Tax Depreciation and Risk

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A major issue under an accretion tax is how to treat depreciable assets, assets that tend to decline in value with time and use. The classic theoretical ideal is economic depreciation, a stream of deductions that exactly replicates the decline in value of each asset over time. Although some parts of U.S. law aim to replicate economic depreciation, tax depreciation is normally allowed at a rate that is faster than economic depreciation. The goal of "accelerated depreciation" is to provide a general subsidy to investment without distorting the choice between assets. Thus, the ideal is to allow acceleration of tax depreciation for all assets but to tailor the degree of acceleration for each asset so that no asset is favored over another strictly due to the tax depreciation rules.

Risk affects depreciable assets. For example, it is uncertain how long an asset will function. Some assets wear out and fail much earlier than others. This type of risk is "retirement risk." Retirement risk plays a special role in the analysis of depreciation rules.

There are other types of risk that affect depreciable assets. For example, there is "revenue risk." Assuming that an asset survives through a particular future year, the revenues from asset output and the future costs of operating the asset during that year are uncertain.

Risk can have a significant impact on the optimal design of depreciation rules. This Article analyzes that impact in two steps. Before starting the two step analysis, Part I discusses current law and the basic impact of risk on the depreciation system.

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* Professor of Law and Helen L. Crocker Faculty Scholar, Stanford University. I have benefited from comments received when earlier drafts were presented to the Los Angeles Tax Policy Group, Harvard Tax Policy Workshop, and workshops at Yale Law School and Stanford Law School. In addition, I am grateful for helpful comments outside of the workshop context from Bill Andrews, Ellen Aprill, Joe Bankman, Dick Craswell, Tom Griffith, Christopher Hanna, Louis Kaplow, Mark Kelman, Bill Klein, Michael Knoll, Ed McCaffery, David Mills, Eric Ramseyer, Roberta Romano, Matt Spitzer, Al Warren, and from various Treasury Department personnel. Finally, I am grateful for valuable comments and thorough research assistance from Bill Scarff. All errors are my own responsibility.


In the framework of an annual accounting period, economic depreciation of an asset calls for a deduction each year equal to the decline in value of the asset during that year.

2. No attempt will be made to separate risk rigorously into mutually exclusive and exhaustive categories. Such a categorization is not necessary for the analysis in the Article. It is convenient at several points, however, to treat retirement risk as a distinct kind of risk.
Part II contains the first step: an analysis under the assumption that the future price path of surviving units is known when an asset is put in service. This assumption is restrictive because the future price path for surviving units normally is uncertain. Nonetheless, this assumption implies that an especially simple set of rules results in a tax treatment that matches economic depreciation. In particular, the statutory schedule should consist of the expected price path for surviving assets. When an asset is retired, appropriate treatment is assured by allowing a deduction for the adjusted basis at the time of retirement. There is no need to adjust the statutory schedule to take retirement risk into account.

Given this simple rule as a baseline, Part II examines two prominent approaches for replicating economic depreciation: the leading academic approach originating in a paper by Charles Hulten and Frank Wykoff, and the government approach used by the Treasury Department to determine class lives. Part II concludes that the Hulten and Wykoff approach results in depreciation rates that are much faster than economic depreciation. The government approach is conceptually sound, but one of the two major ways that the government uses to implement that approach results in large errors. Although the government position was formed too recently to have had much impact on actual depreciation rules, the Hulten and Wykoff approach has had a substantial impact on current law.

Part II goes on to point out that even though retirement risk need not be taken into account in designing a schedule that results in a treatment equivalent to economic depreciation, retirement risk must be taken into account in designing an accelerated schedule that does not favor some assets over others. If two assets are subject to different rates of retirement risk, very different degrees of acceleration may be appropriate for the assets even though they would have the same depreciation schedule under a scheme that replicates economic depreciation.

Part III considers the complications that arise in the more general and realistic case where the asset price path for surviving units is not known in advance. Given that it is still necessary to establish a single ex ante depreciation schedule for each asset, “strategic loss-taking” comes prominently into play: There is an incentive to trade depreciable assets to establish price paths that fall below the statutory schedule. Part III discusses potential responses to this phenomenon.

The most promising general responses consist of changes in the “disposition” rules for depreciable assets combined with adjustments in the ex ante depreciation schedule. Under current law, dispositions result in an immediate tax on loss or gain. It is this feature that allows taxpayers to take losses strategically. Part III considers several alternative disposition

4. See infra note 24 and text accompanying notes 64-66.
5. See infra note 46.
rules such as not taxing dispositions at all and levying a tax on the gross proceeds (without subtracting adjusted basis). Applying these rules requires an accelerated ex ante depreciation schedule since the disposition rules no longer allow a deduction for the loss upon retirement of an asset. None of the combinations of disposition rules and ex ante schedule adjustments emerges as a dominant approach. Part III explores the trade-offs inherent in choosing one alternative over another.

Part IV applies the results from Part III to “group accounting” rules for depreciable assets. These rules permit groups of assets to be depreciated as if they were a single asset. The rules are important because group accounting can significantly lower administrative and compliance costs for some taxpayers. However, strategic loss-taking is a serious problem for some types of group accounts. As a result, a study of group accounting policy is an important and natural application of the theory developed in Part III. Part IV concludes that two of the special disposition rules studied in Part III are very promising approaches for the most troublesome types of group accounts.

Part V summarizes the main conclusions reached in the Article. These conclusions and results are relevant not only for the task of reforming the general depreciation rules in the tax code but also for addressing any tax problem involving assets that tend to depreciate in value.

I. BASIC DEPRECIATION POLICY CONSIDERATIONS AND CURRENT LAW

Before engaging in substantive analysis of depreciation and risk, it is important to set the stage by discussing current law and some general properties of tax depreciation in a risky environment. Section A discusses the basic choices that must be made in designing a depreciation system composed of depreciation schedules set in advance. Attention is devoted to asset retirement patterns and strategic loss-taking since these phenomena may have an important impact in such a depreciation system. Section B discusses the depreciation methods used under both individual item accounting and group accounting, the two major accounting approaches employed under current law.

A. BASIC DEPRECIATION POLICY CHOICES, ASSET RETIREMENT PATTERNS, AND STRATEGIC TRADING

Current tax depreciation rules contain both ex ante and ex post elements. Instead of taking the ex post approach of observing the actual decline in value suffered by each asset each year, a tax depreciation schedule is set in advance at the time the asset is placed in service. This schedule is the ex ante element of the tax depreciation rules. The two crucial features of the schedule are the total time period over which depreciation deductions will be allowed and the pattern of deductions during that “recovery period.” A taxpayer facing constant marginal tax rates and a positive interest rate over time will prefer to have a short recovery
period and a front-loaded pattern of deductions that shifts deductions into earlier years.

Although the tax depreciation schedule is set ex ante, ex post adjustments occur if the taxpayer sells or disposes of the asset. Two sale and disposition events are of particular interest: asset retirements and strategic trades. Asset retirement occurs when the asset reaches the end of its economic life and is discarded. Strategic trading occurs when a taxpayer chooses to sell a depreciating asset in order to achieve a favorable tax result. To illustrate the nature of these events and their impact on the tax depreciation schedule, two examples follow.

In both examples, we will assume that the tax treatment of dispositions will be the one currently applied under “individual item accounting.” That treatment consists of the taxpayer realizing a gain or a loss equal to the amount realized, if any, on disposition minus the adjusted basis. Since this treatment is linked to individual item accounting, we shall refer to it as the “individual item method” when discussing alternative treatments of asset dispositions. Individual item accounting itself will be de-

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6. I assume that salvage value is zero throughout the Article. This assumption simplifies the analysis, and the main results would not be affected by assuming a nonzero salvage value.

7. Dispositions of depreciable assets may result in capital gains and losses or ordinary gains and losses. The rules are moderately complex. Because most depreciable assets are used in a trade or business, these assets normally are subject to the provisions in section 1231. See I.R.C. § 1231(a)(3). That section aggregates gains and losses for assets covered by the section, and if there is a net gain in aggregate, it is treated as a capital gain. See id. § 1231(a)(1). If there is a net loss in aggregate, the loss is treated as an ordinary loss. See id. § 1231(a)(2).

Section 1231 is not the end of the story. The “recapture” provisions in sections 1245 and 1250 require that a portion of the gain on some depreciable assets be treated as ordinary gain. See id. §§ 1245, 1250. Under section 1245 any amount of gain up to the sum of all past depreciation allowances will be ordinary income. See id. § 1245(a)(1) and (2). The rules under section 1250 shift gain to ordinary gain to the extent past depreciation deductions have exceeded the deductions that would have been allowed under the straight-line method. Id. § 1250(a)(1) and (b)(1).

Although the Tax Reform Act of 1986 equalized capital gains and ordinary income rates, those rates have gradually diverged since that time, at least for high-income individuals subject to the highest statutory rates. At the present time, the highest federal statutory rate on ordinary income of individuals is 39.6% while capital gains for high-income taxpayers may be at rates as low as 20%. See id. §§ 1(a)-(d) and (h).

It is worth noting that even in tax environments where capital gains rates equal ordinary income rates, characterization as capital or ordinary is still important. Capital gains are valuable because they tend to free taxpayers from the limitation on capital losses that allows capital losses to be taken only to the extent there are offsetting capital gains. This limitation hinders the ability to take losses early and therefore diminishes the potential value of strategic loss-taking. That potential value is very high, about 10% of value for the typical stock market asset. For a comprehensive discussion of the relation between strategic loss-taking and the limitation on capital losses, see Jeff Strnad, Periodicity and Accretion Taxation: Norms and Implementation, 99 Yale L.J. 1817, 1870-72, 1879, 1882-84, 1885-1889 (1990).

The rest of this Article will ignore the characterization of gain or loss upon disposition as ordinary or capital. Under section 1231, losses from the disposition of depreciable assets will tend to be ordinary losses. The limitation on capital losses does not come into play. This fact means that not much is lost by ignoring the capital loss versus ordinary loss distinction when studying depreciable assets. Ignoring the distinction also simplifies the exposition of the major points in the Article.
fined precisely and contrasted with the major alternative, "group accounting," in section B.\textsuperscript{8}

To begin the first example, assume that the statutory tax depreciation schedule is the smooth curve shown in Figure 1.\textsuperscript{9} Thus, in the first year the taxpayer deducts $20 of the initial $100 value of the asset. The depreciation period is 10 years. Suppose, however, that the asset breaks down right after reaching age three and is discarded as worthless at that time. The adjusted basis at the beginning of that year was $51, and the taxpayer can take a loss in that amount upon discarding the asset.\textsuperscript{10} The ex post depreciation schedule is the one indicated in Figure 2. This schedule tracks the statutory schedule for the first three years and then abruptly drops off to zero in the third year.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Double Declining Balance with Switch to Straight Line}
\end{figure}

One variety of strategic trading is "strategic loss-taking." Like asset retirements, strategic loss-taking results in an ex post adjustment of the pattern of deductions specified ex ante by the statutory depreciation

\textsuperscript{8} See infra text accompanying notes 17-19.

\textsuperscript{9} This curve corresponds to MACRS depreciation for a ten-year recovery period under the Code except that the half-year convention is ignored. See I.R.C. § 168(d)(1) (requiring half-year convention). Thus, depreciation is at a declining balance rate of 20% per year and then switches to straight-line after five years have elapsed.

\textsuperscript{10} To take a loss in the year the asset is abandoned, the taxpayer must show abandonment. I assume throughout the Article that this showing is not an obstacle. For most depreciable assets, this assumption is innocuous because the assets are tangible. The real problems in showing abandonment or worthlessness have been associated with intangible assets such as securities or legal claims. See Michael J. Graetz, supra note 1, at 414-15.
schedule. Suppose that the statutory schedule is in annual increments representing the same annual decrements in value as the statutory sched-

Figure 2
Ex Post Depreciation Schedule

![Ex Post Depreciation Schedule](image)

ule in the Figure 1. This annualized schedule is represented by the step function in Figure 3. First year depreciation is $20 and initial cost was $100 so that the depreciation schedule is at $80 during the first year.\(^\text{11}\) In the same figure, the market value of the asset appears as an irregular curve.

Suppose that the taxpayer can costlessly sell the asset for its market value and replace it with a similar used asset of the same value.\(^\text{12}\) If the

\(^{11}\) The reason for using this particular step function is that asset value as a function of time is plotted on the same graph. Strategic loss-taking possibilities arise if asset value during any given year falls below the adjusted basis after depreciation is deducted at the end of the year. It is easier to see on a graph whether asset value during a year falls below adjusted basis as of the end of the year if the depreciation schedule is represented during each year as equal to the adjusted basis at the end of the year.

Figures 3 and 4 take this approach. Thus, for the first year in Figure 3, the depreciation schedule is represented by a constant $80 value. This $80 value is the adjusted basis at the end of the first year after the depreciation deduction of $20 for that year is taken into account.

\(^{12}\) The wash sale rules of section 1091 deny a loss deduction when sale is accompanied by purchase of a “substantially identical” asset within 30 days preceding or following the sale. I.R.C. § 1091(a). But section 1091 applies only to “shares of stock or securities” and thus does not apply to depreciable assets. \textit{Id.} Sale and repurchase of the same asset might be treated as a sham transaction so that the loss on sale would be denied. See \textit{Knetsch v. United States}, 364 U.S. 361 (1960) (holding a court can invoke sham transaction doctrine to ignore the form of a transaction with “no economic substance” and treat it as something else for tax purposes). However, many depreciable assets such as cars and computers are quite distinct but functionally fungible, and the taxpayer could probably avoid
The taxpayer faces a constant marginal tax rate during the entire ten-year life of the asset, then the taxpayer will want to "accelerate" depreciation by selling the asset during year three. Depreciation along the statutory schedule would have brought the adjusted basis of the asset down to $51 at the end of that year. By selling at the low of $28 during the third year, the taxpayer can take an additional loss of ($51 - $28) = $23 for that year. Assuming that the replacement asset is depreciated at the same rate as the original one but over the shorter life appropriate for a used asset, the effective depreciation schedule will be the step function in Figure 4. This schedule is the same as the one in Figure 3 except that it shifts down to a lower level in year three. In effect, the taxpayer has accelerated statutory depreciation by strategic loss-taking.

Figure 3
Asset Market Value Versus Depreciation Schedule

the sham transaction doctrine by buying a replacement asset that is similar but not identical to the asset that was sold. As a result, it would probably be difficult to design effective wash sale rules for depreciable assets.

13. The ability to trade costlessly means that it is possible to sell an asset when it reaches its lowest market value during some period. The trader need only sell and repurchase whenever the asset drops an infinitesimal amount. The last sale and repurchase will take place at the lowest market value during the period. This strategy involves a large number of trades, but each trade costs nothing.

14. In fact, used assets are treated the same as new assets under MACRS: IRC § 168 does not apply a different schedule or a shorter life to used assets. Although the replacement asset in the text example would have a ten-year life rather than a shorter life reflecting that it is used and not new, initially it would be depreciated at the same double declining balance (20% per year) rate as the original asset.
Two important characteristics of strategic loss-taking are worthy of mention. First, profitable strategic loss-taking is only possible for a particular asset if, at some point in time, the market value of the asset falls below the adjusted basis dictated by the depreciation schedule. Second, the viability of strategic loss-taking depends on transaction costs. If trading is costly enough, the benefits of "accelerating" depreciation will be less than the after-tax trading costs required to secure those benefits.

Strategic loss-taking is not the only form of strategic trading that is important for depreciation policy. But, because strategic loss-taking allows the taxpayer to alter the depreciation schedule set by the government, it must be considered carefully in the design of depreciation policy. As a result, strategic loss-taking looms large in the discussions that follow, and especially so in Part III.

15. If statutory depreciation is accelerated, the scope for strategic loss-taking is lessened. For example, if depreciation for the asset depicted in Figures 3 and 4 were accelerated so that adjusted basis were at or below $28 by the end of year three, then there would be no reason to trade in year three. The extreme instance of accelerated depreciation occurs under a cash flow income tax where the entire cost of an asset is deducted at the time of purchase. In that case, assuming the asset cannot fall below zero in value, there is no scope for strategic loss-taking at all. An example of this treatment in current law is I.R.C. § 179, which allows the immediate deduction of up to $19,000 in 1999 of equipment used in a trade or business and placed into service in that year. For subsequent years, the $19,000 limit gradually goes up to $25,000.

16. See infra note 128 (discussing taking long-term capital gains in a wash sale to enhance depreciable basis); infra text accompanying notes 173-174 (discussing "churning").
B. Current Law and Policy

1. Individual Item Accounting and Group Accounting

Under current law, the taxpayer has a choice for each asset or group of assets between two general approaches to depreciation accounting: individual item accounting and group accounting. The approaches differ in two significant ways. First, as the name suggests, group accounting typically involves the combination of several assets into a single account. At the election of a taxpayer, a group of assets is depreciated as if it were a single asset. Second, the tax treatment of asset dispositions under group accounting typically differs from the treatment under individual item accounting.

As mentioned above, under individual item accounting, a disposition results in a gain or loss equal to the amount realized less the adjusted basis.\textsuperscript{17} Under group accounting, the rules for dispositions are more complicated, and a detailed discussion of these rules is deferred until Part IV to accompany the main policy discussion of group accounting. It is important to note that divergent treatment of dispositions under individual item and group accounting is not inevitable. Indeed, later portions of this Article consider several group accounting disposition rules as policy options for individual item accounting.\textsuperscript{18}

The next subsection of this Part discusses the depreciation schedules that are applied under current law. These schedules are the same regardless of whether the taxpayer uses individual item accounting or group accounting.\textsuperscript{19}

2. Depreciation Schedules

There are two separate major schedules for tax depreciation in the Internal Revenue Code: the Modified Accelerated Cost Recovery System (MACRS) and the Alternative Depreciation System (ADS).\textsuperscript{20} These two

\textsuperscript{17} See supra text accompanying notes 7-8.

\textsuperscript{18} See infra text accompanying note 156.

\textsuperscript{19} For example, Treas. Reg. § 1.168(i)-1 (as amended in 1994), paragraph (d)(1) states that “[d]epreciation allowances are determined for each general asset account by using the applicable depreciation method, recovery period, and convention for the assets in the account.” “General asset accounts” are the group accounts for the Modified Accelerated Cost Recovery System (MACRS) applicable to tangible property placed into service after 1986. Similar provisions applied to previous regimes. For instance, “mass asset accounts” are the group accounts for the Accelerated Cost Recovery System (ACRS) applicable to tangible property placed into service from 1981-86, and the Proposed Regulations applicable to these accounts simply state that “a taxpayer may elect to account for mass assets... in the same mass asset account, as though such assets were a single mass asset.” Prop. Treas. Reg. § 1.168-2(h)(1), 49 Fed. Reg. 5940 (1984). See also infra text accompanying notes 243-247 (suggesting use of special depreciation schedules for some group accounts and expressing doubt that Treasury can create and impose those schedules in Regulations without explicit Congressional authorization).

\textsuperscript{20} In addition to these two major systems, there is a plethora of special rules in the Code. See, e.g., I.R.C. § 169 (five year amortization of pollution control facilities); I.R.C. § 174(b) (five-year amortization of certain research and development expenditures); I.R.C. § 194 (seven-year amortization of certain qualified reforestation expenditures); I.R.C. §§ 59(e) and 263(c) (five-year amortization for certain intangible drilling expenditures).
schedules differ in their relationship to economic depreciation, the theoretical ideal of matching the actual pattern of decline in asset value exactly by depreciation deductions. Generally speaking, the ADS system is meant to mimic economic depreciation while the MACRS system is designed to allow depreciation at a faster rate than economic depreciation in order to stimulate investment. Thus, the Code requires the ADS to be used whenever the goal is to measure income or to depreciate assets under a non-accelerated schedule. In addition, the Department of the Treasury, operating under Congressional mandate to study the class lives to be used as a basis for MACRS and ADS, explicitly assumes that ADS is meant to serve as a surrogate for economic

21. See supra text accompanying note 1.

22. For example, when a corporation distributes money to its shareholders, it is necessary to decide whether the money represents a nontaxable return of capital to the shareholders or is taxable as a distribution of corporate profits to the shareholders. The key question that must be answered to address this question is whether the corporation has profits to distribute. If it does, the distributions will be considered to be profits since the capital contributed by the shareholders is still intact and at work in the corporation after the distribution.

To determine whether a distribution is from profits, the Code sets up an "earnings and profits" account consisting of the net undistributed earnings of the corporation. To compute net earnings, an accurate measure of depreciation must be subtracted from gross earnings. The Code requires that the ADS be used for this depreciation computation. See I.R.C. § 312(k)(3) (1994).

23. For example, taxpayers must use ADS rather than MACRS to depreciate equipment used outside of the United States, equipment leased to tax exempt entities and equipment financed by tax exempt bonds. See I.R.C. § 168(g)(1)(A)-(C).

In each of these three cases, Congress did not want depreciation to be accelerated at all. Acceleration under MACRS was meant to stimulate investment in the domestic economy. Section 168 ensures that MACRS will not be allowed for investments used outside of the United States. Tax exempt entities already receive a total exemption from taxation. Section 168 ensures that they will not be able to receive tax benefits from accelerated depreciation through leases. Tax exempt entities already receive a total exemption from taxation. See GRAETZ, supra note 1, at 261. Section 168 prevents the double benefit that would arise from combining tax exempt financing with accelerated depreciation.

24. In the Tax Reform Act of 1986, Congress directed the Treasury Department to study the actual depreciation pattern of "all depreciable assets," and granted the Department authority both to change class lives for assets with existing class lives and to prescribe class lives for assets that do not have any. See I.R.C. § 168(i)(1)(B) (1986). In the Technical and Miscellaneous Revenue Act of 1988, Treasury's authority to alter the class lives of existing assets or to create class lives for assets without any was eliminated, but the directive to study the depreciation pattern of all depreciable assets was maintained. See I.R.C. § 168(i)(1) (1988); Technical and Miscellaneous Revenue Act of 1988, Pub. L. No. 100-647, § 6253, 102 Stat. 3342 (1988). The Senate first passed an amendment barring Treasury from lengthening class lives. This amendment was extended in Conference to cover all Treasury authority to alter class lives, including the authority to establish class lives for assets that currently have none. See H.R. CONG. REP. NO. 100-1104 § 6253 (1988).

depreciation.\textsuperscript{25}

In setting up MACRS in 1986, Congress wanted to allow depreciation that is “accelerated” in comparison to economic depreciation but intended that the degree of acceleration for each asset class be such that no class of assets would be tax-favored over any other class.\textsuperscript{26} MACRS operates by assigning assets to one of ten classes based on the “recovery period” to be used to depreciate the asset.\textsuperscript{27} For some classes of assets, MACRS allows front-loaded depreciation, that is, a pattern of depreciation with larger deductions in the earlier years of life.\textsuperscript{28}

In contrast, for most assets, ADS consists of straight-line depreciation over the “class life” of the asset.\textsuperscript{29} The government attempts to set the class lives of assets so that the ADS system approximates economic de-
Generally speaking, the class life of an asset is significantly longer than its "recovery period" under MACRS. The class life concept plays an important role in MACRS in addition to its central role under ADS. MACRS has ten asset categories and three depreciation patterns. The Code explicitly assigns some asset types to MACRS categories. If an asset has a class life but is not explicitly assigned by the Code to a MACRS category, it is assigned to one of six MACRS categories depending on its class life. If an asset has no class life and is not explicitly assigned by the Code to a MACRS category, then the Code classifies it as 7-year property for MACRS purposes.

II. OPTIMAL DEPRECIATION WHEN THE AGE-PRICE PROFILE IS KNOWN IN ADVANCE

Part I detailed some basic features of depreciation systems and set out some features of current law. With this background in hand, this Part examines the theory and practice of setting depreciation schedules and assigning recovery periods in a depreciation system. Sections A, B, and C study the situation where the goal is to match tax depreciation with economic depreciation. Section D examines the case of accelerated depreciation.

More specifically, Section A discusses a restrictive assumption, that the age-price profile of surviving assets is known in advance, and sets up an example used throughout the rest of this Part. Section B shows that given the restrictive assumption, there is a way to replicate economic depreciation. Section C shows that the required approach is quite different from the leading academic approach devised by Hulten and Wykoff and that some aspects of the current government approach also deviate from the most significant exceptions are nonresidential real property and residential rental property. These assets are subject to a “mid-month convention.” See I.R.C. § 168(d).

30. This is clear both from the legislative history of the Tax Reform Act of 1986 and from administrative practice. See supra note 25 and accompanying text.

31. See infra note 34.

32. See supra notes 27-28 and accompanying text.

33. This includes automobiles, light trucks, railroad grading and tunnel bores, residential rental property, nonresidential real property, railroad track, and a few other asset types. See I.R.C. §§ 168(e)(3), 168(c)(1).

34. See I.R.C. § 168(e)(1). The classification scheme is as follows:

<table>
<thead>
<tr>
<th>Class Life (in years)</th>
<th>MACRS Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 or less</td>
<td>3-year property</td>
</tr>
<tr>
<td>more than 4 but less than 10</td>
<td>5-year property</td>
</tr>
<tr>
<td>10 or more but less than 16</td>
<td>7-year property</td>
</tr>
<tr>
<td>16 or more but less than 20</td>
<td>10-year property</td>
</tr>
<tr>
<td>20 or more but less than 25</td>
<td>15-year property</td>
</tr>
<tr>
<td>25 or more</td>
<td>20-year property</td>
</tr>
</tbody>
</table>

The MACRS categories indicate the period over which tax depreciation is taken under that system while the class lives are the period under ADS. It is clear from the table set out above that the MACRS period for depreciation of an asset is generally much shorter than the class life of the asset.

required approach. Finally, Section D considers the issue of accelerating depreciation in a neutral manner starting from the baseline of economic depreciation.

In Part III, the restrictive assumption applied in this part is relaxed. The assumption is valuable because it leads to a particularly simple rule for replicating economic depreciation and situates the more complex considerations taken up in the latter Part. Given this rule, it is easy to provide a very direct critique in this Part of the Hulten and Wykoff approach and of the current government approach.

This Part focuses only on individual item accounting. A coherent discussion of group accounting methods requires the more complex setting developed in Part III.

A. An Assumption and an Example

1. The Assumption

As depreciable assets age and wear out, they generally decline in value. This decline reflects the fact that the asset’s revenue earning life is becoming shorter and shorter. The decline can be described by an “age-price profile,” a specification of the value of the asset at each point in its life. This Part studies optimal depreciation under the assumption that the age-price profile for surviving units is known in advance. That is, when the asset is purchased and put into service, the buyer (and the government) know exactly how much surviving units will be worth at each future time.

This assumption is restrictive. It is equivalent to assuming that after the asset is put into service, no new information concerning its durability or revenue earning potential will be discoverable at low cost. To make this point clear, consider racehorses, an example of a depreciable asset that will be used extensively later in the Article. It is clear that racehorses are subject to “revenue risk,” that is, the revenues that a particular horse will earn are uncertain at the time the horse is born. Racehorses are also subject to “retirement risk,” that is, it is uncertain how long a particular horse will be able to race or breed effectively.

36. See, e.g., Treasury Horse Study, supra note 24, at 15-16 (unpredictable and dramatic earnings and value differences between thoroughbred stallions used in breeding).

37. Retirement risk is clearly a major factor to address when considering the impact of risk on tax depreciation policy. Both government policymakers and leading academics are careful to adjust their estimates of economic depreciation to take into account this type of risk. See infra text accompanying notes 51-53. In addition, Treasury Department studies on depreciation patterns find that asset life is highly uncertain for many very different kinds of assets. See Treasury Rental Clothing Study, supra note 24, at 19 (tuxedos); Treasury Scientific Instruments Study, supra note 24, at 15-16 (eight types of scientific instruments); Treasury Fruit Tree Study, supra note 24, at 14 (peach trees); Treasury Horse Study, supra note 24, at 7-9 (thoroughbred geldings, thoroughbred stallions, thoroughbred mares); Treasury Light Trucks Study, supra note 24, at 16-17 (light trucks). These studies suggest that retirement risk is not only theoretically important, but also empirically significant.
The resolution of these risks over a horse's life will cause the value of the horse to differ from the value expected at birth. Thus, a horse's initial races at two years old will reveal a great deal about the horse's future revenue potential. Horses that appear to be Kentucky Derby material will shoot up in value and horses that appear to be losers will plummet in value. Also, after a few years of life some horses may be identifiably less durable or healthy than others. These horses will be expected to have shorter productive lives and will be less valuable than other horses. The result of risk resolution will be that the age-price profile of individual horses will deviate from the age-price profile anticipated when the horse is put into service.

Although the assumption that the age-price profile is known in advance is unrealistic for many assets, the assumption allows the analysis in this Part to be clear and simple. Part III then explores the implications for depreciation policy of relaxing the assumption.

2. The Example

This Part uses an example set up in this section to demonstrate many points. The example employs particular assumptions about the net revenue stream and retirement risk of the asset under study. Nevertheless, the basic conceptual results that underlie the entire Part will be shown to be independent of the particular example chosen. These conceptual results will hold so long as the asset's age-price profile is known in advance. The example combines a statistical distribution of asset lives and a curve representing net revenues at each point in time to yield asset value at each point in time. The net revenues at each future time for an asset that is still operating are assumed to be fixed and known in advance. We also assume that no new statistical information will be discovered about an asset's durability beyond the originally given statistical distribution of lives. Thus, the asset's prospects for additional life at any given later age can be specified at the time the asset is put into service.

Under these assumptions, it is easy to construct in advance an "age-price profile" representing the value of surviving assets at any given time. For each asset age, one simply computes the present value of future revenues taking into account the likelihood that the asset will survive to any particular later year. For the net revenue stream, we will make

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38. Consider a cohort of assets purchased at the same time. With an exponential life distribution, the proportion of assets in the cohort surviving to any particular age will decline in the same way that adjusted basis declines under a declining balance method of depreciation. See infra note 55. Each year the same proportion of the surviving assets fail, just as adjusted basis falls by the same proportion each year under the declining balance method.
a similar assumption: net revenues decline exponentially. For convenience, in this example, as in the rest of the Article, we also will assume that there is no inflation and that assets have no salvage value. When an asset fails, its value falls to zero.

With exponentially declining revenues and an exponential distribution of asset lives, the age-price profile for surviving assets will also be exponential. That is, the value of the asset will decline at a constant rate with age. With a real interest rate of 4%, an expected asset life of 10 years, and revenues that decline at a rate of 20% per year, the age-price profile is as shown in Figure 5. If the asset fails at some point during its life, the age-price curve will follow the curve in Figure 5 up until the time of failure and then will drop off instantaneously to zero.

For convenience, we leave the example in terms of “continuous time.” That is, we follow asset value changes as they occur from moment to moment. An alternative approach would be to annualize the changes in value. The curve in Figure 5 would then look like the “step function” in Figures 3 and 4.

This approach would not change the results but would make the analysis and the graphs awkward.

B. Replicating Economic Depreciation

When the age-price profile for surviving assets is known in advance, there is a simple rule that replicates economic depreciation. This rule calls for using a depreciation schedule based on that age-price profile combined with allowing a loss deduction when an asset fails.

For our example, the age-price profile for surviving assets is the curve shown in Figure 5. Basing the depreciation schedule on this curve means...
that the taxpayer should receive a dollar of depreciation deductions whenever the curve declines by a dollar.\textsuperscript{43} Under this rule, the curve represents the adjusted basis of surviving assets as well as their market value. So long as the asset survives, a tax depreciation schedule based on the curve will exactly match the decline in market value of the asset.

When an asset fails, the asset's value falls from the market value for surviving assets indicated by the curve to zero instantaneously. Economic depreciation requires an instantaneous deduction reflecting exactly that decline in value. But the rule we have specified calls for exactly this treatment. The taxpayer is allowed to deduct the remaining adjusted basis of the asset at the time of failure. This remaining adjusted basis is equal to market value right before failure.

In summary, basing the statutory depreciation schedule on the age-price profile for surviving assets and allowing a loss deduction when an asset fails results in a system that replicates economic depreciation perfectly. So long as an asset survives, tax depreciation is equal to economic

\textsuperscript{43} The continuous time nature of this example means that the taxpayer continuously receives depreciation deductions and the ensuing tax benefits. The accounting period is effectively zero since any decrease in market value results in an instantaneous deduction. A strong case can be made that this accounting period of zero is the appropriate one to use for an accretion tax. See Strnad, supra note 7, at 1821-22, 1832-47, 1853-63.

In a context such as the text example where interest rates and the decline in asset value are known with certainty, it is possible to collect taxes periodically (e.g., annually) but replicate the result that would occur under continuous taxation. Tax liabilities and benefits that arise before the end of the year can be paid or credited with interest at the end of the year. The result will be financially equivalent to continuous taxation. See id. at 1828-29.
depreciation because the tax depreciation schedule is exactly the schedule of the decline in market value of the asset. If the asset fails and is discarded, the statutory schedule no longer represents economic depreciation. However, deduction of the loss at the time of failure exactly corrects the schedule to correspond to economic depreciation.

Note that under this set of rules, there is no scope for profitable strategic loss-taking. Strategic loss-taking is potentially profitable only if market value falls below adjusted basis under the statutory depreciation schedule. In the tax scheme just presented, that situation occurs only when the asset fails and is discarded. But upon failure and disposition of the asset, the taxpayer receives a deduction equal to the loss suffered. There is no need to trade the asset to receive the loss. Furthermore, far from being improper, this deduction is required in order to replicate economic depreciation.

C. Government and Academic Approaches

The rule set forth in the previous section is a rule that results in a tax system based exactly on economic depreciation. Furthermore, this rule is impervious to strategic loss-taking. Using this rule as a benchmark, this section studies the Hulten and Wykoff approach and the government approach designed to achieve that same result. Because of improper treatment of retirement risk, the Hulten and Wykoff approach generally results in depreciation that is greatly accelerated compared to economic depreciation. The government's approach is conceptually sound, but one of the two major variants it uses to implement its approach results in significant errors. In addition, even if the correct variant is employed, the government's approach will fail if strategic loss-taking is viable. Subsection 1 discusses the Hulten and Wykoff approach while subsections 2, 3, 4 and 5 examine the government approach.

1. The Hulten and Wykoff Approach

The leading academic work on the measurement of economic depreciation is a 1981 article by Charles Hulten and Frank Wykoff. Hulten and

44. See supra text accompanying notes 12-15.
45. The taxpayer need only show that the asset has been discarded or abandoned. For depreciable assets that are discarded, it normally should be easy to show abandonment. See supra note 10.
46. See Charles Hulten and Frank Wykoff, supra note 3.

This work is held in very high esteem by the academic community. See, e.g., John B. Shoven, Commentary on Investment Incentives in Theory and Practice, HULTEN & WYKOFF inUNEASY COMPROMISE: PROBLEMS OF A HYBRID INCOME-CONSUMPTION TAX 342 (1988). Treasury depreciation studies also cite Hulten and Wykoff favorably and rely on Hulten and Wykoff for guidance. See, e.g., Treasury Rental Clothing Study, supra note 24, at 20; Treasury Scientific Instruments Study, supra note 24, at 19.

Hulten and Wykoff's work has had significant impact on the actual depreciation rules in the Internal Revenue Code. A 1984 Treasury tax reform study, generally referred to as "Treasury I," proposed the Real Cost Recovery System (RCRS), a set of depreciation rules meant to replicate economic depreciation. These rules were based directly on Hulten and Wykoff's results. A later administration proposal (Treasury II) used the recovery periods
Wykoff's central claim is that economic depreciation can be measured precisely enough so that a tax system based on economic depreciation is a practical option. They establish this claim by showing that it is possible to calculate plausible economic depreciation rates for a wide range of assets using existing data. It is not clear whether Hulten and Wykoff intended that the government use the derived rates directly as the applicable statutory rates for tax depreciation, but the government has done just that in some important settings. We therefore use the term “Hulten and Wykoff approach” even though Hulten and Wykoff may have been well aware of the problems inherent in using their derived rates directly as a tax depreciation schedule.

To compute a curve representing economic depreciation, Hulten and Wykoff begin with market price data. This data gives them an age-price profile, that is, a curve that specifies the market value of surviving assets at each age. Hulten and Wykoff note that this profile “miss[es] an essential point” because it represents “only the value of assets which have survived long enough to be eligible for sampling.”

In response, they adjust the price at each age by multiplying by the probability of survival to that age. The resulting product is an average price where the average is over both surviving assets and assets that have failed earlier and are now worthless. In their own words, this average

and depreciation patterns set out in RCRS as the baseline for proposing a system of accelerated depreciation. With some slight modifications these periods and patterns became MACRS, the accelerated depreciation system currently in effect. See U.S. TREASURY DEP’T, TAX REFORM FOR FAIRNESS, SIMPLICITY, AND ECONOMIC GROWTH, volume 2, at 160 (1984); Brazell et al., A History of Federal Tax Depreciation Policy, OTA Paper 64, at 22-23 (1989).

47. See Hulten & Wykoff, supra note 3, at 82-83, 112.

48. See id. at 94-96.

49. They discuss proposals for statutory tax depreciation schedules, propose their own depreciation rates, and then conclude that their derivation of rates shows that it is not necessary for tax depreciation schedules to be “politically determined.” See id. at 81-82, 94-96, 112. Because the central focus of their article is the ability to derive tax depreciation schedules, it is easy to interpret their article as suggesting that their depreciation rates are meant to be tax depreciation schedule rates.

As discussed below, the problem with directly applying Hulten and Wykoff’s rates as a tax depreciation schedule is that the adjustments that Hulten and Wykoff make for retirements should not be incorporated in the schedule. Having to adjust for retirements makes Hulten and Wykoff’s argument that it is possible to derive tax schedules based on empirical rather than political considerations more difficult. It is not clear why they would include that adjustment if they did not believe the adjustment was appropriate for computing tax rate schedules for actual application.

50. See supra note 46.

51. Hulten & Wykoff, supra note 3, at 91. Hulten and Wykoff note that this type of problem is called "censored sample bias" in econometrics. Id. The sample of prices at each age has been "censored" by removal of the assets that have not survived to be that old.

52. Suppose, for example, that at age 10, surviving units are worth $100 but that only one half of all units survive to this age. Under the Hulten and Wykoff approach, the price at age 10 of $100 is multiplied by the probability of survival which is .50. The result is $50. But this is the average value of all units (both surviving and retired) at age 10. Half of the units are retired and have value $0 and half are functioning and have value $100.
price "takes into account both survivors and nonsurvivors."\footnote{53} Hulten and Wykoff also find that, on average, age-price profiles are nearly exponential in form.\footnote{54} That is, assets tend to decline in value at a constant rate per time interval.\footnote{55} Hulten and Wykoff declare this result "the most significant finding" of their research.\footnote{56} However, they also note that the exponential form for the age-price profiles only holds approximately as an average over many types of assets. In a rigorous statistical test for the shape of these profiles, the exponential form, along with its chief rivals, fails to be statistically acceptable.\footnote{57}

Hulten and Wykoff propose approximating economic depreciation by assuming an exponential decline in value and by using age-price profiles corrected to include nonsurviving assets in computing the depreciation rate for each asset.\footnote{58} The adjustment for retirement risk under the Hulten and Wykoff approach results in a new age-price profile that is lower than the old profile that applies only to surviving assets. At each age, the price in the new profile is the probability of survival up to that age multiplied by the price for that age in the old profile.

The effect of the Hulten and Wykoff approach can be illustrated using the example developed in the earlier sections. Consider Figure 6. The upper exponential curve is the age-price profile for surviving assets. The lower exponential curve is the age-price profile adjusted in the way that Hulten and Wykoff advocate.

Using the lower exponential curve instead of the upper curve will result in depreciation that is much faster than economic depreciation. A surviving asset moves along the upper curve but is depreciated as if the price falls off more sharply along the lower curve. When an asset fails, there is a loss deduction equal to the height of the lower curve. This amount is less than the actual loss, which is the height of the upper curve. The diff-

\begin{table}[h]
\centering
\begin{tabular}{lll}
\hline
Year & Adjusted Basis at Beginning of Year & Deduction \\
\hline
1 & $100.00 & $20.00 = .20 \times$100 \\
2 & $80.00 & $16.00 = .20 \times$80 \\
3 & $64.00 & $12.80 = .20 \times$64 \\
4 & $51.20 & $10.24 = .20 \times$51.20 \\
\hline
\end{tabular}
\caption{Adjusted Basis and Deduction Example}
\end{table}

If the asset actually declines at a constant 20% rate per year, then this declining balance method will correspond to economic depreciation.\footnote{56} Hulten and Wykoff use the term "geometric" instead of "exponential." See id. The terms have the same meaning: curves that change at a constant instantaneous rate.\footnote{54} The correlate of constant exponential decline among depreciation methods in the Code is the declining balance method. Under this method, a constant proportion of adjusted basis is deducted each year. For example, if the declining balance rate is 20% and the asset is initially worth $100, then the pattern of deductions for the first four years will be as follows:

53. Id. at 91.
54. See id. at 93.
55. The correlate of constant exponential decline among depreciation methods in the Code is the declining balance method. Under this method, a constant proportion of adjusted basis is deducted each year. For example, if the declining balance rate is 20% and the asset is initially worth $100, then the pattern of deductions for the first four years will be as follows:

<table>
<thead>
<tr>
<th>Year</th>
<th>Adjusted Basis at Beginning of Year</th>
<th>Deduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$100.00</td>
<td>$20.00 = .20 \times$100</td>
</tr>
<tr>
<td>2</td>
<td>$80.00</td>
<td>$16.00 = .20 \times$80</td>
</tr>
<tr>
<td>3</td>
<td>$64.00</td>
<td>$12.80 = .20 \times$64</td>
</tr>
<tr>
<td>4</td>
<td>$51.20</td>
<td>$10.24 = .20 \times$51.20</td>
</tr>
</tbody>
</table>

56. Id. at 93.
57. This means that there must have been many age-price profiles in their study that deviate significantly from the exponential form.\footnote{57} See id. at 94-96.
The Effect of the Hulten & Wykoff Adjustment

Figure 6

The difference in heights between the curves represents an amount of loss at failure that the taxpayer has been permitted to deduct prematurely, that is, before failure actually occurred.

It is possible to express the acceleration effect under the Hulten and Wykoff approach in terms of class lives using a Treasury Department method described in subsection 2.59 Figure 6 includes the straight-line schedules that are equivalent in present value to the age-price profiles in the figure. These schedules translate the upper and lower age-price profiles into class lives of 8.40 and 5.87 respectively. Using the Hulten and Wykoff approach has reduced class life by about 30%.60

59. See infra text accompanying notes 62-85. Treasury's approach involves adjusting both the age-price profile and the approximating straight-line schedule for retirement loss deductions. It turns out that the proper adjustment for the age-price profile for surviving assets transforms it into the Hulten and Wykoff age-price profile. See infra text accompanying notes 89-91. If one starts out with the Hulten and Wykoff schedule, correcting it for retirement losses (as called for in the government approach) is redundant since that correction is built into the schedule to begin with. This "double correction" results in class lives being much too short.

60. To the extent that this particular example is atypical, it may underestimate the acceleration effect of using the Hulten and Wykoff approach. In the example, net revenues per unit of time for surviving assets fall off at the sharp rate of 20% per year while the asset survival rate is 90% per year. The steep rate of decline in revenues causes the value of surviving assets to fall sharply with age. Combined with the high asset survival rate, this sharp decline means that losses at retirement will not figure as prominently as they would normally. Put another way, the revenue drop causes the curve to drop so sharply that the additional drop due to multiplying by the probability of survival is relatively small. This additional drop is the adjustment required by the Hulten and Wykoff approach.

To illustrate these points with a concrete example, suppose that we change the example so that net revenues per unit of time fall off at only a 10% rate and the asset survival rate is
Hulten and Wykoff's estimates and methods have strongly influenced statutory and administrative recovery periods. These recovery periods may be much too short.

2. Description of the Government Approach

The current tax depreciation treatment of assets depends heavily on their class lives. These class lives determine ADS treatment in almost all cases and determine MACRS treatment for all assets except for the assets specifically assigned to a MACRS category by the Code.

Not surprisingly, the Treasury Department's ultimate goal in studying depreciation for each asset is to determine an appropriate class life for the asset. In particular, Congress has mandated that the Treasury Department study the actual depreciation history of assets so that Treasury might propose revisions to existing class lives and might propose class lives for assets that currently have none. The legislative history concerning this mandate includes the following directive to Treasury about how class lives are to be calculated: "Class lives . . . [should] be determined such that the present value of straight-line depreciation deductions over the class life, discounted at an appropriate real rate of interest, is equal to the present value of what the estimated decline in value of the asset would be in the absence of inflation." Treasury has termed the class life that emerges from this type of calculation the "equivalent economic life" of the asset.

In the six reports issued to date under its study mandate, Treasury reveals its approach to establishing depreciation schedules that replicate economic depreciation. Some of the reports are a bit vague on the details of the approach, but one of them, the study of the depreciation of scientific instruments, is very explicit. That study uses two slightly different

only 80% per year. In this case, the class life corresponding to the age-price profile for surviving assets is 13.64 years. Using the Hulten and Wykoff approach lowers the class life to 5.50, a reduction of almost 60%. This reduction is about twice as large in percentage terms as the reduction in the text example.

61. See supra note 46.
62. See supra note 29 and accompanying text.
63. See supra text accompanying notes 33-34.
64. See supra note 24.

Throughout this Article, the effects of inflation on the tax depreciation system are ignored. Adding inflation into the examples and analysis would complicate them and no real conceptual gain would result.

It seems clear that indexing adjusted basis for inflation would be a good solution to the problems inflation presents to the depreciation system. Other solutions, such as providing accelerated depreciation to offset the taxation of inflationary gains, have the weakness that they are a precise cure only when inflation is at a particular rate. If the actual inflation rate differs from that rate, these solutions simply distort investment incentives. See Graetz, supra note 1, at 398-400.

66. See, e.g., Treasury Rental Clothing Study, supra note 24, at 1; Treasury Scientific Instruments Study, supra note 24, at 1.
67. See supra note 24.
68. A mathematical appendix in that study sets out one variant of Treasury's approach. See Treasury Scientific Instruments Study, supra note 24, at 49-52 (Appendix C).
approaches, and the approaches taken in the other five studies either are, or appear to be, similar to these two approaches.

The goal of the government approach is to produce a straight-line schedule that will result in the same present value of depreciation as economic depreciation. The derived straight-line schedule then indicates the class life that should be assigned to the asset for the ADS, the depreciation system in the tax code that is meant to approximate economic depreciation.69

The government approach includes two stages. First, a retirement adjusted age-price profile is generated. Second, the straight-line schedule that would result in the same present value of depreciation taking into account retirements is computed.70 The total life spanned by the straight-line schedule is the class life for the asset.

a. Stage One: Computing a Retirement-Adjusted Age-Price Profile

The first stage begins with the observation or estimation of the decline in value of the assets over time. This stage results in an age-price profile

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69. See supra text accompanying notes 22-25.

70. The Treasury Department uses a 4% discount rate to compute present values in its depreciation studies. See Treasury Rental Clothing Study, supra note 24, at 24; Treasury Scientific Instruments Study, supra note 24, at 22; Treasury Fruit Tree Study, supra note 24, at 23; Treasury Horse Study, supra note 24, at 13; Treasury Passenger Cars Study, supra note 24, at 10; Treasury Light Trucks Study, supra note 24, at 13.

Where the approximating straight-line schedule has a very different shape from the original age-price profile, it is possible that the class life results will be sensitive to the choice of discount rate. It is unclear how solid Treasury's 4% assumption is. Over the past 60 years, U.S. Treasury bills (short-term notes) have averaged only a 0.4% real return. For a discussion of the tax policy implications of this low historical average rate, see Joseph Bankman & Thomas Griffith, Is the Debate Between an Income Tax and a Consumption Tax a Debate About Risk? Does It Matter, 47 TAX L. REV. 377 (1992).

There is a great deal of variance in the ex post real rate of return. During the 1940s and 1970s, Treasury bills had a significant negative ex post real rate of return. During the 1926-1934 period and during the 1980s, bills had a large positive ex post real rate of return. The 4% figure would not have been a bad estimate of the ex post real rate during the late 1970s and 1980s, the period immediately preceding the Treasury depreciation studies. See Richard A. Brealey & Stewart C. Myers, PRINCIPLES OF CORPORATE FINANCE at 126 (table), 546 (chart) (3d ed. 1988).

However, it is not the ex post real rate that is critical but the real rate that is expected ex ante. Investors will base their economic decisions on that rate. Ex post rates may end up being particularly high if investors overestimate inflation.

Another issue makes matters even more complicated. It is unclear whether the ex ante real riskless rate is stable. Some researchers claim that it displays a unit root which implies that the time series of ex ante real riskless rate is nonstationary. See, e.g., Andrew K. Rose, Is the Real Interest Rate Stable?, 43 J. FIN. 1095 (1988). This result is controversial. See Lai, Is the Real Interest Rate Unstable? Some New Evidence, 29 APPLIED ECON. 359 (1997).

If the ex ante real interest rate is unstable, it will not tend to return to any particular average value but will wander around more or less at random. Thus, long runs of negative or positive real rates are to be expected if the past is prologue. Qualitatively, these runs seem to characterize real ex post returns on U.S. Treasury bills. Furthermore, there is no reason for the average real rate over a long time period to be any particular value. Picking an appropriate real riskless rate to use in tax policy studies is no easy matter!

Finally, the 4% figure is not adjusted for taxes. However, the failure to make such an adjustment may be acceptable if estimating economic depreciation for system-wide application is the goal. See infra note 253.
relating the market value of each asset to its age.\footnote{71}

After deriving an age-price profile, the government approach adjusts this profile to take retirements into account and computes the present value of depreciation under the adjusted schedule. It is here that two variants appear, and both are present and clearly described in the scientific instruments study. One variant is equivalent to the Hulten and Wyckoff approach. The age-price profile for surviving assets is adjusted by multiplying the value of surviving units at each age by the probability of survival.\footnote{72} Since this adjusted profile gives average value as a function of age, it is possible to derive from the profile an “age-depreciation profile” describing the rate of decline in value for the average asset at each age.\footnote{73}

The discounted present value of the decline in average value suffered at each age is the present value of economic depreciation that will be used to compute an equivalent straight-line depreciation schedule.

The other variant is more complex and consists of four steps.\footnote{74} The first step is to compute age-price profiles for units with different known lives.\footnote{75} As a second step, the prices in each age-price profile for a unit of known life are divided by the initial price of a unit with that life. This normalization results in a normalized initial price of one for all units, regardless of life.\footnote{76} The third step is to compute the present value of depreciation as a function of the known asset life.\footnote{77} For each particular life, this present value of depreciation is simply the present value of the declines in value described by the normalized age-price profile for an asset with that life. Finally, since asset lives are unknown, a weighted average of the present values of depreciation for each asset life is constructed where the weights are the probabilities that the asset will have any given

\footnote{71} Treasury has employed two different methods to construct age-price profiles: the market data method and the productivity method. See Treasury Rental Clothing Study, \textit{supra} note 24, at 20-22 (productivity method); Treasury Scientific Instruments Study, \textit{supra} note 24, at 19, 21 (mixture of market data method and productivity method); Treasury Fruit Tree Study, \textit{supra} note 24, at 22-23, 40-41 (productivity method); Treasury Horse Study, \textit{supra} note 24, at 11, 14, 19-21 (market data method); Treasury Passenger Cars Study, \textit{supra} note 24, at 10-11 (market data method); Treasury Light Trucks Study, \textit{supra} note 24, at 12 (market data method).

The market data method consists of using data on market price versus age to estimate the age-price profile. The productivity method is used when market data does not suffice for providing an accurate estimate. \textit{See infra} text accompanying notes 250-253 (description of productivity method in Appendix A).

\footnote{72} See Treasury Scientific Instruments Study, \textit{supra} note 24, at 21-22.

\footnote{73} See, e.g., Treasury Rental Clothing Study, \textit{supra} note 24, at 24; Treasury Fruit Tree Study, \textit{supra} note 24, at 23-25.

\footnote{74} These steps are clearly described in Appendix C of the scientific instruments study. See Treasury Scientific Instruments Study, \textit{supra} note 24 App. C, at 49-52.

\footnote{75} See Treasury Scientific Instruments Study, \textit{supra} note 24, at 49 (equations 1 and 6); Treasury Fruit Tree Study, \textit{supra} note 24, at 29 (Figure 5 and accompanying text); Treasury Horse Study, \textit{supra} note 24, at 12 (Figure 4 and accompanying text).

\footnote{76} See Treasury Scientific Instruments Study, \textit{supra} note 24, at 49 (equations 2 and 7); Treasury Fruit Tree Study, \textit{supra} note 24, at 29 (Figure 5 and accompanying text). \textit{See also} Treasury Horse Study, \textit{supra} note 24, at 12 (Figure 4 and accompanying text; horse prices for different lives are normalized to be the same at the start).

\footnote{77} See Treasury Scientific Instruments Study, \textit{supra} note 24, at 50-51 (equations 3, 4, 8 and 9).
This second variant takes retirement risk into account by averaging together the present value of depreciation for assets with different lives.

b. Stage Two: Computing an Equivalent Straight-line Schedule

The result emerging from either variant of the first stage in the government approach is a present value for economic depreciation that is adjusted to include retirement deductions. In the second stage, the government computes an adjusted straight-line depreciation schedule that results in the same present value of depreciation as the present value of depreciation computed in step one. This straight-line schedule is “adjusted” in the sense that loss deductions due to asset retirements are included in the present value computation. The adjustment process takes the retirement loss at each time, multiplies it by the probability that retirement will occur at that time, and discounts the product back to “time 0,” the time the asset is placed in service.

Thus, the government approach adds the present value of retirement losses to the present value of decline in value along each straight-line schedule before picking one with the same present value as the retirement adjusted age-price profile. Retirement losses accelerate depreciation relative to the schedule, and, as a result, adjusting straight-line depreciation for retirement risk in this way increases the present value of deductions for each straight-line schedule. Thus, a straight-line schedule with a longer life will be required to match any given present value derived from the retirement-adjusted age-price profile.

78. See id. (equation 5 and text following equation 9).
79. See id., at 19-20, 22; Treasury Horse Study, supra note 24, at 13-14.
80. In two of the six studies the government also considered asset sales. In the Treasury Passenger Cars Study, supra note 24, sales replaced retirements in the computations. The reason for this approach is that business use passenger cars typically are sold before the end of their useful life for household (non-business) use. See id. at 2, 24. The government assumed that dispositions in the future would match the pattern observed in the study sample from the mid and late 1980s and that this pattern was exogenous to the actual tax treatment applied. See id. at 32. The only new element that arises from using sales instead of retirements is that sales may result in gains as well as losses. The case would be the same if we considered salvage value upon retirement since that value may exceed the adjusted basis of the asset. Taking the disposition patterns to be exogenous means that no explicit allowance for strategic loss-taking is implicit in the approach.

The second study that considers asset sales is the Treasury Light Trucks Study. See supra note 24. This study takes both retirements and dispositions into account. See id. at 2. But as in the Treasury Passenger Cars Study, the disposition pattern is exogenous and assumed to be the same as the pattern that emerges from the data collected for the study. Thus, there is no explicit consideration of strategic loss-taking. In fact, the numerical results presume that gains rather than losses will predominate in asset sales under the equivalent straight-line schedule. See id. at 36 (Table A-4, column 4).

81. When retirement occurs, the remaining adjusted basis is deducted immediately instead of gradually as indicated by the rest of the schedule. Considering retirement risk therefore increases the present value over the level that would follow from the schedule alone.

82. When the present value of depreciation as a percentage of initial value is lower, the straight-line schedule that produces the same present value of depreciation must be less steeply sloped. Since the class life corresponds to the point where the straight-line schedule crosses $0 value, the class life will be longer.
As an illustration of the government's approach, consider Figure 7, which is based on the example developed previously. The exponentially declining line is the retirement adjusted age-price profile for the asset based on the Hulten and Wykoff approach. The present value of depreciation for this schedule is 88.44% of initial value. The lower straight-line schedule results in the same present value of depreciation if deductions for retirements are not included in the computation. Since this line cuts the horizontal axis at 6.40, 6.40 years is the "appropriate" class life for the asset. The higher straight line results in the same present value of depreciation plus retirement losses.

Adjusting a straight-line schedule for retirement risk requires knowing the distribution of lives for the asset under study. In general, the present value of retirement loss deductions will be higher when depreciation is slower. The reason for this result is that when an asset fails and becomes worthless, the ensuing retirement loss is equal to adjusted basis. Adjusted basis is higher at any given time during the recovery period when depreciation is slower.

Thus, for example, allowing for retirement risk would tend to reduce the gap in present value between five-year and ten-year straight-line depreciation. Even though the present value of depreciation under the five-year schedule is higher, the present value of retirement loss deductions is higher for the ten-year schedule.

This reduction cannot entirely eliminate the gap. Suppose that an asset fails and is retired at time \( t \) and that the retirement loss under the ten-year schedule at this time is \( \$X(t) \) higher than under the five-year schedule. Under the five-year schedule this amount \( \$X(t) \) has been deducted earlier than under the ten-year schedule. For this asset, the present value of depreciation plus retirement losses must be greater under the five-year schedule. But this argument does not depend on the particular retirement time that was chosen. As a result, the argument holds for all assets, independent of retirement time.

83. See supra text accompanying notes 38-42.

84. Strictly speaking this class life should be adjusted to reflect the peculiar features of ADS depreciation. Thus, the appropriate convention for the starting time of service, typi-
preciation as the exponential curve if retirement deductions are taken into account. Since taking into account retirement deductions increases the present value of deductions, this straight line is more gradual and represents slower depreciation for surviving assets than the first straight line. The second, more gradual straight line cuts the horizontal axis at 8.40, and thus requires that class life be set at 8.40 years, a considerably longer class life than the 6.40 years dictated by the first straight line. The impact of the adjustment for retirement is quite dramatic: Class life increases by about 24%.

3. Critique of the Government Approach

At first glance the government approach looks quite different from the approach that we identified as optimal, that is, basing the depreciation schedule on the age-price profile unadjusted for retirements. The government approach adjusts the age-price profile for retirements but then makes an offsetting adjustment for retirements in computing an equivalent straight-line schedule.\(^8^5\)

These adjustments raise three questions. First, is the government’s approach appropriate even though it appears to differ from the approach that we argued was optimal above? Second, are the offsetting adjustments for retirements in the government approach necessary? Given that the government has to derive a straight-line schedule that is equivalent to economic depreciation,\(^8^6\) an alternative and apparently simpler approach would be to convert the “optimal” depreciation schedule derived from the age-price profile for surviving units directly into a straight-line schedule. Third, if the adjustments are appropriate, are both of the government’s variants for adjusting the age-price profile correct?

To address these questions, this subsection argues that if we assume away the possibility of strategic loss-taking, the government’s approach using the Hulten and Wykoff variant for adjusting the age-price profile is correct. The other variant, however, results in depreciation that is too fast. The next subsection shows that the offsetting retirement adjustments are necessary. Simply converting the age-price profile for surviving assets to a straight-line schedule with equal present value will not work. The final subsection shows that the government’s approach fails when strategic loss-taking is possible.

\(^8^5\) The two adjustments are offsetting because the age-price profile is “accelerated” while the surrogate straight-line schedule is “decelerated” by taking retirement deductions into account. Compare supra text accompanying notes 71-73 (acceleration due to adjusting age-price profile for retirement deductions) with supra text accompanying note 82 (deceleration of equivalent straight-line schedule to allow for benefits from retirement deductions).

\(^8^6\) See supra note 25 and text accompanying notes 64-65.
If we ignore strategic loss-taking, the argument for the government approach is that the present value of depreciation adjusted to include the loss deductions that arise from retirements is the total present value of all the tax benefits that come from economic depreciation of the asset. A retirement loss is simply part of the asset’s price path, and it should be accounted for as a tax loss at the time of retirement by the tax system.\(^{87}\) The present value of the tax adjustments for economic depreciation should include the present value that arises from retirements.\(^{88}\)

If the goal is to give the taxpayer a straight-line schedule that is equivalent to depreciation using the age-price profile for surviving assets, it is necessary to ensure that the present value of the sum of depreciation deductions and retirement loss deductions is the same under the two schedules. Only then will a taxpayer face the same expected present value of total tax benefits for cost recovery under both schedules.

In the remainder of this subsection, we explore the two variants that the government has used to adjust for retirements. The first variant simply uses the same adjustment that appears in the Hulten and Wykoff approach: the age-price profile for surviving assets is adjusted downward by multiplying the price at each age by the probability of survival to that age.\(^{89}\) The resulting curve consists of the average value of both surviving and retired assets.\(^{90}\) The present value of depreciation based on this curve is the retirement-adjusted present value that emerges from the first variant.

Using this first variant results in the correct outcome. The decline along the derived curve will be precisely the average decline in value experienced by assets with different lives. The present value of depreciation that follows from the curve will be the present value that is expected on average when the asset is purchased.\(^{91}\) This present value is exactly the goal of the calculations.

The second variant does not perform as well. In the example we have been using in this Part,\(^{92}\) the present value of depreciation calculated by

\(^{87}\) See supra text accompanying notes 43-45.
\(^{88}\) The Treasury Department makes exactly this argument in several of its depreciation studies. See, e.g., Treasury Horse Study, supra note 24, at 14; Treasury Scientific Instruments Study, supra note 24, at 23.
\(^{89}\) See supra text accompanying notes 51-53.
\(^{90}\) See supra note 52 and accompanying text.
\(^{91}\) To make this point in a more mathematical way, note that there are three steps in the computation under the Hulten and Wykoff approach:

1. A curve representing the average value of all units (retired and surviving) is derived;
2. The derivative of this curve gives the instantaneous rate of decline in value at each point;
3. The instantaneous decline in value at each point is integrated after being discounted to present value.

Taking the average and taking the derivative commute. That is, the derivative of the average value curve is equal to the average derivative of the individual value curves. Thus, the overall result will be the expected present value of depreciation independent of the order of steps (1) and (2).
\(^{92}\) See supra text accompanying notes 38-42.
using the first variant is 88.44% of initial value compared to 90.51% of initial value when calculated using the second variant. 93 These present values translate into class lives of 8.40 and 6.36 years respectively for the two variants. 94 The second variant results in a class life that is about 24% too short.

The divergence becomes even more extreme if we alter the example to allow a slower rate of decline in revenues. The rate in the example was 20% per year. If we use a rate of 10% per year, the present values of depreciation for the first and second variants are 83.60% and 87.66% of initial value respectively. The corresponding class lives are 15.08 under the first variant and 9.26 years under the second variant. The second variant results in a class life that is almost 40% too short. This discrepancy is much larger than the discrepancy when revenue declined more quickly.

The intuitive reason for the deviation between the two variants and the increase in that deviation for a lower rate of decline in revenues is that the second variant gives too low a weight to assets with long lives. The second variant begins by computing the age-price path and the present value of depreciation for individual cases where the life of the asset is known. Assets with longer lives will be more valuable initially because these assets will continue producing revenues after assets with a shorter life have been retired. Since they start at a higher initial value, longer-lived assets will contribute more total depreciation than shorter-lived assets. In addition, the depreciation for longer-lived assets will be concentrated in the later years compared to shorter-lived assets. The shorter-lived assets will reach zero value and stop depreciating at some point. At this point the longer-lived assets still have value and are still depreciating.

If assets of different lives are averaged together (as in the age-price profiles we have been considering), the longer-lived assets will be a larger component of value, will contribute more total depreciation, and will cause average depreciation to be shifted toward later years. However, the second variant normalizes the initial price of assets of different lives to be one. This normalization artificially diminishes the contribution of longer-lived assets to the average and causes average depreciation to appear to be faster than it really is. 95

It is not surprising then, that the second variant results in much shorter class lives than the first variant. It is also not surprising that the discrepancy is more severe when revenues decline more slowly. A slower decline in revenues means that the later years of an asset’s life are relatively

93. The mathematical derivation of these results is set out in Appendix A. See infra Appendix A.
94. The class lives are calculated using the Treasury Department’s method for translating the present value of depreciation for an asset into a class life for the asset. See supra text accompanying note 78-84.
95. Appendix A discusses this point more comprehensively and proves that the present value of depreciation is higher under the second variant in general and not just for the parameters that govern the text example. See infra Appendix A and text accompanying notes 260-264.
more important. The second variant artificially downplays these later years.

It is now possible to come to some conclusions about the government approach. First, in the absence of strategic loss-taking, the government approach is conceptually correct. In particular, adjusting both the initial age-price profile and the surrogate straight-line schedule for retirements is theoretically sound. Second, implementing the government approach by the first, "Hulten and Wykoff" variant for adjusting the age-price profile for retirements results in the correct values for class lives. However, using the second variant results in class lives that are much too short. Use of this variant should be abandoned in favor of the first variant.

Unfortunately, these positive conclusions will not hold up when strategic loss-taking is viable. The impact of strategic loss-taking on the government's approach is discussed in subsection 5 below. The next subsection shows that the government approach cannot be simplified to eliminate the retirement adjustments without introducing substantial errors.

4. The Need to Adjust for Retirements

The government's approach rests on the Treasury Department's interpretation of language in the legislative history of the Tax Reform Act of 1986 concerning how class lives should be calculated.96 This language could be interpreted differently to mean that class lives should be determined without making any adjustments for retirement risk.97 Thus, the government could simply translate the age-price profile for surviving assets into the straight-line schedule that results in equal present value of depreciation. This approach would eliminate the laborious corrections for retirement risk both for the age-price profile for surviving assets and for the equivalent straight-line schedule.

Unfortunately, this simplified approach can result in significant errors. These errors arise because a depreciation schedule based on the age-price profile for surviving assets will be equivalent to the corresponding straight-line schedule only for the assets that survive for the duration of both schedules. For assets that perish before both schedules reach zero,

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96. See supra text accompanying notes 64-65.
97. This interpretation would not strain the language in the legislative history. On the contrary, perhaps the most natural interpretation of the language is that the age-price profile should be translated into the straight-line method that is equivalent in present value without adjusting for retirement risk. The relevant language in context is as follows:

If resale price data is used to prescribe class lives, such resale price data should be adjusted downward to remove the effects of historical inflation. This adjustment provides a larger measure of depreciation than in the absence of such an adjustment. Class lives using this data would be determined such that the present value of straight-line depreciation deductions over the class life, discounted at an appropriate real rate of interest, is equal to the present value of what the estimated decline in value of the asset would be in the absence of inflation.

retirement deductions generally will differ under the two schedules. As a result, when retirement risk is taken into account, the expected present value of deductions is not the same for the approximating schedule and the schedule based on the age-price profile for surviving assets. The direction of the errors depends on whether the approximations allow faster or slower depreciation than the actual decline in value of surviving assets in the early years of asset life.

Consider the case where the approximation results in slower depreciation in the early years compared to actual depreciation. For the example that we have been considering in this Part, Figure 8 sets out the age-price profile for surviving assets and the straight-line schedule that results in the same present value of depreciation deductions. It is apparent from the figure that in the early years of the asset’s life the straight-line schedule results in depreciation that is slower than the actual decline in value of the asset. In later years the straight-line schedule results in depreciation at a faster rate than the actual rate of decline in asset value.

The straight-line schedule in the figure extends over 9.42 years. If the taxpayer holds the asset for the full 9.42 years, then the taxpayer will experience depreciation that is too slow in the early years but will “catch up” in the later years when depreciation under the straight-line schedule is too fast.

There is a problem, however, that has been identified by Roger Pies
and David Fischer. 98 Many assets will fail and be discarded before the taxpayer can catch up under the straight-line schedule. In the example that generates the diagram, 99 over 50% of assets will fail before the two curves cross in the diagram at approximately 7.1 years. These assets are depreciated at too slow a rate along the straight-line schedule during their entire life. They never have a chance to catch up in the later years when depreciation is faster along that schedule than the actual rate of decline in value.

This phenomenon results in the Pies-Fischer effect: When retirements are taken into account, the straight-line schedule results in a lower present value of deductions than the schedule based on actual depreciation. In the example represented by Figure 8, the present value of all deductions will fall from 88.44% of initial value for depreciation based on the actual age-price profile to 87.52% of initial value for depreciation using the straight-line approximation to that profile. 100

Our example represents only one of two possible cases. Straight-line approximations may involve depreciation that is faster than actual decline in value in the early years followed by depreciation in the later years that is slower than actual decline in value. In this second case, retirement risk can cause an “inverse” Pies-Fischer effect to occur. A larger proportion of assets experience the early part of the approximate schedule than the later part. Since the early part is accelerated relative to the actual decline in value while the later part is decelerated, the expected present value of all deductions will be higher under the approximated schedule than under the schedule that reflects actual decline in value.

In conclusion, it is not possible to eliminate the adjustments for retirements in the government approach without creating significant errors. However painful, those adjustments are necessary. 101


99. See supra text accompanying notes 38-42.

100. As Pies and Fischer point out, present value effects that appear small, such as the one percentage point effect here, may actually represent significant distortions in terms of the degree of adjustment in the law required to correct them. See Pies & Fischer supra note 98, at 89 (class life for use in straight-line depreciation must be shifted from 5.039 years to 3.619 years to overcome 1.64 percentage point gap in present value).

The exact class lives that Pies and Fischer advocate may be much too short. To construct a depreciation schedule for surviving assets, they use the depreciation rates derived by Hulten and Wykoff. See id. at 90. But these rates already have been accelerated to take into account retirement losses. See supra text accompanying notes 58-61. Nonetheless, Pies and Fischer proceed to adjust the schedule based on these rates for retirement losses. In effect, this approach results in a double adjustment for retirement losses. The resulting class lives will be much too short. See supra text accompanying notes 59-61.

101. The use of a straight-line approximation does raise some problems even under the government’s approach. Although the present value of deductions is the same under the straight-line approximation as under the retirement adjusted age-price profile, there is a residual “ex post” equity problem that arises from using an approximation.

Suppose that the approximating straight-line schedule initially is slower than economic depreciation represented by the age-price profile for surviving assets but that an asset will be fully depreciated under the straight-line schedule before the age-price profile reaches zero. Then, taxpayer one who holds an asset that fails early will receive depreciation that is
5. The Impact of Strategic Loss-taking

In subsection 3, we argued that the government approach was conceptually correct so long as the proper variant is used to adjust the age-price profile for retirement risk. Unfortunately, this conclusion does not hold up if strategic loss-taking is feasible.

To see the basic intuition behind this conclusion, let us return to the theoretically ideal treatment consisting of using a depreciation schedule based on the age-price profile for surviving assets. As we have noted, under this schedule there can be no strategic loss-taking. At any point in time, the asset has been depreciated for tax purposes down to exactly its market value. Adjusted basis is equal to market value. The taxpayer cannot obtain a tax loss by selling the asset.

This result disappears, however, if the depreciation schedule is ever above the age-price profile for surviving assets. At such a point in time, the market value of the asset is less than the adjusted basis of the asset, and the taxpayer can trade to obtain a tax loss.

This phenomenon is independent of whether there is any retirement risk. Consider an asset that will produce constant revenues of $1000 per year and will last exactly 10 years. Figure 9 plots the ensuing age-price too slow compared to economic depreciation. In contrast, taxpayer two who holds an asset that fails right after the approximating straight-line schedule reaches zero will receive depreciation that is too fast compared to economic depreciation.

This ex post problem cannot be fixed short of abandoning approximation techniques altogether and using the actual age-price profile of surviving assets as the basis for the depreciation deductions allowed. As long as the approximate schedule deviates in different directions from economic depreciation during different asset age periods, differential treatment based on the holding period for the asset will occur. See also infra text accompanying notes 177-180 (discussing impact of diversified holdings of depreciable assets on equity problem).

102. See supra text accompanying notes 44-45.

103. Since the exact life is known in advance, there is no retirement risk.

This example is almost identical to the example used by Professor Chirelstein to illustrate what he calls the “sinking-fund depreciation method.” See MARVIN A. CHIRELSTEIN, FEDERAL INCOME TAXATION 137-39 (5th ed. 1988). The sinking-fund method is equivalent to the productivity method described above: The age-price profile that serves as a basis for economic depreciation is equal at each moment in time to the present value of future revenues. See infra text accompanying notes 248-251.

Unfortunately, Professor Chirelstein linked the sinking-fund method to the outcome that economic depreciation will be decelerated compared to straight-line depreciation. See id. at 139. But that deceleration result is not the necessary or natural outcome of the sinking-fund method. Instead the result is an artifact of the particular example Professor Chirelstein examines. The asset in the example has constant revenues, no revenue risk, and no retirement risk.

All of the age-price profiles depicted in figures in this paper have been derived by the same method, the productivity method, that Professor Chirelstein used. The profiles in these figures are based on different assumptions about revenue risk and retirement risk. Yet most of them are accelerated rather than decelerated relative to the straight-line method. See, e.g., supra text accompanying notes 42-43 (Figure 5, exponential life distribution and exponentially declining revenues) and infra text accompanying notes 127-128 (Figure 11, Weibull life distribution with shape parameter 2.125, constant revenues).

Whether deceleration or acceleration relative to the corresponding straight-line curve is “typical” is an empirical question. The evidence unearthed by Hulten and Wykoff suggests that acceleration is typical. They term the situation studied by Chirelstein, where revenues are constant and the asset simply collapses at some known future time, the “one-horse-
profile and the straight-line schedule with the same present value of deductions as the schedule based on that profile.

**Figure 9**

Age-Price Profile and Equivalent Straight-line Schedule

The two schedules cross when the asset is 6.77 years old. Up to this point the taxpayer is better off under the straight-line schedule, but afterwards, the taxpayer should trade frequently so as to secure deductions that track the actual decline in value of the asset.

The total present value of deductions under this strategy are 61.23% of initial value.

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But this case is not typical as is indicated by two of Hulten and Wykoff’s empirical findings. First, Hulten and Wykoff found that the one-horse-shay pattern is not statistically acceptable as a model for the large set of depreciable assets that they studied. Second, the average age-price profile for these assets is convex, that is, the rate of decline is more rapid in the early years of the asset’s life. See supra text accompanying notes 56-57.

At about three years of age, the rate of the actual decline in value becomes greater than the rate of depreciation along the straight-line schedule. At this point in time, the slope of the age-price profile in the figure is equal to the slope of the straight line. But it is not optimal for the taxpayer to switch to the actual schedule at this “equal rates” time, even though the actual decline in value will be at a faster rate for all future times. Switching by trading at the “equal rates” time entails paying taxes on the gain equal to the vertical gap between the schedules at that time. This gain will be offset by accelerated deductions along the age-price profile representing actual decline in value. When the point at which the curves cross is reached, the entire amount of this gain will be offset by additional deductions. However, these offsetting deductions are later in time than the gain experienced from selling at the equal rates time. In present value terms, the taxpayer is worse off.

This same argument applies to potential trades at all points in time before the time at which the curves cross. After that time, there is no penalty in the form of taxable gain for shifting to the faster schedule based on the actual decline in asset value.
pared to 59.80% of initial value for the stream of deductions under either
the straight-line schedule or the schedule based on actual decline in
value.\textsuperscript{105}

Congress has required the government to convert schedules represent-
ing economic depreciation into straight-line schedules with the same pre-
sent value of depreciation.\textsuperscript{106} This requirement is not senseless because it
allows ADS, the part of the Code that approximates economic deprecia-
tion, to be very simple: Each asset is depreciated under the straight-line
method over its class life. But the existence of the requirement makes
strategic loss-taking problems inevitable.

This inevitability results from the fact that a schedule that approxi-
mates economic depreciation must result in adjusted basis sometimes be-
ing higher and sometimes lower than actual market value. If adjusted
basis were always higher than market value, depreciation under the ap-
proximating schedule would be too slow.\textsuperscript{107} If adjusted basis were always
lower than market value, the approximating schedule would be too fast.
Since adjusted basis must \emph{sometimes} be above market value under any
approximation, strategic loss-taking is a potential problem.\textsuperscript{108}

Because Congress requires the government to use approximations, it is
important to consider policy responses to the strategic loss-taking prob-
lem. It turns out, however, that strategic loss-taking is a much more per-
vasive problem. The true scope of the problem has been hidden by our
assumption that the age-price profile for surviving assets is known in ad-
vance. A schedule based on this profile precludes strategic loss-taking
since adjusted basis is always equal to market value.

But, in general, the age-price profile for surviving assets is \emph{not} known

\textsuperscript{105} The taxpayer will be even better off if used assets are depreciated under the same
type of straight-line approximation employed for new assets. Then, after selling at the
point where the curves cross, the taxpayer will receive a new straight-line schedule that will
decline faster than the actual decline in value for some period of time. The taxpayer
should stick to that schedule until it crosses the age-price profile. Then the taxpayer should
trade again, receiving a third straight-line schedule. Pursuing this strategy dominates sim-
ply trading to deduct along the age-price profile after the first straight-line schedule crosses
the curve representing that profile.

\textsuperscript{106} See supra note 25 and text accompanying notes 64-65.

\textsuperscript{107} This assertion is true even if we take retirements into account. Under the slower
schedule with consistently higher adjusted basis, retirement loss deductions are more valu-
able since loss is equal to adjusted basis at the time of retirement. However, the increased
losses exactly correspond to \emph{earlier} depreciation deductions given up by the taxpayer on
the slow schedule. Thus, the slower schedule will result in a lower present value of deprecia-
tion even after adjustment for retirement losses. For a more extensive explanation of this
point, see supra note 82.

\textsuperscript{108} In the context of our example, consider the following figure. The curved line is
the age-price profile for surviving assets. This curve represents the market value of surviv-
ing assets at any given time. The straight-line is the approximating schedule under the
correct, Hulten and Wykoff, variant of the government’s approach. See supra text accom-
panying notes 62-84.
As a result, there is the danger that market prices will end up being lower than adjusted basis under whatever schedule is specified ex ante. Part III takes up the potential policy responses to strategic loss-taking in this more general setting. The solutions discussed there apply equally to the strategic loss-taking problems raised by the need for the government to use approximations.

D. RETIREMENT RISK CONSIDERATIONS IN THE NEUTRAL DESIGN OF ACCELERATED DEPRECIATION SYSTEMS

Section B establishes that retirement risk can be ignored in computing a depreciation schedule that results in economic depreciation. However, if the goal is to accelerate depreciation in a neutral way across assets from a benchmark of economic depreciation, then retirement risk must be taken into account. The basic reasoning behind this assertion is straightforward. Two assets with the same age-price profile for surviving units will have the same statutory schedule if economic depreciation is the goal. Suppose, however, that asset number one has a much higher sur-

![Figure](Age-Price Profile and Approximating Straight-line Schedule)

It is clear from the figure that the approximating straight-line schedule results in an adjusted basis above market value for the first six years and an adjusted basis below market value thereafter. Thus, adjusted basis under the approximating schedule is sometimes below and sometimes above market value. During the first six years, the period when adjusted basis is above market value, strategic loss-taking is possible. The taxpayer will want to "trade down" to the market value and thus effectively accelerate depreciation deductions.

109. See supra text accompanying notes 36-37.
110. See infra text accompanying notes 119-198.
111. See supra text accompanying notes 42-45.
vival rate than asset number two. The statutory schedule covers only surviving units. Asset number one is more likely to survive and therefore is more likely to be depreciated along any given portion of the statutory schedule. Accelerating the statutory schedule will benefit asset one more than asset two.

This result can be illustrated using the example developed in earlier sections. Consider Figure 10. The upper curve is the same age-price profile that we have seen in the example all along. Since this curve is the age-price profile for surviving assets, it is the statutory schedule that should be used if economic depreciation is the goal. The lower curve is an accelerated depreciation schedule. The rate of depreciation on the lower schedule is exactly twice the rate on the upper schedule. The difference in the heights of the two curves represents the cumulative amount of "early" tax depreciation that an asset depreciated on the lower curve will receive compared to economic depreciation.

Figure 10
Age-Price Profile Versus Accelerated Depreciation Schedule

Now consider two assets. Asset one has a survival rate of 9/10 annually while asset two survives only at a rate of 2/3 annually. The following table indicates the probability of survival to various ages for the two assets:

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112. See supra text accompanying notes 38-42.
From the table it is clear that at any given age past a few months, it is much more likely that asset one will be subject to the statutory depreciation schedule. Asset two is much more likely to fail early. As a result, accelerating statutory depreciation should be much more of a benefit for asset one than for asset two.

This intuition turns out to be true and can be quantified. The total set of tax benefits consists of statutory depreciation deductions and retirement loss deductions. For the two assets, the present value of these deductions per dollar of asset cost before and after acceleration of depreciation are as follows:

For asset one, the increase in the present value of deductions is 0.081644 per dollar of investment while the increase for asset two per dollar of investment is only 0.02868, about a third as much. The acceleration of depreciation has clearly favored asset one over asset two. In order for accelerated depreciation to be neutral, the statutory schedule for asset one must be accelerated much less than the one for asset two.113 If the

113. There is much work by economists on the issue of how to accelerate depreciation in a neutral manner. See Alan Auerbach, The Taxation of Capital Income 33 (1983) (citing sources). There is more than one test for neutrality. The two major tests are the present value approach and the internal rate of return approach. See id. at 33-34.
The depreciation rate is doubled for asset two as in the example, neutrality will be achieved if the depreciation rate for asset one is increased by a factor of 1.29.\textsuperscript{114}

It is clear that different amounts of acceleration may be required for two assets that have the same statutory depreciation schedule under an economic depreciation scheme if the assets differ in retirement risk characteristics. This result has important implications for current law. Under current law, ADS is meant to simulate economic depreciation, and MACRS is meant to accelerate depreciation in a neutral manner from the baseline of economic depreciation.\textsuperscript{115} But the Code simply translates the class lives under ADS to recovery periods under MACRS.\textsuperscript{116} This approach may result in some serious misclassification if assets vary greatly in retirement risk characteristics.\textsuperscript{117} In fact, it appears that there is a great

\textsuperscript{114}There is no single accepted definition of neutrality. See supra note 113. The two leading approaches are the present value approach and the internal rate of return approach. See id. Under these two approaches the required acceleration factors for asset one are 1.29 and 1.56 respectively. The text uses the 1.29 figure because the example has been cast in present value terms.

\textsuperscript{115}See supra text accompanying notes 20-26.

\textsuperscript{116}See supra note 34 and accompanying text.

\textsuperscript{117}The misclassification arises because ADS is meant to replicate economic depreciation while MACRS is meant to allow accelerated depreciation that is neutral between assets. The point in the text is that neutral acceleration can result in two assets with the same economic depreciation schedule having very different accelerated schedules. Thus, the fact that two assets are in the same ADS category does not mean that they should be in the same MACRS category. Setting up MACRS categories strictly on the basis of class lives, as in the current Code, is a mistake. See supra note 34.

Earlier parts of the Article point out that ADS depreciation may not accurately replicate economic depreciation. See supra text accompanying notes 60-61 (use of the Hulten and
III. OPTIMAL DEPRECIATION WHEN THE AGE-PRICE PROFILE IS NOT KNOWN IN ADVANCE

Part II studies tax depreciation policy in a context where the age-price profile is known in advance. For many depreciable assets, however, information about revenues or the durability of the asset affect the value of the asset as its life unfolds. For these assets, the age-price profile for the asset is not known in advance. This condition qualifies some of the conclusions of Part II and raises some new issues.

The most fundamental result in Part II is that the age-price profile for surviving assets provides the unique statutory depreciation schedule that leads to an exact replication of economic depreciation. Since a depreciation schedule is set in advance, this result depends on knowing the age-price profile for surviving assets in advance. If that profile is not known in advance, the government must use other approaches for setting the depreciation schedule.

Section A explores the new considerations that arise from not being certain about the future prices of surviving assets. Sections B and C discuss potential tax policy responses to that uncertainty. Section B focuses on changes in the ex ante part of depreciation policy, that is, changes in the statutory depreciation schedule that is fixed in advance. Section C considers changes in the ex post features of depreciation law. Under individual item accounting, those features currently consist primarily of "settling up" by assessing a loss or a gain when an asset is sold or discarded. The loss or gain is equal to the amount realized minus the adjusted basis. Section C considers more elaborate ex post adjustments. Some of these are alternative rules for the tax treatment of dispositions that are actually used in group accounting methods. The discussion, however, continues to presume individual item accounting. Part IV discusses group accounting.

Section D states some conclusions. No clear policy winner emerges from the discussion. The stronger alternatives have strengths and weak-
nesses and only more research can determine which of them is actually the best policy.

A. The Basic Implications of Uncertain Future Asset Prices for Tax Depreciation Policy

To illustrate the dilemma created by not knowing the age-price profile of surviving assets in advance, consider an example that roughly approximates the situation for racehorses. At birth it is unclear how successful a racehorse will be. The horses end up in three general categories.\textsuperscript{120} About 40\% never become “starters,” that is, horses that race regularly.\textsuperscript{121} A minority of the 60\% that become starters enjoy spectacular success. Suppose that this class comprises one-sixth of the starters or 10\% of all racehorses.\textsuperscript{122} Spectacularly successful horses are the ones in the big national and international races such as the Kentucky Derby. The other starters race consistently but probably only in local events. We are assuming that these other starters comprise 50\% of all racehorses. They are clearly “successful” but never earn the huge revenues that spectacularly successful horses earn.

Suppose that there is no information about which type a racehorse is until the horse actually begins to race at age two, but that shortly after the horse is two, the horse’s type is discovered with certainty.\textsuperscript{123} The result will be that all racehorses will have a common age-price profile up until age 2: During the first two years of life the owner must assume a 40\% chance that the horse will not be a starter, a 10\% chance that the horse will be a spectacularly successful starter, and a 50\% chance that the horse will start but will not enjoy spectacular success. When racing begins at age two, the common age-price profile splits into three parts. The price of spectacularly successful horses shoots up at that time while the price of nonstarters falls.

Assume that spectacularly successful horses earn $5000 per year, nonstarters earn $500 per year, and starters that are not spectacularly successful earn $2000 per year. In addition, suppose that all three types of horses have the same distribution of useful lives once they are put into service at two years old.\textsuperscript{124} Finally, suppose that before being tested in

\begin{footnotesize}
\begin{enumerate}
\item[120.] Finer gradations are undoubtedly possible. But these three categories do emerge from the Treasury Department’s study on the depreciation of horses if both racing and later breeding uses are considered. See Treasury Horse Study, \textit{supra} note 24, at 5, 14-18.
\item[121.] The data indicate that between 30\% and 40\% never start. See \textit{id.} at 5.
\item[122.] Unlike the 40\% figure, this figure is simply used to generate the example. It is not clear exactly what proportion of racehorses are, in fact, “spectacularly successful.” Some evidence indicates that the figure may be much less than 10\% of all racehorses, including nonstarters. See \textit{id.} at 18 (about 7\% of all \textit{starting} colts become spectacularly successful breeding stallions).
\item[123.] Information undoubtedly arrives at a more gradual rate than this. In addition, some horses may be more likely to be successful based on their genetic heritage. These horses presumably would have a higher value at birth. The goal here is not to model racehorses precisely, but merely to construct a simple but illustrative example.
\item[124.] Instead of choosing an exponential distribution of lives, we use a Weibull distribution with shape parameter 2.125 and life parameter 0.153. This distribution is estimated
\end{enumerate}
\end{footnotesize}
their first races, two-year olds are sold to the owners who will race them.\footnote{125} Since each horse's type is unknown until after the first few races, all of these sales will occur at the same price.

Figure 11 illustrates the age-price profiles for the three types of horses that follow from these assumptions. Until two years of age all three types have the same age-price profile, then, at two years old, the age-price profiles for the three types diverge. The spectacularly successful starters are identified and their price shoots up.\footnote{126} Their age-price profile is the top one in the figure. The second highest curve is the age price profile for horses that become starters but that do not enjoy spectacular success. The lowest curve is the age-price profile for horses that are nonstarters. These horses experience a large drop in price when they are identified as nonstarters at two years old. Finally, the third highest curve is a weighted average of the other three curves. The weights are the proportions of horses that fall into each of the three categories. This “average curve”

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Figure11.png}
\caption{Racehorse Age-Price Profile}
\end{figure}

from data on the useful lives of thoroughbred geldings. For a discussion of the Weibull distribution and the reasons for using it, see \textit{infra} Appendix C.

\footnote{125} This assumption permits use of the common market value just before two years old as the original basis of the horses. The Treasury Department in its study of horses in effect makes a similar assumption since they take the value of horses before two years old to be constant and use this value as the original basis. \textit{See} Treasury Horse Study, \textit{supra} note 24, at 2, 11. Taking the value at birth as the basis instead would complicate the analysis but would not affect any of the major conclusions.

\footnote{126} The treatment of appreciation during portions of the lives of depreciable assets raises interesting tax policy issues. For a discussion of these issues, see \textit{infra} text accompanying notes 141-145.
therefore represents the average age-price profile for all horses.\(^{127}\)

It is clear from Figure 11 that it is no longer possible to replicate economic depreciation perfectly by using a single statutory schedule.\(^{128}\) Even if the schedule is based on the average curve in the figure, many horses will experience actual depreciation that deviates sharply from the schedule. In particular, if the schedule is based on the average curve, depreciation will be much too fast for the spectacularly successful starters and much too slow for nonstarters.\(^{129}\)

In addition, unless the statutory schedule is generated from a curve that lies entirely below the lowest age-price profile, profitable strategic loss-taking will be possible if trading costs are low enough. For example, suppose in Figure 11 that the statutory schedule is the average curve. When a horse is identified as a nonstarter just after two years of age, the owner will want to sell the horse to speed up tax depreciation. By selling, the owner takes the large loss of \(\$6062.85\) that occurs at two years but

\(^{127}\) If an individual owned many racehorses, price performance for the aggregate group of horses would tend to be close to this average curve.

\(^{128}\) One might wonder why the authorities would not delay until right after the beginning of year three to set the schedule for that year and future years. Then the tax authorities would know which outcome occurs and could set the depreciation schedule on the basis of the "correct" age-price profile.

This approach would work in the example. However, a peculiar feature of the example is that all risk is resolved at one point in time. In the real world, risk tends to be resolved gradually. The tax authorities would have to wait until a machine was retired to know the exact age-price profile that occurred. Then, the depreciation system would be entirely ex post in nature.

The device of having all risk resolved at one time in the example is used to make the analysis simple. Analyzing the value of strategic loss-taking in an environment where risk is resolved continuously is mathematically very difficult. See, e.g., George M. Constantinides, Capital Market Equilibrium with Personal Tax, 51 ECONOMETRICA 611, 611 (1983); Michael J.P. Magill & George M. Constantinides, Portfolio Selection with Transactions Costs, 13 J. ECON. THEORY 245 (1976). For a cogent study of the value of strategic loss-taking for depreciable assets in that kind of environment, see Joseph T. Williams, Trading and Valuing Depreciable Assets, 14 J. FIN. ECON. 283 (1985).

Professor Williams' study includes an aspect of strategic trading for depreciable assets not discussed here: the possibility of selling depreciable assets for a gain in a wash sale in order to increase the adjusted basis of the assets. The added basis is equal to the gain. Since recovery of the added basis through deductions will be delayed compared to the gain, this strategy only will be profitable if the tax rate on the gain is significantly lower than the ordinary income rates at which the deductions will reduce future taxes. At some points in history, the tax code has imposed rates on long-term capital gains (on assets owned longer than some "holding period") that are lower than the rates on ordinary income or short term capital gains. At present, a rate gap of as much as 21.6 percentage points exists. As recently as the late 1980s, however, there was no such gap. See supra note 7.

A complete analysis of strategic trading under current law would include consideration of the possibility of taking long-term capital gains in a wash sale to enhance basis. I ignore this possibility in the text, preferring to focus on aspects of strategic trading that would exist in the absence of any favorable rate treatment for long-term capital gains.

\(^{129}\) A schedule based on the average curve will give each taxpayer at each point in time a deduction equal to the average decline in value across the assets in the category (here racehorses). There are mathematical reasons for choosing the average decline in value as an estimate of the true decline in value. See infra note 146 and text accompanying notes 146-147.
that is not reflected in the average curve. The owner can then replace the horse sold with another nonstarter and depreciate this horse roughly along its age-price profile for the remainder of its life.

This strategy increases the present value of deductions by $1765.30 as of year two, since that is the difference in present value between depreciating along the average curve and taking the $6062.85 loss right after the beginning of year two combined with depreciating along the age-price profile for nonstarters for the rest of the asset’s life. The strategy will be attractive if this increase in present value exceeds the present value of the trading costs of engaging in the strategy.

If strategic loss-taking is viable, the taxpayer can subvert a depreciation schedule based on the average age-price profile. The taxpayer will depreciate along that schedule if the actual age-price profile ends up being above the average one. But if the actual age-price profile turns out to be below the average curve, the taxpayer will jump down to the schedule based on that lower age-price profile. So the overall result will be that at worst the taxpayer will be subject to the schedule based on the average curve. Under some outcomes, the taxpayer will be able to depreciate based on a more favorable schedule. The actual average schedule exper-

130. The $6062.85 loss is the difference between $8589.04, the value right before year two, and $2526.19, the value of a horse that is identified as a nonstarter right after the beginning of year two. The example assumes that the owner first purchased the horse at $8589.04, right before the beginning of year two.

131. Presumably, the average curve that generates the statutory schedule would dictate the shape of the schedule. This shape can be described by the percentage of original cost allowable as a deduction for each time period. But the age-price profile for nonstarters has the same shape as the average curve. That age-price profile simply starts at a lower point. Thus, the replacement horse would be depreciated along its actual age-price profile for the remainder of its life.

132. The present value of depreciation along the average curve adjusted for retirements is $6088.19. The present value of depreciation along the nonstarters’ curve adjusted for retirements is $1790.64. The $6062.85 loss deduction has present value of $6062.85 since the loss occurs immediately.

By taking the loss and shifting to the lower nonstarters’ curve, total present value for all deductions increases by $6062.85 + $1790.64 - $6088.19 = $1765.30. The adjustment for retirements used to compute the $6088.19 and $1790.64 figures consists of weighting depreciation along the curve by the probability of survival and adding the present value of retirement loss deductions where retirement at each particular time is weighted by its probability. These figures therefore represent the total present value of all deductions that will result from depreciating according to the schedule as long as the asset lives and then taking a loss deduction at retirement.

133. This statement is strictly true only under certain patterns of tax rates over time. In particular, the statement is true if the taxpayer is subject to the same marginal tax rates during the investment’s entire life. In that case, all of the depreciation deductions and the deduction for trading costs will translate into after tax benefits at that common marginal rate. The taxpayer need only compare the increase in present value for the depreciation deductions to the present value of the trading cost deductions to decide whether to engage in trading.

Note that trading costs are not necessarily entirely deductible at the time trades are made. Trading costs typically will be added to the basis of the asset and, in effect, will be deducted at the time of sale or disposition of the asset. See Woodward v. Commissioner, 397 U.S. 572 (1972). Although the transaction costs of selling the original horse would result in an immediate tax benefit, the transaction costs of buying the replacement horse might not result in a tax benefit for many years.
Anticipating this phenomenon in advance can cause assets subject to significant future price risk to be favored by the depreciation rules over otherwise similar assets. Going back to the example, let us compare the asset in the example to an asset with an age-price profile that is known in advance and that is the same as the average age-price profile in the example. For both assets, suppose that the statutory depreciation schedule is based on that average age-price profile. Then the individual who buys the asset in the example will anticipate the fact that there will be a special tax benefit if the horse ends up being a nonstarter: The taxpayer in effect will be able to accelerate depreciation if this outcome occurs. As mentioned above, this acceleration increases the present value of depreciation and loss deductions by $1765.30 in that case.

Since there is a 40% chance of any given horse being a nonstarter, the present value of depreciation deductions at the time of purchase right before year two will increase by (.40 x $1765.30) = $706.12. As a result, the total present value of all depreciation and loss deductions at time of purchase will increase from $6088.19 to $6794.31. This increase of about 12% represents the tax advantage of the asset subject to an uncertain future price path in the example over an otherwise identical asset that is not subject to such uncertainty. It is clear, then, that substantial investment distortions may follow if the degree of future price path uncertainty is not taken into account in setting up tax depreciation rules.134

More generally, revenue risk may be divided into a portion that can be diversified away costlessly and a portion that is endemic to the entire economic system. This second type of risk cannot be diversified away costlessly. Some economists believe that the presence of systematic revenue risk, that is, revenue risk that cannot be costlessly diversified away, warrants special depreciation subsidies. The author disagrees with this claim. For a discussion, see infra Appendix D.

One type of revenue risk that is interesting from a tax policy standpoint is on the “cost side” of net revenues. Assets that have the same useful life may differ substantially in the amount and pattern of repair and maintenance costs. Net revenues will reflect these differences since the costs of repair and maintenance are subtracted from gross revenues in the netting process.

Repair and maintenance costs are interesting from a tax policy standpoint. There may be a tendency to expense these costs even if they extend asset life by many years so that there is a policy argument for amortization. Excess expensing of maintenance costs may lead to a tax-induced bias in favor of used assets. Extending the life of used assets is accompanied by a 100% up front deduction for the costs while the extended life of new assets must be paid for with dollars that are deducted only over several future years. A
B. Ex Ante Policy Responses to Price Path Uncertainty

The previous section indicates that price path uncertainty can lead to both inefficiency and inequity. Inequity arises when some taxpayers receive treatment that is either more favorable or less favorable than economic depreciation.\(^{135}\) Inefficiency arises if one type of asset is favored over another by the tax system when there is no economic reason to do so. In the previous section we saw that traditional depreciation rules will favor assets with more uncertain age-price profiles.

One way to respond to the potential distortions and inequities that arise from price path uncertainty is to adjust the depreciation schedule that is set ex ante. This section explores this possibility while the next section examines solutions that involve ex post adjustments to the results under the schedule.

Regardless of whether ex ante or ex post adjustments are the focus, it is useful to divide the analysis into three cases: zero trading costs, very high trading costs, and all cases in between these two extremes. Trading costs are central because they determine the viability of strategic loss-taking. If trading costs are zero, then strategic trading has its fullest scope. If trading costs are very high, no strategic trade will be profitable.

If trading costs are zero, an attractive rule is to allow no depreciation deductions at all. This rule will induce the taxpayer to trade whenever value falls below adjusted basis in order to accelerate loss deductions to the earliest possible moment. But the rule does not result in perfect replication of economic depreciation because it does not tax the owner when depreciable assets appreciate.

To see this point consider the value path for “spectacularly successful” racehorses purchased right before age two from Figure 11 in the previous section.\(^{136}\) This path along with a horizontal line indicating original cost are reproduced in Figure 12 below. Given that trading is costless and that there are no depreciation deductions, the taxpayer will hold the asset until it is about six and one-half years old. Then the taxpayer will trade down the curve by continually selling and repurchasing an equivalent substitute asset.

In this case, the taxpayer is treated properly during the period following the six and one-half year point. As losses accumulate during that period, the taxpayer receives a stream of deductions equal to the losses.
However, immediately after purchase at year two, there is a very large gain followed by four and one-half years of losses equal to that gain. With a constant marginal tax rate, this pattern of appreciation followed by an equal amount of depreciation normally would generate taxes with a present value greater than zero. Here, however, there is no tax effect since the taxpayer does not realize the gain from the appreciation and does not deduct the later offsetting depreciation. In total, the taxpayer receives treatment that is more favorable than economic depreciation.

Bouts of appreciation followed by equal depreciation will happen frequently during the lives of most risky depreciable assets. Good news about future revenues may cause an asset to appreciate in value despite the fact that its estimated remaining life is declining. With costless trading and no statutory depreciation deductions, the taxpayer will have traded the adjusted basis down to the low point before the appreciation started. The asset then will appreciate and fluctuate for a while before moving back down to the level of the adjusted basis. That period of apprecia-

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137. The reason for a positive present value is that the gains are earlier than the losses. If the same tax rate applies to both, then the tax benefits from the losses will be equal in amount to the taxes on the gains since the total gain is equal to the sum of the losses. But the tax benefits will be discounted more heavily since they are received later.

138. This phenomenon actually occurs for two of the three types of horses in the example just presented. See Figure 11, supra text accompanying notes 127-128.

139. With costless trading and no depreciation deductions, taxpayers will trade continuously when the asset declines. See supra note 13. As a result, the adjusted basis of the asset at any point in time will be the minimum value of the asset over the period of ownership up until that time.

140. The asset will eventually fail with probability one. Thus, with probability one, the asset will decline below the level of its adjusted basis sometime in the future.
tion and fluctuation will involve equal total amounts of gain and loss as in the example just presented. During the life of an asset subject to substantial price risk, there should be many such periods of appreciation and fluctuation.

There are several ways to deal with the problem that depreciable assets may appreciate over part of their lives. Under accretion tax theory, the ideal approach would be to tax all gains and to allow deductions for all losses as they occur. By the standards of this theory, allowing no depreciation deductions at all when trading is costless is too lenient a rule. The taxpayer should pay a net tax on each of the bouts of appreciation and fluctuation but does not under the rule.

The potential solutions to this problem of unrealized appreciation for depreciable assets are similar to the potential solutions for the more general class of risky assets that are not necessarily depreciable.\(^\text{141}\) The most prominent such solution is "frequent assessment," a solution that is ex post in nature: The asset's actual value path is observed as closely as possible, and the appropriate tax is levied.\(^\text{142}\)

In two of its six recent studies, the Treasury Department faced the problem of depreciable assets that appreciate during part of their lives.\(^\text{143}\) The Treasury Department's solution for the appreciation aspect of these

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\(^{141}\) The category of "depreciable assets" used here has a similar meaning to the class of assets subject to "a reasonable allowance for . . . exhaustion, wear and tear (including a reasonable allowance for obsolescence)" under section 167 of the Code. I.R.C. § 167(a).

\(^{142}\) For a discussion of this solution and its leading competitors, see Strnad, supra note 7, at 1891-1903. Perhaps the strongest general competitor for frequent assessment is ex post approximation in linear or exponential form. This approach assumes that the asset has followed a linear or exponential path between any two known values on the actual path. However, regardless of whether the exponential or linear variant is used, the approach fails completely to address the problem presented here. That problem is that in between sales at roughly the same price there is a period of appreciation followed by equal amounts of depreciation. The linear and exponential approximations in this case are both the same: Price remains flat during the entire period between sales. As a result, no tax is due. But some tax should be paid to reflect the fact that appreciation is followed by an equal amount of depreciation.

\(^{143}\) One of the studies focused on racehorses. Spectacularly successful racehorses display an age-price profile very similar to the profile that we have been studying. See Treasury Horse Study, supra note 24, at 16. The other study examined fruit and nut trees. See Treasury Fruit Tree Study, supra note 24. Fruit and nut trees are "put into service" for purposes of tax depreciation when they start producing fruit and nuts. See id. at 1, 11, 23-24. But at that point in time they are still growing. Fruit and nut output per tree increases at a rapid rate and does not reach a peak until several years after the first season with significant yield. See id. at 1, 23, 27, 33, 36, 39, 42. This effect is pronounced enough that the trees increase in value during the early years in their lives. In these early years, the fact that remaining life is shorter as time goes on is more than offset by the fact that peak revenues associated with peak production are closer in time and therefore discounted less heavily. In Treasury's fruit and nut tree study this result emerged even though Treasury used a 4% real after-tax discount rate. See id. at 23.
depreciable assets is to ignore the appreciation and the associated offsetting depreciation.\textsuperscript{144} In terms of the example, this means that the appreciation and matching depreciation from the two year point to the six and one-half year point would be ignored. Depreciation would be allowed only starting at the six and one-half year point when the asset first declines below its original cost.

A possible theoretical rationale for this approach is that it would be unfair and perhaps also inefficient to tax the unrealized gain from the appreciation on these depreciable assets when the Code generally allows unrealized gains to go untaxed.\textsuperscript{145} Given that the appreciation is not taxed, the offsetting depreciation should not be deductible either.

When trading is costless, the proposed rule of permitting no statutory depreciation will perfectly implement the Treasury Department approach. The taxpayer will take loss deductions only when and to the extent that value declines below adjusted basis. An initial spurt of appreciation above original cost followed by offsetting depreciation will be totally ignored.

Now consider the opposite case from the case of zero trading costs. If trading costs are very high, strategic trading will never be economically viable. As a result, choosing and imposing a depreciation schedule based on the expected decline in value for surviving assets is an attractive approach. This approach means that at each moment during the asset’s life, the taxpayer will receive a depreciation deduction that is the average of the actual depreciation that would occur over all possible paths. Choosing the average minimizes the variance,\textsuperscript{146} a measure of how much actual depreciation tends to deviate from the amount allowed.\textsuperscript{147}

\textsuperscript{144} See Treasury Horse Study, supra note 24, at 18-19; Treasury Fruit Tree Study, supra note 24, at 1-2, 24-27.

\textsuperscript{145} The Treasury Department did not rely on this rationale. It arrived at its position based on its belief that Congressional intent with respect to the treatment of depreciable assets requires not taking periods of appreciation and matching depreciation into account. See Treasury Fruit Tree Study, supra note 24, at 18; Treasury Horse Study, supra note 24, at 18.

\textsuperscript{146} This point is easy to demonstrate using a little calculus. Suppose that one wants to know the true value of some random variable $X$. There are $n$ possible values, each of which is equally likely. Denote these values $x_i$ where $i = 1, 2, \ldots, n$. The variance of $X$ from some estimate $b$ is simply

$$V(X) = \frac{1}{n} \sum_{i=0}^{n} (x_i - b)^2.$$ 

The second derivative of $V$ with respect to $b$ is 1, which is positive, and the first derivative is zero when $b = \left(\frac{1}{n}\right) \sum x_i$, the average value of $X$. This average value is the minimum variance estimate of $X$.

\textsuperscript{147} The average value here is based on information available at the time the asset is placed into service. Call that moment "time 0." In between time 0 and the time, "$t$," at which depreciation is being approximated, there may be additional information that would change the estimate of average depreciation. The estimate of the average based on time 0 information will not necessarily be a minimum variance estimate of time $t$ depreciation given information available at time $t$.

However, we are considering a scheme where the depreciation schedule is set ex ante, at time 0. Thus, if this schedule is meant to minimize the variance of actual depreciation from
Having considered the polar cases of zero trading costs and very high trading costs, there remains the case where trading costs are “moderate.” In this case, strategic trading will only be viable when the tax timing benefits outweigh the trading costs. This case is considerably more complicated, and the optimal solution does not necessarily lie “in between” the attractive approaches in the other two cases. With very high trading costs, setting the depreciation schedule on the basis of the average age-price profile was appealing. In the case of zero trading costs, the ultimate in “decelerated” depreciation, namely, no depreciation deductions at all, appeared to be a good rule. But it is not necessarily true that a schedule somewhere in between the one dictated by the average age-price profile and one that allows no deductions at all is optimal in the case of moderate trading costs.

To see this point, consider the three outcomes in Figure 11 where the statutory schedule is the second curve from the bottom, the average age-price profile. For convenience, that Figure is reproduced here as Figure 13.

Figure 13
Racehorse Age-Price Profile

<table>
<thead>
<tr>
<th>Price</th>
<th>Age</th>
</tr>
</thead>
<tbody>
<tr>
<td>30000</td>
<td>16</td>
</tr>
<tr>
<td>25000</td>
<td>14</td>
</tr>
<tr>
<td>20000</td>
<td>12</td>
</tr>
<tr>
<td>15000</td>
<td>10</td>
</tr>
<tr>
<td>10000</td>
<td>8</td>
</tr>
<tr>
<td>5000</td>
<td>6</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
</tr>
</tbody>
</table>

Schedule depreciation, the average depreciation at time t based on time 0 information is the best estimate.

It should be noted that the normative goals of tax policy may dictate estimation objectives that are more complicated than simply minimizing variance. For example, if most depreciable assets are held by wealthy taxpayers, then errors on the side of leniency (depreciation that is too fast) may be more serious than errors in the other direction. See Strnad, supra note 7, at 1850 n. 89 (stating that equity/efficiency tradeoffs may mean that at an optimum, additional transfers from high income to low income individuals are socially desirable). As a result, an estimate might be made that weights errors on the side of leniency more heavily. Such an estimate will not necessarily be a minimum variance estimate. In statistical terminology, other central moments such as skewedness will also be socially relevant.
Taxpayers facing the top two outcomes will welcome this schedule since it gives them faster depreciation than economic depreciation. On the other hand, taxpayers facing the lowest outcome will want to engage in strategic trading. This trading will result in depreciation deductions that roughly replicate the actual age-price profile that they will experience. On the other hand, the trading costs incurred by these taxpayers are a social cost.

In terms of accretion tax theory, the taxpayers with the two high outcomes are receiving an improper tax benefit while taxpayers facing the low outcome are treated exactly right. If the depreciation schedule is decelerated by moving it up from the average age-price profile, the improper benefits to the high and middle outcome taxpayers will be reduced.

The central issue is how much deceleration is optimal. If the schedule is based on a curve that is far enough above the middle outcome age-price profile, then strategic trading will be viable for middle outcome taxpayers. At this point, the middle outcome taxpayer will experience tax benefits that are roughly correct, and the additional deceleration in depreciation will reduce the improper benefits experienced by high outcome taxpayers. However, these tax policy benefits are not free. The middle outcome taxpayer now pays for trading, and these trading costs are a social cost of the tax policy.

It is possible that the induced trading costs from decelerating the schedule exceed any gains in efficiency or equity from that deceleration. And here is the rub: The same reasoning applies to the taxpayer

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148. Thus, giving the figure its original racehorse interpretation, these taxpayers take the loss that occurs right after the horse is two years old and then depreciate along the age-price profile for nonstarters. See supra text accompanying notes 129-133.

149. If trading costs were zero, these taxpayers would engage in trading if any part of the schedule-setting curve was even an infinitesimal amount above the actual age-price profile. If trading costs are moderate, the schedule-setting curve must be high enough above the actual age-price profile so that the increase in tax benefits from trading outweighs the trading costs.

150. In our example, there is a upper limit to how much deceleration is desirable. Once the schedule-setting curve reaches the highest age-price profile, there is no reason to go higher. All taxpayers will receive approximately the correct tax treatment. Further deceleration will only induce strategic trading by the high outcome taxpayer. This trading will result in social costs, and there is no corresponding social benefit since the high outcome taxpayer is already being treated correctly.

151. This possibility is enhanced because the efficiency and equity gains will tend to be “second order,” that is, small compared to the increased present value of the taxes collected by the government. For example, it is well known that the efficiency costs of taxation tend to be proportional to the tax rate squared and thus are of lower order than the taxes themselves, which are proportional to the tax rate. See, e.g., Fullerton et al., Replacing the U.S. Income Tax with a Progressive Consumption Tax: A Sequenced General Equilibrium Approach, 20 J. PUB. ECON. 3, 17 (1983). However, the trading that is induced by a slight deceleration in the schedule is likely to involve trading costs approximately equal to the tax savings caused by the trades. In other words, at the margin, trading costs may be “first order” in magnitude.

It is unclear whether this point would hold up for large decelerations of the schedule. In that case, for some taxpayers trading costs will be small compared to the potential gains in tax savings, while others will be at the margin as we have described. Unfortunately, it is
facing the low age-price profile who trades strategically to take losses as measured against the "normal" depreciation schedule based on the average age-price profile. In particular, it may be true that the trading costs induced for that taxpayer are larger than the efficiency and equity costs that would result from *accelerating* the schedule to eliminate that trading. Thus, the optimal policy may be to provide *accelerated* depreciation to eliminate the costs of trading induced by a schedule that simulates economic depreciation on average. Indeed, one of the virtues of an accelerated depreciation system such as MACRS is that it reduces strategic loss-taking. Under an accelerated schedule adjusted basis is lower at any given point in time. It is less likely that adjusted basis will ever exceed market value by enough to make strategic loss-taking profitable net of trading costs.\textsuperscript{152}

The overall picture under moderate trading costs now emerges. As depreciation is decelerated, the improper benefits that flow to high outcome taxpayers are reduced. In addition, more low and moderate outcome taxpayers trade strategically. This trading makes their tax treatment roughly correct, but the sum of their costs of trading is a social cost. Tax equity, as defined by tracking wealth changes as closely as possible,\textsuperscript{153} is enhanced at the price of additional trading costs.\textsuperscript{154} However, the same reasoning applies to the low outcome taxpayer. It may be optimal to *accelerate* the depreciation schedule because the reduction in trading costs for the low outcome taxpayer may exceed the social costs in the form of reduced equity.

In conclusion, clear guidelines for setting ex ante depreciation schedules in the face of uncertain future price paths emerge only when trading costs are very high or very low. With high trading costs, a good approach is to base the depreciation schedule on the average age-price profile. With zero trading costs, providing no depreciation deductions at all is an attractive strategy. In the case of moderate trading costs, not much can be said in general about optimal ex ante depreciation schedules.

difficult to develop models that specify optimal trading strategies and the ensuing total trading costs for various taxpayers. See infra text accompanying notes 289-290.

\textsuperscript{152} It is also the case that failing to index adjusted basis for inflation reduces the scope for strategic loss-taking. Market value is in nominal dollars and increases with general price inflation. Opportunities for strategic loss-taking depend on adjusted basis exceeding market value. Given that the rate of inflation has generally been positive, this condition will be less likely to occur if adjusted basis is not indexed for inflation.

\textsuperscript{153} See supra note 135.

\textsuperscript{154} Efficiency may also be enhanced. Using a statutory schedule based on the average age-price profile tends to improperly favor depreciable assets with high future price risk. See supra text accompanying notes 133-134. There will be an amount of deceleration that will exactly cancel out the present value improper benefits as of the time of purchase. The trade-off between the efficiency gains of deceleration and the added induced trading costs is similar to the trade-off between equity gains and the added trading costs.
C. EX POST POLICY RESPONSES TO PRICE PATH UNCERTAINTY

An alternative or additional way to respond to the complications that arise from revenue risk is to alter the ex post adjustment aspects of the depreciation rules. Under individual item accounting, these aspects consist primarily of "settling up" at the time of disposition of an asset by taxing the difference between amount realized and adjusted basis.¹⁵⁵

Unfortunately, unless the government actually observes the price path of depreciable assets, it is difficult to design purely ex post adjustments that improve on an optimally designed ex ante system. It may be that the only additional information that is available ex post is the actual life or holding period of the asset. Subsection 1 shows that this information cannot be exploited in a way that is free from effective taxpayer manipulation.

This pessimistic conclusion is not the end of the story. It may be possible to combine ex post and ex ante rules in a way that effectively addresses the situation of uncertain price paths and the associated problem of strategic loss-taking. One set of methods of this sort combines disposition rules that discourage strategic loss-taking with changes in the ex ante depreciation schedule that adjust for the disposition rules.

Subsections 2, 3, and 4 consider three such combinations of disposition rule and ex ante schedule: the depreciation bond approach, the proceeds tax rule, and the account adjustment rule. Two of these rules are used presently for certain types of group accounts.¹⁵⁶ Subsection 5 concludes with a summary of the strengths and weaknesses of policies that combine alternative disposition rules with ex ante schedule adjustments.

1. EX POST ADJUSTMENT IN GENERAL

One potential ex post approach is periodic valuation and assessment of taxes. If the period used in this approach is very short, the approach will come close to taxing all gains and losses as they occur. This result is ideal under accretion tax theory.¹⁵⁷

The potential disadvantage of this approach is that frequent valuation may be very expensive. Public markets may not exist for many depreciable assets, and market price series that do exist may not reflect the value

¹⁵⁵. The current Code contains instances of ex post adjustment other than taxing gain or losses at disposition. One such instance involves depreciation under the income forecast method. Under this method, the depreciation deduction for a given year is equal to cost minus salvage value for the asset multiplied by a fraction: current year net income divided by total anticipated income from the asset. See, e.g., Rev. Rul. 78-28 1978-1 CB 51; Rev. Rul. 60-358 1960-2 CB 68 (describing the nature and availability of the income forecast method in some specific instances). The Code requires that the depreciation deduction computed under the original forecasted income stream be adjusted to allow for the actual (ex post) stream for certain assets. The adjustment involves charging or paying the taxpayer interest depending on the nature of the deviation of actual income from forecasted income. See I.R.C. § 168(g).

¹⁵⁶. See infra text accompanying note 207.

¹⁵⁷. See Strnad, supra note 7, at 1821-22, 1832-47, 1853-63.
of assets retained by owners.\textsuperscript{158} One important advantage of taking an ex ante approach to depreciation by setting a schedule in advance is that this approach avoids the need to value depreciable assets frequently.

If observing actual price series is ruled out, it is hard to come up with ex post adjustments that improve a well-designed ex ante system. Consider first the case of very high trading costs. In this case, sales are prohibitively costly so that the only dispositions are retirements. It is hard to see how to improve on the approach suggested in the previous section of using the average expected age-price profile to design a statutory schedule and then allowing losses to be taken upon retirement. Unless more information is observed about actual asset paths, the average expected path as of asset purchase will still be a good estimate of the path that the asset took up until failure and retirement.\textsuperscript{159}

The only additional information that is available costlessly in this case is the time of retirement. If the time of retirement correlates with the path up to retirement, then additional ex post adjustment might be fruitful. Consider the racehorse example once again. If high revenue horses also tend to have longer lives, then a tax surcharge might be levied on horses that are retired after long years of service. Unfortunately, there are problems with this type of ex post rule. As a horse nears the end of its life, value drops off sharply.\textsuperscript{160} If the surtax were high enough to capture the huge excess tax benefit inherent in the high outcome case,\textsuperscript{161}

\textsuperscript{158} Assets that are sold may have different characteristics than those that are retained. A well-known theoretical example of this phenomenon is the so-called “market for lemons.” See George A. Akerlof, The Market for Lemons, 85 Q. J. Econ. 488 (1970). If quality is not observable at low cost, the only equilibrium price in a market may be a price that would be paid for a low quality asset, a “lemon.” If the market price were higher, owners of lemons would unload them. Buyers then would assume that they will receive a lemon and would refuse to pay the higher market price. Thus, assets that are sold rather than sold will be of higher quality.

If market prices reflect only the value of lemons, depreciation schedules based on those prices will tend to be accelerated relative to the age-value profile of the average asset. Academic commentators and the Treasury Department are very aware of this phenomenon and have tried to adjust for it in their analyses. See, e.g., Hulten & Wykoff, supra note 3, at 96-99; Treasury Horse Study, supra note 24, at 27.

In the context of this Article, a market subject to the lemons phenomenon is a “high trading cost” market. To sell an asset that is not a lemon requires expending significant costs to convince buyers of that fact. If these costs were small, there would be no “lemons” problem.

\textsuperscript{159} “Good estimate” here means minimum variance estimate. See supra note 146 and text accompanying notes 146-147. The conclusion that enriching the ex post adjustment rules is not very fruitful would not change if “good estimate” meant an estimate that aimed at goals other than minimizing variance. See supra note 147. All that might change is that the best ex ante estimate would not be the average age-value profile.

\textsuperscript{160} This drop off occurs for horses in all revenue-earning categories. See Figure 11 supra text accompanying notes 127-128.

\textsuperscript{161} In the high outcome case, the present value of depreciation deductions adjusted for retirements based on the decline in value of the average horse is $6088.19. But the present value of depreciation adjusted for retirements for the high outcome horse using its actual age price profile is only $1233.59. This $1233.59 figure is much lower than $6088.19 because the high outcome horses experience a huge spurt of appreciation at age two. See supra text accompanying notes 142. The excess tax benefit for these horses is ($6088.19 - $1233.59) = $4854.60 multiplied by the marginal tax rate.
owners would simply send horses to the glue factory early.\textsuperscript{162}

Suppose, on the other hand, that highly successful horses tend to have shorter useful lives and that a surtax is imposed on early retirements. Owners might then respond by keeping horses around after their racing days are over and claiming that they are still in service. This strategy would be even easier for non-living assets such as machines. Owners of such assets do not have to contend with physical death as a fact that rebuts the claim of continued service.

If trading costs are zero, then the goal is simply a rule that induces the owner to trade frequently. Frequent trades establish a flow of taxes appropriate to the actual value path of the asset. By trading, the taxpayer does the government's work for it. The previous section discussed ex ante rules designed to induce trading. To the extent these rules are unsatisfactory, it seems that actual observation of the asset path is the best palliative.\textsuperscript{163}

In the case of "moderate" trading costs, the favored ex ante approach results in the tax treatment of some assets being established by trading as in the zero trading cost case and the tax treatment of the remaining assets being in accord with an ex ante schedule as in the case of very high trading costs. The same reasons for being skeptical of improving matters by embellishing the ex post rules in the case of zero and very high trading costs apply equally to the moderate trading cost case. The tax treatment of the traded assets is probably already close to being correct. For the non-traded assets not much improvement is possible short of gathering information on the actual value path over time.

This point is strengthened by considering an ex post solution to strategic loss-taking devised by Alan Auerbach.\textsuperscript{164} Auerbach constrains the information available to be the amount realized on disposition, the holding period, the interest rates applicable during the holding period, and the investor's tax rate. He shows that there is one and only one tax approach that will not affect the investor's decision of whether to hold or sell any asset.\textsuperscript{165}

\textsuperscript{162} For the same reasons, early retirement would be the rule for less talented horses. These horses have a lower revenue base so that the surtax would loom even larger. As of 1990, the salvage value of a horse was about $450. See Treasury Horse Study, supra note 24, at 11.

\textsuperscript{163} In particular, both of the most prominent practical ex post approximation techniques fail to improve on the proposed ex ante rule at all. See supra note 171.


\textsuperscript{165} More formally, Auerbach requires that the realization-based tax system satisfy a condition he calls "holding period neutrality." This condition requires that the before tax certainty equivalent return on any asset be independent of the length of the holding period or the asset's past pattern of returns. See id. at 169. When such a condition holds, strategic loss-taking will not play any role. Holding period neutrality will ensure that investors will view assets held for a period of time as equivalent to assets just purchased. The basis of the assets, their price history, and their holding period will not have any impact on the decision to sell or continue holding an asset.

Auerbach shows rigorously that there is only one tax approach that will meet this condition given that the available information at sale or disposition is limited to the amount
To make this approach clear, suppose that the taxpayer holds an asset for two years during which the applicable interest rate was 10% and then sells it for $121. Assuming that the asset did not produce any cash flows, the required approach is to apply an ideal accretion tax under the assumption that the asset appreciated at the applicable interest rate during the holding period. In particular, we would presume that the investor purchased the asset for $100 so that it grew at a 10% compound annual rate to $121 after two years. If the investor were in the 40% bracket and the accretion tax was annualized, $4 would be due at the end of one year on the $10 gain during that year and $4.40 at the end of the second year on the $11 gain during that year. Since all the tax is paid at the end of the second year, the investor would pay interest at the investor's after-tax rate of 6% per year on the $4 due at the end of the first year.

As Auerbach points out, this approach produces ex ante results that are equivalent to a perfect accretion tax but deviates from the ex post results that would follow from such a tax. Ex ante equivalence means that an investor's decision to buy, hold, or sell assets will be the same as if a perfect accretion tax applied. The difference in ex post treatments is easy to see in the case of the asset sold for $121. The tax due is independent of the amount that the investor paid for the asset. If the asset cost $10,000, the investor has a huge loss but is required to pay a tax upon disposition. Similarly, if the asset cost only $1, the investor earned $120 but pays less than $9 in tax despite being subject to a 40% rate.

Applying the Auerbach approach to depreciable assets requires examining how the approach applies to an asset that produces cash flows as well as proceeds upon disposition. In this case, the Auerbach scheme is equivalent to taxing each cash flow as if it were an asset disposition. In other words, a cash flow of $X that occurs time T after a project begins is taxed under an accretion tax approach as if the taxpayer had purchased an asset at time 0 that grew at the prevailing riskless interest rate to $X at time T. For depreciable assets, one would tax all the cash flows that result from the asset, including any realization at disposition or retirement in this manner. This approach would succeed in eliminating strategic loss-taking problems but at the cost of discarding important ex post aspects of an accretion tax.

realized, the holding period, interest rates during the holding period, and the applicable tax rate. See id. at 172 (Proposition 3).
166. See id. at 172.
167. See id. at 169, 176.
168. I use the term “Auerbach approach” and similar language for convenience and to attribute proper credit for what is a very clever and interesting idea. In his original work concerning the approach, Auerbach notes and addresses weaknesses in the approach in addition to discussing its advantages.
169. See id. at 175.
170. The same elements are present as in the example of the asset that was sold for $121. The Auerbach approach entails ignoring the depreciable asset's cost so that tax outcomes are disconnected from the accretion tax ideal of taxing all wealth changes as they occur. See supra notes 43 and 135. Auerbach states this point and responds to it by argu-
In the ensuing subsections, we will consider approaches that have less severe effects on the ex post properties that characterize an accretion tax. It is important to keep in mind Auerbach's uniqueness result: Absent moving to his approach, there is no way to tinker with the accretion tax system in a way that will preserve the ex ante properties of that system and also eliminate incentives to trade or hold assets based only on tax considerations. Once we give up on observing the past price path of assets, we are in a world of trade-offs. Auerbach's approach trades off the ex post properties of accretion taxation to eradicate strategic trading. All of the approaches in the next subsections will involve some sacrifice of accretion tax norms in order to achieve some diminution in strategic trading opportunities.

2. The Depreciation Bond Approach

Allowing for coordinated changes in the ex post and ex ante portions of the tax depreciation rules makes a whole additional set of alternative policies available. In particular, the disposition rules can be altered to penalize strategic loss-taking and the ex ante schedule can be adjusted to take into account the new disposition rules.

One simple policy of this type is the depreciation bond approach. This approach consists of three basic elements. First, the original owner of an asset receives all the depreciation benefits from the asset regardless of how long the original owner retains the asset. Depreciation is therefore similar to a nontransferable bond. Original ownership of the asset entitles the owner to a fixed stream of deductions, just as a bond entitles the owner to a fixed stream of payments.¹⁷¹ Second, dispositions of all kinds are not taxable events. There is no loss deduction upon retirement, and sale does not result in taxable gain or in a deduction for loss. Third, the depreciation schedule would be set using the Hulten and Wykoff approach. Instead of simply basing the schedule on the age-price profile for surviving assets, that profile would be first adjusted by multiplying the price at each age by the probability of survival to that age.¹⁷²

¹⁷¹ Given that future deductions are set in advance, a variant of this step would be to give the taxpayer a single deduction equal to the present value of depreciation at the time the asset is purchased. Such an approach would be identical to the “first-year capital recovery system” proposed by Professors Alan Auerbach and Dale Jorgenson in 1980. See Alan J. Auerbach & Dale W. Jorgenson, Inflation-proof Depreciation of Assets, HARV. BUS. REV. Sept.-Oct. 1980 at 113. This approach has the advantage that the effect of inflation is removed from the depreciation rules. The present value of depreciation and the ensuing tax savings is in current dollars. Thus, there is no need to correct for inflation by indexing adjusted basis or by using other similar adjustments. Auerbach and Jorgenson’s approach, however, has a different disposition rule than the depreciation bond approach. They propose that the seller take the present value of depreciation for the used asset being sold into income at the time of sale. See id. at 114. This disposition rule is similar to the “account adjustment rule” discussed infra in subsection 4. See infra note 190 and accompanying text.

¹⁷² For a discussion of the Hulten and Wykoff approach, see supra text accompanying notes 46-61.
The depreciation bond approach cleanly eliminates the possibility of strategic loss-taking. Since dispositions are not taxable events, trading has no tax effect. Whatever ex ante depreciation schedule is set will "stick" since taxpayers cannot alter the pattern of deductions under the schedule by realizing losses.

Use of this disposition rule requires that a different depreciation schedule be set ex ante. Given that dispositions are not taxed, a depreciation schedule based on the age-price profile for surviving assets would be inappropriate. The argument for using that schedule in the face of retirement risk is that loss deductions granted upon disposition automatically adjust for asset retirements.\(^1\) To replicate economic depreciation under the depreciation bond approach, retirement losses must be built into the schedule as in the Hulten and Wykoff approach.

Although the depreciation bond approach eliminates strategic loss-taking, it makes another kind of strategic trading viable: the "churning" of depreciable assets. If each new owner were entitled to a depreciation bond based on the purchase price, a new asset could be traded back and forth to generate deductions many times the original cost of the asset. This churning cannot happen under current law because excess depreciation is recaptured as gain upon disposition. The original cost of each new asset can only be deducted once.

A potential response to this problem would be to make used assets nondepreciable. Under that rule, used assets would trade at a discount reflecting the absence of depreciation deductions. For example, if a brand new asset were traded right after it was first purchased and if every taxpayer faced the same marginal tax rate, the discount would equal the present value of the depreciation bond multiplied by that tax rate. This result would ensue because the purchaser could buy a new asset instead and obtain a depreciation bond at the same time.

This price environment would ensure that no individual's investment decisions would be biased for or against used assets versus new assets. Sale of used assets would be at a price reduced by the present value of the tax benefits that would result from buying a new asset with the same expected life and productivity. Ignoring trading costs, an owner would be indifferent between holding the old asset and replacing it with the new asset. Similarly, a purchaser would be indifferent between purchasing a new asset accompanied by a depreciation bond or a similar used asset without a depreciation bond but at a discount reflecting the absence of the bond. The purchase and sale of the depreciable assets would turn strictly on business considerations and not on tax benefits or detriments.

Making used assets nondepreciable is not a perfect solution. This rule puts tremendous pressure on the distinction between new and used assets. An asset would sell for a higher price if it was a "new" asset. The taxpayer therefore would have an incentive to "rebuild" or otherwise al-

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\(^1\) See supra text accompanying notes 43-44.
ter the old asset and claim that it is a new asset. A substantial and complex set of rules might be required to maintain the distinction between old and new assets.

There are two other problems with the depreciation bond approach. First, the approach fails to track wealth changes that happen as the value of the asset fluctuates. Thus, for example, if the asset fails early in its life, deductions are not accelerated to reflect the early loss. Eliminating the ex post corrections triggered by dispositions in the current system weakens the accuracy of the system in tracking wealth changes.

There is a “diversification” argument that suggests this problem is not pressing. Most of the depreciable assets in the United States are held by large corporate taxpayers. A typical large corporation undoubtedly holds a large number of depreciable assets. In addition, individuals who hold ownership positions in the corporations often will invest in many different corporations. In effect, these individuals hold a small ownership interest in a very large number of very different depreciable assets. Early retirement of some assets is likely to be offset by unexpectedly long useful lives for other assets. Assets used in one industry that go up in value because industry revenues will be higher than expected are likely to be offset by assets used in another industry that are falling in value because industry revenues will be lower than expected. If depreciation schedules are set in advance to mimic average expected economic depreciation, diversification may bring most investors’ actual experience close to this average.

The problem with this argument is that revenue risk is correlated across the aggregate of all depreciable assets. A significant portion of this risk cannot be eliminated costlessly by holding a diversified portfolio. Thus, there is a substantial probability that actual asset values in aggregate will deviate significantly from the performance that is expected on average. The problem that eliminating ex post adjustments reduces the ability of the tax system to track actual changes in wealth remains despite the ability to hold a diversified portfolio.

174. A kind of tax arbitrage is possible by taking the following steps: buy a new asset, alter it, and resell it as “new” for a price equal to the purchase price plus the cost of alterations. This maneuver is tax arbitrage in the sense that the taxpayer receives depreciation tax benefits without having any ongoing investment.

175. For example, value added accounts might be set up for depreciable assets. When an asset is purchased, only the “new value” added by refurbishing or altering the asset would be depreciable. The potentially large administrative and taxpayer compliance costs for operating this system are obvious.

176. Under accretion tax theory, tracking wealth changes as they occur is normatively desirable. See supra note 135.


178. Portfolio diversification, dividing one’s wealth into many different investments, is a superior strategy for most investors. See BREALEY & MYERS, supra note 70, at 131-34.

179. See id. at 132.

180. There is also an efficiency problem when dispositions are not taxable events: riskier assets are favored by the depreciation rules. The causes of this phenomenon are complicated. For a discussion, see infra Appendix D.
An additional potential problem with the depreciation bond approach arises if not all taxpayers are subject to the same marginal tax rate. In that case, the approach permits depreciation tax benefits to be sold by low bracket taxpayers to high bracket taxpayers. Instead of purchasing new depreciable assets directly, the low bracket taxpayer can arrange for a high bracket taxpayer to buy the assets and immediately resell them to the low bracket taxpayer. The high bracket taxpayer retains the depreciation benefits in the form of the depreciation bond and can pass most of the after-tax value of the benefits on to the low bracket taxpayer in the form of a reduced price.

This maneuver is reminiscent of safe harbor leasing where a high bracket taxpayer leases an asset to a low bracket taxpayer. In this case, the high bracket taxpayer formally retains ownership of the asset and therefore is entitled to the stream of depreciation deductions generated by the asset. The depreciation bond, however, is much simpler since the high bracket taxpayer does not have to retain ownership of the asset to enjoy the tax benefits that flow from depreciating the asset.181

The potential problem with this result is that it may be undesirable to allow low bracket taxpayers to benefit from depreciation deductions to the same degree as high bracket taxpayers. In a scheme that attempts to simulate economic depreciation, these larger benefits compensate for a higher tax rate on revenues so that all taxpayers, independent of marginal rate, perceive the same market value for all investments.182 This neutral outcome will be violated if low bracket taxpayers pay tax on revenues at their low rate but then receive benefits from depreciation deductions based on a higher rate. In that case, low bracket taxpayers will be willing to pay more in pre-tax dollars for investment assets than high bracket taxpayers who are equally efficient users of the assets.183

181. Soon after the addition of safe harbor leasing to the tax code in 1981, Professors Warren and Auerbach in fact suggested direct transferability of tax benefits as an alternative method for achieving the same goals. Direct transferability eliminates tax and other legal problems associated with the requirement that the high bracket taxpayer retain ownership while the low bracket taxpayer uses the asset. See Alvin C. Warren, Jr. & Alan J. Auerbach, Transferability of Tax Incentives and the Fiction of Safe Harbor Leasing, 95 HARV. L. REV. 1752, 1774-78 (1982).

182. See infra note 253.

183. If the tax code does not adhere to economic depreciation, then the transferability of tax benefits may be desirable from a neutrality perspective. In fact, the existence of accelerated depreciation provided a series of rationales for the transfer of tax benefits inherent in safe harbor leasing. For a good discussion, see Warren & Auerbach, supra note 181, at 1753-62, 1768-72; Alvin C. Warren & Alan J. Auerbach, Tax Policy and Equipment Leasing After TEFRA, 96 HARV. L. REV. 1579, 1583-85 (1983).

We have assumed in the text example that most of the benefits of purchase of depreciable assets by high bracket taxpayers for resale to low bracket taxpayers are passed on to the low bracket taxpayers. Thus, high bracket taxpayers will not reap very much of a reduction in taxes from the deals. If the incidence is different and high bracket taxpayers receive much of the benefit, then these taxpayers will receive transfers from the government in their role as intermediary between the asset manufacturer and asset user. In the absence of arguments of the sort that Warren and Auerbach put forward, this intermediary role has no economic purpose and neither do the transfers.
3. The Proceeds Tax Rule

The proceeds tax rule is an approach used for some group accounts.\textsuperscript{184} The rule addresses two of the weaknesses of the depreciation bond approach: The proceeds tax rule stops churning dead in its tracks and makes tax deals between taxpayers in different brackets unprofitable. Using the proceeds tax rule does have a disadvantage. The rule interferes with trading of depreciable assets that is motivated by business rather than tax goals.

The proceeds tax rule is so named because it requires the entire proceeds of an asset sale to be taken immediately into income. There is no deduction for the adjusted basis of the asset. The remaining adjusted basis is deducted as if the asset had been retained.\textsuperscript{185} Thus, the proceeds tax approach is similar to the depreciation bond approach in that depreciation, once set at the start, continues unadjusted until exhaustion even if the taxpayer sells or retires the asset before it is fully depreciated. The difference between the two approaches is that the depreciation bond approach ignores dispositions while the proceeds tax rule taxes the gross proceeds from the disposition.

The proceeds tax rule clearly blocks strategic loss-taking. There is no tax benefit from selling an asset that has a "loss" on it. Recovery of the cost of the asset will proceed on the original schedule even in the face of a sale or other disposition. In fact, sale results in \textit{additional} tax in a present value sense for the "loss" asset. In effect, the remaining value is taxed (even though it is below adjusted basis for a "loss" asset) and then recovered as \textit{future} deductions along the original depreciation deduction schedule. For both "gain" and "loss" assets, there is therefore a penalty for trading because adjusted basis is ignored at the time of disposition.

The proceeds tax rule also makes churning a losing strategy. Purchase of the asset followed by immediate resale results in income equal to the value of the asset plus a series of future depreciation deductions that sum to that value. In effect, the taxpayer receives income and an equal deduction, but the deduction is delayed. For a taxpayer facing constant marginal tax rates and positive interest rates, this combination has negative net present value. Because of the time value of money, the reduction in future taxes from the delayed deductions is worth less than the immediate tax on the income.

The fact that the proceeds tax rule punishes asset sales by ignoring adjusted basis means that it is effective in controlling both strategic loss-taking and churning. However, this feature of the rule also leads to its major disadvantage. The rule will penalize sales that are motivated by business rather than tax considerations. For instance, the taxpayer who

\textsuperscript{184} See \textit{supra} text accompanying notes 318-319.

\textsuperscript{185} For a more complete description of the proceeds tax rule in the context of group accounts, see \textit{infra} text accompanying note 318.
wants to sell some depreciable assets in order to finance a new production technology will have to pay a tax penalty to do so.

Another problem with the proceeds tax rule is that it may be difficult to compute the appropriate ex ante depreciation schedule. Moving from a schedule based on the age-price profile for surviving assets to a Hulten and Wykoff schedule is appropriate when asset dispositions are untaxed as in the depreciation bond approach. But this degree of adjustment is inadequate when dispositions are subject to a punitive tax. It is difficult to specify how much additional adjustment is warranted. The appropriate amount depends on the frequency of "legitimate" business sales of assets that are not fully depreciated. That frequency would be difficult to forecast with any accuracy.

Finally, the inherent penalty on asset sales generated by ignoring adjusted basis results in the same equity problem as under the depreciation bond approach. In particular, the system fails to track and tax wealth changes as they occur.

4. The Account Adjustment Rule

The account adjustment rule is used for some types of group accounts, and the strict definition of the rule is embedded in a group accounting framework. Group accounts treat a group of assets as if they were a single asset for purposes of depreciation. The account adjustment rule halts the tax depreciation of assets that are sold or retired from group accounts and requires that the gain or loss on the disposition be computed. In contrast to individual item accounting, this gain or loss is not recognized immediately. Instead, gain is subtracted from the "adjusted basis" of the group account while loss is added to that "adjusted basis." In effect, the tax on the gain or the reduction in tax on the loss is delayed since gain reduces and loss increases future depreciation deductions.

Translating this rule to the framework of individual item accounting, it is as if gain or loss is taxed or credited under the schedule used to depre-

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186. We have not had to take this special category of sales into account under the previous depreciation methods that we have studied. If the depreciation schedule is based on the age-price profile for surviving assets and that profile is correct, the market value of an asset will be equal to its adjusted basis. There are no tax consequences of sale. See supra text accompanying notes 43-45. The depreciation bond approach exhibits this trait even more strongly. Under any depreciation schedule that is set in advance, sale has no tax consequences. That is, the failure of sale to have tax consequences does not depend on the depreciation schedule being "correct."

The Treasury Department did take asset sales into account in two of its studies, but its approach was to assume that past patterns of sale would persist into the future. See supra note 80. This approach treats the sales as exogenous to the tax system, and, to the extent sales are not tax motivated, assumes that the business conditions that dictated sale during the sample period will be the same in the future.

187. For a more complete description of the rule in the context of group accounting, see infra text accompanying note 317.

188. Adjusted basis is placed in quotation marks because group accounting does not formally assign an adjusted basis to the group account. However, the results of the account adjustment rule are accurately described using the concept of the group account having an "adjusted basis." See infra text accompanying note 313.
iate the asset before it was sold.\textsuperscript{189} Suppose, for example, that the asset was being depreciated under a ten year, straight-line schedule. If the asset is sold for a $100 gain at the end of five years, the account adjustment rule calls for $20 of income for each of years six through ten inclusive.\textsuperscript{190}

The account adjustment rule blocks churning as an effective strategy but allows strategic loss-taking. Churning will not work because the taxpayer who buys an asset and then immediately resells it for the same price will not be allowed to depreciate the asset. The account adjustment rule cuts off future depreciation deductions at the time of sale or disposition. On the other hand, strategic loss-taking is still a viable strategy. Loss is recognized, although somewhat delayed compared to the individual item method. This allows the taxpayer to accelerate depreciation when it turns out to be too slow ex post. Concurrently, the taxpayer can retain depreciable assets with gains on them and thus need not decelerate depreciation at all when it turns out to be too fast ex post.

It is true that strategic loss-taking is not as profitable a strategy under the account adjustment rule as under the individual item method. Losses have delayed effect under the account adjustment rule, while they reduce taxes "immediately," that is, in the year of disposition, under the individual item method.

The impact of the account adjustment rule on "normal" business sales is similar to the impact of the individual item method. If the depreciation schedule is based on the age-price profile for surviving assets and that profile is correct, the market value of an asset will be equal to its adjusted basis. There are no tax consequences of sale, and the taxpayer decides whether or not to sell based strictly on business considerations. This happy outcome is consistently attainable only if the correct age-price profile is known in advance. In the more general context adopted in this Part, there is price path uncertainty. An ex ante depreciation schedule based on the expected price path may prove to be too fast for some assets. In this case, assets will have gains on them and there will be a lock-in effect: The taxpayer may forgo sales that make business sense in order to avoid recognizing the gains.\textsuperscript{191}

The account adjustment rule does require an accompanying adjustment to the ex ante depreciation schedule. The schedule based on the age-price profile for surviving assets is appropriate for individual item ac-

\textsuperscript{189} This translation to individual item accounting will be \textit{exact} if the group account consists entirely of assets placed into the service at the same time and depreciated under the same schedule. There are currently two different group accounting schemes applicable to two different classes of assets. Only one of these schemes restricts group accounts to aggregations of such a uniform set of assets. \textit{See infra} text accompanying notes 306-311.

\textsuperscript{190} An alternative method would be to take the present value of the entire stream of gains into income at the time of sale. If it is true that the purchaser of the used asset would depreciate under a five year, straight-line schedule, then this present value treatment would be identical to the disposition rule proposed by Professors Auerbach and Jorgenson under their "first-year capital cost recovery system." \textit{See supra} note 171.

\textsuperscript{191} The depreciation bond approach avoids this lock-in effect. \textit{See supra} note 186. The proceeds tax rule exacerbates it. \textit{See supra} text accompanying notes 185-186.
counting because that schedule presumes that retirement loss deductions will have immediate effect. When assets are retired, treatment in accord with "economic depreciation" is assured because the loss deduction is exactly equal to the loss in value when the asset is retired. Where the loss deduction is delayed, the associated reduction in taxes is delayed and the taxpayer receives too small a benefit under the standard of economic depreciation. Thus, if the account adjustment rule applies to dispositions, the ex ante depreciation schedule will have to be accelerated compared to the schedule appropriate for individual item accounting.

5. A Comparison of the Alternative Disposition Rules

We are now in a position to summarize the strengths and weaknesses of various disposition rules. These strengths and weaknesses lie along five dimensions:

(1) the strategic loss-taking problem;
(2) the churning problem;
(3) the impact on "normal" business sales of depreciable assets;
(4) the need to know adjusted basis in order to compute tax treatment on sale;
(5) the difficulty in adjusting the ex ante depreciation schedule.

The following table summarizes the performance of the various disposition rules along these dimensions. In the table, the "individual item..."
method” refers to the policy of taxing gains and losses on dispositions immediately.195

<table>
<thead>
<tr>
<th>TABLE III: PROS AND CONS OF DISPOSITION RULES</th>
</tr>
</thead>
<tbody>
<tr>
<td>disposition treatment</td>
</tr>
<tr>
<td>individual item accounting</td>
</tr>
<tr>
<td>depreciation bond approach</td>
</tr>
<tr>
<td>proceeds tax rule</td>
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<tr>
<td>account adjustment rule</td>
</tr>
</tbody>
</table>

Clearly none of the four treatments is dominant. Choosing between them involves trading off one problem against another. Perhaps the most appealing treatment in the group is the depreciation bond approach, but this approach will be viable only if the churning problem can be addressed in a way that does not involve administrative or taxpayer compliance costs that are too large. It is unclear whether the approach suggested earlier of making used assets nondepreciable will meet this goal.196

D. Some Conclusions

Assuming that basing depreciation on annual changes in asset value is infeasible, two general alternatives have emerged for dealing with strategic loss-taking and the fact that the future price path for most depreciable assets is uncertain. One is ex ante adjustment in the depreciation schedule. The problem with this approach is that the desired degree of adjustment depends heavily on the traits of individual assets. In particular, one would need to know whether trading is costly for each asset and also the degree of uncertainty in future prices. The need to classify assets by durability has proven to be difficult enough.197 Factoring in trading costs and price uncertainty would greatly complicate an already arduous task.198

195. The name is derived from the fact that this method is used for individual item accounting under current law. See supra text accompanying notes 7-8.

196. See supra text accompanying notes 174-175.

197. One need only read any of the six Treasury studies of particular assets to be convinced of this fact. See supra note 24 (citing the six studies).

198. Analytic complexity is not the only problem that would come about from a more extensive methodology. Depreciation policy is subject to considerable political pressure from concerned groups. For example, in 1988, Congress withdrew the power to set useful
An alternative approach is to change the disposition rules. These changes would require an accompanying adjustment of ex ante depreciation schedules, but the required adjustments are analytically clear for many of the disposition rules. Unfortunately, there does not appear to be a clear winner among the disposition rules since each rule has some drawbacks. Nonetheless, some of these rules have potential and are worthy of further study.

IV. GROUP ACCOUNTING

Group accounting methods allow a group of assets to be depreciated in a unified account as if they were a single asset. This approach makes tax accounting much easier and less expensive in situations where a taxpayer holds large numbers of identical, low-value assets and where keeping track of these assets on an item by item basis is difficult. This administrative cost saving is undoubtedly the major reason for permitting group accounting.\(^9\)

This Part shows that tax policy toward group accounts turns heavily on the fact that strategic loss-taking is a particularly serious problem for certain types of group accounts. As a result, a powerful analysis of group accounting policy is possible based on the theory developed in the previous Part.

This policy analysis is not merely of theoretical interest. In the face of potentially large administrative cost savings, group accounting is an important aspect of depreciation policy. Controlling strategic loss-taking should not be accomplished at the cost of heavily penalizing taxpayers who choose group accounting.

Section A describes current law governing group accounting. Section B discusses the form that rules addressing group accounting should take.

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\(^9\) See supra note 24. The original proposal restricting Treasury's power merely barred Treasury from lengthening lives. This fact suggests that the withdrawal of power was prompted by industries who feared that Treasury studies of their depreciable assets would result in decelerated depreciation.

As is the case with durability, there is a significant degree of uncertainty about both trading costs and future price volatility for various assets. But the more uncertain the determination of depreciation treatment becomes, the more scope there is for politics. When the political process is factored in, adding more uncertain factors to be considered may make depreciation policy less effective.

Along these lines, it is worth noting the motivation for Hulten and Wykoff's seminal study of economic depreciation. See Hulten & Wykoff, supra note 3, at 82. Their goal was to counter the claim that depreciation policy must be "politically determined" since a theoretically correct depreciation policy cannot be implemented. They intended to accomplish this goal by showing that there is a clear way to estimate economic depreciation. See id.

199. For tangible assets put into service during 1981-1986, group accounting was permitted only for assets where such a rationale was likely to apply. This limitation was motivated by a technical concern about the recapture of the investment tax credit from assets sold out of group accounts. As a result, an attempt was made to narrow the category of assets for which group accounting was allowed as much as possible. See infra text accompanying notes 301-305.
Finally, Section C considers the problem of designing optimal rules under certain constraints set by Congress.

A. Current Law

Group accounting is quite complex. Four major group accounting systems have been in force since 1971. Two remain in force today.

This section provides a qualitative overview of group accounting. Particular attention is devoted to the group accounting rules for the tax treatment of asset dispositions since the disposition rules determine the scope for strategic loss-taking. Appendix E provides a more extensive discussion of group accounting rules under all four post-1971 regimes and provides primary and secondary references for the assertions about current law made in this subsection.

The two group accounting systems that apply to assets put into service currently, that is, in 1999, are a system of "multiple asset accounts" established under the Regulations for section 167, and a system of "general asset accounts" established by the Tax Reform Act of 1986 and delineated in Regulations that apply to assets placed in service in taxable years ending on or after October 11, 1994. The general asset account rules apply to "recovery property," property subject to depreciation under MACRS. Recovery property includes most real and personal tangible property. The multiple asset account rules apply to property that is depreciable but that is not recovery property. This category includes intangible assets such as patents and copyrights.

Both general asset accounts and multiple asset accounts are elective and quite flexible. A taxpayer can set up any number of these accounts and can place any number of assets, including one, in any of the accounts. Use of group accounts for any given asset does not preclude treating other similar assets put into service in the same year or a later year under individual accounting. There is one important difference in the scope of general asset accounts and multiple asset accounts. A single general asset account may contain only assets that were placed into service the same taxable year and that are subject to the same depreciation method, the same convention, and the same recovery period. In contrast, the only limitation on multiple asset accounts is that the same depreciation method (e.g., straight-line or declining balance at a particular...
rate) apply to each account during its entire life.\textsuperscript{203}

The central feature of group accounting is that each group account will be treated as if it is a single asset for accounting purposes. But different types of group accounts are subject to different tax rules upon disposition of assets. In addition, there is a traditional distinction between different types of dispositions. The multiple asset account rules divide dispositions into two categories: "normal" retirements and "abnormal" retirements. The difference between these two categories is somewhat fuzzy,\textsuperscript{204} but some particular cases are clear. For instance, if a few assets are sold from a group account long before they are obsolete, the sales will be treated as abnormal retirements. On the other hand, discarding an asset that wears out in an ordinary way from use will be treated as a normal retirement.

The general asset account rules do not use the normal and abnormal retirement categories but instead define certain dispositions as "qualifying dispositions."\textsuperscript{205} The qualifying disposition category overlaps with "abnormal retirements" in the multiple asset account scheme but is narrower and more precise.\textsuperscript{206}

There are three general methods for treating dispositions under multiple asset accounts and general asset accounts: the individual item method, the account adjustment method, and the proceeds tax method. Part III defines and discusses two of these methods,\textsuperscript{207} and Appendix E contains a more detailed description of all three methods for the interested reader.\textsuperscript{208} For multiple asset accounts, the account adjustment method applies to normal retirements while the individual item method is used for abnormal retirements. For general asset accounts, the individual item method applies on an elective basis to the special category of "qualifying distributions," while the proceeds tax rule applies to all other dispositions.

\textsuperscript{203} See Dooher et al., 412 T.M., Depreciation Methods—Item and Group Accounts (Bureau of National Affairs, 1989), at A-1 to A-2.

\textsuperscript{204} For multiple asset accounts, the definitions of normal and abnormal retirements are set out in Treas. Reg. § 1.167(a)(8). An abnormal retirement is one "due to a cause not contemplated in setting the applicable depreciation rate." Treas. Reg. § 1.167(a)(8). As an example, the Regulation cites an asset destroyed by a casualty or one that "has lost its usefulness suddenly as the result of extraordinary obsolescence." \textit{Id.} Generally speaking, all the "facts and circumstances" must be considered to determine whether a retirement is normal or abnormal. \textit{Id.}

\textsuperscript{205} See Treas. Reg. § 1.168(i)-1(e)(3)(iii)(B) (defining "qualifying dispositions").

\textsuperscript{206} In particular, the "qualifying dispositions" category includes casualty related retirements, Treas. Reg. § 1.168(i)-1(e)(3)(iii)(B)(1), but the category does not incorporate the "extraordinary obsolescence" concept or the idea of "a cause [for retirement] not contemplated in setting the applicable depreciation rate" that defines "abnormal retirements" for purposes of multiple asset accounts. Instead, "qualifying dispositions" explicitly include the narrower cases of charitable contributions of property from general asset accounts and dispositions due to termination of a business. Treas. Reg. § 1.168(i)-1(e)(3)(iii)(B)(2) and (3). The exact distinction between abnormal retirements in the multiple asset account scheme and qualifying dispositions in the general asset account system does not bear on the discussion in this Article and is not explicated further.

\textsuperscript{207} See supra text accompanying notes 184-192.

\textsuperscript{208} See infra text accompanying notes 312-318.
B. Optimal Group Accounting Rules

In discussing the rules that should be used for group accounting, an important threshold question is why group accounting requires rules that are different from the rules for individual asset accounting. Given that the goal of group accounting is to allow aggregate treatment of assets that are costly to account for individually, we need to focus on whether aggregation of several assets into one “group account” asset raises any special problems. Special problems related to strategic loss-taking exist, but only for certain types of aggregation. In particular, problems exist when assets subject to different depreciation schedules or assets put into service at different times are aggregated in the same account. We demonstrate this point in subsection 1 and then discuss possible solutions in subsection 2.

1. Problems Posed By Group Accounting

Some terminology is useful. Accounts that include assets put into service in different years are called “open” accounts while accounts limited to assets put into service in a single year are “closed” accounts. The term “open” captures the idea that the taxpayer may continue to add assets to the account after it has been started in some particular year. Furthermore, accounts that contain assets subject to different depreciation schedules are called “heterogeneous” accounts while accounts that contain only assets subject to a single schedule are “homogeneous” accounts.

Consider first the case of homogeneous, closed accounts, that is, group accounts containing only assets placed into service in the same year and subject to the same depreciation schedule. In this case, there is no real need for separate rules for group accounting. Each asset can be treated as if it were an individual asset.

There are two reasons for this conclusion. First, because all assets in the group would be subject to the same depreciation schedule if accounted for individually, the group can be assigned that same schedule. There is no scope for changing the depreciation schedule for any given asset.

209. See supra note 199 and accompanying text.

210. Group accounting rules raise issues that result from aggregation of depreciable assets. There is also a body of rules that deal with issues that arise from the “disaggregation” of depreciable assets. These are the “repair” rules. Certain components of a depreciable asset may wear out and require replacement before other components. An important issue is how repairs or replacement of components should be treated. See supra note 134.

There is some fascinating history associated with this issue. For example, prior to 1981, the Code allowed taxpayers to “disaggregate” buildings and depreciate the components based on their separate and distinct useful lives. This method was called “component depreciation.” It was abolished in 1981.

Despite the fact that the “repair” rules are intellectually interesting and practically significant, we leave an examination of the rules for another day.

211. This terminology is derived from Dooher et al., supra note 203, at A-2, A-33 to A-39.
asset by aggregating it with other assets in a group account. Second, the
adjusted basis of any particular asset in the group is readily determinable
even though it might be impossible to distinguish one asset in the group
from another. This adjusted basis is simply the adjusted basis for the
group multiplied by the proportion of initial group value represented by
the asset in question.\textsuperscript{212} Thus, the disposition of an asset from the group
account can be treated in exactly the same manner as disposition of that
asset would have been treated under individual asset accounting. Be-
cause neither depreciation of surviving assets nor disposition of assets
need be treated differently, the same depreciation policy considerations
apply as under individual asset accounting.

This happy result falls apart if the law allows heterogeneous or open
accounts. Heterogeneous accounts contain assets subject to different de-
preciation schedules. When such assets are placed into the same group
account, the group account is depreciated under a composite schedule
that is a weighted average of the schedules that would have applied to the
individual assets placed into the account.\textsuperscript{213} Applying the individual item
approach to dispositions will create opportunities for strategic loss-taking
whether the heterogeneous account is open or closed.

The weighted average approach involves computing an initial deprecia-
tion rate based on the assets initially in the accounts and then redeter-
mining the rate “whenever additions, retirements, or replacements
substantially alter the relative proportions of types of assets in the ac-
counts.”\textsuperscript{214} This redetermination rule applies to both closed and open
heterogenous accounts. Typically, such accounts will display some change
in asset mix as assets age and are retired. Nonetheless, leading commen-
tators have noted that “[n]either taxpayers nor the Internal Revenue Ser-
vice personnel have been inclined to go through the involved and
laborious process of recomputing rates very often,” especially for open
heterogeneous accounts, since the “substantial alteration” of asset types
in such an account need be only temporary.\textsuperscript{215} The taxpayer may add

\textsuperscript{212} All assets are placed into service at the same time and are subject to the same
depreciation schedule as they would be if accounted for individually. If the adjusted basis
for the group is X% of initial value, it will be true that the adjusted basis of an individual
asset would have been X% of initial value if this asset had been accounted for individually.
Thus, we can easily convert the adjusted basis of the group into an adjusted basis for an
asset in the group if we know the aggregate initial value of the group and the initial value
of the asset.

\textsuperscript{213} The text below details the major features of the weighted average approach. The
rules for this approach are most clearly set out for “open heterogeneous accounts.” These
are group accounts with assets placed into service at different times as well as assets subject
to different depreciation schedules. This mixture is the most general case, and it is not
surprising that the rules are the most well-developed for this case. For a good description
of these rules for multiple asset accounts, see Dooher et al., \textit{supra} note 203, at A-34 to A-
37. Accounts consisting of assets placed into service at the same time but subject to differ-
ent depreciation schedules are called “heterogeneous accounts closed at the end of the first
year.” The rules for these accounts are not as clear or well developed. \textit{See id.} at A-37 to
A-38.

\textsuperscript{214} Treas. Reg. § 1.167(a)-7(d).

\textsuperscript{215} Dooher et al., \textit{supra} note 203, at A-36.
new assets to the account in the near future, thereby shifting the types of asset in the account back toward the original mix.

Use of the weighted average schedule in group accounts allows the taxpayer to create some lucrative strategic loss-taking opportunities, even if the underlying accounts are subject to redetermination annually. These opportunities differ from the opportunities that exist when individual asset accounting applies. Under individual asset accounting, strategic loss-taking is a potentially serious problem when the future price path for surviving assets is not known in advance: Market price may range below adjusted basis under whatever depreciation schedule is set, and the taxpayer can "accelerate" the schedule by selling the asset for a tax-loss and combining that sale with purchase of a similar replacement asset. In the group accounting context, strategic loss-taking is possible even when there is no uncertainty about asset lives or future price paths. When group accounts use a weighted average schedule rule, the taxpayer can deliberately mix assets with very different depreciation schedules in a group account to achieve more favorable results than the results that would follow from depreciating each asset separately.\footnote{216. This group accounting strategy does not always involve combining a sale for a loss with repurchase of a replacement asset. Nonetheless, it is appropriate to refer to this strategy by the term "strategic loss-taking" since the beneficial results stem from anticipated retirements that accentuate losses. This point is explained in more detail below after developing an example that clarifies the operation of the weighted average method. See infra note 218.}

To see how this manipulation works, it is convenient to start by considering application of the weighted average approach in the absence of any redeterminations. Consider two types of assets with known lives of one year and six years, respectively. Suppose that it also is known in advance that the value of both asset types will decline in a straight-line fashion over their lives. If the Code provides the correct theoretical depreciation schedule for these assets, that is, straight-line depreciation over their lives, there will be no strategic loss-taking opportunities for the assets under individual item accounting. Each asset's adjusted basis will at all times be equal to its market value.

This result disappears if the taxpayer can aggregate the two different asset types into a group account using a weighted average schedule. Suppose, for example, that the taxpayer constructs a group account composed of 40% (by value) of the one year asset and 60% of the six-year asset. This account would be depreciated during the first year as if it were one asset on a two-year straight-line schedule, and absent any redeterminations, the two year straight-line schedule will apply in the second year also.\footnote{217. The weighted average rate of depreciation is the aggregate first year allowance for the assets in the group under individual item accounting divided by the total value of the assets placed into the account. See Treas. Reg. § 1.167(b)-1(b), example (2); Dooher et al., supra note 203, at A-34 - A-35.} But the taxpayer can retire the one-year assets at the end of the first year and claim a loss equal to the remaining adjusted basis of these assets. Assuming that the loss can be taken immediately, as is the case...
under the individual item method, these assets will be depreciated correctly (following economic depreciation) since they have a one-year life. However, the remaining assets with six-year lives will be depreciated for two years on a straight-line basis, thus receiving accelerated treatment. Thus, by mixing long-lived and short-lived assets in the same group account, the taxpayer can accelerate the depreciation of the long-lived assets compared to their individual item treatment.\textsuperscript{218}

One aspect driving this result is the absence of redeterminations. At the end of the first year, the account consists entirely of assets with five years of life remaining. The multiple asset account regulations state that "the average useful life and rate shall be redetermined whenever additions, retirements, or replacements substantially alter the relative proportion of types of assets in the accounts."\textsuperscript{219}

The most rigorous enforcement of this Regulation would be to recompute the average useful life and rate every year. In fact, even with annual redeterminations, the taxpayer can play by the rules and get away with murder. The taxpayer's leeway to do so is especially clear for open accounts. For instance, in the example of the one-year asset and the six-year asset, the taxpayer could annually add smaller and smaller amounts of one-year assets to the account and yet maintain the rate of depreciation on the six-year assets at 50% per year or greater.\textsuperscript{220} By maintaining

\textsuperscript{218} We have previously defined strategic loss-taking to mean the sale of an asset for a tax loss combined with repurchase of a similar asset. See supra text accompanying notes 12-15. In the text example, the taxpayer combines assets in a group account anticipating that some will be retired before they are fully depreciated. The one-year asset is retired for a loss but no asset is sold for a tax loss and replaced with another asset.

We will nonetheless refer to the strategy of accelerating depreciation for long-lived assets by mixing them in a group account with short-lived assets as "strategic loss-taking." The taxpayer is intentionally mixing the assets together in such a way that long-lived assets will receive favorable treatment, accelerated depreciation, while short-lived assets suffer no detriment due to the ability of the taxpayer to form a new "schedule" for those assets by loss-taking trades or retirements.

In fact, the absence of "pure" strategic loss-taking in the text example is largely an artifact of that particular example. If we had mixed two-year assets with six-year assets, the resulting schedule would be straight-line depreciation with a recovery period three and one-third years. The taxpayer would deduct 30\% of initial value in the first three years and 10\% of initial value in the fourth year. In this case, pure strategic loss-taking is called for. After the first year, the two-year assets have fallen 50\% in value, but adjusted basis has only been reduced by 30\%. The taxpayer can trade these costs for a loss and then replace them with equivalent assets with one-year of life remaining.

\textsuperscript{219} Treas. Reg. § 1.167(a)-7(d).

\textsuperscript{220} Consider an example. Suppose that in the first year, there are $1200 worth of six-year straight-line assets and $800 worth of one-year assets. The group account is depreciated on a two-year straight-line schedule so that only $1000 in "adjusted basis" is left after one year. In addition, the one-year assets are retired from the account at a $400 loss, thereby reducing the account "adjusted basis" to $600. Thus, only five-year assets are in the account and these have an aggregate adjusted basis of $600.

Suppose the Regulations are applied strictly so that the taxpayer must recompute the depreciation rate for the account. By adding one-year assets with total value of $360 to the account, the taxpayer would maintain two year straight-line treatment: The first year allowance would be one-fifth of $600 plus $360 which is $480 or one-half of $960, the total value of assets in the account.

The following table details the value in one-year assets that would need to be added each year to maintain a group treatment on a two-year straight-line basis:
the 50% rate, the taxpayer will have replaced six-year straight-line depreciation with 300% declining balance depreciation.\textsuperscript{221} This strategy does not affect the depreciation of the one-year assets since the taxpayer retires them at the end their one-year life, thereby allowing a first year deduction for the entire cost.

Finally, even if the account in question is \textit{closed} and the Regulation is strictly enforced, the taxpayer will still succeed in getting accelerated depreciation. In our example, the taxpayer still deducts one-half of the basis of the six-year assets in the first year. If the regulation is strictly enforced, the taxpayer will have to deduct the remaining half of the basis of these assets under a straight-line schedule over five years,\textsuperscript{222} But the overall result is more favorable to the taxpayer than 200% declining balance depreciation.\textsuperscript{223}

We now turn to the case of open, homogeneous accounts. That is, we focus on the consequences of allowing similar assets placed in service in different years to be in the same account. Special problems arise in these accounts when it is costly to distinguish assets. Consider railroad ties, an asset type that is suggested as a prime candidate for these accounts by the Regulations.\textsuperscript{224}

The railroad company will constantly be replacing ties that have rotted or are damaged with new ones. A group account consisting of all of its railroad ties in service will be a composite of ties placed in service initially in many different years. Labeling each tie in some indelible way and keeping track of it separately in the tax accounts undoubtedly would be very costly.

The treatment of retirements from such a group account poses

<table>
<thead>
<tr>
<th>Year</th>
<th>Remaining Adjusted Basis of Long-Lived Assets</th>
<th>New Value of One-Year Assets that Must be Added</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1200.00</td>
<td>$800.00</td>
</tr>
<tr>
<td>2</td>
<td>$600.00</td>
<td>$360.00</td>
</tr>
<tr>
<td>3</td>
<td>$300.00</td>
<td>$225.00</td>
</tr>
<tr>
<td>4</td>
<td>$150.00</td>
<td>$100.00</td>
</tr>
<tr>
<td>5</td>
<td>$75.00</td>
<td>$0.00</td>
</tr>
<tr>
<td>6</td>
<td>$37.50</td>
<td>$0.00</td>
</tr>
</tbody>
</table>

In year six, the taxpayer can simply deduct the entire $37.50 of adjusted basis since the asset now has only one year of useful life left. The net result is that the taxpayer switches the depreciation of the six-year asset from six-year straight-line depreciation to six-year 300% declining balance depreciation.\textsuperscript{221} The resulting degree of acceleration is more than the Code allows for accelerated depreciation. MACRS permits at most 200% declining balance depreciation compared to the straight-line treatment under ADS. \textit{See supra} note 28 and accompanying text.

\textsuperscript{222} All of the one-year assets have been retired from the account. All that remains is the six-year assets that have five years of life remaining.

\textsuperscript{223} The present value of depreciation taken 50% at the end of year one followed by 10% in each of the following five years is 90.88% of initial value. The present value of depreciation on a 200% declining balance schedule over six years (switching to straight-line when it is optimal to do so) is 90.13% of initial value.

\textsuperscript{224} \textit{See supra} note 199 and \textit{infra} note 302.
problems if the rule for dispositions depends on adjusted basis.\textsuperscript{225} If ties are not identified, the date a tie was placed in service and thus its adjusted basis at retirement are not known. Using the average adjusted basis for ties in the account would be inaccurate if older ties are more likely to be the ones that are retired. In that case, the adjusted basis assigned to retired assets will generally be too high since average adjusted basis will tend to exceed the adjusted basis of the assets actually retired.

2. Some Solutions to the Problems

Subsection 1 shows that the problems specific to group accounting arise when group accounts are allowed to be open or heterogeneous. One obvious potential solution is to require that such accounts be closed and homogeneous.\textsuperscript{226} This restriction is in effect for general asset accounts but does not apply to multiple asset accounts.\textsuperscript{227}

Unfortunately, the restriction vitiates much of the legitimate scope of group accounting. The purpose for allowing group accounts is to address situations in which assets are numerous, indistinguishable, or of low unit value. In these situations individual item accounting would be extremely expensive compared to the value of the assets. This problem is clearest for assets like railroad ties where not permitting open accounts would result in potentially very high administrative and compliance costs. But the same problem can exist if heterogeneous accounts are not permitted. High cost savings may result if the taxpayer is permitted to treat a myriad of small items put into service in the same year in a single account as a single asset. The taxpayer's financial accounts may already be structured in this way, perhaps lumping all the miscellaneous items for particular projects or businesses into one category.

Given the need to allow group accounts that are open or heterogeneous, it would be valuable to establish separate rules for closed, homogeneous accounts. The nice feature of these accounts is that nothing is lost in applying the same rules that apply under individual item accounting.\textsuperscript{228} The Code and the Regulations should permit taxpayers to operate closed, homogeneous accounts under those rules.

The remaining issue is how to treat open, heterogeneous accounts.\textsuperscript{229}

\textsuperscript{225} The individual item method and the account adjustment method both require knowledge of the adjusted basis of the asset being retired (or sold) from a group account. See supra text accompanying note 7; infra text following note 317.

\textsuperscript{226} This requirement in fact applied to "vintage accounts" under the Asset Depreciation Range system in effect from 1971 to 1980. See infra text accompanying note 299.

\textsuperscript{227} See supra text accompanying notes 202-203.

\textsuperscript{228} See supra text accompanying notes 211-212. Ideal individual item accounting rules may be different from the ones in operation today. In particular, they may adjust tax schedules or use a disposition rule other than the individual item method in order to combat strategic loss-taking. See supra text accompanying notes 134-196. But the essential point is that there is no need to establish different rules for closed, homogeneous accounts.

\textsuperscript{229} I choose not to deal with open, homogeneous accounts and closed, heterogeneous accounts separately. The policy adjustments for "openness" and "heterogeneity" are sufficiently similar that it is not worth discussing these two types of accounts separately. Instead the discussion focuses on the most general case: the open, heterogeneous account.
The availability of these accounts raises the danger that they will be used to enhance or create strategic loss-taking opportunities. It is important that the rules that apply to open, heterogeneous accounts aim at controlling strategic loss-taking. At the same time, these rules should not be so severe as to destroy the viability of using these accounts since they can result in large savings of administrative and compliance resources.

Ex ante schedule adjustments alone are unlikely to be a good policy for open, heterogeneous accounts. The big problem with controlling strategic loss-taking by ex ante schedule adjustments is that the required adjustments depend heavily on trading costs and asset volatility. This problem is much worse for open heterogeneous accounts since these accounts mix different kinds of assets and since the asset mix may change over time. Each open, heterogeneous account would require its own special schedule, and setting the appropriate schedule might be very difficult.

Two of the disposition rules discussed in Part III, the depreciation bond approach and the proceeds tax rule, are extremely attractive methods for controlling the strategic loss-taking problems that accompany open, heterogeneous group accounts. Two features of these rules make them particularly suitable. First, under both rules, computing the tax treatment upon disposition of assets from the account does not require knowing the adjusted basis of the assets sold or retired from the account. We have seen that for open accounts, whether homogeneous or heterogeneous, it is difficult to determine the adjusted basis for such assets. Such a determination is possible for closed, heterogeneous accounts. Because these accounts contain assets that are all put in service in the same year, adjusted basis can be computed from scratch knowing the type of asset being sold or retired. Nonetheless, this computation is exactly the kind of asset-by-asset delineation that group accounts are designed to avoid. The computation would be especially burdensome if a group of assets were sold or retired as a block.

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230. See supra text accompanying notes 226-228 (special role for open and heterogeneous accounts).
231. See supra text accompanying notes 135-154.
232. One could not simply start with special schedules for each asset type and then have a formula that mechanically combined those schedules into a single schedule when the assets are mixed in a group account. There may be no danger of strategic loss-taking for two asset types considered separately, but combining them may create a considerable danger. See supra text accompanying notes 217-223 (example). Thus, a special schedule for a group account would have to be tailored to the particular mix of assets in that account. In addition, if the account is open, that mix may change over time so that the schedule would have to be changed along with the mix or would have to be set so as to anticipate changes in mix.
233. See supra text accompanying notes 171, 183-185, and 195-196.
234. It is interesting to contrast the outcome of this sale or retirement of a block of assets with the same transaction for closed, homogeneous accounts. For these accounts, the adjusted basis of a single asset or a block of assets is simply the “adjusted basis” of the account multiplied by the proportion of original account value represented by the asset or block. See supra text accompanying note 212. Thus, it is fairly easy to compute adjusted basis when assets are sold or retired from such an account. This ease of computation
The second advantage of the depreciation bond approach and the proceeds tax rule is that they make strategic loss-taking totally ineffective. Because strategic loss-taking is a particularly serious problem for open, heterogeneous accounts, it makes sense to use a policy that strongly addresses that type of manipulation.

One drawback of using either the depreciation bond approach or the proceeds tax rule is that the ex ante depreciation schedule will have to be adjusted. Unless this schedule is considerably accelerated, the taxpayer will pay a substantial price for shifting to group accounts under these disposition rules. Failure to adjust would discourage use of open, heterogeneous group accounts, and we have seen that using these accounts promises substantial cost savings.

Establishing a separate set of schedules for assets in group accounts that are open or heterogeneous is inelegant and adds to the complexity of the depreciation rules. In addition, unless the government is prepared to define the situations where administrative and compliance costs warrant the use of group accounts, group accounting treatment is destined to be elective. If the special schedules set up for assets in group accounts that are open or heterogeneous overcompensate for the disposition rules applied to those accounts, taxpayers will elect group accounting simply to obtain a more favorable depreciation schedule.

Finally, the problem of designing special schedules differs for the two disposition rules. In the case of the depreciation bond approach, computation of the appropriate accelerated schedule is easy. In contrast, under the proceeds tax rule that computation is difficult. As a result, under the proceeds tax rule the danger of making a schedule adjustment makes it feasible to use disposition rules that hinge on the adjusted basis of the assets that are sold or retired from closed, homogeneous accounts.

235. The other disposition rules are not complete cures. They leave some scope for strategic loss-taking. See supra text accompanying notes 190-191.

236. Of course, adjustment would not be necessary if the depreciation bond approach or the proceeds tax rule were applied generally, that is, to all assets regardless of whether individual item accounting or some form of group accounting applies to the asset.

237. See supra note 60 and text accompanying notes 59-61, and 173-186 (large degree of acceleration required for ex ante depreciation schedule under depreciation bond approach and proceeds tax rule).

238. See supra text accompanying notes 226-228.

239. However, the schedule adjustment rules could operate mechanically. The rules could specify a group of schedules paralleling MACRS and ADS. These schedules would specify the treatment for each type of asset (e.g., by class life or MACRS category). The schedule for a group account would be a weighted average (weighted by initial value) of the schedules for the assets put into the account. If the account were “open,” the schedule would be readjusted when new assets were added.

In contrast, no such mechanical approach will work where the schedule adjustment itself is used to combat strategic loss-taking. See supra note 232. A mechanical method is possible for the depreciation bond approach and the proceeds tax rule since under these approaches the schedule adjustment merely allows for the change in disposition rule. The new disposition rule itself, and not the schedule adjustment, is what stops strategic loss-taking.

240. The Hulten and Wykoff schedule will work. See supra text accompanying note 173.

241. See supra text accompanying notes 185-186.
that overcompensates for the severity of the disposition rule is greater and, in the face of computational difficulty, so is the danger of undercompensating in an effort to avoid overcompensating.

Although the required schedule adjustment for the depreciation bond approach and the proceeds tax rule are a disadvantage, using one or the other of these two disposition rules seems like the best available approach for group accounts that are open or heterogeneous. Using other disposition rules or relying solely on schedule adjustment for such accounts is not practical.

C. Some Suggested Reforms of the Group Accounting Rules

Three policy points emerge from the previous section. First, the same disposition rules and other rules (such as the rules specifying the depreciation schedule) that apply under individual item accounting should apply to group accounting for closed, homogeneous accounts. Second, group accounts that are open or heterogeneous require special treatment. The best policy for these accounts appears to be use of either the depreciation bond approach or the proceeds tax rule. Third, limiting group accounts to those that are closed and homogeneous is undesirable. There are many instances where use of heterogeneous or open accounts can achieve considerable simplification and an ensuing drop in administrative and accounting costs.

Current law does not reflect these three points. The general asset accounts system which applies to the bulk of depreciable assets (all "recovery property") permits only closed, homogeneous accounts and then applies the proceeds tax rule to all dispositions except for the special class of qualifying dispositions. The most important dispositions in that class are retirements or sales associated with closing down a business or a business subunit. Group accounting for the remainder of depreciable assets is subject to the multiple asset account rules. These rules permit open or heterogeneous accounts but impose disposition rules that rely on the adjusted basis of individual assets: The account adjustment rule applies to "normal retirements," while individual item accounting applies to "abnormal retirements."

There is no apparent reason why the group accounting rules for "recovery property" should differ from the rules for other property. Instead, as demonstrated above, different rules should apply to group accounts that are closed and homogeneous from the rules that apply to other accounts. It would make sense to reform the group accounting rules by promulgating one set of rules that apply to all types of property but that differ depending on whether the relevant account is closed and homogeneous. In particular, a good approach would be to apply the same individual item accounting rules that govern a particular type of asset to closed accounts that contain only that asset type. In contrast, it is desirable to subject open or heterogeneous accounts to special disposition rules and perhaps also to an accelerated depreciation schedule.
It is not clear that the Treasury Department could unilaterally mandate this suggested system. There are certain legal roadblocks that stand in the way of Treasury directly implementing these rules for “general asset accounts” by regulation. In addition, adopting an accelerated depreciation schedule for open or heterogeneous accounts may be politically as well as instrumentally difficult for both general asset accounts and multiple asset accounts.

It appears that Treasury would not face any statutory difficulties in extending the general asset account regulations to apply to accounts that are open and/or heterogeneous as well as to closed, homogeneous accounts. In fact, after the passage of the authorizing statutory provisions in 1986 but prior to promulgation of Proposed Regulations in 1992, it was reasonable to expect that the regulations would permit open or heterogeneous accounts.\[242\]

A more difficult problem is that section 168(i)(4), the statutory provision permitting general asset accounts, requires the proceeds tax rule for dispositions from these accounts, “except as provided in regulations.”\[243\] Interpreting this requirement to establish a strong presumption in favor of the proceeds tax approach as the “normal” rule would make it difficult to mandate individual item accounting treatment by regulation for closed, homogeneous accounts. But it is not clear that reading in such a presumption is appropriate. Prior to the promulgation of regulations, some commentators believed that the proceeds tax rule would apply to “normal” or “ordinary” retirements,\[244\] while others believed that the rule properly should apply to “abnormal” or “extraordinary” retirements.\[245\] This lack of consensus about the constraints inherent in the proceeds tax rule mandate of section 168(i)(4) suggests considerable regulatory flexibility for Treasury. In particular, it seems that Treasury could extend general asset account treatment to open and heterogeneous accounts, applying the proceeds tax rule only to such accounts while specifying by

\[242\] See Appendix E, infra note 306 and accompanying text.

\[243\] I.R.C. § 168(i)(4).

\[244\] In particular, one letter to Treasury concerning the Regulations to be promulgated under section 168(i)(4) suggested that the proceeds tax rule be applied only to normal retirements. The letter writer proposed that the individual income method would apply to “substantial abnormal” retirements. This category would be defined to include retirements of more than 20% of the assets (by unadjusted basis) in a general asset account that occur “as a direct result of a cessation, termination, curtailment, or disposition of a business, manufacturing, or other income producing process.” See Public Comments on Proposed Regulations, 88 TAX NOTES TODAY 160-36 (August 4, 1988). The proposed category of “substantial abnormal” retirements would also include retirements due to factors other than business exigencies. The actual Regulations incorporated the cessation of business part of this proposal.

\[245\] The argument relies on the fact that for mass asset accounts, language in the Internal Revenue Code required proceeds tax rule treatment for dispositions. However, Congress changed the language when general asset accounts replaced mass asset accounts, and this change can be construed to mean that normal retirements are not intended to be subject to the proceeds tax rule. For a discussion with the relevant citations, see Megaard and Megaard, infra note 304, at A-115.
regulation that individual item accounting applies to closed, homogeneous accounts.

However, there is another problem with specifying proceeds tax treatment for disposition from multiple asset accounts that are open and/or heterogeneous: Congress did not explicitly authorize use of an accelerated schedule along with the proceeds tax rule in section 168(i)(4) or in the enabling provisions for multiple asset accounts. Without such a schedule, much of the potential usefulness of open or heterogeneous group accounts may be vitiated.\textsuperscript{246} But Treasury would have to rely on a broad reading of Congressional intent to set up accelerated schedules on its own in the Regulations. Congress has proven acutely sensitive to Treasury-determined depreciation schedules in the past, even when Treasury's authority to set the schedules was clear.\textsuperscript{247}

Whether or not Treasury may accelerate depreciation for accounts subject to the proceeds tax rule, an important issue is whether Treasury can take steps in its Regulations to mitigate the worst feature of that rule, the tendency to interfere with sales that have a legitimate business purpose.\textsuperscript{248} One way to accomplish this task would be to apply the proceeds tax rule only to "abnormal" or "extraordinary" retirements and then define "normal" or "ordinary" retirements to include legitimate business transactions in addition to assets that simply wear out. However, designing rules that distinguish between sales motivated by strategic loss-taking and those motivated by an independent business purpose may not be easy.\textsuperscript{249}

It is clear that some of the constraints placed by Congress on the rules for general asset accounts and the lack of Congressional authorization for Treasury to adjust depreciation schedules for group accounts may make it

\textsuperscript{246} See supra text accompanying notes 237-238.

\textsuperscript{247} For example, Treasury promulgated the entire Asset Depreciation Range system by Regulation in 1971. The system generally gave taxpayers the opportunity for accelerated depreciation compared to prior law. Although Treasury arguably had authority to promulgate the Regulations, Congress carefully reviewed them and made some changes before allowing them to go into effect. See Donald, Depreciation: ADR System for Post-1970 Property, 255-3rd Tax Mgmt. - (BNA), at A-2 through A-3 (1988).

It is also instructive that Congress removed the power granted to Treasury in the Tax Reform Act of 1986 to set useful lives before that power was ever exercised. See supra note 24. This revocation may have been prompted by pressure from industries afraid that Treasury would mandate slower depreciation for their assets. See supra note 198.

\textsuperscript{248} See supra text accompanying notes 185-186.

\textsuperscript{249} It is important to note that by combining assets appropriately in group accounts, a strategic loss-taking result can be achieved through simply retiring assets that wear out. See supra text accompanying notes 217-221 (example). These are certainly "normal" retirements. See supra text accompanying note 204. If the proceeds tax rule were not applied to them, some other disposition rule that is effective against strategic loss-taking would have to be applied. The only other candidate we have seen is the depreciation bond approach. But using that rule introduces the danger of churning transactions. See supra text accompanying notes 173-175. Since transactions motivated by legitimate business purposes would count as "normal" retirements, the rules would have to distinguish such transactions from transactions designed to churn depreciable assets. Designing such rules would not be easy. They would have to turn either on motivation tests or on mechanical rules that might well be overinclusive or underinclusive or both.
difficult for Treasury to provide sound rules for group accounts. It may be desirable for Treasury to seek a broader and more flexible mandate from Congress before revisiting these rules.

V. CONCLUSIONS

If the age-price profile for surviving units is known for a particular asset and the goal is to replicate economic depreciation, then the statutory depreciation schedule should be the age-price profile for surviving assets. Contrary to the leading academic approach devised by Hulten and Wykoff, this schedule should not be adjusted for retirement risk. The proper adjustment for that risk occurs automatically if the law allows a loss deduction when assets fail and are discarded.

In contrast, when the future price path for surviving units is known and the objective is to accelerate depreciation in a manner that is neutral across assets, then depreciation schedules must take retirement risk into account. Assets with the same age-price profile for surviving units may require very different degrees of acceleration. The reason for this result is that the assets may be subject to very different survival patterns. If assets have a higher survival rate, then the statutory depreciation schedule is more important because it is more likely to be applied to the assets. Such assets do not need as much acceleration in the statutory schedule to receive the same degree of benefit as assets with lower survival rates.

When the age-price profile for an asset is uncertain, approximating economic depreciation is more complex. Strategic loss-taking becomes an important consideration. Simple and clearly appropriate policies exist only for assets with very high trading costs or zero trading costs. These policies are basing the depreciation schedule on the expected price path in the high trading cost case and allowing no depreciation deductions in the zero trading cost case.

When trading costs are moderate, solutions that combine special disposition rules with ex ante schedule changes become attractive. None of these solutions is dominant. For example, one promising solution, the depreciation bond approach, eliminates strategic loss-taking but permits the churning of depreciable assets. The proceeds tax rule, another possible solution, eliminates both strategic loss-taking and churning but requires a difficult ex ante schedule adjustment and discourages dispositions of depreciable assets for legitimate business reasons.

Finally, some types of group accounts raise special problems. Closed, homogeneous accounts are not in this category. Whatever set of rules turns out to be optimal under individual item accounting will be optimal for these accounts. However, strategic loss-taking is potentially a very severe problem for group accounts that are open or heterogeneous. For these accounts, it may be best to apply rules, such as the depreciation bond approach or the proceeds tax rule, that are particularly effective at curbing strategic loss-taking even if these rules are not used under individual item accounting.
APPENDIX A

A Mathematical Description of the Productivity Method, the Treasury Department Approach, and the Text Examples

This Appendix, along with Appendices B and C, provides mathematical derivations for the results in the text examples and figures. The derivation of the basic example in Section II.A of the text that is used throughout Part II is almost identical to the productivity method. This method is one of the two ways that the Treasury Department uses to arrive at age-price profiles for depreciable assets.

Section 1 of this Appendix provides a verbal description of the productivity method and then uses that method to construct the example used in Section II.A of the text. Section 2 describes the mathematics of the Treasury Department approach. Section 3 discusses the example from Section II.C.4 of the text. Section 4 connects the results in the Appendix with the various figures, tables and numerical examples in Part II of the text.

1. The Productivity Method and the Text Example from Part II

The Treasury Department uses the productivity method to construct an age-price profile when market data does not suffice to estimate the profile directly. The method begins with two empirical inputs: data on asset lives and data on net asset revenues during life. The data on asset lives leads to a useful life distribution for the type of asset under examination. This distribution specifies what proportion of new assets of this type last one year, two years, three years, and so on. The net revenue data allow net revenue per year to be expressed as a function of asset age. This “age-revenue profile” specifies how much profit surviving assets of any given age will earn annually. Given the useful life distribution and the age-revenue profile, it is possible to compute an expected net present value for a surviving asset of any given age. For each future year, the probability that the asset will survive to that year is multiplied by the net revenue that will be earned if the asset survives to that year. This product is discounted back to the present using an interest rate that reflects the time value of money.

The sum of the discounted products from all future years is the net present value of the asset. Calculating a net present value at each age yields a complete age-price profile.

250. See supra note 71.
251. This interest rate should include a risk premium if the revenues are risky.
252. This value is sometimes called the “value-in-use” of the asset. See Treasury Rental Clothing Study, supra note 24, at 20.
253. The productivity method outlined here does not adjust the values derived for taxes. Thus, neither revenues nor the discount rate are reduced to an after-tax amount.

This approach is theoretically sound if the end product will be a system that replicates economic depreciation. If economic depreciation is the rule, then the after-tax present value of all assets will be independent of the tax rate. See Paul A. Samuelson, Tax Deductibility of Economic Depreciation to Insure Invariant Valuations, 72 J. Pol. Econ. 604.
The text example in Part II is derived by applying the productivity method to an assumed pattern of asset lives and an assumed pattern of net asset revenues for surviving assets. In particular, the example assumes an exponential distribution of asset lives so that the probability of survival up to time \( t \) is

\[
S(t) = e^{-\lambda t}.
\]  

(1)

With this distribution of lives, the mean life is \( 1/\lambda \), and assets fail at a constant rate of \( \lambda \) per year.\(^{254}\) In the example, \( \lambda = .10 \) so that the expected life of an asset is 10 years when it is first put into service. Revenues decline exponentially from an initial level at time 0 of \( R \) per year. Thus, the annual revenue rate at time \( t \) is

\[
R(t) = Re^{-\alpha t}.
\]  

(2)

In the example, the parameter \( \alpha \) is set at .20 so that the revenue rate declines at a rate of 20% per year.

In order to compute the value of the asset at any given time, a discount rate is necessary to reduce future revenues to present value. The example assumes a 4% annual rate, equivalent to an exponential rate of \( r = \ln(1.04) = 0.03922 \).

Given the assumed asset life distribution, the assumed age-revenue profile, and the assumed discount rate, it is possible to compute \( V(T) \), the value of the asset at time \( T \):

\[
V(T) = \int_{T}^{\infty} R(t)e^{-r(t-T)}e^{-\lambda(t-T)}dt = \frac{Re^{-\alpha T}}{\alpha + r + \lambda}.
\]  

(3)
Normalizing this value by dividing by value at time 0 yields

\[
\frac{V(T)}{V(0)} = e^{-\alpha T}.
\]  

(4)

Note that the normalized value at time \( t \) does not depend on the expected life of the asset. This fact is a consequence of the assumption that assets fail at a constant rate. That assumption means that the expected life and the distribution of future lives remains constant for surviving assets. If an asset is \( T \) years old, it has the same prospects for survival to year \( T + t \) as it initially did for survival to year \( t \). The only change is that revenues are lower in year \( T + t \) than in year \( t \). Thus, equation (4), the normalized value equation for surviving assets, depends only on the level of revenues.

It is easy to compute the present value of depreciation for the age-price profile generated by this equation. We first differentiate to obtain the instantaneous rate of change in value:

\[
\left[ \frac{V(T)}{V(0)} \right]' = \frac{V'(T)}{V(0)} = -\alpha e^{-\alpha T}.
\]  

(5)

We now integrate the absolute value of this rate of decline discounted to present value to obtain the present value of depreciation:\[255\]

\[
\int_0^\infty \left| \frac{V'(T)}{V(0)} \right| e^{-rT}dT = \frac{\alpha}{\alpha + r}.
\]  

(6)

This present value of depreciation is only valid for assets that survive indefinitely. Under current law, the remaining adjusted basis is deducted at retirement. Including these loss deductions as a component of depreciation, as is theoretically proper,\[256\] the expected present value of depreciation is

\[
PV = \int_0^\infty \left| \frac{V'(T)}{V(0)} \right| e^{-rT}S(T)dT + \int_0^\infty \left| \frac{V(T)}{V(0)} \right| e^{-rT}dT.
\]  

(7)

We call this expected present value of depreciation, “the retirement-adjusted present value of depreciation.” The first integral is identical to the integral in equation (6) except that the integrand is multiplied by the proportion of assets that will survive to each given time. This survival

\[255\] Note that we could have delayed normalizing until after completing this integral. Normalization involves dividing by the initial value, \( V(0) \). This initial value is a positive constant and is carried through as a multiplicative factor from equation (4) to the current equation containing the integral.

\[256\] See supra text accompanying notes 43-45.
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probability is precisely the probability that the asset will be moving down (and depreciating) along the age-price profile for surviving assets at that time. The second integral is the present value of retirement deductions. The term \( 1 - S(T) \) is the cumulative probability of failure up to time \( T \), and the derivative of the term is the density function for asset failure. This derivative is multiplied by the normalized value of the asset at that time and the product is discounted back to time 0 by the discount factor \( e^{-rT} \). We have

\[
P V = \int_{0}^{\infty} ae^{-aT} e^{-\lambda T} e^{-rT} dT + \int_{0}^{\infty} e^{-aT} e^{-\lambda T} e^{-rT} dT = \frac{\alpha + \lambda}{\alpha + r + \lambda} \tag{8}
\]

for the retirement-adjusted present value.

We now turn to the Hulten and Wykoff approach. They adjust the age-price profile for surviving assets at each age by multiplying by the probability of survival to that age. Thus, we replace the normalized value in equation (4) by the new normalized value

\[
NV(T) = \frac{S(T)V(T)}{S(0)V(0)} = \frac{S(T)V(T)}{V(0)} = e^{-(\alpha + \lambda)T}. \tag{9}
\]

The integral in equation (6) becomes

\[
\int_{0}^{\infty} [NV'(T)e^{-rT}dT = \int_{0}^{\infty} [(\alpha + \lambda)e^{-(\alpha + r + \lambda)T}dT = \frac{\alpha + \lambda}{\alpha + r + \lambda} \tag{10}
\]

This value is identical to the value obtained in equation (8) for the retirement-adjusted present value of depreciation. Thus, the present value of depreciation under the Hulten and Wykoff approach is equal to the retirement-adjusted present value of depreciation.

It is important to note that we will obtain a different result for the retirement-adjusted present value of depreciation if the statutory schedule is based on the Hulten and Wykoff age-price profile from equation (9) instead of on the age-price profile for surviving assets from equation (4). When the statutory schedule is the Hulten and Wykoff age-price profile, instead of equation (7) we have

\[
P V = \int_{0}^{\infty} [NV'(T)e^{-rT}S(T)dT + \int_{0}^{\infty} NV(T)[1 - S(T)]e^{-rT}dT \tag{11}
\]
which yields

\[ PV = \int_0^\infty (e^{(\alpha + \lambda)T} - (e^{(\alpha + \lambda)T}e^{-\lambda T} e^{-r T} dT + \int_0^\infty e^{-(\alpha + \lambda)T} \lambda e^{-\lambda T} e^{-r T} dT \]

\[ = \frac{\alpha + 2\lambda}{\alpha + r + 2\lambda}. \] (12)

The term \(2\lambda\) appears on the right hand side here instead of the term \(\lambda\) in equation (8). Thus, the retirement-adjusted present value of depreciation based on the Hulten and Wykoff age-price profile is equivalent to the retirement-adjusted present value of depreciation for the age-price profile for surviving assets if we double the rate at which assets fail when computing that profile.\(^{257}\) It is not surprising that using the Hulten and Wykoff age-price profile as the basis of the statutory depreciation schedule instead of the age-price profile for surviving assets results in dramatically shorter lives.\(^{258}\)

2. The Treasury Department Approaches

The Treasury Department approach requires two steps. First, the Department derives the retirement-adjusted present value of depreciation based on the age-price profile for surviving assets. Second, Treasury constructs a straight-line schedule that has the same present value adjusted for retirements.

The Department has two different ways of computing the retirement-adjusted present value of depreciation. In Section 1 of this Appendix we have already encountered the first way, what the text calls the “Hulten and Wykoff variant.” For our example, the final result of this variant is the retirement-adjusted present value of depreciation computed in equation (10). Section 1 of this Appendix shows that this result is equivalent to the present value of depreciation under the Hulten and Wykoff approach unadjusted for retirement deductions. It is this equivalence that justifies the name “Hulten and Wykoff variant.” The text argues that the retirement-adjusted present value of depreciation computed in this way is correct.\(^{259}\)

The second way that Treasury uses to compute the retirement-adjusted present value of depreciation is more complex and involves four steps:\(^{260}\)

\(^{257}\) This doubling is the mathematical basis for the less precise assertion in the text that using the Hulten and Wykoff age-price profile instead of the age-price profile for surviving assets in effect provides an improper “double correction” for retirements. See supra note 59.

\(^{258}\) Cf. supra text accompanying notes 59-60 (large reduction in class life caused by using Hulten and Wykoff age-price profile).

\(^{259}\) See supra text accompanying notes 86-91.

\(^{260}\) See supra text accompanying notes 74-78.
(1) compute an age-price profile for an asset with known life for each life;
(2) normalize the prices for each profile derived in step (1) by dividing by initial price;
(3) compute the present value of depreciation for each life;
(4) compute an overall present value of depreciation by weighting the present value for each life by the probability that the asset will survive that long.

In our example, to replicate step (1), we need to alter equation (3) to reflect a fixed and known life, $\gamma$:

$$V(T, \gamma) = \int_{T}^{\gamma} R(t) e^{-r(t-T)} dt$$

$$= R e^{\gamma T} \int_{T}^{\gamma} e^{-(\alpha+r)t} dt$$

$$= \frac{R}{\alpha+r} \left[ e^{-\alpha T} - e^{\gamma T} e^{-(\alpha+r)\gamma} \right]. \quad (13)$$

Normalizing by dividing by the initial price, we obtain

$$\frac{V(T, \gamma)}{V(0, \gamma)} = \frac{e^{-\alpha T} - e^{\gamma T} e^{-(\alpha+r)\gamma}}{1 - e^{-(\alpha+r)\gamma}}. \quad (14)$$

Differentiating this expression with respect to $T$, taking the absolute value of the result and integrating that absolute value multiplied by the discount rate $e^{\gamma T}$, yields an expression for the present value of depreciation as a function of the fixed and known life $\gamma$:

$$PV(\gamma) = \gamma \left| \frac{V(T, \gamma)}{V(0, \gamma)} \right| e^{-\gamma T} dT$$

$$= \gamma \int_{0}^{\gamma} \frac{\alpha e^{-(\alpha+r)T} + r e^{-(\alpha+r)\gamma}}{1 - e^{-(\alpha+r)\gamma}} dT$$

$$= \frac{\alpha}{\alpha+r} + \frac{r \gamma e^{-(\alpha+r)\gamma}}{1 - e^{-(\alpha+r)\gamma}}$$

$$= \frac{\alpha}{\alpha+r} + \frac{r \gamma}{e^{(\alpha+r)\gamma} - 1}. \quad (15)$$

where $V_T(T, \gamma)$ denotes $\delta V(T, \gamma)/\delta T$. 

This result is identical to the outcome in equation (6), the present value of depreciation for surviving assets, except for the second term which "corrects" for the fact that the asset has finite life \( \gamma \) instead of an indefinite life. Note that

\[
\lim_{\gamma \to 0} PV(\gamma) = 1
\]

and

\[
PV'(\gamma) < 0 \quad \text{for} \quad \gamma > 0.
\]  \hspace{1cm} (16)

Normalizing initial value to one means that \( PV \), the present value of depreciation, expresses that present value as a proportion of initial value. Equation (16) indicates that the present value of depreciation deductions as a proportion of initial value is lower for assets with longer lives. This outcome follows from the fact that longer asset lives result in later, more heavily discounted depreciation deductions.\(^{261}\)

\(^{261}\) The limit result in equation (16) is obvious, but the derivative result requires some work. Note first that

\[
PV'(\gamma) = \frac{1}{r} \frac{e^{(\alpha+r)\gamma} - 1 - (\alpha+r)\gamma e^{(\alpha+r)\gamma}}{(e^{(\alpha+r)\gamma} - 1)^2}.
\]

For \( \gamma > 0 \), the denominator is always positive. The numerator is 0 when \( \gamma = 0 \) and its sign as \( \gamma \) increases from 0 depends on its derivative. In fact, the derivative of the numerator is

\[- (\alpha+r)^2 \gamma e^{(\alpha+r)\gamma}\]

which is less than zero for all \( \gamma > 0 \). As a result, the derivative of \( PV(\gamma) \) is less than zero for all \( \gamma > 0 \).

\(^{262}\) The outcome may not occur in the absence of normalization. Assets with longer lives have higher initial value, and thus higher total lifetime (undiscounted) depreciation deductions. This higher total may offset the heavier discounting that follows from the fact that longer asset lives mean delayed deductions. If we had not normalized, we would have

\[
NNPV(\gamma) = \int_0^T [V(T, \gamma)e^{-\gamma T} - \frac{R}{\alpha+r} \int_0^T [\alpha e^{-(\alpha+r)\gamma} + re^{-(\alpha+r)\gamma}]dT
\]

\[
= \frac{\alpha}{\alpha+r} \left[ 1 - e^{-(\alpha+r)\gamma} - (\alpha+r)\gamma e^{-(\alpha+r)\gamma} \right]
\]

for the present value of depreciation deductions. Taking the derivative with respect to \( \gamma \) yields

\[
NNPV'(\gamma) = \frac{R}{\alpha+r} \left[ \alpha e^{-(\alpha+r)\gamma} + re^{-(\alpha+r)\gamma} - (\alpha+r)\gamma e^{-(\alpha+r)\gamma} \right]
\]

\[
= Re^{-(\alpha+r)\gamma}(1 - r\gamma) > 0 \quad \text{for} \quad \gamma < \frac{1}{r}.
\]

Thus, for lives less than \( 1/r \) years, the net present value of depreciation deductions increases with asset life.
It follows from equation (13) that

\[
\frac{\delta V(0, \gamma)}{\delta \gamma} = Re^{-(\alpha + r)\gamma} > 0. 
\] (17)

Thus, longer-lived assets will produce more revenues and have a larger initial value. Treasury’s second variant suppresses this difference by treating all assets as if they have the same initial value. As a result, that variant gives more weight to short-lived assets relative to long-lived assets than is warranted.

To compute the present value of depreciation under the second variant, we integrate the expression for PV(\gamma) in equation (15) multiplied by \([1 - S(T)]\)', the probability density function for asset life. The resulting present value is

\[
PV = \int_0^{\infty} PV(\gamma)[1 - S(\gamma)]'d\gamma = \frac{\alpha}{\alpha + r} + \int_0^{\infty} \lambda r \gamma e^{-(\alpha + r + \lambda)\gamma} d\gamma. 
\] (18)

It is difficult to evaluate the integral on the right hand side analytically. The present values in the text under the second variant were derived by computing the integral numerically.

Despite the fact that the integral cannot be easily evaluated, it is not hard to show that

\[
\frac{\alpha}{\alpha + r} + \int_0^{\infty} \lambda r \gamma e^{-(\alpha + r + \lambda)\gamma} d\gamma > \frac{\alpha + \lambda}{\alpha + r + \lambda}. 
\] (19)

That is, the present value of depreciation under the second variant is greater than the present value under the first, “Hulten and Wykoff” variant that we derived in equation (8). To see this point, consider that

\[
1 - e^{-(\alpha + r)\gamma} < (\alpha + r)\gamma \quad \text{for} \quad \gamma > 0. 
\] (20)

---
263. Both sides of equation (20) are zero when \( \gamma = 0 \), but the derivative of the right hand side is greater than the derivative of the right hand side for all values of \( \gamma \) greater than 0. The result in the equation follows.
As a result,

\[ \frac{\alpha}{\alpha + r} + \frac{r}{\alpha + r} \int_{0}^{\infty} \frac{e^{-(\alpha + r + \lambda)\gamma}}{1 - e^{-(\alpha + r)\gamma}} d\gamma \]

\[ > \frac{\alpha}{\alpha + r} + \frac{\lambda r}{\alpha + r} \int_{0}^{\infty} e^{-(\alpha + r + \lambda)\gamma} d\gamma \]

\[ = \frac{\alpha}{\alpha + r} + \frac{r\lambda}{(\alpha + r)(\alpha + r + \lambda)} \]

\[ = \frac{\alpha + \lambda}{\alpha + r + \lambda}. \tag{21} \]

In the numerical example in the text, the second variant resulted in a higher present value of depreciation (and thus a shorter class life) than the first variant.\(^{264}\) Here we have demonstrated that this result is true in general and not just for the particular parameters used in the example.

After computing the retirement adjusted present value of depreciation using either the first or second variant, the Treasury Department computes an equivalent class life. This equivalent class life is the recovery period for a straight-line schedule that results in the same present value of depreciation (after adjusting for retirements) as the retirement adjusted present value of depreciation for the asset.

We now develop a formula for computing equivalent class lives. Straight-line depreciation per dollar of initial asset value over a recovery period of \(T\) years will be at a rate of \(1/T\) per year. Given that \(e^{-\lambda t}\) is the proportion of assets surviving to any time \(t\), the present value of depreciation attributable to an instantaneous interval \(dt\) following time \(t\) will be

\[ \frac{1}{T} e^{-rt} e^{-\lambda t} dt \tag{22} \]

where we have weighted the depreciation \((1/T)dt\) during the time interval \(dt\) by \(e^{-\lambda t}\), the probability of survival to time \(t\), and have discounted the result back to time 0 by multiplying by \(e^{-rt}\). Retirements during the time interval \(dt\) following time \(t\) will contribute

\[ \left[ \frac{T - t}{T} \right] \left[ e^{-rt} \right] \left[ \lambda e^{-\lambda t} dt \right] \tag{23} \]

where the first term in square brackets is the adjusted basis at the time of retirement, the second term discounts value back to time 0, and the third term is the probability of death during the time interval \(dt\) following time \(t\).

\(^{264}\) See supra text accompanying notes 92-94.
Integrating the sum of the terms in equations (22) and (23), we obtain the following expression for the present value of depreciation per dollar of initial value under a straight-line schedule:

\[
\int_{0}^{T} \frac{1 + \lambda(T - t)e^{-(\lambda + r)t}}{T} dt = \frac{1 + \lambda T}{T(\lambda + r)}(1 - e^{-(\lambda + r)T}) - \frac{\lambda}{T}\int_{0}^{T} te^{-(\lambda + r)t} dt.
\]

(24)

And integrating by parts yields

\[
-\frac{\lambda}{T}\int_{0}^{T} te^{-(\lambda + r)t} dt
\]

\[
= -\frac{\lambda}{T}\left[\frac{-T e^{-(\lambda + r)t}}{\lambda + r}\right]_{0}^{T} + \frac{\lambda}{T}\int_{0}^{T} e^{-(\lambda + r)t} dt
\]

\[
= \frac{\lambda}{\lambda + r} e^{-(\lambda + r)T} - \frac{\lambda}{T(\lambda + r)^2}[1 - e^{-(\lambda + r)T}].
\]

(25)

Adding the result from equation (25) to the first term on the right hand side of equation (24), we obtain the retirement-adjusted present value of straight-line depreciation over recovery period T:

\[
PV(T, \lambda) = \frac{1}{\lambda + r} + \frac{r}{T(\lambda + r)^2}[1 - e^{-(\lambda + r)T}].
\]

(26)

To compute the appropriate class life, we equate \( PV(T, \lambda) \) to (8) "\( PV, \)" the retirement-adjusted present value of depreciation in equation and solve for \( T. \)\textsuperscript{265}

Note that if we had failed to adjust for retirements in computing the equivalent straight-line schedule, we would have obtained the same result as in equation but with \( \lambda = 0: \)

\[
PV(T) = \frac{1}{rT}[1 - e^{-rT}].
\]

(27)

This result is smaller than the result in equation whenever \( \lambda > 0. \textsuperscript{266} \) As a

\textsuperscript{265} No easy analytic solution for \( T \) was apparent to the author. The class lives in the text were derived by solving for \( T \) numerically using the MATHCAD engineering scratchpad program.

\textsuperscript{266} This assertion can be verified by taking the derivative of \( PV(T, \lambda) \) in equation (26) with respect to \( \lambda \) and showing that this derivative is positive. But it is easier to simply observe that retirement deductions accelerate depreciation compared to depreciation.
result, a smaller value of $T$ would have to be chosen to reach any particular level of present value.

3. The Known-Life, Fixed Revenue Example in Section II.C.4

Section II.C.4 uses an example that differs from the one used in earlier parts of the text. In particular, this example assumes that asset life is fixed at $T$ years and that revenues flow at a constant rate of $R$ per year. Under these assumptions, the value of the asset at time $t$ will be

$$V(t) = \int_t^T R e^{-r(s-t)} ds = \frac{R}{r} \left[ 1 - e^{-r(T-t)} \right].$$  \hspace{1cm} (28)

Normalizing by the initial value and taking the derivative yields the instantaneous rate of economic depreciation at each time $t$:

$$\frac{V'(t)}{V(0)} = \frac{-re^{-r(T-t)}}{1 - e^{-rT}}.$$  \hspace{1cm} (29)

Taking the absolute value of this depreciation rate, multiplying by the discount factor $e^{rt}$, and integrating yields the present value of depreciation:\footnote{Because we have assumed that the asset will last exactly $T$ years, there is no need to take retirement loss deductions into account. The only retirement is the one that occurs with certainty at the end of $T$ years. At that point the asset has been fully depreciated on the tax accounts. Adjusted basis and value (amount realized) are both zero so that there is no loss.}

$$PV = \int_0^T \frac{re^{-rT}}{1 - e^{-rT}} dt = Tr \frac{e^{-rT}}{1 - e^{-rT}}.$$  \hspace{1cm} (30)

To compute an equivalent class life for this asset, one equates the present value of depreciation from equation (30) with $PV(T)$ from equation (27) and then solves for $T$. This class life translates into an equivalent straight-line schedule, namely the straight-line schedule for a recovery period equal to that class life.

Section II.C.4 also considers the present value of depreciation along the equivalent straight-line curve for the first part of the asset's life followed by depreciation along the age-price profile for the rest of the asset's life. The cross-over time is the time at which the two curves cross.\footnote{See Figure 9 supra text accompanying note 104.}

If this crossover time is $C$ and the straight-line curve is based on a class under the schedule for surviving assets. $PV(T)$ in equation (26) represents exactly the present value of depreciation under this schedule while equation (27) represents present value adjusted for the acceleration that results from retirement deductions.
life of $L$, then the present value of depreciation along this “mixed” curve is

$$PV = \int_0^T \frac{1}{T} e^{-rt} dt + \int_0^T \frac{r e^{-rt}}{1 - e^{-rt}} dt$$

$$= \frac{1}{rT} (1 - e^{-rC}) + (T - C)r \frac{e^{-rT}}{1 - e^{-rT}}.$$  \hfill (31)

4. Linking the Results in this Appendix to the Text

We now connect the results in this Appendix to the numerical results and figures in the text. The following table lists the text figures and numerical results in the left column and the corresponding equations in this Appendix in the right column:

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<th>Equation</th>
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<td>Figure 6</td>
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<td>equation (9) (lower curve)</td>
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<td>equations (8) (12) and (26)</td>
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<tr>
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<td></td>
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<td>equations (26) &amp; (27) (straight line)</td>
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<tr>
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<td></td>
<td>equations (6) and (27) (for straight-line schedule)</td>
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<td>equation (8) (first variant)</td>
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<td>equation (18) (second variant)</td>
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<td></td>
<td>equation (1) (with $\lambda = 9/10, 2/3$)</td>
</tr>
<tr>
<td>Table II in section II.D.</td>
<td>equation (8) (with $a = .10, .20; \lambda = 9/10, 2/3$)</td>
</tr>
</tbody>
</table>
APPENDIX B

Neutrality: Cost of Capital Approaches

This Appendix describes the mathematics of measuring the "neutrality" of accelerated depreciation, that is, the degree to which various accelerated depreciation schemes give the same investment incentive to diverse assets. As discussed in the text, there is more than one way to compare investment incentives for two different assets, and the two principal methods are the present value approach and the rate of return approach.\(^{269}\)

The present value approach is used in Table II of the text.\(^{270}\) In addition, the text uses an example of two assets with different survival probabilities.\(^{271}\) One asset survives each year with a 9/10 probability while the other survives only at a 2/3 rate. These two assets have the same age-price profile for surviving units and thus would have the same depreciation schedule if economic depreciation were the goal. The text points out that if the depreciation rate for the asset with the lower survival rate is doubled compared to economic depreciation, the neutral depreciation rate for the other asset will only be 1.29 as fast as economic depreciation under the present value approach.\(^{272}\)

This Appendix describes the mathematics behind these numerical results. This mathematics is derived largely from a paper by Professor Alan J. Auerbach.\(^{273}\) We begin with the following definitions:

\[
\begin{align*}
t & \quad \text{time in years} \\
r & \quad \text{instantaneous after-tax interest rate} \\
u & \quad \text{the marginal tax rate} \\
z & \quad \text{the present value of depreciation deductions for the asset} \\
c(t) & \quad \text{the cost of renting the asset on an annual basis at time } t \\
q(t) & \quad \text{the per unit value of the asset if placed into service new at time } t \\
D(t) & \quad \text{the depreciation deduction per dollar of asset on an annual basis at time } t \\
\alpha & \quad \text{the instantaneous rate of decline in revenues from the asset} \\
\lambda & \quad \text{the instantaneous rate of failure for the asset}
\end{align*}
\]

Professor Auerbach shows that with these variables, the present value of gross returns per dollar of asset purchased is

\[
P V(\lambda, \alpha) = \int_0^\infty e^{-rt} \frac{c(t) e^{-(\alpha + \lambda)t}}{q} \, dt
\]

(32)

\(^{269}\) See supra note 113.
\(^{270}\) See supra text accompanying notes 112-113.
\(^{271}\) See supra text accompanying notes 111-118.
\(^{272}\) See supra note 114 and accompanying text.
\(^{273}\) See Auerbach, supra note 113.
while the internal rate of return for the asset is

$$\rho(\lambda, \alpha) = \frac{c}{q} - (\lambda + \alpha). \quad (33)$$

To solve these equations, we must derive an expression for q(t) in terms of c(t). We note that rents are taxable, so that after-tax rents from the asset will be (1 - u) times the flow of rents. In addition, depreciation is deductible so that the after-tax benefit to the owner of the asset will be u times the stream of deductions. The present value of the after-tax rents from one unit of the asset combined with the present value of the tax depreciation benefits from that one unit will be q(t), the value of the unit. We write

$$q(t) = \int_{t}^{\infty} e^{-r(s-t)}[(1-u)c(s)e^{-\lambda(s-t)}e^{-\alpha(s-t)} + uq(t)D(s)]ds$$

$$= \int_{t}^{\infty} e^{-r(s-t)}[(1-u)c(s)e^{-\lambda(s-t)}e^{-\alpha(s-t)}]ds + uq(t)z \quad (34)$$

where

$$z = \int_{0}^{\infty} e^{-rs}D(s) \, ds. \quad (35)$$

We now rewrite equation (34) as

$$(1-uz)q(t) = \int_{t}^{\infty} e^{-(r+\lambda+a)(s-t)}(1-u)c(s) \, ds. \quad (36)$$

Since all of our parameters such as t, a and \( \lambda \) are constant over time, q(t) must be constant over time. We therefore take the derivative of q(t) in equation (36) and set it equal to zero to obtain

$$-(1-u)c(s) + (r+\lambda+a)q(1-uz) = 0 \quad (37)$$

so that we can solve for c in terms of q:

$$c = \frac{(r+\lambda+a)q(1-uz)}{(1-u)}. \quad (38)$$

Substituting this value of c into equations (32) and (33), we obtain

$$PV(\lambda, \alpha) = \frac{(1-uz)}{(1-u)} \quad (39)$$
and

$$\rho(\lambda, \alpha) = \frac{(r+\lambda+\alpha)(1-u\lambda)}{(1-u)} - (\lambda+\alpha).$$  \hspace{1cm} (40)$$

for the present value and internal rate of return approaches respectively.

Using equation (39) we note that:

$$\frac{\delta PV}{\delta z} = \frac{-u}{1-u}$$  \hspace{1cm} (41)

so that increasing the present value of depreciation deductions by any particular amount lowers the gross present value of returns required by the proportion u/(1-u). In addition, under the internal rate of return approach we have

$$\frac{\delta PV}{\delta z} = \frac{-u}{1-u}(\alpha+r+\lambda)$$  \hspace{1cm} (42)

so that accelerating depreciation has a similar impact in the internal rate of return case. The only difference is the proportionality factor.

Equations (39) and (40) are the basis for computing the 1.29 value in the text under the present value approach. From equation (4) in Appendix A, the age-price profile for surviving assets is given by the normalized value equation:

$$\frac{V(T)}{V(0)} = e^{-\alpha r}.$$  \hspace{1cm} (43)

A profile with $\alpha = .20$ represents depreciation at twice as fast a rate as a profile with $\alpha = .10$. If, as in the text example, we assume that the depreciation rate is doubled (starting with a profile based on $\alpha = .10$) for the less durable ($\lambda = 2/3$) asset, then we must solve the following equation for $x$ in order to determine the degree of acceleration appropriate for the more durable ($\lambda = .90$) asset:

$$PV(\lambda = .90, \alpha = x) = PV(\lambda = .67, \alpha = .20).$$  \hspace{1cm} (44)

From equation (39) it is clear that equality in equation (44) will occur when $x$ is set such that

$$z(\lambda = .90, \alpha = x) = z(\lambda = .67, \alpha = .20).$$  \hspace{1cm} (45)
But equation (8) in Appendix A gives the value of z, the retirement adjusted present value of depreciation deductions, so that we need to find the value of \( \alpha_1 \) such that

\[
\frac{\alpha_1 + \lambda_1}{\alpha_1 + r + \lambda_1} = \frac{\alpha_2 + \lambda_2}{\alpha_2 + r + \lambda_2}
\]

where

\[
\begin{align*}
\lambda_1 &= .10 \\
\lambda_2 &= .6667 \\
\alpha_2 &= .20.
\end{align*}
\]

This value is .129 so that we must accelerate the curve for the more durable asset 1.29 times compared to our base level of \( \alpha = .10 \) in order to have neutrality when the curve for the less durable asset is accelerated 2.00 times compared to the same base level. Using a similar analysis under the internal rate of return approach would yield 1.56 instead of 1.29.

APPENDIX C

Asset Life Distributions and the Nature of Retirement Risk

The examples in Part II of the text assume an “exponential” asset life distribution while the racehorse examples in Part III assume a “Weibull” distribution of a certain type.\(^{274}\) This Appendix explains these distributions and describes the computations used to derive the figures and numerical results in Part III. The discussion has a value that goes beyond merely explaining the derivation of the text examples. In particular, the discussion serves as a brief introduction to modeling asset life distributions for studying the depreciation pattern of particular assets.\(^{275}\)

As mentioned in one of the footnotes in the text, an exponential distribution is one instance of the more general class of Weibull distributions.\(^{276}\) There is a theoretical reason for considering the class of Weibull distributions in connection with studying asset lives. If continuing functioning of an asset depends on many independent components, then asset failure will occur when the first component fails. For example, a racehorse’s useful life will terminate if the horse breaks its leg, has a heart attack, or succumbs to a virus. Note that each “component” of the horse will have a life with a duration that is greater than or equal to zero years. We can think of the life of each component as a random variable. Since

\(^{274}\) See supra note 124 and text preceding note 38.

\(^{275}\) There is a substantial literature on modeling asset life distributions. These distributions play a critical role in the engineering and design of mechanical and electronic systems. For a good and very accessible introduction to the field, see P. TOBIAS & D. TRINDALE, supra note 38.

\(^{276}\) See id.
the failure of any component means the end of the asset's useful life, the minimum of these random variables will be the useful life of the asset.

We can now describe in statistical terminology the class of distributions that would be useful in modeling asset lives. We seek a "smallest extreme value distribution," the distribution of a minimum of a large number of similar random variables where the random variables are restricted to be positive. Mathematicians have shown that this class of distributions is precisely the class of all Weibull distributions.\(^{277}\)

Each individual Weibull distribution is characterized by two parameters. First, there is the "characteristic life" of the distribution. The characteristic life is the 63.2 percentile so that by the time the characteristic life is reached, 63.2 percent of the assets have failed. Second, there is a shape parameter. This parameter determines whether assets tend to fail around one particular time or whether asset failures are spread out in time.

If we denote the characteristic life as "c," the shape parameter as "m," and time as "t," then the probability density function for the Weibull distribution is:

\[
f(t) = \left[\frac{m}{c}\right] t^{m-1} e^{-(t/c)^m}.
\]

There is another class of distributions that is used to model asset lives, lognormal distributions. These distributions are theoretically better than the Weibull distributions when the asset consists of a homogeneous material that wears out from continued friction or similar stress. A good example would be the pylons that hold up a pier in the ocean. The amount of wear on a pylon each year depends on how rough the ocean was during that year. If the ocean is particular rough during a period of years, pylons installed during those years will have shorter lives. For a discussion of lognormal distributions and their applicability, see id. at 92-98.

Weibull distributions and lognormal distributions clearly differ in their theoretical suitability for modeling the failure rate for particular assets. Nonetheless, for studying the appropriate tax depreciation of most assets, it would not matter very much which class of distributions is used. This conclusion follows from the fact that both classes are very rich, that is, both classes admit many possible patterns for the frequency of asset lives. In a real life problem, we would limit ourselves to one of the two classes and pick the distribution in that class that most closely approximates the data on actual lives. Each class is so rich that the curves selected for the two classes would be very similar in the range over which the data was observed.

The major difference between the two classes occurs where the goal is to extrapolate from the range of the data to values that are smaller or larger than what has been observed. An example would be a component in the space shuttle booster. This component need function only for a very short period of time, perhaps only a minute or so. But data on failure rates may be available only for extended periods such as months or years. In this case it matters a great deal whether one assumes the distribution is from the Weibull class or from the lognormal class. Curves estimated from the two classes would be similar over the range of the data but would diverge sharply outside of that range. When the time period of concern is shorter than those observed in the data, lognormal extrapolations tend to be more optimistic (less failure) than Weibull extrapolations. See id. at 97.

In studying depreciable assets, the need to extrapolate outside of the range of the data will be quite limited. Where data exists, it will tend to cover the time spans of interest.
To understand the shape parameter, consider the following figure which graphs the probability density function against time for three different Weibull distributions.

All three distributions have a characteristic life of 10 years. The sharply peaked curve has a shape parameter of 10, the gently peaked curve has a shape parameter of 2, and the curve that is convex rather than peaked has a shape parameter of 1.

A Weibull distribution with a shape parameter of 1 is an exponential distribution. This distribution is the one that we have used in the examples in Part II of the text. The distribution describes a situation where the rate of death is constant. Since the number of units is higher in the beginning, the number of failures per unit time will start out high and decline gradually if the failure rate is constant. In other words, shorter lives will be more frequent than longer lives. The result is a probability density function that starts out high and gradually declines as shown in the figure.

In contrast, consider the sharply peaked curve. This curve concentrates around the ten year characteristic life and drops off to zero quite sharply around 13 years. This type of curve would characterize an asset that is fairly predictable in its durability. Most units will last around ten years, and almost none will last longer than 13 or less than 5 years. The gently peaked curve represents a case in between the two other curves.

As an illustration, consider the case of thoroughbred racehorses. Using a simple ordinary least squares approach based on Treasury Department
data, the three major categories of thoroughbreds have the following characteristics:

<table>
<thead>
<tr>
<th>Type of Horse</th>
<th>Characteristic Life</th>
<th>Shape Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>geldings</td>
<td>6.555</td>
<td>2.125</td>
</tr>
<tr>
<td>fillies/mares</td>
<td>11.081</td>
<td>1.51</td>
</tr>
<tr>
<td>colts/stallions</td>
<td>7.784</td>
<td>1.292</td>
</tr>
</tbody>
</table>

The corresponding probability density functions are plotted in Figure 15:

![Figure 15: Thoroughbred Starters—Useful Lives](image)

The curve that increases the most rapidly and peaks earliest is the curve for colts/stallions. These horses have a high early death rate. The most gradual curve is for fillies/mares and the sharply peaked curve is for geldings. It is striking that apparently similar assets (different sexes of racehorses) have very different asset life distributions.

The example in Part III of the text is based on the asset life distribution for thoroughbred geldings corresponding to the sharply peaked curve above. The age-price profiles in Figures 11, 12 and 13 in the text are derived under the assumption that revenues are constant at some level R so that the only question is how long the horse will live. The cumulative

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278. The data was derived from bar graphs in the Treasury Horse Study, supra note 24. For estimating Weibull distribution parameters, maximum likelihood methods generally are considered to be superior to ordinary least squares. See P. TOBIAS & D. TRINDALE, supra note 38, at 74-81. However, ordinary least squares methods generally are quite accurate for lives within the range of the data. See id., at 80. In any event, attaining the most precise possible estimates is not critical in this Article.
density function for the Weibull distribution with characteristic life \( c \) and shape parameter \( m \) is
\[
F(t) = 1 - e^{-(tc)^m}
\]
so that the probability of survival up to time \( t \) is
\[
S(t) = e^{-(tc)^m}.
\]

Now the value of the asset at time \( T \) will be:
\[
V(T) = \int_{T}^{\infty} Re^{-r(T-T)}e^{-[\lambda T]^m-[\lambda T]^m} dt
\]
where \( \lambda = 1/c \) and \( r \) is the instantaneous discount rate.\(^{279}\) To derive Figures 11, 12 and 13, the integral in equation (50) was evaluated numerically for many different values of \( T \) and the results were graphed in the figures.\(^ {280}\)

APPENDIX D
Systematic Versus Nonsystematic Risk

Risk divides into two types: systematic risk and nonsystematic risk.\(^ {281}\) Systematic risk is aggregate risk in the economy. Not everyone can diversify away from systematic risk, so this type of risk is "priced," that is, investors will demand a premium to take on systematic risk. Nonsystematic risk is not an aggregate risk. If capital markets are functioning well, all individuals can diversify away from this kind of risk. Nonsystematic risk is not priced because no one is ultimately required to bear it.

Professors Bulow and Summers have argued that depreciable assets subject to systematic risk should be given special depreciation subsidies.\(^ {282}\) This argument is based on distinguishing between two types of

\(^{279}\) In equation (50), the integrand is the revenue rate times the probability that the asset is still operating at a given time discounted to time \( T \) using the instantaneous interest rate \( r \).

\(^{280}\) Numerical simulation was necessary because equation (50) does not have an obvious analytic solution. In a few places, Part III states a present value of depreciation for various values of \( R \). These present values also were computed numerically. The pattern of depreciation was approximated as the change in \( V(T) \) over short intervals. These changes in value were discounted back to time \( 0 \) and added up to provide an estimate of the present value of depreciation. The short intervals were made shorter and shorter until the estimates converged.

\(^{281}\) See BREALEY & MYERS, supra note 70, at 131-34. Systematic risk is sometimes called "market risk" and nonsystematic risk is sometimes called "unique risk." See, e.g., id. at 132.

risk: capital risk and income risk. Income risk is what we have called "revenue risk" in the text. Capital risk is the risk inherent in the fluctuation of asset prices. The government shares in income risk through the tax system. When income is high, taxes are high so that the government shares in the upside. Conversely, when income is low, taxes are low so that the government shares in the downside.

The core of the Bulow and Summers' argument is that this risk-sharing does not occur with respect to capital risk. Since assets are not traded each year and since tax depreciation schedules are fixed in advance and not adjusted annually for actual asset value outcomes, the taxpayer faces the entire brunt of capital risk alone. This phenomenon distorts investment by putting assets subject to systematic risk at a disadvantage.

The basic problem with the Bulow and Summers approach is that it does not take the whole lifespan of the asset into account. If the taxpayer sells or disposes of the asset in the future, then the government will share in the capital risk faced by the taxpayer. For example, suppose that operating an asset becomes unprofitable early in its life due to a sharp drop in revenues, and the asset is discarded. At the time of disposal of the asset, the taxpayer will deduct the remaining basis as a loss. This loss deduction early in life means that the overall pattern of deducting the original cost of the asset will be accelerated relative to the statutory depreciation schedule. The loss taken at retirement of the asset normally would have been deducted over a period of years. The taxpayer receives accelerated deductions, and this treatment partially mitigates the early loss at the expense of the government. Thus, the government does share in capital risk.

It turns out that as long as the tax system takes losses and gains from asset dispositions into account, the effect claimed by Bulow and Summers will not occur. To fully document this result requires technical analysis that appears elsewhere. Instead of repeating that analysis here, I pres-

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283. See supra text accompanying note 2.
284. Capital risk does not coincide exclusively with revenue risk or retirement risk, the categories used in the text. Since the value of an asset depends both on its survivability and on its revenue stream if it does survive, capital risk includes elements of both revenue risk and retirement risk.
285. See Bulow & Summers, supra note 282, at 25.
286. The impact of the tax system on assets subject to nonsystematic risk is not a concern. This type of risk can be diversified away costlessly. Whether or not the government will share in nonsystematic risk will not affect investment incentives.

The basic reasoning that establishes the point is as follows. Suppose that at the beginning of a particular year an asset has an adjusted basis of SX and that for that year $D$ is allowable as a depreciation deduction under the statutory schedule. If the taxpayer takes this deduction, adjusted basis will be ($X - D$) = $Y$ at the end of the year. If, however, the asset falls below $Y$ during the year, the taxpayer can costlessly sell the asset and repurchase a substitute at a price below $Y$. In fact, with zero trading costs, the taxpayer can execute this transaction at the lowest value that the asset attains during the year. See
tent some qualitative arguments of the same nature as the ones in the text.

Consider first the case where trading costs are zero. In this case, strategic trading allows the taxpayer to choose in each time period between depreciation at the statutory rate or depreciation at the actual rate of decline.\textsuperscript{288} Suppose that the statutory depreciation rate for each tax accounting period is set at the average rate over all possible asset value outcomes during that period. Then, the taxpayer will choose the actual rate of decline if the actual rate is faster than the average rate, and the government will share in the downside portion of capital risk. To the extent that decline in asset value happens faster than average, the taxpayer receives a larger deduction and pays the government less in taxes.

In contrast, suppose that the actual decline in value is at a slower rate than the statutory rate. Then, the taxpayer can choose the statutory rate by not trading the asset. In this case, an "upside" outcome occurs since the asset declined less than the average expected decline. But the government does not share in this upside outcome since the taxpayer does not have to decelerate the deductions allowed by the statutory schedule.

It is apparent that in the zero trading cost case, the government shares in the downside of capital risk but not in the upside. This result is even better for the taxpayer than what Bulow and Summers find lacking, namely, a full sharing of risk, both upside and downside, by the government. There certainly is no need to allow more generous depreciation deductions for risky assets in this case. On the contrary, the ability of the taxpayer to share losses with the government but keep all gains suggests less generous depreciation deductions for risky assets. This approach is exactly what is advocated in the text.\textsuperscript{289}

So far the focus has been on the zero trading cost case. Suppose, instead, that the opposite case applies: Trading costs are so high that strategic loss-taking is never profitable. Now the taxpayer knows at the time the asset is purchased that the asset will be held until it is discarded as worthless. As a result, the value of the asset during the time it is held will not affect the net worth of the taxpayer. In finance parlance, the cost of the asset is a "sunk cost." All that matters is the revenue stream that flows from the asset. This revenue stream is taxed. As a result, the government fully shares in the upside and the downside risk for the investment. In essence, capital risk is irrelevant when trading costs are very high.

The "intermediate case" of moderate trading costs is much more complicated. The complications arise from the fact that it is difficult to model optimal taxpayer behavior. To see this difficulty, suppose that the asset's actual value has fallen somewhat below its adjusted basis under the statutory schedule. The taxpayer can sell to "accelerate" deductions relative

\textsuperscript{ supra} note 13. Suppose that this lowest value is $Z. By engaging in "strategic trading," the taxpayer has obtained a deduction in the amount ($X - $Z) which is larger than the statutory depreciation deduction of $D = ($X - $Y).

\textsuperscript{288} This point is established in Part I. See supra text accompanying notes 10-15.

\textsuperscript{289} See supra text accompanying notes 135-146.
to the statutory schedule, but this sale is costly. The taxpayer can also wait in the hope that the asset will continue to decline faster than the statutory schedule. Then the stream of deductions can be accelerated even more for the same trading cost. But the asset may go up in value instead of falling further. Choosing the optimal timing for sales is a difficult problem.

Professor Williams has solved this problem and has embedded the resulting optimal behavior in a simple model with tax depreciation. He finds that the tax system will favor riskier depreciable assets unless trading costs are so high that no trading will occur. Furthermore, where trading costs are not prohibitive, the degree to which riskier assets are favored is higher when trading costs are lower.

These results verify the intuitive story presented above concerning the zero trading cost case and the case of prohibitively high trading costs. In the zero trading cost case, riskier assets are favored. If trading costs are prohibitive, this effect disappears. Professor Williams' results add to this picture by indicating that the case of "moderate" trading costs lies in between the other two cases: Riskier assets will be favored but not as much as in the case of zero trading costs.

Despite these results, there may be an effect of the kind described by Professors Bulow and Summers when asset dispositions have no tax consequences. Some of the disposition rules studied in the text do involve an elimination or reduction of disposition taxes. Consequently, the Bulow-Summers effect must be kept in mind as a potential problem when considering these rules.

APPENDIX E

Group Accounting: A Brief History and Synopsis of the Rules

Over the last thirty years, four systems of group accounting have been in use. These four systems each have a different name for group accounts. Only two of the four systems apply to assets placed into service currently, that is, in 1999. Although the other two apply only to property placed into service in the past, they either affect current law or play a role in the policy analysis in Parts III and IV. As a result, all four systems are described here.

Section A describes the four systems and the assets they covered or cover. Section B discusses the treatment of asset dispositions under the four systems, extending the discussion provided in Part III of the text. Alternative asset disposition treatments is the feature of group account-
ing that is of most interest for this paper.295

A. A Description of the Four Systems and Their Scope

From 1971-1980 a taxpayer could elect to depreciate assets under the Asset Depreciation Range ("ADR") system. This election required that, with a few minor exceptions, all of the taxpayer's property eligible for ADR treatment (in particular, most items of tangible personal and real property) had to be treated under that system.296 The ADR required the use of "vintage accounts," a type of group account.297

The taxpayer had some flexibility in placing assets in vintage accounts. Multiple vintage accounts could be established, and the taxpayer could place as many or as few (including one) assets in each account as desired.298 However, each account could only contain assets that were placed into service the same year, that were subject to the same recovery period and that were subject to the same depreciation method.299

Electing ADR treatment meant that the taxpayer was allowed to select useful lives in a "range" from 20% shorter to 20% longer than the useful lives that otherwise would have applied. The principal motivation for electing ADR treatment was to obtain accelerated depreciation by choosing the shortest lives in the range.300 The administration's main reason for proposing the ADR system was to stimulate investment by allowing taxpayers to obtain depreciation at an accelerated rate compared to previous law.301

For property placed in service from 1981-86, the Accelerated Cost Recovery System was applied as a mandatory system for the items of tangible personal and real property to which the ADR applied. This system replaced both the elective ADR treatment and the treatment that applied when the taxpayer did not elect the ADR.

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295. Group accounting under any of the four systems is complicated and includes many special rules. Only a few features are presented here. Readers interested in more detail are encouraged to consult the sources cited in the description of the systems.
296. See Treas. Reg. § 1.167(a)-11(b)(5)(ii).
297. The property eligible for ADR treatment included all tangible personal or real property that had been assigned a class life and was "section 1245 property" or "section 1250 property." See Treas. Reg. § 1.167(a)-11(b)(2); I.R.C. § 1245(a)(3) (1971); I.R.C. § 1250(c)(1971).
300. See Graetz, supra note 1, at 395.
ACRS allowed the taxpayer to use group accounts called "mass asset accounts" to depreciate certain types of property on an elective basis. Only a limited class of assets called "mass assets" was eligible for this elective treatment. These assets had to be numerous, individual identification had to be impractical, and the value of each asset had to be small compared to the value of the group of assets.\(^{302}\) In addition, assets in a single account had to have the same class life and had to be placed in service in the same year.\(^{303}\) These limitations were motivated largely by a technical concern about recapture of the investment tax credit for assets sold from mass asset accounts.\(^{304}\) Although asset coverage for mass asset accounting under ACRS was much more limited than under the ADR, the Regulations under ACRS did not require the taxpayer to elect mass asset accounting for all mass assets. The taxpayer could pick and choose between group accounting and individual accounting for different mass assets.\(^{305}\)

In the Tax Reform Act of 1986 the investment tax credit was repealed. In addition, MACRS replaced ACRS as the mandatory depreciation system for tangible personal and real property. Congress continued to allow an elective group accounting method, now named "general asset accounts," and mandated that Treasury detail the rules covering these accounts in Regulations. Since the investment tax credit was repealed, Congress directed that the limitations on the assets subject to group accounting for "mass assets" under ACRS be removed so that general asset accounts could contain "diverse assets."\(^{306}\) On its face this directive would seem to suggest that Treasury's Regulations could or should allow the same account to include assets placed in service in different years or subject to different recovery periods or different depreciation methods.

\(^{302}\) More precisely, the Proposed Regulations under ACRS define a "mass asset" to be a mass or group of individual items of recovery property (i) not necessarily homogeneous, (ii) each of which is minor in value relative to the total value of such mass or group, (iii) numerous in quantity, (iv) usually accounted for only on a total dollar or quantity basis, (v) with respect to which separate identification is impracticable, (vi) with the same present class life, and (vii) placed in service in the same taxable year.


This definition seems to have been drawn largely from the existing Treas. Reg. § 1.47-1(e)(4) which is substantially similar but goes on to say:

This term includes portable air and electric tools, jigs, dies, railroad ties, overhead conductors, hardware, textile spindles, and minor items of office, plant, and store furniture and fixtures; and returnable containers and other items which are considered subsidiary assets for purposes of computing the allowance for depreciation.\(^{303}\)


\(^{306}\) The Blue Book for the 1986 Act states that since the investment tax credit was repealed "the Act contemplates that the definition of assets eligible for inclusion in mass asset accounts will be expanded to include diverse assets." See Tax Reform Act of 1986 Blue Book, supra note 25, at 108. This idea is also stated directly in the legislative history of the Act. See Tax Reform Act of 1986 Senate Report, supra note 26 at 104.
Nonetheless, the final Regulations are quite restrictive with respect to the diversity of assets permitted in each account: A single general asset account may only contain assets that are placed into service the same taxable year and that are subject to the same depreciation method, the same convention, and the same recovery period.\textsuperscript{307}

The fourth type of group accounting consists of "multiple asset accounts." These accounts are currently available as an elective alternative to individual item accounting for assets put into service that are not covered by the MACRS system.\textsuperscript{308} The accounts were also available for property not covered by ACRS put into service in 1981-86 and for property put into service from 1971-80 by taxpayers who did not elect ADR.\textsuperscript{309} The taxpayer may set up as many multiple asset accounts as desired, may allocate assets to multiple asset accounts in any desired pattern (including combining assets subject to different recovery periods or depreciation methods), and may choose to depreciate some assets under individual item accounting and others under multiple asset accounts.\textsuperscript{310} In addition, in contrast to vintage accounts under ADR and mass asset accounts under ACRS, multiple asset accounts may be "open," that is, new assets may be continually added to the account, and accounts are not limited to assets placed in service in the same year.\textsuperscript{311}

\textbf{B. THE TREATMENT OF DISPOSITIONS UNDER GROUP ACCOUNTING}

The text mentions four alternative tax treatments for dispositions that have been used in group accounting: the individual item method, the depreciation bond approach, the account adjustment method, and the proceeds tax method.\textsuperscript{312} This section presents the mechanics of these four treatments in a group accounting framework and then specifies the treatments used under the four group accounting regimes presented in Section A.

There is a special accounting terminology for group accounts, but for simplicity the exposition here employs the traditional basis/adjusted basis terminology used in individual asset accounting.\textsuperscript{313} Quotation marks are used for this terminology to indicate that it is technically inappropriate although substantively correct.

Suppose a group of identical assets is placed into service in a particular year as a group account. The "basis" of the account will be the sum of the

\textsuperscript{307} See Treas. Reg. §1.168(i)-1(c)(2)(i), (ii).

\textsuperscript{308} For example, patents and copyrights are intangible assets and therefore are not covered by MACRS.

\textsuperscript{309} See Donald, supra note 247, at A-6 (discussing the availability of multiple asset account election for taxpayer not electing ADR); Megaard and Megaard, supra note 304, at A-111 (discussing the availability of multiple asset accounts for nonrecovery, that is, non-ACRS and non-MACRS, property); Treas. Reg. § 1.167(a)-7(a).

\textsuperscript{310} See Treas. Reg. § 1.167(a)-7(a), (c).

\textsuperscript{311} See Megaard and Megaard, supra note 304, at A-112.

\textsuperscript{312} See supra text accompanying notes 7-8 and 171-192.

\textsuperscript{313} For a rapid and clear introduction to the terminology usually employed for group accounting, see generally Megaard and Megaard, supra note 304.
bases of the assets. In the first year the account will generate a depreciation deduction and that deduction will be subtracted from the “basis” for the account to arrive at an “adjusted basis” for the account. For both multiple asset accounts and general asset accounts, the depreciation rates that apply are the same ones that would be used to depreciate the assets under individual item accounting.314

There are three general methods for treating dispositions under multiple asset accounts and general asset accounts: the individual item method, the account adjustment method, and the proceeds tax method. A fourth method, the depreciation bond approach, was used for some dispositions under the ADR.315 To clarify the operation of these methods, consider an example. Ten assets with an original cost of $1000 each are placed in a group account subject to straight-line depreciation over a 5 year recovery period. Thus, the “basis” of the group account starts at $10,000. Suppose that all of the assets remain in the account for the first two years except that one asset is sold on the last day of the second year. Absent that sale event, the adjusted basis would be $6000 at the end of the two years. We will consider sale of the asset for $800, a “gain” of $200 compared to the “adjusted basis” of $600, and sale for $400, a “loss” of $200 compared to the “adjusted basis” of $600.

Under the depreciation bond approach, the sale has no tax impact on the seller. The seller continues depreciating the account as if the sale asset was still present, and the sale has no effect on the “adjusted basis.”

Under the individual item method, gain or loss is taxed at the time of disposition and the “adjusted basis” attributable to the asset is removed from the group account. The outcomes in the example are as follows:

(1) Gain case. At the end of two years, $600 in “adjusted basis” is attributable to the asset. This amount of adjusted basis is removed from the account, reducing the account “adjusted basis” to $5400. This $5400 is depreciated on a straight-line basis over the remaining three years. Since the sale price of $800 for the asset is $200 more than the $600 of “adjusted basis” attributable to it, a gain of $200 is included in taxable income for the second year.

(2) Loss case. In this case, the outcome is exactly the same except that taxable income is reduced by a $200 loss, the difference between a sale price of $400 and the $600 in “adjusted basis” attributable to the asset.

The outcome under this method is exactly the same as under individual item accounting.316 Gain or loss is realized on the sale of a depreciable  

314. See supra note 19 and accompanying text (general asset accounts); supra note 213 and accompanying text (multiple asset accounts).
315. This terminology for all four approaches is the author’s.
316. This feature motivates use of the name “individual item method.”
asset and the depreciation of other assets is unaffected.\textsuperscript{317}

Under the account adjustment method no gain or loss is realized currently upon disposition of an asset. Instead, any loss is added to the "adjusted basis" of the account and any gain is subtracted. In addition, the "adjusted basis" attributable to the asset is subtracted from the "adjusted basis" of the account. The outcomes in the example are as follows:

(1) \textit{Gain case.} At the end of two years, $600 in "adjusted basis" is attributable to the asset. This amount of "adjusted basis" is removed from the "adjusted basis" of the account, reducing it from $6000 to $5400. Since the sale price of $800 for the asset is $200 more than the $600 of "adjusted basis" attributable to it, a gain of $200 is subtracted from the "adjusted basis" of the account, reducing it to $5200 from $5400. The $5200 is deducted on a straight-line schedule over the remaining three years.

(2) \textit{Loss case.} In this case, the outcome is exactly the same except that "adjusted basis" for the account is increased by the $200 loss (instead of being decreased by a $200 gain). The result is $5600 in "adjusted basis" remaining in the account to be deducted on a straight-line schedule over the remaining three years.

The effect of this method is to delay realization of the gain or loss from disposition of the asset. The gain (or loss) will be recovered as decreased (respectively, increased) depreciation deductions in future years. If a taxpayer faces a constant marginal tax rate over time, this method treats gains more favorably and losses less favorably than the individual item method.

Under the proceeds tax method \textit{no} changes are made in the group accounts due to disposition of an asset from the account, but the amount realized from that disposition is included in taxable income \textit{without} reduction for the "adjusted basis" attributable to the asset. The outcomes in the example are as follows:

(1) \textit{Gain case.} No adjustment is made to the group accounts. An "adjusted basis" of $6000 remains in those accounts to be deducted over the remaining three years using the straight-line method, just as it would have been if no sale had taken place. Since $800 was realized upon sale of the asset, this $800 is included in taxable income for the second year.

(2) \textit{Loss case.} As in the gain case, no adjustment is made to the group accounts. The $400 realized from sale of the asset is included in taxable income for the second year.

In effect, the proceeds tax rule includes not only gain or loss but also the "adjusted basis" attributable to the asset in income in the year of

\textsuperscript{317} Thus, in the example, the $5400 of "adjusted basis" attributable to the other nine assets is deducted at exactly the same rate as it would have been if the tenth asset had not been sold.
disposal and then allows this "adjusted basis" to be recovered as depreciation deductions from the group account in the future. For a taxpayer facing constant marginal rates, this rule is clearly less favorable in its treatment of both gains and losses than any of the other three methods. Although the proceeds tax rule may seem punitive, Parts III and IV show that there may be some justifications for using the rule.\textsuperscript{318}

The following table denotes the treatment of dispositions for each type of retirement and for each of the four regimes sketched in part A of this Appendix.\textsuperscript{319}

<table>
<thead>
<tr>
<th>Type of Retirement</th>
<th>ACRS</th>
<th>ADR</th>
<th>General Asset Accounts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal Retirement</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Abnormal Retirement</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\textsuperscript{318} See supra text accompanying notes 184-186, 232-241.

The proceeds tax rule also has obvious administrative advantages. There is no adjustment of a group account as a result of disposition of an asset in the account, and thus no need to determine what portion of the "adjusted basis" of the group account is attributable to the asset disposed of. In the case of accounts consisting of assets with different useful lives that have been added to the account at different times, this determination could be quite complicated. The only accounting required under the proceeds tax method is to include the amount realized in income.

\textsuperscript{319} For vintage accounts, instead of distinguishing between "normal" retirements and "abnormal" retirements, there is a distinction between "ordinary" retirements and "extraordinary" retirements. This different terminology is accompanied by some substantive differences in the category, although there is a qualitative similarity in the categories. Compare supra note 204 and accompanying text (distinction between normal and abnormal retirements) with Treas. Reg. § 1.167(a)-11(d)(3)(ii) (distinction between ordinary and extraordinary retirements for ADR vintage accounts).

The mass asset accounts rules under ACRS abandoned the distinction between normal and abnormal retirements, applying the same general rule to all dispositions.

The general asset account rules distinguish between "qualifying dispositions" and other dispositions rather than between "normal" and "abnormal" retirements. As in the case of "extraordinary retirements" under the ADR vintage accounts system, the "qualifying disposition" category is similar but not identical to the category of "abnormal retirements." See supra note 206 and accompanying text.

The disposition rules listed in the table are detailed with citations in Megaard and Megaard, supra note 304, except for the rule covering abnormal retirements under ADR and the rules that apply to general asset accounts. These rules are set forth in the Regulations. See Treas. Reg. § 1.167(a)-11(d)(3)(iv)(a) (ADR); Treas. Reg. § 1.168(i)-1(e)(2)(i)-(iii), (e)(3)(iii)(A), (C) (general asset accounts).
### TABLE IV: DISPOSITION RULES UNDER ALTERNATIVE REGIMES

<table>
<thead>
<tr>
<th>Regime</th>
<th>General Rule</th>
<th>Special Rule for Certain Dispositions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>vintage accounts</strong></td>
<td>DB (“ordinary retirements”)</td>
<td>II (“extraordinary retirements”)</td>
</tr>
<tr>
<td>(ADR, 1971-1980)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>mass asset accounts</strong></td>
<td>II or PT (elective by account)</td>
<td>no special rule</td>
</tr>
<tr>
<td>(ACRS, 1981-1986)</td>
<td>(all retirements)</td>
<td></td>
</tr>
<tr>
<td><strong>general asset accounts</strong></td>
<td>PT</td>
<td>II or PT (“qualifying dispositions”)</td>
</tr>
<tr>
<td>(MACRS, 1987-present)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>multiple asset accounts</strong></td>
<td>AA (“normal retirements”)</td>
<td>II (“abnormal retirements”)</td>
</tr>
<tr>
<td>(non-ADR, non-ACRS and non-MACRS property, 1971-present)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Key:
- **AA** = account adjustment method
- **DB** = depreciation bond approach
- **II** = individual item method
- **PT** = proceeds tax method