rate in this case depends on the strategic interaction between the two firms and the intermediary. Given this strategic interdependence, we model this swap as a Nash Bargaining Game and characterize the solution to it.

A swap rate defines the cash flows passing through the intermediary to and from the firms. We identify the condition implied by the competitive nature of the market for intermediary services. This competitivity condition defines the set of feasible swap rates. Each of the feasible swap rates, in turn, determine the shareholder wealth levels for both firms. This is the payoff space in the firms' bargaining game. The actions available to the firms in case of a disagreement as to the final outcome give us the conflict payoffs. Given the payoff space and the conflict payoffs we apply the Nash bargaining solution procedure to arrive at the equilibrium swap rate. Since the equilibrium swap rate is an optimal outcome of the game, any other swap rate would be unacceptable to at least one of the parties. The observed swap rates, therefore would be consistent with our model. In addition to being descriptive of the prevailing swap rates in the market, our model can be used by the intermediaries to quote the rates that are likely to be acceptable to the firms, and by the firms to choose the rates best suited for them.

Footnotes

[1]See Stigum (1990) for a summary of the workings of swap markets. Also, see Arnold (1984).

[2]If we allowed for debts to be priced off two different base rates such as "T-bill rate" and "LIBOR," then we will need to consider the correlation between the two rates.

[3]Our formal analysis assumes that the existing liabilities could be retired with zero cost. The analysis. however, can be easily modified to account for non-zero cost.

[4]This condition is analogous to equation (4) in Cooper and Mello (1991). We restate below their equation (4) in our notation for later reference in our paper.

$$\bar{F}_S^B - \bar{Y}_m^B - PX(\tilde{V}_B,\ \tilde{Y}_m^B,\ F_S^B) = 0 \qquad (3A)$$

where $\bar{F}_S^B$ is the value of a default risk free claim on F_S and Y_m is the value of a default risk free claim on $\tilde{Y}_m^B$. $PX(\tilde{Y}_B,\ \tilde{Y}_m^B,\ F_S^B)$ is the value of a put option on the maximum of $\tilde{V}_B$ and $\tilde{Y}_m^B$ with exercise price equal to F_S^B.

[5]In cases where one of the parties to the swap is a passive intermediary, there is no strategic interdependence and hence there is no necessity of a game theoretic approach. Therefore, in our fixed for floating rate swap model in section A above, and the floating for fixed rate swap model analyzed by Cooper and Mello (1991), it was not necessary to use a game theoretic approach.

APPENDIX

In this appendix we examine the conditions on parameter values that ensure the convexity of payoff space. Convexity is ensured by the condition,

$$\frac{d^2W_A(\tilde{Y}_s^A)}{dW_B(F_s^B)^2} < 0$$

Taking the total derivative of equation (4) in the text, we get

(A-1) $$\frac{dW_A(\tilde{Y}_s^A)}{dW_B(F_s^B)} = \frac{dW_A(\tilde{Y}_s^A)}{d\tilde{Y}_s^A} \cdot \frac{d\tilde{Y}_s^A}{dF_s^B} \cdot \frac{dF_s^B}{dW_B(F_s^B)}$$

Hence:

(A-2) $$\frac{d^2W_A(\tilde{Y}_s^A)}{dW_B(F_s^B)^2} = \frac{d}{dW_B(F_s^B)}\left[\frac{dW_A(\tilde{Y}_s^A)}{d\tilde{Y}_s^A} \cdot \frac{d\tilde{Y}_s^A}{dF_s^B} \cdot \frac{dF_s^B}{dW_B(F_s^B)}\right]$$

which becomes

(A-3) $$\frac{d^2W_A(\tilde{Y}_s^A)}{dW_B(F_s^B)^2} = \frac{d^2W_A(\tilde{Y}_s^A)}{d\tilde{Y}_s^{A2}} \cdot \left(\frac{d\tilde{Y}_s^A}{dF_s^B}\right)^2 \left(\frac{dF_s^B}{dW_B(F_s^B)}\right)^2$$

$$+ \frac{dW_A(\tilde{Y}_s^A)}{d\tilde{Y}_s^A} \left\{ \frac{\partial^2\tilde{Y}_s^A}{\partial F_s^{B2}} + \frac{d\tilde{Y}_s^A}{dF_s^B} \cdot \frac{\partial}{\partial \tilde{Y}_s^A}\left(\frac{\partial \tilde{Y}_s^A}{\partial F_s^B}\right)\right\}\left(\frac{dF_s^B}{dW_B(F_s^B)}\right)^2$$

$$+ \frac{dW_A(\tilde{Y}_s^A)}{d\tilde{Y}_s^A} \cdot \frac{\partial \tilde{Y}_s^A}{\partial F_s^B} \cdot \frac{dF_s^B}{dW_B(F_s^B)} \cdot \frac{d}{dF_s^B}\left(\frac{dF_s^B}{dW_B(F_s^B)}\right)$$

We will analyze each of the three terms in (A-3) above separately.

Consider the <u>first term</u> in equation (A-3).

Since, $W_A(\tilde{Y}_s^A) = C(\tilde{V}_A, \tilde{Y}_s^A)$, using the formula for a call option with stochastic exercise price:

(A-4) $$\frac{\partial W_A(\tilde{Y}_s^A)}{\partial \tilde{Y}_s^A} = -R^{-1}N\left[\{\log\ (V_A/Y_s^A \cdot R^{-1})/\sigma_A\} - 1/2\sigma_A\right]$$

and

$$\text{(A-5)} \qquad \frac{\partial^2 W_A(\tilde{Y}_s^A)}{\partial \tilde{Y}_s^{A^2}} = -R^{-1}n \left[\{\log\ (V_A/Y_s^A \cdot R^{-1})/\sigma_A - 1/2\sigma_A \right] \cdot \frac{1}{\sigma_A} \cdot \frac{Y_s^A \cdot R^{-1}}{V_A} \cdot \frac{R^{-1}}{V_A}$$

This implies that the first term in (A-3) is negative.

Consider the <u>second term</u> in equation (A-3). It can be shown that

$$\text{(A-6)} \qquad \frac{\partial^2 \tilde{Y}_s^A}{\partial F_s^{B^2}} = \frac{d\tilde{Y}_s^A}{dF_s^B} \cdot \frac{\partial}{\partial F_s^B} \log[N(\gamma_1) + N(\gamma_2) - N_2(\gamma_1, \gamma_2, \rho_{Br})]$$

and that

$$\text{(A-7)} \qquad \frac{\partial}{\partial \tilde{Y}_s^A} \left[\frac{\partial \tilde{Y}_s^A}{\partial F_s^B}\right] = \frac{d\tilde{Y}_s^A}{dF_s^B} \cdot \frac{\partial}{\partial \tilde{Y}_s^A} \log[\Delta_1 + \{1 - N(\beta_1)\}]$$

Hence:

$$\text{(A-8)} \qquad \frac{\partial^2 \tilde{Y}_s^A}{\partial F_s^{B^2}} + \frac{\partial \tilde{Y}_s^A}{\partial F_s^B} \cdot \frac{\partial}{\partial \tilde{Y}_s^A} \left(\frac{\partial \tilde{Y}_s^A}{\partial F_s^B}\right) = \frac{d\tilde{Y}_s^A}{dF_s^B} \left[\frac{\partial}{\partial F_s^B}(K_1) + \frac{\partial \tilde{Y}_s^A}{\partial F_s^B} \cdot \frac{\partial}{\partial \tilde{Y}_s^A} \cdot (K_2)\right]$$

Where $K_1 = \log\ [N(\gamma_1) + N(\gamma_2) - N_2(\gamma_1,\ (\gamma_1,\ \gamma_2,\ \rho_{Br})]$

$K_2 = \log\ [\Delta_1 + \{1 - N\ (\beta_1)\}]$.

$$\Delta_1 = N_2(\alpha_1,\ \alpha_2,\ \rho_{Ar}) + N\left(\frac{\alpha_1 - \rho_{Ar}\alpha_2}{\sqrt{1 - e_{Ar}^2}}\right) e^{-\frac{1}{2}\alpha_2^2} \cdot \frac{1}{k^{\frac{1}{2}}}$$

$$- \left(\frac{\tilde{V}_A}{\tilde{Y}_s^A}\right) N\left(\frac{\beta_1 - \rho_{Ar}\beta_2}{\sqrt{1 - e_{Ar}^2}}\right) e^{-\frac{1}{2}\beta_2^2} \cdot \frac{1}{k^{\frac{1}{2}}} > 0 \text{ as shown in Stultz (1982)}$$

$$\gamma_1 = \{\log(V_B/F_s^B \cdot R^{-1})/\sigma_B\} \cdot 1/2\sigma_B$$

$$\gamma_2 = \{\log\ (Y_s^A/F_s^B \cdot R^{-1})/\sigma_r\} \cdot \tfrac{1}{2}\sigma_r$$

$$\alpha_1 = \gamma_1 + \sigma_r$$

$$\alpha_2 = \log\ (V_B/Y_s^A) - \tfrac{1}{2}\sigma^2$$

$$\sigma^2 = \sigma^2_B + \sigma^2_r - 2\,\rho_{Br}\,\sigma_B\,\sigma_r$$

$$\beta_1 = \gamma_2 + \sigma_B$$

$$\beta_2 = \log\,(Y^A_s/V_B) - \tfrac{1}{2}\sigma^2$$

$$k = \min\,(V_B,\ Y^A_s)$$

We will use the total derivative of equation (4) in text to obtain $\dfrac{d\tilde{Y}^A_s}{dF^B_s}$.

But first, we will use the results provided in Stulz (1982) to simplify it to the following:

$$\text{(A-9)} \qquad \bar{e}^{R}F^B_s - \bar{e}^{R}\tilde{Y}^B_m - \bar{e}^{R}F^B_s + MX(\tilde{V}_B,\ \tilde{Y}^B_m,\ 0) - MX(\tilde{V}_B,\ \tilde{Y}^B_m,\ F^B_s)$$
$$+ CM(\tilde{V}_A,\ \tilde{Y}^A_s,\ F^A_m) - P(\tilde{Y}^A_s,\ F^A_m) = 0$$

Further simplification yields:

$$\text{(A-10)} \qquad (1-\bar{e}^{R})\ \tilde{Y}^B_m + \tilde{V}_B - CM(\tilde{V}_B,\ \tilde{Y}^B_m,\ 0) - C(\tilde{V}_B,\ F^B_s) - C(\tilde{Y}^B_m,\ F^B_s)$$
$$+ CM(\tilde{V}_B,\ \tilde{Y}^B_m,\ F^B_s) + CM(\tilde{V}_A,\ \tilde{Y}^A_s,\ F^A_m) - P(\tilde{Y}^A_s,\ F^A_m) = 0$$

Taking the total derivative of this expression yields

$$\text{(A-11)} \qquad \frac{d\tilde{Y}^A_s}{dF^B_s} = \frac{-R^{-1}\ [N(\gamma_1) + N(\gamma_2) - N_2(\gamma_1,\gamma_2,\rho_{Br})]}{[\Delta_1 + \{1-N(\beta_1)\}]} < 0$$

where the last inequality follows from the fact that $\Delta_1 > 0$.

Hence, in equation (A-8) if

$$\text{(A-12)} \qquad \frac{\partial}{\partial F^B_s}(K_1) + \frac{\partial \dot{\tilde{Y}}^A_s}{\partial F^B_s} \cdot \frac{\partial}{\partial \tilde{Y}^A_s}(K_2) < 0$$

then the second term in equation (A-3) is negative.

Consider the <u>third term</u> in equation (A-3)

Since, $W_B(F_s^B) = C(V_A, F_s^B)$,

(A-13)
$$\frac{\partial W_B(F_s^B)}{\partial F_s^B} = -R^{-1}N\left[\{\log\ (V_B/F_s^B \cdot R^{-1})/\sigma_B\} - 1.2\sigma_B\right]$$

and

(A-14)
$$\frac{\partial}{\partial F_s^B}\left[\frac{\partial F_s^B}{\partial W_B(F_s^B)}\right] = R\,\frac{1}{\left[N\left[\{\log\ (V_B/F_s^B \cdot R^{-1})/\sigma_B\} - 1/2\sigma_B\right]\right]^{-2}}$$

$$\cdot\ n\left[\{\log(V_B/F_s^B \cdot R^{-1})/\sigma_B\} - 1/2\sigma_B\right]$$

$$\cdot\ \frac{1}{\sigma_B}\,\frac{F_s^B \cdot R^{-1}}{V_B} \cdot \frac{R^{-1}}{V_B}$$

The terms on the R.H.S. of (A-14) are clearly positive, and (A-13) are clearly negative. Thus, using (A-4) and (A-11) it is clear that the third term in equation (A-3) is always negative. The appendix above characterizes the conditions for the convexity of payoff space.

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TABLE 1

Fixed and Floating Rates

Firm	A	B
Cost of Fixed Rate Funds	10.80%	12.00%
Cost of Floating Rate Funds	8.25%	8.75%

TABLE 2

Interest Obligations of Firms

	Action	Rate for A	Rate for B
1.	Raise fixed rate debt in market	F_m^A	F_m^B
2.	Raise floating rate debt in market	$\tilde{Y}_m^A = \tilde{R} + \delta_m^A$	$\tilde{Y}_m^B = \tilde{R} + \delta_m^B$
3.	Raise floating rate in market and swap with Z as counterparty for fixed rate	F_Z^A	F_Z^B
4.	Raise fixed rate in market and swap with Z as counterparty for floating rate	$\tilde{Y}_Z^A = \tilde{R} + \delta_Z^A$	$\tilde{Y}_Z^B = \tilde{R} + \delta_Z^B$
5.	Raise floating rate in market and swap through Z as intermediary for fixed rate	F_S^A	F_S^B
6.	Raise fixed rate in market and swap through Z as intermediary for floating rate	$\tilde{Y}_S^A = \tilde{R} + \delta_S^A$	$\tilde{Y}_Z^B = \tilde{R} + \delta_S^B$

TABLE 3

Cash Flows to Swap Participants at the Swap Maturity

State	B_Z^A	E_Z^A	Z_Z^A	$C(V_A, \tilde{Y}_Z^A)$	$CM(V_A, \tilde{Y}_Z^A, F_m^A)$	$P(\tilde{Y}_Z^A, F_m^A)$
$V_A > F_m^A > \tilde{Y}_Z^A$	F_m^A	$V_A - \tilde{Y}_Z^A$	$-(F_m^A - \tilde{Y}_Z^A)$	$V_A - \tilde{Y}_Z^A$	0	$F_m^A - \tilde{Y}_Z^A$
$F_m^A > V_A > \tilde{Y}_Z^A$	F_m^A	$V_A - \tilde{Y}_Z^A$	$-(F_m^A - \tilde{Y}_Z^A)$	$V_A - \tilde{Y}_Z^A$	0	$F_m^A - \tilde{Y}_Z^A$
$F_m^A > \tilde{Y}_Z^A > V_A$	$V_A + F_m^A - \tilde{Y}_Z^A$	0	$-(F_m^A - \tilde{Y}_Z^A)$	0	0	$F_m^A - \tilde{Y}_Z^A$
$V_A > \tilde{Y}_Z^A > F_m^A$	F_m^A	$V_A - \tilde{Y}_Z^A$	$\tilde{Y}_Z^A - F_m^A$	$V_A - \tilde{Y}_Z^A$	$\tilde{Y}_Z - F_m^A$	0
$\tilde{Y}_Z^A > V_A > F_m^A$	F_m^A	0	$V_A - F_m^A$	0	$V_A - F_m^A$	0
$\tilde{Y}_Z^A > F_m^A > V_A$	V_A	0	0	0	0	0

FIGURE 1

A Swaps with Z

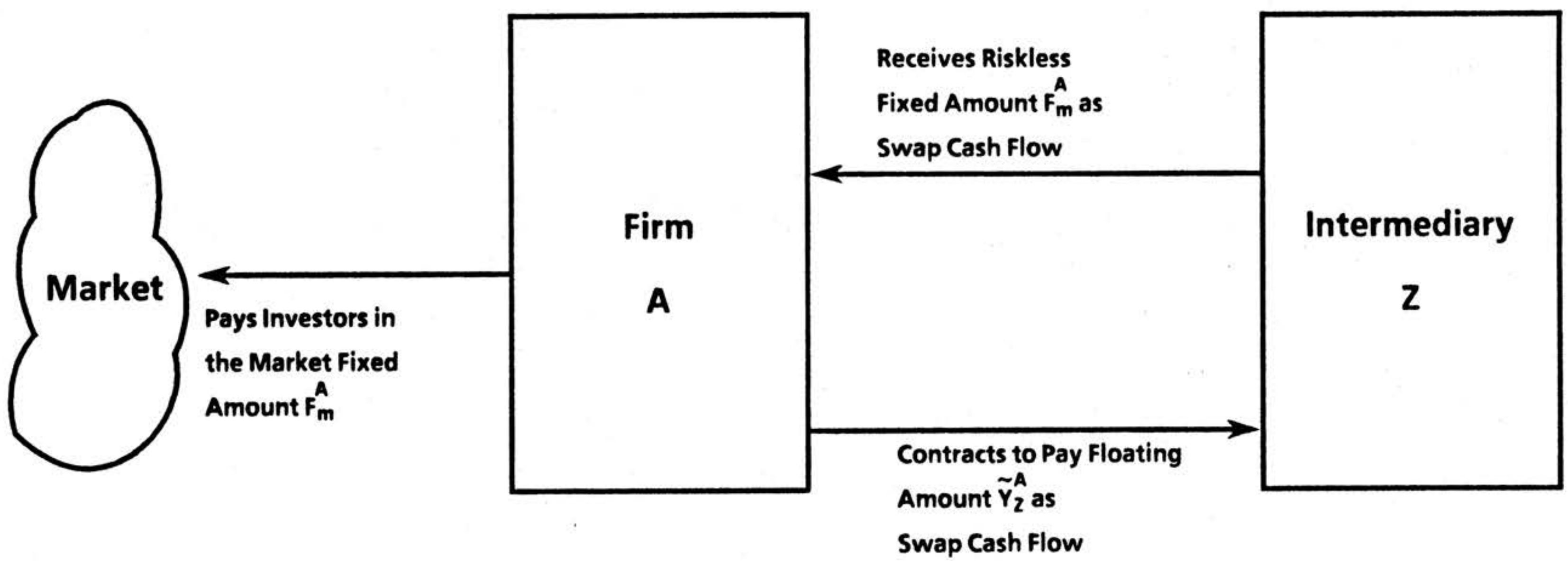


FIGURE 2

A Swaps with B through Z

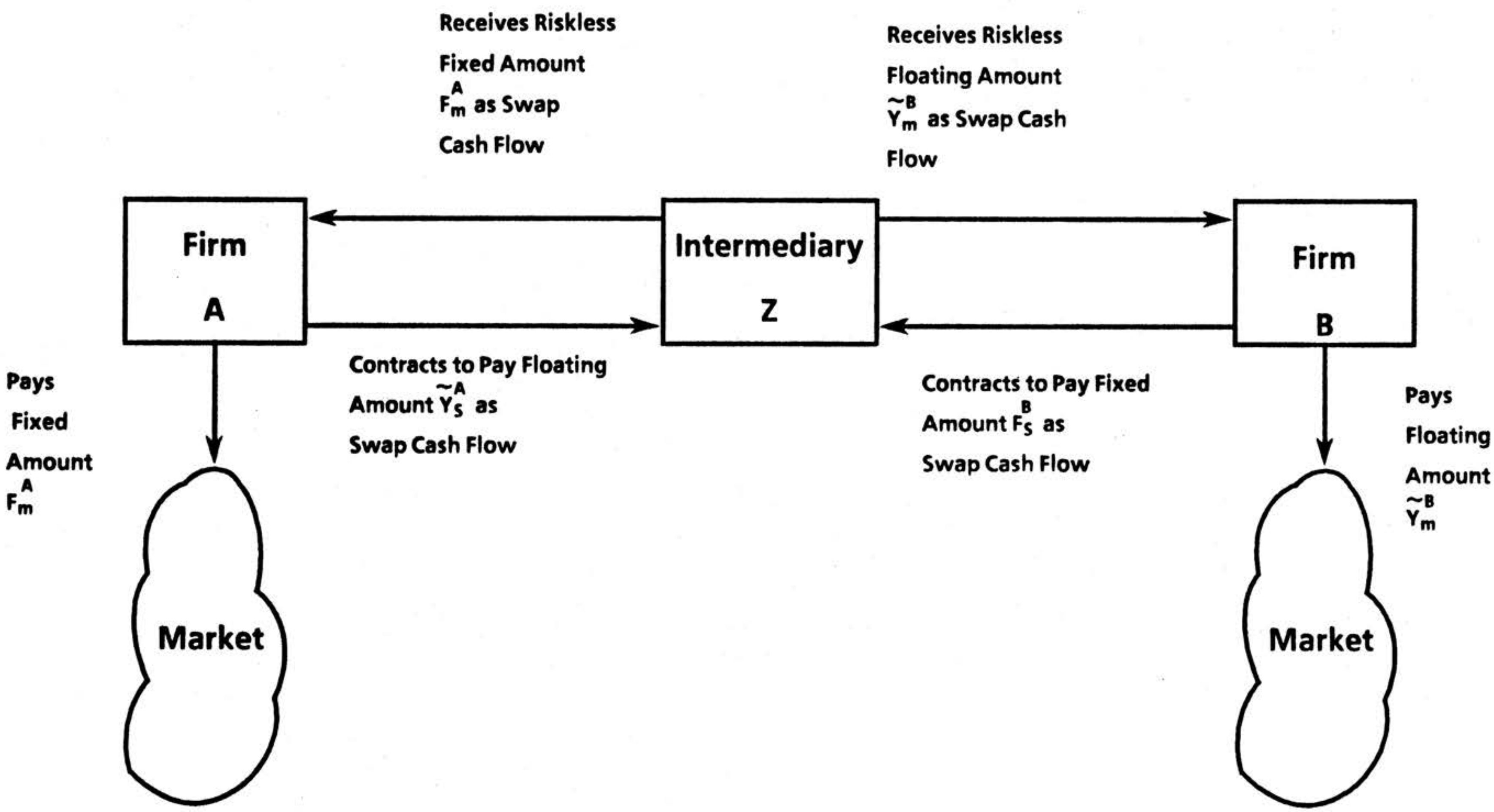


FIGURE 3

Feasible Swap Rates

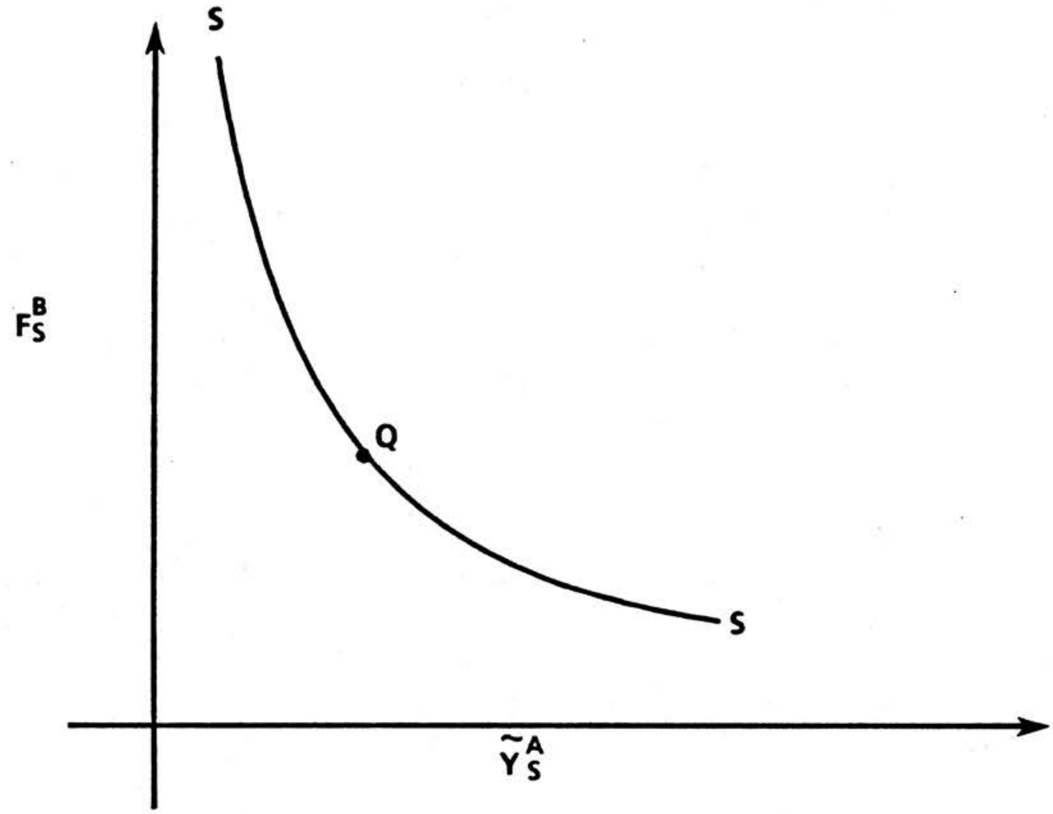


FIGURE 4

Feasible Wealth Levels

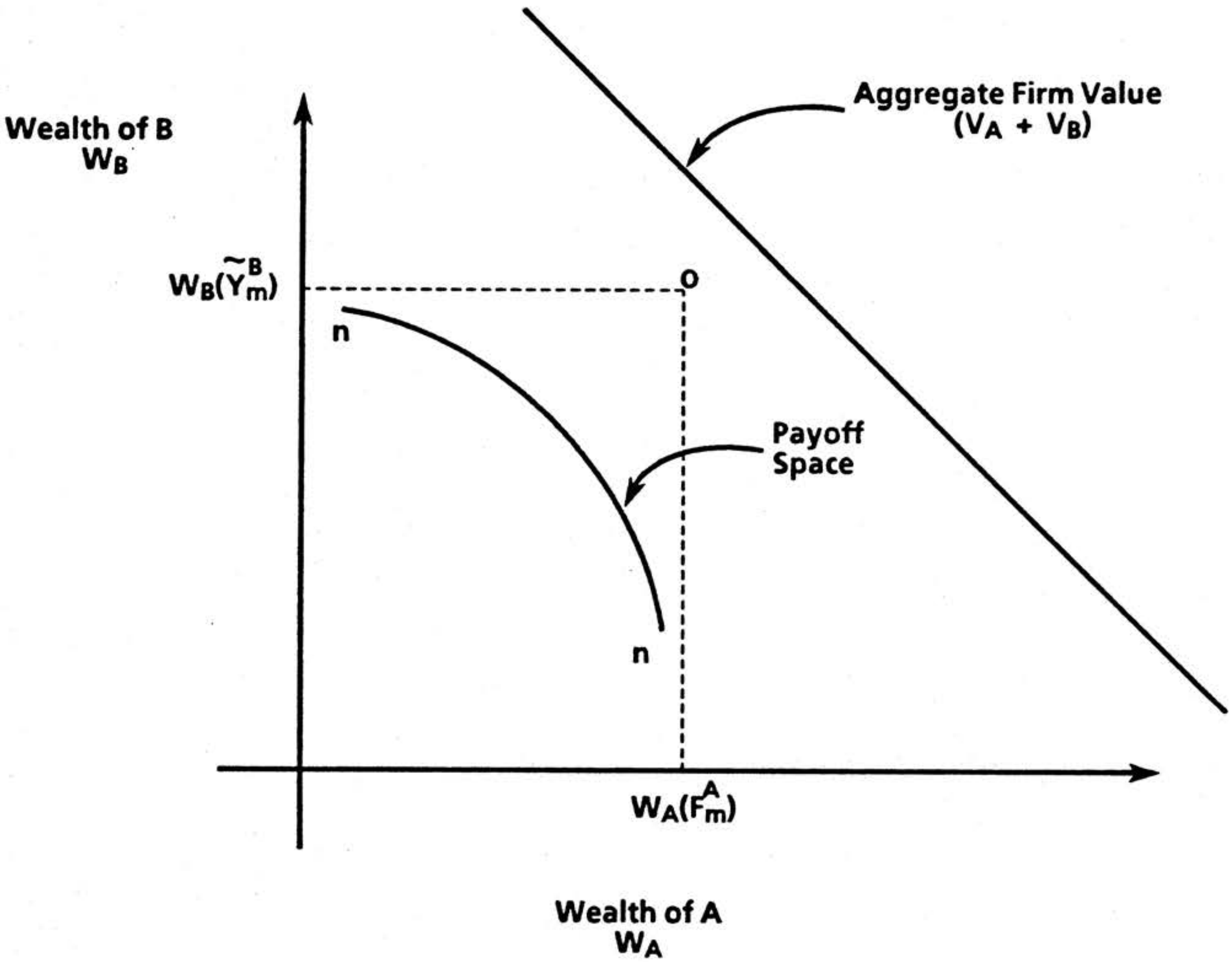
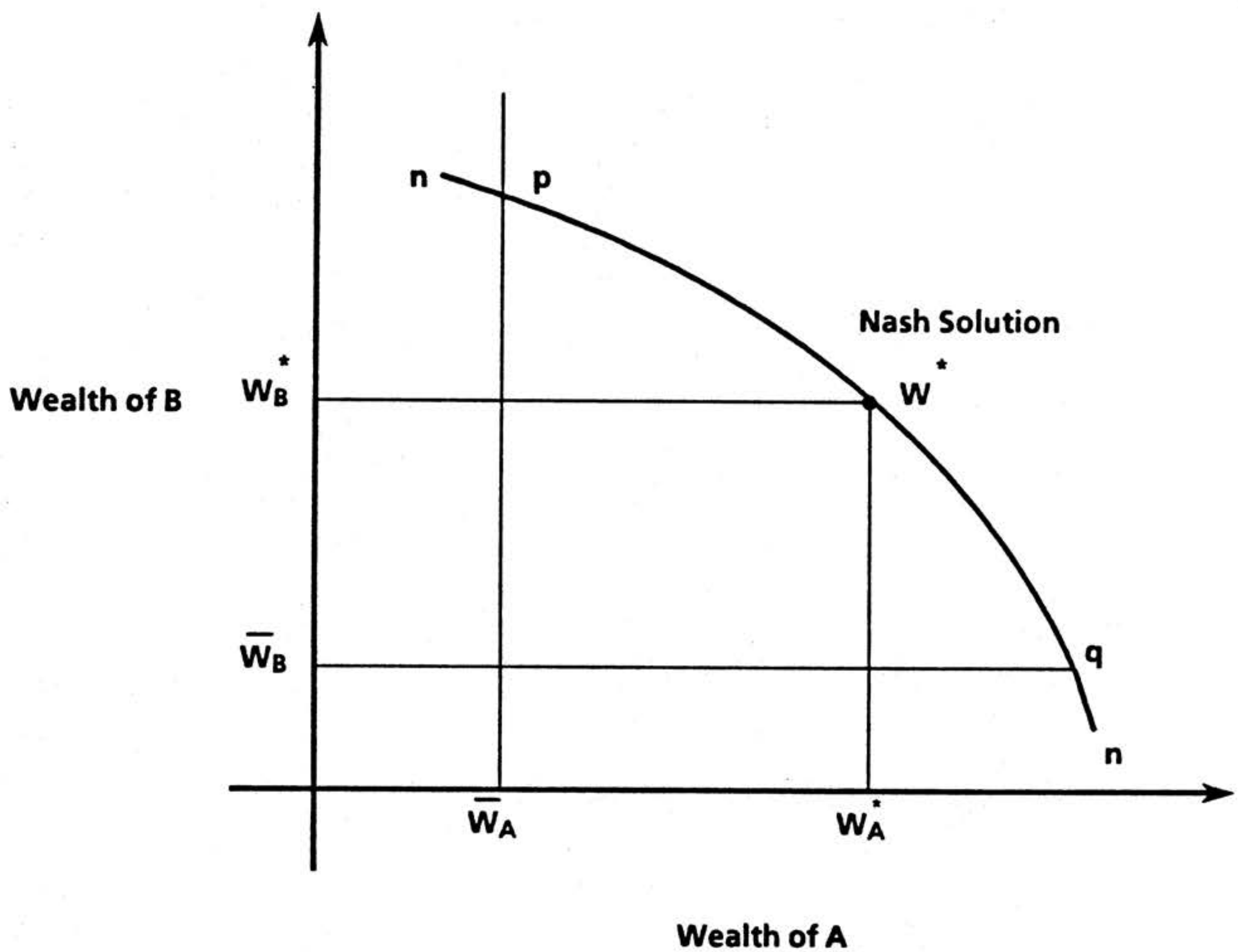


FIGURE 5

Bargaining Game



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