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Abstract

A critical outcome that buyers seek is the timely delivery of the products that they purchase from suppliers. Delivery windows have been proposed as a means to achieve this goal. This paper analyzes the effect of buyer-specified delivery windows on the supplier's flow time variance, flow time allowance, inventory, expected tardiness, and probability of on-time delivery. The results confirm that using delivery windows may have the effect that the supplier's preferred action would be to reduce flow time allowance and variance. Whether on-time delivery performance improves is analyzed for linear and exponential variance cost functions. The results indicate that when the cost of maintaining lower variances grows exponentially, variance reduction does not lead to more timely deliveries.

1. Introduction

A delivery window is a period of time, *specified by the buyer*, in which delivery is desired. Deliveries before the delivery window period begins are forbidden. Deliveries that occur after the delivery window period are considered tardy, and may be subject to penalties. When the delivery occurs within the window the buyer accepts delivery and does not impose any penalties.

The use of delivery windows is documented by Fawcett and Birou [5], Corbett [3], and Kumar and Sharman [14]. Fawcett and Birou use a Likert scale to assess the degree of implementation of various JIT techniques. Use of delivery time windows was ranked fourth out of 14 in implementation status. Delivery time windows were implemented at a level of 5.10 out of 7, where 7 was described as "fully implemented." The highest level of implementation was 5.79. Long-term partnerships, blanket orders, and supplier certification scored higher in level of implementation. They state, "...the use of [delivery] time windows is viewed as a type of insurance against nonperformance and the cost of failures in physical support of JIT sourcing." Reviewing this same data, Fawcett [4] indicates that approximately 10% of suppliers surveyed do not use delivery time windows at all. A little over 41% are at some stage of implementing delivery windows with some of their carriers and suppliers. Close to 48% of the suppliers heavily use them with the majority of their carriers and suppliers, and have relatively well defined delivery time windows.

Corbett [3] describes a case study where the manufacturer quotes an earliest and latest delivery date rather than a single due date. The manufacturer supplies office furniture to buying firms that install it. These buyers typically operate in a project management environment. The early and late start dates for the installation task define the delivery window. Corbett states: "delivery windows offer the ability to improve the on-time delivery performance or dependability." Capacity smoothing methods using the delivery windows are proposed to create a degree of flexibility in scheduling the work.

Kumar and Sharman [14] indicate that delivery windows at supermarkets have decreased from four hours to one hour. They discuss the competitive advantages that suppliers can obtain through on-time delivery and suggest means of achieving on-time delivery. Their recommendation is an application of gap analysis [18]. Kumar and Sharman identify three causes of gaps between delivery expectations and perceived delivery performance. The *calibration* gap occurs when suppliers and buyers measure and evaluate on-time delivery differently. An *organization* gap indicates a lack of incentives or commitment by the supplier to deliver on-time. The *operations* gap is the result of highly variable operations created by complexity in product design, product mix, and supplier processes. The variance reduction considered in this paper addresses this operations gap.

When on-time delivery is critical to operations, two supplier actions are usually assumed to improve delivery timeliness: increasing the flow time allowance (or lead time)†

† The effectiveness of this response may be limited. The ramifications of increasing lead time are discussed by Wight [17].

and reducing the variance of flow time. Of these, many buyers would prefer that suppliers reduce variance rather than increase the allowance. Increasing the allowance decreases the buyer's flexibility in the short-term, and has the effect of increasing the amount of inventory held by the supplier. Kumar and Sharman [14] explicitly indicate that holding additional inventory is an undesirable "quick-fix." Decreasing the variance of flow time is thought to result in more timely deliveries without these negative side effects.

On-time delivery is typically critical when buyers choose to implement Just-In-Time (JIT). JIT usually involves inventory reduction by the buyer. Without the inventory, late deliveries cause the buyer's operations to be very dependent on suppliers to provide on-time delivery. In response, some JIT proponents recommend that the supplier also implement JIT. Suppliers are discouraged from responding to the buyer's need for on-time delivery by holding more inventory. Doing so would defeat the overall purpose of JIT. Inventory would simply be relocated to the supplier's warehouse. Relocating inventory, in general, does not improve the efficiency of the supply chain as a whole.

Despite JIT proponents' assertions that suppliers should not hold more inventory, a growing body of evidence indicates that they often do. A *Wall Street Journal* article titled "Trucks Become 'Warehouses' for Inventories" states that "trucks have become the place of choice for just-in-time stockpiles" [10]. It also suggests that this practice may have an impact on the economic recovery of 1994. Inventories in trucks are not included in the measures of inventory used to track the progression of the recovery. This practice also occurs in Europe. Hill and Vollman [11] state "At one European automobile manufacturer we were told of all the great inventory reductions that had been achieved through the application of JIT. Later, we saw a large construction project that turned out to be a greatly increased parking lot for vendor trucks. The inventory was moved from the factory warehouse to semitrailers!" Not all suppliers use trucks. PPG Industries has created an industrial park for their suppliers' warehouses [13]. A buyer for PPG Industries states "Our company is in the business of producing chemicals—not managing inventory. It's distribution's job to inventory products." Freeland [6] found that nearly half of the single source suppliers he surveyed were required to hold safety stock to insure supply. Other examples have been documented [1,2,7,12].

Buyers implementing JIT need a method of getting suppliers to provide on-time delivery that is brought about through variance reduction instead of holding inventory. This method must take into account how suppliers will respond to it. This paper analyzes how a cost-minimizing supplier's response to buyer-specified delivery windows impacts the variance of flow time, inventory levels, and delivery timeliness. The results are mixed. Using delivery windows may have the desired effect of making flow time variance reduction the preferred action of suppliers. However, depending on the cost functions associated with controlling variance, improved inventory levels and on-time delivery may not result.

The cost of maintaining a given variance is assumed to be positive and is described as a mathematical function of the variance. This function is called the variance cost function. Two variance cost functions are explored: linear and exponential. Both variance cost functions are decreasing with respect to the variance. They increase as variance is reduced. The supplier's optimal (cost minimizing) decisions are noticeably different for the two variance cost functions. These optimal decisions are compared on five dimensions: flow time variance, flow time allowance, inventory, expected tardiness, and probability of on-time delivery (see Table 1). For both variance cost functions, as the window duration is reduced the variance of flow time and flow time allowance decrease. However, the exponential function leads to increased supplier inventories, lower probabilities of on-time delivery and greater average tardiness as the window duration is reduced. For the linear variance cost function, the probability of on-time delivery is constant for all window durations, but the average tardiness and supplier's inventory decrease as the window duration is reduced.

The two functions considered here are not intended to be exhaustive. Rather, they represent two different views of how cost change as variance is reduced. Timeliness of deliveries can be considered to be one aspect of quality. Costs incurred maintaining a low flow time variance can be thought of as part of the prevention component of the cost of quality. The two postulated shapes of the prevention cost curve are finite and roughly linear, and exponential [15,16]. If the prevention cost of quality curve increases in a gradual, linear manner to a finite maximum at zero defects, then zero defects may be the lowest cost of quality. If the curve grows exponentially approaching zero defects (low variance), then the optimal quality level is greater than zero defects. The author's intent is not to argue which cost function accurately describes reality, rather it is to suggest that the two variance cost functions that have been selected for analysis are those under consideration in a broader context of inquiry.

The organization of the paper is as follows. The second section is the formulation of the suppliers total expected cost function with delivery windows. Section three discusses the optimal solution when the variance cost function is exponential. Section four discusses the optimal solution when the variance cost function is linear. Implications of the results for the two variance cost functions and some of the intuitive aspects of the findings are discussed in section five. Section five also identifies the limitations of this research. Conclusions and directions for ongoing research are given in section six.

2. Formulation

The formulation of this problem is a modification of the formulation presented by Grout and Christy [8,9]. The formulation differs from these in two respects. First, a delivery is considered on-time if the supplier delivers the order during the delivery window. In the prior formulations, a delivery was only on-time when it was delivered on the due date. No window or tolerance for variation was provided by the buyer. Appropriate incentives were created to get the supplier to bear all of the risk associated with flow time variation. Using a delivery window, the buyer shares the risk of flow time variation by accepting the cost that result when deliveries are made within the window. The cost that the buyer incurs is the holding cost of the inventory that results when shipments are delivered early in the window. Notice, however, that the buyer does not accept all of the risk, the supplier bares the risk of flow time variation that results from upper and lower tail outcomes. Since the supplier experiences cost in the tails of the flow time distribution, it is hypothesized that using delivery windows will tend to cause the supplier to reduce variation in an effort to avoid the cost in either tail. Second, the fixed-value, all-or-nothing bonus was omitted from this formulation in an effort to simplify the mathematics.

Let

$F =$ The random variable of flow time. Flow time is the duration between the start of work and the completion of work on a given lot or order.

$A =$ The flow time allowance. The flow time allowance is the amount of time budgeted for the completion of work on a lot or order. The allowance is the difference between the start time and the due date.

$g(F) =$ The probability density function of flow time. The uniform distribution is used to model flow times.

$G(A) =$ The cumulative distribution function of flow time,

$\mu =$ the mean flow time,

$\sigma^2 =$ the variance of flow time,

$R =$one half of the range of flow time. For the uniform distribution, $\sigma^2 = \frac{(2R)^2}{12}$.

$X_s(A, \sigma^2) =$ The Supplier's expected cost function.

$\alpha =$ The finished goods holding cost per time period (assumed to be positive).

$\beta =$The cost of tardiness per time period (assumed to be positive).

$W =$ The duration of the window.

$P =$ The per-time period penalty. The penalty is selected by the buyer and is assumed to be non-negative.

$c(\sigma^2) =$The variance cost function. The cost function of maintaining a chosen variance. The function is assumed to be decreasing, $c'(\sigma^2) < 0$. The smaller the variance that is chosen, the more costly the maintenance required.

$\theta =$The cost parameter of the variance cost function.

$M =$ The cost of achieving zero variance when the variance cost function is a linear function of the range.

2.1. Trading Scenario and Assumptions

The trading scenario between buyer and supplier is a long-term contract between a single risk-neutral buyer and a single risk-neutral supplier for multiple deliveries of fixed size lots of a single make-to-order product. The supplier's flow time is stochastic. The contract specifies the selling price per unit, duration of the window, and late penalty. The penalty is collected at the time that delivery occurs and is assumed to be positive and proportional to the amount of time that the order is tardy. The timing and quantity of demand is assumed to be known sufficiently early to allow the supplier to freely select when to initiate production and, hence, the flow time allowance. No expedited, or rush orders are necessary. The supplier manufactures products on a make-to-order basis. The supplier must decide when to initiate work on the buyer's order so that delivery will occur within the delivery window.

In this paper, the buyer's option to hold inventory and the supplier's option to do business elsewhere are not modelled explicitly. It is assumed that the supplier will not do business if delivery windows and contractual penalties reduce expected profits below some minimum acceptable expected profit. Likewise, if the costs of achieving on-time delivery are too high, the buyer will use alternative methods of achieving the desired service level. These alternatives include holding inventory or vertically integrating.

2.2. Supplier's Optimization Problem

Figure 1 shows the relationship between the distribution of flow time, the allowance, and the delivery window. When an order is completed before the window, $A - W > F$, the order is completed early and is held by the supplier. The expression $(A - W) - F$ is the duration that inventory must be held. If $A < F$, then the duration that the order is tardy is $F - A$. The supplier's expected relevant cost function is

$$X_s(A, \sigma^2) = \alpha \cdot \int_0^{A-W} [(A-W)-F] \cdot g(F)dF + (\beta + P) \cdot \int_A^{\infty} (F - A) \cdot g(F)dF + c(\sigma^2). \quad (1)$$

Two variance cost functions $c(\sigma^2)$ are considered. The exponential cost function is formulated as $c(\sigma^2) = \frac{\theta}{\sigma^2} = \frac{3\theta}{R^2}$. This function is convex and approaches infinite cost as the variance approaches zero. The other cost function that is considered increases as a linear function of the range to a maximum, M. This cost function is $c(\sigma^2) = M - \theta R$. Flow times are assumed to be uniformly distributed. This assumptions allows mathematical results to be determined for the two variance cost functions. The generality of these results is considered in section five.

3. Exponential Variance Cost Function

The variance cost function increases exponentially as the variance approaches zero. For a specific instance where the flow time is uniformly distributed and the variance cost function exponential, equation (1) can be restated as

$$X_s(A, R) = \alpha \cdot \int_{\mu-R}^{A-W} [(A-W)-F] \cdot \frac{1}{2R} dF + (\beta + P) \cdot \int_A^{\mu+R} (F - A) \cdot \frac{1}{2R} dF + \frac{3\theta}{R^2}. \quad (2)$$

The optimal solution is found by taking the derivative with respect to A and R, and solving the equations that result from setting the derivatives equal to zero. The optimal allowance and range for the exponential variance cost function are labelled A_E and R_E, and given below:

$$\begin{aligned} A_E = \mu + W - & \frac{W(\beta+P)}{\alpha+\beta+P} - \frac{\alpha^{\frac{1}{3}}(\beta+P)^{\frac{1}{3}}W^2}{2\cdot 3^{\frac{1}{3}}\Phi} \\ & + \frac{\alpha^{\frac{1}{3}}(\beta+P)^{\frac{4}{3}}W^2}{3^{\frac{1}{3}}(\alpha+\beta+P)\Phi} + \frac{(\beta+P)\Phi - \frac{\Phi}{2}(\alpha+\beta+P)}{3^{\frac{2}{3}}\alpha^{\frac{1}{3}}(\beta+P)^{\frac{1}{3}}(\alpha+\beta+P)}, \end{aligned} \tag{3}$$

and

$$R_E = \frac{\alpha^{\frac{1}{3}}(\beta+P)^{\frac{1}{3}}W^2}{2\cdot 3^{\frac{1}{3}}\Phi} + \frac{\Phi}{2\cdot 3^{\frac{2}{3}}\alpha^{\frac{1}{3}}(\beta+P)^{\frac{1}{3}}}, \tag{4}$$

where

$$\Phi = \left[216(\alpha+\beta+P)\theta + \sqrt{3}\sqrt{15552\theta^2(\alpha+\beta+P)^2 - \alpha^2W^2(\beta+P)^2}\right]^{\frac{1}{3}}. \tag{5}$$

The supplier's expected cost function is convex. The analysis of the Hessian matrix that shows that this solution results in minimum cost is provided in the appendix.

The supplier's optimal decision is a function of the duration of the window, W. The effect of the window duration on the supplier's optimal selection of A and R can be shown graphically. Figure 2 shows how the flow time distribution changes as the duration of the window changes for a specific set of parameter values. The shape of the curves shown does not change dramatically when the parameters change. For instance, the curve $\mu + R_E$ is convex and approaches A_E from above as W increases. These characteristic exist for any parameter values within the assumed ranges. The curve showing $\mu - R_E$ is concave and approaches $A_E - W$ from below. The range of the uniform distribution of flow time always exceeds the duration of the delivery window. The variance and allowance decrease.

As the window duration decreases, the bounds of the distribution bend away from the delivery window. The buyer's order could be completed at $\mu - R_E$ and be held by the supplier as inventory until $A_E - W$. As $\mu - R_E$ and $A_E - W$ move further apart, it indicates that the expected amount of time inventory is held by the supplier will increase. The supplier will hold inventory longer. At the same time, The distance between $\mu + R_E$ and A_E increases. For the supplier, this means that the duration that jobs are tardy increases as W decreases. The probability of tardy delivery is

$$1 - G(A) = \frac{(\mu + R_E) - A_E}{2R_E}. \tag{6}$$

As W decreases, the probability of a tardy delivery increases. The probability of on-time delivery decreases. More deliveries will be late.

As the window is reduced the variance is also reduced, yet more deliveries are late. This result occurs because the variance cost function increases rapidly. With increases in the costs of maintaining variance, holding inventory and allowing more orders to be late become more cost-effective alternatives. When $W = 0$, the slopes of the bounds of the distribution are zero. All of the orders are either held in inventory for some period of time or are late. The optimal proportion of on-time deliveries can be increased by increasing the late penalty, P. When $W = 0$, the optimal probability of on- time delivery $G(A)$ can be shown to satisfy the following equation:

$$G(A) = \frac{\beta + P}{\alpha + \beta + P} \tag{7}$$

Prior to implementing delivery windows, a buyer has a delivery window where $W \geq A$. This means that an order can be delivered as soon as it is completed. Implementing delivery windows involves reducing the window duration over time. As this reduction occurs for the exponential cost function, the inventory levels and delivery timeliness deteriorate; however, the length of the allowance and the variance of flow time continue to decrease.

4. Linear Variance Cost Function

The linear variance cost function is defined as $c(\sigma^2) = M - \theta R$. The supplier's expected cost equation is the same as (2), except for the third term. The optimal solution is also found using the same methods used in the exponential case. The optimal allowance and range for the linear variance cost function are labelled A_L and R_L:

$$A_L(P) = \mu + W - \frac{W(\beta+P)}{\alpha+\beta+P} + \left[\frac{-\alpha+\beta+P}{\alpha+\beta+P}\right]\frac{W}{2}\sqrt{\frac{\alpha(\beta+P)}{\alpha(\beta+P)-\theta(\alpha+\beta+P)}}, \quad (8)$$

and

$$R_L(P) = \frac{W}{2}\sqrt{\frac{\alpha(\beta+P)}{\alpha(\beta+P)-\theta(\alpha+\beta+P)}}. \quad (9)$$

In the this case, the supplier's expected cost function is convex. The Hessian matrix is shown to be positive semi-definite in the appendix indicating that the optimal solution is a minimum. However, the solution is a real number only when θ is relatively small. When $\alpha(\beta+P)-\theta(\alpha+\beta+P)$ is non-positive, (8) and (9) either involve division by zero or result in an imaginary number. Variance reduction will not remain attractive when the marginal cost of that reduction is large. For A_L and R_L to be real,

$$\theta < \alpha\frac{(\beta+P)}{(\alpha+\beta+P)}. \quad (10)$$

The value of θ is assumed to be sufficiently small to result in real values of (8) and (9) in the remainder of the paper.

Figure 3 shows how the flow time distribution changes as the duration of the window changes with a linear variance cost function. The allowance and range are linear functions of W. Moreover, the slopes of these lines result in the proportion of late orders remaining constant with respect to W. The probability of on-time delivery is

$$G(A) = \frac{\beta+P}{\alpha+\beta+P} + \frac{\alpha}{(\alpha+\beta+P)\sqrt{\frac{\alpha(\beta+P)}{\alpha(\beta+P)-\theta(\alpha+\beta+P)}}}. \quad (11)$$

Notice that (11) does not include W. Although managers would like the proportion of late orders to decrease, for the linear variance cost function the proportion is constant in W. However, when an order is late, the expected tardiness is reduced. The length of time inventory must be held by the buyer and supplier also decrease. So although the proportion of late deliveries does not decrease, the severity and thus the cost of late deliveries and holding inventory do decrease.

5. Implications, Intuition, and Limitations

The supplier's actions are very different for the two variance cost functions that have been examined. These differences can be seen by examining Figure 2 and Figure 3.

In some ways the comparison of linear and exponential variance cost functions mirrors the discussion in quality management literature about the optimal defect level. When the prevention curve is linear with moderate slope, zero defects is the optimal quality level. When the variance cost function is linear, reducing W to zero continues to have the desired effect of simultaneously improving most performance measures and not harming any. The exponential prevention curve leads to a trade-off in cost of quality that yields a optimal level of quality that allows some defects to be tolerated. The exponential variance cost function leads to a trade off between low variance and higher probability of late deliveries that suggest $W > 0$ may be best.

Reducing the delivery window may make variance reduction the most attractive action for the supplier, but that variance reduction does not always lead to achieving on-time delivery. This research shows that the variance cost function has an important impact on whether implementing delivery windows will actually improve the timeliness of orders.

In those case where windows do not achieve the desired on-time delivery performance, the buyer faces a dilemma. Should the variance be reduced to improve the efficiency of the supply chain or should a high variance be permitted in exchange for more timely delivery? The buyer may trade off between these two objectives by selecting a duration where the bounds of the distribution have not diverged from the window too much yet where the variance is acceptably small. An alternate approach would be to use delivery windows to manage supplier variance and to use monetary incentives like the penalty P to achieve the desired probability of on-time delivery and expected tardiness.

The results of this paper are shown only for uniformly distributed flow times. The equations describing optimal supplier behavior would be different for other flow time distributions. The results shown here do not fully characterize the optimal behavior of suppliers; however, some findings would hold for a variety of flow time distributions.

Some findings that hold for other flow time distributions when the variance cost function is exponential follow. When $W = 0$, the variance will not be zero. This is true because

$$\lim_{\sigma^2 \to 0} c(\sigma^2) = \infty. \tag{12}$$

If $W = 0$ and $\sigma^2 > 0$, then holding inventory until the due date and delivering orders late are the only possible outcomes. Also, the variance cost function approaches zero as W becomes large. When inventory and tardy costs exceed the variance cost function, the bounds of the distribution will tend to approach the window. The probability of holding inventory and delivering orders late will approach zero. This suggests that the bounds bending away from the window will occur for a variety of flow time distributions.

The fact that the variance and allowance converge to the mean flow time for the linear function also seems reasonable for other flow time distributions. If $W = 0$ and $\sigma^2 = 0$, $c(\sigma^2)$ must be finite, and the slope of $c(\sigma^2)$ must have a small enough slope to make variance elimination more attractive than holding inventory or delivering orders late. As long as the marginal variance cost is sufficiently small relative to holding, tardy and penalty costs, variance reduction would minimize cost as the window duration decreased. The fact that the bounds are straight lines when the uniform distribution is used would not be true for other distributions.

6. Conclusions

Buyers, for whom on-time delivery is critical, need a method of creating incentives for suppliers. To be effective, these methods must be analyzed with respect to how suppliers will respond to them. The incentives should ideally result in supplier behavior that has three important attributes. First, the supplier's on-time delivery performance should improve: the proportion of on-time delivery should increase and the expected tardiness should decrease. Second, the supplier's variance of flow time should decrease. Third, the supplier should not hold additional inventory in order to make the increased timeliness possible. These outcomes are sought because they result in improved individual firm performance and improved supply chain efficiency.

This paper considers whether delivery windows achieve these outcomes. A mathematical model of the effect of buyer-specified delivery windows on the supplier's flow time variance, flow time allowance, inventory, expected tardiness, and probability of on-time delivery is presented. Using delivery windows may have the effect of making the supplier's preferred action to reduce flow time allowance and variance. However, depending on the supplier's cost function to control variance, improved inventory levels and on-time delivery may not result. For uniformly distributed flow times and linear variance cost function with moderate slope, all of the desired outcomes occur except the proportion of on-time deliveries stays constant. When the slope is too large the solution becomes either undefined or imaginary. For a uniform distribution and exponential cost functions, only one of the three outcomes occurs. The variance of flow time is reduced, but inventory and on-time delivery actually get worse. In one of the scenario presented here, improving the predictability of flow time may not lead to more timely delivery or inventory reduction.

This research suggests that using delivery windows may be effective for inducing supplier variance reduction but may not result in more timely deliveries with less supplier inventory. Several research questions remain unanswered. What is the typical shape of variance cost functions? Are moderately sloped linear functions common? If not, what method can be used to achieve all three outcomes? Selecting a method to improve the performance and efficiency of buyer-supplier relationships is not simple and could be counter-intuitive. Continued research that attempts to generalize and broaden managerial insight into how buyer-supplier relationships should be structured for efficient outcomes would be of interest.

7. Appendix

To show that the supplier's expected costs result in a minimum, the Hessian matrix, H, must be positive semidefinite. The Hessian matrix for both the exponential and linear variance cost functions are present below.

7.1. Convexity of Expected Costs for Exponential Variance Cost Function

$$H = \begin{array}{cc} & \begin{array}{cc} \qquad\qquad \partial A_E \qquad\qquad & \qquad\qquad \partial R_E \qquad\qquad \end{array} \\ \begin{array}{c} \partial A_E \\[1em] \partial R_E \end{array} & \begin{pmatrix} \frac{\alpha+\beta+P}{2R_E)} & \frac{\alpha W-(A_E-\mu)(\alpha+\beta+P)}{2R_E^2} \\ \frac{\alpha W-(A_E-\mu)(\alpha+\beta+P)}{2R_E^2} & \frac{(\beta+P)[\mu-A_E]^2}{2R_E^3} + \frac{\alpha[W+\mu-A_E]^2}{2R_E^3} + \frac{18\theta}{R_E^4} \end{pmatrix} \end{array} \tag{13}$$

$$|H| = \frac{\alpha(\beta+P)W^2}{4R_E^4} + \frac{9\theta(\alpha+\beta+P)}{R_E^5} \tag{14}$$

For the Hessian matrix to be positive semidefinite the determinant and the diagonal elements of the Hessian matrix must be non-negative. The second partial derivative of $X_s(A, R)$ with respect to A and R are positive since all of the parameters and variables are assumed to be either non-negative or positive. The determinant $|H|$ is non-negative for the same reason.

7.2. Convexity of Expected Costs for Linear Variance Cost Function

The supplier's expected costs are convex in the allowance and the range of flow times. The Hessian matrix and its determinant are

$$H = \begin{array}{cc} & \begin{array}{cc} \qquad\qquad \partial A_L \qquad\qquad & \qquad\qquad \partial R_L \qquad\qquad \end{array} \\ \begin{array}{c} \partial A_L \\[2ex] \partial R_L \end{array} & \begin{pmatrix} \frac{\alpha+\beta+P}{2R_L} & \frac{\alpha W-(A_L-\mu)(\alpha+\beta+P)}{2R_L^2} \\ \frac{\alpha W-(A_L-\mu)(\alpha+\beta+P)}{2R_L^2} & \frac{(\beta+P)[\mu-A_E]^2}{2R_E^3}+\frac{\alpha[W+\mu-A_E]^2}{2R_E^3} \end{pmatrix} \end{array} \tag{15}$$

$$|H| = \frac{\alpha(\beta+P)W^2}{4R_E^4} \tag{16}$$

This Hessian matrix is also positive semidefinite. The determinant and the diagonal elements of the Hessian matrix are non-negative by assumption.

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Table 1: Changes in Delivery Performance Measures as Window Width Decreases for Two Variance Cost Functions

Measures	Exponential Variance Cost Function	Linear Variance Cost Function
Variance	decreases, bounds of distribution remain close to the allowance and window until window is small.	decreases, bounds of distribution remain proportional to the allowance and window.
Duration Inventory is held	increases for supplier decreases for buyer	decreases for supplier decreases for buyer
Flow Time Allowance	decreases	decreases
Probability of On-Time Delivery	decreases, especially for small window	remains constant
Expected Tardiness	increases	decreases

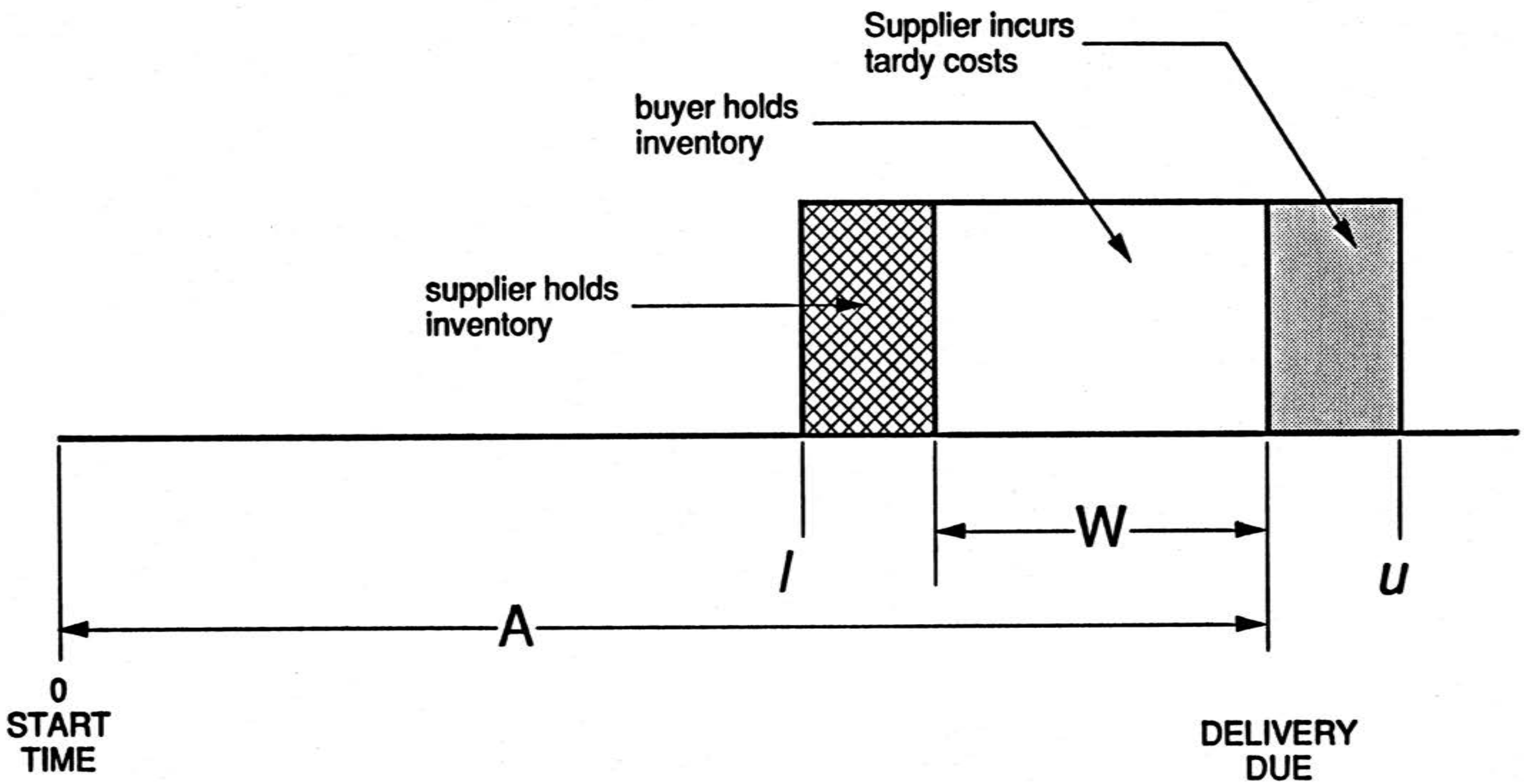

FIGURE 1: How the relationship between a uniform flow time distribution, the allowance **A**, and the window **W** affects the buyer and supplier.

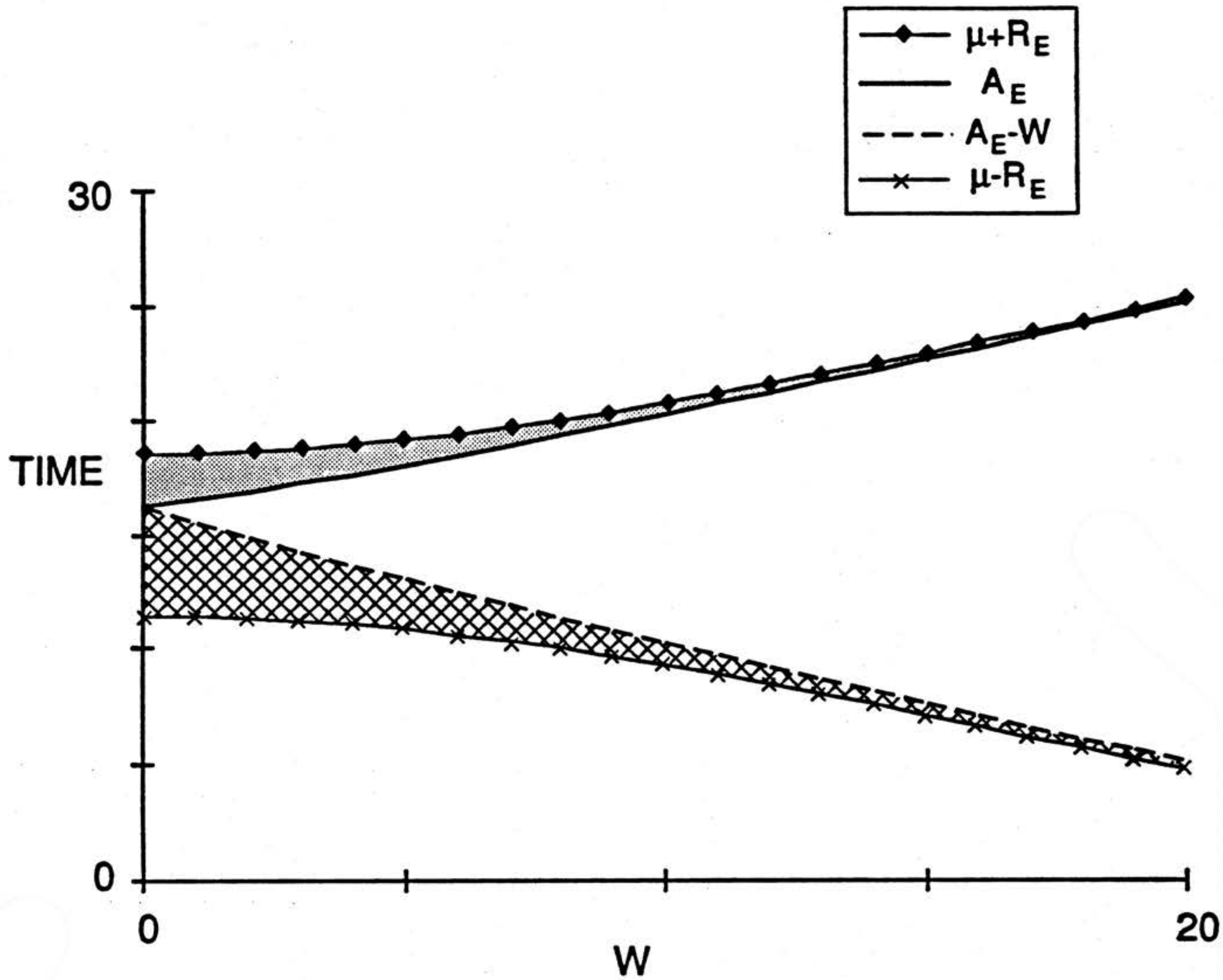

FIGURE 2: As the window duration decreases, the bounds of the distribution bend away from the window

graph uses specific values for equations 3 & 4: $\alpha = 10$, $\beta = 15$, $\theta = 5$, $\mu = 15$, $P = 5$

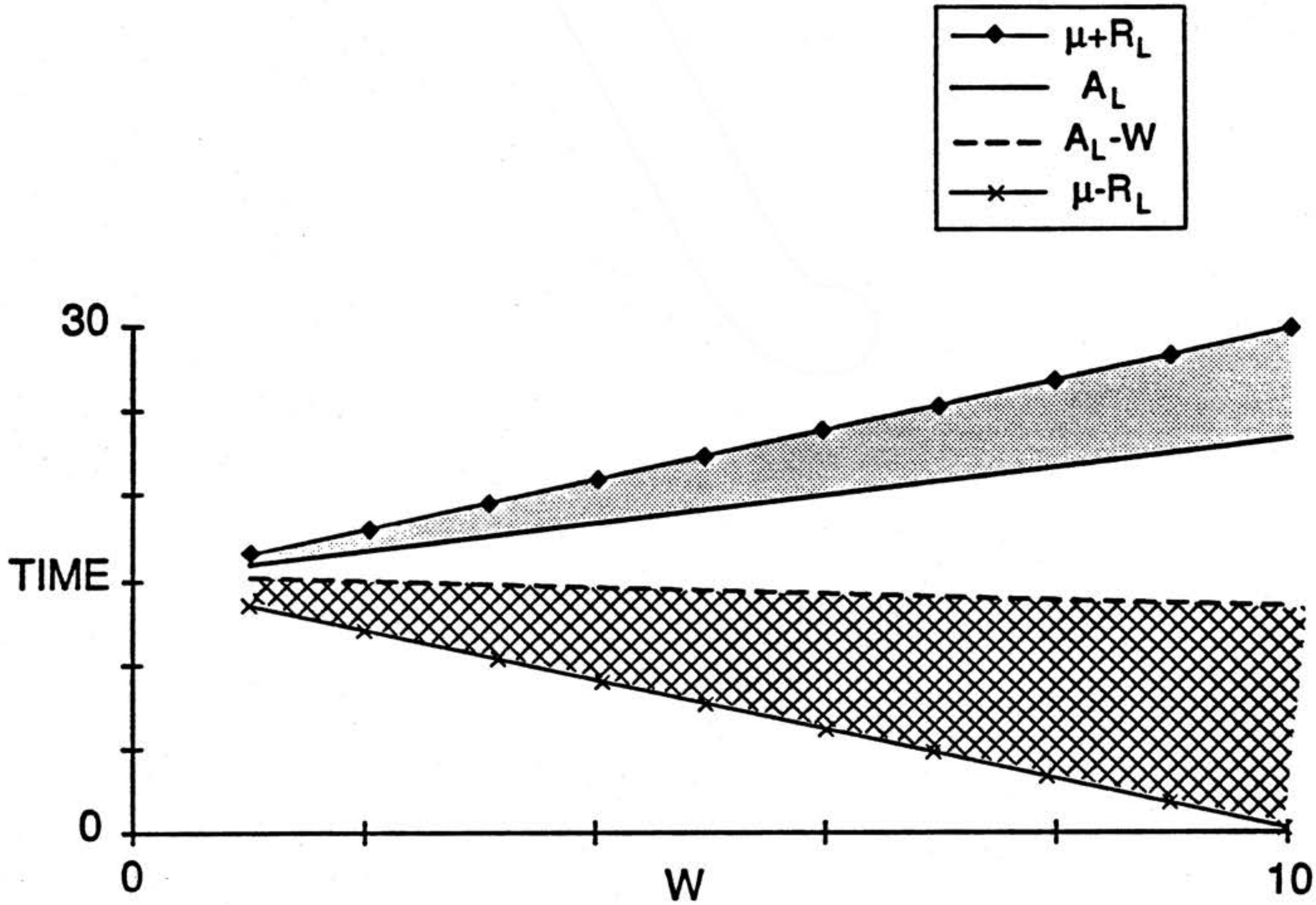

FIGURE 3: As the window duration decreases, the bounds and allowance converge.

graph uses specific values for equations 8 & 9: $\alpha = 10$, $\beta = 15$, $\theta = 5$, $\mu = 15$, $P = 5$, $M = 200$

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