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Hedging Uncertain Foreign Exchange Positions

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This paper looks at a number of issues related to hedging or covering foreign exchange exposure. Some of these issues, including the cost of cover and the hedging decision, have been explored elsewhere. We hope to contribute to those discussions by using a formal utility maximizing framework that differs from the decision processes used in those papers. In addition, by adding an element of uncertainty to the amount of the exposure we are able to explicitly link foreign exchange exposure to business risk and to demonstrate how the availability of hedging opportunities affects productive decisions.

The first section of the paper introduces a hedging problem that includes uncertainty about the existence of the exposed position. The next section offers a solution technique that deals with the uncertain exposure as well as the hedging decision once the degree of exposure is known. Subsequent sections of the paper present empirical estimates of relevant relationships and discuss extensions of the basic model presented in section II.

I. A Case of Uncertain Exposure

One of the most significant trends during the recent period of flexible exchange rates has been the diminishing role of the American dollar as a vehicle for international commerce. The dollar remains the most widely used currency but the percentage of total eurocurrency deposits and bonds denominated in dollars has fallen. Similar patterns have been detected in the choice of currency for third country transactions as well as for imports and exports involving U.S. companies.

This last phenomenon has complicated the task facing American managers. As long as they were able to deal in dollar contracts they were immune to nominal exchange risk. If they import or export using non-dollar terms, then they need to be concerned about exchange fluctuations.
One specific problem arising from doing business in another currency is related to transactions requiring competitive bids with a time delay between the bid submission date and the award date. This type of transaction is particularly important in construction-related industries but also is relevant for general manufacturing concerns. The hedging decision as typically described in texts and articles is complicated by the uncertainty created by the bidding process. Once the contract is awarded the firm faces the standard hedge/no-hedge question, but until that point it does not know for certain whether it will even have an exposure.

The point can be clearly made by example. Assume that an American company submits a bid on January 1 to try to win a contract to supply electronic components to a French firm. The buyer stipulated that the bids must be denominated in francs and that payment, upon contract completion will also be in francs. Each bidder is required to post a performance bond guaranteeing that he will accept the bid if awarded or face forfeiture. The bid is to be awarded on February 1 with delivery and payment March 1.

The U.S. company's bid is 4,000,000 francs or $1,000,000 as of January 1. To keep matters simple assume all of the costs are in dollars, so the potential exposure comes solely from the 4,000,000 franc receivable due March 1 if the contract is won. On February 1, if the U.S. firm is awarded the contract, then it has a straightforward decision to make: hedge all, part, or none of its exposure. However, prior to February 1, when the award is uncertain, the firm has a less clearcut decision even though it is vulnerable to exchange fluctuations.

On January 1 the company commits itself to deliver goods in the future for 4,000,000 francs if it wins the bid. In the interim between submission and receipt of payment, changes in the dollar value of the franc will affect
the profitability of the transaction. The firm could decide to hedge the ex­
pected future receipt on the date the bid is submitted. This would be accom­
plished by selling francs forward. However, if the firm then fails to win the
contract, it will not have the underlying cash flow to match with the forward
position. Any difference between the forward rate and the spot rate at which
the contract is closed at maturity would be a speculative gain or loss. On
the other hand, if the firm chooses not to hedge on January 1 and is awarded
the contract, then the expected value of that contract might have changed sub­
stantially by February 1.

From the above description it becomes evident that the firm confronts
several decisions. On January 1 the amount of the bid must be determined and
the first hedging decision must be made. If the bid is awarded, then a second
hedging decision is necessary on February 1. The three actions are interre­
lated and, as will be demonstrated below, are dependent upon a set of rela­
tively complex exchange rate relationships even when a number of simplifying
assumptions are invoked. A problem similar to this one was posed by Fieger
and Jacquillat [2] although they did not offer a solution which is feasible
with current financial instruments.

III. A Multi-period Hedging Decision Model

The approach we follow is to use the recursive technique of stochastic
dynamic programming that was proposed by Mossin [6] as a solution to this gen­
eral type of problem. This framework recognizes the multi-period nature of
the problem by having the decision maker maximize his expected utility at each
stage of the transaction. Working backward and taking into account the range
of outcomes possible at each point in time and the probability attached to
each outcome, the manager is able to determine the optimal decision to be made
at each preceding stage.
The decision making process can be outlined within the context of the problem described in section II which required decisions to be made on January 1 and February 1. To identify the nature of each of those decisions assume that the manager's objective is to maximize the expected value of end of period wealth subject to a risk constraint; i.e., he has a quadratic utility function with mean and variance as the two arguments. On February 1 the manager will seek to maximize utility as of March 1 when payment is received. Utility will be a function of expected wealth, $W_2$, on that date and the variance related to it.

On February 1 the firm finds out whether it has received the contract so that its decision will depend on the outcome of the bidding competition. Therefore, there are two different decisions to be made: one based on receiving the contract and another reflecting the loss of the contract. Each outcome must be solved for because the January 1 decision requires the manager to maximize the expected utility of wealth as of February 1, $W_1$. That is accomplished by multiplying each outcome by the probability that it will occur. $\pi$ is the probability of winning the bid and $(1 - \pi)$ that of losing it.

In order to focus initially on the hedging activities several simplifying assumptions are made. They will either be relaxed or discussed in more detail when extensions of the basic solution are taken up below. The most important assumption is that the bid price in francs, $R$, and the probability of winning the bid, $\pi$, are given. As a consequence, the only decision variables are the amounts hedged, $Q_0$ and $Q_1$, at time 0, January 1, and time 1, February 1, respectively. It is also assumed that all hedging takes place in the forward market and that forward contracts exist only for a maturity equal in length to the periods of the model. Finally, it is assumed that the costs incurred by the firm, $C$, are dollar costs of a fixed amount.
At time 1 the manager will know whether or not the firm has won the bid.

If the bid is won he will choose $Q_{1A}$ (the subscript A denotes the bid being awarded) so that to maximize the firm's objective function. We can write this as:

(1) $\max L_A = \sigma^2(W_{2A}^f) - 2\lambda E(W_{2A}^f)$

$W_{2A}^f = W_1 + Q_{1A}(P_2 - P_1^f) + R P_2 - C$,

where all the terms are as defined previously and $P_1$ is the dollar/franc exchange rate at time $i$. The $f$ superscript denotes a forward exchange rate for a contract maturing at time $i + 1$. Wealth, measured in dollars is the sum of the beginning wealth, the gain or loss on the forward exchange position and the revenues and cash outflows from the contract.

Substituting the expression for wealth into equation (1)

$$\max = \frac{\sigma^2(W_1) + 2\sigma^2(P_2 - P_1^f) + R\sigma(P_2) + 2Q_{1A}\sigma(W_1, P_2 - P_1^f)}{Q_{1A}}$$

$$+ 2R\sigma(W_1, P_2) + 2Q_{1A}\sigma(W_1, P_2 - P_1^f) - 2\lambda[W_1 + Q_{1A}E(P_2 - P_1^f)]$$

Recalling that at time 1, $W_1$ and $P_1^f$ are known, therefore:

(3) $\frac{\partial L_A}{\partial Q_{1A}} = 2Q_{1A}\sigma^2(P_2) + 2R\sigma^2(P_2) - 2\lambda E(P_2 - P_1^f) = 0$

(4) $Q_{1A} = -R + \lambda e_2/\sigma(P_2)$,

where $e_1 = E(P_1 - P_1^f)$.

If the firm does not receive the bid, then it would have as its wealth function at time 1:

$$W_{2B} = W_1 + Q_{1B}(P_2 - P_1^f).$$
Following the same procedure as before yields:

\[ Q_{1B}^* = \lambda e_2 / \sigma (P_2). \]

* \(Q_{1A}^*\) is the optimal quantity hedged when exposure is \(R\) whereas \(Q_{1B}^*\) is the optimal hedge when exposure is zero. Using the traditional dichotomy, expression (4) is the solution to the optimal hedging decision and (5) the solution for the optimal speculation. In both cases, the size of the hedge is determined by the relationship between the forward rate and the expected future spot rate, the variance of the future spot rate and a risk tolerance factor, \(\lambda\). The only difference is that the hedger begins the period with a position and must determine to what extent he wants to augment or offset it.

If \(e_2\) is zero, that is the forward rate is an unbiased estimate of the future spot rate\(^2\), then neither the speculator nor the hedger sees any advantage to maintaining an exposed position. For the speculator that leads to no position in the forward market, but the hedger must neutralize his commercial position by taking an equal size but opposite one in the forward market. The \(Q_{1A}^* = -R\) in equation (4) reflects that.

The hedging result is diametrically opposed to one sometimes cited in the literature. Giddy [3] argues that when the forward rate is an unbiased estimate of the future spot rate then there is no advantage to hedging. His conclusion can only hold in general if the decision maker is risk neutral. As long as the firm is concerned about risk as well as expected wealth then there will be a trade-off between the expected wealth the manager will give up and the risk reduction he achieves. If \(e_2 = 0\), there is no loss of expected wealth due to hedging; but, as long as \(\sigma^2(P_2) \neq 0\), there is risk reduction. Therefore rather than forego any hedging as Giddy prescribes, the firm should hedge the full amount of its exposure.\(^3\)
A related point is the identification of the cost of hedging. Ignoring transactions costs, the "price" paid by the firm for risk reduction is the difference between the forward rate and the expected future spot rate. That differs from the view held by some (see Calderon-Rossell [1]) that the premium or discount is the appropriate cost of cover.

Once the optimal decisions are arrived at for time 1, it is possible to step back using the recursive technique and solve for $Q_0$, the optimal time 0 hedge. The analytical procedure is to substitute $Q_{1A}$ and $Q_{1B}$ into the utility functions $L_{1A}$ and $L_{1B}$. These are then used along with the probability of winning the bid to form a derived objective function. The objective at time 0 is to maximize the expected value of this function.

\[ \text{(6) Max } E(L_0) = \pi L_{1A}^* + (1 - \pi) L_{1B}^* \]

\[ = \sigma (W_1) - 2 \lambda E(W_1) + \pi [ R \sigma (P_2) + 2 R \sigma (W_1, P_2) \]

\[ - 2 \lambda (R(P_2) - C) + 2 \pi (-R + X) R \sigma (P_2 - P_1, P_2) \]

\[ + 2(-\pi R + X)[2 \sigma (W_1, P_2 - P_1) - 2 \lambda e_2] \]

\[ + [\pi (R - 2RX + X^2) \sigma (P_2 - P_1)] \]

where $X = \lambda e_2 / \sigma^2 (P)$.

Defining $W_1 = W_0 + Q_0(P_1 - P_0)$ and noting that $P_0$ is known at time 0, equation (6) becomes

\[ \text{(7) } E(L_0) = Q_0 \sigma (P_1) - 2 \lambda [W_0 + Q_0 e_1] + \pi [ R \sigma (P_2) + 2 R Q_0 \sigma (P_1, P_2) \]

\[ - 2 \lambda (R(P_2) - C)] + 2 \pi (-R + X) R \sigma (P_2 - P_1, P_2) \]

\[ + (-\pi R + X)[2 Q_0 \sigma (P_1, P_2 - P_1) - 2 \lambda e_2] \]

\[ + [\pi (R - 2RX + X^2) \sigma (P_2 - P_1)] \]
After differentiating it is possible to solve for the optimal zero period forward position,

\[
Q_0^* = -\frac{\pi R \sigma(P_1, P_1) + \lambda \sigma(P_1, P_2 - P_1)}{\sigma^2(P_1)}
\]

In the case when the forward rate is an unbiased estimate of the future spot rate or where the firm has expectations that are the same as the market, equation (8) can be simplified considerably.

\[
Q_0^* = -\frac{\pi R \sigma(P_1, P_1)}{\sigma^2(P_1)}
\]

There are two aspects of equation (9) that are particularly important. First, not surprisingly the hedging decision at time zero is a function of the probability of winning the bid. The second and more interesting result is that the hedging decision at time zero depends upon the covariance between the spot price at time one and the price of a forward contract at time one for delivery at time two. The usual relationship links an existing forward contract and the spot price rather than a forward contract that will be traded in the future.

The reason for the unusual result is that for this solution it was assumed that no contract exists with a maturity as long as the period of exposure. The forward contracts are for one month or period while the bid transaction is for two. Equation (9) can therefore be interpreted as a chaining rule for hedging beyond the length of the longest available forward contract. Since firms are often confronted with that situation, for instance with bond payments, it is important to have a framework for analyzing the hedging decision under those circumstances.
III. Empirical Issues

We began this paper by describing a decision problem which may confront firms engaging in international transactions. Then we proposed a specific type of approach to deriving the optimal solution to the currency hedging aspect of this problem. This solution is not unique. It depends upon the assumptions which we made concerning the markets in foreign currency. Of these assumptions, the most important were that we could deal in forward markets, and that the time interval, over which there was foreign exchange exposure, exceeded the duration of the longest forward contract. We will maintain these assumptions, consider the empirical estimation of the parameters required to implement our solution, and present some illustrative statistics.

Three types of statistics are required by our solution: forecasts of the future spot exchange rates; estimates of the variance of the future spot rates; and an estimate of the covariance between a future spot rate and the rate on the forward contract which comes into existence at that time. We have referred to this last variable as $\sigma(P_1, P_f)$. By referring to equation (9), it can be seen that it determines, along with $\sigma^2(P_f)$, the proportion of the expected value of the foreign exchange exposure which is hedged at time zero.

Forecasting future spot rates is certainly an important subject; and doing it accurately would be valuable. But rate forecasting is a subject beyond the scope of this paper. Therefore we will focus on the estimation of, and likely size of, the ratio $\sigma(P_1, P_f)/\sigma(P_f)$.

To estimate any variance or covariance statistic from an historic sample we require estimates of the expected value of each of the observations. We cannot in general treat the sample as if it possessed a stable mean which could be estimated from the sample. Specifically, the deviations of spot exchange rates from their expected values cannot be found by subtracting the
a 30 day contract and on a 90 day contract. The results are reported in Table 1.

<table>
<thead>
<tr>
<th></th>
<th>30 Day Ratio</th>
<th>90 Day Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pound</td>
<td>1.0158</td>
<td>0.8444</td>
</tr>
<tr>
<td>Mark</td>
<td>1.0269</td>
<td>0.6393</td>
</tr>
</tbody>
</table>

Since neither the 30 day nor 90 day contract is the longest forward contract, we are less interested in the exact magnitudes of the hedge ratio than we are in their relative values. Here we find the relationship to be quite consistent with our prior expectations. For the 30 day example, the hedge ratio is approximately one. In terms of our problem, we would take this to mean we should hedge the full amount of the expected value of our exposure. When the time horizon increases to 90 days the relationship between $P_1$ and $P_f$ is weaker. In this instance we should hedge between 60% and 80% of the expected value of our exposure. We would expect that when the time horizon is 180 days, the relationship between $P_1$ and $P_f$ will be still weaker and the hedge ratio will be lower.  

IV. Further Discussion and Extensions

The two primary extensions relate to the assumptions in the basic model that the probability of winning the bid, $\pi$, is independent of the bid, $R$, and that forward contracts are available for only one period. Relaxing either one or both of the assumptions complicates the solution and increases the information requirements of applying the model.

The case in which $\pi$ is allowed to vary with $R$ is of particular importance because it demonstrates the link between the availability of hedging
opportunities and business decisions. The appendix contains the derivations on which the discussion in this section is based. The procedure used in the appendix is the same as was used in the basic model. The difference is that \( \pi \) is assumed to be inversely related to \( R \) reflecting that the higher the bid, the lower the probability of winning the contract.

Once the interrelationship between the bid and winning the contract is introduced then a third decision must be made: What is the optimal bid? Solving for \( R^* \) is relatively straightforward given the dynamic programming framework. In the appendix it is done for two distinct cases. The first assumes that no forward contracts are available, that is, no hedging is possible. The second case allows for hedging along the same lines of section II.

Comparing the outcomes reveals that the optimal \( R^* \)'s differ. This demonstrates that when hedging is available the manager will bid differently than when the opportunity to hedge does not exist. Without specifying the exact functional relationship between \( \pi \) and \( R \) it is not possible to determine whether \( R^* \) is higher or lower when hedging is allowed.

To understand why the firm might bid a higher price it is necessary to recall the point made earlier that hedging generally offers an opportunity to reduce risk at a cost of lower expected value. With \( R \) and \( \pi \) endogenous to the model then the risk and expected return facing the manager are determined by variance and expected value of getting the bid as well as the uncertainty related to exchange rates. There is an implicit price of risk in both the exchange market and the real goods market. A manager might avail himself of a lower price of risk in the goods market by using a forward exchange contract to offset additional risk taken on in the real goods market. At times that would imply a higher bid than would be submitted if the foreign exchange market option did not exist.
The basic model can also be expanded to include a wider variety of maturities for forward contracts. To accomplish that the objective function at each decision point would include a Qi for each of the i maturities available. The manager would then need to determine the optimal Qi using the same technique applied in section II. The optimal choices will depend upon the relationships among the current prices of the contracts, their expected value at the end of the holding period, and variances and covariances of the i instruments. It should be apparent that the larger number of options available to the firm increases the information requirements for making optimal decisions.

Similarly we could introduce futures contracts with a variety of combinations of maturity structures. These contracts would involve us more explicitly in two issues: intra-period gains and losses on the contract; and basis risk. Futures contracts require daily adjustments to capital to recognize changes in futures prices. This creates an uncertain demand for the firm's financial resources which, if combined with stochastic short-term interest rates, greatly complicates the problem. Basis risk, which depends upon the variance of the difference between the spot price and the futures price, would arise if the maturities of the real asset exposure and the futures contract are not matched. In general, this extra component will also complicate the solution to the problem discussed in this paper.6
Footnotes

1 Although we use the terms synonymously we believe that the distinction made by Rodríguez and Carter [8] is useful. They define hedging as reducing risk related to a balance sheet exposure whereas covering is concerned with transaction risk.

2 The empirical evidence on this point is mixed. Levich [5] and Robichek and Eaker [7] discuss the issues and some of the results.

3 Throughout the analysis transactions costs are neglected. Estimates of these costs as reflected in the bid-ask spreads are small enough that they will not change any of the conclusions.

4 We have examined both the pound and mark for longer intervals and for portions of the time period. In addition, we have data on the Swiss frank and the Canadian dollar for portions of the period. In general, the unreported results for the 30 day interval measures are very similar. The 180 day interval measures exhibit more variability, especially when the number of observations is reduced.

5 It may be possible to estimate the ratio for the longest term forward contract by using the interest parity relationship to estimate the expected value of future spot rates. Whether or not that is worthwhile will depend upon the application intended. We are attempting to provide illustrative measures only.

6 A solution which employs futures markets is available from the authors.
In this appendix we will show that hedging opportunities affect business or production decisions. This interrelationship receives little attention in the literature on hedging. (For a recent exception see Holthausen, [41.]) We are emphasizing it because the role of financial markets is to enhance productive activity. To simplify the demonstration we will assume:

\[ W_0 = C = e_1 = e_2 = 0 \]

The decision maker's objective is to minimize the risk of terminal wealth with respect to the expected value of terminal wealth. In the absence of hedging opportunities this can be expressed as:

\[ \max L = \sigma^2(W_2) - 2\lambda R(W_2) \]

\[ = \frac{2}{\pi R} E(P_2) - \frac{2}{\pi R} E(P_2) - 2\lambda \pi R E(P_2) \]

The effect of the hedging opportunity we considered in the body of the paper can be seen by writing the corresponding equation with a hedging opportunity. The appropriate expression is that which identifies the objective with respect to \( R \), given optimal hedging decisions. This can be found by substituting the optimal value for \( Q_0 \) from equation (9) into equation (7) and rewriting (7) to reflect the simplifying assumptions of equation (A1):

\[ E(L_0) = \frac{2}{\pi R} E(P_1) - \frac{2}{\pi R} E(P_1) - \frac{2}{\pi R} \sigma(P_1) \sigma(P_1) - 2\lambda \pi R E(P_2) \]

Even if the moments of the distributions of \( P_1 \) and \( P_2 \) are identical, which is unlikely, the two expressions (A2) and (A3) differ. The third term of (A3) has no corresponding element in (A2). As we would expect, this term reduces the value of \( E(L_0) \) (increases expected utility), relative to the situation when no hedging occurs. We have examined the partial derivatives of both (A2)
and (A3) with respect to $R_2$. The optimal values of $R$ will, in general, be different in the two cases, but it is not possible to state which will be larger.
References


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