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SEQUENTIAL INFORMATION DISSEMINATION
AND
RELATIVE MARKET EFFICIENCY

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Sequential Information Dissemination
and Relative Market Efficiency

Abstract

Research in the area of capital market efficiency with respect to information generally considers only the two conclusions that the market is efficient or it is not efficient; information is either instantaneously reflected in prices or it is not. We investigate a market structure in which individual traders may require differential time lags to respond to new information, either because of time lags in acquiring the information or because of differences in ability to process received information quickly. In the context of this market, we argue that the dichotomous view of efficiency versus inefficiency is not useful; rather it is meaningful to measure the relative efficiency of the market. We develop a measure of relative market inefficiency and investigate properties of the measure. Then we consider economic determinants of the degree of market inefficiency and review our measure of inefficiency against the criteria proposed by Goldman and Sosin (1979).

1. Introduction

An efficient capital market is a market in which prices adjust 'quickly' to 'fully reflect' all 'available' information. This seems to be the generally accepted interpretation of the pathbreaking work by Fama (1970). Since that time, financial economists have struggled to more clearly define the concept of efficiency. The research efforts of those economists is justified by the fact that the allocation of income and wealth in the financial market and, indeed, the economic properties of the overall economy, are at least to some extent affected by the information efficiency of capital markets.²

This paper represents an effort to better understand the elusive concept of market efficiency. We consider a market structure in which information is gradually diffused into the market. With the exception of Goldman and Sosin (1979), previous research in capital market efficiency has paid little attention to the process by which information is disseminated into the market. We consider a market structure which allows for the gradual diffusion of new information.
data. Our model permits us to explicitly consider the time dimension of the price adjustment process, a factor ignored even by Goldman and Sosin. Within this framework, we argue that the traditional designation of a market as either efficient or inefficient is inadequate. We then develop a measure of relative market efficiency. With this capability we can gauge the degree of inefficiency.

The remainder of the paper is divided into six sections. Following this brief introduction is a review of existing theoretical investigations into information efficiency. Included is a discussion of the perceived limitations of this prior work. Section three describes the market structure employed in our analysis. In the fourth section we offer an intentionally traditional definition of market efficiency which is consistent with gradual information dissemination. After positing this definition we demonstrate the problem associated with its application.

Subsequent to this traditional efficiency analysis, we introduce a measure of relative efficiency. Section five defines the metric and examines some of its characteristics. In the sixth section we investigate the effects of several important economic variables on our measure of inefficiency. The paper closes with a summary of our findings and points towards needed future research.

2. Previous Definitions of Market Efficiency

Much of the theoretical and empirical work on market efficiency in recent years draws upon the framework exposited by Fama (1970). In order to operationalize the notion that prices "fully reflect" information, Fama posits an expectations model to describe the market's use of information:
Thus Fama asserts that the price expected to prevail next period \((t+1)\) for asset \(j\) conditional on the information set this period \(\Phi_t\) is equal to one plus the expected rate of return times the asset's current price. Implicit in this definition is that, whatever the return generating process, the information set \(\Phi_t\) is fully utilized in determining expected returns. Fama's work implies investors should not, on average, be able to earn excess profits by trading on the information in \(\Phi_t\).

Fama also proposes a set of conditions sufficient to imply market efficiency. These are: (1) a frictionless market; (2) costless information which is equally available to all market participants; and (3) homogenous beliefs. Fama points out that these conditions are not necessary and a market may still be efficient with 'some' violation of each condition.

In response to criticism of his earlier definition, Fama (1976a), (1976b) offers an alternative interpretation of market efficiency. He suggests a two step price formation process. On the basis of the information set used by 'the market' to set prices at time \(t\), \(\Phi_t\), 'the market' assesses a joint probability distribution of security prices at time \((t+1)\), \(f_m(P_{t+1}|\Phi_t^m)\), where \(P_{t+1} = (P_{1,t+1}, P_{2,t+1}, \ldots, P_{n,t+1})\) is the vector of security prices at time \(t+1\). From this joint distribution and an assumed model of equilibrium returns, the market determines current prices. Market efficiency implies that the market uses all relevant information in determining prices, \(\Phi_t^m = \Phi_t\), and that the market assessed distribution equals the 'true' distribution, \(f_m(P_{t+1}|\Phi_t^m) = f(P_{t+1}|\Phi_t)\).

Fama's work provides an intuitive description of the notion of information efficiency and the impetus for most of the empirical efficiency work.
His definition of efficiency, however, has some conceptual difficulties. Fama offers no explanation of why the market might ignore some information in determining prices, i.e., why $\phi^m_t \neq \phi^L_t$. In addition, the idea of a 'true' distribution for security returns seems to imply the existence of an 'intrinsic value' for that security which is independent of the beliefs and preferences of the market participants. Finally, Fama's efficiency definition requires some assumption concerning an equilibrium return generating model.

Rubenstein (1975) and Beja (1976) propose action-oriented definitions of market efficiency. Rubenstein uses a three date ($t=0,1,2$) Arrow-Debreu economy to demonstrate his efficiency criterion. He is interested in efficiency with respect to two sets of information. New information is defined as a revelation of the state of nature at $t=1$. This revelation provides the traders with additional insight concerning the $t=2$ state of the world. Rubenstein's definition of all data includes this new information plus the individual's prior (time $t=0$) subjective distribution of states.

An individual perceives new information as fully reflected in price if and only if there is no portfolio revision at $t=1$. The assumption of a complete market removes any incentive for trading except for a revision of beliefs not fully offset by a change in price. Any trading at the intermediate date implies at least one investor does not believe prices fully reflect the new information.

Other scholars have attempted to refine the concept of efficiency. Huber (1978) attributes the following definition to Sharpe:

Prices fully reflect the information set $\phi$, if prices are identical to the prices that would exist if $\phi$ was made public.

This definition (or some variant thereof) is one of the more frequently used definitions in the literature. Note the severity of its implications. If
even a single investor in a large market discovers some information not generally known to the market, equilibrium price must react as if everyone possessed the same data.

The concept of efficiency may be viewed as either an ex post property of a series of prices or an ex ante property of a pricing process. If the profession is to empirically test the efficiency of financial markets any definition must be consistent with ex post efficiency. It is equally important, however, to be able to consider the ex ante efficiency of an equilibrium pricing process. Huber (1978) noted the distinction. He offers the following definition:

A process determining equilibrium prices and allocations is termed efficient with respect to an information structure if, for every possible set of signal outcomes from the information structure, equilibrium prices and allocations are identical to those which would be determined if the same set of signal outcomes had been generated by a public information structure.

This definition is consistent with both ex ante and ex post notions of efficiency.

All of the studies reviewed thus far offer explicit definitions of an efficient market. The research examined in the remainder of this overview is concerned with market settings which either do or do not achieve efficiency. From this work are implications for the concept of information efficiency.

Grossman and Stiglitz (1975), (1976), are interested in the efficiency properties of a market in which only a fraction of the market participants receive additional data and the uninformed investors use market price to infer what they can about the information set of the informed traders. Grossman
and Stiglitz develop a market structure in which information is costless and equilibrium price is a one-to-one correspondence with the information set of the informed group of investors. If, through repeated observations of the (assumed) stationary return generating process, the uninformed learn this correspondence, they can use market price to infer the information set of their better informed trading partners. In this case, since every investor has the same information set, the market is efficient.

Now consider the same market structure except that the new information is costly. If an investor chooses to become informed the traders remaining uninformed invert price and the market is efficient. The fact that uninformed individuals may 'free-ride' eliminates the incentive for any investor to pay to become informed. However, if no one receives the new data, market price cannot be efficient. Thus before a market participant has an incentive to obtain the new information, he must expect to earn a fair return on his information acquisition. This implies that equilibrium prices cannot 'fully reflect' all information.

If we introduce an additional source of randomness (besides the new information) into this basic market structure, say random endowments, there is no one-to-one correspondence between the informed investor's data set and equilibrium price. Price may be high because the news is favorable or because supply is low. The uninformed cannot discover the information set of the informed agent with certainty. In equilibrium a fraction of the market participants choose to become informed, and at the margin, the informed investor earns a fair economic return on his investment in the information.

Figlewski (1978) is interested in investigating the mechanism by which markets become efficient. He has the following scenario by Cootner [(1967), page 80] in mind.
Given the uncertainty of the real world the many actual and virtual investors will have many, perhaps equally many, price forecasts . . . If any group of investors was consistently better than average at forecasting price, they would accumulate wealth and give their forecasts greater and greater weight. In the process, they would bring the present price closer to the 'true' value. Conversely, investors who were worse than average in forecasting ability would carry less and less weight. If this process worked well enough, the present price would reflect the best information about the future in the sense that the present price, plus normal profits, would be the best estimate of future price.

In this view the market price is a weighted average of investors' beliefs where the weights are directly related to the investor's wealth. Assuming some minimum regularity assumptions on demand curves, there is some distribution of wealth for which the wealth weights are identical to the forecasting ability weights and each trader's information is correctly reflected in equilibrium price. Figlewski finds that in the short run the actual distribution of wealth tends towards this efficient market distribution. Traders with information undervalued by price have expected gains while traders whose data is overvalued can expect losses. In the long run, however, random forecasting errors prevent the market from achieving the efficient market wealth distribution. Thus the market is efficient with probability zero.

Figlewski is also interested in assessing the relative efficiency of his market setting under various assumptions about the characteristics of the market participants. Using the variance of current versus future market price
as his measure of inefficiency, Figlewski finds, through numerical analysis, that traders who are more risk averse and more homogeneous in their ability to gather and process information produce more efficient markets.

Goldman and Sosin (1979) partition the market participants into two groups. One group, denoted speculators, enjoys a priority of information receipt over the other group, the investors. The first market setting Goldman and Sosin (GS) examine is one in which there is no uncertainty regarding the number of speculators or the amount of time which elapses before the investors are informed. They find that speculators may 'sit-on' the new data for some period of time until it is profitable to act. This type of behavior is certainly not in the best interests of market efficiency.

The primary focus of GS is the effect of the frequency of transacting on market efficiency. Their measure of inefficiency (MI) is the mean square error between the price that exists when just the speculators are informed and the market price when everyone is informed. GS investigate the conditions under which their MI is minimized with continuous trading. They find that, if 'sufficient' uncertainty surrounds the dissemination process, continuous trading is detrimental to market efficiency.

GS examine two types of dissemination uncertainty. Type I uncertainty (speed of dissemination uncertainty) is the length of time after the speculators receive the new data that the investors remain uninformed and type II uncertainty (path of dissemination uncertainty) is uncertainty concerning the number of speculators. With only type I uncertainty GS find their MI is minimized when there is continuous trading. The addition of a 'sufficient' level of type II uncertainty implies the existence of a unique optimal time interval between tatonnements which minimizes their MI. Furthermore this optimal time interval decreases as the level of type II uncertainty decreases.
In summary, it seems that efficiency is typically defined in terms of the equivalence of certain market characteristics under alternative information settings. In Fama (1976 a,b) it is the equivalence of the market assessed distribution function and the 'true' distribution. Rubinstein (with regard to new information) and Beja define efficiency with regard to identical portfolio positions. The most popular definition involves the equivalency of prices. Sharpe, Goldman and Sosin, Rubinstein, Figlewski, and Grossman and Stiglitz all compare existing market price to 'the efficient market price.' Finally, Huber defines efficiency as the equivalence of both prices and portfolio positions. With the exception of Goldman and Sosin, none of these studies explicitly consider the manner in which new data is disseminated to the market participants. In order to develop a definition of market efficiency in an environment consistent with gradual information dissemination, we introduce our market structure in the following section.

3. Market Structure

Imagine a world in which expected-utility-of-wealth maximizing individuals are faced with three decisions affecting wealth. They must choose a level of consumption, select a quantity of information activities and allocate their remaining funds among various investment opportunities. Some of these investment opportunities offer a stochastic return while others may be riskless. Suppose, as in Verrecchia (1980), the more an individual spends on the acquisition and processing of information, the more rapidly he can expect to react to the release of new data. The new data consists of an (assumed) exogenous signal which the investor views as being informative about the return(s) of some risky security(ies). In such an economy the actions of investors naturally form a continuum along the information expenditure line.
Some traders invest (relatively) large amounts in information with the expectation of being able to react quickly, other individuals spend little and are content with less of an informational advantage. If investors' beliefs about the economy are rational, their expectations about their position in the dissemination process must be fulfilled. 8

Thus individuals make irrevocable consumption and information related expenditure decisions for the current time period and trade to an equilibrium in the financial security market prior to the release of additional information. Each individual does, however, consider the anticipated implications of the forthcoming information event when selecting his optimal consumption, information and investment decisions. Subsequent to this initial equilibrium the information event occurs. This event may consist of either a public signal (everyone receives identical signals) or private signals. Regardless of the type of information investors receive, relative times are determined by the equilibrating process in the market for information (described in the preceding paragraph). The information market equilibrium implies investors react to the information event sequentially until all are informed. 9

In addition to the exogenous signal we assume each investor is capable of using equilibrium price as a 'noisy' signal of the other investors' information sets. This implies that as each investor receives and reacts to the exogenous signal, every investor infers as much as possible about that signal. To avoid problems of informative overefficiency (see Grossman-Stiglitz (1976)), we assume the existence of a second stochastic element in the economy which prevents a one-to-one correspondence between the equilibrium price changes and the exogenous signal. 10

After each exogenous signal those investors gaining information, either by directly observing the signal or making inferences about the signal from
price, alter their prior beliefs about the future value(s) of the risky asset(s) in question. We do not specify the exact process by which investors form their new beliefs.

Given the revised beliefs, individuals find their prior portfolio position to be suboptimal. They use the financial security market to alter this current investment allocation. The actions of these investors create a new equilibrium with a new market-clearing price based on the new set of beliefs.

This sequence of events is repeated as each investor becomes informed and results in a series of temporary equilibria between the initial equilibrium when no one is informed and the final equilibrium when all are informed. The process is illustrated below.

The financial market begins in equilibrium.

Step 1a: The first trader receives the exogenous signal.
1b: Investor(s) alter beliefs and retrade.
1c: The market achieves equilibrium.

Step 2a: The second trader receives the exogenous signal.
2b: Investor(s) alter beliefs and retrade.
2c: The market achieves equilibrium.

Step Na: The Nth (last) trader receives the exogenous signal.
Nb: Investor(s) alter beliefs and retrade.
Nc: The market achieves equilibrium.

Each of the equilibria, with the exception of the last, is merely temporary. These equilibria last only until the next investor becomes informed.

4. Market efficiency with gradual information dissemination.

None of the efficiency concepts mentioned earlier explicitly consider the information dissemination process (although most make the implicit assumption that information is available simultaneously to all investors). Our posited market structure, on the other hand, considers this information diffusion process as the central concern. Before we can address the issue of efficiency in this more general market setting, we must define an efficiency concept which
is consistent with the sequential receipt of information by the market participants. We offer such a definition below. Our interpretation retains the \textit{ex ante} emphasis of Huber and is intentionally traditional in the sense that it requires identical prices and allocations under alternative information structures.

A process determining final equilibrium prices and final allocations is efficient with respect to the information structure if, for every possible set of signal outcomes and for every possible sequence of signals received by the investors, final equilibrium prices and final allocations are identical to those prices and allocations which would be determined by the same process if the entire sequence of signals was received by each investor.

This definition suggests the following scenario. Suppose we generate $N$ signals and number them 1 through $N$. Further, suppose we arbitrarily assign the $N$ investors numbers, 1 to $N$. Assume investor $i$ observes only signal $i$. In an efficient market the final prices and allocations must be independent of the ordering of signals and individuals and identical to the prices and allocations which would exist if each investor observes all $N$ signals. In this definition we require nothing of the intermediate prices generated by the $(N-1)$ temporary equilibria. This is consistent with other traditional efficiency concepts which are concerned only with the property of a stable equilibrium price.

We posit this definition primarily as a 'strawman,' for the purpose of demonstrating the inadequacy of a dichotomous, efficient/inefficient designation in a market characterized by gradual information dissemination. Proper
consideration of this information diffusion process implies only very restrictive (and uninteresting) market settings achieve efficiency. We address this dilemma by proposing a measure of relative efficiency which is based on the sequential receipt of new data by investors.

We demonstrate the restrictiveness of the conditions required to guarantee efficiency in our market setting by focusing on a two date \((t = 0, 1)\) pure exchange economy with one risky and one riskfree asset. Individuals \((i = 1, 2, \ldots, N)\) are assumed to maximize the expected utility of date 1 wealth by selecting holdings of the riskless asset \((b_i)\) and the risky security \((z_i)\). The quantity \(z_i\) is investor \(i\)'s fractional holding of the total fixed supply of risky asset.

Each unit of the riskless security held is a claim to \(r\) units of time \(t=1\) wealth. The uncertain \(t=1\) value of the total supply of the risky asset is \(\tilde{X}\). Individuals enter the economy with an endowment vector \((\tilde{b}_i, \tilde{z}_i)\) which they consider optimal given their prior beliefs. The endowment vector, combined with the normalized price vector \((1, p)\), determines investor wealth,

\[
W_i = \tilde{b}_i + \tilde{z}_i p, \tag{1}
\]

which acts as a constraint in the selection of an optimal portfolio \((b_i^*, z_i^*)\).

Each trader maximizes his expected utility of time \(t=1\) wealth,

\[
E[U_i (\tilde{Y}_i)], \text{ where} \tag{2}
\]

\[
\tilde{Y}_i = r b_i + z_i \tilde{X}
\]

subject to the budget constraint. By substituting for \(b_i\) from equation (1) into equation (2), we may rewrite equation (2) as,

\[
\tilde{Y}_i = r W_i + z_i (\tilde{X} - r p), \tag{3}
\]

where \((\tilde{X} - r p)\) represents the excess return earned for placing wealth at risk.
Further assumptions are:

(A1) The capital markets are frictionless;

(A2) There are no short selling restrictions or taxes;

(A3) All market participants are price takers; and

(A4) Individuals' utility functions are bounded and exhibit nonsatiation and risk aversion.

All of these are fairly standard in asset pricing literature. Assumption (A3) implies individuals behave as if their actions do not affect market price.

The first order condition for the maximization of expected utility [see Mossin (1972)] is,

\[ \frac{dU_i}{dz_i} = E\{U_i'[rW_i + z_i (\bar{X} - rp)] (\bar{X} - rp)\} = 0, \]  

where \( E \{\cdot\} \) is the expectations operator. Formally equation (4) is written,

\[ \int_{-\infty}^{\infty} U_i' [rW_i + z_i (\bar{X} - rp)] (\bar{X} - rp) f_i(\bar{X}) d\bar{X} = 0, \]  

where \( f_i(\bar{X}) \) is the density function determined by investor i's beliefs.

In order to illustrate the inadequacy of the traditional efficient/inefficient designation in a sequential information receipt setting, we make an assumption about the density function in equation (5), \( f_i(\bar{X}) \). We assume that investor i believes that the end-of-period value of the risky security is determined as if it were generated by a random process which is well approximated as a normal distribution, i.e.,

\[ \bar{X} \sim N(E_i, V_i). \]

In this case the optimal holding of the risky security after the final trading opportunity (when everyone is informed) is,

\[ z_{i,N}^* = \frac{[E_{i,N}(\bar{X}) - rp_N]}{[V_{i,N}(\bar{X}) \cdot R_{i,N}(\bar{Y})]}, \]

where \( R_{i,N}(\bar{Y}) = -E(U_i''(\bar{Y}_i)) / E(U_i'(\bar{Y}_i)), \) the investor's risk aversion coefficient, and the subscript N denotes the final trading round values. This assertion is proven in the Appendix.
Our distributional assumption allows us to find closed form expressions for the investors' portfolio positions [equation (6)] and final equilibrium price. To write final market price, $P_N$, we make use of the equilibrium clearing condition,

$$\sum_{i=1}^{N} z_{i,N}^* = 1.$$  \hspace{1cm} (7)

Substituting for $z^*$ from equation (6) into equation (7) and solving for price yields

$$P_N = (1/r) \left[ \frac{\sum_{i=1}^{N} (E_{i,N}(X)/R_{i,N}(Y) \cdot V_{i,N}(X))^{-1}}{\sum_{i=1}^{N} (R_{i,N}(Y) \cdot V_{i,N}(X))^{-1}} \right]$$  \hspace{1cm} (8)

Equation (8) is the price that exists after complete information dissemination.

Our definition of efficiency requires that actual final equilibrium price equation (8), be identical to the price which would exist if each investor received every signal. Suppose we denote this 'efficient-market' price as $P_N^*$ and the investor's beliefs and risk aversion upon observing all $N$ signals as $E_{i,N}^*(\bar{X}), V_{i,N}^*(\bar{X})$ and $R_{i,n}^*(\tilde{Y})$, respectively. Before the market is efficient, i.e., $P_N = P_N^*$, it must be true that:

$$E_{i,N}^*(\bar{X}) = E_{i,N}^*(\bar{X}); \hspace{0.5cm} V_{i,N}^*(\bar{X}) = V_{i,N}^*(\bar{X}); \hspace{0.5cm} and \hspace{0.5cm} R_{i,n}^*(\tilde{Y}) = R_{i,n}^*(\tilde{Y})$$

for all investors.\textsuperscript{12} In addition, this statement must hold true independent of the order of information dissemination. We argue below that the market achieves this efficiency criterion only under very restrictive conditions.

Suppose the information structure is such that the signals generated are not identical and the equilibrium price is not perfectly invertible\textsuperscript{13} on the information set of the informed. Individuals receive only one signal (of $N$) free of noise. In general investors' beliefs, $E_{i,N}(\bar{X}), V_{i,N}(X)$ are not equal to $E_{i,N}(X)$ and $V_{i,N}(X)$. 
Suppose, on the other hand, the market structure is such that

(a) N identical signals are generated (and investors know they are identical), or,

(b) that price is perfectly invertible on the information set of the informed.

Condition (a) or (b) is sufficient to assure investors' beliefs are the same as if they receive each signal. In fact, if condition (a) or (b) holds investors do have access to all N signals.

In addition to identical beliefs equation (8) requires that \( R_{i,N}(\tilde{Y}) = R_{i,N}^*(\tilde{Y}) \) for all investors independent of dissemination order. One situation in which this is true is when investors' utility functions possess constant absolute risk aversion. In this case \( R_{i,N}(\tilde{Y}) = R_i \) for all levels of final wealth. However, if \( R_{i,N}(\tilde{Y}) \) is not constant in \( \tilde{Y} \) then, even with identical signals, \( P_N \neq P_N^* \). The inefficiency occurs because end-of-period wealth depends on the individual's order in the information process.

An individual who recognizes his priority position in the receipt of new data assumes a speculative position which (given reasonable assumptions about the information gathering process) increases, on average, that trader's wealth at the expense of his uninformed trading partners [see Jennings and Barry (1981)]. If we rewrite equation (3) as,

\[
\tilde{Y}_i = rW_{i,N} + z_{i,N}^* (\tilde{X} - r_{P_N}),
\]

the relationship between \( Y_i \) and \( W_{i,N} \) is evident. Since \( W_{i,N} \) depends on the order of information receipt, only condition (b) is sufficient to produce an efficient market.

Condition (b) has problems also. Grossman and Stiglitz (1975), (1976) have demonstrated that perfect price invertibility is incompatible with market efficiency if it is costly to gather information. Therefore, if information
acquisition is costly, complete efficiency cannot hold in the market we have presented.

Even if information is costless, Kihlstrom and Mirman (1975) and Grossman and Stiglitz prove that price is perfectly invertible if and only if a one-to-one correspondence exists between the equilibrium price and the information set, i.e., if there is no noise in the pricing process. Potential sources of noise are suggested by Grossman and Stiglitz (supply uncertainty) and Goldman and Sosin (dissemination uncertainty). Thus, even if information is costless, market efficiency would hold only under the very restrictive condition that prices are fully invertible.16

In this section we developed a traditional definition of efficiency which is consistent with gradual information dissemination. With this definition, price must be perfectly invertible on the information set of the informed before the market is efficient, and perfect invertibility is a highly restrictive condition. Thus, if we insist on the traditional efficient/inefficient dichotomy, the concept of market efficiency is uninteresting. What is needed is a measure of the degree of market efficiency. We suggest such a measure in the following section.

5. A measure of market inefficiency

The basic concept of information efficiency is that price adjusts quickly and completely to new data relevant to an asset's value. Existing research concentrates on the amount of price adjustment, ignoring the amount of time required for this adjustment to occur. We develop a measure of inefficiency which considers both the price and time dimensions. Thus, our metric is sensitive to both how much and how rapidly prices adapt to new information.

The previous section revealed two implications of the gradual dissemination of information for the concept of market efficiency. First we must be
sure our measure of inefficiency recognizes the ex ante stochastic nature of final market price. Each sequence of information receipt by investors may produce a unique value for $P_N$. Therefore, prior to the complete dissemination of the new data, final market price is a random variable, $P_N$.

We also found that ex ante there is no reason to believe final equilibrium price converges to the 'efficient-market' price, $P_N^*$. Given certain assumptions regarding investor behavior and the investors' information gathering system, Jennings (1981) shows, on average, final equilibrium price is 'correct', i.e., $E(P_N) = P_N^*$, but that $P_N = P_N^*$ with probability zero. To be useful in a gradual information dissemination environment, any measure of inefficiency we propose must compare the relative efficiency of different pricing processes which converge at different rates to different final equilibrium prices, none of which is the 'efficient-market' price.

Figure 1 represents a hypothetical price adjustment path through time. Price adjusts from an initial stable equilibrium price existing prior to the release of the information, $P_0$, to a final stable equilibrium price when everyone is informed, $P_N$, over the time interval $0 \leq T_N = t_N - t_0$. In our notation, $P_i$ is the equilibrium price which exists after individual $i$ becomes informed and $t_i$ is the time at which that investor receives the new data. If $0 < i < N$ the point $(P_i, t_i)$ represents a temporary equilibrium. The news becomes available at $t_0$, but the first investor does not become informed until $t_1 > t_0$. In general the quantities $P_{i+1} - P_i$ and $(t_{i+1} - t_i)$ depend on $i$.

In order to demonstrate our proposed measure of inefficiency (MI) we initially examine price adjustment paths which converge to $P_N^*$. This assumption is later relaxed with only minor modifications in the concept of our metric.
Figure 2 illustrates alternative price adjustment paths which represent: (A) smooth and rapid convergence towards $P_N$, and; (B) irregular and slow convergence to the same final price. (In drawing Figure 2 we have assumed the number of investors is large so that the step-function of price path of Figure 1 may be approximated by a continuous curve.) We believe curve 'A' represents a more efficient price adjustment process. Process 'A' is 'close to' the new equilibrium price throughout most of the adjustment period and converges monotonically to $P_N$. Curve 'B', on the other hand, remains 'far from' $P_N$ during most of the adjustment process and wanders about somewhat randomly. In order to objectively gauge the relative efficiency of various price adjustment processes, however, we must develop a rigorous measure of 'close to' and 'far from' as a function of time.

The most extreme concept of efficiency is one in which market price instantaneously changes to the new equilibrium level as soon as the information becomes available. In our model this implies the market price moves from $P_0$ to $P_N^* = P_N$ at time $t_0$ (when the first investor becomes informed) and that this price remains constant as each trader becomes informed, i.e., $P_1 = P_2 = \ldots = P_N$ and $t_0 = t_1$. Figure 3 represents this 'most efficient' price adjustment process. This extreme form of efficiency occurs if the investor first in the know faces so little uncertainty about future price that he assumes a speculative position large enough to force price to the new level or if the actions of the first informed investor tip-off the uninformed and they infer the new data from his actions. We use this 'most efficient' case as the standard against which we measure the relative efficiency of other price adjustment paths.

Figure 4 uses a hypothetical price adjustment path to illustrate the intuition behind our MI. The 'most efficient' price adjustment path is
represented by the line segments connecting point \((P_0, t_0)\), \((P_N, t_0)\) and \((P_N, t_N)\). Our hypothesized price adjustment path is the curve between \((P_0, t_0)\) and \((P_N, t_N)\). The shaded area represents the inefficiency of the actual price path relative to the 'most efficient' price path. Formally,

\[
\text{MI} = \int_{t_0}^{t_N} |P_N - p(t)|\,dt, \tag{10}
\]

where \(p(t)\) is the actual price at time \(t\). Our measure is sensitive to: (1) the differences in the price with only a subset of the market informed, \(p(t)\), and the price with everyone informed, \(P_N\), and; (2) the length of time these price deviations persist.

Our measure of inefficiency possesses the properties which Goldman and Sosin (1979, p.32) consider important:

(P1) Symmetry - equal absolute deviations of actual market price from the efficient market price should cause equal measures of inefficiency,

(P2) Monotonicity - a larger absolute deviation from the efficient market price should cause a larger measure of inefficiency,

(P3) Normalization - a zero deviation should imply a zero measure of inefficiency,

(P4) Convexity - a doubling of the absolute deviation should at least (in our case, exactly) double the measure of inefficiency, and,

(P5) Computational convenience - any measure of inefficiency should be straightforward to calculate.

In addition, our MI extends properties (P2), (P3) and (P4) to the time dimension as well as the price dimension. That is, a price adjustment process which suffers the same price deviation for time \(T' > T\) will be more inefficient.

Another important property is:

(P6) Insensitivity to units of measurement - changing the units of
measurement for either the price or time dimensions should not affect the relative measure of inefficiency.

Both the Goldman and Sosin measure (for the price dimension) and our MI possess this property.

In the above analysis we take \( P_N \) as given independently of the information dissemination process. The fourth section demonstrated that, in general, \( P_N \) is typically not equal to \( P^*_N \), the final equilibrium price that would exist if each investor received every signal. Extending our analysis to consider this additional factor does not alter the intuition behind our MI.

Figure 5 depicts the situation in which there is a permanent divergence between \( P_N \) and \( P^*_N \). Panel 5a illustrates the case where \( P_N \) undershoots the efficient market price and panel 5b represents the case where the final equilibrium price overshoots \( P^*_N \). In either case the intuition behind our measure of inefficiency is captured in the formal definition,

\[
MI = \int_0^T |P^*_N - p(t)| \, dt. \tag{11}
\]

In equation (11) we change the upper limit of integration to reflect the fact that without another information event (or some other reason to induce trading) any divergence between the market price and \( P_N \) at time \( T_N \) persists indefinitely.

There are two problems with the measure of inefficiency defined by equation (11). First, it provides an indefinite MI for any price adjustment path in which \( P_N \neq P^*_N \). This implies we are unable to compare the relative efficiency of two non-converging price adjustment paths.

The second problem is more subtle. The reallocation of wealth which occurs due to an investor possessing an information priority depends on exactly when the investor becomes informed. An investor receiving the news at time \( t_0 \) earns a return on his information gathering activity for the entire period.
The investor informed partially through the dissemination process has a shorter time period over which to earn his information priority return. Equation (11) ignores this reallocation process. We suggest a solution to these problems in the analysis later in this section.

While equation (11) does not allow us to compare the efficiency of any two arbitrary price adjustment paths, it does permit us to make some dominance arguments. Using the dominance idea we are able to judge the relative efficiency of select examples.

First, we find that a price adjustment process that converges to $P_N^*$ is always more efficient than a process which does not converge. If price adjustment process $A$ converges and process $B$ does not then,

$$\int_{t_0}^{t_f} |P_N^* - P_B(t)| dt > \int_{t_0}^{t_f} |P_N^* - P_A(t)| dt.$$ 

It may be, however, that for any finite time period, say $t_f > t_0$, we have

$$\int_{t_0}^{t_f} |P_N^* - P_B(t)| dt < \int_{t_0}^{t_f} |P_N^* - P_A(t)| dt,$$

i.e., $B$ is more efficient than $A$. Thus traditional definitions of market efficiency which require equivalence between the equilibrium price and some efficient-market price are correct only in the limit. In the short run a non-converging price adjustment path may be preferred to one which converges. Figure 6 illustrates this concept.

We may also use a dominance argument to judge the relative efficiency of two non-converging price adjustment paths under certain conditions. In Figure 7 price adjustment path $A$ dominates (in efficiency) path $B$ since $P_A(t)$ is everywhere closer to $P_N^*$ than $P_B(t)$. We are unable to judge, however, the relative efficiency of two non-converging price adjustment paths which intersect at any time $t > t_0$ with the concept represented by equation (11). In order to extend our analysis to any arbitrary information dissemination processes we must modify our MI.
Specifically we must alter the definition of our MI to assure a finite integral and to consider the timing of pricing deviations. One method which overcomes both of these problems is to examine the present value of price deviations per unit time, i.e.,

\[ \text{MI} = \int_0^\infty e^{-\rho t} |p_N^* - p(t)| dt, \]  

where \( \rho \) is an appropriate discount rate. With equation (12) those deviations occurring far in the future receive little (in the limit zero) weight. While (12) resolves the problem that MI might not be comparable over arbitrarily large time intervals, it demands a rational choice of the discount rate \( \rho \).

One could argue that to the extent that \( p(t) \) differs from \( P_N^* \) there is an aggregate misallocation of wealth that is linear in \( |P_N^* - p(t)| \). The misallocated wealth could be invested in the riskless asset, at least, so that the riskless rate of return could be justified in equation (12). Ultimately the choice of an "appropriate" \( \rho \) depends on societal values regarding the choice between inefficiency today versus inefficiency tomorrow.

We have, in equation (12), developed a measure of inefficiency that considers several important factors in determining the relative efficiency of a securities market. Our MI introduces the time dimension into the efficiency concept and handles its implications. In the following section we examine some of the economic variables which influence a market's relative efficiency as measured by (12).


In the previous section we developed our measure of market inefficiency. This section investigates some of the economic variables affecting the degree of efficiency. If the market is unbiased in its estimate of the impact of new
data, relative efficiency is determined by the rapidity of price adjustment. The speed at which prices adapt to the gradual dissemination of additional information is examined in Jennings and Barry (1981). Much of what we have to say here is contained in that paper in more detail.

The rapidity of price adjustment at any point in time is graphically portrayed by the slope of the price adjustment curve. If the slope is 'large' the adaptation of the price to the data is 'rapid'. We can divide the economic factors determining the shape of the price adjustment curve into market characteristics and individual investor attributes.

Individuals perceiving that they possess priority in the receipt of information speculate. The rapidity of price adjustment depends, in part, on the amount of speculative behavior. Significant speculation by the informed investors leads to rapid price adjustment. The size of the speculative position assumed by those traders in-the-know depends on their perceived ability to forecast future price. Many economic variables determine an investor's forecasting ability. Some of these variables are the type of signal, investors' knowledge about the economic attributes of his peers, the perceived reliability of the new data, the degree of dissemination uncertainty and the individual's perceived social weight in the pricing process. A factor not directly affecting the speculative activity of an investor but which affects portfolio positions (and, therefore, equilibrium price) is the information content of the signal for both the investor receiving the data and those traders who must infer the news from observing the price reaction.

Investors with an informational advantage are better off if the signal generating process produces identical signals for each trader. With identical signals the informed market participant can more accurately estimate the impact of the signal on other traders who subsequently receive the signal. This
knowledge permits the informed investor to improve his estimate of future price.

The level of knowledge possessed by an investor about the economic characteristics of his fellow traders also influences the size of the speculative positions taken. A trader with considerable knowledge concerning the current beliefs, risk aversion and the information gathering process of his trading partners assumes a larger speculative position than an individual without such knowledge.

The impact of additional information on an investor's beliefs depends on the perceived accuracy (precision) of the new data. If the additional information is perceived as possessing high (low) precision the investor assumes a large (small) speculative position.

An investor with a good idea of where he stands in the information dissemination process can clearly assume a larger speculative position than if he is unsure about his priority. If an investor believes that he may enjoy an informational advantage over no one he can ill-afford to speculate since he may be unable to rid himself of his extreme portfolio without suffering a loss.

In Jennings and Barry (1981) we show that the perceived social weight of an individual in the process determining future prices is important in deciding the speculative activity of the investor. Those investors who believe they are relatively important in setting price assume large speculative positions, while those who believe they are less crucial to the process are content with less speculation. This social weight is measured by the investor's subjective estimate of the correlation of his belief about future market price with the future price. The larger the absolute value of this correlation coefficient, the larger the speculation and the more rapid the price adjustment.
A signal with substantial information content for the recipient is one which contains information 'different' from the prior beliefs. For example, consider the case in which individuals are Bayesian in their use of new data, the random process of concern is normally distributed with unknown mean \( \bar{\mu} \) and known variance, and the new information can be thought of a sample from a normal process with the same unknown mean and a known sampling variance. An observed sample mean differing greatly from the individual's prior belief about \( \bar{\mu} \) causes significant revision of that prior. If the sample mean is equal to the prior belief the investor's posterior expectation about the unknown mean is identical to the prior. Thus a signal with significant information content causes more rapid price adjustment.

One of the most important factors in determining the rapidity of price adjustment is the ability of an investor not receiving the exogenous signal to learn from price adjusts more rapidly than a market ignoring the information content of equilibrium price. In a sophisticated market every individual responds to each signal. Thus, the receipt of a signal favorable (unfavorable) to future asset value by any investor tends to cause an increase (decrease) in demand for every investor. On the other hand, in a naive setting only the recipient of the signal alters his demand curve.

All of these factors are important in determining the shape of the price adjustment curve. Factors favorable to more rapid price adjustment tend to make the actual price path look like the 'most efficient' price adjustment path.

7. Summary and empirical implications.

In this paper we extend the theoretical examination of market efficiency to a market structure in which information is disseminated gradually to the investors. We demonstrate that the traditional definition of market
efficiency as the equivalence between equilibrium price and an 'efficient-market price' is not useful in this more general market setting. In place of the traditional efficient/inefficient dichotomy we developed a measure of relative efficiency. Our measure of inefficiency considers both the price and the time dimension in the price adjustment process. Specifically our MI is the present value of the deviations in the actual price adjustment path from the 'most efficient' price adjustment path per unit time. Formally this is,

\[ MI = \int_{t_0}^{\infty} e^{-\rho t} | P_N^* - p(t) | dt, \]

where \( t_0 \) is the time at which the information becomes available,

\( \rho \) is an appropriate discount rate,

\( P_N^* \) is the final equilibrium price which would exist if every investor received each signal.

and

\( p(t) \) is the actual market price at time \( t \).

This measure is sensitive to both how much and how quickly prices adjust to new data. Finally we discuss economic factors, both individual investor attributes and market characteristics, which influence the relative efficiency of a market place.

The next step in the process of learning more about the concept of efficiency is to attempt to use some of the concepts of this paper in empirical efficiency tests. One obvious drawback is that to calculate our MI you must know the equilibrium price that would exist with complete dissemination of all data, \( P_N^* \). Patell and Wolfson (1979) and Hillmer and Yu (1979) provide empirical techniques which estimate the point at which a market reaches a new equilibrium. Empirical research may be able to assume that \( P_N^* \) is an unbiased estimator of \( P_N^* \) or that signals are identical for certain information events which may allow us to estimate the relative efficiency of the market's reaction to various information events.
FOOTNOTES

1. The unmodified use of the term "efficient" in this paper refers to information (as opposed to economic) efficiency.

2. Despite the conclusions of Stiglitz (1981), many economists consider the competitiveness of the financial market to be linked to the economic efficiency of the economy as a whole.

3. See Fama (1970) for a metaphorical description of "the market."

4. Thus the name action efficiency. If the market is efficient the investor takes no portfolio action.

5. See Kihlstrom and Mirman (1975) for a formal derivation of the necessary and sufficient conditions for the uninformed to learn the information set of the informed from equilibrium price.

6. The efficiency concept introduced in Diamond and Verrecchia (1981) appears to be the first definition allowing a price adjustment following the public announcement of information. We further demonstrate the inappropriateness of a strict price equivalency by introducing sequential information dissemination.

7. The investor first receiving the new information has a competitive advantage from being first in the know. Jennings and Barry (1981) demonstrate this advantage.

8. Patell and Wolfson (1979) and Hilmer and Yu (1979) offer empirical evidence that the reaction of investors to an information event is indeed sequential.

9. Allowing for the possibility that investors choose to remain uninformed then investors are informed one at a time until all who wish to be informed are informed.

10. Typically it is assumed that endowments of the various assets are random. In this market setting we could also allow non-information related trades, i.e., trades due to consumption, shifting, preferences or liquidity. Either approach would add the required noise to the price system.

11. Thus we take the consumption and information expenditures as given.

12. We dismiss as unlikely the possibility of exactly offsetting differences in $E_{i,t}(\bar{X}), V_{i,t}(\bar{X})$ and $R_{i,t}(\bar{Y}_t)$ from their appropriate values such that $P_N = P^*_N$.

13. In the Grossman and Stiglitz and Kihlstrom and Mirman sense.

15. We are ignoring the (assumed) minor real time it takes for information to be disseminated in the market place. If an investor's planning horizon is one year and the information is fully disseminated within a day or two then, multiplying $W_{i,N}$ and $P_N$ by the annual riskless rate of return, $r$, is clearly an approximation. For illustrative purposes we believe it is close enough.

16. Barry (1979) has argued that the information requirements for fully invertible prices in the Kihlstrom-Mirmen market are extreme.

17. Speculative behavior is defined as an individual assuming a portfolio position he would not hold if there were not other trading opportunities prior to the distribution of final wealth.
BIBLIOGRAPHY


Beja, A., 1976, The limited information of market processes, Unpublished manuscript.


Jennings, R., and C. Barry, 1981, Speculation, market price adjustment and investor priority of information receipt, Unpublished manuscript.
Jennings, R., 1981, Efficiency in a financial market characterized by sequential information arrival, Unpublished manuscript.


FIGURE 1

Hypothetical Price Adjustment Path

Price

\[ P_n \]
\[ P_{n-1} \]
\[ P_5 \]
\[ P_4 \]
\[ P_3 \]
\[ P_2 \]
\[ P_1 \]
\[ P_0 \]

Time

\( t_0 \)  \( t_1 \)  \( t_2 \)  \( t_3 \)  \( t_4 \)  \( t_5 \)  \( t_{n-1} \)  \( t_n \)

Price

\[ P_n \]
\[ P_{n-1} \]
FIGURE 2

Two Alternative Price Adjustment Paths
FIGURE 3

The 'Most Efficient' Price Adjustment Path
FIGURE 4

Measure of Inefficiency
FIGURE 5

Non-Converging Price Adjustment Paths

5a - Undershooting

5b - Overshooting
FIGURE 6

Relative Efficiency

6a - Converging

6b - Non-Converging
FIGURE 7

Efficiency Dominance

Price

\( P_0 \) \( \rightarrow \) \( P_n \)

\( t_0 \) \( t_1 \) \( t_n \) Time

A \( \rightarrow \) B
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