1-1-1993

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A MATHEMATICAL PROGRAMMING METHOD FOR GENERATING ALTERNATIVE MANAGERIAL PERFORMANCE GOALS AFTER DATA ENVELOPMENT ANALYSIS

Working Paper 93-0601*

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Abstract

Data Envelopment Analysis (DEA) is an application of linear programming which allows a comparison of Decision Making Units (DMU) which share the same operating charter but function in different environments. DEA allows the identification of a "peer group" so that an individual DMU is compared only with DMU's that operate in a similar environment. From the "best" in this peer group, an efficient frontier can be identified. In this paper, we discuss, via mathematical programming, a way of determining alternative possible courses of action for the manager of a DMU that has been deemed inefficient.

Keywords: Data envelopment analysis, linear programming, quadratic programming, efficiency analysis, managerial performance goals.

I. Introduction[1]

Data Envelopment Analysis (DEA) is a linear programming based technique which has gained wide acceptance as a means of measuring the efficiency of decision making units (DMU). Using empirical data consisting of vectors of the inputs and outputs of a group of similar DMUs, the method provides a piece-wise linear estimate of an empirical production or cost function. DMUs which are not on the estimated production function are deemed to be inefficient. A group of other DMUs, called the peer group, or reference set, is identified. The peer group forms the nearest facet of the estimated production function. A point is then identified which is a projection of the vector of inputs and outputs onto that facet. This new efficient point is then used as a target for the future performance of the DMU.

Since the original work by Charnes, Cooper and Rhodes [1978], several different DEA models have been developed and applied in a variety of settings (see Seiford [1989]). DEA has been used successfully, for example, to study the efficiency of hospitals (Banker, Conrad and Strauss [1982], Morey, Fine and Loree [1990]), courts (Lewin, Morey and Cook [1982]) and banks (Parkan [1987]).

While there have been numerous articles on the extensions of the original models and applications, relatively little attention has been given to the managerial reaction required when a DMU is deemed inefficient. The typical prescription is that a

[1]The authors would like to thank Professor Richard Morey for his helpful comments and suggestions.

proportional contraction of the input vector be made while holding the vector of outputs fixed, or alternatively, that a proportional expansion of the output vector be made while holding the vector of inputs fixed. The result of each analysis is a single performance target represented as a vector of inputs and outputs that is guaranteed to be efficient. Other than the works described below, research has generally not focused on finding alternative efficient performance targets that also satisfy the objectives particular to a specific DMU.

Golany [1988] proposes an interactive procedure for finding alternative efficient points for an inefficient DMU to use as performance targets. The maximum attainable level of each output is determined by solving a linear program for each output. This information is then used in a series of linear programs to construct a set of alternative efficient solutions, each one giving priority to a particular output, while the input vector is held constant. These solutions are then presented to the DMU for evaluation. If a satisfactory solution is found, the procedure stops. Otherwise the preferences of the DMU are incorporated into a new set of alternatives by resolving a new set of linear programs with revised lower bounds on the outputs, and the evaluation process is repeated.

Charnes, et. al. [1992] discussed sensitivity analysis in the context of the additive model of DEA (Charnes, et. al. [1985]). Linear programming formulations were presented which found a "region of stability," a symmetrical region within which a DMU's current classification of efficient or inefficient will not change. Two formulations were suggested, one using the ∞-norm and one using the 1-norm, the latter being suggested when one desires to evaluate the sensitivity of a DMU's classification relative to a perturbation of a subset of the input-output vector. While the method does not provide for the explicit construction of performance objectives, the "radius of stability" it identifies is minimum change required for an inefficient DMU to become virtually efficient.

In this paper we discuss possible courses of action for the inefficient DMU. In particular, we use a mathematical programming approach to obtain alternative efficient input-output vectors. These vectors are feasible and consistent with the goals of the inefficient DMU, and may be considered for use as managerial performance objectives. Our discussion focuses on the DMU's possible courses of action after being scored inefficient by a study using the model of Banker, Charnes and Cooper [1984]. However, the general concepts which follow are equally applicable to other DEA models.

II. The BCC Model

Banker, Charnes and Cooper (BCC) [1984] extended the original model of Charnes, Cooper and Rhodes to include the concept of economic returns to scale. The primal formulation of the BCC model is

$$\text{MAX } h_k = \sum_{r=1}^{s} \mu_r Y_{rk} - u_k \tag{1}$$

$$\text{s.t.} \quad \sum_{i=1}^{m} \upsilon_i x_{ik} = 1 \tag{2}$$

$$\sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} \upsilon_i x_{ij} - u_k \leq 0 \qquad j = 1,2,...n \tag{3}$$

The term u_k was interpreted by Banker, Charnes and Cooper as an indicator of returns to scale ($u_k<0$ implies increasing returns to scale. etc.). The variables υ_i are weights applied to the sum of the inputs x_{ik} of DMU_k, which are constrained to be unity in (2). The μ_r are weights on the outputs y_{rj} of the other DMU's in the analysis. Thus, (3) can be seen as a linearization of the fractional constraint

$$\frac{\sum_{r=1}^{s} \mu_r y_{rj}}{\sum_{i=1}^{m} \upsilon i x_{ij} - u_k} \leq 1 \tag{4}$$

which is the "Engineering ratio" of efficiency discussed in Charnes, Cooper and Rhodes [1987] extended to include the scale efficiency term. Our interest is in the dual of this linear program:

$$\text{MIN } h_k = \Theta_k - \varepsilon \left(\sum_{r=1}^{s} S_r^+ + \sum_{i=1}^{m} S_i^-\right) \tag{5}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} y_{rj}\lambda_j - S_r^+ = y_{rk} \qquad r=1,2,...s \tag{6}$$

$$\sum_{j=1}^{n} x_{ij}\lambda_j - \Theta_k x_{ik} + S_i^- = 0 \quad i=1,2,..m \tag{7}$$

$$\sum_{j=1}^{n} \lambda_j = 1 \tag{8}$$

$$\lambda_j \geq 0 \qquad j=1,2,...n \tag{9}$$

The most significant difference between the above problem (BCC) and the earlier CCR formulation is the addition of (8). The $\lambda_j>0$ are the multipliers which allow identification of the peer group. ε is a small non-Archimedean constant. By requiring the λ_j to sum to one, the linear program creates a composite DMU from a convex combination of the vectors of inputs and outputs of the DMU's in the envelopment. This composite DMU will possess a vector of inputs and outputs as good or better than the vector of inputs and outputs of DMU_k (i.e., fewer inputs and the same or more output). Since (6) requires the sum of the outputs y_{rj} over j=1,2,...,m DMU's to be at least equal to y_{rk}, the level of output r of DMU_k, (7) in turn provides a multiplier Θ_k which is a measure of efficiency of DMU_k relative to its peer group. We can define the peer group for DMU_k as the set:

$$P_k = \{ \ j \ | \ \lambda_j > 0 \ \} \tag{10}$$

If DMU_k is efficient, then $h_k = 1$, all S_r^+ and $S_i^- = 0$ with $\Theta_k = 1$. If $0<\Theta_k<1$, then DMU_k is inefficient and $\mathbf{X} = \Theta_k\mathbf{X_k} - \mathbf{S^{-*}}$ and $\mathbf{Y} = \mathbf{Y_k} + \mathbf{S^{+*}}$ represent the projection of DMU_k on to the efficient frontier. Set P_k describes a facet of a piece-wise linear frontier; and therefore, the set of DMU_j where x_j, $j \ \varepsilon \ P_k$ are the points of nondifferentiability on the frontier.

<u>Proposition 1</u>:

Suppose DMU_k is a member of some DMU's peer group; that is, $k \ \varepsilon \ P_t$ for some t. DMU_k must be efficient.

<u>Proof</u>:

Consider DMU_o's evaluation:

$$\Pi_o:\ h_o = \text{Min } \Theta_o - \varepsilon \sum_{i=1}^{m} S_i^- + \sum_{r=1}^{s} S_r^+ \tag{11}$$

s.t.

$$\sum_{j=1}^{n} x_{ij}\lambda_j + S_i^- = \Theta_o x_{io} \quad i=1,2,\ldots m \tag{12}$$

$$\sum_{j=1}^{n} y_{rj}\,\lambda_j - S_r^+ = y_{ro} \quad r=1,2,\ldots s \tag{13}$$

$$\sum_{j=1}^{n} \lambda_j = 1 \tag{14}$$

$$\Theta_o,\ \lambda_j,\ S_i^+,\ S_r^- \geq 0 \qquad \begin{array}{l} i=1,2\ldots.m \\ j=1,2,\ldots n \\ r=1,2,\ldots s \end{array} \tag{15}$$

Suppose DMU_k is a member of DMU_o's peer group (i.e., $k \varepsilon P_o$) and that DMU_k is not efficient. DMU_k's evaluation is given by:

$$\Pi_k:\ h_k = \text{Min } \Theta_k - \varepsilon \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ \tag{16}$$

s.t.

$$\sum_{j=1}^{n} x_{ij}\ \gamma_j + s_i^- = \Theta_k x_{ik} \quad i=1,\ldots m \tag{17}$$

$$\sum_{j=1}^{n} y_{rj}\ \gamma_j - s_r^+ = y_{rk} \quad r=1,\ldots s \tag{18}$$

$$\sum_{j=1}^{n} \gamma_j = 1 \tag{19}$$

$$\Theta_k,\ \gamma_j,\ s_i^+,\ s_r^- \geq 0 \qquad \begin{array}{l} i=1,2,..m \\ j=1,2,..n \\ r=1,2,..s \end{array} \tag{20}$$

Then, $h_k^* < 1$ and either $\Theta_k^* < 1$ or some s_i^{+*} or s_i^{-*} is strictly positive (or both). Therefore, at optimality, we know from (17) - (19) that:

$$\sum_{j\varepsilon P_k} x_{ij}\ \gamma_j^* \leq x_{ik} \qquad i=1,2,..m \tag{21}$$

$$\sum_{j\varepsilon P_k} y_{rj}\ \gamma_j^* \geq y_{rk} \qquad r=1,2,..s \tag{22}$$

$$\sum_{j\varepsilon P_k} \gamma_j^* = 1 \tag{23}$$

Also, we know that strict inequality holds for at least one of the constraints (21) - (22). Now, consider the optimal solution to Π_o, in which λ_k^* is basic. From (12) and (21) we know,

$$\sum_{\substack{j \in P_o \\ j \neq k}} x_{ij} \lambda_j^* + (\sum_{j \in P_k} x_{ij}\gamma_j^*)\gamma_k^* \leq$$

$$\sum_{\substack{j \in P_o \\ j \neq k}} x_{ij} \lambda_j^* + x_{ik}\lambda_k^* + S_i^{-*} = \Theta_o^* x_{io} \quad i=1,2,..m \tag{24}$$

and from (13) and (22),

$$\sum_{\substack{j \in P_o \\ j \neq k}} y_{rj} \lambda_j^* + (\sum_{j \in P_k} y_{rj}\gamma_j^*)\lambda_k^* \geq$$

$$\sum_{\substack{j \in P_o \\ j \neq k}} y_{rj} \lambda_j + y_{rk}\lambda_k^* - S_i^{+*} = y_{ro} \qquad r=1,2,..s \tag{25}$$

and from (14) and (19):

$$\sum_{\substack{j \in P_o \\ j \neq k}} \lambda_j^* + (\sum_{j \in P_k} \lambda_j^*)\lambda_k^* = 1 \tag{26}$$

Results (24) - (26) provide a solution to Π_o that is feasible, and since at least one of (21) - (22) will be a strict inequality, so will at least one of (24) - (25). Therefore, we have constructed a solution which is "better" than the optimal solution to Π_o. Therefore, by contradiction DMU_k must be efficient. ■

If some DMU, say DMU_o, is inefficient, then the linear program used to evaluate it will find a convex combination of the vectors of inputs of some group defined by the set P_o whose sum is equal to $\Theta^*_o \mathbf{X}_o$ - $\mathbf{S}^{-*}$ with $\mathbf{X}_o$ the vector of inputs for DMU_o.

Proposition 2:

Suppose DMU_o is inefficient, with peer group $\{(X_jY_j)\ j\varepsilon P_o\}$. Any DMU constructed as a convex combination of these peer group members must be efficient.

Proof:

Let DMU_c be a convex combination of DMU_j, $j\varepsilon P_o$, that is

$$x_{ic} = \sum_{j\varepsilon P_o} w_j x_{ij} \qquad i=1,2,\ldots m \tag{27}$$

$$y_{rc} = \sum_{j\varepsilon P_o} w_j y_{rj} \qquad r=1,2,\ldots s \tag{28}$$

$$\sum_{j\varepsilon P_o} w_j = 1 \tag{29}$$

$$w_j \geq 0 \qquad j\varepsilon P_o \tag{30}$$

DMU_c's evaluation is given by:

$$\Pi_c:\ h_c = \text{Min } \Theta_c + \varepsilon \sum_{i=1}^{m} v_i^- + \sum_{r=1}^{s} v_r^+ \tag{31}$$

s.t.

$$\sum_{j=1}^{n} x_{ij}\gamma_j + v_i^- = \Theta_c x_{ic} \qquad i=1,2,\ldots m \tag{32}$$

$$\sum_{j=1}^{n} y_{rj}\gamma_j - v_r^+ = y_{rc} \qquad r=1,2,\ldots s \tag{33}$$

$$\sum_{j=1}^{n} \gamma_j = 1 \tag{34}$$

$$\Theta_c,\ \gamma_j,\ v_r^+,\ v_i^- \geq 0 \quad \begin{array}{l} i=1,2,\ldots m \\ j=1,2,\ldots n \\ r=1,2,\ldots s \end{array} \tag{35}$$

Suppose DMU_c is not efficient. Then for γ_j^*,

$$\sum_{j\varepsilon P_c} x_{ij}\ \gamma_j^* \leq x_{ic} \qquad i=1,2,\ldots m \tag{36}$$

$$\sum_{j\varepsilon P_c} y_{rj}\ \gamma_j^* \geq y_{rc} \qquad r=1,2,\ldots s \tag{37}$$

$$\sum_{j\varepsilon P_c} \gamma_j^* = 1 \tag{38}$$

where P_c is the index set of peer group members.

We can write the solution to Π_o as a function of peer group members in P_o and DMU_c as follows:

$$\sum_{j\varepsilon P_o} \lambda_j^*(X_j, Y_j) = \sum_{j\varepsilon P_o}(X_j, Y_j)\,\delta_j + ß(X_c, Y_c)$$

$$= \sum_{j\varepsilon P_o}(X_j, Y_j)\,\delta_j + ß \sum_{j\varepsilon P_o}(X_j, Y_j)\,w_j$$

$$= \sum_{j\varepsilon P_o}(\delta_j + ß\ w_j)\ (X_j Y_j) \tag{39}$$

by letting

$$ß = \underset{j}{\text{Min}}\ \frac{\lambda_j^*}{w_j} \qquad \text{for } w_j \neq 0.$$

then $\qquad \delta_j = \lambda_j^* - ßw_j$

(It can be shown that $ß < 1$.) Also,

$$ß + \sum_{j\varepsilon P_o} \delta_j = ß + \sum_{j\varepsilon P_o} \lambda_j^* - ß \sum_{j\varepsilon P_o} (w_j) = \sum_{j\varepsilon P_o} \lambda_j^* = 1. \tag{40}$$

Hence from (36) and (12)

$$\sum_{j\varepsilon P_o} x_{ij}\ \delta_j + ß \sum_{j\varepsilon P_c} x_{ij}\ \gamma_j^* + S_i^{-*} \leq$$

$$\sum_{j\varepsilon P_o} x_{ij}\ \delta_j + ßx_{ic} + S_i^{-*} = \Theta_o^* x_{io} \quad i=1,2,\ldots m \tag{41}$$

Also, from (37) and (13)

$$\sum_{j\varepsilon P_o} y_{rj}\ \delta_j + ß \sum_{j\varepsilon P_o} y_{rc}\ \gamma_j^* - S_r^{+*} \geq$$

$$\sum_{j\varepsilon P_o} y_{rj}\ \delta_j + ß\ y_{rc} - S_r^{+*} = y_{ro} \qquad r=1,2,..s \tag{42}$$

Results (40) - (42) provide a solution Π_o that is feasible and since at least one of (36) - (37) is a strict inequality so will at least one of (41) - (42). Therefore, we have constructed a solution "better" than the optimal solution to Π_o. Therefore, by contradiction, DMU_c must be efficient. ■

Propositions 1 and 2 will be useful in the discussion of managerial reaction to inefficiency. In particular, these propositions suggest that in addition to the radial contraction prescribed by the BCC model, peer group members and convex combinations of these members might also be worthy operational targets for consideration.

III. Reaction to Inefficiency

Consider the manager of a DMU that has been scored inefficient in a study using the BCC model as previously described in (11) - (15). The solution yields $\Theta_o^* < 1$ which indicates the fraction of the current level of inputs to which the manager should be able to reduce and still maintain current output levels. Reducing each input by the fraction $(1-\Theta_o^*)$ maintains the current mix of inputs as prescribed in microeconomic theory. Operationally, of course, the flexibility in the manager's real environment might be greater or less than that allowed by radial contraction. Indeed, many inputs might be more easily controlled for one manager than another so that the concept of separating controllable and non-controllable inputs on a system-wide basis proposed by Banker and Morey [1986], might be applicable to a particular DMU. If the manager is being evaluated using the DEA techniques, then his or her goal must be to become efficient as easily and cost effectively as possible within that DMU's operational constraints.

If input prices are available to the manager, reaching the frontier in the most cost effective manner might be the goal. There is no reason why the most cost effective achievable efficient point should correspond to the point obtained by radial contraction. One could argue that if input prices are available, allocative DEA (see Morey, Fine and Loree [1990]) should be used, but it is certainly possible that the BCC model was used because of inconsistences in prices or price availability over all DMU's in the system.

Mathematically, one approach would be to find the closest (in a cost sense) achievable efficient point from those efficient points described by Propositions 1 and 2. If these costs or prices are not available, another approach would perhaps be to find the closest efficient point in the sense of Euclidean distance, minimizing the total amount of change in inputs required. The idea is shown graphically in Figure 1. If point E meets the output requirements, it might serve as a better target for DMU than point D. For both scenarios, the objective is to find the "best" peer group member or convex combination of peer group members relative to the DMU's goal which is within the operational constraints of DMU_o. Furthermore, since the BCC model is based on the premise that maintaining output levels is desirable, it is prudent to assume that this newly constructed target should also maintain the previous output levels. In general, the mathematical program to be used to prescribe the optimal course of action for inefficient DMU_o can be written

$$\text{Max or Min} \quad f(\hat{x}_i, \hat{y}_r) \tag{43}$$

s.t.

$$\sum_{j \varepsilon P_o} \psi_j \, x_{ij} = \hat{x}_i \qquad i=1,2,\ldots m \tag{44}$$

$$\sum_{j \varepsilon P_o} \psi_j \, y_{rj} = \hat{y}_r \qquad r=1,2,..s \tag{45}$$

$$\hat{x}_i \leq x_{io} \qquad i=1,2,..m \tag{46}$$

$$\hat{y}_r \geq y_{ro} \qquad r=1,2,\ldots s \tag{47}$$

$$\sum_{j \varepsilon P_o} \psi_j = 1 \tag{48}$$

$$\psi_j, \; \hat{x}_i, \; \hat{y}_r \geq 0 \qquad \begin{array}{l} i=1,2,\ldots m \\ j=1,2,..n \\ r=1,2,\ldots s. \end{array} \tag{49}$$

In addition, it may be necessary to append constraints of the form

$$(\hat{\mathbf{x}}, \hat{\mathbf{y}}) \in \Omega \tag{50}$$

where Ω describes the restrictions and limitations of the operating environment unique to DMU_o.

By Propositions 1 and 2 any DMU constructed by (43) - (48) will be efficient. Also, by Proposition 2, even if (46) and (47) are not imposed; that is, even if we relax the assumption of fewer or the same levels of inputs and maintaining levels of output, an efficient DMU will result.

If the objective is minimizing the Euclidean distance to the peer group facet, (43) is replaced with

$$\text{Min} \sqrt{\sum_{i=1}^{m} (\hat{x}_i - x_{io})^2 + \sum_{r=1}^{s} (\hat{y}_r - y_{ro})^2} \qquad (51)$$

This objective is equivalent to minimizing the sum of squares; that is, we can ignore the square root. If the operational constraints are linear, we have an easily solved quadratic program. Any linear cost minimizing or profit maximizing objective with linear operational constraints leads to an easily solved linear program.

Finally, note that the operational constraints appended to the mathematical program (43) - (49) may render the program infeasible. Since the solution to Π_o is feasible to (43) - (49), an infeasible program indicates that even the constructed DMU from the BCC model is not achievable when the DMU's real operational constraints are considered.

IV. An Example

Bessent, et. al. [1983] used DEA to study the efficiency of occupational-technical programs in a comprehensive community college. Each DMU had "an administrative head responsible for supervising the teaching staff, curriculum, and expenditures" (p.88). It is worth noting that since the study was done in 1983, the BCC formulation was not available and the original CCR formulation was used. The four inputs used in the analysis were:

1. Student contact hours generated by each program (lecture and laboratory hours for one course per week times the number of students times the number of weeks of instruction).

2. Number of full-time equivalent instructors in each program.

3. Facilities allocated as determined by square feet assigned to each program for classroom, office and laboratory use.

4. Direct instructional expenditures in each program.

The three outputs were:

1. Revenue earned by contact hours through state funding formulas.

2. The number of students completing the program who are employed directly in their field of training.

3. Employer satisfaction with occupational training of students employed using a 25-point scale.

The data from their analysis and the BCC efficiency scores are listed in Table 1.

<< Table 1 about here >>

DMU 1 was an occupational training program in advertising art. With an efficiency score of $\Theta = 0.6694$ and with slacks $S_2^+ = 0.504$, $S_1^- = 0.368$, and $S_3^- = 1.260$, the DMU shows both technical and scale inefficiency. The radial contraction suggested by Banker, Charnes and Cooper [1984] requires that all four inputs be reduced to slightly less than 67% of their current level, and that student contact hours and facility allocation be reduced by the additional amount of the slack variables, while simultaneously increasing the number of students employed by the amount of the surplus variable. A thoughtful examination of the potential difficulties DMU 1 will face in making this adjustment suggests that alternative courses of action may be more desirable. For example, reducing the number of FTE staff may be difficult if some or all of the staff are tenured faculty. Similarly, a reduction in facilities allocation may be dependent upon the availability of alternative space. Partitioning off a fraction of the existing space may reduce the amount of allocated space, but it is not a sensible approach to the problem. Both of the inputs mentioned are examples of inputs that, while not strictly uncontrollable, may be more difficult to reduce than others.

Alternative efficient points were found using (43)-(49), with the following objective function:

$$\text{Min } f(\hat{x},\hat{y}) = \sum_{i=1}^{m} (\hat{x}_i - x_{io})^2 + \sum_{r=1}^{s} (\hat{y}_r - y_{ro})^2 \tag{52}$$

which is the squared $\|\cdot\|_2$ for the input-output vector. In other words, we seek to find the shortest Euclidean distance (ignoring the square root) to the efficient frontier. Since the objective of this research is to provide a method for generating alternative performance targets for an inefficient DMU, the determination of a "best" or optimal alternative is a subjective one to be made by the DMU. Therefore, minimizing $\|\cdot\|_2$ is only one of many possible objectives for which plausible arguments can be made. The minimization of $\|\cdot\|_1$ or the maximization of $\|\cdot\|_\infty$ are some alternatives. The $\|\cdot\|_2$ is appealing because, if the constraints are linear, the model is easily solved.

Four different scenarios were used to explore four alternatives. The results are summarized in Table 2. The first alternative is the result of solving (51) and (44)-(49). Notice that for three of the four inputs, smaller reductions in the input vector are required. However, in order to enjoy this benefit, all three outputs must be increased. Notice that f^*, the squared norm for this alternative, is substantially less than that produced by the radial contraction. Perhaps the most attractive feature of this alternative might be that the DMU is allowed to retain a substantially higher level of expenditures.

<< Table 2 about here >>

Now suppose that a reduction of FTE staff below three is highly undesirable because of the problem of reassigning tenured faculty. Operational constraints of the form:

$$\Omega_1 = \{\hat{x}_2 \geq 3.0\} \tag{52}$$

are appended the problem and the model is resolved. The new alternative maintains the FTE staff at the minimum level of three, while other levels of inputs and outputs are adjusted, most notably that a higher level of revenue generated is now required. Again the squared norm of $f^* = 25.0739$ is less than that suggested by the radial contraction.

We take the analysis a step further by supposing that any staff reduction is highly undesirable. The first set of operational constraints is replaced with:

$$\Omega_2 = \{\hat{x}_2 \geq 4.0\} \tag{53}$$

and the model is again resolved. The new alternative preserves the staffing level, but at a great cost. While the DMU is allowed to keep its staff and expenditure level, this must be bought by more than more than doubling both the number of students employed and the level of customer satisfaction. Thus the DMU can explore trade offs among alternatives. Here the suggestion is that the current staffing level is too high and that it may not be maintained without dramatic increases in productivity.

A final alternative to be explored was the case where the DMU might be reluctant to alter or relocate from the current facilities. The operational constraints:

$$\hat{\Omega}_3 = \{x_3 \geq 4.02\} \tag{54}$$

replaced Ω_2 and the model was found to be infeasible. Thus the DMU finds that there is no efficient point on the facet defined by its peer group which will allow it to keep its facilities in their current form. This suggests that the advertising art occupational educational-training program under evaluation may have to find alternative facilities, or that the use of facility allocation as an input may not capture the peculiar space and equipment requirements of the individual programs.

We have illustrated the flexibility of our approach by proposing several different plausible scenarios in the context of a real problem. Other approaches could be used to generate an even wider variety of alternative efficient points. For example, one could apply the model with a linear objective of minimizing the weighted sums of inputs required (or a combination of them with changes in outputs required) and apply parametric programming to the objective function to yield a variety of solutions. The solutions would show courses of action as the relative importance of inputs varies.

V. Conclusion

The DEA literature contains very little discussion of how a DMU that has been found to be inefficient can take corrective action using information obtained from the DEA study. The various DEA models prescribe one course of action via the constructed DMU used for comparison. Our point is that DMU's need not limit the action to that prescribed by the particular model used. A particular DMU might have more or less control over certain inputs and outputs than the evaluation study assumes. Indeed, it might even be the case that certain inputs really are completely controllable (or not) for a particular DMU, when, because of the consensus of the majority of the DMU's, it was declared uncontrollable for the evaluation study.

We have described a set of valid targets for one particular DEA model. Propositions 1 and 2 provide the information needed for a mathematical programming approach to the inefficient DEA's problem of achieving efficiency. Through a mathematical program which restricts the feasible set to a particular facet (any peer group), any other restrictions, relaxations, or goals of that particular DMU can be incorporated to determine a course of action. Unlike other methods, our approach allows for the simultaneous adjustment of both inputs and outputs. This allows for greater flexibility in choosing alternatives, as well as providing some insight into the trade offs in the input-output vector along the efficient frontier. Not only is this more useful, but it also provides a more realistic environment for the decision maker.

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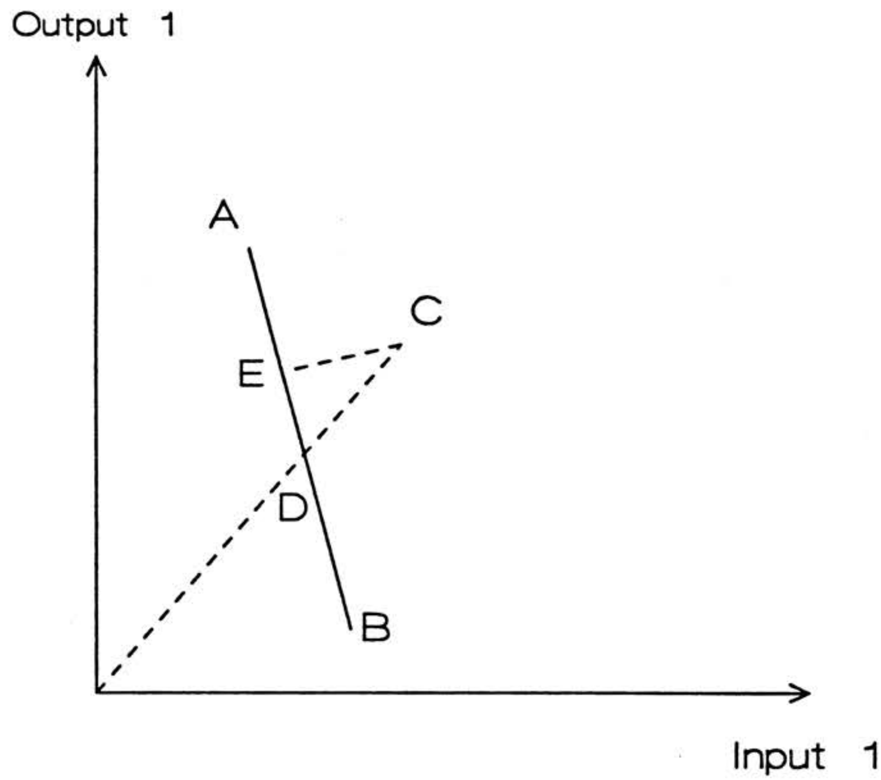

Figure 1 Alternative Efficient Points

Table 1
Data from Bessent, et. al. [1983]

DMU	BCC Efficiency	Peer Group	Revenue Generated ($10,000)	Students Employed	Employer Satisfaction	Contact Hours (10,000)	Number of FTE Staff	Facility Allocation (1,000 ft^2)	Expenditures ($10,000)
1	0.6694	4,9,13,19	12.5	15	2.85	6.65	4.0	4.02	11.02
2	1.0000	--	100.6	180	34.20	48.88	32.0	9.52	70.07
3	0.9856	4,9,14,22	28.2	52	10.56	10.28	13.5	4.02	19.13
4	1.0000	--	158.5	114	23.26	25.93	25.0	4.44	93.36
5	0.9350	4,9,13,18,19	10.1	16	3.52	3.62	3.1	1.75	7.30
6	0.6600	4,11	32.4	28	5.82	14.06	11.0	8.31	27.62
7	0.8307	4,9,11,22	30.1	40	8.80	12.21	8.1	8.40	22.23
8	1.0000	--	5.2	11	1.90	2.03	2.5	0.56	4.44
9	1.0000	--	4.3	35	8.75	3.51	3.0	0.15	6.00
10	0.8378	4,11,22	26.2	29	6.38	15.76	10.1	3.44	17.55
11	1.0000	--	6.0	12	2.64	3.83	2.5	0.13	2.50
12	1.0000	--	1.1	3	0.45	0.64	3.0	0.31	2.27
13	1.0000	--	4.1	12	2.64	1.99	1.1	1.65	4.63
14	1.0000	--	39.5	89	20.11	25.52	16.5	0.43	25.67
15	1.0000	--	2.5	8	1.68	1.07	2.0	1.12	4.82
16	1.0000	--	72.7	71	12.71	23.55	4.5	12.19	82.44
17	1.0000	--	13.6	51	12.75	11.99	7.2	0.77	18.28
18	1.0000	--	4.5	0	0.00	1.31	3.0	0.77	2.48
19	1.0000	--	2.6	7	0.91	2.66	1.1	1.25	1.07
20	0.6436	4,8,13,18,19	5.2	3	0.54	2.81	4.0	1.25	6.06
21	1.0000	--	6.3	22	3.75	2.49	2.5	9.60	9.08
22	1.0000	--	7.2	27	5.40	5.18	4.0	0.10	3.65

Table 2
Alternative Performance Targets for DMU 1

Inputs	Radial Contraction	$\lVert\cdot\rVert_2$	$\lVert\cdot\rVert_2$ Ω_1	$\lVert\cdot\rVert_2$ Ω_2	$\lVert\cdot\rVert_2$ Ω_3
1	4.083	4.010	4.453	4.965	---
2	2.678	2.732	3.000	4.000	
3	1.431	1.610	1.528	0.935	
4	7.378	8.637	8.671	11.02	
Outputs					
1	12.500	13.776	15.058	15.970	---
2	15.504	16.083	15.987	30.094	
3	2.850	3.051	2.850	6.767	
Weights					
ψ_4	0.062	0.068	0.079	0.080	---
ψ_9	0.050	0.000	0.005	0.519	
ψ_{13}	0.094	0.356	0.077	0.000	
ψ_{19}	0.794	0.576	0.839	0.401	
f^*	28.310	22.908	25.0739	267.571	*Inf.*

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